A Survey on Mathematical models of Swarm Robotics

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Abstract

This article provides a survey of available literature of some of the methodologies employed by researchers to utilize swarm robotics for modelling purposes. The article envisages the development of mathematical models involving in swarm systems. Suitable application with respect to the mathematical models is also reported. The extensive bibliography provided with the article will be of help to the new entrant as well as researchers working in this field.

1. Introduction

In the last decades, researchers have motivated in swarm systems consisting of multiple autonomous agents. Such systems can exhibits complex behaviours which appears transcend the abilities of the relatively constituent individuals. Swarm systems are inspired from the observation of social insects [10][9]. Such observations have tried to extract ideas, models and philosophies underlying natural systems and apply them to artificial problems. When applying rules extracted from natural systems to artificial problems we need specific mathematical models to understand the internal mechanism of the systems with respect to real and simulation environments. Natural systems have flexibility, scalability and reliability but artificial systems are not. This is the reason why the mathematical model plays a vital role in modelling of swarm systems. In principally, swarm robotics has wide range of applications with various domains, but we restrict our discussion to follow the significant issues concerning the topics like swarm intelligence, distributed systems, self-organized systems, and concentrated towards the mathematical models of swarm robotics.

Perspectives of Swarm Robotics

In the late 1940s Grey Walter and his collaborators pioneered the first experiment on turtle-like robots equipped with light and touch sensors; these simple robots interacted in a seemingly social manner and exhibited “complex behaviour” [17], but the rapid progress of swarm robotics research is actively studied in the 1990s. The concept of swarm robotics was first initiated in the early 1989 by G.Beni [6] with the discussion of cellular robotics systems. Deneubourg and his colleagues studied the first experiment on stigmergy in simulated and physical “ant-like robots” [5] [15] in the early 1990s. Since then, numerous researchers have developed robot collectives, self-organized systems [13] and have used robots as models for studying social insect’s behaviour [33]. Swarm robotics is difficult to define properly due to wide range of applications, so we mention the particular one defined by E.Sahin [63] “Swarm robotics is the study of how large number of relatively simple physically embodied agents can be designed such that a desired collective behaviour emerges from the local interaction among agents and between the agents and the environment.”

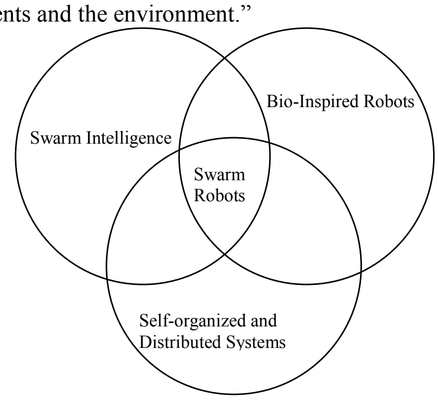

Figure.1. Swarm intelligence, bio-inspired robots, self-organized systems and distributed systems are overlap when the individuality plays a lesser role and swarm robots are intersected with these systems.

The rest of the paper is organized as follows. In the Section 2 we present the related works of swarm systems. Then, Section 3 explains a mathematical models. In section 4 suitable application with respect to mathematical model is posed. Finally we report the conclusions in Section 5.

2. Related Works

Baynidir and Sahin [4] proposed a review studies on swarm robotics. They classified the taxonomy in five modelling axis namely, modelling, behaviour design, communication, analytical studies and problems. Dudek et al., [18] also classified the swarm robotics literature in various domains like communication range, communication bandwidth, communication topology, and swarm size. And Lerman et al., [39] proposed review studies on macroscopic probabilistic models. Other hand, some of the Key research domains in swarm robotic applications is mention as follows:

Aggregation- This task is normally refers to gather a group of randomly scattered robots in the environment and forms the robot clusters. An application of aggregation behaviours is studied by Kube and Zhang [32]. Melhuish et al., [53] implemented an aggregation algorithm in several ways. And also Trianni et al., [78] used genetic algorithms to evolve neural networks controllers for a swarm of robots in an aggregation task. Similarly, Bahgeci and Sahin [2] tried to achieve the aggregation behaviour by evolving weights of neural networks. In contrast, Soyal and Sahin [67] also studied the probabilistic aggregation strategy of swarms systems. Garnier et al., [23] tried to reproduce the aggregation behaviour observed in cockroaches.

