Adapting a Single Filament Model for a Three-Filament Experiment

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The Project Outline

Main goal

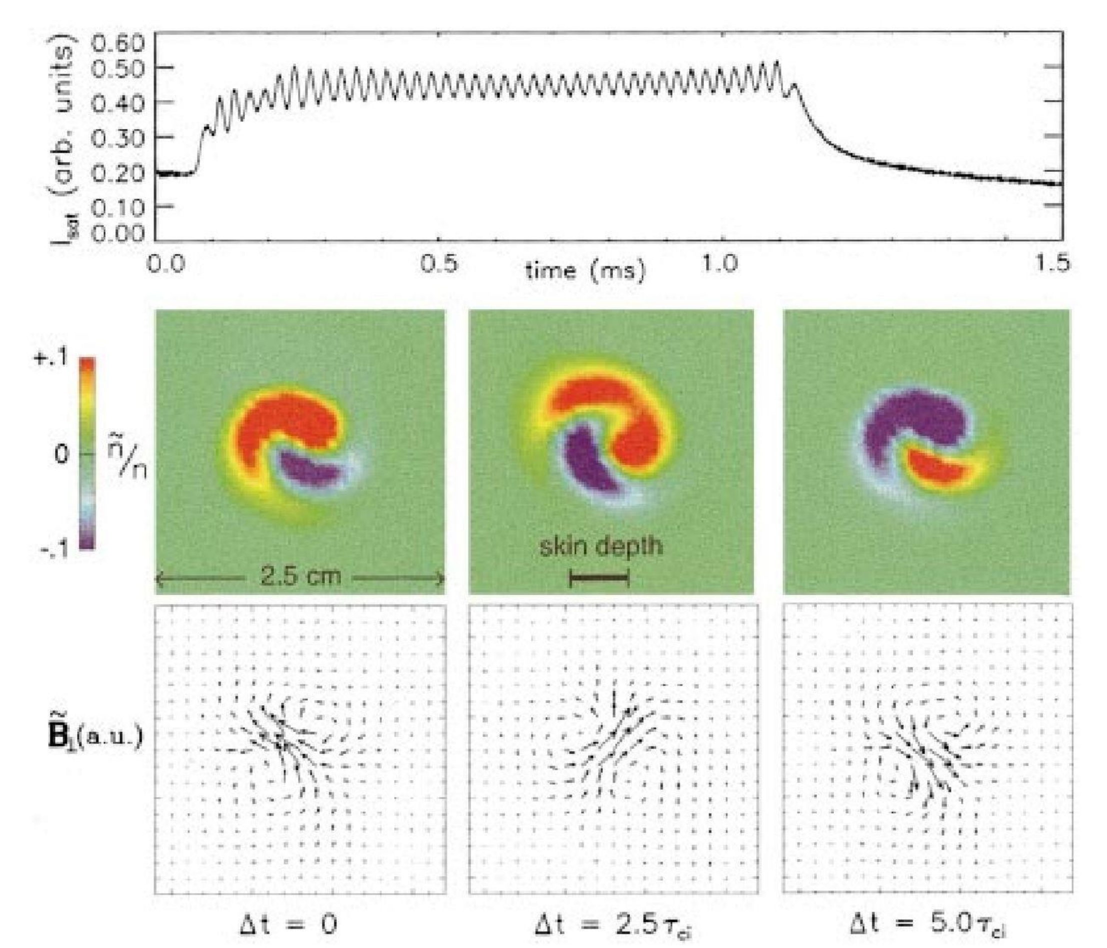
- Adapt a single filament potential model to represent a three-filament plasma experiment
- Numerically simulate the model in MATLAB
- Compare simulation of the adapted potential model to ion saturation data to determine model accuracy

A few other additions

- Calculate the plasma flow fields from the adapted potential model
- Add potential well to model to account for the induced background ion saturation of close filaments and compare to potential data

What is a Temperature Filament?

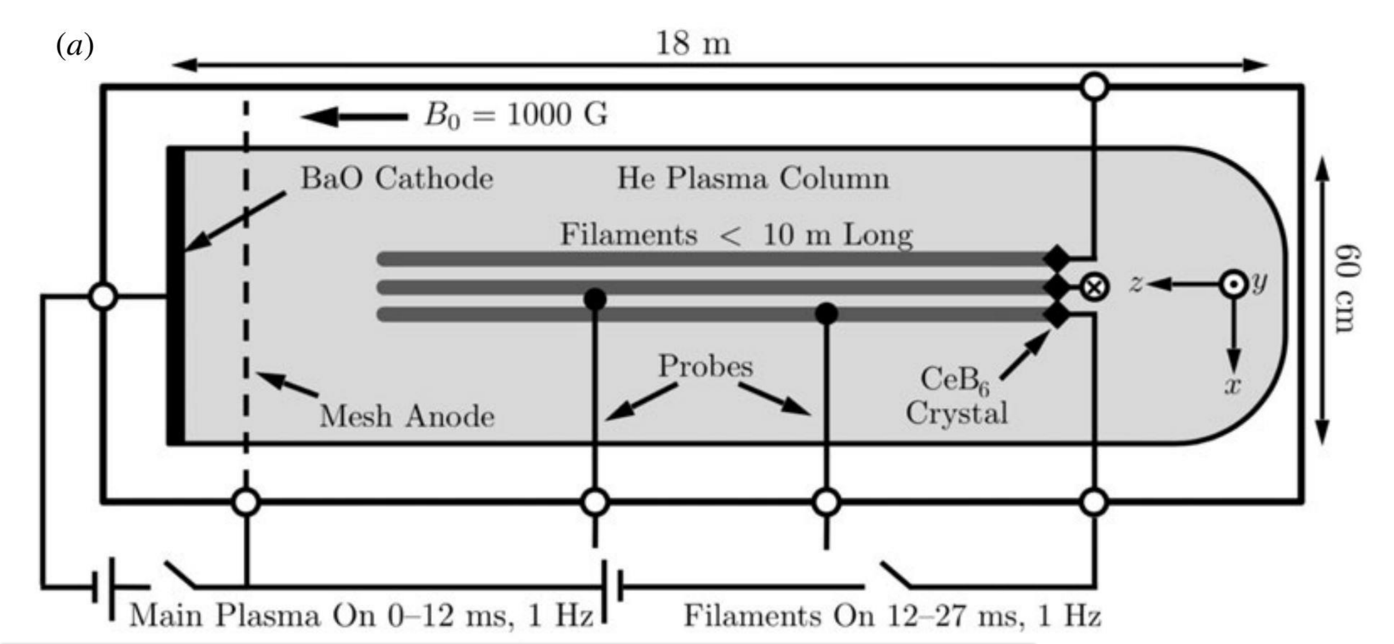
- Temperature structure/dispersion in a plasma surrounding a heat source
- Filaments evolve and rotate with time due to movement in the plasma caused by diffusion, convection, and E X B energy transport.
- Rotate/fluctuate extremely quickly ~μs scale

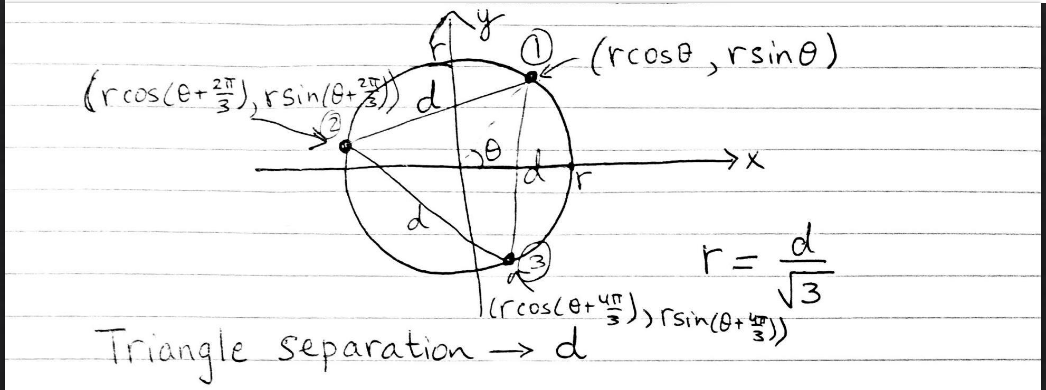


Burke et al. 2000

Three Filament Experiment

- Helium plasma
- Three electron heat sources in triangular formation
- Magnetized by uniform magnetic field



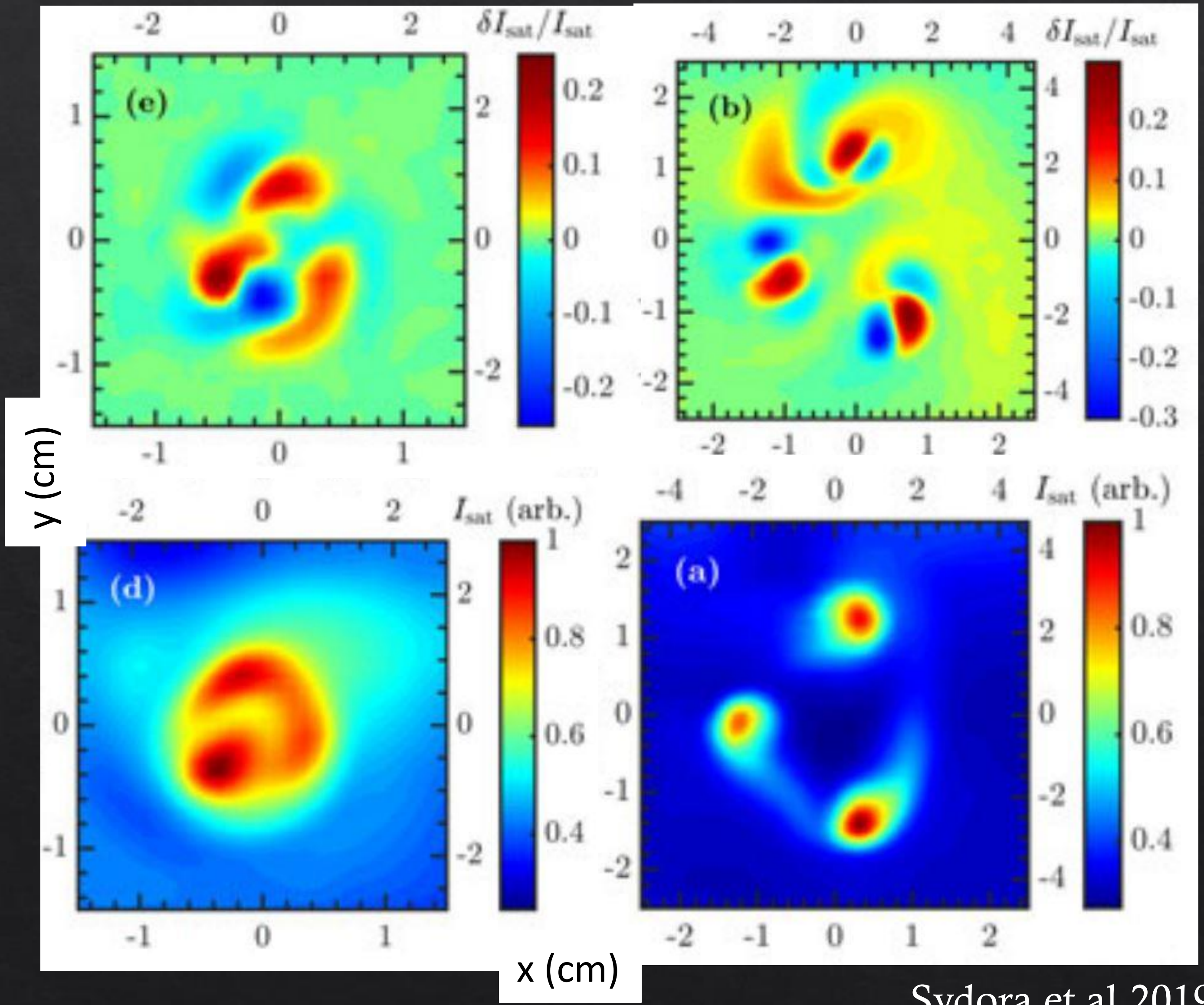


Sydora et al.2019

Schematic drawn by Dr. Richard Sydora

Data

- Data collected at 0.5cm and 1.5cm distance with and without 25kHz bandpass
- Filaments interact at close distances
- High ion saturation = high temperature = high potential in plasma medium



Sydora et al.2019

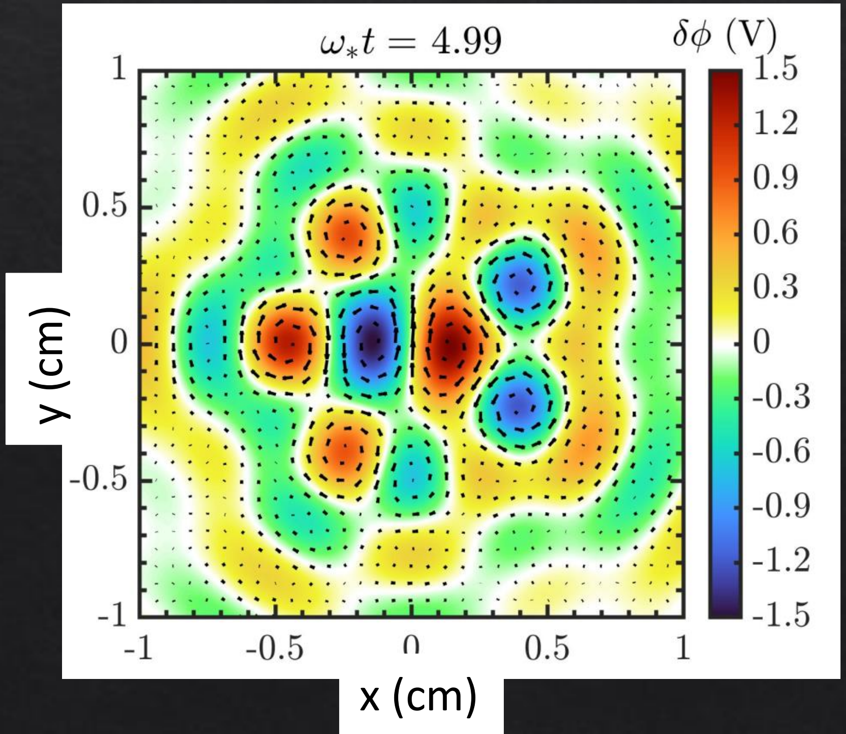
Modeling the Experiment

- Used single filament potential model from previous study (Shi et al. 2009)

$$\delta(r,\theta,t) = A_1 J_1(k_1 r) e^{i m_1 \theta} e^{-\alpha r} e^{-iwt} + A_6 J_6(k_6 r) e^{i m_6 \theta} e^{-\alpha r} e^{-iwt}$$

- Adapted model to represent a bimodal and monomodal structures in our experiment.

$$\delta(r,\theta,t) = A_n J_n(k_n r) e^{i m_n \theta} e^{-\alpha r} e^{-iwt}$$



Modeling the Experiment

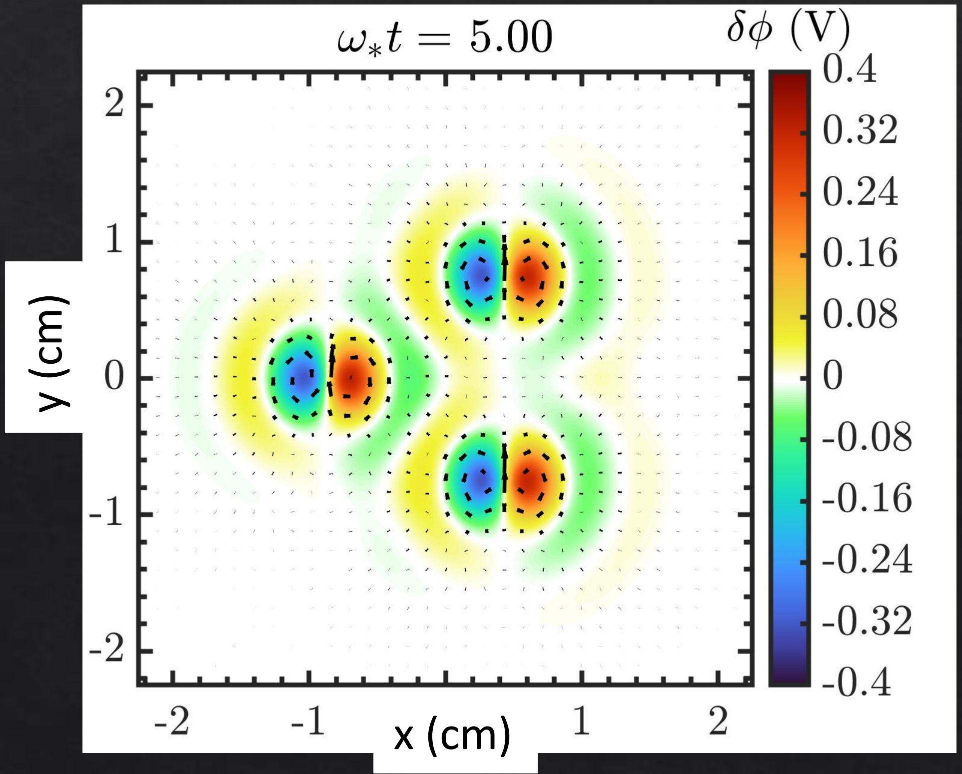
- Further modified model to represent three filaments

$$\boldsymbol{\delta}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) = \sum_{\boldsymbol{n}=1}^{3} \boldsymbol{\delta_n}\left\{\boldsymbol{r}\left((\boldsymbol{x}-\boldsymbol{x'}_{\boldsymbol{n}}),\left(\boldsymbol{y}-\boldsymbol{y'}_{\boldsymbol{n}}\right)\right),\boldsymbol{\theta}\left((\boldsymbol{x}-\boldsymbol{x'}_{\boldsymbol{n}}),\left(\boldsymbol{y}-\boldsymbol{y'}_{\boldsymbol{n}}\right)\right),\boldsymbol{t}\right)\right\}$$

- Calculated flow field using drift velocity to approximate movement

$$0 = q(\boldsymbol{E} + v\ X\ \boldsymbol{B})$$

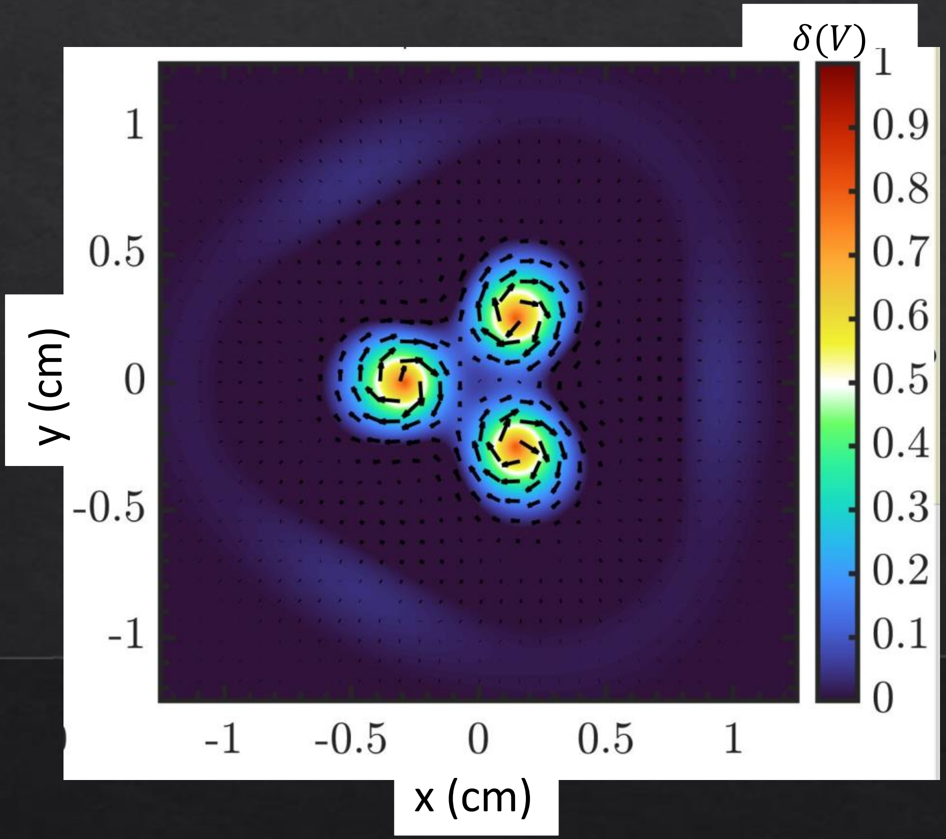
$$\mathbf{U} = \sum_{\boldsymbol{n}=1}^{3} \frac{\boldsymbol{E_{nx}}}{\boldsymbol{B}} \quad \mathbf{V} = \sum_{\boldsymbol{n}=1}^{3} \frac{\boldsymbol{E_{ny}}}{\boldsymbol{B}}$$



Numerical Model and Simulation

Monomodal model

- Single modes with curling flow fields



Bimodal model

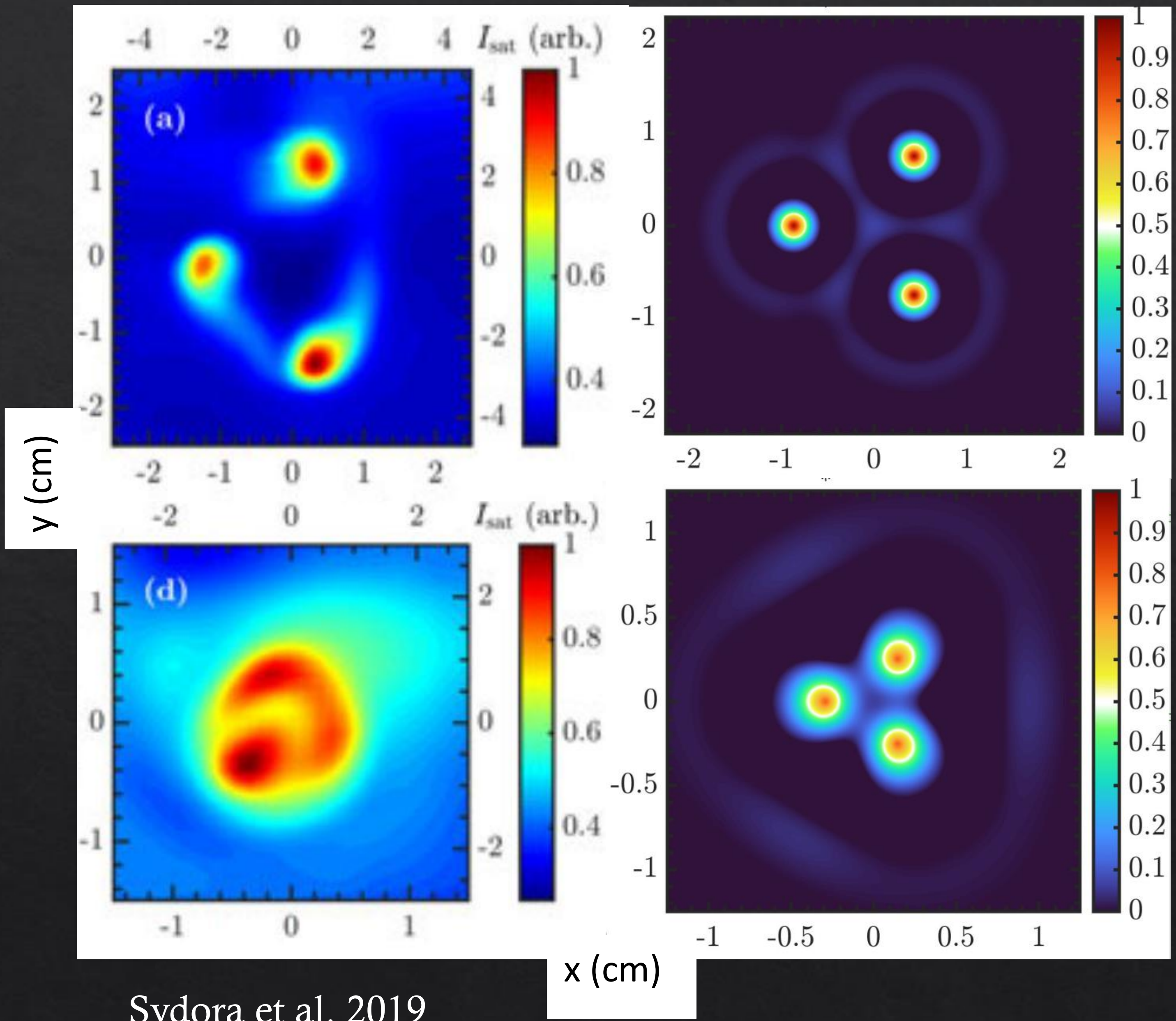
- Rotating dipole filament structures
- Modes interact and create stretching and compressed structures and movement vortices

Unfiltered data

Single mode structure => single mode model

Accurately models 1.5cm distance

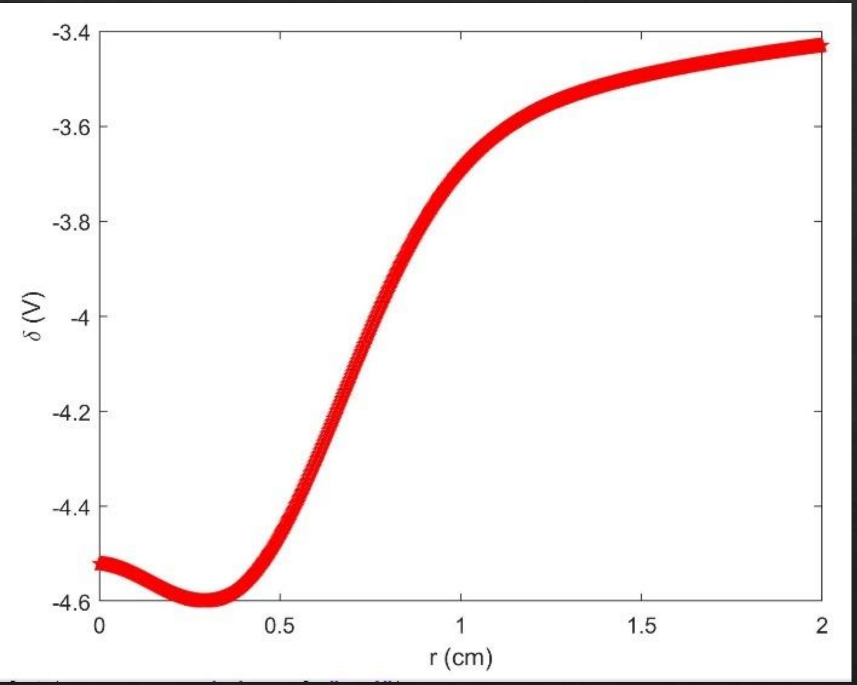
Need to account for background current for .5



Sydora et al. 2019

Potential Well

- When filaments are placed close together, a background ion saturation is induced
- This can be solved by adding a potential well to the model

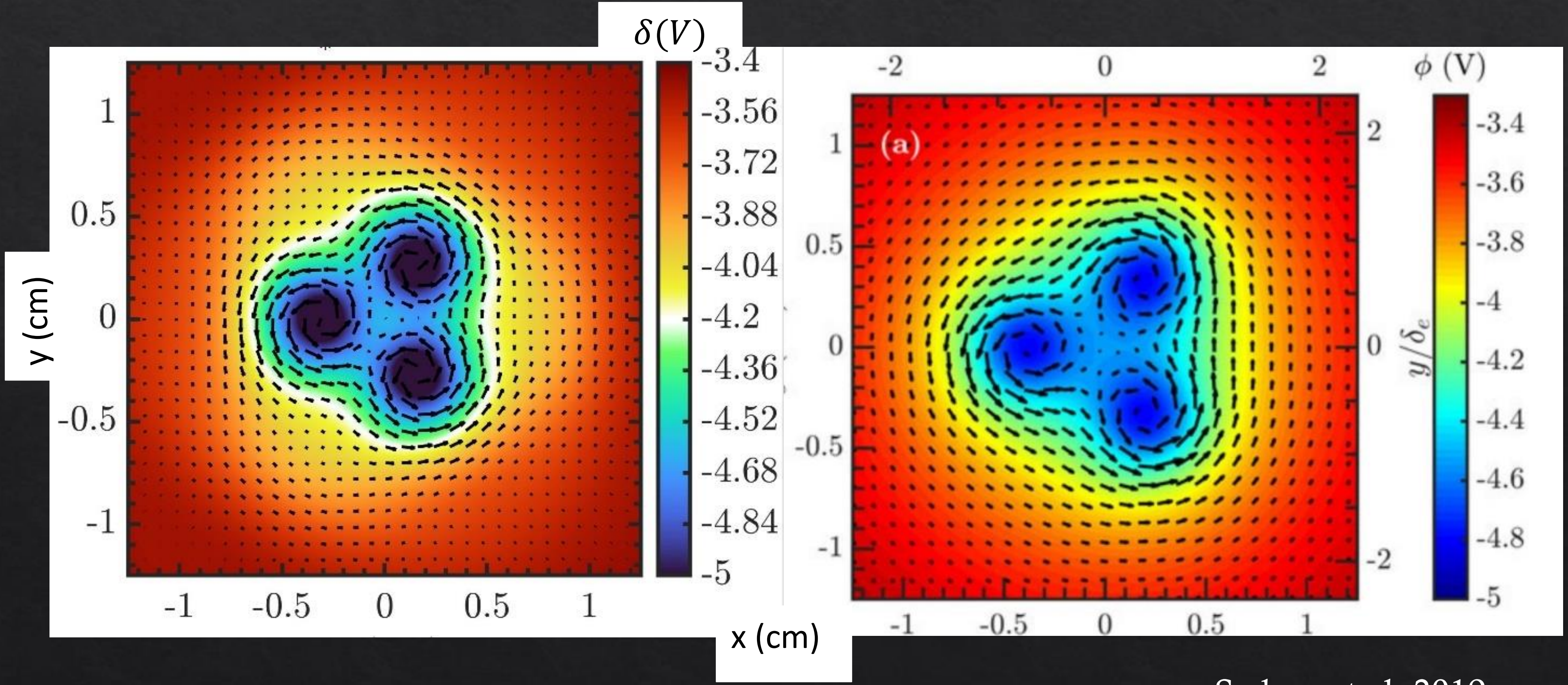


$$\delta_{br}(r) = C_1 + C_2 e^{-C_3(r-C4)\text{^}2} + C_5(r + 3\ cm)^{-4}$$

C_1=-3.309 V, C_2=-0.690 V, C_3=5.712 cm^{-2}, C_4=.397 cm, $C_5 = -75.410$ eV cm^4

