Deterministic Operations Research

Sections 3.1-3.3:
Modeling - Linear Programs

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Version Compilation date: July 6, 2022

3. Modeling: Linear Programming

Example: Toy Maker Excel PuLP Gurobipy

Consider the problem of a toy company that produces toy planes and toy boats. The toy company can sell its planes for $10 and its boats for $8 dollars. It costs $3 in raw materials to make a plane and $2 in raw materials to make a boat. A plane requires 3 hours to make and 1 hour to finish while a boat requires 1 hour to make and 2 hours to finish. The toy company knows it will not sell anymore than 35 planes per week. Further, given the number of workers, the company cannot spend anymore than 160 hours per week finishing toys and 120 hours per week making toys. The company wishes to maximize the profit it makes by choosing how much of each toy to produce.

We can represent the profit maximization problem of the company as a linear programming problem. Let x_1 be the number of planes the company will produce and let x_2 be the number of boats the company will produce. The profit for each plane is $\$10 - \$3 = \$7$ per plane and the profit for each boat is $\$8 - \$2 = \$6$ per boat. Thus the total profit the company will make is:

$$z(x_1, x_2) = 7x_1 + 6x_2 \tag{3.1}$$

The company can spend no more than 120 hours per week making toys and since a plane takes 3 hours to make and a boat takes 1 hour to make we have:

$$3x_1 + x_2 \leq 120 \tag{3.2}$$

Likewise, the company can spend no more than 160 hours per week finishing toys and since it takes 1 hour to finish a plane and 2 hour to finish a boat we have:

$$x_1 + 2x_2 \leq 160 \tag{3.3}$$

Finally, we know that $x_1 \leq 35$, since the company will make no more than 35 planes per week. Thus the complete linear programming problem is given as:

$$\begin{cases} \max \ z(x_1, x_2) = 7x_1 + 6x_2 \\ s.t. \ 3x_1 + x_2 \leq 120 \\ \quad x_1 + 2x_2 \leq 160 \\ \quad x_1 \leq 35 \\ \quad x_1 \geq 0 \\ \quad x_2 \geq 0 \end{cases} \tag{3.4}$$

3.1 Modeling and Assumptions in Linear Programming

Outcomes

1. *Address crucial assumptions when chosing to model a problem with linear programming.*