Fold and Transcritical Bifurcations in Temperature Data

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Overview

Extreme temperature transitions, such as heat waves and cold snaps, have the potential to wreak havoc on infrastructure, human life, and the functioning of local ecosystems (Wang et al. 2020). Today's climate models offer a high resolution picture of climate patterns and how the climate evolves over time. The drawbacks are that these models are often very complicated, requiring computationally intensive simulation to generate numerical solutions and predictions. Although these models offer high spatial resolution, it is still notoriously difficult to predict the temporally abrupt transitions in temperature that define heatwaves and cold snaps. An alternative approach to modeling and predicting these extreme temperature transitions involves using the mathematical theory behind dynamical systems. A dynamical system consists of one or more ordinary differential equations that mathematically represent how particular quantities, for instance temperature or other weather indices, change over time. When parameters in the system change, it is possible for a dynamical system to undergo a bifurcation, in which the stability of its equilibrium points change. This can alter the trajectories of the quantities being modelled by the dynamical system, causing phenomena like large jumps or oscillations. Because of the potential for bifurcations to cause unexpected and disruptive changes, early warning signals have been developed so that upcoming bifurcations can be spotted before stability changes occur, based on patterns in data (Bury et al. 2021). There is an abundance of publicly accessible local climate data from many cities around the world, and hence previously-observed heat waves and cold snaps will be detectable in this data. We theorize the same transitions in time series data that characterize heat waves and cold snaps also characterize bifurcations in dynamical systems, therefore early warning signals (EWS) should exist before these extreme temperature transitions. This means the approach pioneered by Bury et al, that used machine learning to construct early warning signals for bifurcations in paleoclimate data, should also be able to used with heat waves and cold snaps. This would allow for an accurate, low computational cost method to be generated to predict these extreme temperature transitions in advance.

Research Goals and Methodology

The objective of this study are...

1. Prove both Fold and Transcritical Normal Form bifurcations exist in temperature data.
2. Identify early warning signals before detected bifurcations.

Fold Normal Form Bifurcation: $\frac{\partial f}{\partial x} = \mu - x^2$

- Bifurcation occurs at $\mu = 0$ where system transitions from a stable to a chaotic state.
- Can be identified as a sudden transition to a very cold, very hot, or chaotic temperature state.

Transcritical Normal Form Bifurcation: $\frac{\partial f}{\partial x} = x\mu - x^2$

- Bifurcation occurs at $\mu = 0$ where system transitions from a stable state to another stable state.
- Can be identified as a transition from a constant temperature state to a quickly increasing or decreasing temperature state.

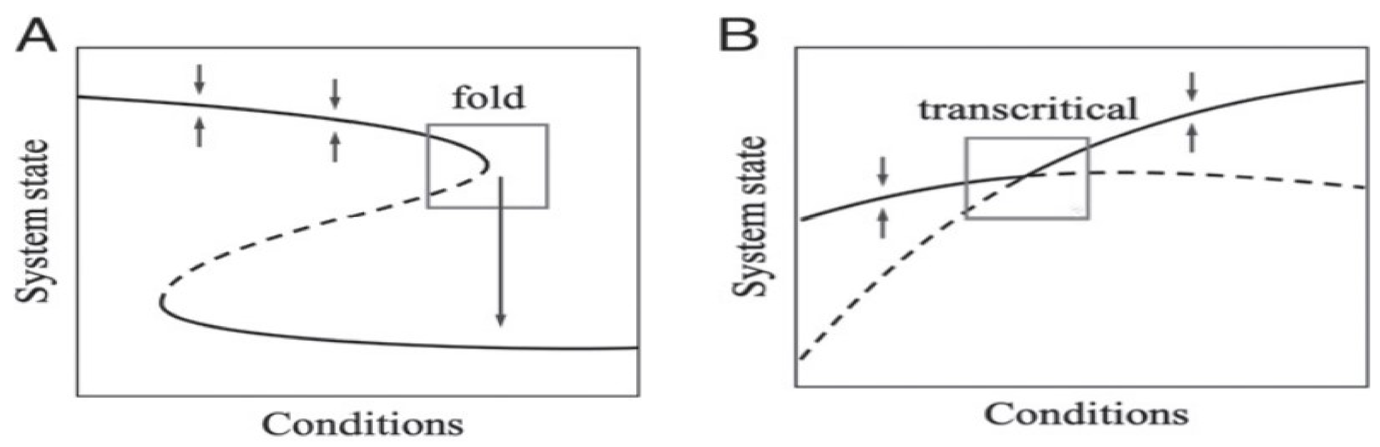

Figure 1: Fold Normal Form bifurcation (A) and Transcritical Normal Form bifurcation (B). Figures from Bury et al. 2021.

