Ant colony Optimization Algorithms : Introduction and Beyond

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Outline

What is Ant Colony Optimization?

ACO

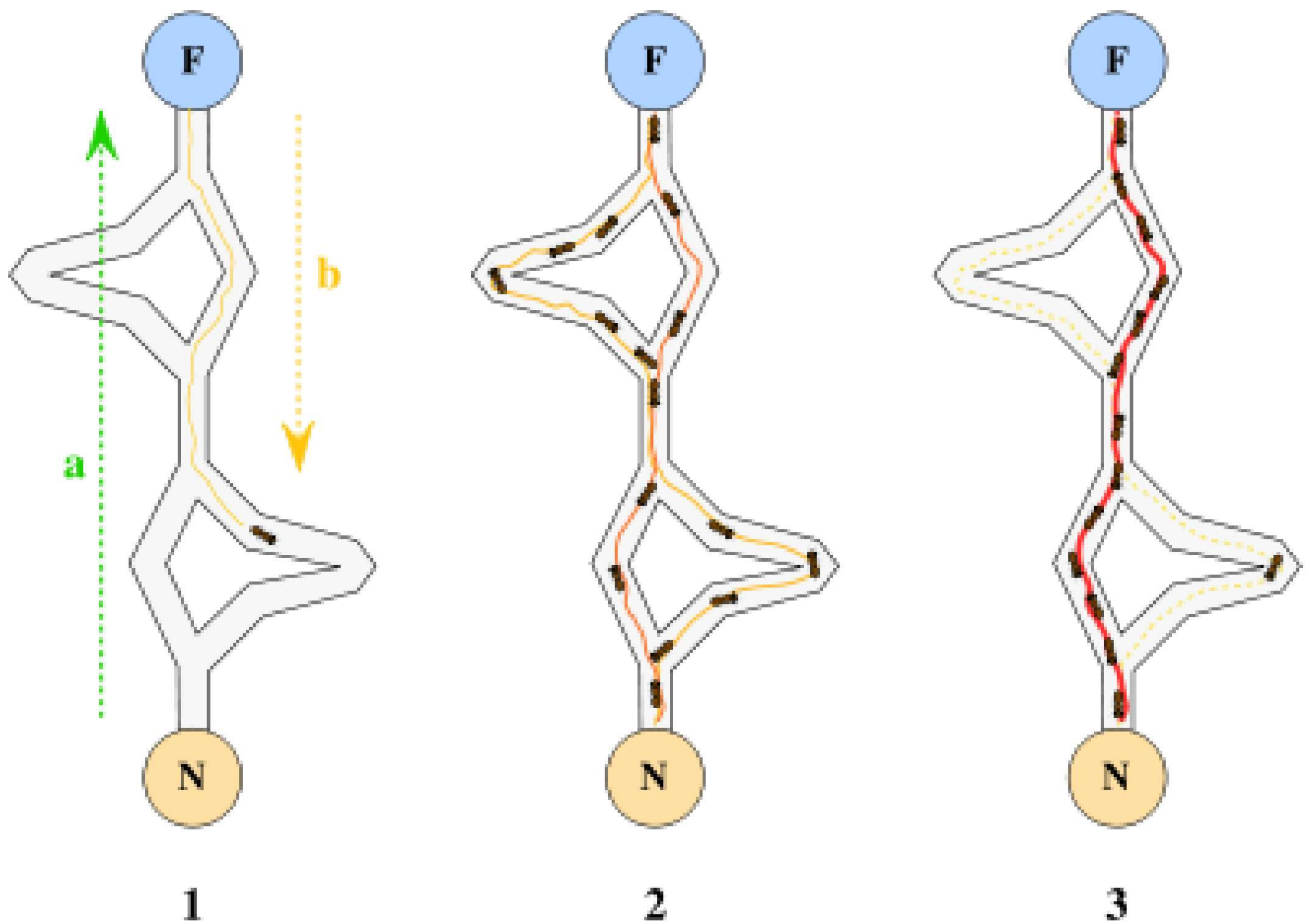
- Probabilistic technique.
- Searching for optimal path in the graph based on behaviour of ants seeking a path between their colony and source of food.
- Meta-heuristic optimization

