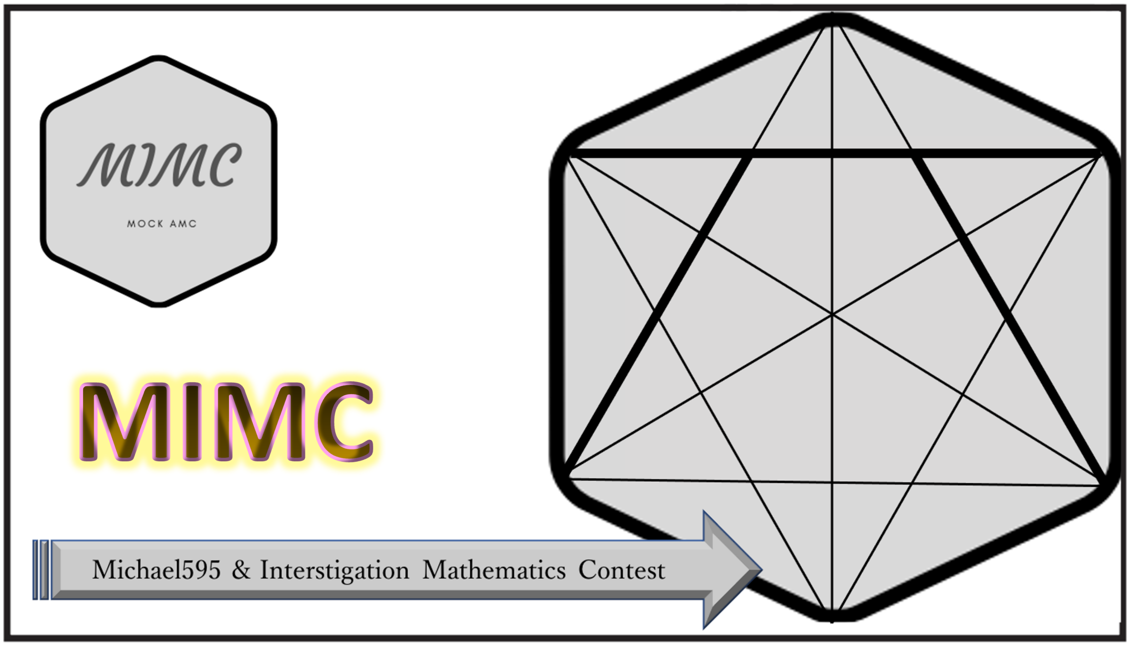

2023 Spring MIMC Contest!

Please read the instructions carefully before you begin.

INSTRUCTIONS

1. DO NOT SCROLL DOWN TO THE PROBLEMS UNTIL YOU ARE READY.
2. This is a free-response test. **You must prove all answers written in order to get full credit.**
3. You **must** simplify your answers to the simplest possible – that includes rationalizing the denominator, reducing a fraction, etc. You will **not** receive credit for an unsimplified answer.
4. SCORING: Point value for each question is labeled after each question.
5. No aids are permitted other than scratch paper, rulers, compass, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. When you are ready to start the test, you can begin working on the problems. You will have 30 minutes to complete the test.
8. When you finish the exam, upload your submission as an attachment to Michael.Interstigation on Art of Problem Solving as soon as possible.
9. Enjoy the problems!

Let a_n be a sequence such that $a_n = 2a_{n-1} + 2^{n-1}$ and $a_0 = 0$.

1. Find a_1, a_2, a_3, a_4, a_5. (5 points)
2. Find a_{2023}. You may leave your answer in exponential form. (10 points)
3. Find and prove an explicit formula for a_n in general. (15 points)
4. Let k be an integer such that $0 \le k < n$. Express a_n in terms of a_k. (10 points)
5. Define a sequence $b_{n,c} = cb_{n-1,c} + c^{n-1}$ for which $c \in \mathbb{N}$ and $b_{0,c} = 0$. Find an explicit formula for $b_{n,c}$ or prove that it does not exist. (15 points)

   Let $f : \mathbb{R} \to \mathbb{R}$ such that $f(n) = a_n$ for all nonnegative integers such that f is continuous and differentiable for all $x \in \mathbb{R}$.

6. How many different functions f are there satisfying the conditions? (10 points)
7. Is there a polynomial f satisfying the conditions? Give an example or prove otherwise. (15 points)

   Let $S_n = \sum\limits_{i=0}^{n} a_n$.

8. Find an explicit formula for S_n. (10 points)
9. Find an explicit formula for $\frac{a_n}{S_n}$. (5 points)
10. Find $\lim\limits_{n\to\infty} \frac{a_n}{S_n}$. (5 points)

ADDITIONAL INFORMATION

1. **The Committee on the Michael595 & Interstigation Math Contest (MIMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The MIMC also reserves the right to disqualify score from a test taker if it is determined that the required security procedures were not followed.**
2. **The publication, reproduction or communication of the problems or solutions of the MIMC 10 will result in disqualification. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules except the private discussion form.**

Sincerely, the MIMC mock contest cannot come true without the contributions from the following testsolvers, problem writers and advisors:

Michael595 (Problem Writer)

Interstigation (Problem Writer)