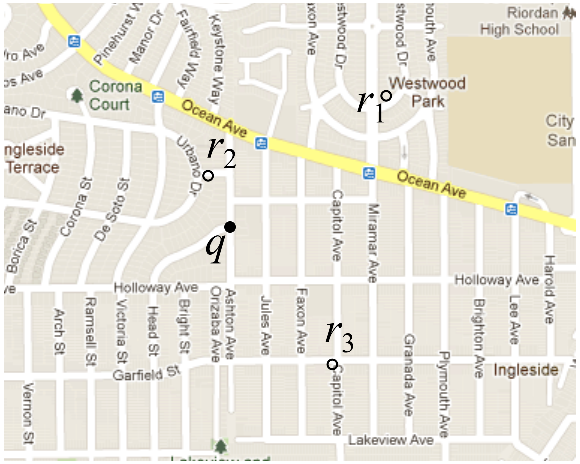
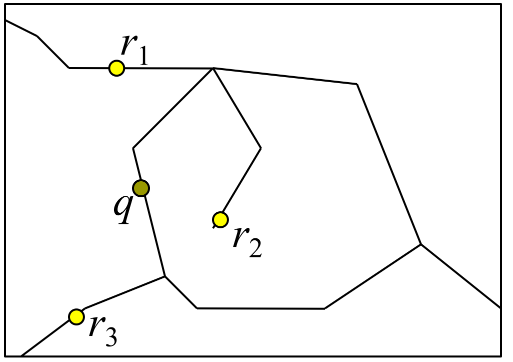
Spatial Networks

- Modeling and storing spatial networks
- Shortest path search
- Spatial queries over spatial networks
- Advanced indexing techniques for spatial networks



Network Distance

- In many real applications accessibility of objects is restricted by a spatial network
  - Examples
    - Driver looking for nearest gas station
    - Mobile user looking for nearest restaurant



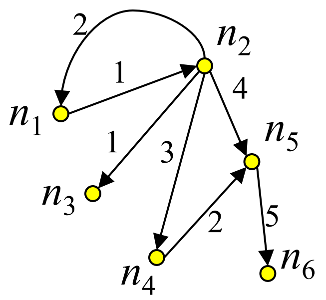
- Shortest path distance used instead of Euclidean distance
- SP(a,b) = path between a and b with the minimum accumulated length

Challenges

- Euclidean distance is no longer relevant
  - R-tree may not be useful, when search is based on shortest path distance
- Graph cannot be flattened to a one-dimensional space
  - Special storage and indexing techniques for graphs are required
- Graph properties may vary
  - directed vs. undirected
  - length, time, etc. as edge weights

Modeling Spatial Networks

- Adjacency matrix only appropriate for dense graphs
- Spatial networks are sparse: use adjacency lists instead



graph

	n_1	n_2	n_3	n_4	n_5	n_6
n_1	0	1	∞	∞	∞	∞
n_2	2	0	1	3	4	∞
n_3	∞	∞	0	∞	∞	∞
n_4	∞	∞	∞	0	2	∞
n_5	∞	∞	∞	∞	0	5
n_6	∞	∞	∞	∞	∞	0

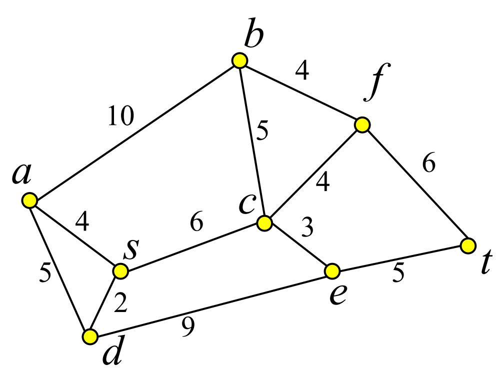
adjacency matrix

n_1	(n_2, 1)
n_2	(n_1, 2), (n_3, 1), (n_4, 3), (n_5, 4)
n_4	(n_5, 2)
n_5	(n_6, 5)

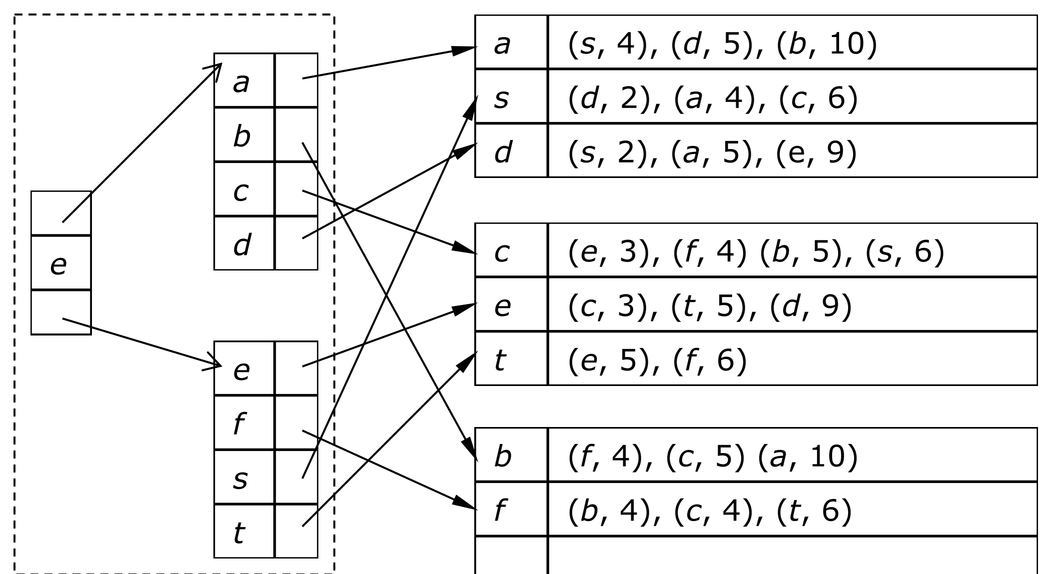
adjacency lists

Storing Large Spatial Networks

- Problem: adjacency lists representation may not fit in memory if graph is large
- Solution:
  - partition adjacency lists to disk blocks [based on proximity]
  - create B$^+$-tree index on top of partitions [based on node-id]



