Should We Give Up after Solyndra? Optimal Technology R&D Portfolios under Uncertainty

Mort Webster, Karen Fisher-Vanden, David Popp, Nidhi Santen

Abstract: Climate change and other environmental challenges require the development of new energy technologies with lower emissions. In the near term, R&D investments, either by the government or the private sector, can reduce the costs of these lower-emitting technologies. However, the returns to R&D are uncertain, and there are many potential technologies that may emerge to play an important role in the future energy mix. In this paper, we address the problem of allocating scarce R&D resources across technologies when uncertainties exist. We develop a multistage stochastic dynamic programming version of an integrated assessment model of the climate and economy that represents endogenous technological change through R&D decisions for two substitutable noncarbon backstop technologies. We demonstrate that near-term R&D investment in the higher cost technology is justified and that the optimal R&D investment in the higher cost technology increases with both higher variance and higher skewness in the distribution of returns to R&D.

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A RESPONSE TO global climate change will necessarily include reducing carbon emissions from energy production in the future. There are many potential fuels and technologies that may contribute to future energy production that generate lower greenhouse gas emissions. In the case of electric power, low-emitting options include nuclear power, carbon capture and sequestration from coal or natural gas combustion, solar power (photovoltaic [PV] or concentrated solar thermal), wind power (onshore or offshore), and biomass-fueled combustion. Similarly, in the case of transportation, lower-carbon-emitting technologies include efficient diesel engines, electric vehicles, hybrid electric vehicles, compressed natural gas (CNG) vehicles, and biofuels.

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Low-carbon-emitting technologies vary widely in their current relative costs, technological maturity, and commercial viability. An important policy challenge is how to efficiently allocate scarce resources for research and development (R&D). Public and private sources of R&D funding must choose how to allocate these funds across these disparate technologies. For example, the US Department of Energy (US DOE) is a major source of public funding for energy R&D (US DOE 2015). Each year, US DOE allocates funding for R&D across technology areas as part of the budget and appropriations process. However, there is limited objective or quantitative guidance for agencies on how to choose among possible allocations (Pugh et al. 2011).

To focus the discussion, consider the simple case of two low-carbon energy technologies that are relatively easily substitutable. Although no technology is a perfect substitute for another, the ease of substitution in some circumstances may be significant. For example, solar PV and onshore wind power constitute one such pair within the electric power system. Both are intermittent renewable sources of electricity that emit no carbon. Although their relative potential in any specific location may vary depending on the solar and wind resources that exist, they play a similar role within the power system. Other possible examples include nuclear power and coal-fired power with carbon capture for electricity production (both are baseload technologies) or hybrid, battery electric, and CNG vehicles for transportation.

In each of these examples, one technology is currently significantly lower cost than the alternative(s). For example, the 2015 estimates of levelized cost from the US Energy Information Administration is \$73.6 per megawatt hour (MWh) (2012 US\$) for onshore wind as compared to \$125.3 per MWh for solar PV (EIA 2015). In this case, solar PV is still higher cost than wind despite decades of significant public and private R&D investment in many industrialized countries in solar PV technology. This has resulted in lowering the cost of solar PV by several orders of magnitude (Barbose, Weaver, and Darghouth 2014).

A reasonable question, therefore, is whether continued R&D investment in solar PV is justified and, if so, at what level, given that wind technology is significantly lower cost and can fill a similar role in the power system. This question, as applied to solar, has become particularly relevant since the high-profile bankruptcies of two solar technology companies in the United States that had previously received substantial public investment. Solyndra, a California-based company that manufactured solar panels, received \$539 million in guaranteed government loans in 2009, but by August 2011 was bankrupt (Weiner 2012). Around the same time, Evergreen Solar, a Massachusetts-

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based solar panel manufacturer, also declared bankruptcy (Church 2011). In both cases, a number of factors contributed to the firm's failure, including competition from lower-cost manufacturers in China. Nevertheless, these failures have been cited in debates over public R&D investments, and some continue to argue that further investments in solar technology are not cost effective (e.g., U.S. News & World Report 2012) or at least that public investment in solar should be scaled back.

In this motivating example of wind versus solar PV, a critical decision factor that is often neglected is the difference in underlying uncertainty between the two technologies—the distribution of uncertain returns to R&D investments in solar technologies is likely to be highly positively skewed (i.e., a small probability of a very large cost reduction) and more skewed than for wind technologies, the more commercially established technology. Baker et al. (2015) provide a survey and aggregation of several expert elicitations characterizing the uncertainty in solar energy costs and the impacts of R&D investment and find the resulting distributions to be highly skewed. In the past decade, R&D investment from programs such as Sunshot—a US Department of Energy program (Sunshot 2012)—resulted in numerous potentially revolutionary technologies for PV, including innovations in advanced materials and manufacturing processes (e.g., Green 2000). In the past several years, solar PV prices have in fact declined sharply, driven primarily by factors such as economies of scale in production rather than new advanced materials (Barbose et al. 2014). Does this mean that these recent R&D investments were not wise, given the information available, and that continued investments are unwarranted? Or do the results of solar R&D investments to date merely reflect a highly skewed distribution of uncertain returns, and therefore continued R&D in solar PV is a wise decision?

In this paper, we investigate the factors that influence the economically efficient allocation of R&D investment across two substitutable energy technologies. In particular, we seek to answer the following question: under what circumstances is some level of R&D investment to a higher-cost technology warranted when a lower-cost substitute exists? Further, how should the allocation change if the returns to R&D investment in the higher-cost technology is uncertain and positively skewed?

Here we explicitly model R&D as a stochastic process. Uncertainty is a central feature of the R&D process, since a priori one does not know what will be gained from a given level of investment. Moreover, the relevant characteristics of the underlying uncertainty include more than simply mean and variance. Some technologies may have a very low probability of significant cost reduction from R&D investment and a very high probability of little or no cost reduction; this implies skewness in the distribution of returns to R&D investment. When decision makers must allocate R&D funding among technologies with qualitatively different risk profiles (e.g., more skewed but lower mean returns vs. less skewed and higher mean), how should this allocation be made? Using our stochastic framework, we weigh the influence of the shape of the probability distribution of uncertain returns to R&D on the optimal level of R&D investment in the higher-cost technology.

Numerous studies exist that have addressed the empirical, theoretical, and applied numerical modeling of innovation and the R&D process. Several excellent reviews exist (e.g., Loschel 2002; Gillingham, Newell, and Pizer 2008; Pizer and Popp 2008; Popp, Newell, and Jaffe 2010), so we only briefly review the most relevant studies here. Many theoretical and numerical models assume an innovation possibilities frontier (IPF) (Kennedy 1964; Kamien and Schwartz 1968; Acemoglu 2002), which is a function of the stock of "knowledge capital" (Romer 1990; Acemoglu 1998). Examples of numerical models that include endogenous directed technological change include Goulder and Schneider (1999), Goulder and Mathai (2000), Buonanno, Carraro, and Galeotti (2003), Popp (2004, 2006b), and Sue Wing (2006). The majority of these models are deterministic and focus on assessing, for instance, the welfare gain in a climate change policy scenario when technological change is assumed to be endogenous (e.g., Goulder and Mathai 2000; Buonanno et al. 2003; Popp 2004); the relative effectiveness of alternative policy instruments on inducing innovation (e.g., Fischer and Newell 2008; Fischer, Newell, and Preonas 2013); how climate change uncertainty affects optimal R&D (e.g., Baker, Clarke, and Weyant 2006), and the effects of international R&D spillovers (e.g., Bosetti et al. 2009).