Foraging- In this scenario, the robot is able to collect the objects and deliver them in to some pre-specified home location. Early works in robot foraging focused on the design of behaviours to fulfil the task. The first implementation of foraging using a group of real robots is believed to be done by mataric and marjanovic [51]. Mataric [50] also reveal the reinforcement learning [49] approaches in designing the foraging behaviour in a group of physical robots. Lerman et al., [41] [42] developed a macroscopic probabilistic model to quantitatively analyse the effect of swarm size and interference among the robots on the overall performance based on mataric's experiments. They reveal the group performance improves as the system size grows at first, but declines for large group sizes due to the effects of interference. Sugawara et al., [71] [68] proposed a virtual pheromone system for robots foraging task. Theraulaz et al., [74] investigated the division of labour in social insects, is mostly used the task allocation mechanism.

Clustering and Sorting- These works are mainly influenced by the nest building behaviour of termites and wasps, In a nest of wasps, the larvae, cocoons and eggs are arranged as separate clusters in entirely different part of the nest. Deneubourg et al., [15] proposed a simple model to explain this phenomenon. The first implementation of collective clustering using a group of robots is presented by Beckers et al., [5]. Martinoli, et al., [46] have carried out more experiments using the same algorithm, they found that the pucks cluster into several piles, instead of a big cluster as observed in Beckers et al [5]. Melhuish et al., [52] made a step further by extending the collective clustering to collective sorting.

Exploration-Is closely related to dispersion, as robots must distribute themselves to maximize the rate at which environment is explored. Exploration is typically used in unknown environments where robots are not endowed with a map. Zolt et al., [80] experimentally studied a market-based coordination scheme for exploration using a team of ten robots exploring an odour environment and dividing the task using auction algorithm [81]. Robot task allocation using the exploration work is presented in Berhault et al., [8]. Burgard et al., [11] presented a centralized architecture for explicit collaboration by trading off travel cost and information gain in order to distribute the robots in the environment.

Flocking- Flocking and Schooling are examples of highly coordinated group behaviours exhibited by large group of birds and fish. The flocking phenomenon is first initiated by Reynolds [62] boids in the simulation environment. The success of the boids model has influenced the attention of researchers in the robotics domain. Mataric [49] explored emergent behaviour and group dynamics by combination of predefined simple behaviour such as aggregation and dispersion. Brogan and Hogdins [12] evaluated the flocking behaviour in the simulated environment.

3. **Mathematical models**

Concerning swarm system, individual robot behaviour is simple but they exhibit the complex behaviours for the desired tasks. The interaction among the robots, between the robots and its environment is very important to achieve the overall group performance. We need specific parameters to understand the whole behaviours of the system. Real and Sensor-based experiments are the well known aspects to observe the behaviours of the system with different parameters. On the other hand, real and simulation experiments are very expansive and time-consuming, and also have some problems in size, noisy and other environmental issues for the over all system performance. Using the mathematical analysis we can rapidly and efficiently study the swarm systems in order to understand the behaviour of the system with real and simulation environments. Mathematical models of swarm systems come in two flavours: microscopic and macroscopic. In the microscopic modelling each robot is modelled individually and their interaction mathematically and in the macroscopic modelling whole system is modelled as a single representation.

