Problem Set 7

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ECON815: Topics in Microeconomics

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Discrete-Time Dynamic Model

1 Firm Problem

A utility-maximizing firm, in this case an infinitely-lived hospital, gets utility from the profits they generate and the quality of care they provide.

$$\begin{aligned} &\max_{\{I_t+1\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U\left(\pi_t, k_t\right) \\ &\text{s.t. } I_{t+1} \leq R_t\left(k_t\right) - C_t + I_0 + w_0 \end{aligned} \tag{1}$$

The infinitely-lived hospital chooses only their investment in equipment. Prices are set exogenously (ex: by the government). For simplicity, revenue is determined solely as a function of quality ($U'(k) > 0, U''(k) < 0$). The hospital has a fixed cost every period (from operating at capacity). Investment affects quality ($k' > 0, k'' < 0$), but equipment depreciates over time, lowering quality. Depreciation δ has a constant factor ρ for the aging of equipment, and a stochastic factor ϵ for equipment breaking. The value of money is constant.

1.1 Population

There is one hospital in this model. There are enough consumers (patients) with great enough demand that the hospital operates at capacity in every period. Demand does not affect prices.

1.2 Preferences

The hospital has a time preference, $0 < \beta < 1$, and weight preferences α and γ for profit and quality respectively.

Utility: $U(\pi_t, k_t) \Rightarrow ln(\alpha\pi_t + \gamma k_t)$

log utility provides these features:

- $U'(\cdot) > 0, U''(\cdot) < 0$
- Inada conditions: $U'(0) = \infty$

1.3 Technology

There is no production in this model. Revenue is merely a function of quality, and quality is a measure of investment in equipment stock.

Profit: $\pi_t = R_t(k_t) - C_t - I_{t+1}$

Revenue: $R_t(k_t) \equiv$ revenue as a function of quality.

Cost: C_t is a fixed cost of operting at full capacity in a period.

Quality: $k_t \equiv$ depreciated $(\delta_t = \rho + \epsilon_t)$ stock $= I_t + \sum_0^{t-1}(1 - \delta_t) I_{t-1}$

The hospital has full information, including the exogenous price. However, the hopsital does not know exactly what the depreciation on its equipment will be. Depreciation has a stochastic component.

Depreciation: $\delta_t = \rho + \epsilon_t$ where $\epsilon_t \sim U(0, 0.1)$

$\rho \equiv$ a constant factor of depreciation

1.4 Endowments

$w_0 > 0 \equiv$ initial monetary endowment from donor

$I_0 > 0 \equiv$ initial equipment stock from donor

The donor only exists to provide the initial endowments. They never expect any return.

2 Bellman Equation

The Bellman equation for the hopsital's problem is given by:

$$V(C, K) = \max_{I'} ln(\alpha\pi_t + \gamma k_t) + \beta E[V(C', k')] \tag{2}$$

The state variables are:

$C \Rightarrow$ the fixed cost of operating at full capacity

$k \Rightarrow$ the hospital's quality as determined by their prior depreciated equipment investment decisions

The control variables are:

$I \Rightarrow$ the hospital's investment in equipment

The hospital is choosing how much to invest, which dictates what the quality of their care is and what their profits are, dependent on depreciation.

3 The Original Problem...

In my first attempt of Problem Set 7 and subsequently Problem Set 8, I had a much more complicated model. Despite my attempts to structure the model and paper, I was not able to fully express everything necessary. This made translating the model to code impossible. After struggling for a period with the original problem, and then failing to complete the assignment, I decided to approach something much more simple. I recognize that this is not my forte, but it is representative of my best work given the time constraints. The original problem, in words, is as follows...

There is a fixed population, *M*, of three types of patients each with their own insurance structures. These are Medicare, private insurance, and the uninsured. A portion of each type of patient is ill and requires medical care. Each patient *i* of an insurance type has a different income distribution and medical care repayment structure. Each hospital *j* sets their own price. The utility for patient *i* of type *T* at hospital *j* is equal to that hospital's quality plus (that patient's income minus what the price they pay) plus an error term.

To calculate market equilibrium, I would need to know what hospital profits are. These would be dependent on market shares. Market shares would be calculated using the probability that a utility-maximizing patient chooses to receive care from a hospital.

There are N_t hospitals in the market. N_t varies period to period based on hospitals' entry and exit decisions. Hospitals can be of two types: for-profit and non-profit. Each hospital has some quality, fixed cost, and variable cost that varies with the quality of the hospital. Non-profit hospital get utility from profits, like for-profit hospitals, but also from their quality of care. For-profit hospitals must pay an income tax.

At the start of each time period, hospitals may choose to exit the market, liquidating their equipment in the process. Their scrap value is drawn at random from a uniform distribution. Liquidation is considered net profit, so for-profit hospitals only keep the after-tax portion. Each hospital that chooses to remain in the market then chooses how much to invest. Investment increases efficiency and quality is a function of efficiency. Efficiency would be concave so that investment does not lead to unbounded efficiency. Gained efficiency from investment is also stochastic. After investment decisions are made, two hospitals may choose to enter the market, one non-profit and one for-profit. Each receives a sunk cost of entry from a distribution.