

Safe Concurrent Programming

Abstract

The safe parallelization of sequential programs is an important and challenging task. A parallelization is safe if it ensures that no new behaviors are introduced; the semantics of the parallel program is identical to that of the sequential program. In particular the parallel program must not exhibit scheduler indeterminacy or deadlock.

This is a challenging problem. Imperative memory locations are famously indeterminate under concurrent read/write operations. Further, most concurrent programming languages support synchronization mechanisms that enable a thread to wait until some appropriate condition is true, thus leading to the possibility of deadlock.

In this paper we develop a very rich fragment of the concurrent, imperative programming language X10 in which *all* programs have sequential semantics. The advantages of such a language are striking: the programmer simply cannot introduce determinacy or deadlock-related errors. All programs can be debugged as if they are sequential programs, without having to reason about all possible interleaving of threads.

The fragment includes all of X10's most powerful constructs – (clocked) *finish* and *async*, *places* and *at*. We introduce two new abstractions – *accumulators* and *clocked values* – with lightweight compiler and run-time support. Accumulators of type τ permit multiple concurrent writes, these are reduced into a single value by a user-specified reduction operator. Accumulators are designed to be determinate and deadlock-free at run-time, under all possible uses. Clocked values of type τ operate on two values of τ (the *current* and the *next*). Read operations are performed on the current value, write operations on the next. Such values are implicitly associated with a clock and the current and next values are switched (determinately) on quiescence of the clock. Clocked values capture the common “double buffering” idiom. Accumulators and clocked values are designed in such a way that they are deadlock-free and determinate even in the presence of arbitrary aliasing and patterns of concurrency.

We combine these notions with a lightweight framework for detecting disjointness of access via static effects checking.

We show all programs written in this fragment of the language are guaranteed to have sequential semantics. That is, programs in this fragment may be developed and debugged as if they are sequential, without running into the state explosion problem.

Further, many common patterns of concurrent execution can be written in this language in an “obvious” way. These patterns include histograms, all-to-all reductions, stencil computations, master/slave execution, and most of the important HPC computing idioms. We show that there are simple statically-checkable rules that can estab-

Figure 1. A program that is not sequential

lish for many common idioms that some run-time synchronization checks can be avoided.

The key technical problem addressed is unrestricting aliasing in the heap. We develop the idea of an *implicit ownership domain* (first introduced in the semantics of clocks in X10 in a paper in Concur 2005) that newly created objects may be registered with the current activity so that they can be operated upon only by this activity and its newly spawned children activity subtree. This permits determinacy and deadlock freedom to be established locally, independent of the context in which the code is being run.

1. Introduction

A *safe* parallel program is one that can be executed with a sequential (depth-first) schedule S and for which every schedule produces the same result as S . Such a program is semantically a sequential program, hence it is scheduler-determinate and deadlock-free.

A *safe programming language* is an imperative parallel programming language in which every legal program is a safe program. Programmers can write code in such a language secure in the knowledge that they will not encounter a large class of parallel programming problems. Such a language is particularly useful for parallelizing sequential (imperative) programs. In such cases (*contra* reactive programming) the desired application semantics are sequential, and parallelism is needed purely for efficient implementation.

The key property of a safe programming language is that the *same* program can be developed and debugged as a sequential program and then safely run in parallel. Parallel execution is guaranteed to effect only performance, not correctness. Safety is a very strong property.

In this paper we establish that a very rich fragment of X10 is safe. The fragment supports multi-place, fine-grained concurrency over an arbitrary heap, clocked computations, concurrent accumulators, and *clocked types* that safely capture the “red black” idiom for iterative computations. It also uses a very lightweight effects mechanism to reason about disjointness of access. We show that a large variety of concurrency and communication idioms can be naturally expressed in Safe X10.