graph



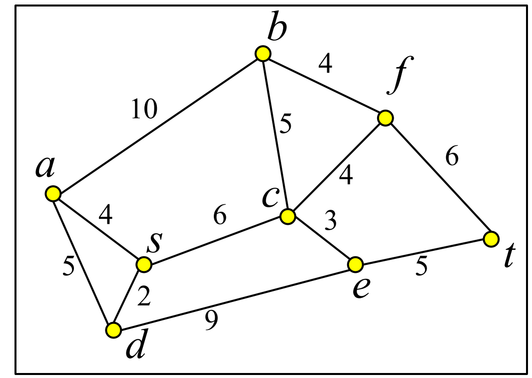
B$^+$-tree

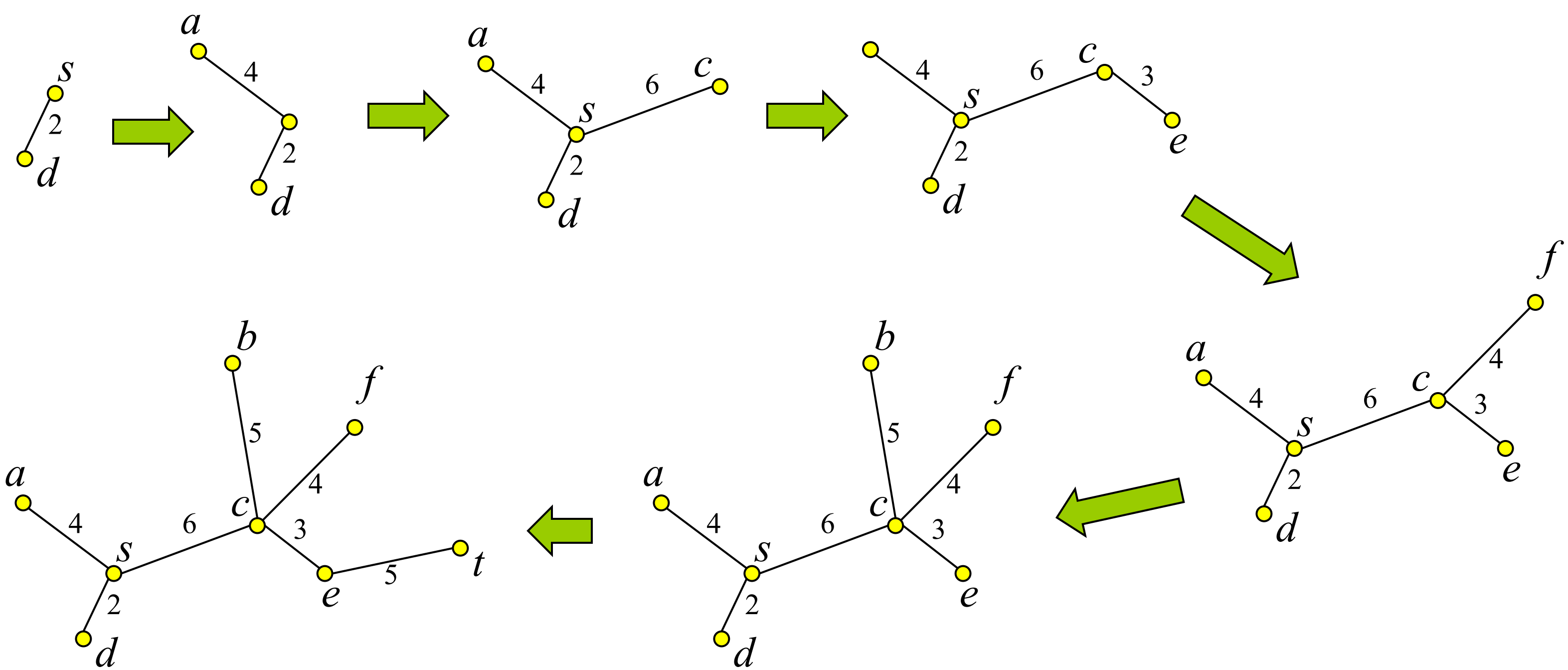
Shortest Path Computation

- Given a graph G(V,E), and two nodes s,t in V, find the shortest path from s to t
- A classic algorithmic problem
- Studied extensively since the 1950's
- Several methods
  - Dijkstra's algorithm
  - A*-search
  - Bi-directional search
  - *Reach* preprocessing heuristic

Dijkstra's Shortest Path Search

- Idea: incrementally explore the graph around s, visiting nodes in distance order to s until t is found (like NN)





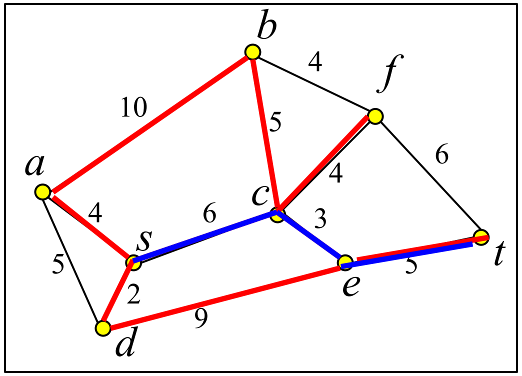
Dijkstra's Shortest Path Search

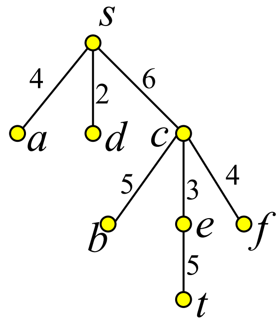
```
function Dijkstra_SP(source node s, target node t)
    for each graph node v
        SPD(s, v) := ∞; /* initialize shortest path distance */
        path(s, v) := null; /* initialize shortest path */
        mark v as unvisited;
    initialize a priority queue Q;
    SPD(s, s) := 0; add s to Q;
    while not empty(Q)
        v := top(Q); /* node v on Q with smallest SPD(s, v) */
        remove v from Q;
        mark v as visited;
        if v = t then return path(s, t);
        for each neighbor u of v
            if u is not marked as visited
                if SPD(s, u) > SPD(s, v) + weight(v, u)
                    SPD(s, u) := SPD(s, v) + weight(v, u);
                    path(s, u) := path(s, v) + (v, u);
                    add or update u on Q;
```

Dijkstra's Shortest Path Search

Current node	Queue
s (0)	*d* (2), *a* (4), *c* (6)
d (2)	*a* (4), *c* (6), *e* (11)
a (4)	*c* (6), *e* (11), *b* (14)
c (6)	*e* (9), *f* (10), *b* (11)
e (9)	*f* (10), *b* (11), *t* (14)
f (10)	*b* (11), *t* (14)
b (11)	*t* (14)
t (14)	

t is de-queued: shortest path has been found!

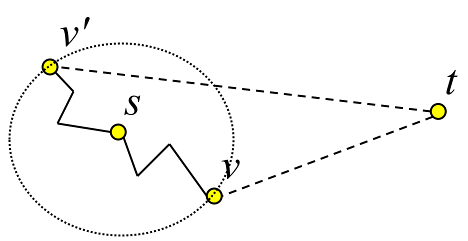




shortest path tree of node *s*

A*-search

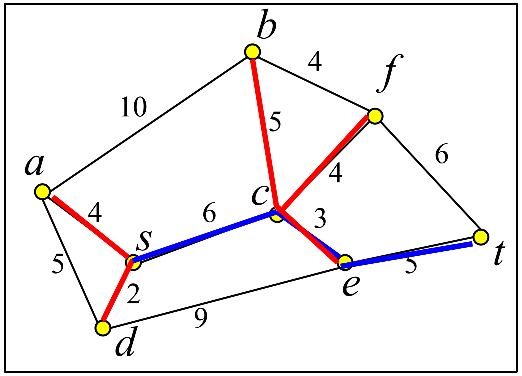
- Dijkstra's search explores nodes around *s* without a specific search direction until *t* is found
- Idea: improve Dijkstra's algorithm by directing search towards *t*
- Due to triangular inequality, Euclidean distance is a lower bound of network distance
- Use Euclidean distance to lower bound network distance based on known information:
  - Nodes are visited in increasing SPD(*s*,*v*)+dist(*v*,*t*) order
    - SPD(*s*,*v*): shortest path distance from *s* to *v* (computed by Dijkstra)
    - dist(v,t): Euclidean distance between *v* and *t*
  - Original Dijkstra visits nodes in increasing SPD(*s*,*v*) order



A*-search: Example

SPD(s,c)+dist(c,t)

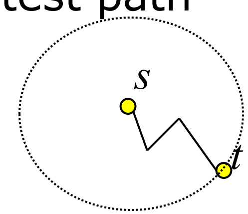
Current node	Queue
s (0)	d (2+14), a (4+15), c (6+7)
c (6)	a (4+15), d (2+14), b (11+9), f (10+6), e (9+5)
e (9)	a (4+15), d (2+14), b (11+9), f (10+6), t (14)
t (14)	

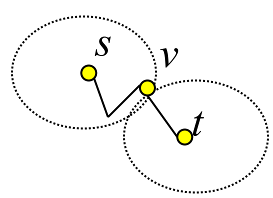


t is de-queued: shortest path has been found!