In studies addressing uncertain returns to R&D, the focus is typically on R&D investments in a single backstop that substitutes for a conventional (i.e., fossil) energy source. Theoretical models of optimal R&D under uncertainty include Hung and Quyen (1993), who show that uncertainty can delay R&D investment and accelerate conventional resource depletion under some conditions, and Tsur and Zemel (2005), who explore a range of dynamic growth paths with and without R&D investment and show that the growth path depends on an economy's production technology and learning ability, and by its knowledge-capital endowment. Goeschl and Perino (2009) develop a model where the backstop technology has uncertain environmental characteristics in a multipollutant context, but deterministic cost impacts of R&D. Baker and Adu-Bonnah (2008) develop a two-stage stochastic model where the impacts of R&D are uncertain with a probability distribution over three possible cost outcomes (target cost, breakthrough, or failure). They show that the effect of increasing the mean-preserving variance on R&D investment is ambiguous and depends on the type of technology targeted. Bosetti and Tavoni (2009) present a stochastic two-stage version of the WITCH model (Bosetti et al. 2006) where the investment cost of the backstop is drawn from a symmetric distribution that depends on the cumulative knowledge stock. In contrast to Baker and Adu-Bonnah (2008), they demonstrate from both an analytical and numerical approach that uncertainty in the returns to R&D unambiguously increases the optimal level of R&D. Most recently, Santen and Anadon (2016) present a multistage stochastic model for joint R&D and capital investments under uncertain returns to R&D for the electric power sector, comparing results from deterministic, scenario-based, and fully stochastic versions of their model. As with previous studies, their focus is on uncertainty and R&D into a single technology (solar).

There are fewer studies that explicitly consider multiple technologies and uncertainty. One notable example is Gritsevski and Nakicenovic (2000), who represent more than 100 energy technologies in the MESSAGE model (Messner, Golodnikov, and Gritsevskyi 1996). However, their approach is focused on learning by doing rather than explicit R&D investment. Using a large number of scenarios of technology dynamics, they show that there are many distinct paths with similar energy costs. Pugh et al. (2011) also use a scenario-based approach to explore the energy R&D portfolio question. Using results from a range of scenarios generated by the GCAM model (Brenkert et al. 2003) to estimate the benefits of alternative technologies and a probability distribution for the likelihood of achieving the maximum emissions reductions, they generate a set of possible 5-year R&D investment plans.

Most similar to this study, Baker and Solak (2011) develop a two-stage stochastic model with R&D investments in three alternative backstop electric power technologies: solar, coal with carbon capture, and nuclear. R&D investment decisions are made in stage 1 while abatement decisions are made in stage 2. The probability distribution of the uncertain returns to R&D for each of these technologies is estimated from expert elicitation (Baker, Chon, and Keisler 2008, 2009a, 2009b). In general, they find that the R&D investment portfolio is diverse across technologies and is robust to the uncertainty associated with climate damages. However, they do not investigate the sensitivity to different shapes of the distribution of uncertain returns to R&D, nor do they consider more than two stages. Webster, Santen, and Parpas (2012) show that for some problems, multistage ($N > 2$) models may lead to very different stage 1 optimal strategies compared to two-stage formulations.

Several questions and challenges remain unaddressed by the extant literature. First, given two technologies with uncertain returns to R&D, how should R&D investments be allocated? In particular, the relative impacts of mean, variance, and higher-order moments in the distribution of uncertain returns on the R&D allocation decision have not been methodically explored. Second, in a multistage context with repeated opportunities for R&D investment and observed outcomes, what is the conditionally optimal R&D investment portfolio, given failures to reduce cost in previous stages? Finally, calibrating models of technological change to be consistent with the empirical literature on the depreciation of knowledge remains a challenge. Representations of the innovation possibilities frontier (IPF) assume a depreciation rate, based on findings in the empirical literature that knowledge decays over time in terms of its effectiveness at generating new knowledge. However, a deficiency in current formulations is the assumption that cost reductions achieved also decay, implying that costs increase over time without a minimum threshold of new knowledge generated each period.

We build on the previous studies of optimal R&D investment under uncertainty by investigating the optimal ratio of R&D investment between two generic backstop technologies. We develop a multistage stochastic dynamic model, based on the ENTICE-BR model (Popp 2006b) with R&D-induced technological change, incorporating innovation possibility frontiers and cumulative knowledge capital stocks. Using this model, we explore the factors that determine the level of R&D investment allocated to the higher-cost backstop when a lower-cost technology exists. In particular, by varying the shape of the distributions of returns to R&D, we show that the optimal allocation of R&D investment between the two technologies depends not only on the relative variance but also on the skewness of the distributions.

The specific contributions of this work are:

- A model of multistage R&D investment decisions between two low-carbon backstop technologies with uncertain returns to R&D;
- A novel formulation of technological change that distinguishes between two knowledge stocks: one that generates new knowledge and one that generates perpetual cost reductions; and
- A decomposition of the relative importance of the following three factors on the optimal R&D investment allocation decision: (*a*) variance in the distribution of uncertain returns, (*b*) skewness of the distribution, and (*c*) initial relative costs of the backstop technologies.

Two key results of this work are of central relevance to current policy discussions on R&D investment portfolios. First, our results show that optimal R&D investment in the more expensive backstop technology should be greater the more positively skewed the distribution of returns to R&D is. Additionally, even below-average outcomes from first-stage R&D investment in the high-cost technology do not imply that future R&D should be focused exclusively on the lower-cost technology.

The remainder of the paper proceeds as follows. In section 1, we develop a simple analytical two-stage model of R&D investment allocation decision making under uncertainty. This model frames the qualitative result that we later explore with the numerical model. Section 2 presents the methodology and key assumptions underlying the stochastic dynamic version of ENTICE-BR model with two substitutable backstops. We present the results from the numerical model in section 3, showing both the deterministic results (no uncertainty in R&D), and then the stochastic results for an illustrative probability distribution. Section 4 presents a sensitivity analysis of the numerical model, distinguishing between the effects of increasing variance and increasing skewness, as well as the effect of the number of decision stages. The model is applied to a case study of solar versus wind in section 5. Finally, section 6 provides concluding remarks.

1. R&D UNDER UNCERTAINTY AND THE EFFECT OF SKEWNESS

To motivate and frame the numerical model and its results, we begin by illustrating the key concepts with a simple stylized analytical model. Although this model omits many features of the problem in order to retain analytical tractability and transparency, it does provide important insights relevant for the numerical model. The numerical model presented in the subsequent sections includes many of the complicated features omitted here and produces qualitatively similar results.

Consider a two-stage model, $t = 1, 2$, and two possible backstop energy technologies, $i \in \{1, 2\}$. In the first stage ($t = 1$), the decision maker must choose levels of R&D investment in each of the two technologies, R_1, R_2. For simplicity, assume that the total budget is exogenously constrained:

$$R_1 + R_2 <= B,$$

and that the two technologies are perfect substitutes.

We denote the cost of technology i at time t by C_i^t. Assume that technology 1 has a lower cost in the initial period than technology 2:

$$C_1^1 < C_2^1$$

In this model, the cost in stage 2 can be reduced through R&D investment according to some increasing, concave function f:

$$C_i^2 = C_i^1 - f(R_i); f' > 0, f'' < 0.$$

The concavity assumption reflects the diminishing marginal returns of R&D investment in a given technology at a specific time, which has been demonstrated empirically (e.g., Popp 2002).

We model uncertainty in the returns to R&D investment in technology 2 using a simple multiplicative shock to the cost reduction function. Specifically, we assume that with probability π, there is a multiplicative shock θ to $f(R_2)$, where $\theta > 1$. To model a "failure" in R&D, we assume that with probability $(1 - \pi)$ the cost reduction in period 2 from R_2 is 0. This is a stylized approach, for simplicity and clarity, where the positive shock θ represents a realized cost reduction; in reality there is a sequence of conditions that must occur before the cost reduction is realized, including successful innovation, diffusion, and so on, which we do not explicitly treat here. The failure outcome of zero cost reduction is also stylized—in fact, any $\theta < 1$ would have the same effect that technology 1 is preferred—but this assumption enables a mean-preserving sensitivity analysis presented below. Furthermore, we assume that the magnitude of the exogenous cost shock, $\theta > \theta^c$, where θ^c is large enough to make technology 2 the lower cost and therefore the preferred technology in the second period. For ease of explication, we neglect the case where $\theta < \theta^c$; this case has a trivial result since, under these conditions, all R&D investment should be allocated to technology 1.