Determinism in parallel programming is a very active area of research. Guava [?] introduces restrictions on shared memory Java programs that ensure no data-races primarily by distinguishing monitors (all access is synchronized) from values (immutable) and objects (private to a thread). However Guava is not safe since Guava programs may use *wait/notify* for arbitrary concurrent signalling and hence may not be executable with a sequential schedule. The Revisions programming model [2] guarantees determinism by isolating asynchronous tasks but merging their writes determinately. However, the model explicitly does not require that a sequential schedule be valid (c.f. Figure 1 in [2]).

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.

DPJ develops the “determinacy-by-default” slogan using a static type-and-effects system to establish commutativity of concurrent actions. The deterministic fragment of DPJ is safe according to the definition above. Safe X10 offers a much richer concurrency model which guarantees the safety of common idioms such as accumulators and cyclic tasks (clocks) without relying on effects annotations. The lightweight effects mechanism in X10 can be extended to support a much richer effects framework (along the lines of DPJ) using X10’s constrained type system. We leave this as future work.

The SafeJava language [?] is unfortunately not safe according to our definition, even though it guarantees determinacy and deadlock freedom, using ownership types, unique pointers and partially ordered lock levels. Again, a sequential scheduler is not admissible for the model.

Some data-flow synchronization based languages and frameworks (e.g. Kahn style process networks [? ?], concurrent constraint programming [?], [3]) are guaranteed determinate but not safe according to our definition since they do not permit sequential schedules. Indeed they permit the possibility of deadlock. (The notion of safety is also not quite relevant since these frameworks do not support shared mutable variables.)

Determinizing run-times support coarse-grained fork-join concurrency by maintaining a different copy of memory for each activity and merging them determinately at finish points ([4], [?], [5], [6],[1]). Safe X10 can run on such systems in principle, but does not require them. To execute Safe X10, such systems need to support fine-grained asynchrony (with some form of work-stealing or fork-joining scheduler), clocks and accumulators.

Desiderata:

- Data-structures should by design be dynamically determinate and deadlock-free.
- They should be first-class – they can be stored in data-structures/read from them, passed as arguments to procedures/returned etc with no restrictions.
- Usable – common idioms should be naturally and elegantly expressible.
- Additional static type-checking can provide extra guarantees (e.g. no concurrency related run-time exceptions) that may aid efficient implementation.

We discuss three examples

* clocks * accumulators * clocked types

Challenge Arbitrary nature of object graphs.

1. Use activity registration as a mechanism to tame object graphs.
2. Focus on structured concurrency. Using scoping and block-structure to delimit regions of code that may execute in parallel and affect the data structure.
3. Accumulation can be defined safely by delaying. However, the delay operation is guaranteed to be deadlock-free.
4. Clocked types support phased computation, another common idiom particularly for stencil computations.

Key contributions:

1. Identification of determinate, deadlock-free data-structures.
2. Discussion of design alternatives which points out the difficulty of integrating these ideas in a modern OO language.
3. Discussion of various idioms expressible using these data-structures.
4. Proof of determinacy and deadlock-freedom in an abstract version of the language.

These constructs are implemented in X10, available as open source from SVN head and will be in the next release of X10.

Semantics and theorems for an abstract version of the language.

2. X10

Discussion of X10.

2.0.1 sync

We introduce a new derivative synchronization concept, that of syncing.

We introduce `Runtime.sync()`. It returns only after all asyncs spawned by the current activity have terminated or stopped at an advance on a clock registered on the current activity.

Programs using `finish/async/at/atomic/clocks/Runtime.sync()` cannot deadlock.

3. Accumulators and Clocked Types

3.1 Accumulators

Each async (dynamically) has a set of registered `@Sync` accumulators and `@Async` accumulators.

- The registered `@Async` accumulators for an activity are the registered `@Sync` and `@Async` accumulators of its parent activity.
- The registered `@Sync` accumulators for an activity are the ones it has created.

This permits computations to be determinate even though accumulators can be stored in heaps, since no async other than the async that created the accumulator or one of its progeny can actually operate on them. One can still use the flexibility of the heap to arrange for complex data-dependent transmission pathways for the accumulator from point of creation to point of use. e.g. arrays of accumulators, hash-maps, etc. In particular, accumulators can be passed into arbitrary method invocations, returned from methods etc, with no restrictions.

