Probabilistic Techniques and Randomized Algorithms

Homework 2024-2025

Problem 1 (1.0)

Provide the sample points of the $G_{3,1/3}$ sample space and the probability of each point.

Problem 2 (1.0)

In $G_{12,p}$ random graphs find a) the distribution and the average degree of any vertex b) the distribution and the expectation of the number of edges.

Problem 3 (1.0)

Prove that if there is a real p, $0 \le p \le 1$, so that

$$\binom{n}{k} p^{\binom{k}{2}} + \binom{n}{t} (1-p)^{\binom{t}{2}} < 1,$$

then the Ramsey number $r(k,t)$ satisfies $r(k,t) > n$.

Problem 4 (1.0)

Prove that, for every integer n, there exists a coloring of the edges of the complete graph K_n by two colors so that the total number of monochromatic K_4 subgraphs is at most $\binom{n}{4} 2^{-5}$.

Problem 5 (1.5)

Find the threshold probability for the existence, with high probability, of a K_5 in $G_{n,p}$.

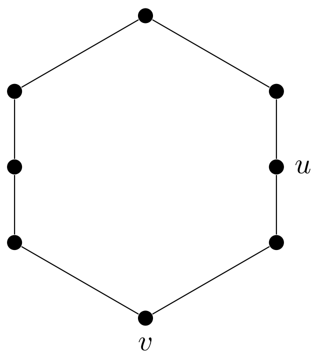
Problem 6 (1.5)

Prove that any $k-$SAT formula in which no variable appears in more than $\frac{2^{k-2}}{k}$ clauses is satisfiable.

Problem 7 (1.0)

Provide the details of the proof that Top-in-at-Random converges to a uniform distribution. Why this is important in the card shuffling context?

Problem 8 (2.0)



For a symmetric random walk on the following graph a) find the stationary distribution, if one exists b) estimate the expected time to move from u to v, c) estimate the expected time between the successive visits to u. For all the above questions use i) the general Markov Chain methods of lecture 9, ii) the random walk tools of lecture 10.