



# Tutorial 2

## K-Map

COMP2120B Computer organization

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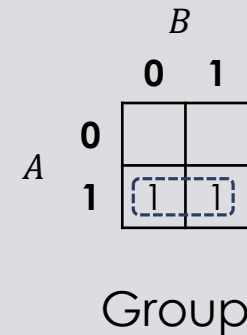
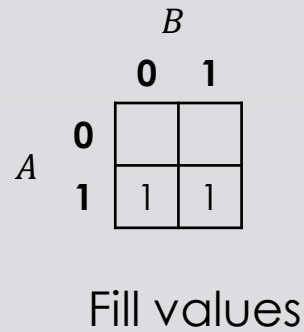
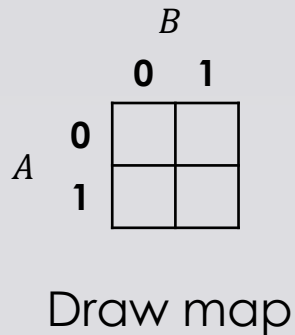
# Overview



- Karnaugh Map (K-Map) is a method of representing Boolean function of up to 4 variables.
  - It provides an easy way to simplify Boolean function in the sum-of-product form.

# Steps

$A$	$B$	$F = AB + A\bar{B}$
0	0	0
0	1	0
1	0	1
1	1	1



Result:  $F = A$

We will examine these steps one by one.

# Maps



Always 1-bit difference for adjacent codes

2 variables

		<i>B</i>	
		0	1
<i>A</i>	0		
	1		

3 variables

		<i>BC</i>			
		00	01	11	10
<i>A</i>	0				
	1				

4 variables

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00				
	01				
	11				
	10				

Note the order

Alternative forms

<i>AB</i>				
00	01	11	10	

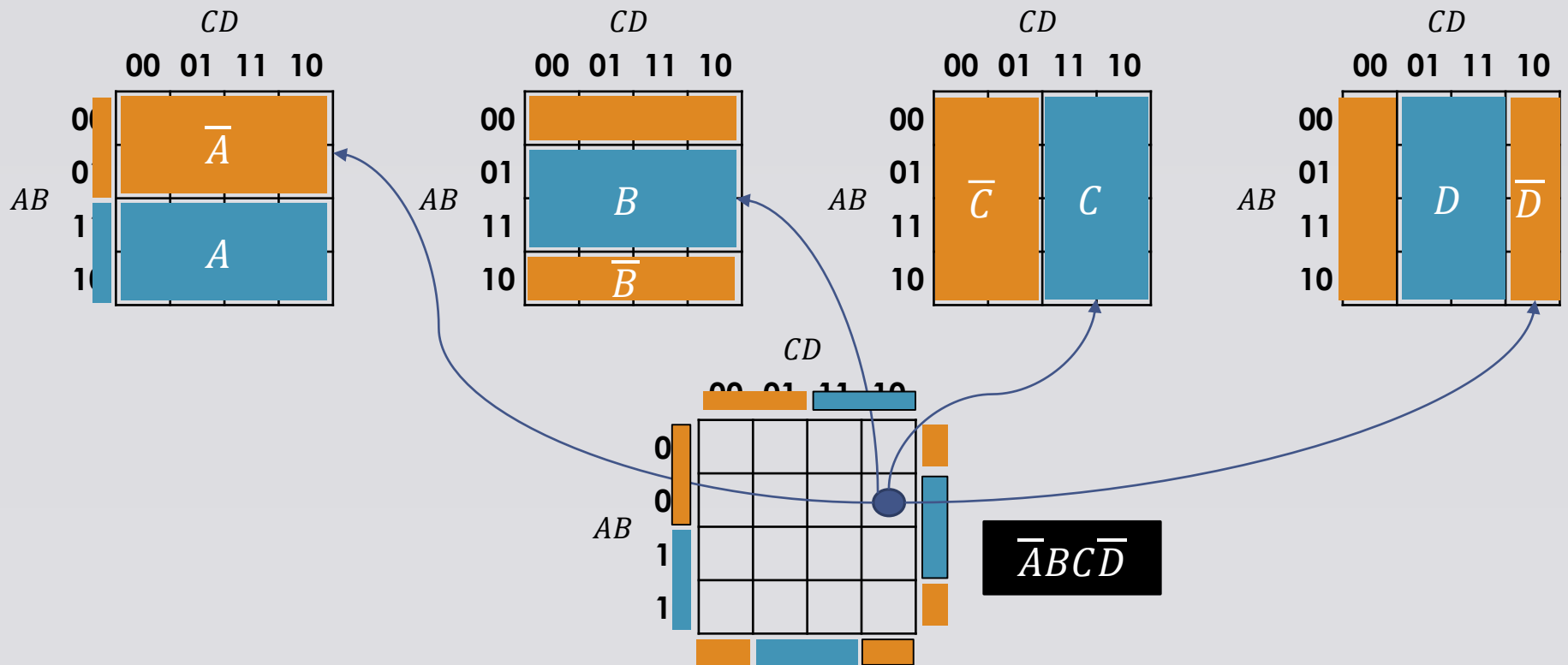
		<i>C</i>	
		0	1
<i>AB</i>	00		
	01		
	11		
	10		