Since the goal of this research is to assess strategies to reduce carbon emissions, we assume that there is an emissions target in stage 2 that must be met. Because we have two noncarbon energy technologies that are perfect substitutes, the decision maker will adopt the technology that has the lowest cost in stage 2. We can thus frame the objective function for the R&D investment decision as minimizing the expected cost of whichever technology is adopted. The expected costs depend both on whether a positive shock θ occurs and the amount of R&D invested (R_2), as R&D is necessary to take advantage of the shock.

The case where R&D fails, which occurs with probability $(1 - \pi)$, is trivial. Here, the cost in period 2 is simply the cost of technology 1. The portion of the R&D budget not allocated to technology 2 is used to lower the cost of technology 1. Thus, with probability $(1 - \pi)$ technology 1 is adopted with a cost of $C_1^1 - f(B - R_2)$.

A positive shock occurs with probability π. Here, the final decision is more complicated. The cost of technology 2 will be $C_2^1 - \theta f(R_2)$. However, this may not be lower than the cost of technology 1, as the resulting cost also depends on the amount of R&D invested in technology 2, R_2. Consider an extreme case in which $R_2 = 0$. The cost of technology 2 does not fall at all, so technology 1 remains cheaper. Thus, if a positive outcome to R&D is realized, the expected cost at $t = 2$ will be

$$\min\left(C_2^1 - \theta f(R_2), C_1^1 - f(B - R_2)\right).$$

To capture this in the cost minimization problem, we define an additional decision variable a, such that if $a = 1$, then R_2 is sufficiently large enough to reduce the cost of technology 2 below that of technology 1, and if $a = 0$, then R_2 is not sufficient regardless of θ. The objective function is therefore:

$$\begin{aligned} \min_{R_2, a} \pi \left\{ a\left[C_2^1 - \theta f(R_2)\right] + (1 - a)\left[C_1^1 - f(B - R_2)\right] \right\} \\ + (1 - \pi)\left[C_1^1 - f(B - R_2)\right]. \end{aligned} \tag{1}$$

To obtain the first-order conditions, we set the derivative with respect to both R_2 and a equal to zero. The relevant derivative here with respect to R_2 is:

$$-\theta\pi a f'(R_2) + \pi(1 - a)f'(B - R_2) + (1 - \pi)f'(B - R_2) = 0.$$

Collecting similar terms and rearranging, we can express the optimality condition for R&D investment in technology 2 as:

$$f'(R_2) = f'(B - R_2) \cdot \frac{1 - \pi a}{\pi a \theta}.$$

The case where $a = 0$ is trivial, since all investment should therefore go to technology 1, which will have the lowest cost with probability 1. When $a = 1$, the above expression simplifies to:

$$f'(R_2) = f'(B - R_2) \cdot \frac{1 - \pi}{\pi\theta}. \tag{2}$$

By definition, the mean of the distribution for θ is:

$$\mu = \pi\theta + (1 - \pi)(0) = \pi\theta. \tag{3}$$

Substituting (3) into (2), we obtain:

$$f'(R_2) = f'(B - R_2) \cdot \frac{1 - \pi}{\mu}. \tag{4}$$

The intuition for the expression in equation (4) is as follows. Relative to a solution where R&D investment is allocated such that the marginal productivities of R&D are equal (as would be implied by $f'(R_2) = f'(B - R_2)$), the effect of the second term on the right-hand side is that the optimal allocation will consist of R_2 with a lower marginal productivity than R_1. Consider the effect of increasing the successful outcome θ while preserving the mean μ. To hold the mean constant, as θ increases, π must decrease, and consequently $(1 - \pi)$ increases. This leads to an increase in the optimal R&D investment in technology 2. The effect of increasing θ in this simple case will increase both the variance and the (positive) skewness. Using numerical simulations below from continuous distributions, we will separate the effects of variance and skewness and demonstrate that the optimal R&D investment into the higher-cost technology is increasing in both variance and in skewness.

The intuition behind this result is that the downside of investment in R&D for technology 2 is bounded since we can always use technology 1 if the cost of technology 2 does not fall. However, the upside of such investment (success in reducing technology 2's cost) has a much weaker bound (costs cannot go below zero). As a result, a mean-preserving spread around θ is a mean-decreasing spread in expected future costs in optimal R&D investment as θ increases. This asymmetry between upside and downside of R&D is what drives the relationship in equation (4): as π decreases and θ increases, preserving the mean but increasing variance and skewness, more R&D into technology 2 is optimal.

The model presented above is necessarily simplified and omits many critical features. Such features include the presence of a cumulative, depreciating knowledge capital stock; a nonlinear formulation for the innovation possibilities frontier; a dependence of the benefits of R&D on the exact amount of future adoption of each technology; less than perfect substitutability; multiple R&D decision stages; and continuous probability density functions to represent uncertainty in returns to R&D. Because simple analytical closed-form solutions are not possible in a model with these features, we focus the remainder of the paper on a numerical model of R&D investment under uncertainty.

2. NUMERICAL MODEL METHODOLOGY

The numerical model used in our analysis is based on the ENTICE (Popp 2004) and ENTICE-BR models (Popp 2006a, 2006b), both of which are extensions of the DICE model (Nordhaus and Boyer 2000). ENTICE and ENTICE-BR are integrated assessment models of climate change that represent the global economy and environment and include endogenous technological change. ENTICE includes R&D investments in energy efficiency as decision variables, and ENTICE-BR additionally includes R&D investments in a noncarbon-emitting energy backstop technology.

We modify the ENTICE-BR model to solve for the optimal R&D investment allocation between two substitutable energy backstops under uncertainty. The original ENTICE-BR model represents R&D investment as additions to the two knowledge capital stocks—one that increases the energy efficiency of the economy and one that lowers the cost of a noncarbon energy backstop—according to an innovation possibilities frontier. The original model is deterministic, and the parameters in the model were calibrated to be consistent with historically observed patterns of R&D investment, energy patents, and energy consumption (Popp 2001, 2002).

In order to carry out our analysis, the following three major modifications are made to the original ENTICE-BR model: (1) we replace the single noncarbon energy backstop with two backstops that can substitute for each other; (2) we modify the representation of the knowledge stock to distinguish between the dual roles for knowledge capital—generating new knowledge versus reducing technology cost; and (3) the model is reframed as a stochastic dynamic programming problem and is solved using approximate dynamic programming. Finally, for the case study of wind versus solar in the following section, we calibrate parameters in this model to be consistent with results from Popp et al. (2013), which used forward patent citation counts by energy technology type to characterize the distribution of uncertain returns to R&D and the decay rate of knowledge capital by technology. Each of the above modifications is described in detail below.

2.1. Two Backstop Technologies

The DICE/ENTICE family of models is similar in that the objective of each model is to maximize the present value of discounted utility, subject to economic and climate system constraints. Here we discuss only the relevant constraints that are modified. A full model description is provided in the appendix, available online. In the original ENTICE-BR model, output Q_t is produced by employing physical capital stock K_t, labor L_t, and effective energy units E_t. Labor grows according to an exogenous population growth trend, and physical capital stock is the result of cumulative investment over time—that is, the current period's investment plus the previous period's capital stock that is not depreciated. Overall technological progress is captured through changes in the total factor productivity term, A_t. Effective energy units are produced from three possible energy inputs: fossil fuels F_t, a carbon-free backstop B_t, and knowledge pertaining to energy efficiency $H_{E,t}$. The prices (i.e., cost) of fossil fuels and backstop energy, respectively, are $p_{F,t}$ and $p_{B,t}$, and are subtracted from total output.[1] Thus the production function for the economy is:

$$Q_t = A_t K_t^{\gamma} L_t^{1-\gamma-\beta} E_t^{\beta} - p_{F,t} F_t - p_{B,t} B_t. \tag{5}$$

Effective energy units are produced from a nested constant elasticity of substitution (CES) representation that aggregates fossil fuels F_t, backstop energy B_t, and knowledge stock for energy efficiency $H_{E,t}$:

$$E_t = \left[\alpha_H H_{E,t}^{\rho_H} + \left(\left(\frac{F_t}{\alpha_\phi \Phi_t} \right)^{\rho_B} + B_t^{\rho_B} \right)^{\rho_H/\rho_B} \right]^{1/\rho_H}. \tag{6}$$

In equation (6), Φ_t represents the exogenous changes in carbon emissions per unit of carbon energy services, α_ϕ represents the fraction of the exogenous trend in carbon intensity that is retained in the model after endogenous R&D is added, and α_H is a scaling factor representing the amount of energy saved from increases in the energy efficiency knowledge stock. The elasticity of substitution between fossil fuels and backstop energy is $1/(1-\rho_B)$, and the elasticity of substitution between the fossil/backstop bundle and energy efficiency is $1/(1-\rho_H)$.

