Automobile Passenger Comfort Assured Through LQG/LQR Active Suspension

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Abstract: An analytical investigation of a half-car model including passenger dynamics, subjected to random road disturbances is performed, and the advantage of active over conventional passive suspension systems are examined. Two different performance indices for optimal controller design are proposed. The performance index is a quantification of both ride comfort and road handling. Due to practical limitations, all the states required for the state-feedback controller are not measurable, and thus must be estimated with an observer. Stochastic inputs are applied to simulate realistic road surface conditions, and statistical comparisons between passive system and the two controllers, with and without state estimator, are carried out to gain a clearer insight into the performance of the controllers.The simulation results demonstrate that an optimal observer-based controller, when including passenger acceleration in the performance index, retains both excellent ride comfort and road handling characteristics.

Key Words: Active suspension, observer-based control, ride comfort, passenger dynamics

NOMENCLATURE

Notation	Description	Units	Values
I_p	Body inertia	$Kg \cdot m^2$	3443.05
m_{p1}	Driver mass	Kg	75
m_{t1}	Front axle mass	Kg	87.15
k_1	Front main stiffness	N/m	66824.2
k_2	Rear main stiffness	//	18615.0
k_{p1}	Front seat stiffness	//	14000
k_{p2}	Rear seat stiffness	//	14000
k_{t1}	Front tire stiffness	//	101115
k_{t2}	Rear tire stiffness	//	101115
b_1	Dimension	m	1.271
d_1	Dimension	m	0.481
m	Body mass	Kg	1794.4
m_{p2}	Passenger mass	Kg	75
m_{t2}	Rear axle mass	Kg	140.04

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Notation	Description	Units	Values
c_1	F.m. damping	$\frac{N \cdot s}{m}$	1190
c_2	R.m. damping	"	1000
c_{p1}	F.s. damping	"	50.2
c_{p2}	R.s. damping	"	62.1
c_{t1}	F.t. damping	"	14.6
c_{t2}	R.t. damping	"	14.6
b_2	Dimension	m	1.713
d_2	Dimension	m	1.313

1. INTRODUCTION

Demand for better ride comfort and controllability of road vehicles has motivated many automotive industries to consider the use of active suspensions. These electronically controlled suspension systems can potentially improve the ride comfort as well as the road handling of the vehicle. Generally, a vehicle suspension system may be categorized as either passive, semi-active, or fully-active.

Passive suspension systems consist of conventional springs and shock absorbers used in most cars. The springs are assumed to have almost linear characteristics, while most of the shock absorbers exhibit a nonlinear relationship between force and velocity. In passive systems, these elements have fixed characteristics and, hence, have no mechanism for feedback control (Miller, 1988).

Semi-active suspensions provide controlled real-time dissipation of energy (Crosby and Karnopp, 1973). For an automotive suspension this is achieved through a mechanical device called an *active damper* which is used in parallel to a conventional spring. The main feature of this system is the ability to adjust the damping of the suspension system without any use of actuators. This type of system requires some form of measurement with a controller board to properly tune the damping.

Active suspension employs pneumatic or hydraulic actuators which in turn create the desired force in the suspension system (Esmailzadeh, 1979; Wright and Williams, 1984). The actuator is secured in parallel with a spring and shock absorber. Active suspension requires sensors to be located at different points of the vehicle to measure the motions of the body, suspension system, and/or the unsprung mass. This information is used in the online controller to command the actuator in order to provide the exact amount of force required. Active suspensions may consume large amounts of energy in providing the control force, and therefore, in the design procedure for the active suspension the power limitations of actuators should also be considered as an important factor.

In any vehicle suspension system, there are a variety of performance parameters that need to be optimized. Among them are four important quantities that should be considered carefully in designing a suspension system; namely, *ride comfort, body motion, road handling*, and *suspension travel*. The trade-off between ride comfort and road handling characteristics is usually a trial and error procedure. Moreover, no suspension system can simultaneously minimize all four of the above mentioned parameters. The advantage of controlled suspension is that a better set of design trade-offs are possible in comparison to passive systems (Miller, 1988). State-feedback control for active suspension is a powerful tool for designing a controller (Esmailzadeh and Bateni, 1992; Shannon and Vanderploeg, 1989). In this approach a mathematical quantification of ride comfort and road handling will be optimized considering the actuator limitations. Since body motion and suspension travel are also functions of the system states, they will also be optimized during the design.

Linear optimal control theory provides a systematic approach to design the active suspension controllers (Chen, 1970), and has been used by several investigators. Sinha, Wormely, and Hedrick (1987) and Caudill, Sweet, and Oda (1982) have used this method to design active suspension controllers for railroad vehicles. Esmailzadeh and Bateni (1992) investigated a pneumatic controlled active suspension for automobiles. Hrovat (1990, 1992) appends this method with the concept of dynamic absorber for improved performance for quarter-car and half-car models. Elmadany (1991) considered using integral and state feedback controllers for active suspension for a half-car model. Shannan and Vanderploeg (1989) considered the lateral and longitudinal motions for a full-car model, and implemented active controller with linear optimal control. Observer-based controller design is discussed by Alleyne and Hedrick (1992), Elmadany and Samaha (1992), and Rajamani and Hedrick (1993).

