Circa Geometrica Generalia

G.W. Leibniz, 1678-80?
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[Typescript]

On General Geometrics and the Calculus of Situs or a characteristical picture *Miscellaneous observations* prefatory to constituting a plainly new Geometric Analysis.

(1) A point is the simplest of the things that are in extension. Hence:

((1)) A point is similar to a point, $a \sim b$

(2) A point is equal to a point $a = b$

(3) A point is congruent to a point $a \simeq b$

These will have use for demonstrating similarities, equalities or congruences of other things that are determined by certain points. See also §60 below.

(4) A point coincides to a point which it is assumed to be in, or if b is in a then $a \infty b$. In addition to these paragraphs 1.2.3.4 see §60 below.

((4)) In fact, in general whatever is situated in a point coincides with the point itself. If multiple points have some common property, and so each one of them is called by the common name X, then the locus common to all and proper to them alone we will call $\overline{X}$. Or $\overline{X}$ will signify:

(5) that every point X is in $\overline{X}$, and

(6) that every point in $\overline{X}$ is X.

(7) If every X is Y, then $\overline{X}$ *will be in* $\overline{Y}$.

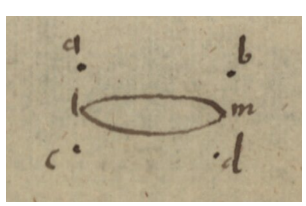
(8) If $\overline{X}$ is in $\overline{Y}$ every X will be Y.

(9) If $\overline{X}$ is in $\overline{Y}$ and $\overline{Y}$ is in $\overline{X}$, then $\overline{X}$ and $\overline{Y}$ will *coincide*.

(10) If $\overline{X}$ and $\overline{Y}$ coincide, then $\overline{X}$ will be in $\overline{Y}$ and $\overline{Y}$ will be in $\overline{X}$.

(11) If A is in $\overline{X}$ and $\overline{X}$ is in $\overline{Y}$ then A will be in Y. This is demonstrated thus. If A is in $\overline{X}$ of course A is X (by article 6). Now since $\overline{X}$ is in $\overline{Y}$ by hypothesis, every X will be Y (by 8). Therefore (by common Logic) A will also be Y. Therefore (by 5) A will be in $\overline{Y}$. QED. This proposition could also be stated like this: what contains the container contains the contained.

(12) If the situs of points a and b to each other is the same as the situs of points c and d, then points l and m of a rigid body that can be applied to a and b can also be applied to c and d.



[Fig. 1]

(13) And conversely, if this can be done, then the situs of the points is the same.

(14) The situs of the point a to b is the same as that of the point b to a.

(15) If points l and m of a rigid body can be applied to points a and b, and of course l to a and m to b, then in turn l can be applied to b and m to a. Indeed the situs of the point a to the point b is the same as that of the point b to the point a (by 14). Therefore (by 12) what was claimed can be done.

(16) If points of a determined rigid body are determined, which given points can touch at the same time, the situs of the given points to each other will be determined. On the determined see also §.25.65.

(17) $a.b.$ signifies the situs of points a and b to each other.[1] And $a.b.c.$ signifies the situs of the three points a and b and c to each other.

(18) If $a.b.c.$ is given, $a.b.$ is given.

(19) If $a.b.$ and $a.c.$ and $b.c.$ are given, $a.b.c.$ is given.

(20) $a.b. \simeq c.d.$ signifies that the situs between points a and b is the same as that between points c and d, or some rigid thing can be understood whose extremes are a and b, congruent to a rigid thing whose extremes are c and d. Or the points a and b can be congruent to the points c and d, while preserving the situs that a and b have to each other.

(21) If $a.b. \simeq l.m.$, and $a.c. \simeq l.n.$ et $b.c. \simeq m.n.$, then $a.b.c. \sim l.m.n.$

(22) If $a.b.c. \simeq l.m.n.$ then $a.b \simeq l.m.$ and so forth, each pair to the corresponding pair.

(23) If $a.b.c. \simeq l.m.n.$ and $a.b.d. \simeq l.m.p.$ and $a.c.d. \simeq l.m.p.$ and $b.c.d. \simeq m.n.p.$[2] then $a.b.c.d. \simeq l.m.n.p.$

(24) If, of two extensions having some common nature, points of the one that are of a sufficient number for determining this nature to a particular individual[3] have the same situs to each other as the same number [of points] in the other, then those extensions are congruent to each other. Let their common nature be $\odot$ and suppose that with four points determined in $\odot$, the individual of this $\odot$ is determined, and let there be two things F and G of which F is $\odot$ and G is $\odot$ as well, and let four points a, b, c, d in F be assumed, and similarly four points l, m, n, p in G, and let $a.b.c.d. \simeq l.m.n.p.$; then $F \simeq G$. For example, if there are two Elliptical circumferences and four points in one are situated to each other in the same way as four points in the other, then these two Elliptical circumferences will be congruent. Because given four points the ellipse is given. About determination see also §. 65 below.

(25) Those things are similar that, considered separately, cannot be distinguished, or in which, considered in themselves, no discriminating attributed can be noted, but one needs to either compare them to each other, or some third thing to both. Thus if two figures are similar, no proposition (that does not assume anything from outside) can be declared about the one that cannot be declared also about the other. If an eye is stationed successively in two rooms made from the same material, if they are dissimilar, it will note some difference, in situs or order, or even the proportions of the parts or lines to each other, and the compared angles with a straight line. But if no such thing can be noted, then the eye will have nothing from which to distinguish the one from the other, unless it looks at both at the same time from outside and compares them, or brings some measure with it (such a natural measure in humans are the limbs; in fact even the back of the eye if there is a notable difference in magnitude). Hence, e.g., two circles are similar; for examine each separately, draw lines where you like, consider the ratios of angles at lines and the ratios of straight lines to each other; you will note nothing in the one that you are not going to note in the other. But if you compare two Ellipses you will easily note a difference. For draw a straight line out from the center to the circumference at some assumed angle to the axis, and note the ratio of that line to the axis of the Ellipse, do the same in the other Ellipse at the same angle,

[1] Leibniz wrote $a.b$ rather than $a.b.$ here, and in general was inconsistent about whether or not to include the period at the end of these expressions. We have chosen to include the periods consistently.

[2] Here Leibniz repeated $a.c.d. \simeq l.m.p.$ twice; we corrected this to follow the presumable pattern.

[3] Leibniz has multiple edits and insertions in this sentence. At the location of this footnote we omit an extra *illa puncta* "those points" which would have interfered with the sentence structure by introducing a new subject. Otherwise, we follow Leibniz's intended final version.

and very often you will obtain another ration; and thus you will easily distinguish one from another.

(26) If the determiners are similar, and the mode of determination itself is similar, the things determined will also be similar. About determination §.65.75 below.

(27) Hence equilateral triangles are similar, since a triangle is determined given one side and two (and so also three) angles; if therefore all three angles are the same in both, since a side is similar to a side, a line to a line of course, nothing appears in the determiners from which one could elicit an attribute for the one that could not also be elicited for the other.

(28) Now similar triangles have proportional sides, else some proportion of the sides could be noted in the one that could not be noted in the other. Therefore by the preceding equiangular triangles have proportional sides.

(29) Conversely, triangles whose sides are proportional are equiangular. Indeed, a triangle is determined given three sides; if the sides are already proportional, no discriminating attribute in the determiners, namely the sides, can be found. Therefore they are similar; therefore in both there is the same ratio of the angles of the same triangle to each other as well as to their sum; but the sum of the angles is the same in both (since it makes two lines) therefore also the angles (having in both the same ratio to this sum, else a difference could be noted) are the same in both.

(30) Things that are similar according to one mode of determining are also the same with respect to another mode of determining. Thus, if two triangles are similar with respect to the sides, or have the same ratio of single sides to the sum in both, they will be similar with respect to angles as well, or will have the same ratio of single angles to the sum of the angles in both.

(31) Things are *homogenous* that either are similar, or can be rendered similar by transformation, as a straight and a circular curve, a convex and a planar surface. Indeed since every curve can be extended to a straight line, every surface flattened out and converted into a square, every solid converted into a cube, and a line is similar to a line, a square to a square, a cube to a cube, it is clear that all curves, surfaces, and solids are homogeneous to each other. In fact the definition of Homogeneous that Euclid uses cannot be accommodated here, since not even the smallest congruent portion can be found, and so also no common measure approximating it as exactly as one likes. They are also seen to be comparable from a generating cause, for if two points are moved at equal speed and time, the described curves, though dissimilar, will still be equal; but if the speed is the same and the time unequal, they will be as the times,[4] and so things are homogeneous for which there is a ratio. However, the Euclidean definition could also be accommodated here if the curved [lines] and gibbous [surfaces] were considered as polygons or polyhedra of infinitely many angles. See also §.38 below.

((31)) A transformation is a change that is done such that the simplest things that are inside in both are the same. Indeed, although sometimes parts remain, as when a square is changed into a right isosceles triangle, still sometimes no part remains, but only the points, as when a circle is changed into an equal square.

(32) *Equals* are those which either are congruent or can be rendered congruent by transformation.

(33) The *greater* is that whose part is equal to another whole (the lesser).

(34) The *lesser* is that which is equal to the part of another (the greater).

(35) Hence it is demonstrated that the part is less than the whole, or the whole is greater than the part. For the part is equal to a part of the whole (namely itself), therefore less than the whole.

(36) If A is $\ulcorner B$, then $B \urcorner A$.

(37) If something is neither greater nor less, but still homogeneous, it will be equal. Indeed since it is homogeneous, it can be rendered similar, therefore let it be similar, and since all similar things can be understood to be made from each other by continued increment

[4] That is, their ratio will be the same as the ratio of the times.

or decrement, or they have a common generation, of course that which is generated first in increment (resp. decrement) will be lesser (resp. greater). But those that will be generated at the same time will be *equal*, which proposition can be regarded as a new definition of equality. The same can be demonstrated also from the definition above; when two proposed things are similar, let the corresponding things be applied to the corresponding things, then they will either be congruent and will be equal, or one will exceed the other everywhere, else they would not be similar; indeed, if it does not exceed everywhere, their boundaries will intersect each other somewhere, and not intersect each other somewhere, which is absurd, for only the corresponding things ought to coincide; but if necessary, these things can demonstrated more precisely. Briefly, whatever things are similar cannot be distinguished except by magnitude. Hence I conclude: if A is not $\ulcorner B$ and A is not $\neg B$ and A Homog. B, then $A = B$.

Notes