We extend this model to include two carbon-free energy backstops, $B_{1,t}$ and $B_{2,t}$. Thus, equation (6) is replaced with:

$$E_t = \left[\alpha_H H_{E,t}^{\rho_H} + \left(\left(\frac{F_t}{\alpha_\phi \Phi_t} \right)^{\rho_B} + \left(B_{1,t}^{\rho_\kappa} + B_{2,t}^{\rho_\kappa} \right)^{\rho_B/\rho_\kappa} \right)^{\rho_H/\rho_B} \right]^{1/\rho_H}. \tag{7}$$

The elasticity of substitution between the two backstop technologies is $1/(1-\rho_\kappa)$. Similarly, we denote the respective prices of the backstops with $p_{B1,t}$ and $p_{B2,t}$ and subtract these from total output. Equation (5) is therefore replaced by:

$$Q_t = A_t K_t^{\gamma} L_t^{1-\gamma-\beta} E_t^{\beta} - p_{F,t} F_t - p_{B1,t} B_{1,t} - p_{B2,t} B_{2,t}. \tag{8}$$

2.2. Knowledge Stock Representation of Technological Change

In ENTICE-BR, the price of the backstop can be reduced by investing in knowledge stock, $H_{B,t}$. The knowledge stock increases with the accumulation of investment in research and development, $R_{B,t}$, that is,

1. In a Ramsey-type model there are not prices in the general equilibrium sense. For the remainder of this paper, we use the term "price" of a backstop to represent its cost. Reducing the backstop price in the numerical model is equivalent to reducing the cost (C) in section 1.

$$H_{B,t} = h(R_{B,t}) + (1 - \delta_H)H_{B,t-1}. \tag{9}$$

The parameter δ_H represents knowledge decay. The function $h(R_{B,t})$ is the innovation possibility frontier, which models the process of new knowledge creation from R&D and the existing knowledge stock:

$$h(R_{B,t}) = aR_{B,t}^{b_B}H_{B,t}^{\phi_B}. \tag{10}$$

Both b_B and ϕ_B are assumed to be between 0 and 1, implying diminishing returns to R&D both within a given period (b_B) and across time (ϕ_B) (Popp 2004). Finally, the relationship between accumulating knowledge stock and the price of the energy backstop is represented by:

$$p_{B,t} = \frac{P_{B,0}}{H_{B,t}^{\eta}}. \tag{11}$$

The parameter η converts the level of knowledge stock to a reduction in backstop price in the initial period and is calibrated so that $1 - 2^{-\eta}$ is the price reduction from a doubling of the knowledge stock. Consistent with other treatments of endogenous technological change, we assume here that an increase in the knowledge stock necessarily leads to a reduction in the price of the technology. In reality, new knowledge can sometimes increase cost, or even increase uncertainty, although such cases may be fairly nuanced, with accompanying increases in quality. For purposes of clarity, we do not include the possibility of increased cost from new knowledge here but leave this to future work.

One of the challenges in calibrating representations of technological change, such as the representations described above, using empirical data is that although there is evidence that knowledge decays over time, using nonzero depreciation rates requires a nonzero level of R&D to simply maintain the backstop price at a constant level over time. Without new R&D investments in a model like that described by equations (9)–(11), the price of the backstop will rise over time as the knowledge stock decays. This behavior is not intuitive. Despite evidence that knowledge's effectiveness at creating new knowledge decays, there is no comparable evidence showing that the resulting cost reductions from new knowledge fade away over time. Once a cost-reducing innovation occurs and is adopted, it remains part of the technology.

Here we introduce a modification to the prevailing representation of knowledge stock to address the challenge just described. Instead of a single knowledge stock for each technology, we separately track two distinct but related knowledge stocks for each technology. One knowledge stock, $H_{B,t}^K$, is combined with new R&D investment to create new knowledge (as in eq. [10]). This first type of stock depreciates over time. The second knowledge stock, $H_{B,t}^C$, only reduces technology price (as in eq. [11]). This latter stock does not depreciate over time; therefore, if R&D investment falls to zero after a specific amount of time, the technology price will remain constant. This modeling innovation allows us to calibrate the model to fit historical observations of both wind and solar technologies for the case study presented in section 5. The dynamics and constraints for both stocks are defined for each of the two backstop technologies below.

The model used here therefore includes the following constraints, which replace equations (9)–(11). The knowledge stock dynamics are:

$$H^K_{B,t} = h\left(R_{B,t}\right) + (1 - \delta_B)H^K_{B,t-1} \quad B \in \{B_1, B_2\}, \tag{12a}$$

$$H^C_{B,t} = h\left(R_{B,t}\right) + H^C_{B,t-1}. \tag{12b}$$

The innovation possibilities frontiers are only influenced by the knowledge stock for creating new knowledge, $H^K_{B,t}$. As in equation (10), both b_{B} and ϕ_B are assumed to fall between 0 and 1, implying diminishing returns to both R&D and to knowledge stock:

$$h\left(R_{B,t}\right) = aR^{b_B}_{B,t}\left(H^K_{B,t}\right)^{\phi_B} \quad B \in \{B_1, B_2\}. \tag{13}$$

The price of each backstop is a function of the price-reduction knowledge stock:

$$p_{B,t} = \frac{P_{B,0}}{\left(H^C_{B,t}\right)^{\eta}} \quad B \in \{B_1, B_2\}. \tag{14}$$

2.3. Stochastic Dynamic Formulation

As described above, the objective function for the deterministic version of the model is to maximize the present value of discounted utility, subject to the economic and climate system constraints, or mathematically:

$$\max_{X(t)} \sum_{t=0}^{T} U(c(t), L(t))(1 + \rho(t))^{-1}, \tag{15}$$

where $X(t)$ represents the decision vector over time, which includes investment in physical capital stock as well as investment in R&D, and energy produced from fossil fuels and backstop sources. In general, we assume that utility is a function of per capita consumption $c(t)$ and population $L(t)$, and $\rho(t)$ is the social rate of time preference.

To model decision making under uncertainty, we reformulate the model as a stochastic dynamic program. We use a formulation consistent with Webster et al. (2012), where the DICE model is reformulated as a seven-stage stochastic dynamic program. Here, we also assume seven decision stages for the reference version of the model $t = \{1, 2, \ldots, 7\}$.[2] The decision or action space of the model consists of R&D investment in each of the

2. As shown in Webster et al. (2012) and also in the sensitivity analysis in the appendix, the impact on the first-stage decision of the set of decision stages is roughly the same for any number of stages T greater than five. The choice of seven stages is for computational tractability. Given sufficient computing power, the results for $T = 35$ (decadal decisions) would be qualitatively the same.

two backstop energy technologies and the quantity of energy produced from each backstop in the current stage, $X = [R_1, R_2, B_1, B_2]$. Uncertainty affects the returns to R&D investment as captured by the innovation possibilities frontier (eq. [13]). We introduce a multiplicative shock θ to the new knowledge function $h(R_{B,t})$:

$$h\left(R_{B,t}\right) = \theta\left[aR_{B,t}^{b_B}\left(H_{B,t}^{K}\right)^{\phi_B}\right] \quad B \in \{B_1, B_2\}, \tag{16}$$

and assume that θ is drawn i.i.d. from a distribution with an expected value of 1, so that the expected value replicates the original deterministic model. We consider many different distributions in the analysis and describe them in the next section.