3.1 Probabilistic models

A probabilistic model is involving in both microscopic and macroscopic approaches. Martinoli et al., [43] Presents a modelling technique based on rate equations which was successfully applied to several scenarios. Many works can be found followed this approach by Martinoli himself but also by Lerman et al., [39] [44] [40] A methodologically very sophisticated approach is reported in [41] where parameters of the rate equations are determined by the use of the system identification techniques. Galstyan et al [22] present a generalization of Master equation for robot densities and continuous space .And used microscopic approach to study collective behaviour of a swarm of robots engaged in cluster aggregation and collaborative stick-pulling [47] in which a robot's interactions with other robots and the environment are modelled as a series of stochastic events, with probabilities determined by simple geometric considerations and systematic experiments with one or two real robots. In contrast, Lerman et al [42] developed a macroscopic model, as widely used in physics, chemistry, biology and the social sciences, to directly describe the collective behaviour of the swarm robotics. A class of microscopic models have been used to study the effect of interference in a swarm of foraging robots [40] and collaborative stick-pulling [45] have successfully expanded the macroscopic probabilistic model to study dynamic task allocation in a group of robots engaged in a puck collecting task, in which the robots need to decide whether to pick up red or green pucks based on observed local information. Jeanson et al., [31] presents the aggregation strategies in cockroaches using the probabilities of stopping and moving the aggregation point of view. Ijspreet et al., [29] demonstrated the probabilistic models to control the robots in stick-pulling experiments. Apart from these studies population dynamics of the swarm system using the probabilistic models also exists. In this case the average number of robots in a certain state at some time, instead usually do not take into account the spatial distribution of the robots. Nevertheless, such models have shown strong quantitative agreement with a series of real-world robotic case studies where the performance metric is non-spatial. Object clustering [35], collaborative manipulation [45], inspection [42] and robot aggregation [48] good agreement has been obtained for model prediction and realistic simulation. Probabilistic modelling has also been used to address the robot task allocation problems by Agassounon et al., [1]. Here the contribution from Agassounon et al, is of particular interest for swarm robotic coordination as task allocation in their work is based on threshold-based algorithms, which are inspired by division of labour in ant colonies [74] and can be competitive to market-based simulation solution [34]. Probabilistic model related works can be found in [57] [61] [48] [73] [26].

3.2 Differential equations

Differential and difference equations are plays a vital role in swarm robotics, in order to understand the analytical and control behaviour of the system. The Markov chains can be transcribed in to a system of difference or differential equations [72] (one for each state) that summarize the average state transitions and thus track the average number of robots in each state. In many cases, interactions among the robot lead to the transition probabilities that are a function of the number of the number of robots in other states, and thus yield a system of differential/difference equations that are non-linear.Lerman et al., [40] presents a mathematical model of foraging in a homogeneous group of robots using the behaviour based control [41] [40]. They demonstrated the interference technique by using the series of coupled differential equation [43] [40] [41], In contrast some of the researchers used the difference equations for their foraging studies. For example, Liu et al [38] proposed the difference equation to study the collective foraging experiments. Martinoli et al., [44] also applied macroscopic modelling to stick pulling problems. For each stage, a difference equation (DE) is developed and the steady state of the DE system is analyzed to obtain the average number of robots in each state at the end of the experiments. A part from these studies some of the researchers used Partial differential equations with Fokker-plank phenomenon for pheromone studies of swarm systems [61] [65] [64]. In the mathematical biology point of view some of the works on swarm systems are exists and researchers have used integro-differential equations with non-local continuum model for biological aggregation [77], [19].And also Mogilner and Edelstien-Keshet [55] are proposed a non-local model for a swarms using integro-differential equations. And this type of problems are studied by several researchers in various domains [54] [75] [76] [79] [69] [70] [20].

3.3 Dynamical systems

Most of the process that occurs in nature can be described as dynamical systems, as demonstrated by Okubo [58] and Berder [7]. Dynamical systems are widely used in ecology, economy, social science, and biology. In swarm systems, it's a new field and that has become popular in recent years. At present there is not much work is in progress, but we try to show some works towards this topic for new entrants. In dynamical systems whose behaviour is determined by individual interactions and the elements are changed over time. Gazi and Passino [25] [24] studied the stability analysis of swarms using the dynamical systems and similarly lemanon and Sun [36] studied the cohesive swarm under consensus [37] [19] [3] by using the integral actions and also these techniques used by Li et al., [37] for the Stable swarming by mutual interaction of Attraction/alignment/repulsion process. Related works can be found in [59] [60] [14] [3].