This cell represents the value of  $\bar{A}B\bar{C}\bar{D}$ :

$AB$  as 01  $\rightarrow \bar{A}B$   
 $CD$  as 10  $\rightarrow \bar{C}\bar{D}$

# More visualization

- Each variable split the map in half in a different way



# Fill in values

- It is straight-forward if we already have a truth table.

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00		1	1	1
	01			1	1
	11	1	1	1	1
	10		1	1	

- You may omit the 0's.
- However, in many cases, you have to derive a truth table yourselves.

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	$F(A, B, C, D)$
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

# Exercise: truth table



- Construct the truth table for Boolean function  $F(A,B,C,D)$  as specified below:

$$F(A, B, C, D) = \begin{cases} 1, & 1 \text{ is the majority} \\ 0, & 0 \text{ is the majority} \\ d, & \text{otherwise} \end{cases}$$

- Where  $d$  stands for “don’t care”.
- Draw the K-Map afterwards.

# Truth table: solution

$A$	$B$	$C$	$D$	$F(A,B,C,D)$
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	d
0	1	0	0	0
0	1	0	1	d
0	1	1	0	d
0	1	1	1	1
1	0	0	0	0
1	0	0	1	d
1	0	1	0	d
1	0	1	1	1
1	1	0	0	d
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

	$CD$			
	00	01	11	10
$AB$ 00			d	
01		d	1	d
11	d	1	1	1
10		d	1	d

You may omit the 0's.  
You **must** keep the d's



# Grouping – group size



- Grouping size must be power of 2 (1, 2, 4, 8, or 16)
- Groups must be rectangular.

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00		1	1	1
	01			1	1
	11	1	1	1	1
	10		1	1	

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00		1	1	1
	01			1	1
	11	1	1	1	1
	10		1	1	

Valid

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00		1	1	1
	01			1	1
	11	1	1	1	1
	10		1	1	

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00		1	1	1
	01			1	1
	11	1	1	1	1
	10		1	1	

Invalid

# Group size

What is the relationship between group size and the number of variables in a term?



- A valid group represents a single term (you cannot derive a single term from an invalid group)

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00		1	1	1
	01			1	1
	11	1	1	1	1
	10		1	1	

*AB* as 11 → *AB*  
*CD* as 11 → *CD*

Term: *ABCD*

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00		1	1	1
	01			1	1
	11	1	1	1	1
	10		1	1	