The method

```
Runtime.isRegistered[T](x:Acc[T]):Int
```

returns 0 if `x` is not registered with the current activity, 1 if it is `@Sync` registered, and 2 if it is `@Async` registered.

An invocation `e.m(e1, ..., en)` of an `@Sync` method on an `Acc` is translated to:

```
{
  val x = e;
  if (Runtime.isRegistered(x) != 1)
    throw new IllegalAccAccess(x);
  Runtime.sync();
  x.m(e1, ..., en)
}
```

The `@Sync` methods on an `Acc` are ones that return its current value (`@Read`) and ones that reset it (`@Write`).

An invocation `e.m(e1, ..., en)` of an `@Async` method on an `Acc` is translated to:

```
{
  val x = e;
  if (Runtime.isRegistered(x) == 0)
    throw new IllegalAccAccess(x);
  x.m(e1, ..., en);
}
```

The only `@Async` method on an `Acc` is the one that offers an update to its value (`@Write`).

In many cases the compiler can statically evaluate whether `Runtime.isRegistered(x) > 0` and/or whether a call to `Runtime.sync()` will suspend.

It may then appropriately simplify the above code. e.g. in the code below

```

val x:Acc[Int] = new Acc[Int](0, Int.+);
finish for (i in 0..100000) async
  x <- i;
Console.OUT.println("x is " + x());

```

the compiler can infer that `x()` won't suspend, due to the `finish`. Hence it may eliminate the run-time suspension check. Further it can establish that `x` is `@Sync` registered with the current activity, hence it can eliminate the access check.

Notes:

- Accs are first-class values. There are no restrictions in storing them in data-structures, reading them, passing them as arguments to methods, returning them from methods etc.
- However, any attempt to use it will fail unless the `Acc` is registered with the current activity.
- The runtime checks in `Runtime.sync()` and `Runtime.registered(...)` ensure that the operations on an `Acc` are determinate.

Proposition 3.1. *Acc's are determinate under arbitrary usage.*

3.2 Example use of Acc

Example 3.3 (Distributed word-count). // A `DistHashMap` is used because the input is a `DistStream`

```

@det
def
  wordCount (m:DistStream[Word]) :DistHashMap[Word,Int] (m.dist)
  {
    val a = new DistHashMap[Word, Acc[Int]] (m.dist,
      (w:Word)=> new Acc[Int] (Int.Sum));
    finish for (p in m.dist.places()) async at (p) {
      for (word in m(p).words())
        a(word) <- 1;
    }
    return new DistHashMap[Word, Int] (m.dist,
      (w:Word)=> a(w));
  }

```

Accumulators can be used to implement collective operations such as all-to-all reductions in a straightforward “shared memory” style.

Here we show the single-sided, blocking version.

Example 3.4. @det

```

def reduce[T] (in:DistArray[T], red:Reducible[T]) :T {
  val acc = new Acc[T] (red);
  val temp = new GlobalRef[Acc[T]] (acc);
  finish for (dest in in.dist.places()) async
    at (dest) {
      val local = new Acc[T] (red);
      for (p in in.dist | here) {
        local <- in(p);
      }
      val x = local();
      async at (origin) temp() <- x;
    }
  return acc();
}

```

An `allReduce` can be implemented by following the above operation with a broadcast:

```

@det
def
  allReduce[T] (in:DistArray[T] {self.dist==Dist.UNIQUE},
    red:Reducible[T], out:DistArray[T] (in.dist)) :void {
    val x = reduce(in, red);
  }

```

```

finish for (dest in out.dist.places()) async at
  (dest) {
    for (p in out.dist | here)
      out(p)=x;
  }
}

```

One can write this code using a clock (to avoid two finish nests).

The collective style requires extending clock so the advance method takes arbitrary args and performs collective operations on them, mimicking the MPI API.

Collecting finish The collecting `finish` construct is of the form `finish(r) S`, where `r` is an instance of `Reducible[T]` and `S` is a statement. Within the dynamic execution of `S`, any execution of the statement `offer t`; results in a value `t` being accumulated in a set. (`t` must be of type `T`.) The result of the reduction of this set of `T` values with `r` is then returned.

```

{
  val x = new Acc[T] (r);
  finish {
    S [ x <- t / offer t; ]
  }
  x()
}

```

An attractive aspect of collecting finish is that nesting is used quite naturally to reflect the relationship between the parallel computation performing the accumulation and the value returned. In particular there is no need to explicitly introduce the notion of an accumulator, or to register it with the current block, or to check that other activities in the current block have quiesced.

On the other hand, this strength is also a limitation. It is not possible to use the same idiom for clocked code, i.e. collect values being offered by multiple clocked asyncs in the current phase. Further, using this idiom safely across method calls requires the addition of `offers T` clauses (similar to `throws T` clauses, specifying the type of the value being offered. Otherwise at run-time a value of an incompatible type may be offered leading to a run-time exception. Finally, the lack of a name for the result being collecting means that it is difficult to use the same computation to accumulate multiple separate values without more contortions. e.g. one could set the return type to be a tuple of values, but then the `offer` statement would need to specify the index for which the `offer` was intended. Now it is not clear that the index type should be arithmetic – why should not one be able to collect into the range of a `HashMap` (so the index type could be an arbitrary type `Key`)?

We feel that these decisions are all orthogonal to the actual process of accumulating and should be dealt with by the data-structuring aspects of the language design.

3.3 Clocked types

The central idea behind clocked data-structures is that read/write conflicts are avoided using “double buffering.” Two versions of the data-structure are kept, the *current* and the *next* versions. Reads can be performed simultaneously by multiple activities – they are performed on the current version of the data-structure. Writes are performed on the next version of the data-structure. On detection of termination of the current phase – when all involved activities are quiescent – the current and the next versions are switched.

`Clocked[T]` and `ClockedAcc[T]` are distinguished in that unlike the former the latter permits accumulation operations.

Clocked objects are registered with activities, just like accumulators. This permits computations to be determinate even though objects can be stored in heaps, since no `async` other than a child of the `async` that creates the clocked object can actually operate on them.

Each async (dynamically) has a set of registered clocked values. The registered clocked values for an activity are the clocked values it has created, and the ones registered to its parent activity.

`Clocked[T]` has a constructor that takes two `T` arguments, these are used to initialize the now and next fields. These arguments should be “new” (that is, no other data-structure should have a reference to these arguments).

For `x:Clocked[T]` the following operations available to any activity on which `x` is registered:

- `x()` – this returns the value of the current field.
- `x() = t` – This is translated to `x.next()=t`. That is, the value of the next field is set. Note: write-write conflicts are possible since multiple activities may try to set the value at the same time.
- `x.finalized()` – this returns the value of the now field but modifies the internal state so that any subsequent attempt to use `x()=t` will result in a runtime exception.

`ClockedAcc[T]` has a constructor that takes two `T` values and a `Reducer[T]` as argument. The two `T` values are used to initialize the current and next fields. These arguments should be “new” (that is, no other data-structure should have a reference to these arguments). The reducer is used to perform accumulate operations.

Operations for `x:ClockedAcc[T]`:

- `x()` – this returns the value of the now field.
- `x() <- t` – this accumulates `t` into the next field. Note: No write-write conflicts are possible.
- `x() = t` – this resets the value of the next field to `t`. To avoid read/write and write/write conflicts, this operation should be invoked only by the closure argument of `Clock.advanceAll(closure)`. (See below.)
- `x.finalized()` – this returns the value of the now field but modifies the internal state so that any attempt to use `x()=t` or `x() <- t` will result in a runtime exception.

We add the following method on `Clock`:

```
public static def advanceAll(x: ()=>void) {...}
```

If all activities registered on the clock invoke `advanceAll(f)` (for the same value `f`), then `f` is guaranteed to be invoked by some activity `A` registered on the clock at a point in time when all other activities have entered the `advanceAll(f)` call and the current/next swap has been performed for all registered clocked values. At this point – also called the *clock quiescent point* – it is guaranteed that none of the other activities are performing a read or write operation on user-accessible memory.

(A possible implementation of `Clocked[T]` and `ClockedAcc[T]` is that a system-synthesized closure (that performs the current/next swap) is run at the clock quiescent point before the user specified closure is run.)

Example 3.5. `\input {examples/stencil.x10}`

In the example above there is no need to reinitialize the value of `err()` between phases since the value will monotonically decrease. However for some other computations this may be necessary. For example, suppose the error metric was the sum of all errors. The the above code would change as follows. We would pass in a different reduction operation `red` that sums rather than returns a max. Further, we would replace the `Clock.advanceAll()` call with

```
Clock.advanceAll(()=>{err()=0.0D;});
```

This resets the next value of `err` to be 0.0D before the accumulations start to happen.

4. Effects

We represent effects in X10 as follows.

We use types to classify objects, i.e. to specify sets of objects. A *location set* is of the form `T.f` where `T` is a type and `f` is a mutable field of type `T`. It stands for the set of locations `x.f` where `x` is of type `T`. We do not need to introduce any distinct notion of regions into the language.

The key insight is that two memory locations are distinct if they are fields in two objects `m` and `n` such that for some property `P`, `m.p != n.p`. Therefore we introduce enough properties into classes to ensure that we can distinguish between objects that are simultaneously being operated on. Specifically, the location sets `S{c}.f` and `S{d}.f` are disjoint if `c[x/self], d[x/self]` is not satisfiable for any `x:S`.

Methods are decorated with `@Read(L)` and `@Write(M)` annotations, where `L,M` are location sets. The annotation is *valid* if every read (write) of a mutable location `o.f` in the body of the method it is the case that `o.f` lies in (the set described by) `L (M)`.

We permit location sets to be named:

```
static locs Cargos = Tree.cargo;
static locs LeftCargo(up:Tree) =
  Tree{self.up==up, self.left==true}.cargo;
```

Unlike the region system of [?] we do not need to introduce explicitly named, intensional regions, rather we can work with extensional representations of location sets (the set of all locations satisfying a certain condition). There is no need for a separate space of region names with constructors. Two location sets are disjoint if their constraints are mutually unsatisfiable, not because they are named differently. In particular, we do not need to assume that the heap is partitioned into a tree of regions.

Two key properties of the X10 type system are worth recalling. The run-time heap cannot contain cycles involving only properties. This in turn depends on the fact that the X10 type system prevents this from escaping during object construction[?].

We mark a property as `@ersatz` to indicate that space is not allocated at run-time for this property. Therefore the value of this property is not accessible at run-time – it cannot be read and stored into variables. It may only be accessed in constraints that are statically checked. Dynamic casts cannot refer to `@ersatz` properties.

We enrich the vocabulary of constraints. First, we permit existentially quantified constraints `x:Tê` – this represents the constraint `c` in which the variable `x` of type `T` is existentially quantified. Second, we permit the *extended field selector* `e.f$î` where `e` is an expression, `f` a field name and `î` is a `UInt`.