3.4 Automata theory

The automata theory in swarm systems are mainly inspired from the cellular robotics systems [30]. The key idea of these applications is a way to represent finite state spaces, and the transition between these states is move from one state to another state in different ways. Dudek et al.,[18] presents unbounded number of finite state automata with the ability of communicate their states to their neighbours in arbitrary Turing machines. They explain automata and turning machines by using the conventions and notations of [28]. Soyal and Sahin [67] performed the aggregation task by using the probabilistic finite state automata (PFSA). Ijspreet et., [29] also performed stick pulling experiment with PFSA. Shen et al., [66] also presented cellular automata with swarm robotics with a computational model for self-organization studies. Ermentrout and Edelstein-Keshet [21] shown some work in their review studies and automata theory in biological aspects can be found in [27] [56] [16]. Automata theory in swarm systems has some advantageous in unknown exploration, because environment can be split in to a cell based systems, to choose the significant finite state machines.

4. Suitable applications with respect to model

Probabilistic models: 1. Foraging, chain formation, stick-pulling, aggregation tasks, cooperation-transportation, clustering and sorting. 2. Nest building of wasps and termites. 3. Flocking and schooling of fish.
Automata theory: 1. Collaborative-mapping, unknown exploration, lattice formation, graph-based theory. 2. Space systems. 3. Pattern formations, Stick-pulling problems.
Differential and Partial differential equations: 1. Pheromone, trophallaxis and stigmergic studies, collective foraging, space systems, clustering and sorting, flocking and schooling of fish. 2. Attraction , alignment and repulsion systems.
Dynamical systems: 1. Cohesion and stability analysis, beacon and odour localization, bifurcation models, attraction, alignment and repulsion systems. 2. Aggregation tasks. 3. Population dynamics of swarms.

Table 1. Mathematical model related to applications

Markov's Chain Process: $\Delta p(n,t) = p(n,t+\Delta t) - p(n,t)$ $= \sum_{n'} p(n,t+\Delta t/n',t)p(n',t) - \sum_{n'} p(n',t+\Delta t/n,t)p(n,t)$	*P(n,t)*= Probability of an agent to be in the state n at time t, Δt is an Transition probabilities, probability of state n', $\sum_{n'}$ is sum of all possible states.
Rate-Equation (Time-Continuos): $\frac{dNn(t)}{dt} = \sum_{n'} W(n/n',t)Nn'(t) - \sum_{n'} W(n'/n,t)Nn(t)$ $W(n/n';t) = \lim_{\Delta t \to 0} \frac{p(n,t+\Delta t/n',t)}{\Delta t}$	n , n' = all possible states at each instant, *Nn*=average fraction of agents in state n at time t.
Rate-Equation (Time-Discrete): $Nn((k+1)T = Nn(kT) + \sum_{n'} TW(n/n',kT)Nn'(kT)$ $- \sum_{n'} TW(n'/n,kT)Nn(kT)$	*W* is an Transition rates(Linear, nonlinear) Time discrete: *k* is an iteration index, *T* time step
Geometric Probabilities: $r_i = \frac{vWs}{As} g_i$ $p_i = r_i T$	*As* is detection area of the object, *v* is a mean robot speed, *Ws* robot's detection width, g_i geometric probabilities, $r_{i,}$ encountering rate. In time discrete p_i is encountering probabilities (per time step).
Linear Model-Probabilistic Delay: $Ns(k+1) = Ns(k) - PaNs(k) + PsNa(k)$ $Na(k+1) = N_0 - Ns(k+1)$ $Ns(0) = N_0; Na(0) = 0$	*Pa*=Probability of obstacle avoids, *Ps*= Probability of resuming search, *Ns* is average robots in search, *Na* is average robots in obstacle avoidance, *No* is no of robots ,*k*=iteration index.
Non-Linear Model-Probabilistic Delay: $Ns(k+1) = Ns(k) - P_{g1}[M_0 - N_g(k)]Ns(k) + P_{g2}N_g(k)Ns(k)$ $+ P_{g1}[M_0 - N_g(k-Tg)]\Gamma(k;0)Ns(k-Tg)$	M_0=no of sticks, Γ=fraction of robots that abandon pulling, *Tg* gripping parameter at time t. and other terms represented same as linear model.
Dynamics of Random walk: $\frac{dp(n,t)}{dt} = W_{(n-1)\to n}P(n-1,t) + W_{(n+1)\to n}P(n+1,t) - W_{n\to(n+1)}P(n,t)$ $- W_{n\to(n-1)}P(n,t)$ $\frac{dp(n,t)}{dt} = \frac{1}{2}[P(n-1,t) + P(n+1,t)] - P(n,t)$	*W* is an Transition rate, *P(n,t)* probability of given state at time t

