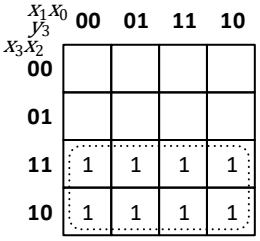
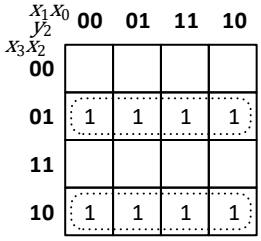
COMP2120B Computer organization

Assignment 1 Solution

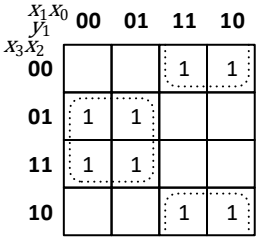
a) K-maps:



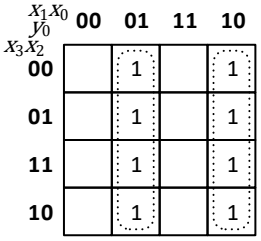
$y_3 = x_3$



$y_2 = x_3\overline{x_2} + \overline{x_3}x_2$



$y_1 = x_2\overline{x_1} + \overline{x_2}x_1$



$y_0 = x_1\overline{x_0} + \overline{x_1}x_0$

Using XOR only:

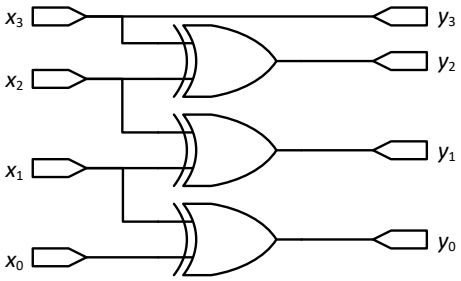
$y_3 = x_3$

$y_2 = x_3\overline{x_2} + \overline{x_3}x_2 = x_3 \oplus x_2$

$y_1 = x_2\overline{x_1} + \overline{x_2}x_1 = x_2 \oplus x_1$

$y_0 = x_1\overline{x_0} + \overline{x_1}x_0 = x_1 \oplus x_0$

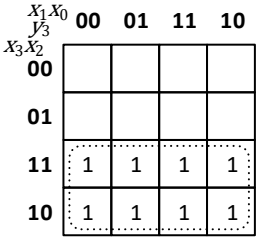
Logic circuit:



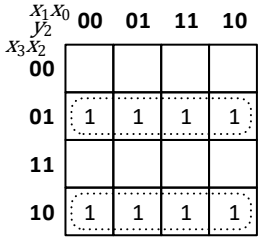
b) Truth table:

Inputs				Outputs			
y_3	y_2	y_1	y_0	x_3	x_2	x_1	x_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	1
1	0	0	0	1	1	1	1
1	0	0	1	1	1	1	0
1	0	1	0	1	1	0	0
1	0	1	1	1	1	0	1
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	1
1	1	1	0	1	0	1	1
1	1	1	1	1	0	1	0

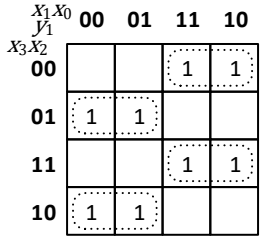
K-maps:



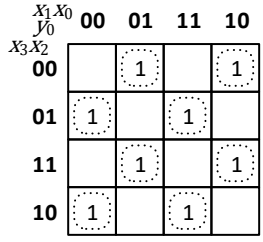
$y_3 = x_3$



$y_2 = x_3\overline{x_2} + \overline{x_3}x_2$



$y_1 = x_3x_2x_1 + x_3\overline{x_2}\,\overline{x_1} + \overline{x_3}x_2\overline{x_1} + \overline{x_3}\,\overline{x_2}x_1$



$$y_0 = x_3x_2x_1\overline{x_0} + x_3x_2\overline{x_1}x_0 + x_3\overline{x_2}x_1x_0 + x_3\overline{x_2}\,\overline{x_1}\,\overline{x_0} + \overline{x_3}x_2x_1x_0 + \overline{x_3}x_2\overline{x_1}\,\overline{x_0} + \overline{x_3}\,\overline{x_2}x_1\overline{x_0} + \overline{x_3}\,\overline{x_2}\,\overline{x_1}x_0$$

Using XOR only:

$y_3 = x_3$

$y_2 = x_3\overline{x_2} + \overline{x_3}x_2 = x_3 \oplus x_2$

$y_1 = x_3x_2x_1 + x_3\overline{x_2}\,\overline{x_1} + \overline{x_3}x_2\overline{x_1} + \overline{x_3}\,\overline{x_2}x_1 = (x_3x_2 + \overline{x_3}\,\overline{x_2})x_1 + (x_3\overline{x_2} + \overline{x_3}x_2)\overline{x_1}$

Observe that $(x_3x_2 + \overline{x_3}\,\overline{x_2}) + (x_3\overline{x_2} + \overline{x_3}x_2) = 1$ and $x_3\overline{x_2} + \overline{x_3}x_2 = y_2$, so $x_3x_2 + \overline{x_3}\,\overline{x_2} = \overline{y_2}$.

Therefore, $y_1 = \overline{y_2}x_1 + y_2\overline{x_1} = y_2 \oplus x_1$

$$\begin{aligned} y_0 &= x_3x_2x_1\overline{x_0} + x_3x_2\overline{x_1}x_0 + x_3\overline{x_2}x_1x_0 + x_3\overline{x_2}\,\overline{x_1}\,\overline{x_0} + \overline{x_3}x_2x_1x_0 + \overline{x_3}x_2\overline{x_1}\,\overline{x_0} + \overline{x_3}\,\overline{x_2}x_1\overline{x_0} \\ &\quad + \overline{x_3}\,\overline{x_2}\,\overline{x_1}x_0 \\ &= (x_3x_2\overline{x_1} + x_3\overline{x_2}x_1 + \overline{x_3}x_2x_1 + \overline{x_3}\,\overline{x_2}\,\overline{x_1})x_0 + (x_3x_2x_1 + x_3\overline{x_2}\,\overline{x_1} + \overline{x_3}x_2\overline{x_1} + \overline{x_3}\,\overline{x_2}x_1)\,\overline{x_0} \end{aligned}$$

Observe that $(x_3x_2\overline{x_1} + x_3\overline{x_2}x_1 + \overline{x_3}x_2x_1 + \overline{x_3}\,\overline{x_2}\,\overline{x_1}) + (x_3x_2x_1 + x_3\overline{x_2}\,\overline{x_1} + \overline{x_3}x_2\overline{x_1} + \overline{x_3}\,\overline{x_2}x_1) = 1$ and $x_3x_2x_1 + x_3\overline{x_2}\,\overline{x_1} + \overline{x_3}x_2\overline{x_1} + \overline{x_3}\,\overline{x_2}x_1 = y_1$, so $x_3x_2\overline{x_1} + x_3\overline{x_2}x_1 + \overline{x_3}x_2x_1 + \overline{x_3}\,\overline{x_2}\,\overline{x_1} = \overline{y_1}$.

Therefore, $y_0 = \overline{y_1}x_0 + y_1\overline{x_0} = y_1 \oplus x_0$

Logic circuit: