Name: Solutions

Worksheet 7.7 – Approximate Integration

1) a) Calculate S_6 for $\int_0^3 x^5\,dx$. Then compute A, the actual value of the integral. Then compute E_S. You may use a calculator. (LT: 1i)

$S_6 = \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6)$ where $y_i = x_i^5$

$\Delta x = \frac{3-0}{6} = \frac{1}{2}$

$S_6 = \frac{\left(\frac{1}{2}\right)}{3}\left(0^5 + 4\left(\frac{1}{2}\right)^5 + 2(1)^5 + 4\left(\frac{3}{2}\right)^5 + 2(2)^5 + 4\left(\frac{5}{2}\right)^5 + 3^5\right)$

$= 121.6875$

$A = \int_0^3 x^5\,dx$

$= \frac{x^6}{6}\Big|_0^3$

$= \frac{3^6}{6}$

$= 121.5$

$E = A - S_6$

$= 121.5 - 121.6875$

$= -0.1875$

$S_6 =$ 121.6875	A = 121.5	$E_S =$ −0.1875

b) Find the smallest value of n for which $Error(S_n) \le 10^{-9}$ for the integral, $\int_0^3 x^5 dx$. Given:

$|E_S| \le \frac{K(b-a)^5}{180n^4}$. You may use a calculator. (LT: 1j)

$f(x) = x^5$

$f'(x) = 5x^4$

$f''(x) = 20x^3$

$f^{(3)}(x) = 60x^2$

$f^{(4)}(x) = 120x$

$K = 360$

$|E_S| \le \frac{360(3-0)^5}{180n^4} \le 10^{-9}$

$n^4 \ge 4.86 \times 10^{11}$

$n \ge 834.947$

$n \ge 836$ (must be an even integer)

$\boxed{836}$

2) Use Simpson's Rule to estimate the average temperature in a museum over a 3-hour period, if the temperature (in degrees Celsius), recorded at 15-minute intervals, are

Elapsed Time (min)	0	15	30	45	60	75	90	105	120	135	150	165	180
Temp (ºC)	21	21.3	21.5	21.8	21.6	21.2	20.8	20.6	20.9	21.2	21.1	21.3	21.2

You may use a calculator. (LT: 1i)

Hint: $f_{AVE} = \frac{1}{b-a}\int_a^b f(x)\,dx$

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + 2y_6 + 4y_7 + 2y_8 + 4y_9 + 2y_{10} + 4y_{11} + y_{12}\right)$$

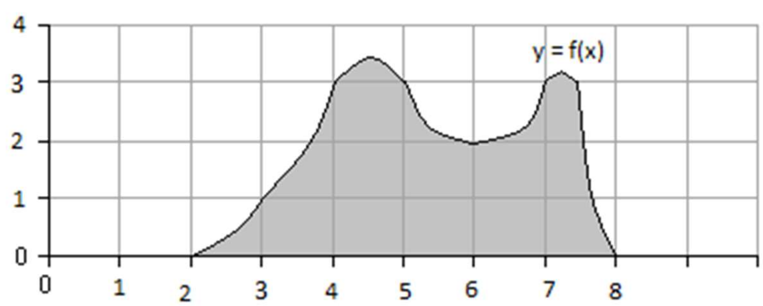
$$\approx \frac{15}{3}\left(21 + 4(21.3) + 2(21.5) + 4(21.8) + 2(21.6) + 4(21.2) + 2(20.8) + 4(20.6) + 2(20.9) + 4(21.2) + 2(21.1) + 4(21.3) + 21.2\right)$$

$$\approx 3818$$

$$f_{ave} \approx \frac{1}{180-0}(3818) = 21.211\,°C$$

$\boxed{21.211\,°C}$

3) Use S_6 to approximate the volume of the solid region obtained by revolving the shaded plane region below *f(x)* around the *y-axis*. Try not to use a calculator. (LT: 1i)



$$V = \int_2^8 2\pi x f(x)\,dx$$

$$V \approx S_6$$

$$\approx \frac{\Delta x}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6\right)$$

where $y_i = 2\pi x_i f(x_i)$

and $\Delta x = \frac{8-2}{6} = 1$

$$V \approx \frac{1}{3}(2\pi)\left(2(0) + 4(3)(1) + 2(4)(3) + 4(5)(3) + 2(6)(2) + 4(7)(3) + 8(0)\right)$$

$$\approx \frac{2\pi}{3}(204)$$

$$\approx 136\pi$$

$$\approx 427.257$$

$\boxed{136\pi}$