2023 Summer MIMC Contest!

Please read the instructions carefully before you begin.

INSTRUCTIONS

1. DO NOT SCROLL DOWN TO THE PROBLEMS UNTIL YOU ARE READY.

2. This is a free-response test. **You must prove all answers written in order to get full credit** unless otherwise stated.

3. You <u>**must**</u> simplify your answers to the simplest possible – that includes rationalizing the denominator, reducing a fraction, etc. You will <u>**not**</u> receive credit for an unsimplified answer.

4. SCORING: Point value for each question is labeled after each question.

5. No aids are permitted other than scratch paper, rulers, compass, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.

6. Figures are not necessarily drawn to scale.

7. When you are ready to start the test, you can begin working on the problems. You will have 30 minutes to complete the test.

8. Note your answer should be in the simplest form. Any other answer may be accepted as long as it is correct. There may be multiple valid formulas.

9. You **may** reference from previous questions, but not the other way around. For example, you may use the result from question 2 in question 7, but you may not use the result from question 7 in question 2.

10. When you finish the exam, upload your submission as an attachment to Michael.Interstigation on Art of Problem Solving as soon as possible.

11. Enjoy the problems!

In a circle of n people labeled $1, 2, \ldots, n$ counterclockwise, starting from the first person, he would slay the person to the right of him (counterclockwise) and pass the sword to the next alive person to continue the slaying. That is, person 1 would slay person 2 and give the knife to person 3, and person 3 would slay person 4 and give the knife to person 5, and so on. (At some point it looks back, so when person n has the knife, he would slay person 1 and give the knife to person 3.)

For problem 1*, participants are allowed to only submit a diagram without explanations.*

1. (a) Through a diagram/graph, find the person who will survive at the end when $n = 5$. (5 points)

   (b) Through a diagram/graph, find the person who will survive at the end when $n = 7, 9$. (5 points)

   (c) Through a diagram/graph, find the person who will survive at the end when $n = 4, 8, 16$. (10 points)

2. Boyu claims that in order to survive the game, the following strategy will always work:

   (a) When there is an even number of players, always stand in the first position.

   (b) When there is an odd number of players, always stand in the exact middle position. That is, if there are $2n + 1$ players, Boyu would occupy the $(n + 1)$st position.

   Is he correct? If he is correct, prove his claim. Otherwise, give a counterexample and *an example that he is correct coincidentally.* (15 points)

3. Find the person who will survive at the end when $n = 25$ by using the observation from problem, and check your answer with a diagram. (10 points)

   Let $f(n)$ be the position of the player who survived when there are n players.

4. Find and prove an explicit formula for $f(n)$ that applies to all positive integers n. (20 points)

5. In a circle of 1285 people labeled $1, 2, \ldots, 1285$ counterclockwise, starting from the first person, he would slay the person to the right of him (counterclockwise) and pass the sword to the next alive person to continue the slaying. That is, person 1 would slay person 2 and give the knife to person 3, and person 3 would slay person 4 and give the knife to person 5, and so on. (At some point it looks back, so when person 1285 has the knife, he would slay person 1 and give the knife to person 3.) If you want to survive but must play the game, where would you sit? In other words, find $f(1285)$. (5 points)

   Now, the game is revamped! In a circle of n people labeled $1, 2, \ldots, n$ counterclockwise, starting from the first person, he would slay the person that is 2 spaces away from him (person 1 would slay person 3 instead of 2), and the revamped game has all other procedures identical to the old game. Let $g(n)$ be, instead, the number of the person who survives the new game. (Note that if there are only two people in a circle, then the person would slay hirself. That is, $g(2) = 2$.) If the last person is killed, then the next person that is alive will obtain the knife.

6. Find $g(7)$, $g(9)$, $g(23)$. (10 points)

7. Find and prove an explicit or a recursive formula for $g(n)$ that applies to all positive integers n. (20 points)

ADDITIONAL INFORMATION

1. The Committee on the Michael595 & Interstigation Math Contest (MIMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The MIMC also reserves the right to disqualify score from a test taker if it is determined that the required security procedures were not followed.

2. The publication, reproduction or communication of the problems or solutions of the MIMC 10 will result in disqualification. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules except the private discussion form.

Sincerely, the MIMC mock contest cannot come true without the contributions from the following testsolvers, problem writers and advisors:

Michael595 (Problem Writer)

Interstigation (Problem Writer)