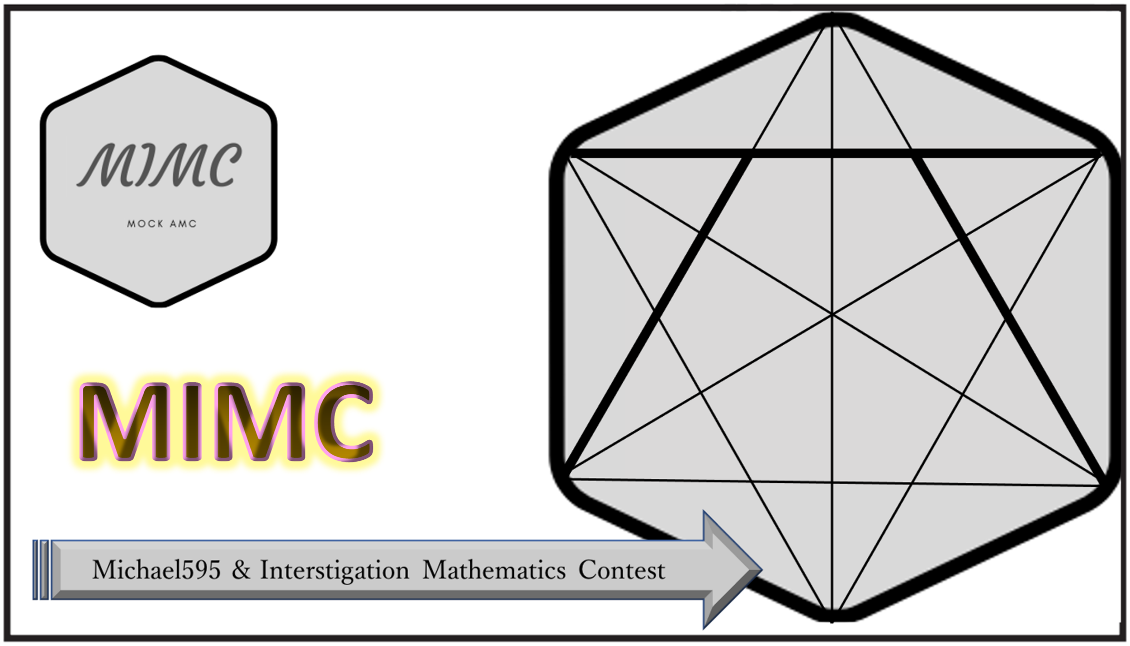

2023 Spring MIMC Contest!

Please read the instructions carefully before you begin.

INSTRUCTIONS

1. DO NOT SCROLL DOWN TO THE PROBLEMS UNTIL YOU ARE READY.
2. This is a ten question short answer test. Each answer is unique and is not limited to integers.
3. You **must** simplify your answers to the simplest possible – that includes rationalizing the denominator, reducing a fraction, etc. You will **not** receive credit for an unsimplified answer.
4. SCORING: You will receive 1 point for each correct answer and 0 points for each incorrect or blank answer.
5. No aids are permitted other than scratch paper, rulers, compass, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. When you are ready to start the test, you can begin working on the problems. You will have 30 minutes to complete the test.
8. This is a qualification test, and that being said, every participant with score higher than the threshold would qualify to round 1.
9. When you finish the exam, fill in and submit the Google Form.
10. Enjoy the problems!

Defn. Let $\gcd(x, y)$ denote the greatest common divisor of x and y. Note that you **must** show your work even for questions starting with "find".

1. Find $\gcd(37, 19)$. (5 points)
2. Find $\gcd(123456789061, 123456789011)$. (10 points)
3. Describe the method you used to find the gcd of two large numbers. For example, describe how you found the gcd for the numbers in problem 2. (10 points)
4. Find $\gcd(37, 19)$ using the method you described above. (5 points)

Defn. Let the continued fraction be a fraction of the form

$$a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \cdots}}.$$

For example, the continued fraction for $\frac{7}{4}$ is

$$\frac{7}{4} = 1 + \frac{1}{1 + \frac{1}{3}}.$$

5. Find the continued fraction representation of $\frac{37}{19}$. Compare that with your result from problem 4. (10 points)
6. Find the continued fraction representation of $\frac{123456789061}{123456789011}$. (5 points)

   Beyond rewriting fractions, continued fractions can also be used to approximate irrational numbers. For example, very famously, the continue fraction of the golden ratio, $\varphi = \frac{1+\sqrt{5}}{2}$ is

   $$\varphi = 1 + \frac{1}{1 + \frac{1}{1 + \cdots}}.$$

   You may check that is indeed the golden ratio by setting the denominator to be φ, as it repeats infinitely. Now we get that $\varphi = 1 + \frac{1}{\varphi}$. Solving it gives $\varphi = \frac{1+\sqrt{5}}{2}$.

7. Find the continued fraction approximation of $\sqrt{2}$ up to 5 places (that is, until a_5; see definition). (7 points)
8. Find the continued fraction approximation of $\sqrt[3]{2}$ up to 5 places. (10 points)
9. Does the approximation for $\sqrt{2}$ and $\sqrt[3]{2}$ terminate? Does it repeat? If it terminates or repeats, find the exact form of the continued fraction and prove it. (11 points)
10. Prove or disprove that the continued fraction representation of a number terminates if and only if the number it approximates is rational. (12 points)
11. Find and prove any conditions for the continued fraction representation of a number to repeat indefinitely. Compare that to when decimal representation of a number terminates, repeats, or infinitely without repeat. Can you define a new notion of rationality? (15 points)

ADDITIONAL INFORMATION

1. **The Committee on the Michael595 & Interstigation Math Contest (MIMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The MIMC also reserves the right to disqualify score from a test taker if it is determined that the required security procedures were not followed.**
2. **The publication, reproduction or communication of the problems or solutions of the MIMC 10 will result in disqualification. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules except the private discussion form.**

Sincerely, the MIMC mock contest cannot come true without the contributions from the following testsolvers, problem writers and advisors:

Michael595 (Problem Writer)

Interstigation (Problem Writer)