Bi-directional search

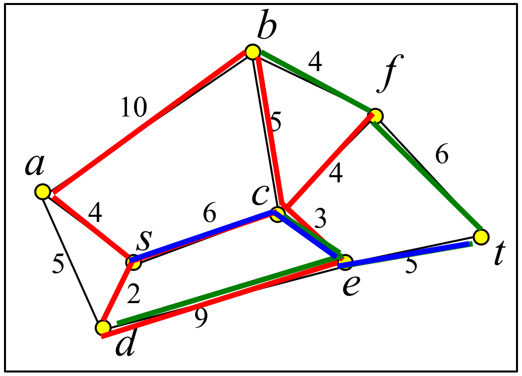
- Dijkstra's search explores nodes around *s* without a specific search direction until *t* is found
- Idea: search can be performed in concurrently from *s* and from *t* (backwards)
- The shortest path tree of s and the (backward) shortest path tree of t are computed concurrently
  - One queue Q_s for forward and one queue Q_t for backward search
  - Node visits are prioritized based on min(SPD(s,v), SPD(v,t))
  - If v already visited from s and v in Q_t, then candidate shortest path: p(s,v)+(v,u)+p(u,t) [if v already visited from t and v in Q_s symmetric]
    in Q_t
  - If v visited by both s and v terminate search; report best candidate shortest path





Bi-directional search: Example

Current node	Q_s	Q_t
s (0), t (0)	*d* (2), *a* (4), *c* (6)	*e* (5), *f* (6)
d (2) [s]	*a* (4), *c* (6), *e* (11)	*e* (5), *f* (6)
a (4) [s]	*c* (6), *e* (11), *b* (14)	*e* (5), *f* (6)
e (5) [t]	*c* (6), *e* (11), *b* (14)	*f* (6), *c* (8), *d* (14)
c (6) [s]	*e* (9), *f* (10), *b* (11)	*f* (6), *c* (8), *d* (14)
f (6) [t]	*e* (9), *f* (10), *b* (11)	*c* (8), *b* (10), *d* (14)
c (8) [t]		



candidate shortest path:
s ➔d ➔e ➔t (16)

candidate shortest path:
s ➔c ➔e ➔t (14)

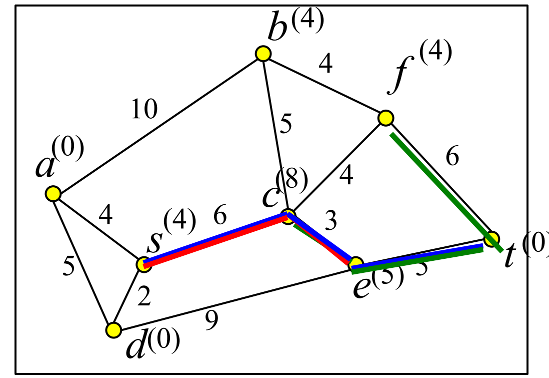
c is visited from both *s* and *t*!
terminate and report shortest path

REACH: A Preprocessing Heuristic

- For every node v in the network find all pairs of nodes s, t such that SP(s,t) includes v
  - Local reach $r_{s,t}(v) = \min\{SPD(s,v), SPD(v,t)\}$
  - **Reach** $r(v) = \max\{r_{s,t}(v)\}$, for all s,t such that SP(s,t) includes v
  - r(v): in the worst case, what is the minimum of SPD(s,v), SPD(v,t) given that v is in SP(s,t)?
- Assume r(v) is precomputed for every v in the graph
- In bi-directional search, asssume v is visited (e.g. from s)
  - For every unvisited neighbor u of v (by FWD and BK search):
    - $SPD(s,v) \leq SPD(s,u)$ and $SPD(s,v) \leq SPD(u,t)$
    - If $r(u) < SPD(s,v)$ then
      - $r(u) < SPD(s,u)$ and $r(u) < SPD(u,t)$ =>
      - u cannot be in SP(s,t) =>
      - u needs not be enheaped (pruned)
- **Reach** reduces the nodes to be added to the heap
  - However, it may require high preprocessing cost (all SPs computed)

Bi-directional w/ Reach: Example

Current node	Q_s	Q_t
s (0), t (0)	*c* (6)	*e* (5), *f* (6)
e (5) [t]	*c* (6)	*c* (8)
c (6) [s]	*e* (9)	*c* (8)
c (8) [t]		



candidate shortest path:
s ➔*c* ➔*e* ➔*t (14)*

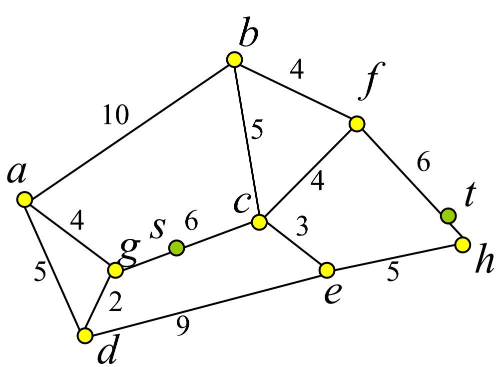
c is visited from both *s* and *t*!
terminate and report shortest path

Combination of Techniques

- A*, bi-directional search, and **Reach** can be combined to a powerful search technique
- A* can only be applied if lower distance bounds are available
- **Reach** can only be applied after pre-processing
  - expensive
  - high maintenance cost
- All versions of Dijkstra’s search require non-negative edge weights
  - Bellman-Ford is an algorithm for arbitrary edge weights (some of them could be negative)

Source/Destination on Edges

- We have assumed that points s and t are nodes of the network
- In practice s and t could be arbitrary points on edges
  - Mobile user locations
- Solve problem by introducing 2 more nodes

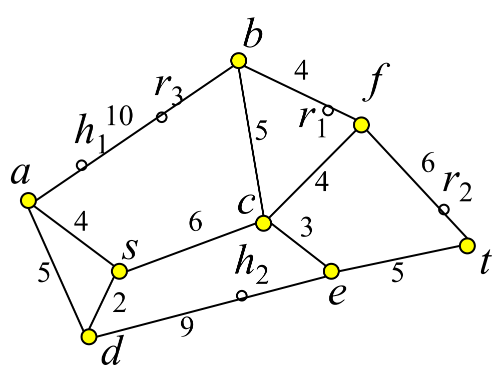


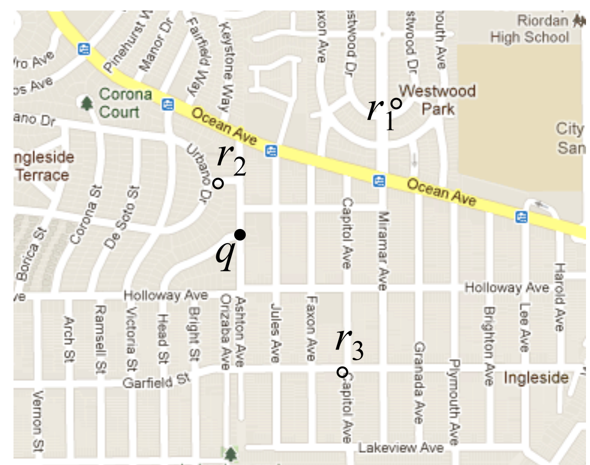
- en-heap g (2) and c (4) first

- t is reached from f or h

Spatial Queries over Spatial Networks

- Data:
  - A (static) spatial network (e.g., city map)
  - A (dynamic) set of spatial objects
- Spatial queries based on network distance:
  - Selections. Ex: find gas stations within 10km driving distance from here
  - Nearest neighbor search. Ex: find k nearest restaurants from present position
  - Joins. Ex: find pairs of restaurants and hotels at most 100m from each other





