

# COMP2120B Computer organization

## Assignment 1 Solution

a) K-maps:

$x_3x_2$	$x_1x_0$	00	01	11	10
00					
01					
11		1	1	1	1
10		1	1	1	1

$$y_3 = x_3$$

$x_3x_2$	$x_1x_0$	00	01	11	10
00					
01		1	1	1	1
11					
10		1	1	1	1

$$y_2 = x_3\bar{x}_2 + \bar{x}_3x_2$$

$x_3x_2$	$x_1x_0$	00	01	11	10
00				1	1
01		1	1		
11		1	1		
10				1	1

$$y_1 = x_2\bar{x}_1 + \bar{x}_2x_1$$

$x_3x_2$	$x_1x_0$	00	01	11	10
00			1		1
01			1		1
11			1		1
10			1		1

$$y_0 = x_1\bar{x}_0 + \bar{x}_1x_0$$

Using XOR only:

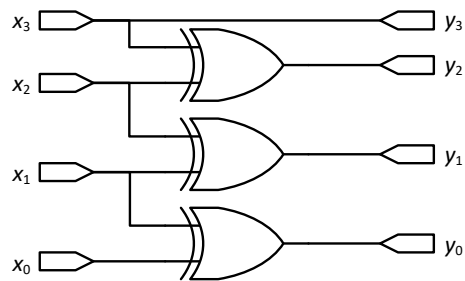
$$y_3 = x_3$$

$$y_2 = x_3\bar{x}_2 + \bar{x}_3x_2 = x_3 \oplus x_2$$

$$y_1 = x_2\bar{x}_1 + \bar{x}_2x_1 = x_2 \oplus x_1$$

$$y_0 = x_1\bar{x}_0 + \bar{x}_1x_0 = x_1 \oplus x_0$$

Logic circuit:



b) Truth table:

Inputs				Outputs			
$y_3$	$y_2$	$y_1$	$y_0$	$x_3$	$x_2$	$x_1$	$x_0$
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	1
1	0	0	0	1	1	1	1
1	0	0	1	1	1	1	0
1	0	1	0	1	1	0	0
1	0	1	1	1	1	0	1
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	1
1	1	1	0	1	0	1	1
1	1	1	1	1	0	1	0

K-maps:

$x_1x_0$ $y_3$	00	01	11	10
$x_3x_2$ 00				
01				
11	1	1	1	1
10	1	1	1	1

$y_3 = x_3$

$x_1x_0$ $y_2$	00	01	11	10
$x_3x_2$ 00				
01	1	1	1	1
11				
10	1	1	1	1

$y_2 = x_3\bar{x}_2 + \bar{x}_3x_2$

$x_1x_0$ $y_1$	00	01	11	10
$x_3x_2$ 00			1	1
01	1	1		
11			1	1
10	1	1		

$y_1 = x_3x_2x_1 + x_3\bar{x}_2\bar{x}_1 + \bar{x}_3x_2\bar{x}_1 + \bar{x}_3\bar{x}_2x_1$

$x_1x_0$ $y_0$	00	01	11	10
$x_3x_2$ 00		1		1
01	1		1	
11		1		1
10	1		1	

$$y_0 = x_3x_2x_1\bar{x}_0 + x_3x_2\bar{x}_1x_0 + x_3\bar{x}_2x_1x_0 + x_3\bar{x}_2\bar{x}_1\bar{x}_0 + \bar{x}_3x_2x_1x_0 + \bar{x}_3x_2\bar{x}_1\bar{x}_0 + \bar{x}_3\bar{x}_2x_1x_0 + \bar{x}_3\bar{x}_2\bar{x}_1x_0$$

Using XOR only:

$$y_3 = x_3$$

$$y_2 = x_3\bar{x}_2 + \bar{x}_3x_2 = x_3 \oplus x_2$$

$$y_1 = x_3x_2x_1 + x_3\bar{x}_2\bar{x}_1 + \bar{x}_3x_2\bar{x}_1 + \bar{x}_3\bar{x}_2x_1 = (x_3x_2 + \bar{x}_3\bar{x}_2)x_1 + (x_3\bar{x}_2 + \bar{x}_3x_2)\bar{x}_1$$

Observe that  $(x_3x_2 + \bar{x}_3\bar{x}_2) + (x_3\bar{x}_2 + \bar{x}_3x_2) = 1$  and  $x_3\bar{x}_2 + \bar{x}_3x_2 = y_2$ , so  $x_3x_2 + \bar{x}_3\bar{x}_2 = \bar{y}_2$ .

$$\text{Therefore, } y_1 = \bar{y}_2x_1 + y_2\bar{x}_1 = y_2 \oplus x_1$$

$$\begin{aligned} y_0 &= x_3x_2x_1\bar{x}_0 + x_3x_2\bar{x}_1x_0 + x_3\bar{x}_2x_1x_0 + x_3\bar{x}_2\bar{x}_1\bar{x}_0 + \bar{x}_3x_2x_1x_0 + \bar{x}_3x_2\bar{x}_1\bar{x}_0 + \bar{x}_3\bar{x}_2x_1x_0 + \bar{x}_3\bar{x}_2\bar{x}_1x_0 \\ &= (x_3x_2\bar{x}_1 + x_3\bar{x}_2x_1 + \bar{x}_3x_2x_1 + \bar{x}_3\bar{x}_2\bar{x}_1)x_0 + (x_3x_2x_1 + x_3\bar{x}_2\bar{x}_1 + \bar{x}_3x_2\bar{x}_1 + \bar{x}_3\bar{x}_2x_1)\bar{x}_0 \end{aligned}$$

Observe that  $(x_3x_2\bar{x}_1 + x_3\bar{x}_2x_1 + \bar{x}_3x_2x_1 + \bar{x}_3\bar{x}_2\bar{x}_1) + (x_3x_2x_1 + x_3\bar{x}_2\bar{x}_1 + \bar{x}_3x_2\bar{x}_1 + \bar{x}_3\bar{x}_2x_1) = 1$  and  $x_3x_2x_1 + x_3\bar{x}_2\bar{x}_1 + \bar{x}_3x_2\bar{x}_1 + \bar{x}_3\bar{x}_2x_1 = y_1$ , so  $x_3x_2\bar{x}_1 + x_3\bar{x}_2x_1 + \bar{x}_3x_2x_1 + \bar{x}_3\bar{x}_2\bar{x}_1 = \bar{y}_1$ .

$$\text{Therefore, } y_0 = \bar{y}_1x_0 + y_1\bar{x}_0 = y_1 \oplus x_0$$

Logic circuit:

