Stewart Series Testing

The following problems are drawn directly from p781 of the official textbook by J. Stewart.
They are for optional, additional practice only.
They are not necessarily representative of problems on APMA 1110 exams.

Two similar-looking series are given. Test each one for convergence or divergence:

(1)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5^n + n}$

(2)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/2}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

(3)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{3^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n}$

(4)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n}$ (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n}$

(5)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n^2+1} \right)^n$

(6)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$ (b) $\displaystyle\sum_{n=10}^{\infty} \frac{1}{n \ln n}$

(7)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+n!}$ (b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n} + \frac{1}{n!} \right)$

(8)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$

Test the series for convergence or divergence:

(9) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{n^3+1}$

(10) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n^3+1}$

(11) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2-1}{n^3+1}$

(12) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2-1}{n^2+1}$

(13) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^2}$

(14) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{2n}}{(1+n)^{3n}}$

(15) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

(16) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^4}{4^n}$

(17) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n}}{(2n)!}$

(18) $\displaystyle\sum_{n=1}^{\infty} n^2 e^{-n^3}$

(19) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^3} + \frac{1}{3^n} \right)$

(20) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k\sqrt{k^2+1}}$

(21) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$

(22) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin 2n}{1+2^n}$

(23) $\displaystyle\sum_{k=1}^{\infty} \frac{2^{k-1} 3^{k+1}}{k^k}$

(24) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^4+1}}{n^3+n}$

(25) $\displaystyle\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 5 \cdot 8 \cdots (3n-1)}$

(26) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}-1}$

(27) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$

(28) $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt[3]{k}-1}{k(\sqrt{k}+1)}$

(29) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cos\left(1/n^2\right)$

(30) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{2+\sin k}$

(31) $\displaystyle\sum_{n=1}^{\infty} \tan(1/n)$

(32) $\displaystyle\sum_{n=1}^{\infty} n \sin(1/n)$

(33) $\displaystyle\sum_{n=1}^{\infty} \frac{4-\cos n}{\sqrt{n}}$

(34) $\displaystyle\sum_{n=1}^{\infty} \frac{8+(-1)^n n}{n}$

(35) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$

(36) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{5^n}$

(37) $\displaystyle\sum_{k=1}^{\infty} \frac{k \ln k}{(k+1)^3}$

(38) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$

(39) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\cosh n}$

(40) $\displaystyle\sum_{j=1}^{\infty} (-1)^j \frac{\sqrt{j}}{j+5}$

(41) $\displaystyle\sum_{k=1}^{\infty} \frac{5^k}{3^k+4^k}$

(42) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}}$

(43) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^{n^2}$

(44) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+n\cos^2 n}$

(45) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$

(46) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}}$

(47) $\displaystyle\sum_{n=1}^{\infty} (\sqrt[n]{2}-1)^n$

(48) $\displaystyle\sum_{n=1}^{\infty} (\sqrt[n]{2}-1)$