Methodology

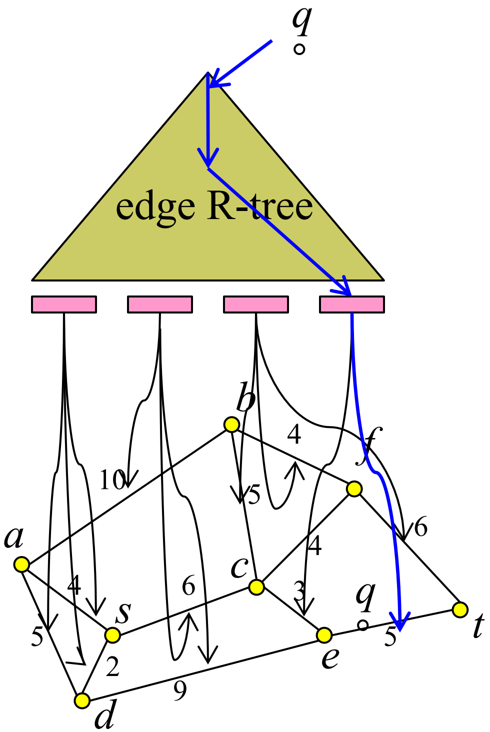
- Store (and index) the spatial network
  - Graph component (indexes connectivity information)
  - Spatial component (indexes coordinates of nodes, edges, etc.)
- Store (and index) the sets of spatial objects
  - Ex., one spatial relation for restaurants, one spatial relation for hotels, one relation for mobile users, etc.
- Given a spatial location p, use spatial component of network to find the network edge containing p
- Given a network edge, use network component to traverse neighboring edges
- Given a neighboring edge, use spatial indexes to find objects on them

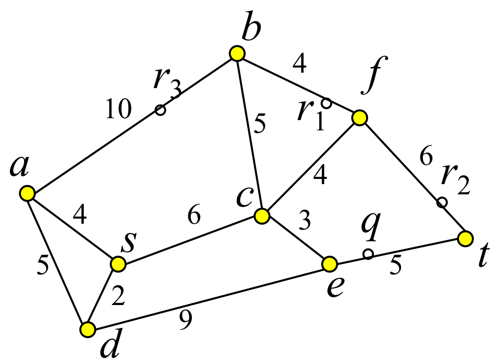
Evaluation of Spatial Selections (1)

- Query: find all objects in spatial relation R, within network distance ε from location q
- Method:
  - Use spatial index of network (R-tree indexing network edges) to find edge n_1n_2, which includes q
  - Use adjacency index of network (graph component) and apply Dijkstra's algorithm to progressively retrieve edges that are within network distance ε from location q
  - For all these edges apply a spatial selection on the R-tree that indexes R to find the results

Evaluation of Spatial Selections (1)

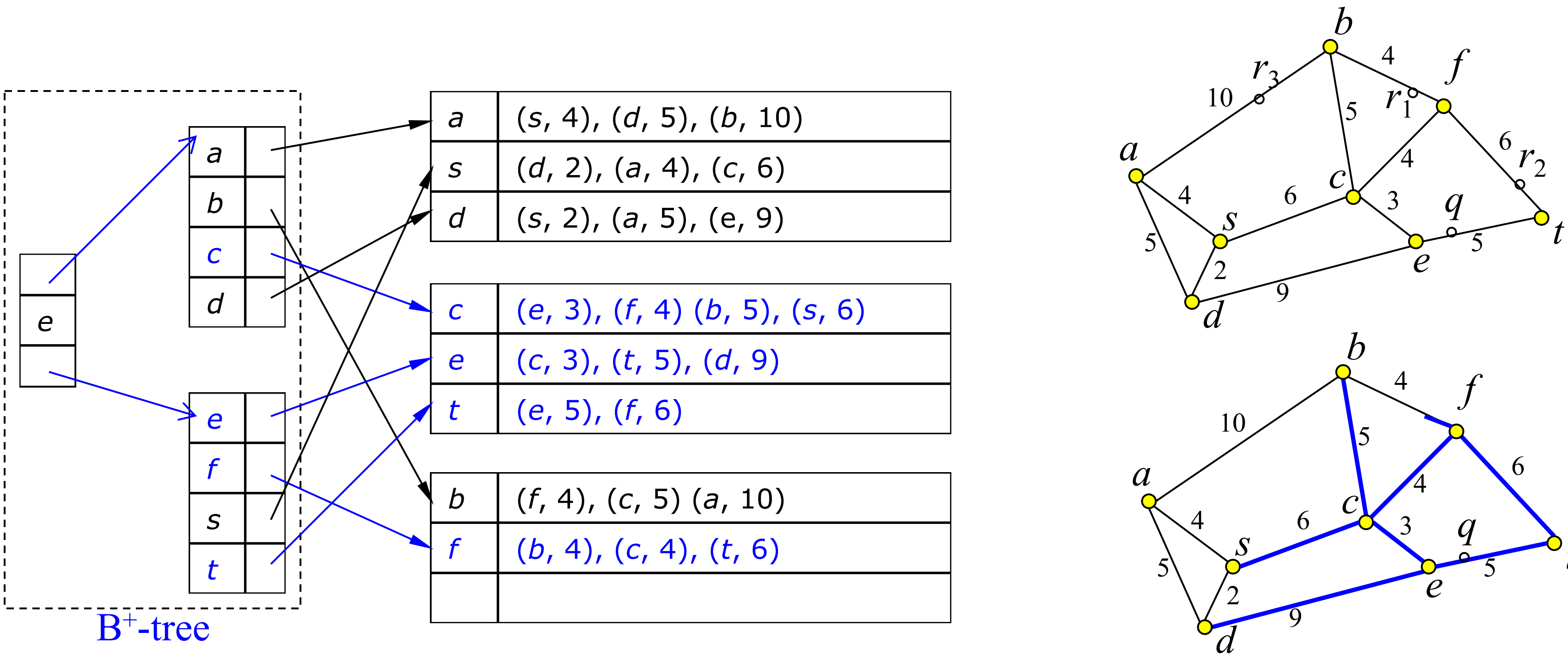
- Example: Find restaurants at most distance 10 from q
- Step 1: find network edge which contains q





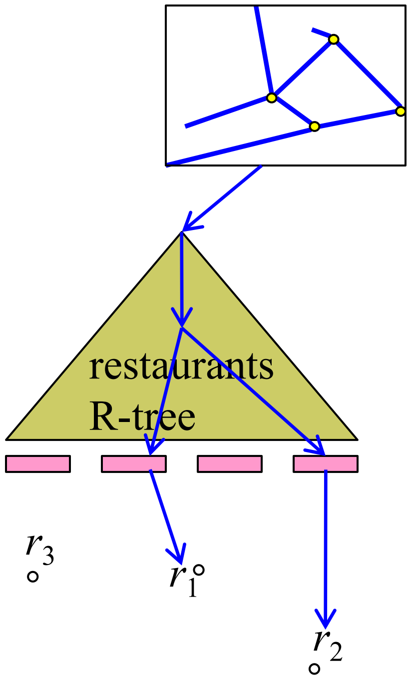
Evaluation of Spatial Selections (1)

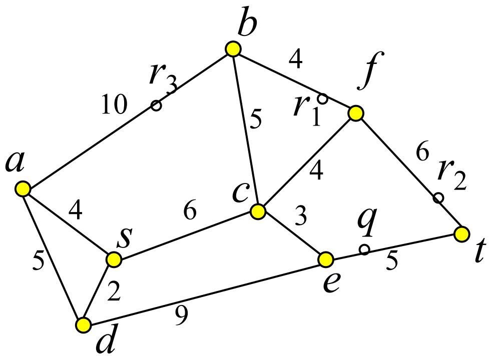
- Example: Find restaurants at most distance 10 from q
- Step 2: traverse network to find all edges (or parts of them within distance 10 from q)



Evaluation of Spatial Selections (1)

- Example: Find restaurants at most distance 10 from q
- Step 3: find restaurants that intersect the subnetwork computed at step 2



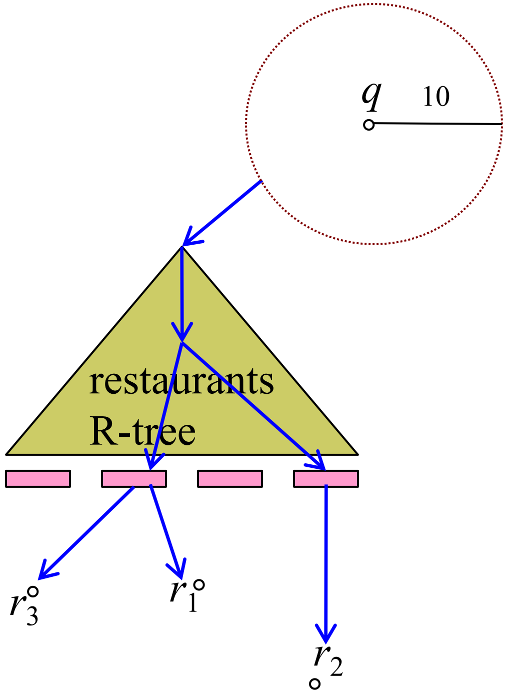


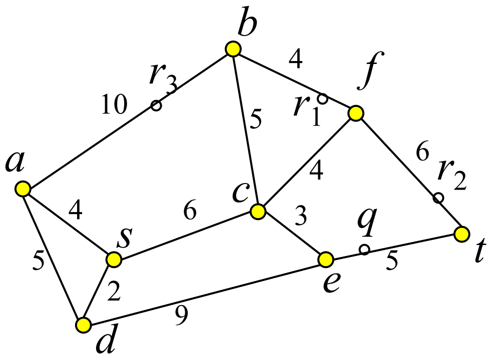
Evaluation of Spatial Selections (2)

- Query: find all objects in spatial relation R, within network distance ε from location q
- Alternative method based on Euclidean bounds:
  - Assumption: Euclidean distance is a lower-bound of network distance:
    - dist(v,u) ≤ SPD(v,u), for any v,u
  - Use R-tree on R to find set S of objects such that for each o in S: dist(q,o) ≤ ε
  - For each o in S:
    - find where o is located in the network (use Network R-tree)
    - compute SPD(q,o) (e.g. use A*)
    - If SPD(q,o) ≤ ε then output o

Evaluation of Spatial Selections (2)

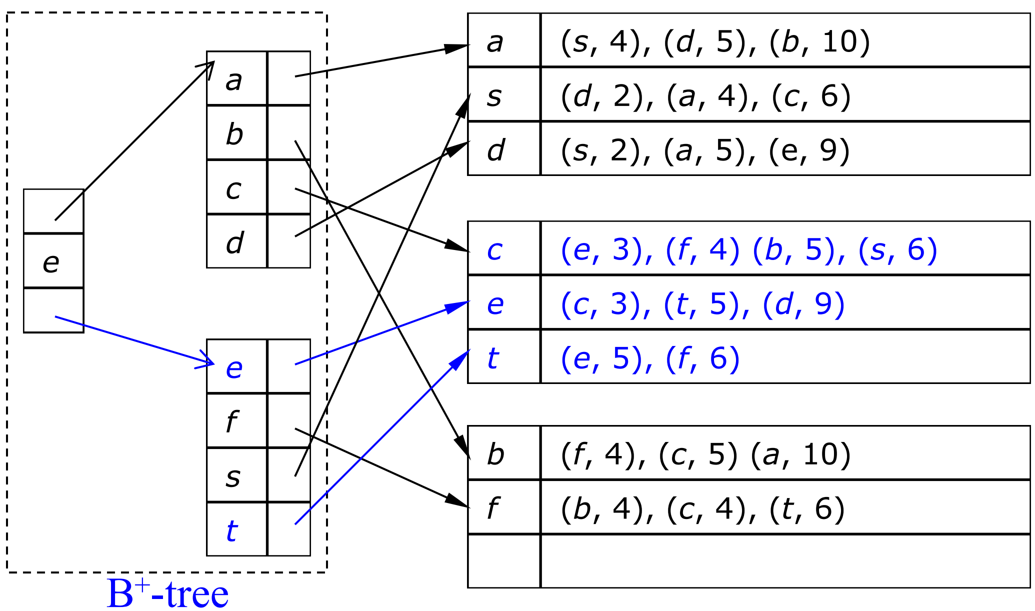
- Example: Find restaurants at most distance 10 from q
- Step 1: find restaurants for which the Euclidean distance to q is at most 10: S={r_1,r_2,r_3}

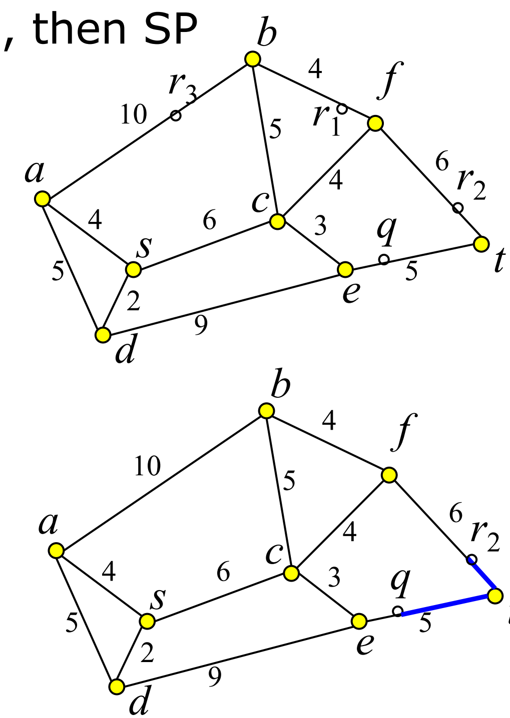




Evaluation of Spatial Selections (2)

- Example: Find restaurants at most distance 10 from q
- Step 2: for each restaurant in S, compute SPD to q and verify if it is indeed a correct result
  - Ex. for r_2, first find where r_2 is located (edge f,t), then SP



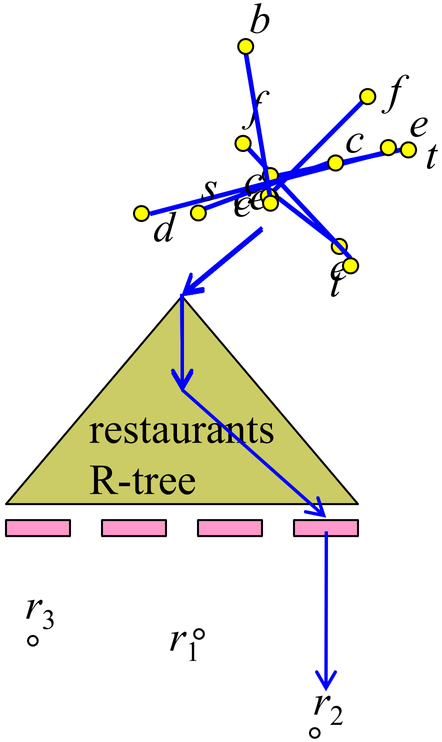


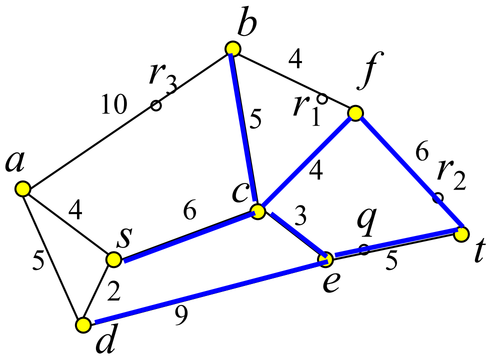
Evaluation of NN search (1)

- Query: find in spatial relation R the nearest object to a given location q
- Method:
  - Use spatial index of network (R-tree indexing network edges) to find edge n_1n_2, which includes q
  - Use adjacency index of network (graph component) and apply Dijkstra's algorithm to progressively retrieve edges in order of their distance to q
  - For each edge apply a spatial selection on the R-tree that indexes R to find any objects
  - Keep track of nearest object found so far; use its shortest path distance to terminate network browsing

Evaluation of Spatial Selections (1)

- Example: Find nearest restaurant to q





current NN = r_2

$SPD(q, r_2)$=5

√

Evaluation of NN search (2)

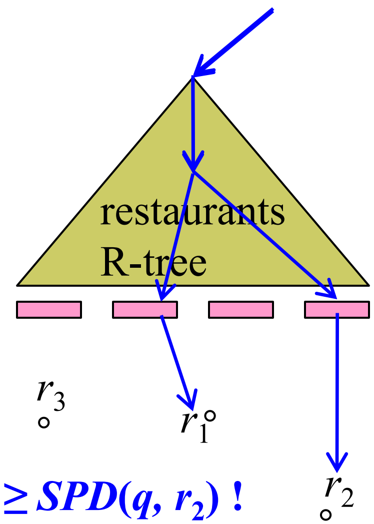
- Query: find in spatial relation R the nearest object to a given location q
- Alternative method based on Euclidean bounds:
  - Assumption: Euclidean distance lower-bounds network distance:
    - dist(v,u) ≤ SPD(v,u), for any v,u

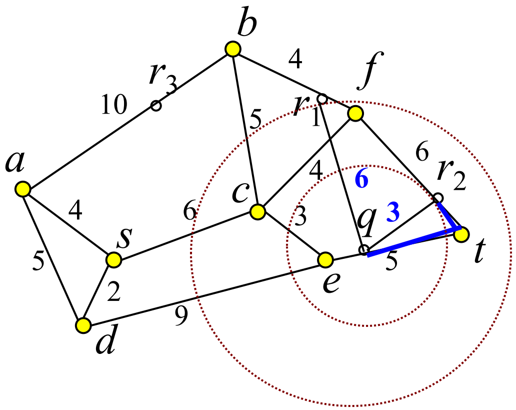
1. Use R-tree on R to find Euclidean NN p_{E1} of q;
2. CurrentNN= p_{E1}; bound=SPD(q,p_{E1}); //e.g. use A*
3. Find next Euclidean NN p_{Ei} of q
4. If dist(q,Ei)≥bound, then
   report (CurrentNN,bound) as result;
5. Compute SPD(q,p_{Ei}); if bound>SPD(q,p_{Ei}) then
   CurrentNN= p_{Ei}; bound=SPD(q,p_{Ei});
6. Goto step 3

Evaluation of Spatial Selections (2)

- Example: Find nearest restaurant to q

find 2nd Euclidean NN of q





***dist*(q, r_1) ≥ *SPD*(q, r_2) !**

current NN = r_2 √
SPD(q, r_2)=5

Spatial Join Queries

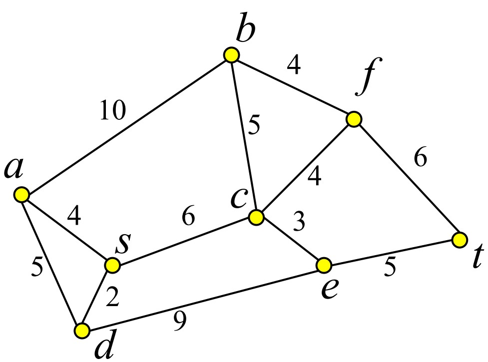
- Query: find pairs (r,s), such that r in relation R, s in relation S, and SPD(r,s)≤ε
- Methods:
  - For each r in R, do an ε-distance selection queries for objects in S (Index Nested Loops)
  - For each pair (r,s), such that Euclidean dist(r,s)≤ε compute SPD(r,s) and verify SPD(r,s)≤ε

Notes on Query Evaluation based on Network Distance

- For each query type, there are methods based on network browsing and methods based on Euclidean bounds
- Network browsing methods are fast if network edges are densely populated with points of interest
  - A limited network traversal can find the result fast
- Methods based on Euclidean bounds are good if the searched POIs are sparsely distributed in the network
  - Few verifications with exact SP searches are required
  - Directed SP search (e.g. using A*) avoids visiting empty parts of the network

Shortest Path Materialization and Indexing in Large Graphs

- Dijkstra's algorithm and related methods could be very expensive on very large graphs
- (Partial) materialization of shortest paths in static graphs can accelerate search



graph

	a	b	c	s	...
a	0, -	10, *ab*	10, *asc*	4, *as*	...
b	10, *ba*	0, -	5, *bc*	11, *bcs*	...
c	10, *csa*	5, *cb*	0, -	6, *cs*	...
s	4, *sa*	11, *scb*	6, *sc*	0, -	...
...	...	...	...	...	...

brute-force materialization
$O(n^3)$ space, $O(1)$ time

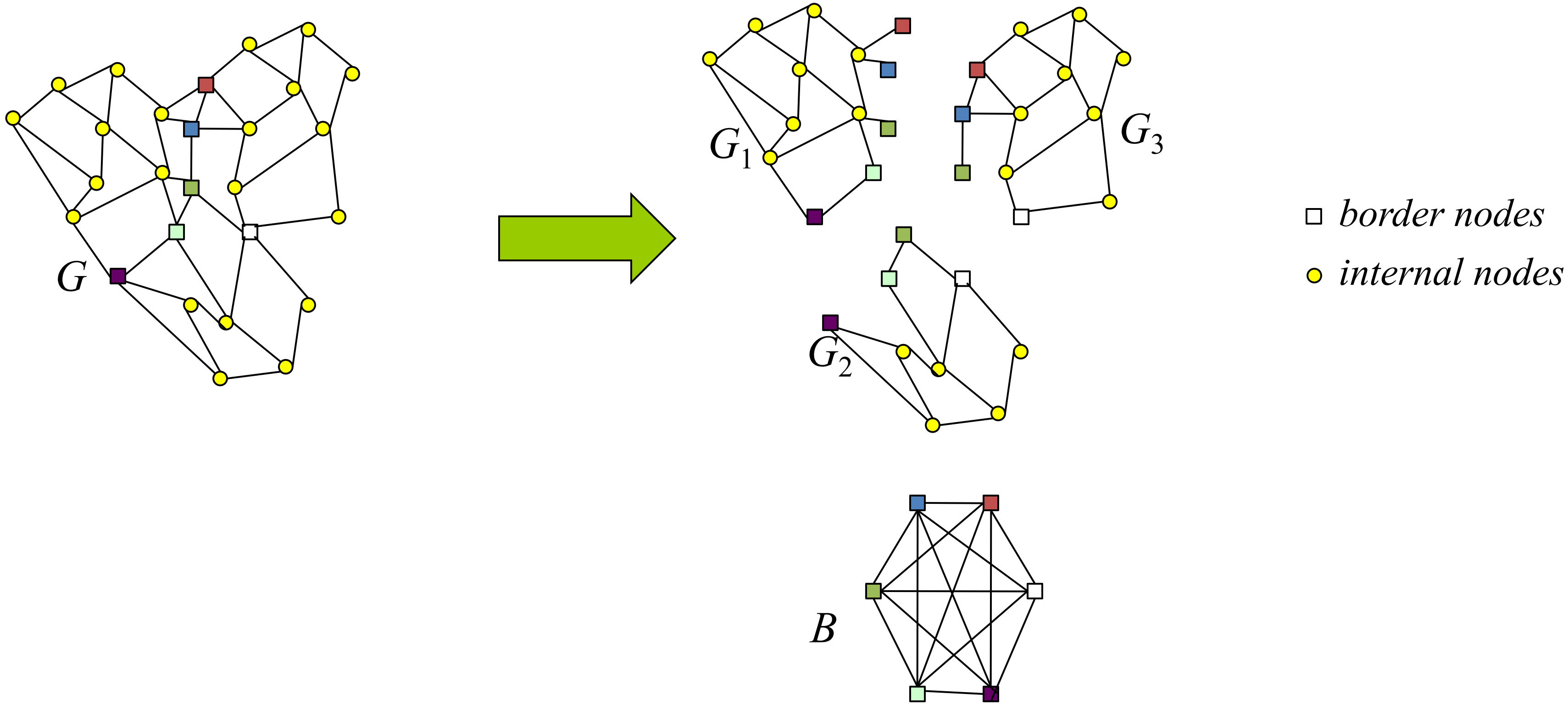
	a	b	c	s	...
a	0, -	10, *b*	10, *s*	4, *s*	...
b	10, *a*	0, -	5, *c*	11, *c*	...
c	10, *s*	5, *b*	0, -	6, *s*	...
s	4, *a*	11, *c*	6, *c*	0, -	...
...	...	...	...	...	...

distance matrix with successors
$O(n^2)$ space, $O(n)$ time

Hierarchical Path Materialization

- Idea: Partition graph G into $G_1, G_2, G_3, \ldots$ based on connectivity and proximity of nodes
- Every edge of G goes to exactly one G_i
- Border nodes belong to more than one G_i's
- For each G_i compute and materialize SPs between every pair of nodes in G_i (matrix M_i)
  - Partitions are small enough for materialization space overhead to be low
- Compute and materialize SPs between every pair of border nodes (matrix B)
  - If border nodes too many, hierarchically partition them into 2^{nd}-level partitions

Hierarchical Path Materialization

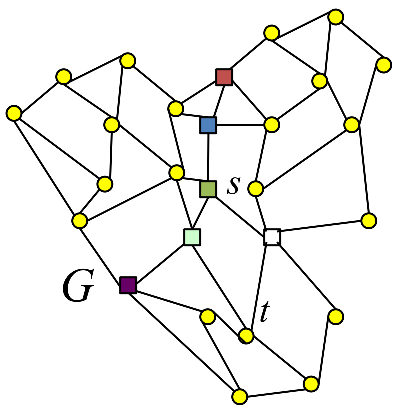


border nodes SP materialization

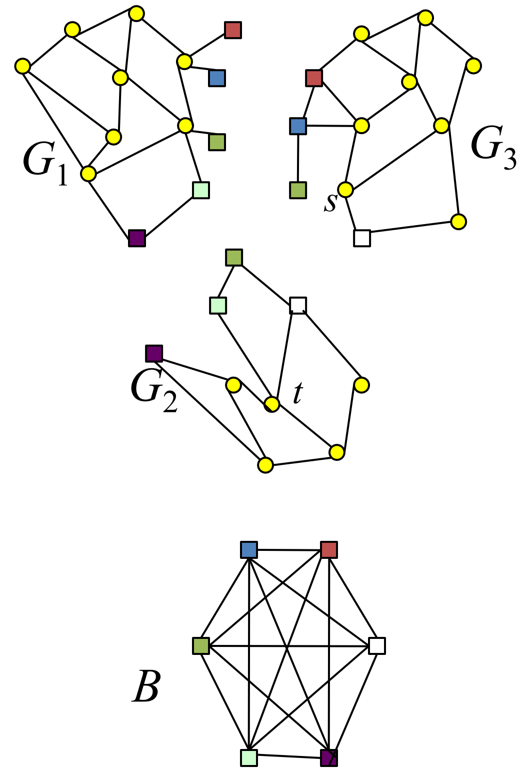
Hierarchical Path Materialization

- Shortest path search using HPM:
- If s,t border nodes directly use matrix B
- If s border, t non-border
  - $SP(s,t) = \min\{p(s,u)+p(u,t) \mid u \text{ in } B \text{ and } u \text{ in } G_t\}$
- If s non-border, t border
  - $SP(s,t) = \min\{p(s,u)+p(u,t) \mid u \text{ in } B \text{ and } u \text{ in } G_s\}$
- If s,t non-border nodes
  - If s,t in same G_{st} then SP(s,t) materialized in M_{st}
  - If s in G_s, t in G_t then
    - $SP(s,t) = \min\{p(s,u)+p(u,v)+p(v,t) \mid u,v \text{ in } B,\ u \text{ in } G_s,\ v \text{ in } G_t\}$

Hierarchical Path Materialization



SP(s,t) = ?