*AB* as 11 → *AB*  
*CD* as 11 and 10 → *C*

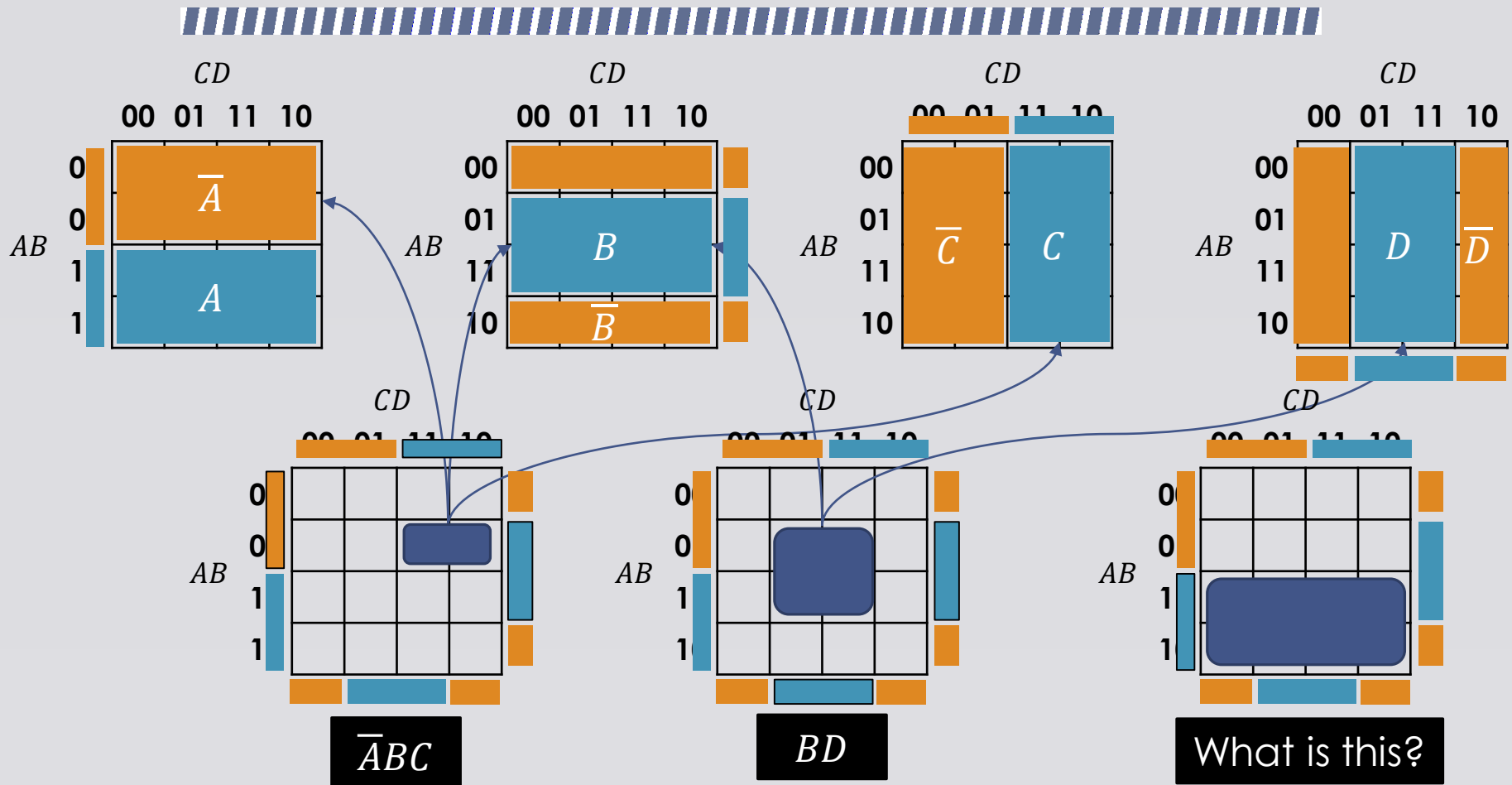
Term: *ABC*

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00		1	1	1
	01			1	1
	11	1	1	1	1
	10		1	1	