In this study, we are again considering state-feedback control for active suspension; however, our intention is not to compare this method with other control approaches, as done by Yue, Butsuen, and Hedrick (1989) and Sharp and Hassan (1986). We intend to emphasize the effective methodology of controller design in order to satisfy a preassigned set of design criteria. In the numerical example given in this paper, the parameters of a typical mid-size car are used with the road information extracted from ISO, in order to illustrate how the methodology presented here can be utilized to satisfy a specified performance within the natural limitations of actuators and controllers.

There are two main contributions of the present research. First, we present a model of vehicle including passenger dynamics, which to our knowledge is given little attention in the existing literature. This has been accomplished in a framework of a half-car model, which may be generalized further to a full-car model. This research illustrates the importance of passenger dynamics, when the objective would be to improve the passenger ride comfort as well as to satisfy the road handling requirements. Second, we developed a statistical technique for quantitative comparison of the system. This method extracts useful quantitative data from the time history of important variables, which results into relative ease in comparing different methods. Here, this approach is used to compare the two controller designs, with and without state estimation, to the passive system. However, it can well be utilized as an effective platform for comparison of different controller designs.

2. MATHEMATICAL MODELING

This section is devoted to the mathematical modelling of the vehicle, considering the passenger dynamics and road disturbance. A linear model is considered to represent the vehicle-passenger dynamics, while a normal random profile is used to model the road roughness.

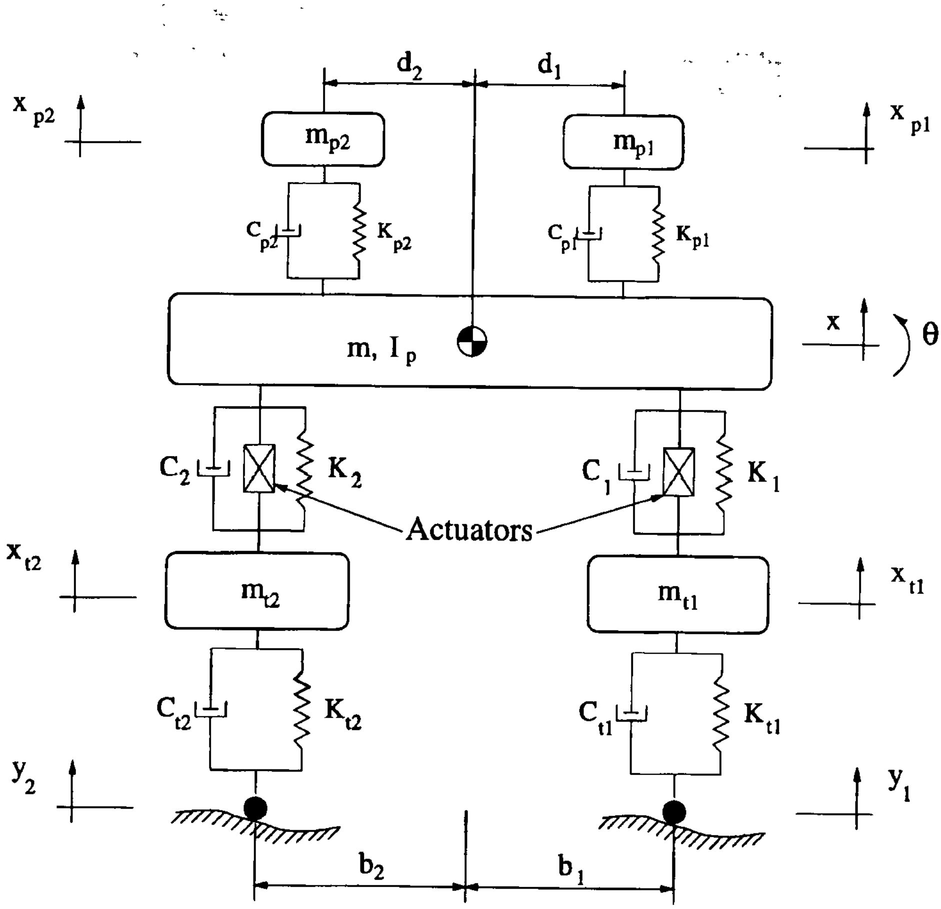

Figure 1. Half-car dynamical model of road–vehicle–passenger with six degrees of freedom.

2.1. Vehicle-Passenger Model

Figure 1 illustrates the half-car model of a passenger car, which has six degrees of freedom (6 dof). The model consisted of a body, two axles, and two passengers. Body motions are considered to be bounce and pitch, with every axle having its own bounce. The passengers are considered to have only vertical oscillations. The suspension, tire, and passenger seats are modeled by linear springs in parallel to viscous dampers. The actuators are considered to be a source of controllable force, and are located parallel to the suspension spring and shock absorber.

The system variable notations with their corresponding values are presented in Nomenclatures. The parameters related to the tires are denoted with subscript t, while the passenger parameters have subscript p. The system with six degrees of freedom is represented by the following states: body bounce, x; body pitch, θ; tire deflections, x_{t1}, x_{t2}; and passenger vertical motions, x_{p1}, x_{p2}.