Example 4.1. type

```
Tree(up:Tree, l:Boolean) = Tree{self.up==up, self.left==l};
class Tree (@ersatz up:Tree, @ersatz left:Boolean) {
  var left:Tree(this, true);
  var right:Tree(this, false);
  var cargo:Int;
  static def make(up:Tree,
    left:Boolean):Tree(up, left) {
    val x = new Tree(up, left);
    x.left = new Tree(x, true);
    x.right = new Tree(x, false);
    return x;
  }
  def makeConstant(x:Int)
    @Write(Tree{î:UInt^self.up$î=this}.cargo)
    @Read(Tree{î:UInt^self.up$î=this}.left)
    {
      @Read(Tree{î:UInt^self.up$î=this}.right)
    }
  finish {
    @Write(Tree{self==this}.cargo)
    this.cargo = x;
  }
}
```

```

    if (left != null)
      async left.makeConstant(x);
    if (right != null)
      async right.makeConstant(x);
  }
}

```

In this example the fields `left`, `right` and `cargo` are mutable. It is possible to mutate a tree `p` – replace `p`'s left child with another tree `q`. However, `q` cannot be in `p`'s right subtree, because then its `up` field or `left` field would not have the right value. That is, once a `Tree` object is created it can only belong to a specific tree in a specific position.

Note that a `Tree` can be created with null parent and children. This is how the root is created: `new Tree(null, true)` or `new tree(null, false)`.

In checking the validity of the effect annotation on `makeConstant` the compiler must check subtyping relations. That is, for the `@Write` annotation it must check:

```

Tree{i:UInt ^ (self.up$! = this, self.up$! . left == true)}
  <: Tree{i:UInt ^ (self.up$! = this)}
Tree{i:UInt ^ (self.up$! . up == this, self.up$! . left == false)}
  <: Tree{i:UInt ^ (self.up$! = this)}
Tree{self == this} <: Tree{i:UInt ^ (self.up$! = this)}

```

This is easily verified. The notion of “distinctions from the left” and “distinctions from the right” of `[?]` are not needed. These arise naturally through the use of constraints (distinct access paths, vs distinct fields).

The machinery of “region path lists” of DPJ is not needed, it is provided for by the constraint language in X10.

Example 4.2. `class Tree (@ersatz up:Tree, @ersatz left:Boolean) {`
 `var left:Tree(this, true);`
 `var right:Tree(this, false);`
 `var mass:Double;`
 `var force:Double;`
 `static def make(up:Tree,`
 `left:Boolean):Tree(up, left) {`
 `val x = new Tree(up, left);`
 `x.left = new Tree(x, true);`
 `x.right = new Tree(x, false);`
 `return x;`
`}`
`def makeConstant(mass:Double, force:Double)`
`@Write(Tree{i:UInt ^ self.up$! = this}. (mass, force))`
`@Read(Tree{i:UInt ^ self.up$! = this}.left)`
`{`
 `@Read(Tree{i:UInt ^ self.up$! = this}.right)`
`{`
 `finish {`
 `@Write(Tree{self == this}.mass)`
 `async this.mass = mass;`

 `@Write(Tree{self == this}.force)`
 `async this.force = force;`

 `if (left != null)`
 `async left.makeConstant(mass, force);`
 `if (right != null)`
 `async right.makeConstant(mass,`
 `force);`
 `}`
`}`
`}`

Example 4.3. `class Tree (@ersatz up:Tree, @ersatz left:Boolean) {`
 `var left:Tree(self.up == this, self.left == true);`
 `var right:Tree(self.up == this, self.left == false);`
 `var mass:Double;`
 `var force:Double;`
 `var link:Tree;`
 `static def make(up:Tree,`
 `left:Boolean):Tree(self.up == up, self.left == left) {`
 `val x = new Tree(up, left);`
 `x.left = new Tree(x, true);`
 `x.right = new Tree(x, false);`
 `return x;`
`}`
`def computeForce()`
`@Write(Tree{i:UInt ^ self.up$! = this}.force)`
`@Read(Tree{i:UInt ^ self.up$! = this}.left)`
`{`
 `@Read(Tree{i:UInt ^ self.up$! = this}.right)`
`@Read(Tree.mass)`
`{`
 `finish {`
 `@Write(Tree{self == this}.mass)`
 `async this.force =`
 `(this.mass * link.mass) * G;`

 `if (left != null)`
 `async left.computeForce();`
 `if (right != null)`
 `async right.computeForce();`
 `}`
`}`
`}`

5. Programming Examples

We now demonstrate that several common idioms of concurrency and communication lie in the semantically sequential subset of X10.