*AB* as 01 and 11 → *B*  
*CD* as 11 and 10 → *C*

Term: *BC*

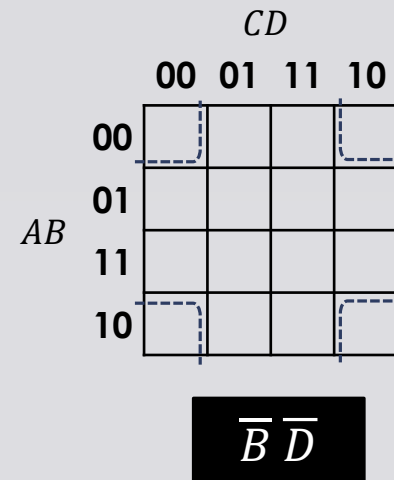
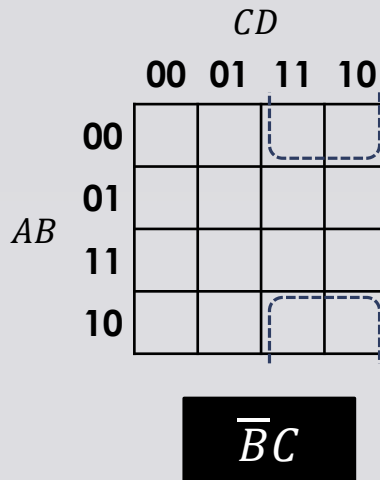
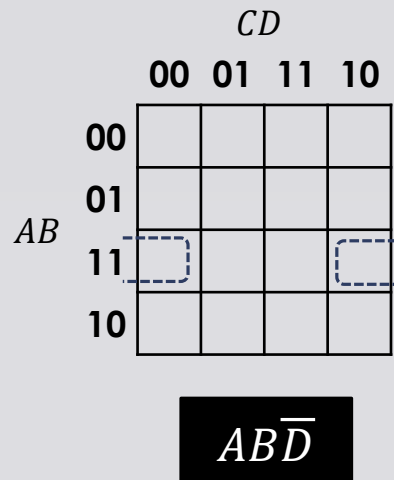
# More visualization (for groups)



# Grouping - wrapping



- Groups can be wrapped around the edges



# Grouping – choices



- Group all 1's
- Prefer **bigger** groups and **fewer** groups
- Overlap is allowed

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00		1	1	1
	01			1	1
	11	1	1	1	1
	10		1	1	

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00		1	1	1
	01			1	1
	11	1	1	1	1
	10		1	1	

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00		1	1	1
	01			1	1
	11	1	1	1	1
	10		1	1	

There is only one possible group of 4 for each of these two 1's.

The remaining 1's can be grouped in another group of 4.

$$F = AB + \bar{A}C + \bar{B}D$$

# Don't care



- Don't care can be included in any group, but there is no need to group all of them or any of them.
- This allow us to form larger groups.

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00			d	
	01		d	1	d
	11	d	1	1	1
	10		d	1	d


$$F = ABC + ABD + ACD + BCD$$

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00			d	
	01		d	1	d
	11	d	1	1	1
	10		d	1	d

$$F = AB + CD$$

Note that there are many possible groupings for this K-Map

# Exercise

- 
- Given the truth table as shown, find the Boolean function for  $P$  and  $Q$  using K-Map.

$A$	$B$	$C$	$D$	$P$	$Q$
0	0	0	0	1	1
0	0	0	1	0	1
0	0	1	0	1	1
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	0	0
0	1	1	0	1	1
0	1	1	1	1	0
1	0	0	0	d	d
1	0	0	1	d	d
1	0	1	0	d	d
1	0	1	1	d	d
1	1	0	0	0	1
1	1	0	1	0	0
1	1	1	0	d	d
1	1	1	1	0	0



# Solution

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>P</i>	<i>Q</i>
0	0	0	0	1	1
0	0	0	1	0	1
0	0	1	0	1	1
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	0	0
0	1	1	0	1	1
0	1	1	1	1	0
1	0	0	0	d	d
1	0	0	1	d	d
1	0	1	0	d	d
1	0	1	1	d	d
1	1	0	0	0	1
1	1	0	1	0	0
1	1	1	0	d	d
1	1	1	1	0	0

*CD*

	00	01	11	10
00	1			1
01			1	1
11				d
10	d	d	d	d

*AB*

$$P = \overline{B} \overline{D} + \overline{A} B C$$

*CD*

	00	01	11	10
00	1	1		1
01	1			1
11	1			d
10	d	d	d	d

*AB*

$$Q = \overline{B} \overline{C} + \overline{D}$$