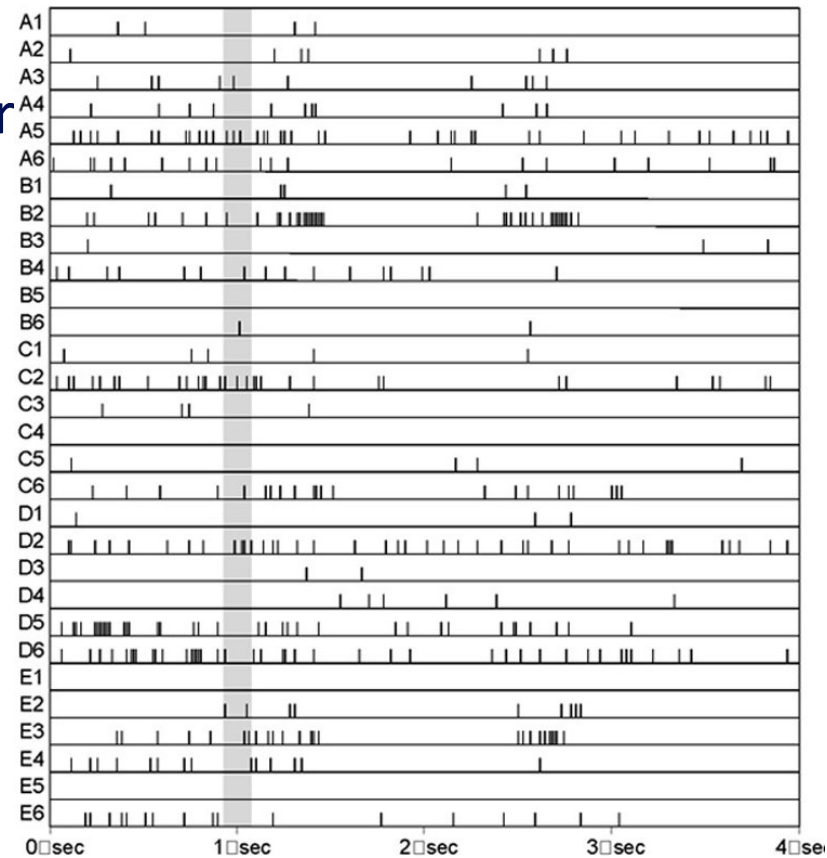


# Spiking Neurons

# Spike Train

- Action potentials convey information through their timing or frequency
- Spikes ← Abstraction from action potential:
  - ignore the brief duration of an action potential.
  - ignore the shape and amplitudes
  - Just keep track on when neurons are fired.
- High-dimensional time series
  - the number (frequency) and timing of spikes matter

