

Mathematical Programming

gray 50% complete. Goal 80% completion date: August 20

Notes: This chapter is meant to be an introduction to all the types of deterministic problems that we might discuss. It should

[inline]Add intro that explains the format of problems, i.e., what the complexity comment means in each problem and a

Identify reasons for studying operations research

Define "Mathematical Programming"

Learn about different applications of the tools in this book

Explore the different types of optimization models and what types we will see in this book.

Why study operations research?

What is Mathematical Programming?

Applications

Types of Optimization problems We will state main general problem classes to be associated with in these notes. These

[width = 1][h]problem-class-diagram

Along with each problem class, we will associate a complexity class for the general version of the problem. See sec:comp

Linear Programming (LP) [inline]Describe applications and add images Some linear programming background, theory

Linear programming can come in several forms, whether we are maximizing or minimizing, or if the constraints are \leq , $=$, \geq .

[inline]Can shrink figure if there are not a lot of numbers of greek letters involved. [Linear programming constraints and ob

[inline]Move this to simplex chapter Start with a problem in form given as eq:LP and convert it to standard form eq:st

Binary Integer programming (BIP) Given a matrix $A \in m \times n$, vector $b \in m$ and vector $c \in n$, the *binary integer programming*

A slightly more general class is the class of *Integer Linear Programs* (ILP). Often this is referred to as *Integer Program* (IP)

[Comparing the LP relaxation to the IP solutions.][scale = 0.3][h]wiki/File/integer-programming.png

Integer Linear Programming (ILP) Given a matrix $A \in m \times n$, vector $b \in m$ and vector $c \in n$, the *integer linear programming*

An even more general class is *Mixed-Integer Linear Programming* (MILP). This is where we have n integer variables x_1, \dots, x_n

Below, the matrix A now has $n + d$ columns, that is, $A \in m \times n + d$. Also note that we have not explicitly enforced non-ne

Non-Linear Programming (NLP) NLP Given a function $f(x): \mathbb{R}^n \rightarrow \mathbb{R}$ and other functions $f_i(x): \mathbb{R}^n \rightarrow \mathbb{R}$ for $i = 1, \dots, m$, the *non-linear programming*

Nonlinear programming can be separated into convex programming and non-convex programming. These two are very

Observe that convex programming is a generalization of linear programming. This can be seen by letting $f(x) = c^T x$ and

Non-Convex Non-linear Programming [inline]Move this to later chapter on complexity of NLP When the function f or

IP as NLP As seen above, quadratic constraints can be used to create a feasible region with discrete solutions. For exam

has exactly two solutions: $x = 0, x = 1$. Thus, quadratic constraints can be used to model binary constraints. Binary Integer

Alternatively, consider the transformation where $x_i \in \{-1, 1\}$.

This can be reformulated with a single nonconvex constraint as

Machine Learning [inline]Many machine learning problems fall into this category. Todo: describe applications, give refer

Machine learning problems are often cast as continuous optimization problems, which involve adjusting parameters to m

*Loss Function Minimization

In supervised learning, this objective is typically a loss function L that quantifies the discrepancy between the prediction

where N is the number of data points, l is a per-data-point loss (e.g., squared error for regression or cross-entropy for classi

*Clustering Formulation

Clustering, on the other hand, seeks to group or partition data points such that data points in the same group are mor

where C_j represents the j -th cluster and c_j is the centroid of that cluster.

This formulation converts a clustering problem into a MILP problem. In practice, numerous algorithms