



2023 Spring MIMC Contest!

Please read the instructions carefully before you begin.

INSTRUCTIONS

1. DO NOT SCROLL DOWN TO THE PROBLEMS UNTIL YOU ARE READY.
2. This is a ten question short answer test. Each answer is unique and is not limited to integers.
3. You **must** simplify your answers to the simplest possible – that includes rationalizing the denominator, reducing a fraction, etc. You will **not** receive credit for an unsimplified answer.
4. SCORING: You will receive 1 point for each correct answer and 0 points for each incorrect or blank answer.
5. No aids are permitted other than scratch paper, rulers, compass, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. When you are ready to start the test, you can begin working on the problems. You will have 30 minutes to complete the test.
8. This is a qualification test, and that being said, every participant with score higher than the threshold would qualify to round 1.
9. When you finish the exam, fill in and submit the Google Form.
10. Enjoy the problems!

The MIMC Committee reserves the right to disqualify scores from an individual if it determines that the required security procedures were not followed.

A binary tree is a data structure in Computer Science which there is a single root, and for each node, there are two or less children for each parent. In this case, define a *perfect* binary tree to be a binary tree with n layers such that each node has two children nodes except the ones in the last layer, which have none. For an n layer binary tree, Josh randomly chooses a node in the tree and a different node in the tree. Then, he computes the shortest distance between the two nodes he chose. Define the distance between the two nodes to be the shortest number of branches need to reach the other node from one node.

1. Consider a 3-layer *perfect* binary tree. How many nodes are there in total and what is the maximum distance between two nodes in this binary tree? (5 points)
2. Derive a general form for a n layer binary tree for question 1. That is, find the number of nodes and the maximum distance between two nodes in a *perfect* binary tree of n layers. (10 points)

For a n layer *perfect* binary tree, mark each node with a unique positive integer that is between 1 and the number of nodes, N . Define $\text{dist}(a, b)$ to be the distance between node a and node b , and let

$$F(n) = \sum_{i=1}^N \sum_{j=1}^N \text{dist}(i, j)$$

3. Find $F(2)$ and $F(3)$. (10 points)
4. Find $F(4)$ and $F(5)$. (20 points)
5. For any specific branch in the binary tree, find the total number of times it will be counted to calculate $F(n)$. A branch is counted if that branch is used in the shortest distance to travel between two nodes. (15 points)
6. Find a general form of $F(n)$. (20 points)
7. Can you extend some of the problems above to ternary tree? (20 points)

ADDITIONAL INFORMATION

1. The Committee on the Michael595 & Interstigation Math Contest (MIMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The MIMC also reserves the right to disqualify score from a test taker if it is determined that the required security procedures were not followed.
2. The publication, reproduction or communication of the problems or solutions of the MIMC 10 will result in disqualification. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules except the private discussion form.

Sincerely, the MIMC mock contest cannot come true without the contributions from the following testsolvers, problem writers and advisors:

Michael595 (Problem Writer)

Interstigation (Problem Writer)