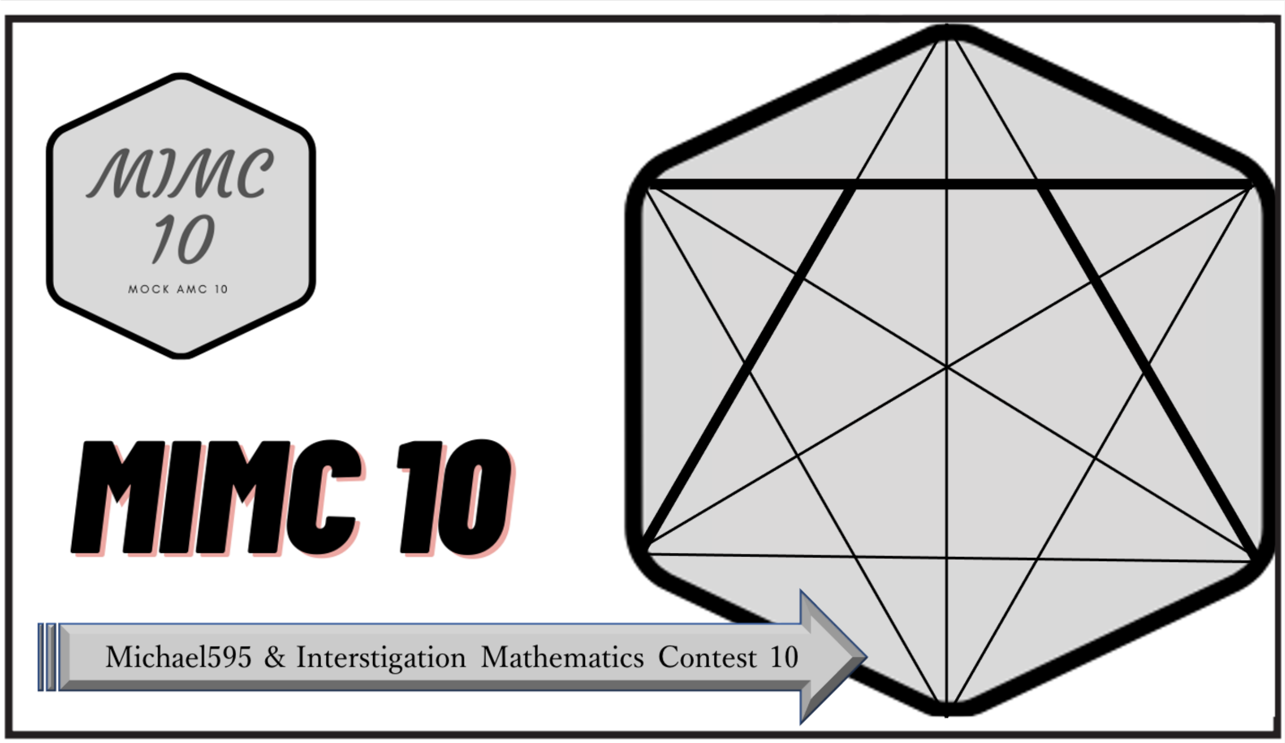


INSTRUCTIONS

1. DO NOT SCROLL DOWN TO THE PROBLEMS UNTIL YOU ARE READY.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil if you would like to get one from here. Check the blackened circles for accuracy and erase errors and stray marks completely. However, only answers on the MIMC Google Form found on the AoPS community page or here will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, rulers, compass, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. When you are ready to start the test, you can begin working on the problems. You will have 75 minutes to complete the test.
8. When you finish the exam, fill in and submit the Google Form.
9. Enjoy the problems!

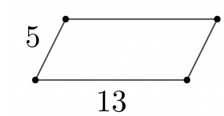
The MIMC Committee reserves the right to disqualify scores from an individual if it determines that the required security procedures were not followed.

The Committee will publish a projected AIME floor, Distinction and Distinguished Honor Roll, however, there will not be a mock AIME hosted by MIMC Committee.

1. Calculate $1^1 + 2^2 + 3^3 + 4^4$.

   (A) 10 **(B)** 38 **(C)** 96 **(D)** 286 **(E)** 288

2. If the height of the parallelogram below is 7, find its area.



   (A) 35 **(B)** 60 **(C)** 65 **(D)** 91 **(E)** This parallelogram cannot exist.