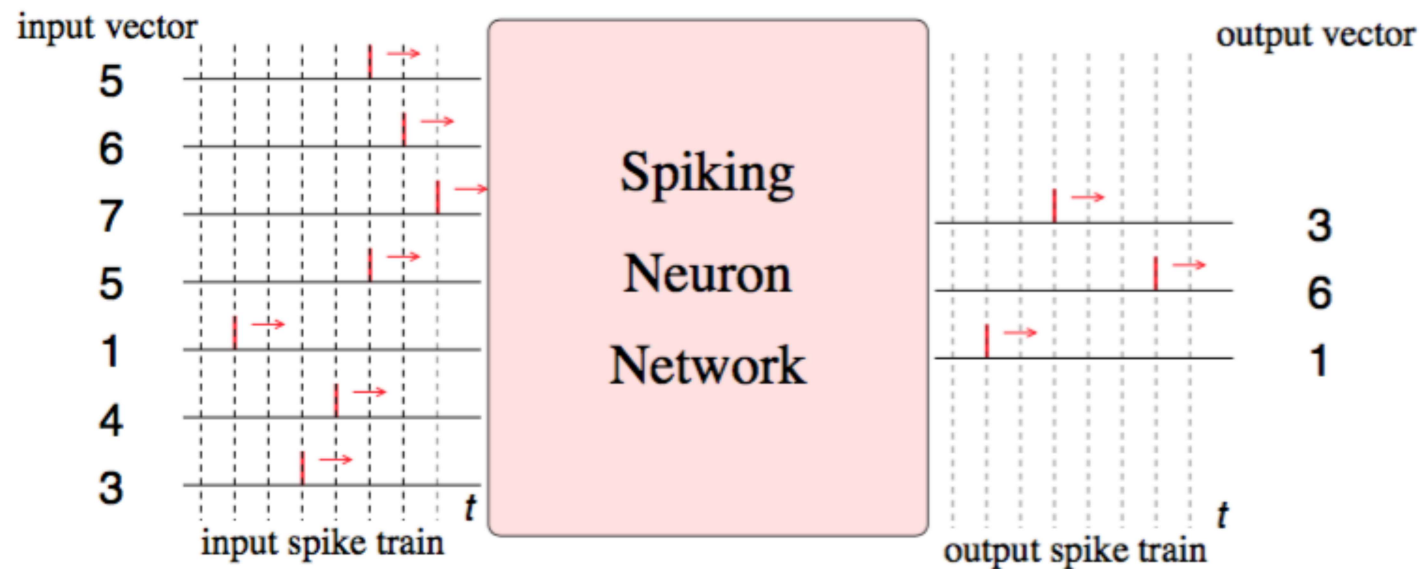


Spike trains from 30 neurons in the visual cortex of a monkey.

# Towards SNN

- In the traditional model, neuron ‘fires’ at each iteration in the most cases.
- In the traditional model, the output is real-valued; no coding scheme considered.
- The investigations in neuroscience suggest that the information is transmitted and processed as spike trains in the brain.
- It is possible to make ANN much more biological plausible?

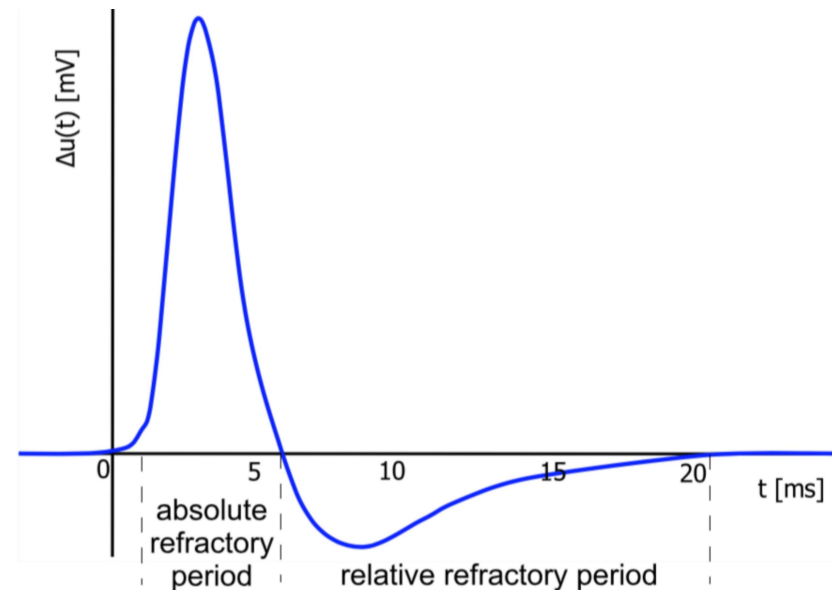
# From ANN to SNN



Encoding: An integer value is encoded as the 'time-to-first-spike'