Adding Background Potential and Comparing

Model was compared to potential and flow velocity calculated from data



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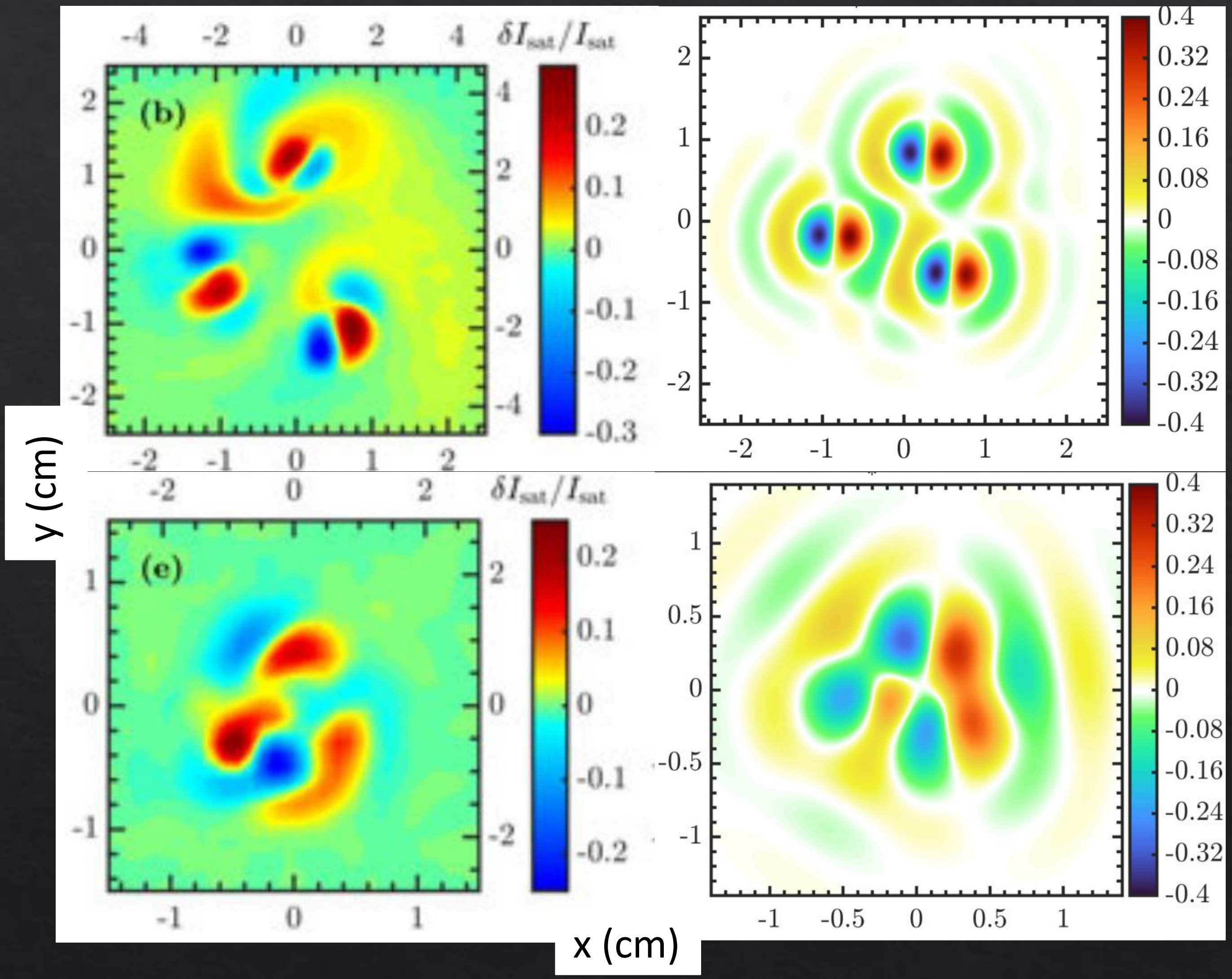
Filtered Data

- Much less accurate
- Doesn't account for evolving modes
- Doesn't account for uneven rotation speed with time
- Doesn't account for uneven mode amplitudes
- E x B?



Conclusion

Model can be adapted to represent higher filament numbers

Model accurately models monomodal structures for unfiltered data

Model adapts well to background potential

Model less accurately models' bimodal filaments

Model needs to be adapted to have parameters that evolve with time

Model needs to account for residual E X B

More research needs to be done

: thank you gif - Bing images

References

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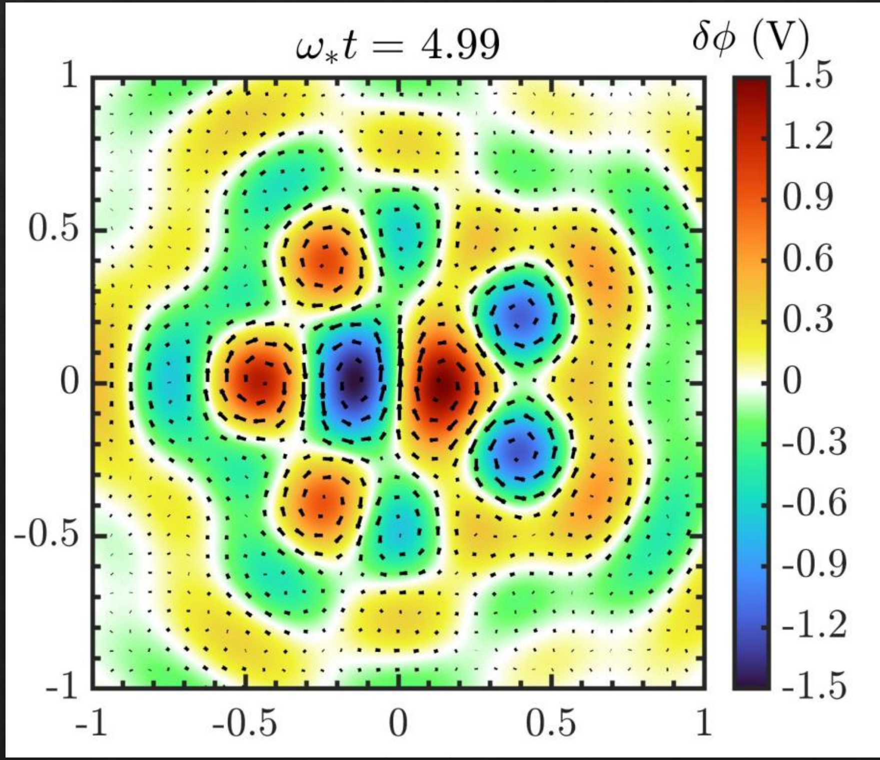
- Burke, A. T., Maggs, J. E., & Morales, G. J. (2000). "Experimental study of fluctuations excited by a narrow temperature filament in a magnetized plasma", *Physics of Plasmas,* **7**, 1397-1407 https://doi.org/10.1063/1.873957
- Shi, M., Pace, D. C., Morales, G. J., Maggs, J. E., & Carter, T. A. (2009). Structures generated in a temperature filament due to drift-wave convection. *Physics of Plasmas*, ***16***(6), 062306.
- Pace, D. C., Shi, M., Maggs, J. E., Morales, G. J., & Carter, T. A. (2008). Exponential frequency spectrum and Lorentzian pulses in magnetized plasmas. *Physics of Plasmas*, ***15***(12), 122304.
- : thank you gif - Bing images
- Lecture 3 -Guiding centre, E X B drift, drift in a general force – YouTube For Lorentz field derivation

