Implements Example 7.1 in Wachsmuth and Wachsmuth (2011), https://doi.org/10.1051/cocv/2010027

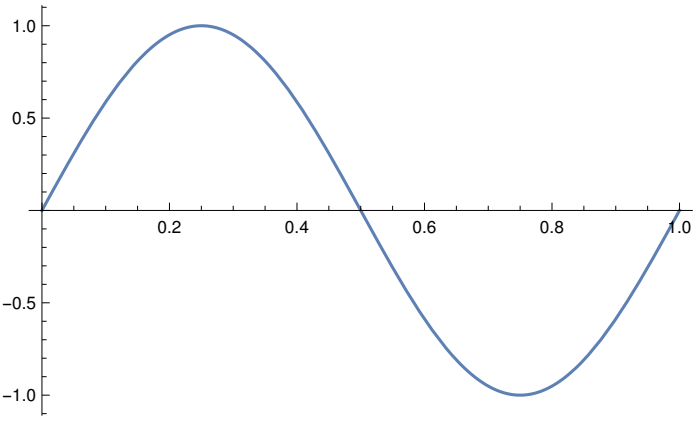
In[89]:=
```
a = -30; b = 30; β = 1/2;
```

In[90]:=
```
z[x_] := Sin[2 Pi x]
```

In[91]:=
```
u[x_] := Piecewise[{{a, 1/12 < x < 5/12}, {b, 7/12 < x < 11/12}}, 0]
```

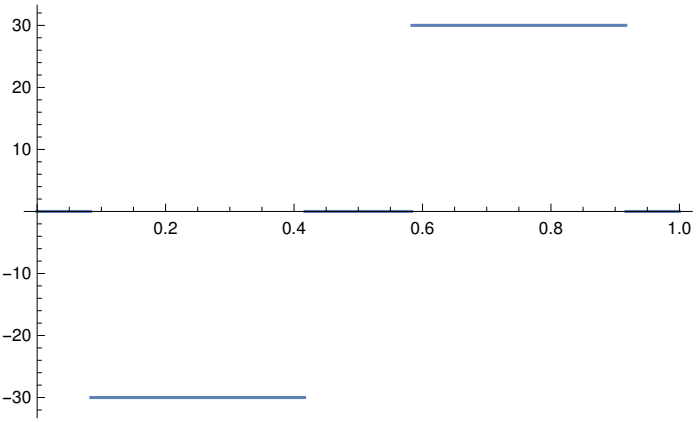
In[92]:=
```
Plot[z[x], {x, 0, 1}]
```

Out[92]=



In[93]:=
```
Plot[u[x], {x, 0, 1}]
```

Out[93]=



In[94]:=
```
state = DSolve[{-y''[x] == u[x], y[0] == 0, y[1] == 0}, y[x], x]
```

Out[94]=

$$\left\{\left\{y[x] \to -5x + \begin{cases} 0 & x \le \frac{1}{12} \\ \frac{5}{48} - \frac{5x}{2} + 15x^2 & \frac{1}{12} < x \le \frac{5}{12} \\ -\frac{5}{2} + 10x & \frac{5}{12} < x \le \frac{7}{12} \\ -\frac{365}{48} + \frac{55x}{2} - 15x^2 & \frac{7}{12} < x \le \frac{11}{12} \\ 5 & \text{True} \end{cases}\right\}\right\}$$

In[95]:=
```
FullSimplify[%]
```

Out[95]=
$$\left\{\left\{y[x] \to \begin{cases} 5 - 5\,x & 12\,x > 11 \\ -5\,x & 12\,x \le 1 \\ -\frac{5}{2} + 5\,x & \frac{5}{12} < x \le \frac{7}{12} \\ \frac{5}{48}\left(-73 + 72\left(3 - 2\,x\right)x\right) & \frac{7}{12} < x \le \frac{11}{12} \\ \frac{5}{48} + \frac{15}{2}\,x\left(-1 + 2\,x\right) & \text{True} \end{cases}\right\}\right\}$$

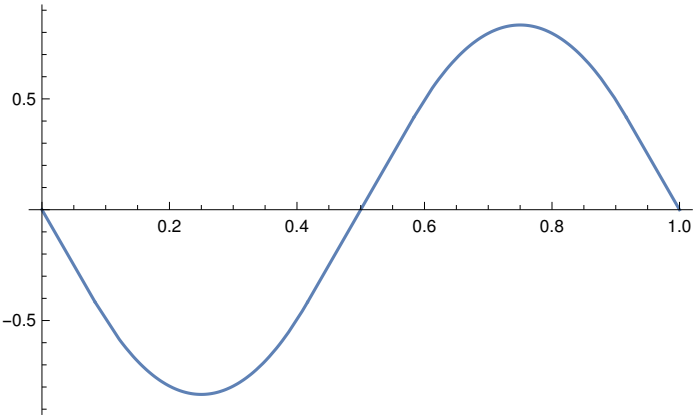
In[96]:=
```
statesol[x_] := y[x] /. state[[1]]
```

In[97]:=
```
statesol[x]
```

Out[97]=
$$-5\,x + \begin{cases} 0 & x \le \frac{1}{12} \\ \frac{5}{48} - \frac{5\,x}{2} + 15\,x^2 & \frac{1}{12} < x \le \frac{5}{12} \\ -\frac{5}{2} + 10\,x & \frac{5}{12} < x \le \frac{7}{12} \\ -\frac{365}{48} + \frac{55\,x}{2} - 15\,x^2 & \frac{7}{12} < x \le \frac{11}{12} \\ 5 & \text{True} \end{cases}$$

In[98]:=
```
Plot[statesol[x], {x, 0, 1}]
```

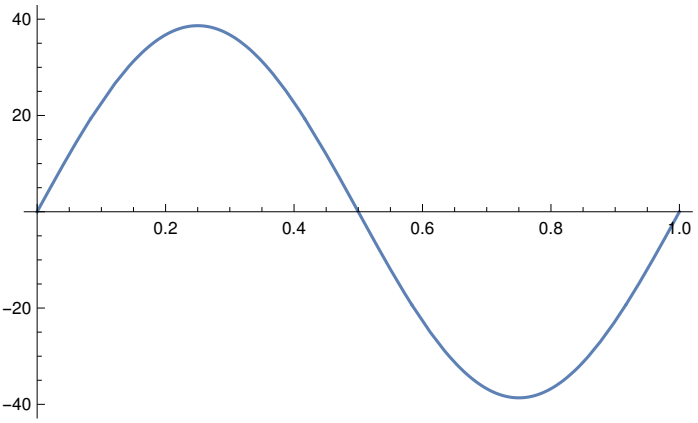
Out[98]=



In[99]:=
```
yd[x_] := statesol[x] + 4 Pi Pi Sin[2 Pi x]
```

In[100]:=
```
Plot[yd[x], {x, 0, 1}]
```

Out[100]=



Optimal objective function value

In[101]:=
```
1/2 Integrate[(4 Pi Pi Sin[2 Pi x])^2 , {x, 0, 1}] +
 β/2 Integrate[ Abs[u[x]], {x, 0, 1}]
```

Out[101]= $5 + 4\,\pi^4$

In[102]:=
```
N[%, 20]
```

Out[102]= 394.63636413600974895