

Problem 1

$$\mu \sim N(\mu_0, \sigma_0^2)$$

$$P(\mu) = \frac{1}{\sqrt{2\pi}\sigma_0} \cdot e^{-\frac{(\mu-\mu_0)^2}{2\sigma_0^2}}, \ln P(\mu) = -\ln \sqrt{2\pi}\sigma_0 - \frac{(\mu-\mu_0)^2}{2\sigma_0^2}$$

$$\ln L(\mu) = -\ln \sqrt{2\pi} - \frac{\sum_{i=1}^n (x_i - \mu)^2}{2}$$

$$-l = \frac{(\mu-\mu_0)^2}{2\sigma_0^2} + \frac{\sum_{i=1}^n (x_i - \mu)^2}{2}$$

$$l' = \frac{(\mu-\mu_0)}{\sigma_0^2} + \sum_{i=1}^n (x_i - \mu) = 0$$

$$\mu - \mu_0 - \sum_{i=1}^n (x_i - \mu)\sigma_0^2 = 0$$

$$\mu = \frac{\mu_0 + \sigma_0^2 \cdot \sum_{i=1}^n x_i}{\sigma_0^2 \cdot n + 1}$$