# Elements of Neural Dynamics

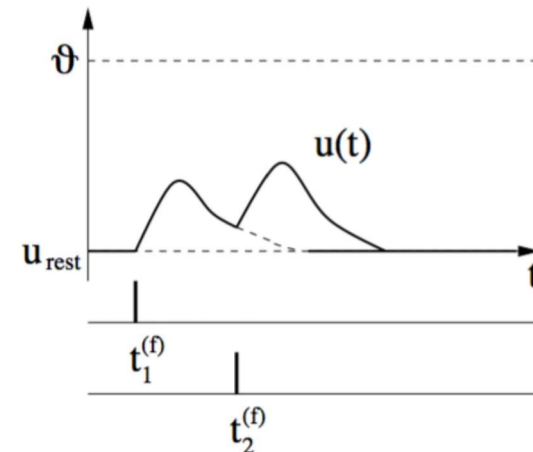
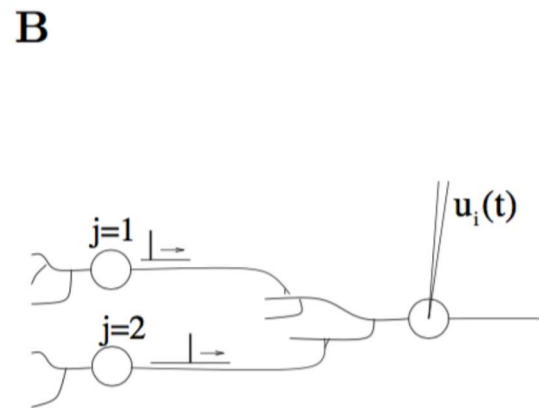
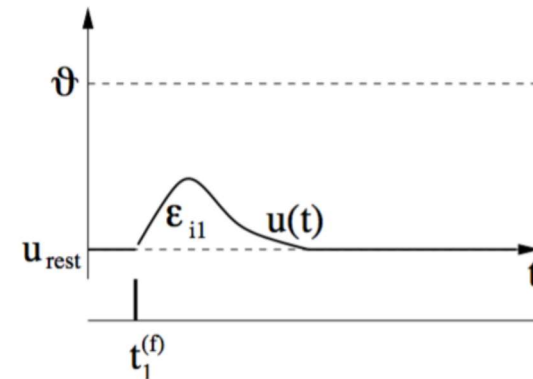
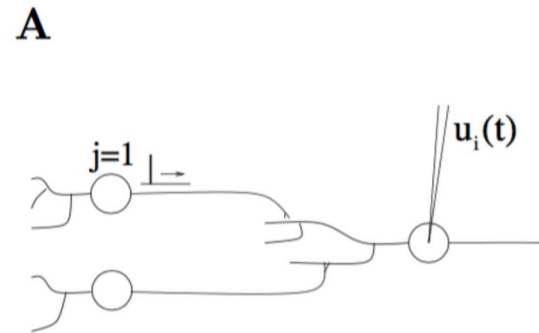
- Absolute refractory period
  - Minimal temporal distance between two spikes
- Relative refractory period
  - where it is difficult, but not impossible to excite an action potential.
- Negative Spike-after-potential



Dynamics of spike firing

# Elements of Neural Dynamics

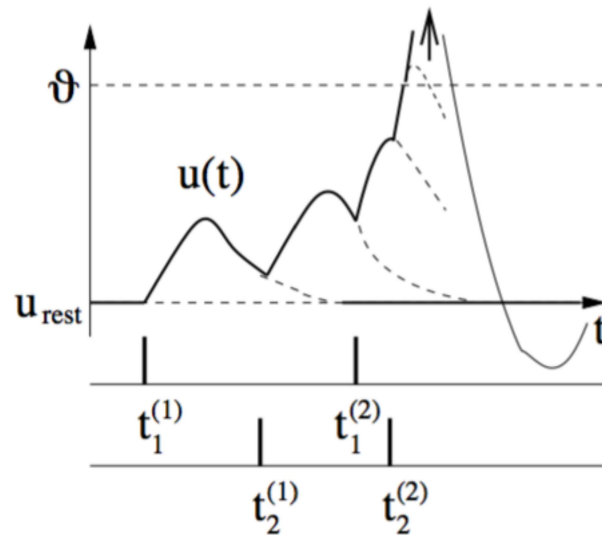
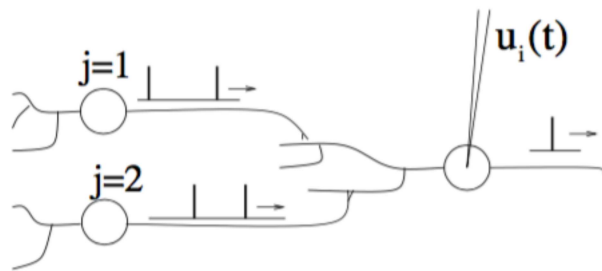
- Postsynaptic potential (PSP)
  - Transient impact of incoming spikes
- Attenuate over time
- Additive in this model