The following equations of motion can be derived using the Newton-Euler method:

$$
\begin{aligned}
\ddot{x} = &-\frac{1}{m}(k_1+k_2+k_{p1}+k_{p2})x - \frac{1}{m}(k_1b_1 - k_2b_2 + k_{p1}d_1 - k_{p2}d_2)\theta \\
&+ \frac{k_{p1}}{m}x_{p1} + \frac{k_{p2}}{m}x_{p2} + \frac{k_1}{m}x_{t1} + \frac{k_2}{m}x_{t2} + \frac{c_{p1}}{m}\dot{x}_{p1} + \frac{c_{p2}}{m}\dot{x}_{p2} + \frac{c_1}{m}\dot{x}_{t1} + \frac{c_2}{m}\dot{x}_{t2} \\
&- \frac{1}{m}(c_1+c_2+c_{p1}+c_{p2})\dot{x} - \frac{1}{m}(c_1b_1 - c_2b_2 + c_{p1}d_1 - c_{p2}d_2)\dot{\theta} + \frac{f_1+f_2}{m}
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
\ddot{\theta} = &-\frac{1}{I_p}(k_1b_1 - k_2b_2 + k_{p1}d_1 - k_{p2}d_2)x - \frac{1}{I_p}(k_1b_1^2 + k_2b_2^2 + k_{p1}d_1^2 + k_{p2}d_2^2)\theta \\
&- \frac{k_{p1}d_1}{I_p}x_{p1} + \frac{k_{p2}d_2}{I_p}x_{p2} - \frac{k_1b_1}{I_p}x_{t1} + \frac{k_2b_2}{I_p}x_{t2} \\
&- \frac{1}{I_p}(c_1b_1 - c_2b_2 + c_{p1}d_1 - c_{p2}d_2)\dot{x} - \frac{1}{I_p}(c_1b_1^2 + c_2b_2^2 + c_{p1}d_1^2 + c_{p2}d_2^2)\dot{\theta} \\
&- \frac{c_{p1}d_1}{I_p}\dot{x}_{p1} + \frac{c_{p2}d_2}{I_p}\dot{x}_{p2} - \frac{c_1b_1}{I_p}\dot{x}_{t1} + \frac{c_2b_2}{I_p}\dot{x}_{t2} + \frac{1}{I_p}(f_1b_1 - f_2b_2)
\end{aligned} \tag{2}
$$

$$
\ddot{x}_{p1} = \frac{1}{m_{p1}}\left(k_{p1}x + k_{p1}d_1\theta - k_{p1}x_{p1}\right) + \frac{1}{m_{p1}}\left(c_{p1}\dot{x} + c_{p1}d_1\dot{\theta} - c_{p1}\dot{x}_{p1}\right) \tag{3}
$$

$$
\ddot{x}_{p2} = \frac{1}{m_{p2}}\left(k_{p2}x - k_{p2}d_2\theta - k_{p2}x_{p2}\right) + \frac{1}{m_{p2}}\left(c_{p2}\dot{x} - c_{p2}d_2\dot{\theta} - c_{p2}\dot{x}_{p2}\right) \tag{4}
$$

$$
\begin{aligned}
\ddot{x}_{t1} = &\frac{1}{m_{t1}}\left(k_1x + k_1b_1\theta - (k_1+k_{t1})x_{t1} + k_{t1}y_1 - f_1\right) \\
&+ \frac{1}{m_{t1}}\left(c_1\dot{x} + c_1b_1\dot{\theta} - (c_1+c_{t1})\dot{x}_{t1} + c_{t1}\dot{y}_1\right)
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\ddot{x}_{t1} = &\frac{1}{m_{t2}}\left(k_2x - k_2b_2\theta - (k_2+k_{t2})x_{t2} + k_{t2}y_2 - f_2\right) \\
&+ \frac{1}{m_{t2}}\left(c_2\dot{x} - c_2b_2\dot{\theta} - (c_2+c_{t2})\dot{x}_{t2} + c_{t2}\dot{y}_2\right).
\end{aligned} \tag{6}
$$

These equations can be simply written as a matrix equation:

$$
\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{G}\mathbf{w}, \tag{7}
$$

where the state vector $\mathbf{x}$ is composed of

$$
\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \dot{\mathbf{x}}_1 \end{bmatrix}; \qquad \mathbf{x}_1 = \left[x\ ,\ \theta\ ,\ x_{p1}\ ,\ x_{p2}\ ,\ x_{t1}\ ,\ x_{t2}\right]^T. \tag{8}
$$

The input vector $\mathbf{u}$ represents the two actuator forces, while the disturbance vector $\mathbf{w}$ consists of road disturbance.

$$
\mathbf{u} = [f_1\ ,\ f_2]^T; \qquad \mathbf{w} = [y_1\ ,\ y_2\ ,\ \dot{y}_1\ ,\ \dot{y}_2]^T. \tag{9}
$$

The matrix representation of equation (7) would be the basis for linear optimal controller design.

Table 1. Road roughness values classified by ISO.

Degree of Roughness $S(\Omega) \times 10^{-6}$		
Road Class	*Range*	*Geometric Mean*
A (Very good)	< 8	4
B (Good)	8 – 32	16
C (Average)	32 – 128	64
D (Poor)	128 – 512	256
E (Very poor)	512 – 2048	1024
F	2048 – 8192	4096
G	8192 – 32768	16384
H	32768 <	

2.2. Road Roughness Model

In the early days of studying the performance of vehicles on rough roads, simple functions such as sine waves, step functions, or triangular waves were generally applied as disturbances from the ground. While these inputs provide a basic idea for comparative evaluation of designs, it is recognized that the road surface is usually not represented by these simple functions, and therefore the deterministic irregular shapes cannot serve as a valid basis for studying the actual behavior of the vehicle.