The solution of the finite horizon stochastic dynamic program described above can be obtained by solving the Bellman equations (Bellman 2003) for all states and all stages:

$$V_t = \max_{X_t}[U_t + E\{V_{t+1}(X_t, \theta_t)\}], \tag{17}$$

The value function V_t is a mapping from state to optimal value, and the conventional solution method for a finite-horizon problem is backward induction. However, backward induction requires that the state space is defined such that the process is Markov. For the ENTICE-BR model which includes the modifications described above, this would require a 12-dimensional state space (see appendix). For a seven-stage model and any reasonable resolution of discretization in each dimension of state space, the computational cost of exact dynamic programming via backward induction would be prohibitive.

As a result, we solve the model using approximate dynamic programming (ADP) (Bertsekas and Tsitsiklis 1996; Powell 2007). ADP methods find an approximate solution to the dynamic programming problem using adaptive random sampling to estimate an approximation of the value function. ADP uses basis functions defined on a set of "features," typically a subset of the dimensions of the state space. The specific algorithm is similar to that applied in Webster et al. (2012), but with a few key differences, described here. A full description is provided in the appendix. In this study, we approximate the value function as a linear function of the prices of the two backstops:

$$\hat{V}_t = \beta_{0,t} + \beta_{1,t}p_{B1,t} + \beta_{2,t}p_{B2,t}.$$

The vector β is solved for using least squares on an initial set of sample paths and then is iteratively updated using the Bellman error method (Bertsekas and Tsitsiklis 1996) until convergence is achieved.

2.4. Key Assumptions

There are several key assumptions in the model formulation and parameters, which have important implications for the analysis. One critical assumption is that the probability distribution of shocks to returns to R&D is known. We recognize that, in reality, R&D decision makers also face the more difficult problem of allocating R&D

across technologies with uncertain outcomes, where the distributions of the uncertainty are not known. In this situation, the R&D investment allocation decision in each period must account for an additional consideration: by investing in R&D in a particular technology, is it possible to gain knowledge about the distribution of returns to R&D in this technology? Formally, this is defined as a partially observable Markov decision process (POMDP) (see, e.g., White 1993, chap. 6). There is a sizable literature on the theory and computational solution methods for POMDPs (e.g., Gittens, Glazebrook, and Weber 2011; Powell and Ryzhov 2012). However, the tractable application of these methods to a model of the complexity analyzed in this study remains as an area for future work. Nonetheless, as recent expert elicitations such as Baker et al. (2015) show, technology experts are able to make educated projections about the true underlying uncertainty.

A second assumption with important implications is that technologies are highly substitutable, and we do not assume capital to be immobile. As a result, the model here is not designed to address the problem of technology "lock-in," that is, the case where if one technology appears superior at first but later turns out not to be, that it is not possible to substitute this technology for the more superior one due to immobile capital. An analysis of this question would require a different model; in particular, one that assumes vintaged capital stock such as McFarland, Reilly, and Herzog (2004). Similarly, it is assumed that the model adapts to innovations that lower the price of either backstop. Although the assumed elasticities in the nested production function (7) do not imply an infinite elasticity of substitution between the two backstop technologies, this approach is also not designed to explicitly address the challenges of adapting infrastructure for radically new technologies. An analysis of this question is better pursued with sectoral models that resolve engineering constraints.

3. NUMERICAL RESULTS: OPTIMAL ALLOCATION OF R&D INVESTMENT BETWEEN TWO BACKSTOP TECHNOLOGIES

In this section, we explore the factors that determine the optimal allocation of R&D investment between two substitutable energy backstop technologies in the ENTICE-BR model. We begin with the dynamics when the returns to R&D are deterministic. We then show how the results change when the returns to R&D are stochastic, using an illustrative example distribution for the shock to R&D returns. In subsequent sections, we perform sensitivity analysis, and we use parameters and distributions informed by Popp et al. (2013) to simulate an illustrative case study of wind versus solar R&D investment.

3.1. Deterministic Results

In the absence of uncertainty, the major determinant of optimal R&D investment shares is the ratio of backstop prices. The benefit of R&D in each period is lower expected future energy prices (i.e., lower future costs). Therefore, backstop energy use in future periods largely determines the level of R&D investment in that technology in each period. To illustrate this effect, we present the optimal level of R&D investment and backstop energy production for both technologies for three different initial prices of noncarbon technology 2: \$1,300, \$1,500, and \$1,800 per ton of carbon equivalent (CTE). In the results below, we set the initial price of technology 1 to \$1,200, to represent the lowest price alternative to fossil energy. As a reference, the price of conventional fossil energy in the ENTICE model is \$276 CTE (Popp 2006b). The elasticity of substitution in the CES production function for energy between the two backstops, ($1/(1-\rho_\kappa)$ in eq. [6]), is assumed to be 5.0, that is, the technologies are highly substitutable.

Table 1 shows energy production from the two noncarbon backstops for each of the three initial prices of technology 2. Note that although technology 1 has a lower price in all cases, it is always optimal to use both nonfossil technologies. However, the ratio of energy production from the two technologies depends on relative prices; for example, if the initial price of technology 2 is \$1,300, only slightly higher than technology 1, then the ratio of energy production from the two technologies is 0.67 (equivalently, technology 2 produces 33% of the noncarbon energy). If the initial price is \$1,500, then the ratio of energy production from the two technologies drops to 0.29, and for an initial price of \$1,800, the ratio is ~0.13. In the absence of uncertainty, for a given price level, this ratio remains roughly constant over time, except for initial and terminal period effects.

In a forward-looking model like ENTICE-BR, future energy production from technology 2 determines the optimal level of R&D investment. Each period, the model chooses the level of R&D investment in each technology to equate the marginal opportunity cost of investment to the discounted reduction in future costs (e.g., the dis-

Table 1. Backstop Energy Production under Deterministic Returns to R&D for Three Different Initial Prices of Technology 2

	\$1,300			\$1,500			\$1,800		
Decision Period	Tech 1 (CTE)	Tech 2 (CTE)	Ratio	Tech 1 (CTE)	Tech 2 (CTE)	Ratio	Tech 1 (CTE)	Tech 2 (CTE)	Ratio
1	.33	.19	.57	.37	.09	.25	.42	.05	.11
2	.67	.45	.67	.79	.22	.29	.90	.11	.13
3	1.24	.83	.67	1.44	.41	.29	1.65	.21	.12
4	2.07	1.38	.67	2.42	.69	.29	2.76	.35	.13
5	3.28	2.19	.67	3.83	1.09	.29	4.38	.55	.12
6	4.83	3.22	.67	5.63	1.61	.29	6.44	.80	.13
7	4.91	3.93	.80	6.87	1.96	.29	7.85	.98	.13

Note. Technology 1 has initial price \$1,200. Energy production is in units of tons of carbon equivalent (CTE), and ratios represent (energy produced from technology 2)/(energy produced from technology 1).

counted marginal benefit of investment). Santen et al. (2017) discuss similar results from a deterministic energy R&D and capital investment portfolio planning model. The optimal levels of R&D investment in both noncarbon technologies are shown in table 2. Because technology 2 is used less when its initial price is higher (table 1), optimal R&D investment in technology 2 is lower for higher initial prices. If the initial price is \$1,300, it is optimal to invest \$700 million in the first period, but only \$500 million if the initial price is \$1,500, and \$300 million if the initial price is \$1,800.

The results here for the deterministic version are consistent with those of the original ENTICE-BR model (Popp 2006b). However, our objective in this paper is to explore the impact of uncertain returns to R&D, which we explore in the remainder of the paper.