6. Implementation Considerations

Registration of accumulators with activities needs to be implemented efficiently. This may require the implementation of an activity stack, with registration information being looked up lazily and cached, rather than pushed eagerly. Also this information is clearly not needed in the body of `async`'s that can be statically analyzed to not contain accumulator operations.

7. Semantics

Featherweight X10 (FX10) is a formal calculus for X10 intended to complement Featherweight Java (FJ). It models imperative aspects of X10 including the concurrency constructs `finish` and `async`.

Overview of formalism

7.1 Syntax

Fig. 2 shows the abstract syntax of FX10. ϵ is the empty statement. Sequencing associates to the left. The block syntax in source code is just `{S}`, however at run-time the block is augmented with (the initially empty) set of locations π representing the write capabilities of the block.

Expression `val x = e; S` evaluates `e`, assigns it to a new variable `x`, and then evaluates `S`. The scope of `x` is `S`.

The syntax is similar to the real X10 syntax with the following difference: FX10 does not have constructors; instead, an object is initialized by assigning to its fields. X10 uses `acc` and not `val` for new accumulator declarations. These are minor simplifications intended to streamline the formal presentation.

$$\begin{array}{c}
\frac{}{\mathbf{e}, \mathbf{H} \rightsquigarrow_{\pi} \mathbf{H}} \text{ (R-EPSILON)} \quad \frac{S, \mathbf{H} \rightsquigarrow_{\pi} S', \mathbf{H}' \mid \mathbf{H}' \mid \Diamond}{(\pi_1) \{S\}, \mathbf{H} \rightsquigarrow_{\pi_2} (\pi_1) \{S'\}, \mathbf{H}' \mid \mathbf{H}' \mid \Diamond \quad \pi = \pi_1, \pi_2} \text{ (R-TRANS)} \\
\frac{}{S \mid S_1, \mathbf{H} \rightsquigarrow_{\pi} S' \mid S_1, \mathbf{H}' \mid \Diamond} \\
\frac{B, \mathbf{H} \rightsquigarrow_{\pi} B', \mathbf{H}' \mid \mathbf{H}' \mid \Diamond}{[\text{clocked}] [\text{finish} \mid \text{async}] B, \mathbf{H} \rightsquigarrow_{\pi} [\text{clocked}] [\text{finish} \mid \text{async}] B', \mathbf{H}' \mid \mathbf{H}' \mid \Diamond} \text{ (R-TRANS-B)} \\
\frac{S, \mathbf{H} \rightsquigarrow_{\pi} S', \mathbf{H}'}{[\text{clocked}] \text{ async } B \mid S, \mathbf{H} \rightsquigarrow_{\pi} [\text{clocked}] \text{ async } B \mid S', \mathbf{H}'} \text{ (R-ASYNC)} \\
\frac{\mathbf{e}, \mathbf{H} \rightsquigarrow 1, \mathbf{H}'}{\text{val } x = \mathbf{e}; S, \mathbf{H} \rightsquigarrow_{\pi} S[1/x], \mathbf{H}'} \text{ (R-VAL)} \quad \frac{1' \notin \text{dom}(\mathbf{H})}{\text{new } C, \mathbf{H} \rightsquigarrow_{\pi} 1', \mathbf{H}[1' \mapsto C()]} \text{ (R-NEW)} \quad \frac{\mathbf{H}(1') = C(\dots) \quad \text{mbody}(\mathbf{m}, C) = \bar{x}.S}{1'.\mathbf{m}(1), \mathbf{H} \rightsquigarrow_{\pi} S[1/\bar{x}, 1'/\text{this}], \mathbf{H}} \text{ (R-INVOKE)} \\