Further Explanation

- The slides following this are to further explain concepts briefly talked about in the presentation. These slides will not be presented unless topics are asked about

Single Filament Model

- Base of the model is scaler potential on 2D plane perp to b field
- $\boldsymbol{\delta(r,\theta,t) = A_1 J_1(k_1 r) e^{im_1\theta} e^{-\alpha r} e^{-iwt}} +$
  $\boldsymbol{A_6 J_6(k_6 r) e^{im_6\theta} e^{-\alpha r} e^{-iwt}}$
- Positive value = hot, Negative value = cold
- Can derive electric fields $\boldsymbol{E = -\nabla V}$ where
- $\boldsymbol{E_r(r,\theta,t) = -A_1\left(\frac{\partial J_1(k_1 r)}{\partial r} - \alpha J_1(k_1 r)\right) e^{im_1\theta} e^{-\alpha r} e^{-iwt}} + .$
- $\boldsymbol{E_\theta(r,\theta,t) = -\frac{im_1}{r} A_1 J_1(k_1 r) e^{im_1\theta} e^{-\alpha r} e^{-iwt}} + \ldots..$
- **Important for calculating plasma flow**



Shi et al. 2009

Plasma Flow

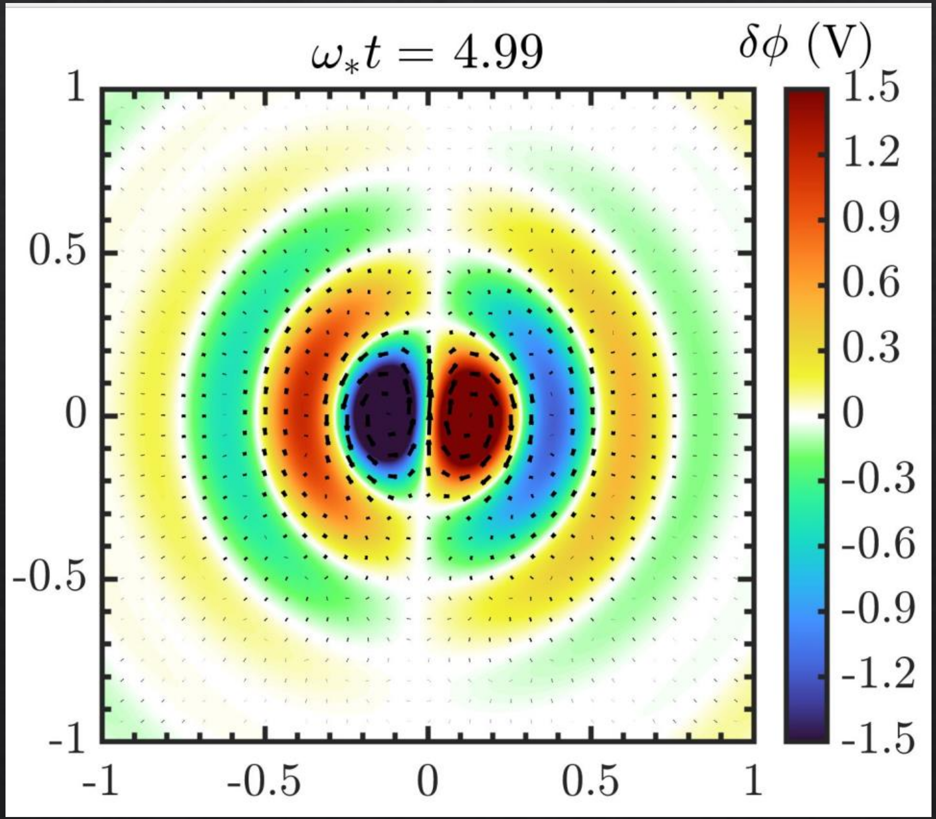
- Solve Lorentz force equation for velocity to calculate average drift velocity
- $F = m\frac{dv}{dt} = q(\boldsymbol{E} + v\ X\ \boldsymbol{B})$
- $0 = q(\boldsymbol{E} + v\ X\ \boldsymbol{B})$ Because we only care about average flow dude to E x B (from our potential)
- $\boldsymbol{E}$ = -($v\ X\ \boldsymbol{B}$)
- Curl B both sides and use a vector identity we get
- $\boldsymbol{E}\ X\ \boldsymbol{B} = vB^2 - \boldsymbol{B}(v \cdot \boldsymbol{B})$ End term is zero
- $v = \frac{\boldsymbol{E}\ X\ \boldsymbol{B}}{B^2}$ and because they are always perp $v = \frac{E}{B}$ field strength scalier

$\mathbf{U} = \frac{\boldsymbol{E_r}}{\boldsymbol{B}}$, $\mathbf{V} = \frac{\boldsymbol{E_\theta}}{\boldsymbol{B}}$

Modifying for three filaments

- $\delta(r,\theta,t) = A_1 J_1(k_1 r) e^{im_1\theta} e^{-\alpha r} e^{-iwt} + A_6 J_6(k_6 r) e^{im_6\theta} e^{-\alpha r} e^{-iwt}$
- $\delta(r,\theta,t) = A_n J_n(k_n r) e^{im_n\theta} e^{-\alpha r} e^{-iwt}$
- $E_r(r,\theta,t) = -A_n \left(\frac{\partial J_n(k_n r)}{\partial r} - \alpha J_n(k_n r)\right) e^{im_n\theta} e^{-\alpha r} e^{-iwt}$
- $E_\theta(r,\theta,t) = -\frac{im_n}{r} A_n J_n(k_n r) e^{im_n\theta} e^{-\alpha r} e^{-iwt}$
- $E_x = E_r \cos\theta - E_\theta \sin\theta,\ E_y = E_r \sin\theta + E_\theta \cos\theta$