3. Find $\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1}}}} - \frac{1}{1+\frac{1}{1+\frac{1}{1}}}$.

   (A) $-\frac{2}{5}$ **(B)** $-\frac{1}{15}$ **(C)** 0 **(D)** $\frac{1}{15}$ **(E)** $\frac{2}{5}$

4. There are 100 locked boxes placed in a 10×10 grid. There are keys for adjacent boxes in each box (diagonally touching does not count as adjacent). There may be more than one key for each box, but you only need one key to unlock the corresponding box. If you only have the key to the bottom left box to start with, what is the minimum number of boxes you have to open in order to obtain the key to the top right box?

   (A) 9 **(B)** 10 **(C)** 18 **(D)** 19 **(E)** 20

5. Let $ABCD$ be a square with diagonal length d, and let $EFGH$ be a square with diagonal length d^2. Find the ratio of the area of the square $EFGH$ to the area of the square $ABCD$. Express your answer in terms of d.

   (A) $d\sqrt{2}$ **(B)** d **(C)** $2d$ **(D)** d^2 **(E)** $2d^2$

6. How many ordered triples of positive integers (a, b, c) are there satisfying both $a^2 + b^2 = c^2$ and $a^6 + b^6 = c^6$?

   (A) 0 **(B)** 3 **(C)** 18 **(D)** 54 **(E)** infinitely many

7. Find the smallest possible positive integer k such that $11^{99} - k$ is a multiple of 15.

   (A) 1 **(B)** 6 **(C)** 11 **(D)** 13 **(E)** 14

8. Define an operation $a \star b = 2ab + a + b$ for any integers a, b. How many ordered pairs of integers (x, y) are there such that $x \star y = 10$?

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 12

9. An odd integer is called *uniform* if the remainder when the integer is divided by $2, 3, 4, 5, 6, 7, 8, 9$ are all the same. For example, 1 is *uniform* because the remainder is always 1, but 3 is not *uniform*. What is the minimum positive difference between two *uniform* integers?

   (A) 1 **(B)** 2 **(C)** 1260 **(D)** 2520 **(E)** 362880

10. In a group of soccer tournament, four teams, A, B, C, and D play in a round-robin style where each team plays every other team exactly once. For example, if the final score between A and B is $x : y$, then A wins if $x > y$, B wins if $x < y$, and they draw otherwise. x goals are scored for A and conceded for B, and y goals are scored for B and conceded for A. In a score table, all of the statistics from all games for one team is added up. Sometime in the middle of the tournament, the score table looks like this:

Team	Games Played	Wins	Draws	Losses	Goals Scored	Goals Conceded
A	3	1	0	2	1	3
B	3	2	1	0	5	2
C	2	1	1	0	1	0
D	2	0	0	2	2	4

Then what is the final score of the match between B and D?

(A) $B\ 1 : 0\ D$ **(B)** $B\ 2 : 1\ D$ **(C)** $B\ 3 : 0\ D$ **(D)** $B\ 3 : 2\ D$
(E) The match has not happened yet.

11. Two real numbers x, y such that $-4 \le x \le y \le 4$ are chosen at random. What is the probability that $|x + y| = |x| + |y|$?

(A) $\frac{1}{4}$ **(B)** $\frac{25}{64}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{9}{16}$ **(E)** $\frac{25}{32}$

12. There are three circles. ω_1 is inscribed in a regular octagon with side length 1, ω_2 is inscribed in a regular hexagon with side length 1, ω_3 is inscribed in a square with side length 1. Let $[\omega]$ denote the area of the circle ω. If $\frac{[\omega_1]}{[\omega_2][\omega_3]} = \frac{a+b\sqrt{c}}{d\pi}$ in the simplest form, find $a + b + c + d$ (some of a, b, c, d may be 0).

(A) 4 **(B)** 7 **(C)** 15 **(D)** 25 **(E)** 30

13. Given real numbers x, m, find the minimum value of

$$\sqrt{\sqrt{x^2 - 2mx + 2x + m^2 - 2m + 1} + m - 1}$$

given that it is defined in the real numbers.

(A) 0 **(B)** $\sqrt{\sqrt{2} - 1}$ **(C)** $\sqrt{2}$ **(D)** $\sqrt{\sqrt{2} + 1}$ **(E)** $\sqrt{3}$

14. For all positive integers $n > 3$, what is the minimum number of positive divisors of

$$n(n^2 - 1)(n^2 - 4)(n^2 - 9)?$$

(A) 20 **(B)** 30 **(C)** 40 **(D)** 60 **(E)** 120

15. Koal is playing a game of buttons! Koal starts with 1, and there are eight buttons in front of them that takes an the number k that Koal has and updates the number to $2k, 3k, 4k, 5k, 6k, 7k, 8k, k + 1$, respectively. Given that Koal presses each button once, what is the probability that Koal will get an odd number?