# Elements of Neural Dynamics

- A spike is generated if the cumulated membrane potential reaches the threshold from below.
- Generalization of threshold unit

C



# Spike Response Model

- Modeling the membrane potential over time  $u(t)$ :
  - accumulation of PSP + internal membrane potential
  - $\hat{t}$ : the last firing time
  - $t_i^{(f)}$ : the presynaptic neuron fires at  $t_i^{(1)}, t_i^{(2)}, \dots, t_i^{(f)}$

$$u(t) = \underbrace{\eta(t - \hat{t})}_{\text{action potential}} + \underbrace{\sum_i w_i \sum_f \epsilon_i(t - t_i^{(f)})}_{\text{Input}}$$

- Kernel function:
  - Spike emission  $\eta(t)$  and
  - postsynaptic potential  $\epsilon(t)$
- Generalization of threshold unit
- Synaptic plasticity  $\rightarrow$  weights

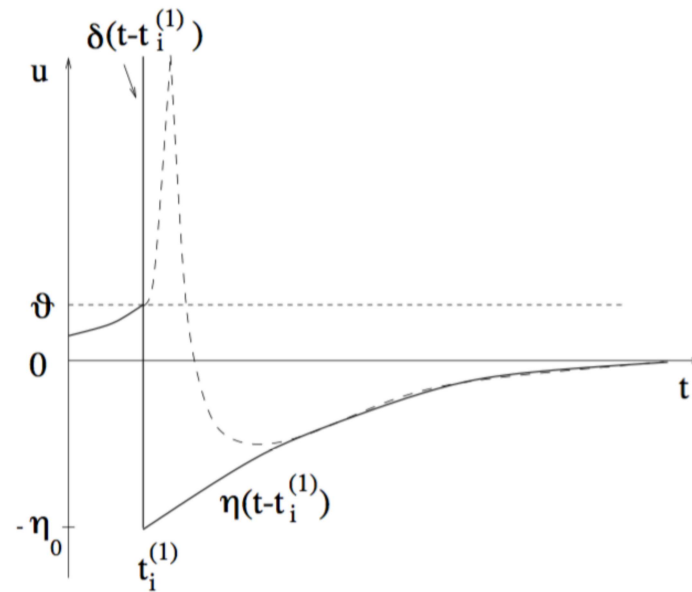


# Spike Response Model

- Common Kernel form:

-  $\eta(t)$ :

$$\eta(t - t_i^{(f)}) = \begin{cases} 1/\Delta t & \text{for } 0 < t - t_i^{(f)} < \Delta t \\ -\eta_0 \exp\left(-\frac{t - t_i^{(f)}}{\tau}\right) & \text{for } \Delta t < t - t_i^{(f)} \end{cases}$$



