W11 - Homework

Stepwise problems - Thu. 11:59pm

Power series as functions

01

Modifying geometric power series

Consider the geometric power series $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$.

(a) By modifying the series $\frac{1}{1-x}$, write $\frac{1}{5-x}$ as a power series centered at $c = 0$ and determine its radius of convergence.

(b) By modifying the series $\frac{1}{1-x}$, write $\frac{1}{16+2x^3}$ as a power series centered at $c = 0$ and determine its radius of convergence.

Tayler and Maclaurin series

02

Maclaurin series I

For each of these functions, find the Maclaurin series.

(a) $x \ln(1 - 5x)$ (b) $x^2 \cos(x^3)$

Applications of Taylor series

03

Approximating $1/e$

Using the series representation of e^x, show that:

$$\frac{1}{e} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \cdots$$

Now use the alternating series error bound to approximate $\frac{1}{e}$ to an error within 10^{-3}.

Regular problems - Sat. 11:59pm

Power series as functions

04

Power series of a derivative

Suppose that a function $f(x)$ has power series given by:

$$f(x) = x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+2}}{n+1}$$

The radius of convergence of this series is $R = 1$.

What is the power series of $f'(x)$ and what is its radius of convergence?

05

Finding a power series

Find a power series representation for these functions:

(a) $f(x) = \dfrac{x^2}{x^4 + 81}$ (b) $g(x) = x^2 \ln(1 + x)$

06

Modifying and integrating a power series

(a) Modify the power series $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$ to obtain the power series for $f(x) = \frac{1}{1+x^4}$.

(b) Now integrate this series to find the power series for $\int f(x)\,dx$.

Tayler and Maclaurin series

07

Maclaurin series II

For each of these functions, find the Maclaurin series.

(a) $\sin 3x^2$ (b) $x^2 e^{5x}$

08

Tayler series of $1/x$

Find the Taylor series for the function $f(x) = \frac{1}{x}$ centered at $c = 1$.

09

Discovering the function from its Maclaurin series I

For each of these series, identify the function of which it is the Maclaurin series.

(a) $\displaystyle\sum_{n=0}^{\infty} (-1)^n 2^n x^n$ (b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{3n}}{n!}$

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Discovering the function from its Maclaurin series II

For each of these series, identify the function of which it is the Maclaurin series, and evaluate the function at an appropriate choice of x to find the total sum for the series.

(a) $\sum_{n=0}^{\infty}(-1)^n \frac{\pi^{2n+1}}{4^{2n+1}(2n+1)!}$ (b) $\sum_{n=0}^{\infty} \frac{2^{2n}}{n!}$ (c) $\sum_{n=0}^{\infty}(-1)^n \frac{\pi^{2n+2}}{3^{2n+1}(2n)!}$

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Summing a Maclaurin series by guessing its function

For each of these series, identify the function of which it is the Maclaurin series:

(a) $\sum_{n=0}^{\infty}(-1)^n \frac{5x^{4n+2}}{(2n+1)!}$ (b) $\sum_{n=0}^{\infty} \frac{(-5x)^{n+1}}{n+1}$

Now find the total sums for these series:

(c) $\sum_{n=0}^{\infty} \frac{(-5)^n}{n!}$ (d) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{9^n (2n)!}$

(Hint: for (c)-(d), do the process in (a)-(b), then evaluate the resulting function somewhere.)

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Data of a Taylor series

Assume that $f(3)=1$, $f'(3)=2$, $f''(3)=12$, and $f'''(3)=3$.

Find the first four terms of the Taylor series of $f(x)$ centered at $c=3$.

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Evaluating series

Find the total sums for these series:

(a) $\sum_{n=0}^{\infty}(-1)^n \frac{\sqrt{3}^{2n+1}}{3^{2n+1}(2n+1)}$ (b) $\sum_{n=0}^{\infty}(-1)^{n+1} \frac{1}{5^{n+1}(n+1)}$

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Large derivative at $x=0$ using pattern of Maclaurin series

Consider the function $f(x)=x^2 \sin\left(5x^3\right)$. Find the value of $f^{(35)}(0)$.

(Hint: find the rule for coefficients of the Maclaurin series of $f(x)$ and then plug in 0.)

Applications of Taylor series

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Some estimates using series

For each of these estimates, use the error bound formula for alternating series.

Without a calculator, estimate $\cos(0.02)$ (angle in radians) with an error below 1×10^{-6}.

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Some estimates using series

For each of these estimates, use the error bound formula for alternating series.

Find an infinite series representation of $\int_0^1 \sin(x^2)\,dx$ and then use your series to estimate this integral to within an error of 10^{-3}.