Are you a robot? ☐

Robot Control - Theory Exam - 2024/25

Every problem has to be solved on a separate sheet of paper. Write your student ID and name on every sheet of paper. You can use a calculator and a ruler. You have 60 minutes to solve the problems.

Problem 1 (6 points)

A mass-spring system is subject to damping and a nonlinear restoring force, modeled by the equation:

$$\ddot{z} + 2\beta\dot{z} + \alpha z + \gamma z^2 = 0$$

where: $z(t)$ is the displacement, $\beta = 0.5$ is the damping coefficient, $\alpha = 1$ is the linear stiffness coefficient, $\gamma = 1$ is a nonlinear coefficient.

1. (4 points) Linearize the system around the fixed points.
2. (1 point) Perform two simulation steps using the Forward Euler method and a timestep $\Delta t = 0.1$ starting at the fixed points.
3. (1 point) Discuss the results. Will the system stay at the fixed points during the simulation? Why yes or why not? Does the result depend on the choice of the timestep? How the behaviour at the fixed points compares to any other initial state?

Hint: Let $\mathbf{x}(t)$ be the system state. If the system evolves according to $\dot{\mathbf{x}} = f(\mathbf{x})$, the Forward Euler integration scheme is given by:

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \Delta t f(\mathbf{x}_i)$$

Problem 2 (6 points)

Consider a planar (2D) robot with three revolute joints and three links. The table below provides some configurations $(\theta_1, \theta_2, \theta_3)$ of the robot, in degrees, along with the corresponding positions of the end effector (x, y), in meters.

θ_1	θ_2	θ_3	x	y
0	0	0	6	0
90	0	0	0	6
0	90	0	3	3
0	0	90	5	1
270	270	270	?	?
?	?	?	-5	-1

Tasks:

1. **Part 1: Robot Structure and Missing Values (3 points)**

   (a) Draw a diagram of the robot, labeling the links l_1, l_2, and l_3, and the joint angles θ_1, θ_2, and θ_3. Indicate the direction of positive rotation for each joint. Based on the provided data, determine the lengths of the links l_1, l_2, and l_3. Explain your reasoning.

   (b) Calculate the missing values in the table above.

2. **Part 2: Inverse Kinematics (3 points)**

   (a) How many solutions does the inverse kinematics problem have for this robot (ignoring self-collisions but keeping joint limits $\theta_x \in [0, 360)$)? Briefly explain your reasoning.

   (b) Find **one** set of joint angles $(\theta_1, \theta_2, \theta_3)$ that solves the inverse kinematics problem. You may express your solution using inverse trigonometric functions (e.g., arcsin, arccos, arctan).

Problem 3 (6 points)

A pinhole camera with the following parameters is used to capture images:

- Focal length: f = unknown (assumed equal for x and y)
- Principal point: $c = (320, 240)$ pixels
- Image resolution: 640 x 480 pixels
- Camera pose (in world coordinates): Position $P_c = (0, 0, 0)$, oriented along the positive z-axis (i.e., looking in the +z direction).

The camera, in world frame, is located at the position p = (0, 0, 0) and is oriented in the direction of the z axis.

Tasks:

1. **Part 1: Focal Length Calculation (3 points)** A 3D point P_w = (100, 100, 10) in world coordinates is projected onto the image plane at pixel coordinates p = (370, 290). Determine the focal length f of the camera. Assume the lens distortion is negligible.

2. **Part 2: Camera Translation (3 points)**

   The camera is translated (without rotation) in world coordinates. Determine the translation vector T such that the 3D point P_w = (100, 100, 10) is projected onto the **center** of the image. Is this translation unique? Explain your reasoning.