In this study a real road surface, taken as a random exciting function, is used as the input to the vehicle-road model. Power spectral density (PSD) analysis is used to describe the basic properties of random data. Several attempts have been made to classify the roughness of a road surface. In this study, classifications are based on the International Organization for Standardization (ISO). The ISO has proposed road roughness classification using the PSD values (ISO, 1982), as shown in Table 1.

To make use of the above mentioned classification, a normal random input is generated with a variable amplitude. Using fast Fourier transform (FFT), a trial and error attempt is proposed in order to obtain the desired PSD characteristics of the random input. Table 2 illustrates the stochastical characteristics of the final random input design, which corresponds to the *poor* road condition as being classified by ISO.

3. OPTIMAL CONTROLLER DESIGN

The performance characteristics that are of most interest when designing the vehicle suspension are passengers' ride comfort, body motion, road handling, and suspension travel. The passenger acceleration has been used here as an indicator of ride comfort. Suspension travel and body motion are the states of the system, but road handling is related to the tire deflection. The controller should minimize all these quantities.

The linear time-invariant system, (LTI), is described by equation (7). For controller

Table 2. Stochastical characteristic of road random disturbance.

	Random Input Characteristics	
	Spatial (mm)	*PSD* $(m^3/cycle)$
Min	−49	0
Max	40	2.06×10^{-3}
Mean	0	3.08×10^{-4}
STD	12	3.26×10^{-4}

design it is assumed that all the states are available and also can be measured exactly. First of all let us consider a state variable feedback regulator:

$$\mathbf{u} = -\mathbf{K} \cdot \mathbf{x}, \tag{10}$$

where $\mathbf{K}$ is the state feedback gain matrix. The optimization procedure consists of determining the control input $\mathbf{u}$, which minimizes the performance index. The performance index $\mathbf{J}$ represents the performance characteristic requirement as well as the controller input limitations.

In this paper two different approaches are taken in order to evaluate the performance index, and hence design the optimal controller. The first approach is the conventional method, in which only the system states and inputs are penalized in the performance index. However, in the second approach special attention is paid to the ride comfort, and hence the passenger acceleration terms are also included in the performance index.

3.1. Conventional Method (CM)

In this method, the performance index $\mathbf{J}$ penalizes the state variables and the inputs; thus, it has the standard form of

$$\mathbf{J} = \int_0^\infty \left(\mathbf{x}^T \mathbf{Q}\, \mathbf{x} + \mathbf{u}^T \mathbf{R}\, \mathbf{u}\right) dt, \tag{11}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are positive definite, being called weighting matrices. Here the passenger acceleration, which is an indicator of ride comfort, is not being penalized.

To obtain a solution for the optimal controller introduced in equation (10) the LTI system must be stabilizable (Bryson and Ho,1975). This condition, unlike controllability, is rather more accessible. A system is defined stabilizable when only the unstable modes are controllable. Therefore, for a system with no unstable mode, being the case considered in this paper, the optimal solution is guaranteed.

Linear optimal control theory provides the solution of equation (11) in the form of equation (10). The gain matrix $\mathbf{K}$ is computed from

$$\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}, \tag{12}$$

where the matrix $\mathbf{P}$ is evaluated, being the solution of the Algebraic Riccati Equation (ARE).

$$\mathbf{AP} + \mathbf{A}^T\mathbf{P} - \mathbf{PBR}^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q} = \mathbf{0}. \tag{13}$$

Equation (7) for the optimal closed-loop system, being used for computer simulation, can be written in the form of

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{BK})\mathbf{x} + \mathbf{Gw}. \tag{14}$$

3.2. Acceleration Dependent Method (ADM)

In this method, the two passenger accelerations are included in the performance index. Suppose that the vector $\mathbf{z}$ represents the passengers' acceleration, in the form of

$$\mathbf{z} = \begin{bmatrix} \ddot{x}_{p1} \\ \ddot{x}_{p2} \end{bmatrix}. \tag{15}$$

The performance index can be written in the following form:

$$\mathbf{J} = \int_0^\infty \left(\mathbf{x}^T\mathbf{Q}\,\mathbf{x} + \mathbf{u}^T\mathbf{R}\,\mathbf{u} + \mathbf{z}^T\mathbf{S}\,\mathbf{z}\right) dt. \tag{16}$$

The weighting matrix for acceleration terms in the simple case may be assumed diagonal:

$$\mathbf{S} = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}. \tag{17}$$

Therefore, equation (16) becomes

$$\mathbf{J} = \int_0^\infty \left(\mathbf{x}^T\mathbf{Qx} + \mathbf{u}^T\mathbf{Ru} + \ddot{x}_{p1}^T S_1 \ddot{x}_{p1} + \ddot{x}_{p2}^T S_2 \ddot{x}_{p2}\right) dt. \tag{18}$$