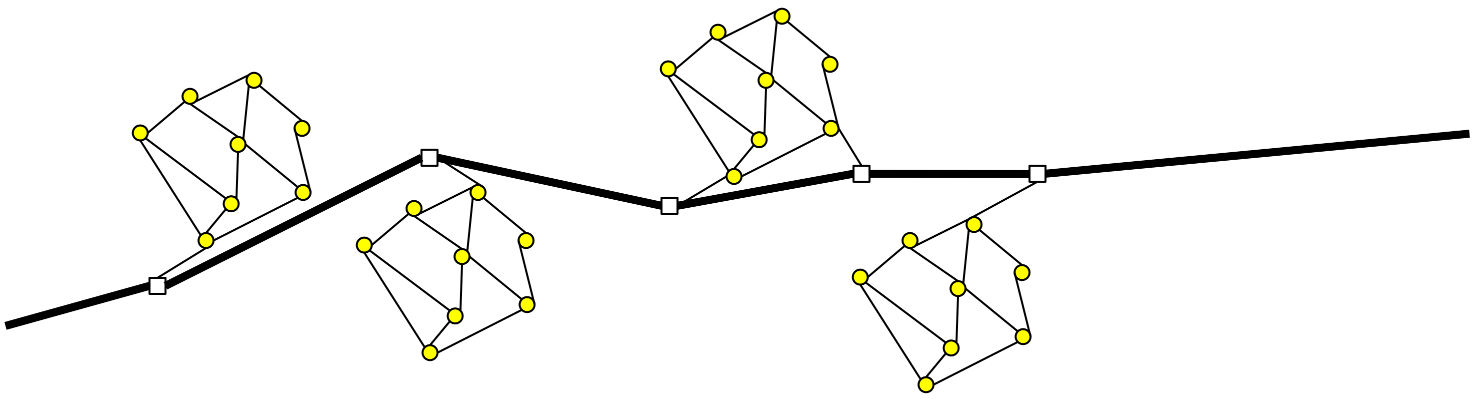


border nodes SP materialization

- *border nodes*
- *internal nodes*

Hierarchical Path Materialization

- Good partitioning if:
  - small partitions
  - few combinations examined for SP search
- Real road networks:
  - Non-highway nodes in local partitions
  - Highway nodes become border nodes



Compressing Materialized Paths

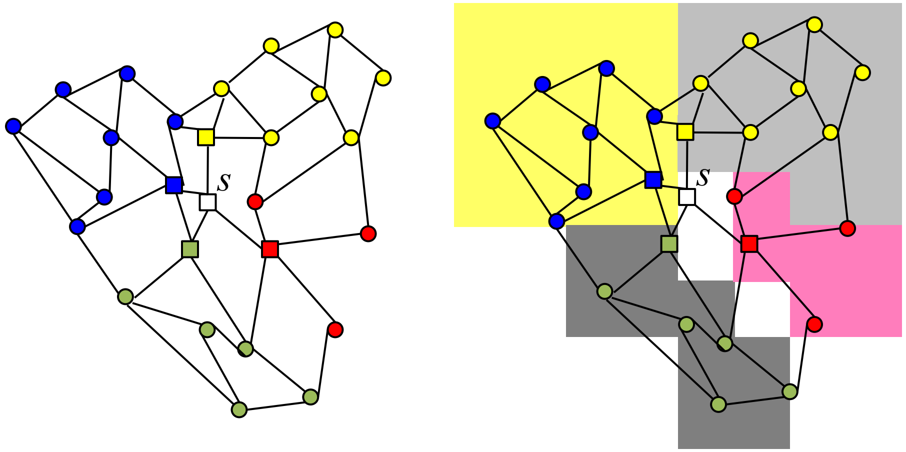
- Distance matrix with successors has $O(n^2)$ space cost
- Motivation: reduce space by grouping targets based on common successors

	a	*b*	*c*	*s*	…
a	0, -	10, *b*	10, *s*	4, *s*	…
b	10, *a*	0, -	5, *c*	11, *c*	…
c	10, *s*	5, *b*	0, -	6, *s*	…
s	4, *a*	11, *c*	6, *c*	0, -	…
…	…	…	…	…	…

distance matrix with successors
$O(n^2)$ space, $O(n)$ time

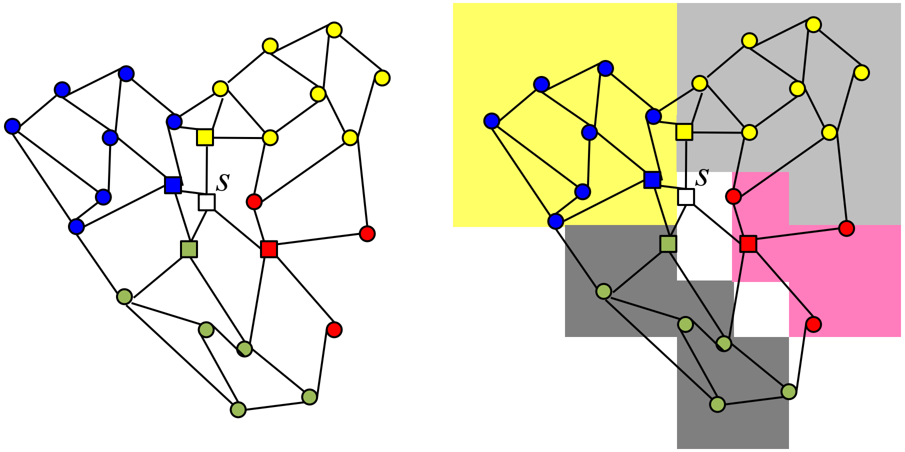
Compressing Materialized Paths

- Create and encode one space partitioning defined by targets of the same successor
- For each node s, index I_s a set of <succ,R> pairs:
  - succ: a successor of s
  - R: a continuous region, such that for each t in R, the successor of s in SP(s,t) is succ



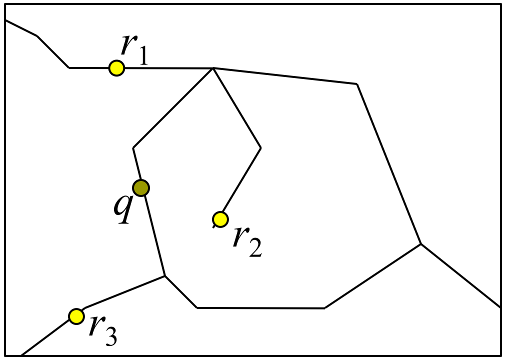
Compressing Materialized Paths

- To compute SP(s,t) for a given s, t:
  1. SP=s
  2. Use spatial index I_s to find <succ,R>, such that t in R
  3. SP = SP + (s,succ)
  4. If succ = t, report SP and terminate
  5. Otherwise s=succ; Goto step 2



Embedding Methods for SP

- Select a small subset L of nodes in G
- Nodes in L are called landmarks
- For each l in L, for each v in G:
  - pre-compute SPD(v,l), SPD(l,v)



- To compute SP(s,t) use A* search
  - To compute the lower bound distance between any node v and t we can use (triangular inequality):
    - D=max{|SPD(v,l)-SPD(l,t)|, for each l in L}
  - Prioritization of node visits is now done using SPD(s,v)+D

- Ex: if r_3 is used as landmark, then $|SPD(q,r_3)-SPD(r_3,r_2)|$ is a better LB for $SPD(q,r_2)$ compared to Euclidean distance

Summary

- Indexing and search of spatial networks is different than spatial indexing
  - Shortest path distance is used instead of Euclidean distance, to define range queries, nearest neighbor search, and spatial joins
- Spatial networks could be too large to fit in memory
  - Disk-based index for adjacency lists is used
- Several shortest path algorithms
- Spatial queries can be evaluated using Euclidean bounds
- Advanced indexing methods for shortest path search on large graphs