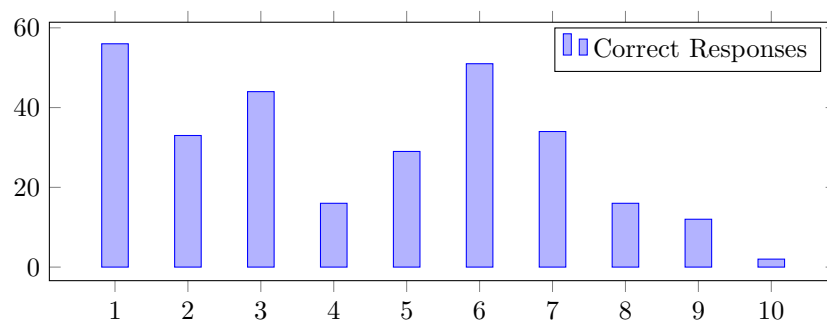


## ANSWERS

#1	#2	#3	#4	#5
16393	$\frac{16\pi\sqrt{3}}{3}$	$\frac{10}{7}$	$4\sqrt{2} + 2\sqrt{2}$	3
#6	#7	#8	#9	#10
384	1	\$25	$k(1, 10, 25, -134, -774, 5945)$ , $k \in \mathbb{R}$	926640

## STATISTICS

Number of Responses:	59	Minimum:	0
Mean:	4.95	First Quartile:	4
Standard Deviation:	2.08	Median:	5
Mode:	5	Third Quartile:	6
Range:	10	Maximum:	10



The MIMC Committee reserves the right to disqualify scores from an individual if it determines that the required security procedures were not followed.

1. The answer is just

$$3 \oplus 4 = 3^2 + 4^7 = 9 + 16384 = \boxed{16393}.$$

2. Divide 16 on both sides to get

$$\frac{x^2}{16/3} + \frac{y^2}{16} = 1,$$

which is an ellipse. The area of the ellipse is  $\pi ab$  where  $a$  and  $b$  are the length of semi-major and minor axes of the ellipse, respectively. Therefore,  $a = \sqrt{16} = 4$  and  $b = \sqrt{16/3} = \frac{4}{\sqrt{3}}$ , so the area of the ellipse is

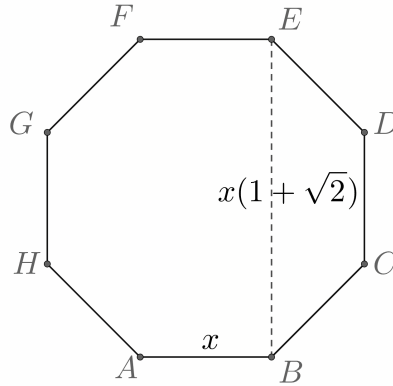
$$\pi ab = \pi(4) \left( \frac{4}{\sqrt{3}} \right) = \frac{16\pi}{\sqrt{3}} = \boxed{\frac{16\pi\sqrt{3}}{3}}.$$

3. Start by solving the first equation. Cross-multiply to get  $a + \sqrt{3} = 3a - 3\sqrt{3}$ , so  $a = 2\sqrt{3}$ .

In the second equation, we can do a bit of simplification by factoring out  $\sqrt{3}$  to get  $\frac{2+k}{2-k} = 6$ , which

$$2 + k = 12 - 6k \implies k = \boxed{\frac{10}{7}}.$$

4. Let the side length of the octagon be  $x$ . Then  $BE = x + \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}x = x(1 + \sqrt{2})$ .



We can break up the image into two trapezoids and a rectangle. Each trapezoid has area

$$\frac{1}{2} [x + x(1 + \sqrt{2})] \left( \frac{x\sqrt{2}}{2} \right) = \frac{(2 + \sqrt{2})\sqrt{2}}{4} x^2 = \frac{2\sqrt{2} + 2}{4} x^2 = \frac{1 + \sqrt{2}}{2} x^2,$$

and the rectangle has area

$$x [x(1 + \sqrt{2})] = (1 + \sqrt{2})x^2.$$

Therefore, the total area is

$$(1 + \sqrt{2} + 1 + \sqrt{2})x^2 = 64,$$

solving for  $x$ , we get

$$x^2 = \frac{32}{1 + \sqrt{2}} \implies x = \frac{4\sqrt{2}}{\sqrt{1 + \sqrt{2}}},$$

which means that  $BE = x(1 + \sqrt{2}) = \frac{4\sqrt{2}}{\sqrt{1 + \sqrt{2}}}(1 + \sqrt{2}) = 4\sqrt{2}\sqrt{1 + \sqrt{2}} = \boxed{4\sqrt{2 + 2\sqrt{2}}}$ .

5. (a): This obviously works because  $1 > 0.9999$ .  
 (b): This does not work because 1.7 is barely less than the longest distance between any two points within the unit cube since the space diagonal is  $\sqrt{1 + 1 + 1} = \sqrt{3}$ , which is about 1.72, and all these space diagonals pass through the center, so it wouldn't be possible to form a triangle with these three lengths.  
 (c): It is easy to prove that it is possible to include a tetrahedron with side length  $\sqrt{2}$  when all the sides are one of the diagonals on a surface, which is a unit square.  
 (d): Notice that the base radius of 0.001 is very small, so we can treat the cylinder as a line of length 1.7. The longest diagonal in a unit cube is  $\sqrt{3} > 1.7$ , so it is possible.