# Spike Response Model

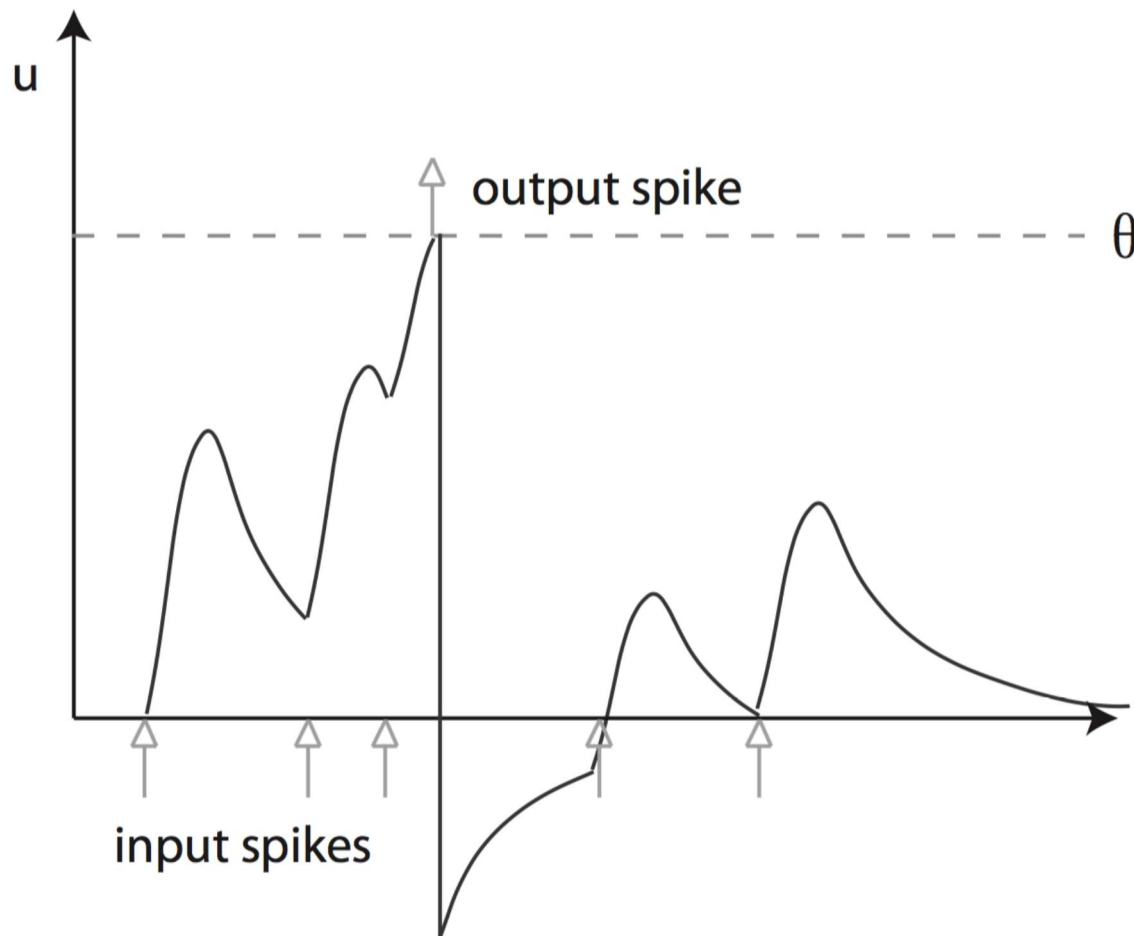
- Modeling the membrane potential over time  $u(t)$ :
  - accumulation of PSP + internal membrane potential
  - $\hat{t}$ : the last firing time
  - $t_i^{(f)}$ : the presynaptic neuron fires at  $t_i^{(1)}, t_i^{(2)}, \dots, t_i^{(f)}$

$$u(t) = \eta (t - \hat{t}) + \sum_i w_i \sum_f \epsilon_i(t - t_i^{(f)})$$

- When to fire? Reaches the threshold from below

$$u(t) = v \bigwedge \frac{du(t)}{dt} > 0 \xrightarrow{\text{yields}} \text{spike at time } t$$

# Spike Response Model



# Other models...

- Hodgkin-Huxley Model

$$C \frac{du}{dt} = -g_{Na} m^3 h (u - E_{Na}) - g_K n^4 (u - E_K) - g_L (u - E_L) + I(t)$$