Data: Data was obtained from The National Oceanic and Atmosphere Administration (NOAA) for the following thirteen cities across Canada; Vancouver, Prince George, Edmonton, Calgary, Saskatoon, Winnipeg, Yellowknife, Thunder Bay, Toronto, Montreal, Bagotville, Halifax, and St. Johns. The data had to have three hour gaps between measurements and pass all quality control checks to be analysed. The good data was organized into unbroken time series measurements of length 500. The temperature data was detrended using Gaussian smoothing to account for noise within the data and normal temperature fluctuations caused by time of day.

Data Analysis for Existence of Bifurcations: A Long Short-Term Memory (LSTM) Machine Learning Model was used to identify the probability of both Fold and Transcritical Bifurcations in each 500 point time series. The model was trained with 500,000 bifurcating time series to insure a high probability of correct bifurcation identification. Each time series where the model predicted 90 percent and above probability that a bifurcation existed within the time series was analysed by inspection for evidence of a bifurcation(s).

Data Analysis for Existence of EWS: We analysed the one month period, if the temperature data existed, before every properly identified bifurcation (or bifurcation cluster). We looked to identify an increase of variance in temperature and bifurcation probability, as well as an increase in bifurcation probability leading up to the bifurcation.

Results

Fold Bifurcation Detection:

- 21 independent clusters of Fold Normal Form bifurcations identified in the temperature data.
- Transitions to very cold and very hot temperature states, including cold snaps, can be described as Fold Normal Form bifurcations.
- Flickering between low and high temperature states identified as Fold Normal Form bifurcations.
- Pitfalls: Backwards detection.

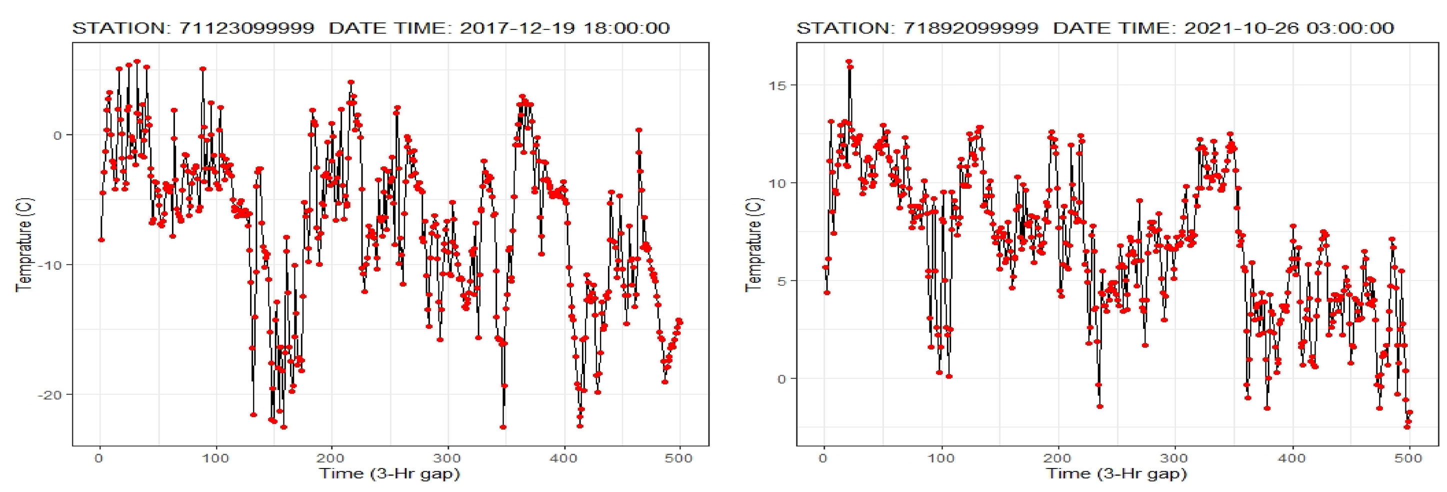

Figure 2: Fold Normal Form bifurcation cluster (left) and flickering between temperature states (right).