$\mathbf{U} = \frac{E_x}{B}, \quad \mathbf{V} = \frac{E_y}{B}$

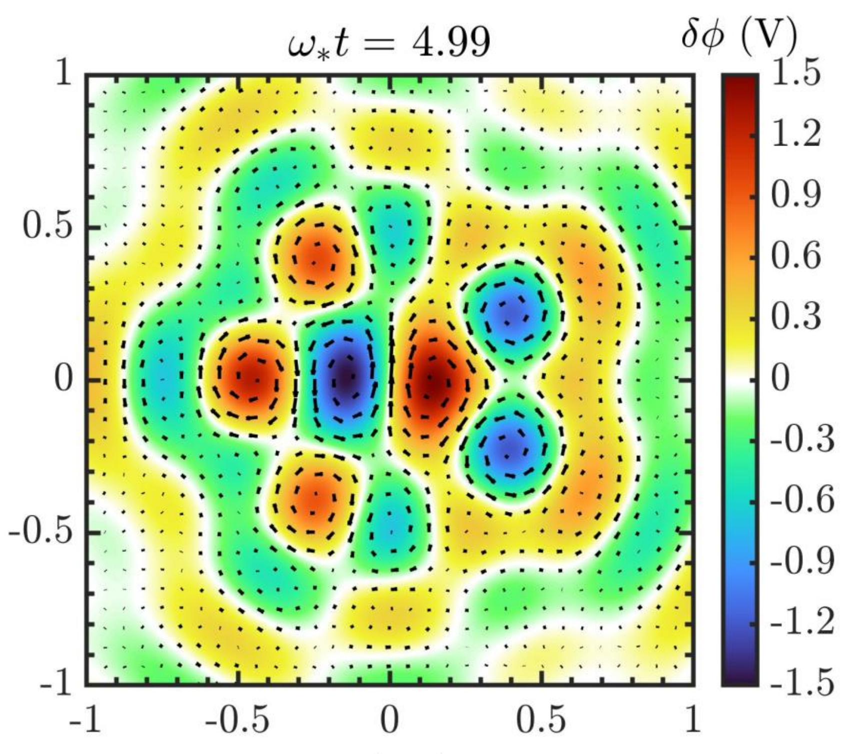


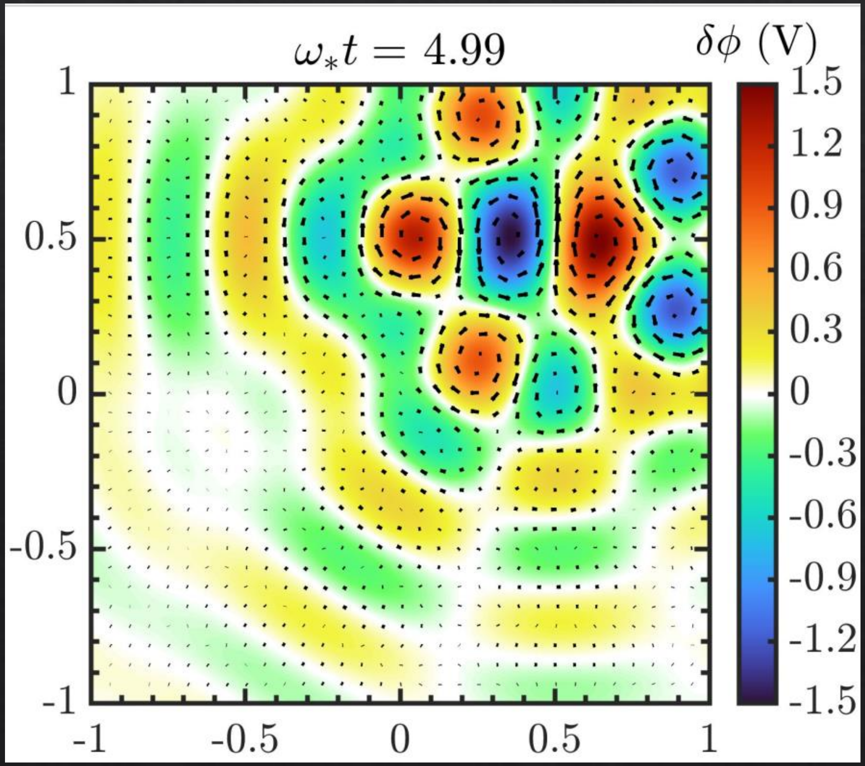
Cartesian change of frames

$$\delta(r,\theta,t) = A_n J_n(k_n r) e^{i m_n \theta} e^{-\alpha r} e^{-iwt}$$

$$r = \sqrt{x^2 + y^2} \quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$r = \sqrt{(x-x')^2 + (y-y')^2} \quad \theta = \tan^{-1}\left(\frac{y-y'}{x-x'}\right)$$





Adding filaments

$$\delta(r,\theta,t) = A_n J_n(k_n r) e^{i m_n \theta} e^{-\alpha r} e^{-iwt}$$

$$r = \sqrt{(x-x')^2 + (y-y')^2} \quad \theta = \tan^{-1}\left(\frac{y-y'}{x-x'}\right)$$

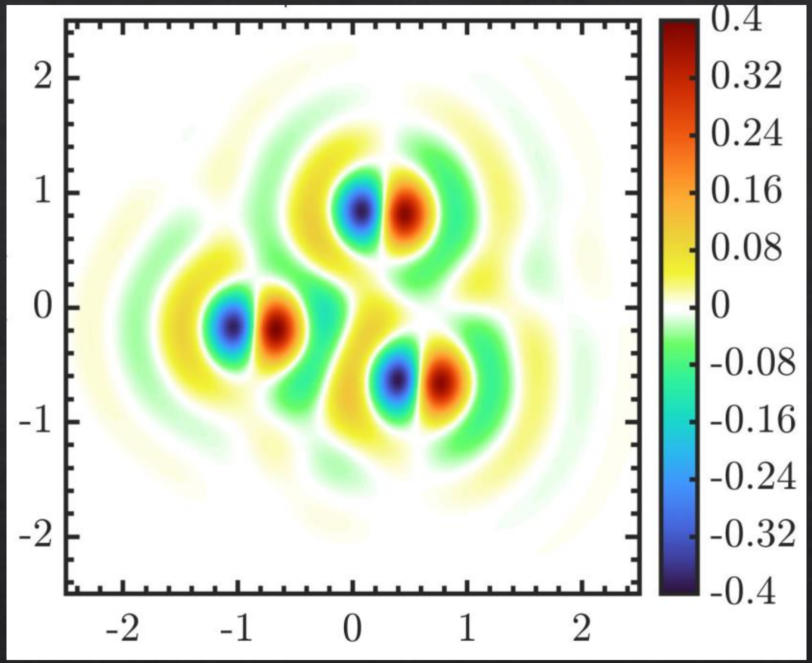
$$\delta(x,y,t) = \sum_{n=1}^{3} \delta_n \left\{ r\left((x-x'_n),\left(y-y'_n\right)\right), \theta\left((x-x'_n),\left(y-y'_n\right)\right), t \right\}$$

$$E_x(x,y,t) = \sum_{n=1}^{3} E_{xn} \left\{ r\left((x-x'_n),\left(y-y'_n\right)\right), \theta\left((x-x'_n),\left(y-y'_n\right)\right), t \right\}$$

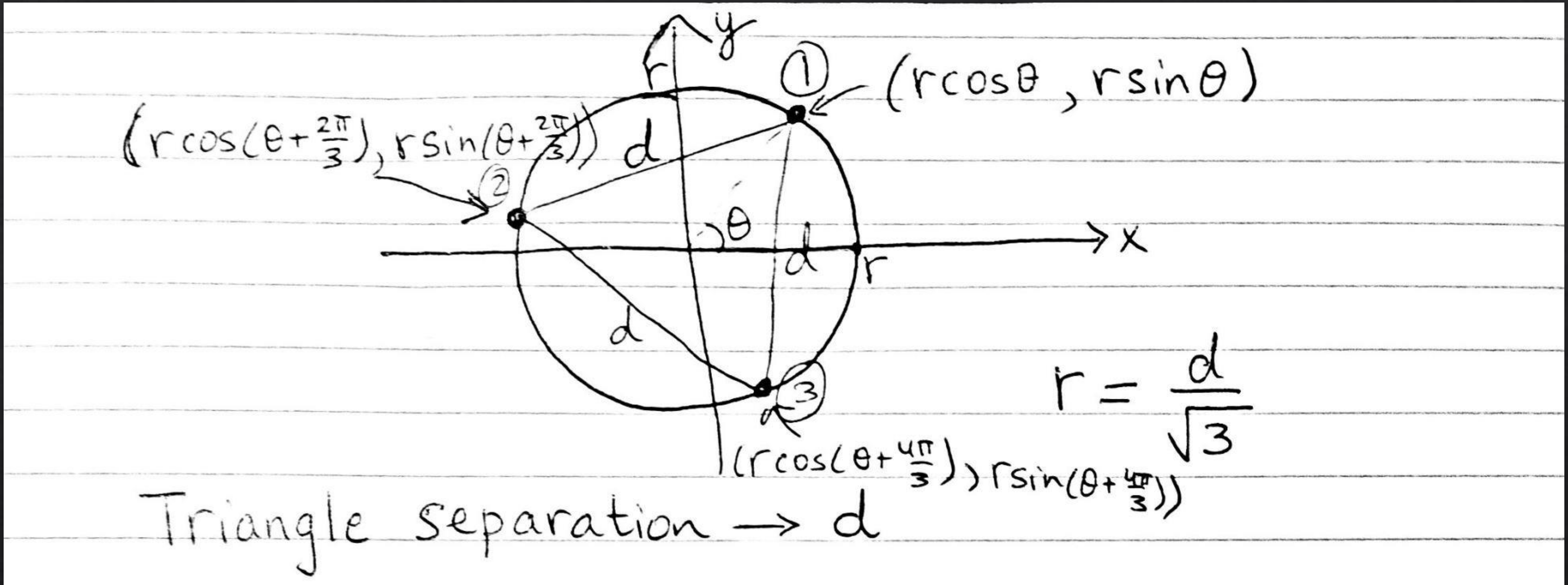
$$E_y(x,y,t) = \sum_{n=1}^{3} E_{yn} \left\{ r\left((x-x'_n),\left(y-y'_n\right)\right), \theta\left((x-x'_n),\left(y-y'_n\right)\right), t \right\}$$