Overall,  $\boxed{3}$  of the four options can fit inside a unit cube.

6. We can either brute force calculate the problem:

$$2^2 + 3^2 + \cdots + 10^2 = 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 + 100 = \boxed{384}.$$

Or we can use the formula that  $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ . That is,

$$2^2 + 3^2 + \cdots + 10^2 = 1^2 + 2^2 + \cdots + 10^2 - 1^2 = \frac{10 \cdot 11 \cdot 21}{6} - 1 = 385 - 1 = \boxed{384}.$$

7. We just set  $f(x) = 1$ :

$$\begin{aligned} \frac{-x^2 + 2x + 1}{1 - x^2} + 2x\sqrt{1 - x^2} &= 1 \\ \frac{-x^2 + 2x + 1 - 1 + x^2}{1 - x^2} + 2x\sqrt{1 - x^2} &= 0 \\ \frac{2x}{1 - x^2} + 2x\sqrt{1 - x^2} &= 0 \\ 2x \left( \frac{1}{1 - x^2} + \sqrt{1 - x^2} \right) &= 0 \end{aligned}$$

The only case for this equation to hold is when  $2x = 0 \implies x = 0$  because the expression  $\frac{1}{1-x^2} + \sqrt{1-x^2} = 0$  is impossible to achieve:  $\frac{1}{1-x^2} > 1$  and  $\sqrt{1-x^2}$  is always greater than or equal to 0, so 0 is the only solution, which there is only  $\boxed{1}$  zero.

8. Let  $H$  denote a head and  $T$  denote a tail. Note that if  $HT, TH, TT$  occur during any time, then Newton is guaranteed to win because Newton has already won with  $THH$  in order for two heads in a row to appear to make the  $HHT$  in Farad's winning position as there is always a tail before a head. Only when  $HH$  is the first two flips would Farad be guaranteed to win.

Thus the probability of Farad to win is  $\frac{1}{4}$ , and since he would get \$100 per win, he would be willing to pay  $\frac{1}{4}(\$100) = \boxed{\$25}$ .

9. We will use the well-known point-distance formula that the distance between  $(x_1, y_1)$  and a line  $ax + by + c = 0$  is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

From the definition of a parabola, let the coordinates of the focus be  $(p_1, p_2)$ , then the distance from the focus to a point on the parabola is equal to the distance from that point to the directrix.

Thus, the equation below holds:

$$\begin{aligned}\sqrt{(x-2)^2 + (y-15)^2} &= \frac{|5x - y + 3|}{\sqrt{5^2 + 1^2}} \\ (x-2)^2 + (y-15)^2 &= \frac{(5x - y + 3)^2}{26} \\ 26(x^2 - 4x + 4 + y^2 - 30y + 225) &= (5x - y)^2 + 3^2 + 6(5x - y) \\ 26x^2 - 104x + 104 + 26y^2 - 780y + 5850 &= 25x^2 + y^2 - 10xy + 9 + 30x - 6y \\ x^2 - 10xy + 25y^2 - 134x - 774y + 5945 &= 0\end{aligned}$$

Therefore, the answer is  $\boxed{(1, 10, 25, -134, -774, 5945)}$ . Note that the equation is set to zero on the right side, so any multiple of this sextuple would work. Thus any answer here would work:

$$k(1, 10, 25, -134, -774, 5945), \quad k \in \mathbb{R}.$$

10. We need to make sure that

1,2,3	2,4,6	3,6,9
4,8,12	5,10	6,12

are in this specific ordering. Note that we can group the thirteen numbers into five groups:

$$(1, 2, 3, 4, 6, 8, 9, 12) \quad (5, 10) \quad (7) \quad (11) \quad (13)$$

such that there are no requirements between two different groups. After ordering the numbers internally within each group, there are  $\frac{13!}{8!2!1!1!1!} = 77220$  ways to order the five groups into an overall ordering.

There are 12 ways to order the first group:

$$\begin{array}{ll}(1, 2, 3, 4, 6, 8, 9, 12) & (1, 2, 4, 3, 6, 8, 12, 9) \\ (1, 2, 3, 4, 6, 8, 12, 9) & (1, 2, 4, 3, 6, 9, 8, 12) \\ (1, 2, 3, 4, 6, 9, 8, 12) & (1, 2, 4, 3, 8, 6, 9, 12) \\ (1, 2, 3, 4, 8, 6, 9, 12) & (1, 2, 4, 3, 8, 6, 12, 9) \\ (1, 2, 3, 4, 8, 6, 12, 9) & (1, 2, 4, 8, 3, 6, 9, 12) \\ (1, 2, 4, 3, 6, 8, 9, 12) & (1, 2, 4, 8, 3, 6, 12, 9)\end{array}$$

There is 1 way to order each of the last four groups, so overall there are  $77220 \cdot 12 = \boxed{926640}$  orderings.