Table 2: Probabilistic models

Differential euqations: $\frac{dNs(t)}{dt} = \alpha_r Ns(t-\tau)[Ns(t-\tau)+N_0] - \alpha_r Ns(t)[Ns(t)+N_0],$ $\frac{dM(t)}{dt} = -\alpha_p Ns(t)M(t)$	*Ns(t)* be the no of robots in search state at time t, $Ns(t)+Na(t)=N_0$ be the total no of robots, *M(t)* be the no of uncollected pucks, t-τ exit the avoiding and resume search.α_r be the rate of detecting another robot, α_p be the rate of detecting the puck.
Fokker-Plank equation(difusión with drift): $\frac{\partial}{\partial t} S(r,t) = D\nabla^2 S(r,t) - \alpha\nabla[\nabla P(r,t)S(r,t)]$	*S(r,t)* density of agents at position r at time t. *P(r,t)* intensity of the pheromone, *D* diffusion with drift, α>0 is greediness of following gradient. $\nabla=(\frac{\partial}{\partial x},\frac{\partial}{\partial y})$
Langevin Equation: $v = \frac{dR}{dt}, m\frac{dv}{dt} = \psi(t) - \gamma v,$ $\frac{dR}{dt} = A(R,t) + B(R,t)F(t)$	The drift *A* and diffusion coefficient *B* might be time and/or space-dependent, *F(t)* is an uncorrelated time, *Velocity* $v=(v_1, v_2)^T$ and Position $R=(R_1, R_2)^T$. m is an mass ,γ is an friction constant , and a noise term $\psi=(\psi_1, \psi_2)^T$ and *B* is an Brownian motion with drift.
Master Difference Equation: $N_0 = N_s(k) + N_R(k) + N_G(k) + N_D(k) + N_H(k) + N_A(k)$ $+ N_{Ah}(k) + N_{Ag}(k) + N_{Ad}(k)$	Let $N_s(k)$,$N_G(k)$,$N_D(k)$,$N_H(k)$ and $N_R(k)$ be the average no of robots in states searching grabbing, deposit, homing and resting respectively in time step k. Also $N_A(k)$,$N_{ag}(k)$,$N_{Ad}(k)$ and $N_{Ah}(k)$ are the average no of robots in states avoidance which transfer from states searching, grabbing, deposit and homing .
Integro-differential Equation: $S(x) = \int_{R^n} K(x-y)\rho(y)dy \equiv K * \rho$ $\rho t + \nabla.(Vl^{n+1}\rho K * \nabla\rho - Vrl^{2n+1}\rho^2\nabla\rho) = 0.$ $\int_{R^n} K(x)dx = l^n, x, v \in R^n, \rho(x,t) \geq 0 and Vl = \frac{va\alpha}{\nabla(K+\rho)}.$	*S(x)* is the sensing function of the population at position x, Velocity *v(x,t)*. The *K* incorporates the sensing range and degradation for the particular species. *l* is an character range and degrade over the distance,ρ which moves the velocity at time t. ∇ is an del operator.