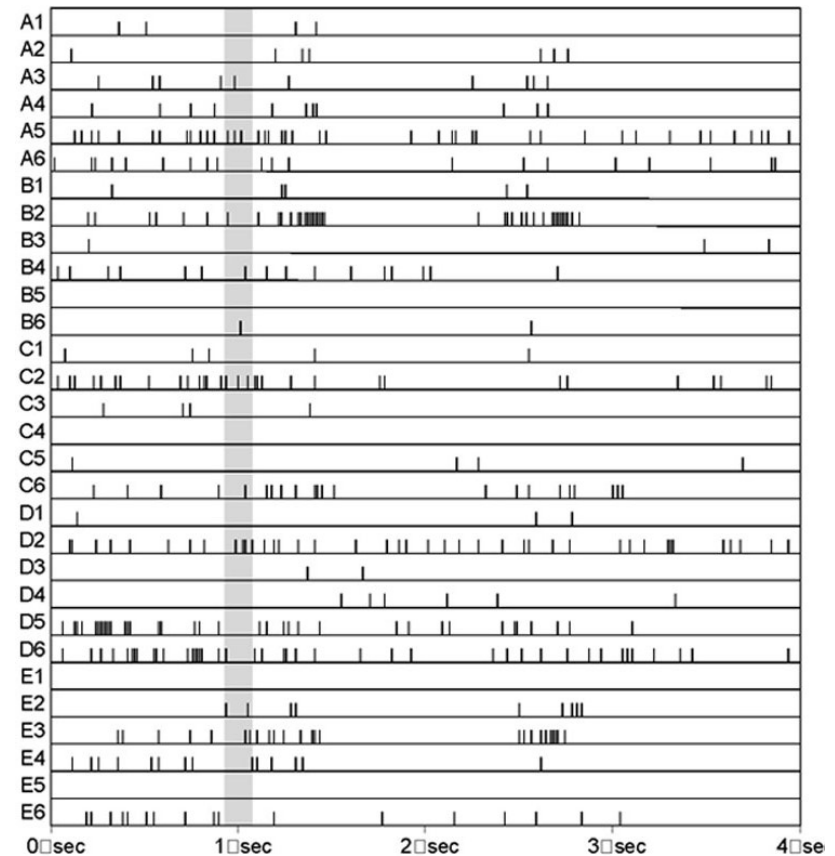
$$\tau_n \frac{dn}{dt} = -[n - n_0(u)], \quad \tau_m \frac{dm}{dt} = -[m - m_0(u)], \quad \tau_h \frac{dh}{dt} = -[h - h_0(u)]$$

- Derived from dynamics in electrical circuits
  - Match the biological data reasonably well
  - Complexity is quite high
- 
- Integrate-and-fire model
    - Derived from Hodgkin-Huxley model
    - More computational tractable

