Unit 01 - Essential problems

Shells

Shells volume - offset graph, y-axis

Consider the region in the first quadrant bounded by the lines $x = 0$, $x = 2$, $y = 0$, and the curve $y = \frac{1}{\sqrt{x^2+1}}$. Revolve this about the y-axis.

Find the volume of the resulting solid.

IBP

Integration by parts - A and T

Compute the integral:

$$\int x^2 \sin x \, dx$$

Integration by parts - A and L

Compute the integral:

$$\int x^3 \ln x \, dx$$

Integration by parts - A and I

Compute the integral:

$$\int \tan^{-1}(x) \, dx$$

Trig power products

Somewhat odd power product

Compute the integral:

$$\int \sin^2 x \cdot \cos^3 x \, dx$$

Tangent and secant both even

Compute the integral:

$$\int \tan^2 x \cdot \sec^2 x \, dx$$

All even power product

Compute the integral:

$$\int \sin^4 x \cdot \cos^2 x \, dx$$

Tangent and secant mixed parity

Compute the integral:

$$\int \tan^3 x \, \sec^2 x \, dx$$

- (a) Using $du = \sec^2 x \, dx$.
- (b) Using $du = \sec x \, \tan x \, dx$.

Trig subs

Trig sub

Compute the definite integral:

$$\int_0^{1/2} \frac{x^2}{\sqrt{1-x^2}} \, dx$$

Trig sub

Compute the integral:

$$\int \frac{dx}{x^3\sqrt{x^2-4}}$$

Trig sub

Compute the integral:

$$\int \frac{x^2}{(x^2+1)^{3/2}}\,dx$$

Partial fractions

Distinct linear factors

Compute the integral:

$$\int \frac{1}{(x+2)(x-3)}\,dx$$

Long division first

Compute the integral:

$$\int \frac{2x^3+3x^2+7x+4}{x+1}\,dx$$

Partial fractions - linear and quadratic

Compute the integral:

$$\int \frac{5x^2-5x+14}{(x-2)(x^2+4)}\,dx$$

Partial fractions - repeated factor

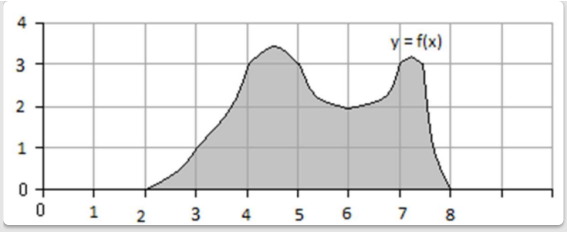
Compute the integral:

$$\int \frac{1}{x(x-1)^3}\,dx$$

Simpson's Rule

Simpson's Rule for volume by shells

Use Simpson's Rule with $n=6$ to compute the volume of the solid obtained by revolving the pictured region about the y-axis. Can you do it without using a calculator?



Area of a garden bed

The width of a garden bed is measured every 2 feet as shown. How much mulch (in cubic yards) should I buy to cover this garden bed with a 6-inch layer of mulch?