Table 3:Differential and Partial differential Equations

Difusión-advection-reaction Equation: $\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}(D\frac{\partial f}{\partial x} - vf)$	Here x is one-dimensional coordinate, t is time, $f(x,t)$ is the swarm density, constant coefficient D(the consequences of the density dependent diffusion).
One set of Aggregation model for swarm: $V(f) = a_e f + A_a(K_a * f) - A_r f(K_r * f), where$ $K_a(x) = -\frac{x}{2a^2}\exp(-\frac{x^2}{2a^2}), K_r(x) = -\frac{x}{2r^2}\exp(-\frac{x^2}{2r^2})$	$Ka(x).Kr(x)$ are normalized attraction and repulsion kernels, f is constant terms containing convolutions.
One set parameter Bifurcation model: $\dot{x} = f(x,\alpha), x \in R^n, g(x) = \frac{1}{1+\lvert f(x),\rvert}, \dot{x} = \mu - x^2$ $\dot{x} = (\mu - x)x, \dot{x} = (\mu - x^2)x, \dot{x} = (\mu + x^2)x$	$G(x)$ is transformation function, x is particle swarm, α is parameter, and μ - interval
Cohesive swarms: $\dot{x}_i(t) = u_i(t) + \sum_{j \approx i} g(\lVert x_i(t) - x_j(t) \rVert)(x_i(t) - x_j(t))$ $u_i(t) \in R^n, g : R^+ \rightarrow R$	The physical state of ith agent at time is $x_i(t)$, the equation represents as interaction between agents.
Consensus filter equation: $\dot{\hat{x}}_i(t) = (x_0(t) - \hat{x}_i(t)) + \sum_{j \approx i} A_{ij}(\hat{u}_j(t) - \hat{x}_i(t))$ $+ \sum_{j=1}^{n} A_{ij}(\hat{x}_j(t) - \hat{x}_i(t))$	Consensus ith agent at time t, the coefficient A_{ij} is the ij-th components of the adjacent matrix, u and x are the same as cohesive swarming.
Probabilistic finite state automata: $P_w = A_w / A_A, P_R = N_R.A_R / A_A, P_{G1}(t) = N_{G1}(t).A_S / A_A$ $P_{G2}(t) = N_{G2}(t).R_{G2}(t).A_S / A_{A,} P_N(t) =$ $1 - (P_w + P_R + P_{G1}(t) + P_{G2}(t))$	At each iteration, no of probabilities P_N,P_W encountering a wall. P_R encountering robot,P_S finding a stick, P_{G1}, and P_{G2} holding stick and another robots respectively,,A_W surrounding wall, A_A whole arena A_R one robot.
Turing Machines: $M = (Q_M, \Sigma_M, \Gamma_M, \delta_M, q_{0M}, B_M, F_M),$	M is an arbitrary Turing machine, δ_M is the program of Turing machine,Γ_M set of tape symbols,Σ_M finite set of input tape symbols, q_{0M} start state, B_M input symbols and F_M set of final states.

Table 4: Dynamics systems and Automata theory

The applications are not limited and we believe that readers can quickly pick-up the suitable model for their research, and posed these applications in table 1.

In this section we expose three more tables showing suitable applications with respect to mathematical models. In each table the reader can see a brief explanation about the equations and the significance of the components. We organized the models into three tables namely "Probabilistic models (table 2), "Differential and Partial differential Equations" (table 3), and "Dynamical Systems and Automata Theory" (table 4).

5. Conclusions

This article reports the various mathematical models of swarm systems, and provides significant equations with brief explanation helpful to the reader in order to choose specific models. Also, the extensive bibliography provided with this article will be of help to the new entrants as well as researchers working in this field.

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