Equation (18) can be further modified, since both passenger accelerations are linearly dependent on the state variables. Hence,

$$\ddot{x}_{p1} = \mathbf{v}_1\,\mathbf{x} \quad ; \quad \ddot{x}_{p2} = \mathbf{v}_2\,\mathbf{x}, \tag{19}$$

where $\mathbf{v}_1$ and $\mathbf{v}_2$ are two constant row vectors depending on system parameters. Thus, equation (18) can be written as

$$\mathbf{J} = \int_0^\infty \left(\mathbf{x}^T\left(\mathbf{Q} + \mathbf{v}_1^T S_1 \mathbf{v}_1 + \mathbf{v}_2^T S_2 \mathbf{v}_2\right)\mathbf{x} + \mathbf{u}^T\mathbf{Ru}\right) dt \tag{20}$$

or in the simple form of

$$\mathbf{J} = \int_0^\infty \left(x^T\mathbf{Q}_n\,x + u^T\mathbf{R}\,u\right) dt, \tag{21}$$

where

$$\mathbf{Q}_n = \mathbf{Q} + \mathbf{v}_1^T S_1 \mathbf{v}_1 + \mathbf{v}_2^T S_2 \mathbf{v}_2. \tag{22}$$

The optimal solution for equation (21) can be found in a similar manner to that of equation (11). Equation (13) shows that the optimal solution and its final performance of the closed-loop system are directly related to the initial values of weighting matrices, **Q** and **R**.

4. OPTIMAL OBSERVER DESIGN

In the previous section we assumed that the state vector is available for use in the optimal state feedback law. It might appear that the state feedback method is not applicable in practice, since the whole state feedback is not available. However, it is possible to estimate the states of the system, provided the system is detectable. This is done by designing a dynamical system called *State Observer*. It is possible to recover much of the behavior of the state feedback law by using state estimates instead of states in the feedback law.

The input to the state observer is the measured outputs of the system **y**, which generally can be expressed in the form

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{v}, \tag{23}$$

where **x** is the system states and **v** is the measurement noise. A typical sensor arrangement for passenger cars can provide the relative suspension travel, and the relative passenger bounce. For this case,

$$\mathbf{C} = \begin{bmatrix} -1 & -b1 & 0 & 0 & 1 & 0 \\ -1 & +b2 & 0 & 0 & 0 & 1 \\ -1 & -d1 & 1 & 0 & 0 & 0 \\ -1 & +d2 & 0 & 1 & 0 & 0 \end{bmatrix}. \tag{24}$$

The observer structure is in the form of

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{L}(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}}), \tag{25}$$

where **L** is the optimal observer gain matrix, which produces an LQG optimal estimate of x, denoted as $\hat{\mathbf{x}}$. Similar to Section 3.1, **L** can be evaluated from

$$\mathbf{L} = \mathbf{P}\mathbf{C}^T\mathbf{V}^{-1}, \tag{26}$$

where **P** is the positive definite solution of the Riccati equation,

$$\mathbf{A}\mathbf{P} + \mathbf{P}\mathbf{A}^T - \mathbf{P}\mathbf{C}^T\mathbf{V}^{-1}\mathbf{C}\mathbf{P} + \mathbf{W} = \mathbf{0}, \tag{27}$$

and $\mathbf{W} = E[\mathbf{w}\mathbf{w}^T]$, $\mathbf{V} = E[\mathbf{v}\mathbf{v}^T]$ are the plant disturbance and measurement noise covariances. Notice in equation (25) that $\hat{\mathbf{x}}$ is a function of output **y** and input **u**, but the input **u** itself is generated by a state feedback law as

Table 3. Variable limits assigned for the controller design.

Variables	x	θ	x_{p1}	x_{p2}	x_{t1}	x_{t2}	$\ddot{x}_{p1}$	$\ddot{x}_{p2}$	f_1	f_2
Limits	0.01	0.05	0.01	0.01	0.05	0.05	0.05 g	0.05 g	500	500

$$\mathbf{u} = -\mathbf{K}\hat{\mathbf{x}}. \tag{28}$$

Therefore, for simulation purposes we can augment the system dynamic equation (equation (7)) with that of the observer (equation (25)), and solve the whole at once as illustrated by the following equation:

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\hat{\mathbf{x}}} \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{A} & -\mathbf{BK} \\ \hline \mathbf{LC} & \mathbf{A}-\mathbf{BK}-\mathbf{LC} \end{array}\right] \cdot \begin{bmatrix} \mathbf{x} \\ \hat{\mathbf{x}} \end{bmatrix} + \left[\begin{array}{c|c} \mathbf{G} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{L} \end{array}\right] \cdot \begin{bmatrix} \mathbf{w} \\ \mathbf{v} \end{bmatrix}. \tag{29}$$

5. SIMULATION RESULTS

A program has been written using Matlab, to handle the controller and observer design and simulation. For controller design equation (12), for observer design equation (26) and for simulation equation (29) are the basis of the program.

5.1. Statistical Consideration

To have a quantitative comparison between the different simulation results, a statistical approach is followed. Since the input to the system is in the form of normal random distribution, it is expected to have normal distributed outputs. Therefore, we can calculate useful probability values for the signals.