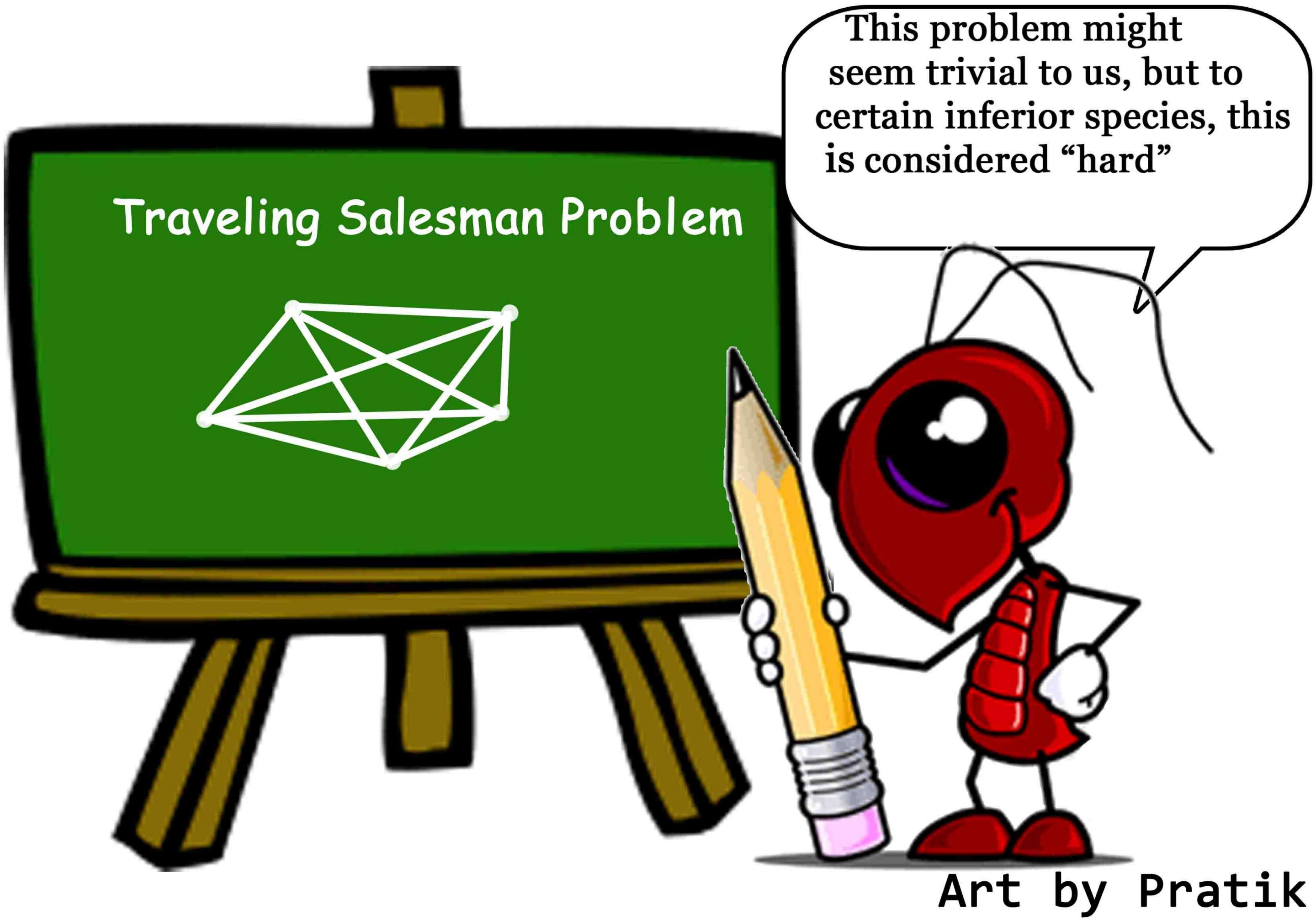
ACO Concept

Overview of the Concept

- Ants navigate from nest to food source. Ants are blind!
- Shortest path is discovered via pheromone trails.
- Each ant moves at random
- Pheromone is deposited on path
- More pheromone on path increases probability of path being followed

Ant Colony Optimization




Traveling Salesman Problem
This problem might seem trivial to us, but to certain inferior species, this is considered “hard”
Art by Pratik

ACO System

Overview of the System

- Virtual trail accumulated on path segments
- Path selected at random based on amount of "trail" present on possible paths from starting node

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- Virtual trail accumulated on path segments
- Path selected at random based on amount of "trail" present on possible paths from starting node
- Ant reaches next node, selects next path
- Continues until reaches starting node
- Finished tour is a solution.
- Tour is analyzed for optimality

Meta-heuristic

1. Heuristic method for solving a very general class of computational problems by combining user-given heuristics in the hope of obtaining a more efficient procedure.