3.2. Stochastic Results for a Reference Distribution

Here we investigate the optimal allocation of R&D investment between the two backstops when the returns to R&D for technology 2 are uncertain. Returns to R&D for technology 1 are assumed to be deterministic in this section (and are stochastic in sec. 5). As described in section 2, we model uncertain R&D returns by multiplying the generation of new knowledge produced from R&D investment in technology 2 (eq. [13]) by a random shock θ. As a base case, we begin by assuming that θ is normally distributed with a mean of 1.0 and a standard deviation of 0.4. In section 4, we perform sensitivity analysis on the probability distribution of the shock. We continue to assume that the initial price for technology 1 is \$1,200 and that the elasticity of substitution between the technologies is 5.0 (highly elastic).

For all stochastic results shown here and in the rest of the paper, the solution strategy is as follows. First, the ADP algorithm is used to solve for the approximate value

Table 2. R&D Investment under Deterministic Returns to R&D for Three Different Initial Prices of Technology 2

	\$1,300			\$1,500			\$1,800		
Decision Period	Tech 1 (\$B)	Tech 2 (\$B)	Ratio	Tech 1 (\$B)	Tech 2 (\$B)	Ratio	Tech 1 (\$B)	Tech 2 (\$B)	Ratio
1	.90	.70	.78	.90	.50	.56	1.00	.30	.30
2	1.50	1.20	.80	1.60	.90	.56	1.70	.50	.29
3	2.40	2.00	.83	2.50	1.30	.52	2.70	.80	.30
4	3.40	2.70	.79	3.70	1.90	.51	3.90	1.10	.28
5	4.30	3.40	.79	4.80	2.20	.46	5.30	1.30	.25
6	4.30	3.10	.72	5.30	1.90	.36	6.00	1.10	.18
7	1.60	1.40	.88	2.30	.80	.35	2.60	.50	.19

Note. Technology 1 has initial price \$1,200. Investments are in units of billions of US\$, and ratios represent (R&D investment in technology 2)/(R&D investment in technology 1).

functions. (Details on the algorithm, including pseudo code, are provided in the appendix.) The approximate value functions can then be used to choose the optimal decision in each stage as a function of the sequence of shocks to R&D returns that are observed. The final step is to perform a Monte Carlo simulation, sampling many possible sequences of shocks and choosing the decisions based on the approximations, in order to obtain an estimate of the expected value of the outcomes. The results of the Monte Carlo also provide an optimal first-stage decision and a probability distribution over decisions in stages 2–7, since those decisions depend on the particular sample path. We will present most results in terms of the 5th percentile, median, and 95th percentile for backstop energy production and R&D investment decisions in stages 2 through 7. Because our focus is on the relative allocation of R&D investment and energy production between the two technologies, we present results in terms of the ratio technology 2/technology 1, rather than absolute magnitudes.

Introducing uncertain returns to R&D investment in noncarbon energy technologies leads to significant variability across samples in the relative share of energy production. As shown in table 3, by design, the ratio of energy produced by the two technologies in the initial period and the median ratio over remaining periods are equivalent to the deterministic results. However, the 5th and 95th percentile results are lower and higher, respectively, than the deterministic results. In the case of technology 2, for an initial price of $1,300, the ratio of energy production ranges from 0.3 (in the 5th percentile case) to 1.5 (in the 95th percentile case), which implies that technology 2 produces more energy than technology 1. Equivalently, in the 95th percentile case, technology 2 produces 60% of noncarbon energy while technology 1 produces 40%. At higher initial prices, the results are less dramatic but qualitatively the same. For an initial price of $1,500, the ratio of energy production ranges from 0.13 to 0.5. For an initial price of $1,800, the ratio of energy production remains largely unchanged across the different sample paths of R&D outcomes; in this case, the price differential is sufficiently large enough that there will not be any substitution between the

Table 3. Ratio (Technology 2/Technology 1) of Backstop Energy Production under Uncertainty

	Initial Price = $1,300			Initial Price = $1,500			Initial Price = $1,800		
Stage	.05	.5	.95	.05	.5	.95	.05	.5	.95
1	.57	.57	.57	.25	.25	.25	.11	.11	.11
2	.50	.67	1.00	.29	.29	.50	.13	.13	.13
3	.43	.67	1.20	.13	.29	.43	.13	.13	.13
4	.29	.67	1.20	.13	.29	.43	.13	.13	.29
5	.29	.67	1.20	.13	.29	.50	.13	.13	.13
6	.29	.67	1.20	.13	.29	.50	.13	.13	.13
7	.29	.80	1.50	.13	.29	.50	.13	.13	.13

two technologies. However, as we will show below, even in this case the optimal R&D investment allocation may still vary across sample paths depending on the actual R&D outcomes in the early decision periods.

Table 4 presents the ratios of optimal R&D investment in technology 2 relative to R&D in technology 1. More R&D resources in the initial period are allocated to technology 2 under uncertainty than are allocated to this technology in the deterministic case. For an initial price of $1,300, the optimal R&D ratio in period 1 increases from 0.78 without uncertainty to 0.88 with uncertainty. For an initial price of $1,500, the R&D ratio in stage 1 is 0.56 without uncertainty and 0.64 with uncertainty, while for an initial price of $1,800, this ratio in stage 1 is 0.30 without uncertainty and 0.38 with uncertainty. In all of the cases presented here, the absolute magnitude of R&D investment in technology 1 does not change in response to the shock to R&D, so the higher ratios result solely from an increase in R&D investment in technology 2.

At later decision stages, the R&D investment decisions vary with the observed shock, and the range depends on the initial price of the backstop. With an initial price of $1,300, the R&D investment ratio ranges from 0.3 (5th percentile) to 1.2 (95th percentile), which implies more R&D investment in technology 2 than in technology 1 by the last decision stage. While this range is narrower with an initial price of $1,800, the R&D investment ratio still ranges from 0.13 to 0.24 (compared to 0.13 in the deterministic case). This occurs despite the fact that energy production from technology 2 does not vary across the sample paths of uncertainty with this higher initial price.

There are two key reasons why R&D investment in technology 2 is higher under uncertainty. The first and more obvious one is that as the price of technology 2 falls because of better than expected returns to R&D, there is substitution away from technology 1 to technology 2. This substitution enhances the expected total benefit of additional cost savings from further R&D investment in technology 2, because technology 2 is now producing more. Second, even if no substitution occurs and the ratio of energy production remains constant, the amount of energy produced by technology 2 is never

Table 4. Ratio (Technology 2/Technology 1) of Backstop R&D Investment under Uncertainty

	Initial Price = $1,300			Initial Price = $1,500			Initial Price = $1,800		
Stage	.05	.5	.95	.05	.5	.95	.05	.5	.95
1	.88	.88	.88	.64	.64	.64	.38	.38	.38
2	.78	.87	.94	.52	.57	.66	.28	.33	.38
3	.74	.82	.91	.50	.57	.63	.27	.31	.39
4	.69	.77	.89	.43	.50	.58	.25	.29	.37
5	.63	.73	.86	.41	.48	.58	.21	.26	.33
6	.56	.67	.82	.31	.38	.48	.16	.20	.25
7	.31	.73	1.19	.13	.33	.58	.13	.16	.24

zero in any of the cases considered. Because production from technology 2 is growing over time, there is still an expected benefit to reducing the price of technology 2. Therefore, reductions in the backstop price will reduce future costs, even if they do not influence the quantity of future energy production. When returns to R&D are uncertain, the possibility of larger reductions in future cost justify higher levels of R&D investment.

4. SENSITIVITY ANALYSES

The previous section presented the results for a single illustrative probability distribution of returns to R&D. However, the results will vary depending on the specific probability distribution assumed. In this section, we perform several sensitivity analyses to explore how the characteristics of uncertain R&D returns affect the optimal share of R&D investment in the higher-price technology. As discussed in the introductory section, returns to R&D investment in many of the currently high-price low-carbon energy technologies, such as solar PV or coal with carbon capture and sequestration, also tend to be more uncertain (i.e., higher variance) and/or appear to be positively skewed, according to results from expert elicitations (e.g., Baker et al. 2015). Results from the analytical model of section 1, although stylized, suggest that more R&D investment should be channeled to the higher price technology where returns to R&D investment are more uncertain (i.e., distributions with higher variance/skewness). However, this simple analytical model could not distinguish between the impacts of variance and skewness. Here, we utilize the numerical model to decompose these impacts.