For a Gaussian normal distribution, the probability function of the random signal $x(t)$ can be written (Thomson, 1988):

$$\begin{aligned} &\mathrm{Prob}\,[-\lambda\sigma \leq x(t) \leq \lambda\sigma] = \\ &\frac{1}{\sigma\sqrt{2\pi}} \int_{-\lambda\sigma}^{\lambda\sigma} e^{-\frac{x^2}{2\sigma^2}} dx = \mathrm{Erf}\left(\frac{\lambda}{\sqrt{2}}\right) \end{aligned} \tag{30}$$

and

$$\begin{aligned} &\mathrm{Prob}\,[|x(t)| > \lambda\sigma] = \\ &1 - \mathrm{Prob}\,[-\lambda\sigma \leq x(t) \leq \lambda\sigma] = \mathrm{Erfc}\left(\frac{\lambda}{\sqrt{2}}\right), \end{aligned} \tag{31}$$

where σ is the standard deviation (STD), λ is a real number, Erf denotes error function, and Erfc denotes complementary error function.

In this study, some limits are assigned for all the states, both passenger acceleration and actuator limits in order to satisfy the required ride comfort, road handling, and the

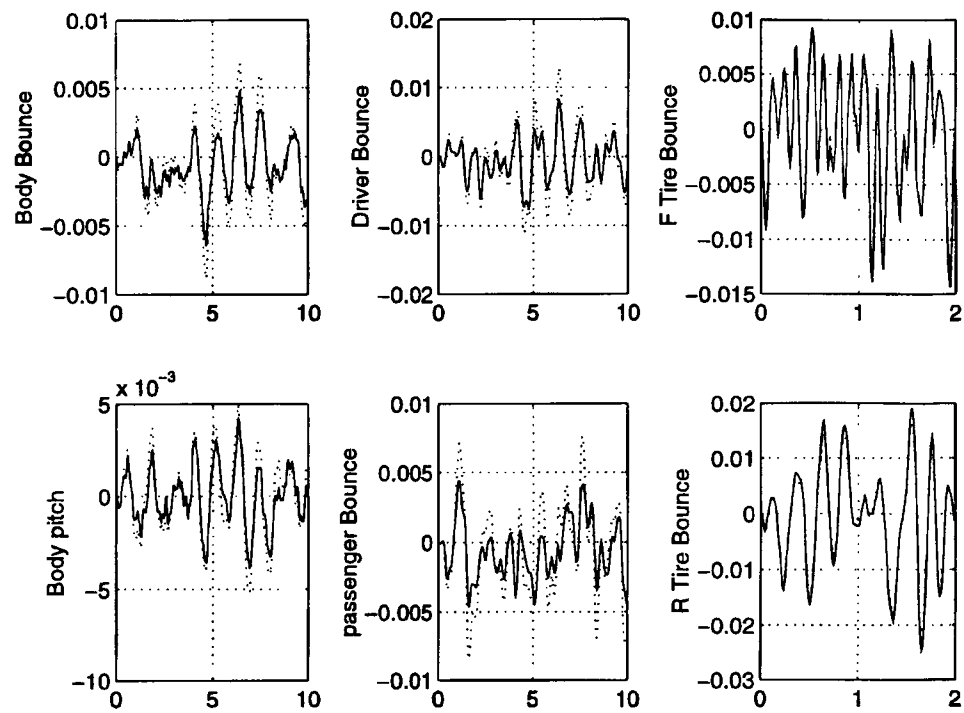

Figure 2. Comparison of real and estimated states. Dotted = estimated; solid = real.

design restriction. These limits are illustrated in Table 3. The first condition in designing the controller is to satisfy these limits. This can be examined by checking the probability values of the outputs.

For a quantitative comparison between the two controllers, for each variable the amount of bounding limit with 90% probability is calculated. This quantity can be easily obtained using standard deviation of the signal together with equation (30). Let Erf $^{-1}$ represent the inverse error function. Then, equation 30 will transform to

$$\lambda_{90} = \sqrt{2}\ \mathrm{Erf}^{-1}(0.9) \tag{32}$$

and

$$x_{90} = \lambda_{90} \times \sigma + \mathrm{mean}\,[x(t)], \tag{33}$$

where x_{90} represents the bounding limit of the random signal x with 90% probability. This quantity can be used to compare different designs quantitatively.

The problem of controller design is then a challenge of finding suitable weightings that satisfy the design performances. This can be done by trying an arbitrary weighting matrix **W** and comparing the resultant x_{90} of the closed loop system to the prescribed limits, and adjusting the weighting elements due to this comparison. This methodology