# Neural Coding

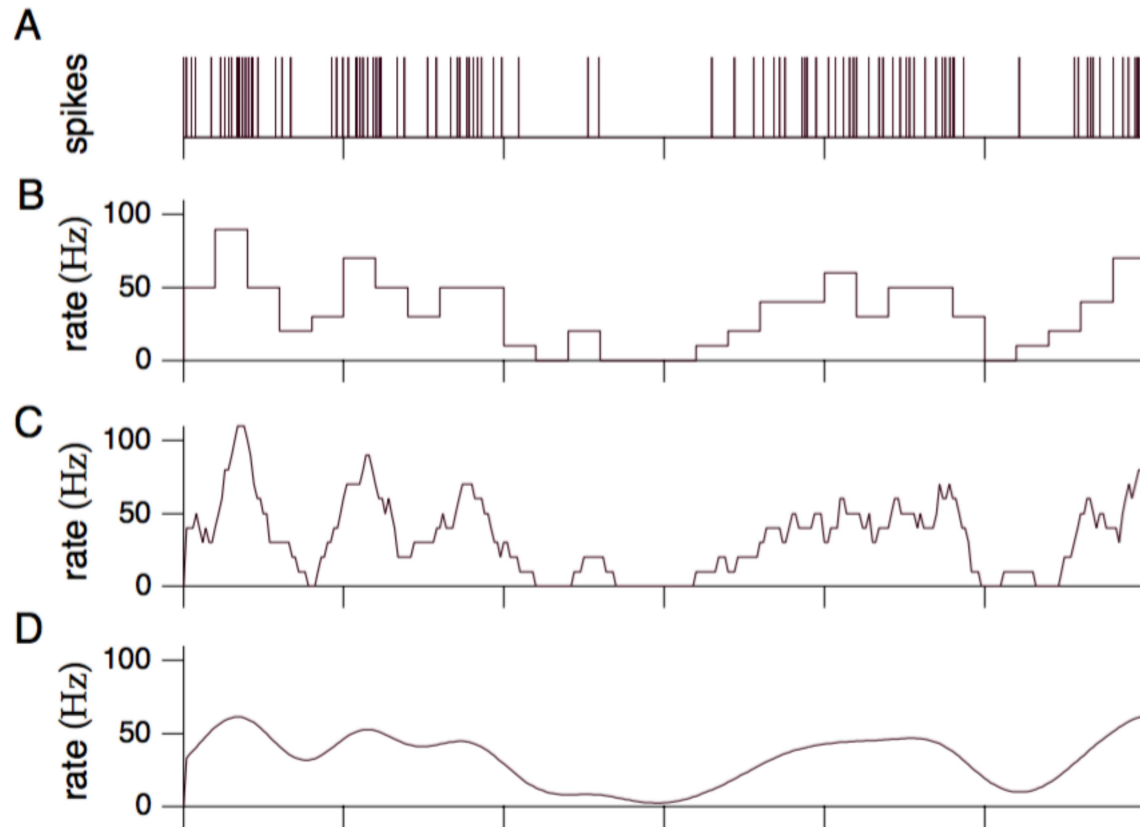
# Neural coding

- How can we interpret spike trains?
  - Rate coding → average firing rate
  - Temporal coding → temporal patterns in spike trains
  - Population coding → joint patterns of a collection of neurons
  - ...



