

# W04 - Homework

## Stepwise problems - Thu. 11:59pm

---

### Arc length

01

✍ Arc length - reversed  $x$  and  $y$  roles

Find the arc length of the curve that satisfies the equation  $x = \frac{1}{12}y^3 + y^{-1}$  over  $y \in [1, 2]$ .

### Surface areas of revolutions - thin bands

02

✍ Surface area: revolved cubic

The curve  $y = x^3$  over  $x \in [0, 2]$  is revolved around the  $x$ -axis.

Find the area of the resulting surface.

## Regular problems - Sat. 11:59pm

---

### Arc length

03

✍ Arc length - tricky algebra

Find the arc length of the curve  $y = \frac{x^4}{8} + \frac{1}{4x^2}$  for  $x \in [1, 2]$ .

(Hint: expand under the root, then simplify, then factor; now it's a square and the root disappears.)

04

✍ Arc length - tricky integration

Find the arc length of the curve  $y = e^x$  for  $x \in [0, 1/2]$ .

(Hint: the integral can be done using either: (i)  $u$ -sub then trig sub, or (ii) 'rationalization' then partial fractions.)

### Surface areas of revolutions - thin bands

05

✍ Surface area: cone

A *cone* may be described as the surface of revolution of a ray emanating from the origin, revolved around the  $x$ -axis.

Let  $f(x) = mx$  for some  $m > 0$ . Find the surface area of the cone given by revolving the graph of  $f$  over  $x \in [0, h]$ .

Can you also calculate this area using geometry? And verify the two methods give the same formula? (Hint: ‘unroll’ the cone into a sector.)

06

☑ **Surface area: parabolic reflector**

A parabolic reflector is given by rotating the curve  $y = x^2$  around the  $y$ -axis for  $x \in [0, 2]$ .

What is the surface area of this reflector?

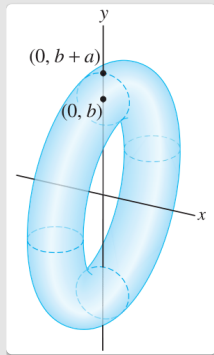
07

☑ **Surface area: torus**

A torus is created by revolving about the  $x$ -axis the circle with this equation:

$$x^2 + (y - b)^2 = a^2$$

Find the surface area of this torus.



(Hint: compute for the top and bottom of the circle separately and add the results.)