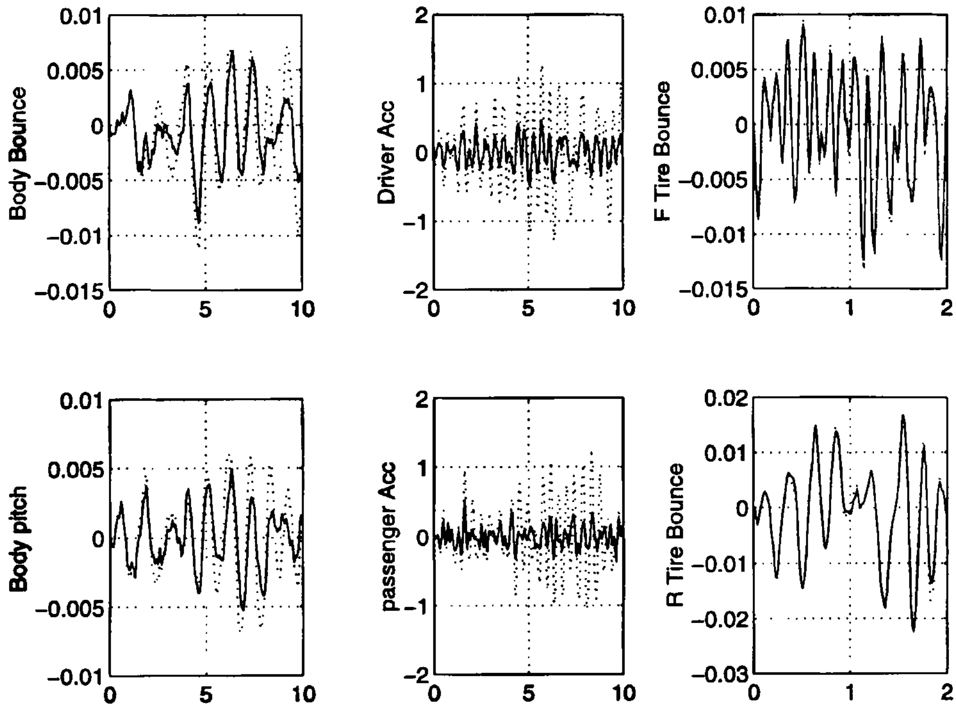

Figure 3. Comparison between passive and active systems. Dotted = passive; solid = active.

has been forwarded for a typical mid-size car and the results are illustrated in Figures 2 through 5.

5.2. *Observer Performance*

Figure 2 compares the estimated states to the real ones. The numerical values used in the simulation for system parameters are given in Nomenclature. As is clearly illustrated, the estimated states, despite the randomness of the signal, are quite close to the real states. This performance is achieved by employing optimal observer design, where both the speed of the response and its initial error are penalized in the optimization cost function.

5.3. *Active and Passive Comparison*

Let us now examine the effect of the active system to remedy the drawbacks of the passive system. Figure 3 illustrates how the active suspension can effectively absorb the vehicle vibration in comparison to the passive system. There are the body motions, passenger acceleration, and tires deflection compared in this figure, acceleration dependent method (ADM) is used for the controller design in the active system. The passenger accelerations in the active system are reduced significantly, which guarantee better ride comfort. Moreover, the tire deflection is also smaller in the active suspension system; therefore, it is concluded

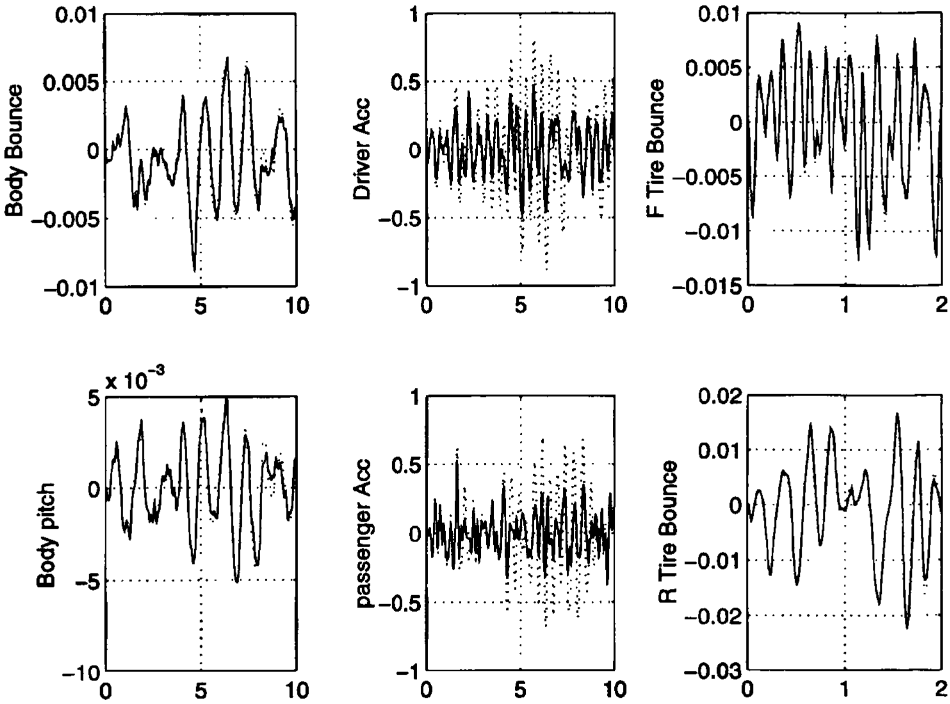

Figure 4. Comparison of two controller designs. Dotted = conventional; solid = acceleration dependent.

that the active system retains both better ride comfort and road handling characteristics compared to the passive system.

Table 4 gives quantitative comparison of these systems, which illustrates the system variables bounds with 90% probability for passive and both active systems. In this statistical comparison it is shown that the body bounce and passenger acceleration in the active case are reduced to about half of their values in the passive system, and the tire deflection is also reduced to at least 40%. This confirms the efficiency of the active suspension in both ride comfort and road handling performance.