4.1. Sensitivity to Variance of the Uncertainty in Returns to R&D

We begin by assessing the impacts of increasing the variance of the distribution of R&D returns while holding both the mean and the skewness constant. Specifically, we compare the results from alternative normal distributions associated with the shock θ to new knowledge in equation (16). Table 5 compares the results for the deterministic model and four versions of the stochastic model, assuming normal distributions for the shock with standard deviations ranging from 0.1 to 0.4. All distributions have a mean of 1.0 and a skewness equal to zero, by definition.

Table 5. Impact of Variance in the Returns to R&D on the Ratio (Technology 2/Technology 1) of R&D Investment, Energy Production, and Energy Price

Distribution	Mean	Variance	R&D Ratio ($t = 1$)	95% Back Ratio ($t = 7$)	5% Price Ratio ($t = 7$)
Deterministic	NA	NA	.56	.29	1.29
Normal	1.00	.01	.58	.29	1.25
Normal	1.00	.04	.59	.44	1.21
Normal	1.00	.09	.61	.50	1.17
Normal	1.00	.16	.64	.50	1.12

As the variance in the distribution of uncertain returns to R&D increases, higher levels of R&D investment into technology 2 are optimal. The ratio of R&D investment in the two technologies increases monotonically from 0.56 to 0.64 as the variance increases, and is roughly linear in the standard deviation. This trend is driven by the possibility of much higher utilization of technology 2 in future energy production. The median ratio of energy production between the two technologies is essentially the same as in the deterministic results. However, it is informative to look at the upper bound (95th percentile) to see how much of technology 2 could be utilized. By the last decision stage, the 95th percentile of the ratio of energy production increases from 0.3 to 0.5 as the variance is increased. Greater energy production by technology 2 in some samples occurs because higher variance makes a larger reduction in the price of technology 2 possible in some cases. The price ratio that drives greater use of technology 2 is best seen by examining the corresponding lower bound of the price ratio (the 5th percentile). The last column of table 5 shows the 5th percentile of the price ratio of technology 2 to technology 1 in the last stage (also see fig. D1 in the appendix for the price levels over time for these probability distributions). This ratio falls from 1.3 to about 1.1 as variance is increased. As discussed in section 1, symmetric uncertainty in the returns to R&D (in terms of reductions in the price of the technology) has an asymmetric impact on R&D investment decisions. This is because if the backstop price for technology 2 in one sample falls by less than the average reduction, it does not change the investment or energy production decisions; we already were not using technology 2 and we still do not. In contrast, if in one sample the price of technology 2 falls by much more than the average reduction, the subsequent decisions are more responsive; we use much more of technology 2 than in the average case.

4.2. Sensitivity to Skewness of the Uncertainty in Returns to R&D

Here we explore whether the increase in optimal investment is only a function of increased variance, or whether positive skewness has an additional distinct impact. To investigate this, we compare the results assuming (*a*) no uncertainty; (*b*) uncertain returns to R&D with a normal ($\mu = 1.0$, $\sigma = 0.4$) distribution; (*c*) uncertain returns with three alternative skewed distributions. Across the three skewed distributions, we hold the mean constant at 1.0 and the standard deviation constant at 0.4 (and a variance of 0.16), but assume different levels of skewness. To do this, we construct the following three distributions:

- A lognormal distribution, with mean $\mu = 3$ and standard deviation $\sigma = 0.39$. These refer to the mean and standard deviation of the natural log of the support. Because this distribution does not have a mean of 1.0 as desired, we normalize by dividing every random sample from this distribution by its expected value, which is 1.67. In this way, the mean of the samples of R&D returns shock is 1.0.

- A generalized extreme value distribution, with parameters $K = 0.2$ (the shape parameter), $\sigma = 1.0$ (the scale parameter), $\mu = 3.7$ (the location parameter). These samples are also normalized by the expected value of this distribution, 4.52, so that the mean shock is 1.0.
- A generalized Pareto distribution, with parameters $K = 0.01$ (the shape parameter), $\sigma = 0.065$ (the scale parameter), $\theta = 0.1$ (the location parameter). These samples are also normalized by the expected value of the distribution, 0.166, so that the mean shock is 1.0.

The probability distributions assumed for this sensitivity analysis are summarized in table 6, along with their mean, variance, skewness, and the resulting R&D investment ratio in the first decision stage. Note that as the skewness increases, with variance held constant, the initial R&D investment share into technology 2 increases. In the case of the lognormal distribution, with a skewness coefficient of 1.26, the increase in the R&D investment share is negligible. However, for the extreme value distribution, with a skewness coefficient of 3.02, the optimal R&D ratio increases from 0.64 to 0.66. For the Pareto distribution, with a skewness coefficient of 13.8, the optimal R&D ratio increases to 0.78.

Table 6. Impact of Skewness on the Ratio (Technology 2/Technology 1) of R&D Investment, with Constant Mean and Variance

Distribution of Shock and Parameters	Mean	Variance	Skewness	R&D Ratio at $t = 1$
Deterministic:				
NA	NA	NA	NA	.56
Normal:				
$\mu = 1$	1.00	.16	.00	.64
$s = .4$				
Lognormal:				
$\mu = 3$	1.00	.16	1.26	.64
$s = .39$				
NORM = 21.67				
Generalized extreme value:				
$K = .2$ (shape)	1.00	.16	3.02	.66
$\mu = 1.0$ (scale)				
$s = 3.7$ (loc)				
NORM = 4.52				
Generalized Pareto:				
$K = .01$ (shape)	1.00	.16	13.78	.78
$\mu = .065$ (scale)				
$\theta = .1$ (loc)				
NORM = .166				

The intuition behind these results also follows from the asymmetric response of R&D investment to uncertainty in returns to R&D. By increasing the positive (right-tailed) skewness of the distribution, while holding mean and variance constant, not only do very high positive outcomes have a higher probability (in the upper tail), but the probability mass of the distribution also increases below the mean (i.e., below 1.0 in this case). This is mathematically necessary for the distribution to still integrate to unity and have the same expected value. Nonetheless, the higher probability of worse than expected outcomes (probability mass below the mean) is more than outweighed by the benefit of possible very high positive outcomes (the long upper tail of the distribution). The net effect in this numerical example is to increase the optimal R&D investment in technology 2 in the initial stage.

An additional sensitivity analysis is presented in the appendix, exploring the implications for different numbers of decision periods in the model. Those results show qualitatively little difference between 5 and 12 decision periods. The reference version of this model using seven decision periods is based on this analysis.

5. CASE STUDY: WIND VERSUS SOLAR

In this section, we apply our numerical model to a real-world example of two technologies that are prominent in the public debate over low-carbon energy: wind and solar. These technologies are often compared because they are both intermittent noncarbon electricity sources. Current estimates show that solar photovoltaic technologies cost approximately 60% more than onshore wind technologies. As discussed in the introductory section, some have argued, based on recent "failures" in solar, that further R&D investment in solar technology is not prudent. Using our numerical model, we examine whether continued R&D investment in solar technology is indeed a poor decision. To do this, we calibrate our two-backstop model to approximate the critical characteristics of wind and solar technologies. We then use the numerical model to determine the optimal investment strategy in later stages, conditional on experiencing below average R&D investment returns in previous stages.

We use probability distributions for the returns to R&D for wind and for solar that are qualitatively based on the empirical results of Popp et al. (2013), and we calibrate the other technical change parameters in the model to correspond to wind and solar based on recent estimates. For space considerations, we describe the calibration procedure and other assumptions in detail in the appendix and refer the interested reader there.

As in the previous sections, we present the results in terms of the ratio of energy production and the ratio of R&D investment between the two technologies (solar/wind). Reporting ratios rather than levels allows us to assess the impacts on relative R&D investment allocation. We therefore explicitly report the ratio of backstop energy production and R&D investment for each sample path after convergence and summarize the results with the median and 90% range of the ratio over time.

