W07 - Homework

Stepwise problems - Fri. 11:59pm

Sequences

01

L'Hopital practice - converting indeterminate form

By imitating the technique in from the L'Hopital's Rule example, find the limit of the sequence:

$$a_n = \sqrt{n}\,\ln\left(1 + \frac{1}{n}\right)$$

02

Squeeze theorem

Determine whether the sequence converges, and if it does find its limit:

(a) $a_n = \dfrac{\cos^2 n}{2^n}$ (b) $b_n = (2^n + 3^n)^{1/n}$

(Hint for (b): Verify that $3 \leq b_n \leq (2 \cdot 3^n)^{1/n}$ by comparing the 2^n to another copy of 3^n.)

Series

03

General term of a series

Write this series in summation notation:

$$\frac{1}{1} - \frac{2^2}{2 \cdot 1} + \frac{3^3}{3 \cdot 2 \cdot 1} - \frac{4^4}{4 \cdot 3 \cdot 2 \cdot 1} + \cdots$$

(Hint: Find a formula for the general term a_n.)

04

Geometric series

Compute the following summation values using the sum formula for geometric series.

(a) $\displaystyle\sum_{n=0}^{\infty} 5^{-n}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{2 + 3^n}{5^n}$ (c) $\displaystyle\sum_{n=-4}^{\infty} \left(-\frac{4}{9}\right)^n$ (d) $\displaystyle\sum_{n=0}^{\infty} e^{3-2n}$

Regular problems - Mon. 10:00am

Sequences

05

Computing a sequence by terms

Calculate the first four terms of each sequence from the given general term:

(a) $\cos \pi n$ (b) $\dfrac{n!}{2^n}$ (c) $(-1)^{n+1}$ (d) $\dfrac{n}{n+1}$ (e) $\dfrac{3^n}{n!}$ (f) $\dfrac{(2n-1)!}{n!}$

06

General term of a sequence

Find a formula for the general term (the n^{th} term) of each sequence:

(a) $\dfrac{1}{1}, \dfrac{-1}{8}, \dfrac{1}{27}, \ldots$ (b) $\dfrac{2}{6}, \dfrac{3}{7}, \dfrac{4}{8}, \ldots$ (c) $\dfrac{3}{5}, -\dfrac{4}{25}, \dfrac{5}{125}, -\dfrac{6}{625}, \ldots$

07

Limits and convergence

For each sequence, either write the limit value (if it converges), or write 'diverges'.

(a) $b_n = \dfrac{5n-1}{12n+9}$ (b) $b_n = (-1)^n \left(\dfrac{5n-1}{12n+9}\right)$ (c) $a_n = \sqrt{4 + \dfrac{1}{n}}$ (d) $a_n = \cos^{-1}\left(\frac{n^3}{n^3+1}\right)$

(d) $a_n = 10 + \left(-\dfrac{1}{9}\right)^n$ (e) $a_n = 10 + \left(-\dfrac{1}{9}\right)^n$ (f) $c_n = 1.01^n$ (g) $a_n = 2^{1/n}$

(h) $c_n = \dfrac{n!}{9^n}$ (i) $a_n = \dfrac{3n^2+n+2}{2n^2-3}$ (j) $a_n = \dfrac{\cos n}{n}$ (k) $d_n = \ln 5^n - \ln n!$

(l) $a_n = \left(2 + \dfrac{4}{n^2}\right)^{1/3}$ (m) $c_n = \ln\left(\dfrac{2n+1}{3n+4}\right)$ (n) $y_n = \dfrac{e^n}{2^n}$ (o) $a_n = \dfrac{(\ln n)^2}{n}$

(p) $a_n = \dfrac{(-1)^n(\ln n)^2}{n}$ (q) $b_n = \dfrac{3-4^n}{2+7\cdot 4^n}$ (r) $a_n = \left(1 + \dfrac{1}{n}\right)^n$ (s) $a_n = \dfrac{1}{\ln\left(1+\frac{1}{n}\right)}$

(t) $a_n = n \sin \dfrac{\pi}{n}$

Series

08

Repeating digits

Using the geometric series formula, find the fractional forms of these decimal numbers:

(a) $0.\bar{2} = 0.222222\ldots$ (b) $0.4\bar{9} = 0.4999999\ldots$

09

Series from its partial sums

Suppose we know that the *partial sums* S_n of a series $S = \sum_{n=1}^{\infty} a_n$ are given by the formula $S_n = 5 - \frac{2}{n^2}$.

- (a) Compute a_3.
- (b) Find a formula for the general term a_n.
- (c) Find the sum S.

10

Geometric series - partial sums and total sum

Consider the series:

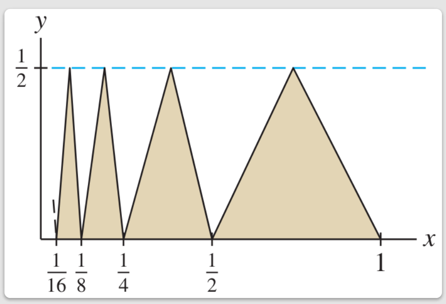
$$\sum_{n=1}^{\infty} \frac{(-8)^{n-1}}{9^n}$$

- (a) Compute a formula for the N^{th} partial sum S_N. (You may apply the known formula or derive it again in this case using the "shift method.")
- (b) By taking the limit of this formula as $N \to \infty$, find the value of the series.
- (c) Find the same value of the series by computing a_0 and r and plugging into $S = \frac{a_0}{1-r}$.

11

Total area of infinitely many triangles

Find the area of all the triangles as in the figure:



(The first triangle from the right starts at 1, and going left they never end.)