\frac{\mathbf{H}(1) = C(\bar{f} \mapsto 1') \quad g = f_i \mid g \notin \bar{f}}{1.g, \mathbf{H} \rightsquigarrow_{\pi} 1'_i, \mathbf{H} \mid \Diamond} \text{ (R-ACCESS)} \quad \frac{\mathbf{H}(1) = C(F)}{1.f = 1', \mathbf{H} \rightsquigarrow_{\pi} \mathbf{H}[1 \mapsto C(F[\bar{f} \mapsto 1'])]} \text{ (R-ASSIGN)} \\
\frac{1 \notin \text{dom}(\mathbf{H})}{(\pi_1) \{\text{acc } x = \text{new Acc}(r, z); S\}, \mathbf{H} \rightsquigarrow_{\pi} (\pi_1, 1) \{S[1/x]\}, \mathbf{H}[1 \mapsto \text{Acc}(r, z)]} \text{ (R-ACC-N)} \\
\frac{}{\mathbf{e} \downarrow} \text{ (R-Q-CA)} \quad \frac{B \downarrow}{\text{clocked async } B \downarrow} \text{ (R-Q-CA)} \quad \frac{S \downarrow}{(\pi) \{S\} \downarrow} \text{ (R-Q-B)} \quad \frac{S_1 \downarrow \quad S_2 \downarrow}{S_1 \mid S_2 \downarrow} \text{ (R-Q-S)} \\
\frac{\mathbf{H}(a) = \text{Acc}(r, v) \quad S \downarrow \quad a \in \pi_1 \mid a \notin \pi_1}{(\pi_1) \{S \text{ val } x = a(); S_1\}, \mathbf{H} \rightsquigarrow_{\pi} (\pi_1) \{S \mid S_1[v/x]\}, \mathbf{H}' \mid \Diamond} \text{ (R-ACC-R)} \quad \frac{\mathbf{H}(a) = \text{Acc}(r, v) \quad w = r(v, p) \quad a \in \pi \mid a \notin \pi}{a \leftarrow p, \mathbf{H} \rightsquigarrow_{\pi} \mathbf{H}[a \mapsto \text{Acc}(r, w)] \mid \Diamond} \text{ (R-ACC-W)} \\
\frac{S_1 \downarrow \quad S, \mathbf{H} \rightsquigarrow_{\pi} S', \mathbf{H}'}{S_1 \mid S, \mathbf{H} \rightsquigarrow_{\pi} S_1 \mid S', \mathbf{H}'} \text{ (R-Q)} \\
\frac{B \overset{+}{\rightsquigarrow} B'}{\text{clocked async } B \overset{+}{\rightsquigarrow} \text{clocked async } B'} \text{ (R-ADV-S)} \quad \frac{}{(\text{async} \mid [\text{clocked}] \text{ finish}) B \overset{+}{\rightsquigarrow} (\text{async} \mid [\text{clocked}] \text{ finish}) B \quad \text{advance}; S \overset{+}{\rightsquigarrow} S} \text{ (R-ADV-A, CAF)} \\
\frac{S \overset{+}{\rightsquigarrow} S'}{(\pi) \{S\} \overset{+}{\rightsquigarrow} (\pi) \{S'\}} \text{ (R-ADV-B)} \quad \frac{S_1 \overset{+}{\rightsquigarrow} S'_1 \quad S_2 \overset{+}{\rightsquigarrow} S'_2}{S_1 \mid S_2 \overset{+}{\rightsquigarrow} S'_1 \mid S'_2} \text{ (R-ADV-S)} \quad \frac{B \overset{+}{\rightsquigarrow} B'}{\text{clocked finish } B, \mathbf{H} \rightsquigarrow_{\pi} \text{clocked finish } B', \mathbf{H}} \text{ (R-ADV)}
\end{array}$$

Figure 3. FX10 Reduction Rules ($S, \mathbf{H} \rightsquigarrow_{\pi} S', \mathbf{H}' \mid \mathbf{H}'$ and $\mathbf{e}, \mathbf{H} \rightsquigarrow_{\pi} 1, \mathbf{H}'$).