(A) $\frac{1}{20}$ **(B)** $\frac{3}{40}$ **(C)** $\frac{1}{10}$ **(D)** $\frac{1}{8}$ **(E)** $\frac{1}{5}$

16. Let A be a sequence of positive integers in increasing order such that all elements in A can be expressed as
$$3^{2^{a_1}} + 3^{2^{a_2}} + \cdots + 3^{2^{a_n}}$$
for which $a_1, a_2, \ldots, a_n$ are distinct nonnegative integers. Given that all integers a that can be written in that form are in the sequence A, find the remainder when A_{30} is divided by 1000.

(A) 213 **(B)** 372 **(C)** 375 **(D)** 492 **(E)** 495

17. Kidderminster has five 4-sided dice, and he rolls the dice as follows. He starts by rolling all five dice at once. Whenever he rolls a 1, he will stop rolling that die. Then he throws the remaining dice again until there are none left. Let P_n be the probability that Kidderminster rolls exactly n dice at once at some point. Find P_3.

(A) $\frac{351}{781}$ **(B)** $\frac{18198}{27335}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{1856}{2783}$ **(E)** $\frac{3}{4}$

18. It's well known that $1 + 3 + \cdots + (2k-1) = k^2$. What is the number of positive divisors of
$$1^2 + 3^2 + 5^2 + \cdots + 99^2?$$

(A) 12 **(B)** 18 **(C)** 24 **(D)** 36 **(E)** 48

19. Find the number of ordered triples (x, y, z) such that $x, y, z \in \{1, 2, 3, 4, 5, 6\}$ and that 5 divides the expression
$$x^2 - xy + y^2 - yz + z^2 - zx.$$

(A) 0 **(B)** 6 **(C)** 12 **(D)** 15 **(E)** 18

20. Let a_n be a sequence defined as $a_n = 2a_{n-1} + 4a_{n-2}$. Given that $a_0 = 0$ and $a_1 = 1$, find the number of factors of 2 in a_{2023}.

(A) 0 **(B)** 2022 **(C)** 2023 **(D)** 2024 **(E)** 4046

21. How many values of n are there such that $0 \le n < 300$ and $n^4 - 1$ is a multiple of 30?

(A) 8 **(B)** 15 **(C)** 30 **(D)** 80 **(E)** 150

22. There is a point A on a circle α with center O and radius 1. Draw a circle β with center A and radius OA. The let the points of intersection of α and β be B and C. Then draw circle γ with center B and radius BC where γ intersects α again at point D. Let the line OB intersect γ at points E, F. Then let line DE intersect α again at G, line DF intersect α again at H. Find the area of the quadrilateral $AGDH$.

(A) $\frac{\sqrt{6}-\sqrt{2}}{2}$ **(B)** $\sqrt{2}$ **(C)** $\sqrt{3}$ **(D)** $\frac{\sqrt{2}+\sqrt{6}}{2}$ **(E)** 2

23. If I choose a positive integer n less than 1000, the probability that the decimal expansion of $\frac{1}{n}$ is a purely repeating decimal with a period of 6 can be written as $\frac{a}{b}$, find $a+b$.

(A) 51 **(B)** 511 **(C)** 1018 **(D)** 1019 **(E)** 1024

24. Let $a_1, a_2, \ldots, a_{2023}$ be the roots of the polynomial

$$x^{2023} - 2023 \cdot 2022x^{2021} + c_{2021}x^{2020} + c_{2020}x^{2019} + \cdots + c_2x + c_1$$

where each c_i is a real constant. Given that $a_1, a_2, \ldots, a_{2023}$ form an arithmetic sequence, what is the square of the common difference of the arithmetic sequence?

(A) 0 **(B)** $\frac{3}{253}$ **(C)** $\frac{4}{337}$ **(D)** $\frac{6}{253}$ **(E)** $\frac{8}{337}$

25. Let ABC be a triangle such that $AB = 15, AC = 4\sqrt{13}$ and $BC = 17$. Let D be a point on BC such that $AD \perp BC$, and let E a point on AC such that $DE \perp AC$. Let the intersection of lines AB and DE be B', and let C' be the point such that $B'C' \| BC$ and $\triangle AB'C' \sim \triangle ABC$. Let ω be the circumcircle of $\triangle AB'C'$, and define P as the intersection point of line AD and ω, and $P \neq A$. Furthermore, let M_1 be the intersection closer to B of line BC with ω. Find M_1P.

(A) 20 **(B)** $10 + 12\sqrt{5}$ **(C)** 30 **(D)** $2\sqrt{235}$ **(E)** 32