Transcritical Bifurcation Detection:

- 2 Distinct Transcritical Normal Form bifurcations identified.
- Both bifurcations model a transition to cold temperature states, where one accurately models the transition to a cold snap.
- Both bifurcation identifications happen in the same city (Prince George BC).

Early Warning Signals:

Clear early warning signals identified in the data before bifurcation.

- EWS 1: Increase in bifurcation probability leading up to a bifurcation.
- EWS 2: Increase of variance of bifurcation probability leading up to a bifurcation.
- Increase in bifurcation probability variance detectable before consistent increase in bifurcation probability.
- Simultaneity of both early warning signals occur on average six days in advance.

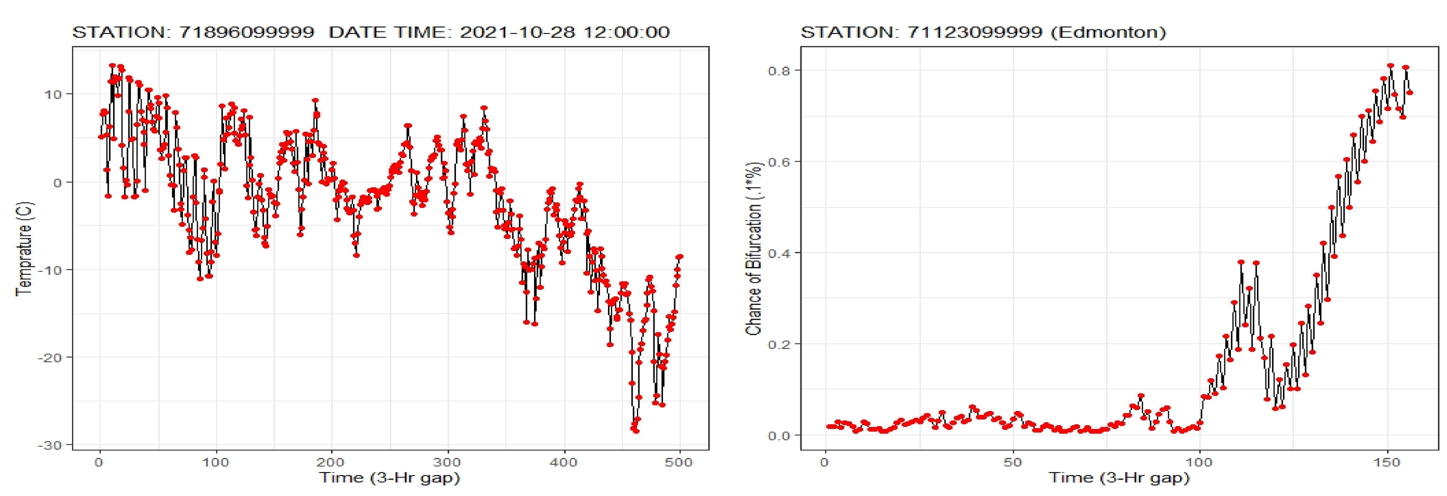

Figure 3: Transcritical Normal Form bifurcation (left) and early warning signals in bifurcation probability data (right).

Future Research

- Conduct study on large data set of 300 global cities, analysing for bifurcations and early warning signals in climate system covariates in addition to temperature.
- Derive a model to represent climate system bifurcations or uses SINDY (sparse identification of nonlinear dynamical systems) to extract a model from the data.
- Use results from the large scale study and our model to create a standardized method to predict extreme temperature transitions using local climate data.

Key References

(1) Bury, T.M., Sujith R. I., Pavithran I., Scheffer M., Lenton T.M., Anand M., Bauch C.T. (2021). *Deep learning for early warning signals of tipping points.* Proceedings of the National Academy of Sciences of the United States of America. 118(39)

(2) Wang, C., Wang, Z.-H., Sun, L. (2020). *Early-warning signals for critical temperature transitions.* Geophysical Research Letters, 47