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1. Heuristic method for solving a very general class of computational problems by combining user-given heuristics in the hope of obtaining a more efficient procedure.
2. ACO is meta-heuristic
3. Soft computing technique for solving hard discrete optimization problems

History

1. Ant System was developed by Marco Dorigo (Italy) in his PhD thesis in 1992.
2. Max-Min Ant System developed by Hoos and Stützle in 1996
3. Ant Colony was developed by Gambardella Dorigo in 1997

The ACO Meta-heuristic

ACO

Set Parameters, Initialize pheromone trails

SCHEDULE ACTIVITIES

1. Construct Ant Solutions
2. Daemon Actions (optional)
3. Update Pheromones

Virtual trail accumulated on path segments

ACO - Construct Ant Solutions

ACO - Construct Ant Solutions

An ant will move from node i to node j with probability

$$p_{i,j} = \frac{(\tau_{i,j}^{\alpha})(\eta_{i,j}^{\beta})}{\sum(\tau_{i,j}^{\alpha})(\eta_{i,j}^{\beta})}$$

where

$\tau_{i,j}$ is the amount of pheromone on edge i,j

α is a parameter to control the influence of $\tau_{i,j}$

$\eta_{i,j}$ is the desirability of edge i,j (typically $1/d_{i,j}$)

β is a parameter to control the influence of $\eta_{i,j}$

ACO - Pheromone Update

ACO - Pheromone Update

Amount of pheromone is updated according to the equation

$$\tau_{i,j} = (1 - \rho)\tau_{i,j} + \Delta\tau_{i,j}$$

where
$\tau_{i,j}$ is the amount of pheromone on a given edge i, j
ρ is the rate of pheromone evaporation
$\Delta\tau_{i,j}$ is the amount of pheromone deposited, typically given by

$$\Delta\tau_{i,j}^{k} = \begin{cases} 1/L_k & \text{if ant } k \text{ travels on edge } i,j \\ 0 & \text{otherwise} \end{cases}$$

where L_k is the cost of the k^{th} ant's tour (typically length).

ACO

ACO

- Many special cases of the ACO metaheuristic have been proposed.
- The three most successful ones are: Ant System, Ant Colony System (ACS), and MAX-MIN Ant System (MMAS).
- For illustration, example problem used is Travelling Salesman Problem.

ACO - Ant System

ACO - Ant System

- First ACO algorithm to be proposed (1992)
- Pheromone values are updated by all the ants that have completed the tour.

$$\tau_{ij} \leftarrow (1-\rho)\cdot\tau_{ij} + \sum_{k=1}^{m}\Delta\tau_{ij}^{k},$$

where
ρ is the evaporation rate
m is the number of ants
$\Delta\tau_{ij}^{k}$ is pheromone quantity laid on edge (i,j) by the k^{th} ant

$$\Delta\tau_{i,j}^{k} = \begin{cases} 1/L_k & \text{if ant } k \text{ travels on edge } i,j \\ 0 & \text{otherwise} \end{cases}$$

where L_k is the tour length of the k^{th} ant.

ACO - Ant Colony System

ACO - Ant Colony System

- First major improvement over Ant System
- Differences with Ant System:
  1. Decision Rule - Pseudorandom proportional rule
  2. Local Pheromone Update
  3. Best only offline Pheromone Update

ACO - Ant Colony System

ACO - Ant Colony System

- Ants in ACS use the pseudorandom proportional rule
- Probability for an ant to move from city i to city j depends on a random variable q uniformly distributed over $[0, 1]$, and a parameter q_0.
- If $q \leq q_0$, then, among the feasible components, the component that maximizes the product $\tau_{il}\eta_{il}^{\beta}$ is chosen, otherwise the same equation as in Ant System is used.
- This rule favours exploitation of pheromone information

ACO - Ant Colony System

ACO - Ant Colony System

- Diversifying component against exploitation: local pheromone update.
- The local pheromone update is performed by all ants after each step.
- Each ant applies it only to the last edge traversed:

$$\tau_{ij} = (1 - \varphi) \cdot \tau_{ij} + \varphi \cdot \tau_0$$

where
$\varphi \in (0, 1]$ is the pheromone decay coefficient
τ_0 is the initial value of the pheromone (value kept small *Why?*)

ACO - Ant Colony System

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- Best only offline pheromone update after construction
- Offline pheromone update equation

$$\tau_{ij} \leftarrow (1-\rho)\cdot\tau_{ij} + \rho\cdot\Delta\tau_{ij}^{best}$$

where

$$\tau_{ij}^{best} = \begin{cases} 1/L_{best} & \text{if best ant } k \text{ travels on edge } i,j \\ 0 & \text{otherwise} \end{cases}$$

- L_{best} can be set to the length of the best tour found in the current iteration or the best solution found since the start of the algorithm.

ACO - MAX-MIN Ant System

ACO - MAX-MIN Ant System

- Differences with Ant System:
  1. Best only offline Pheromone Update
  2. Min and Max values of the pheromone are explicitly limited
     - τ_{ij} is constrained between τ_{min} and τ_{max} (explicitly set by algorithm designer).
     - After pheromone update, τ_{ij} is set to τ_{max} if $\tau_{ij} > \tau_{max}$ and to τ_{min} if $\tau_{ij} < \tau_{min}$