$$\mathbf{U} = \sum_{n=1}^{3} \frac{E_{nx}}{B}$$

$$\mathbf{V} = \sum_{n=1}^{3} \frac{E_{ny}}{B}$$



Formulation of Triangular Structure



cr: Dr. Richard Sydora

Including Triangular Structure

$$x'(d, \theta in) = R(d)\cos(\theta in + \theta_n)$$

$$y'(d, \theta in) = R(d)\sin(\theta in + \theta_n)$$

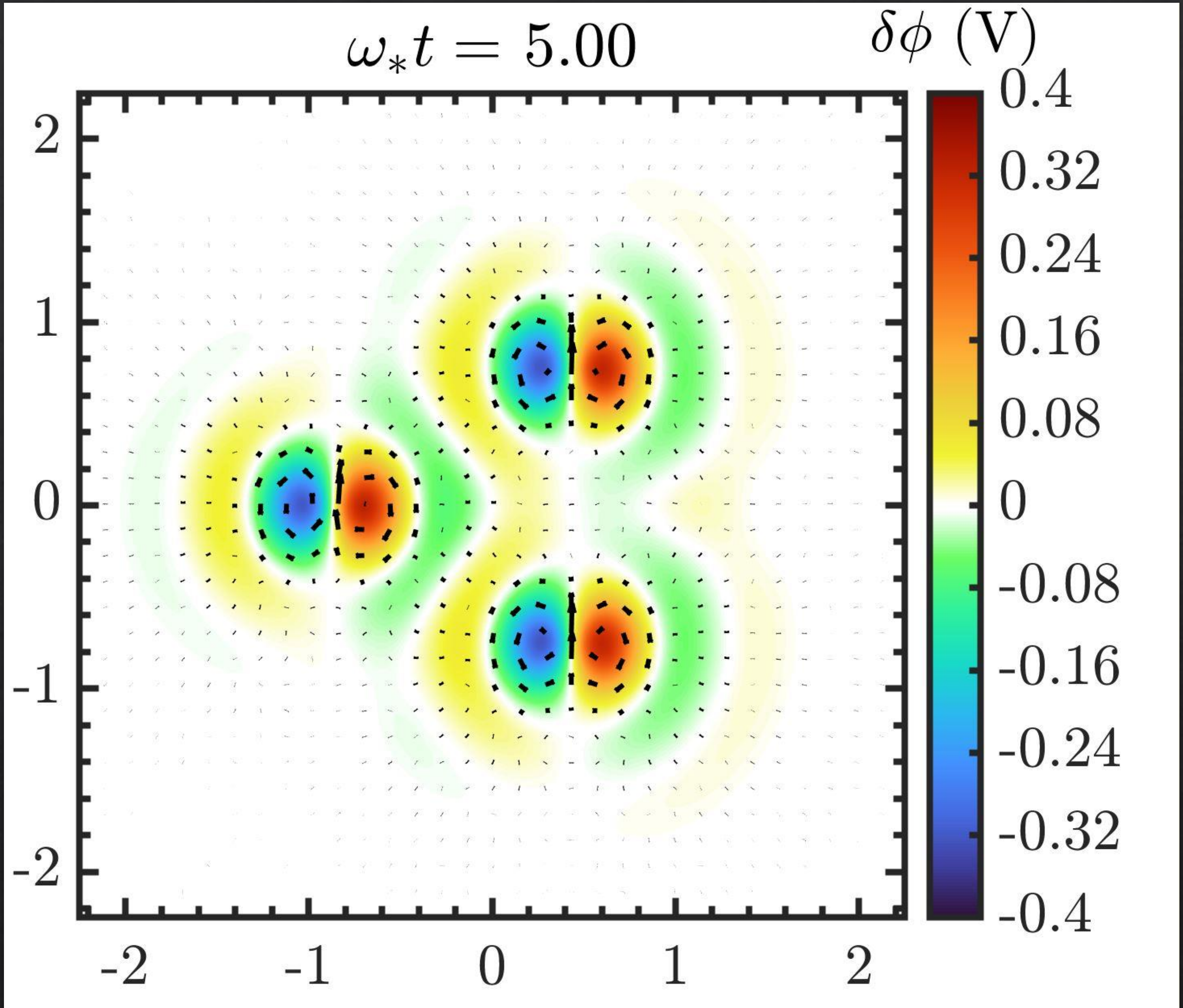
$$\theta_1 = 0, \qquad \theta_2 = \frac{2\pi}{3} \qquad \theta_3 = \frac{4\pi}{3}$$

$$\delta(x, y, t) = \sum_{n=1}^{3} \delta_n \left\{ r\left(\left(x - x'_n(d, \theta in\ \theta_n)\right), \left(y - y'_n(d, \theta in\ \theta_n)\right) \right), \theta\left(\left(x - x'_n(d, \theta in\ \theta_n)\right), \left(y - y'_n(d, \theta in\ \theta_n)\right) \right), t \right\}$$

$$E_x(x, y, t) = \sum_{n=1}^{3} E_{xn} \left\{ r\left(\left(x - x'_n(d, \theta in\ \theta_n)\right), \left(y - y'_n(d, \theta in\ \theta_n)\right) \right), \theta\left(\left(x - x'_n(d, \theta in\ \theta_n)\right), \left(y - y'_n(d, \theta in\ \theta_n)\right) \right), t \right\}$$

$$E_y(x, y, t) = \sum_{n=1}^{3} E_{yn} \left\{ r\left(\left(x - x'_n(d, \theta in\ \theta_n)\right), \left(y - y'_n(d, \theta in\ \theta_n)\right) \right), \theta\left(\left(x - x'_n(d, \theta in\ \theta_n)\right), \left(y - y'_n(d, \theta in\ \theta_n)\right) \right), t \right\}$$

$$\mathbf{V} = \sum_{n=1}^{3} \frac{E_{ny}}{B} \qquad \mathbf{U} = \sum_{n=1}^{3} \frac{E_{nx}}{B}$$



$\omega_* t = 5.00$
$\delta\phi$ (V)
0.4
0.32
0.24
0.16
0.08
0
-0.08
-0.16
-0.24
-0.32
-0.4
2
1
0
-1
-2
-2
-1
0
1
2