The results are presented graphically in figure 1. Without uncertainty, energy produced from solar is only 12% of the energy produced by wind, in all seven stages. With uncertain returns to R&D investment, the median energy production ratio is the same as in the deterministic solution; however, the 5th percentile of the energy ratio is zero after stage 3 (no energy production from solar), while the 95th percentile ratio is approximately 0.7 (energy produced by solar is a 41% share of the noncarbon total). The difference between the deterministic and stochastic results for the R&D investment ratio is more striking (fig. 1*B*). The optimal R&D investment in solar without uncertainty is roughly 20% of the R&D investment in wind. Under uncertainty, however, the optimal first-stage R&D investment ratio is significantly higher, nearly 50%. The median R&D investment ratio remains well above the deterministic solution in all stages except the last stage. The 90% range for the R&D investment ratio is quite wide, ranging from zero to 60%.

Finally, we examine the optimal response to a less than average return from R&D investment in solar. Because the random returns to R&D investment and decisions are all continuous, we need to define what a "poor outcome" from R&D investment is and what "eventual success" of a technology is. Different threshold values could be used, but the qualitative results remain the same. As an illustration, we define a random draw of the return to R&D investment in solar of less than 0.5 (where 1.0 defines

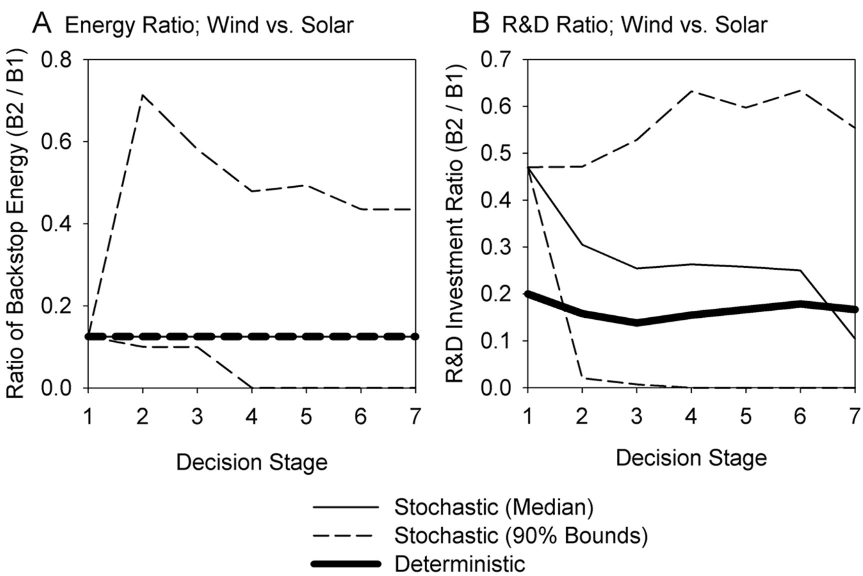

Figure 1. Optimal ratios of backstop energy production and R&D investment (solar/wind). Median and 90% bounds in the stochastic case are shown in thin lines, and the deterministic results are shown in thick lines.

the mean outcome) as a "poor outcome." We also define "eventual success" of solar as a sample path for which the ratio of energy produced from solar to energy produced by wind in stage 7 is greater than 1 (i.e., a share of more than 50% of the noncarbon energy production). This subset of sample paths consists of those for which more energy is produced from solar than from wind.

Using these thresholds, from a sample set of 1,000, there were 387 simulations where the random shock to returns to R&D investment in solar was less than 0.5 in the first stage, or a probability of 0.387. We wish to determine the optimal R&D investment in solar in stage 2, conditional on observing this "poor outcome" in stage 1, and also the conditional probability of eventual success given this outcome in stage 1. The conditional optimal R&D investment in solar in stage 2 is $634 million, not significantly less than the unconditional optimal R&D investment in solar of $710 million. Therefore, one poor draw is not sufficient to justify major reductions in R&D investment in solar. Conditional on a poor outcome in stage 1 as defined here, the probability of eventual success of solar is 0.018. This is considerably less than the unconditional probability of success across all 1,000 samples, 0.071. Nevertheless, the expected future amount of energy produced from solar is sufficiently large that continued R&D is justified for these cases.

Of course, repeated poor outcomes over many stages should not lead to continued R&D investment at the same level. In this sample of model runs, if the returns to R&D investment in solar in the first three stages are poor (less than half the expected value), then the optimal R&D investment in solar from stage 4 onward is effectively zero.

6. CONCLUDING DISCUSSION

This paper addresses the question of optimal R&D investment portfolios in low-carbon energy technologies in the context of climate change. We develop both simple analytical and detailed numerical models to represent two substitutable low-carbon energy technologies, where each can have costs reduced through investment in R&D. Using both models, we show explicitly how uncertain returns to R&D can change the optimal R&D investment decision. Given two technologies with different initial costs and uncertain R&D investment returns in the higher cost technology, the optimal R&D investment in the higher cost technology is higher the larger the variance and positive skewness of the distribution of uncertain returns.

In contrast with other approaches that model specific technologies in more detail to examine the optimal portfolio question, we have deliberately focused on a stylized model of two technologies and performed sensitivity analyses to explore the general conditions that justify R&D investment in the higher-cost technology. Nevertheless, we do conduct an analysis where model parameters are calibrated to be consistent with the empirical data on wind and solar technologies. Using data on wind and solar patent citations to characterize the nature of uncertain R&D investment returns for each technology, we illustrate that significant R&D investment into solar is justified—the optimal portfolio would allocate roughly 2/3 of the R&D investment budget to wind and 1/3 to solar in the first stage. Moreover, a poor first-stage return to R&D investment in solar technology does not lead to a large reduction in the optimal solar R&D investment in stage 2. Repeated poor outcomes over three or more stages must be observed before abandoning solar is justified.

There are a number of related questions that we do not explicitly address, due to the inappropriateness of the modeling approach used here. One key question is the socially optimal division of R&D investment between the public and private sectors. This question requires more detailed modeling of spillovers and knowledge externalities than is captured in the ENTICE model. Nor do we address how R&D investment in energy technologies should be, or will be, divided among different nations. Clearly, energy technologies exist in a global market, and technological advances achieved in one country reduce the costs of that technology in other countries. However, this issue involves complex issues of economic competitiveness. The results here have focused only on the socially optimal total global R&D investment, without consideration of public/private or international divisions of effort. Additionally, we have examined here the case of two easily substitutable energy technologies. Some technologies are complementary, with both technologies needed in order to make the energy system function reliably, and the implication for R&D portfolios over a more general set of technologies is left for future work.

Finally, the decision problem faced by R&D investment decision makers often requires choosing an R&D investment portfolio without knowing the "true" distribution of the uncertainty in R&D investment returns for each technology. In this study, we have assumed that the probability distribution is known, and we demonstrate how optimal R&D investment will change for alternative distributions. When the probability distribution is not known, there is an additional expected benefit to R&D: researchers gain information about the true distribution. The question of how to allocate R&D investment across technologies when the true distribution of the uncertainty is not known is another important area for future research. The appropriate formulation of this problem is as a partially observable Markov decision problem (POMDP), which are computationally challenging to solve.

Our main contribution to the debate over energy technology R&D investment is conceptual. Given two substitutable technologies, where one costs more today, greater uncertainty about future R&D outcomes justifies more R&D into the higher-cost technology. In particular, if one believes that the distribution of R&D outcomes is highly positively skewed, then considerable R&D may be justified. A direct consequence of a positively skewed distribution is that the vast majority of outcomes from individual R&D projects will be well below average. Thus a poor outcome from R&D may not necessarily be a reason to abandon future efforts; rather many failures may be expected along the road to long-shot technology breakthroughs. This perspective lends support to several recent efforts by the US Department of Energy, including ARPA-E and the Sunshot Program. An efficient R&D portfolio will therefore allocate R&D both to currently competitive technologies and also to long shots, in proportion to the relative expected benefits if successful.

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