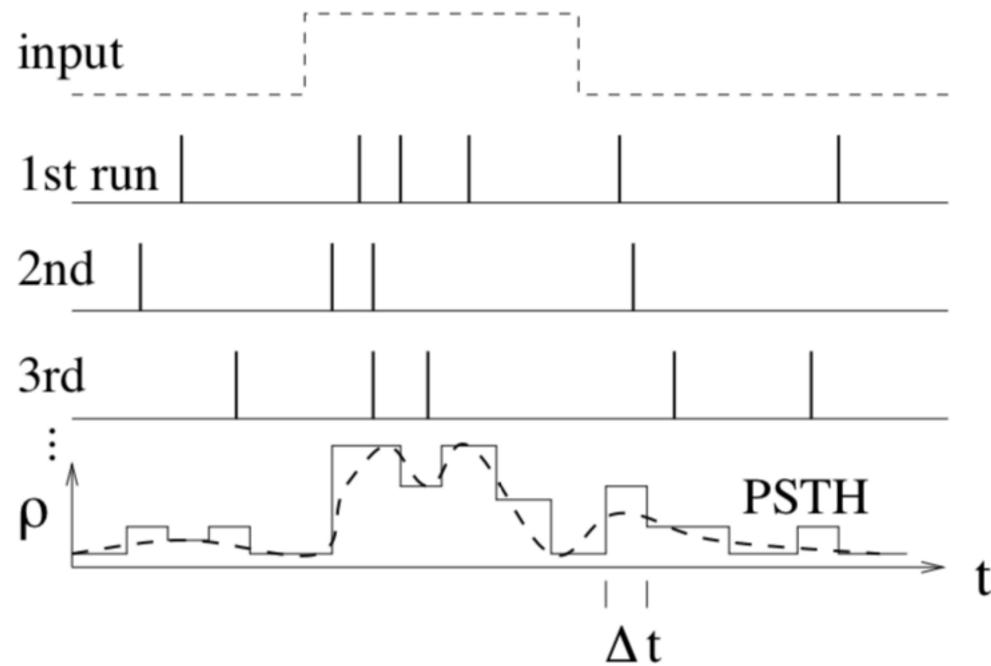
# Spike-Rate coding: Spike Count

B: discrete-time binning; C: Moving average; D: Gaussian window



# Spike-Rate coding: Spike Density

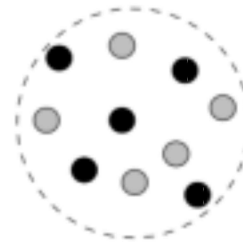
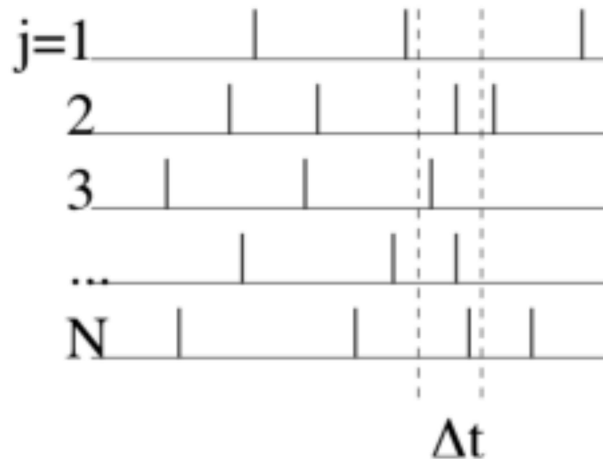
rate = average over several runs  
(single neuron, repeated runs)





# Spike-Rate coding: Population Activity

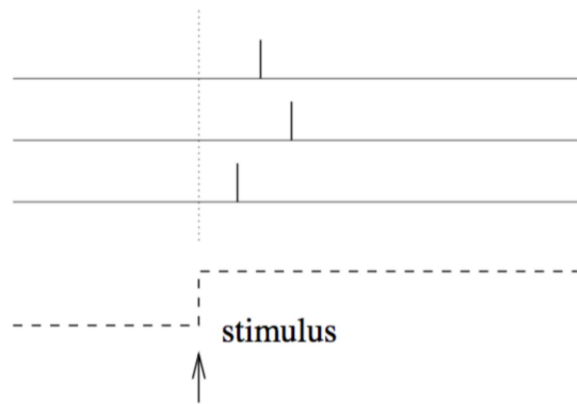
rate = average over pool of equivalent neurons  
(several neurons, single run)



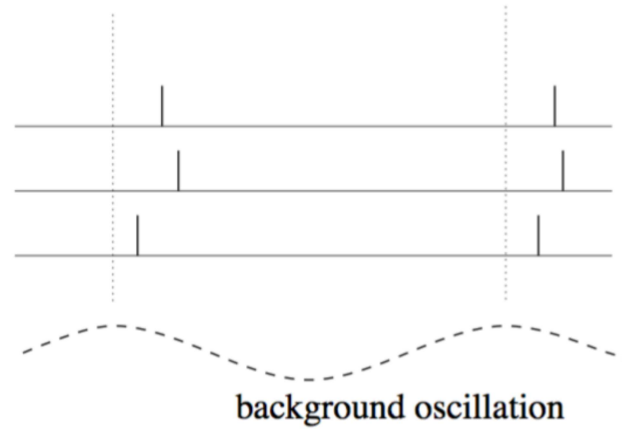
activity

$$A = \frac{1}{\Delta t} \frac{n_{\text{act}}(t; t + \Delta t)}{N}$$

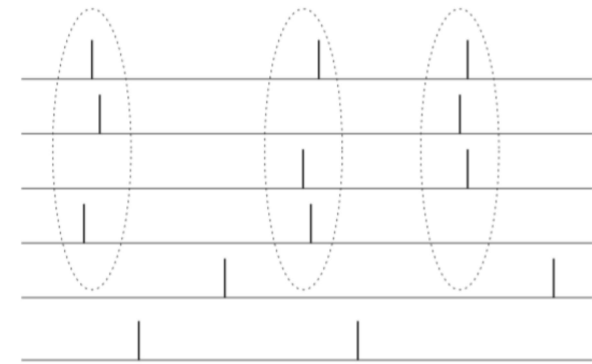
# Temporal coding



Time-to-first spike



Phase



Temporal correlation

# Neural coding