5.4. Active Controllers Comparison

Now let us compare the results of two different methods of controller design in detail. Figures 4 and 5 show the simulation results for the final controllers' design. In Figure 4 body bounce and pitch, passenger acceleration and the tires' deflection are compared. The body motions and the tires' deflection are approximately the same in both methods; however, the passenger accelerations are significantly lower in the ADM approach. This implies that ADM can improve the ride comfort, while retaining the road handling performance.

Figure 5 compares the actuator forces of the two different methods. The actuator forces are well below the limits and practically implementable. In the ADM approach, gaining

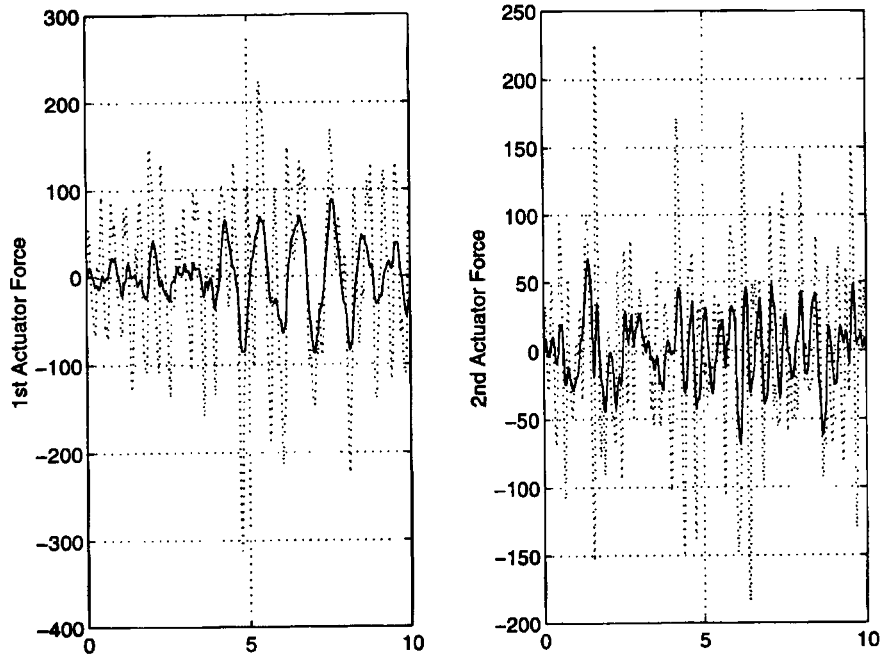

Figure 5. Comparison of the actuator forces for two controller designs. Solid = conventional; dotted = acceleration dependent.

better passenger acceleration is possible at the cost of larger actuator forces. However, optimal controller design can limit the actuator forces in some realistic bounds.

Table 4 gives a quantitative comparison of the above mentioned methods. This table gives the variable limits with 90% probability. Comparing the variable limits with the prescribed limits in both methods, the controllers are able to satisfy all the limits. However, the ADM approach is more successful in reducing the passengers' acceleration. The body motion and tire deflection limits do not have significant difference in both methods. These quantitative values can be an effective tool for the designer to satisfy the required performance or to compare different designs.

6. CONCLUSION

The objective of this paper has been to examine the use of optimal state-feedback controllers for improving the ride comfort and stability performance of the road vehicles. The potential for improved vehicle ride comfort, and road handling resulting from controlled actuator forces, are examined. The performance characteristics of such suspension systems are evaluated by two methods, and compared with a passive suspension system. Optimal observer was employed to estimate the system states, from few measured signals, hence making state-feedback implementable.

Table 4. Comparison of variable bounds with 90% probability for passive and active systems.

	90% Probable Bound		
States	*Passive*	*CM*	*ADM*
x	7.601×10^{-3}	2.150×10^{-3}	2.029×10^{-3}
θ	4.957×10^{-3}	2.368×10^{-3}	2.151×10^{-3}
x_{p1}	1.160×10^{-2}	4.925×10^{-3}	4.019×10^{-3}
x_{p2}	8.777×10^{-3}	3.175×10^{-3}	2.922×10^{-3}
x_{t1}	1.109×10^{-2}	4.348×10^{-3}	3.971×10^{-3}
x_{t2}	1.268×10^{-2}	7.315×10^{-3}	7.520×10^{-3}
$\ddot{x}_{p1}$	8.294×10^{-1}	5.260×10^{-1}	3.092×10^{-1}
$\ddot{x}_{p2}$	7.261×10^{-1}	4.483×10^{-1}	2.383×10^{-1}
f_1	0	$5.859 \times 10^{+1}$	$1.430 \times 10^{+2}$
f_2	0	$4.019 \times 10^{+1}$	$1.013 \times 10^{+2}$

The result of comparison presented in this paper lead to the conclusion that the optimal control theory provides a useful mathematical tool for the design of active suspension systems. Using random inputs for road surface disturbances applied to the vehicle make it possible to have a more realistic idea about the vehicle dynamic response to the road roughness.

The suspension designs which may have emerged from the use of optimal state-feedback control theory proved to be effective in controlling vehicle vibrations and achieve better performance than the conventional passive suspension. The stochastical comparison of the final designs can well be used in further improvement of the controller performance. Moreover, it provides either a detailed quantitative comparison between different designs, or a better degree of satisfaction for the required performances.

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