Statistical language models

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Statistical properties of languages

We will view words as *random events* generated by some *random process*.

Some high-level observations:

Heap's law: The number of unique words in a text is proportional to the square root of the number of tokens

Zipf's law: The number of occurrences of a word is inversly proportional to the word's rank in a frequency table.

Statistical language model

A (statistical) language model predicts the probability of the next word, based on the preceding words.

- *He wrote a letter to his* _______.

Applications of language models

- Word prediction
  - *I'm going* ______ ↙

to
for
on

- Spelling correction
  - *Flights form Boston.*
- Speech recognition
  - P("recognize speech") $>$ P("wreck a nice beach")
- Translation
  - P("tall building") $>$ P("high building")
- ... and many more

Some notation

$\mathbf{P}(\mathbf{w})$ = the probability of the word w.

- Picking a random word from a text, what is the probability that that word will be w?

$\mathbf{P}(\mathbf{w_1 w_2 \ldots w_n})$ = the probability of the sequence $w_1\ w_2\ \ldots\ w_n$.

- Picking a random sequence of i words from a text, what is the probability that that sequence will be $w_1\ w_2\ \ldots\ w_n$?

Chain rule for probabilities

The *chain rule* rewrites a joint probability into a product of conditional probabilities.

$$P(A, B) = P(B|A)P(A)$$

In general:

$$P(A_1, \ldots, A_n) = \prod_{i=1}^{n} P(A_i|A_1, \ldots, A_{i-1})$$

For example: $P(\text{I like ants}) = P(\text{ants}|\text{I like})P(\text{like}|\text{I})P(\text{I})$

How do we estimate the probabilities $P(w_i|w_1, \ldots, w_{i-1})$?

Maximum likelihood estimation (MLE)

Count word strings in a text corpus, compute fraction:

$$P(w_i | w_1, \ldots, w_{i-1}) = \frac{c(w_1, \ldots w_i)}{c(w_1, \ldots w_{i-1})}$$

In particular:

$P(w) = \frac{c(w)}{N}$, where N is the number of tokens in the corpus

Problems with Maximum likelihood estimation

Data sparsity is a problem for the straightforward MLE method.

E.g. $\frac{c(\text{“Strange things have happened”})}{c(\text{“Strange things have”})}$

- What if there are no occurrences of “Strange things have happened”?
- What if there are no occurrences of “Strange things have”?

Regardless of how much data you have, this will happen over and over.

Maximum likelihood estimation (MLE)

Count word strings in a text corpus, compute fraction:

$$P(w_i|w_1,\ldots,w_{i-1}) = \frac{c(w_1,\ldots w_i)}{c(w_1,\ldots w_{i-1})}$$

In particular:

$P(w) = \dfrac{c(w)}{N}$, where N is the number of tokens in the corpus

Example

- Corpus with 19 tokens (including punctuation):

  I live in Boston.
  I like ants.
  Ants like honey.
  Therefore I like honey too.

- What is $P(\text{like}|\text{I})$ based on the above corpus?
- What is $P(\text{honey}|\text{I like})$?
- What is $P(\text{Boston}|\text{I like})$?

Example

- Corpus with 19 tokens (including punctuation):

  I live in Boston.
  I like ants.
  Ants like honey.
  Therefore I like honey too.

- What is $P(\text{like}|\text{I})$ based on the above corpus? 2/3
- What is $P(\text{honey}|\text{I like})$? 1/2
- What is $P(\text{Boston}|\text{I like})$? 0

Markov assumption

Assume that a word only depends on the previous n words.

$$P(w_i|w_1,\dots,w_{i-1}) = P(w_i|w_{i-n},\dots,w_{i-1})$$

$n = 0$ (*unigram* model)

$$P(\text{I have a unicorn}) = P(\text{I})P(\text{have})P(\text{a})P(\text{unicorn})$$

$n = 1$ (*bigram* model)

$$P(\text{ I have a unicorn}) = \\ P(\text{I})P(\text{have}|\text{I})P(\text{a}|\text{have})P(\text{unicorn}|\text{a})$$

In general: An *n-gram model* takes the $n-1$ preceding words into account. Typically such models are estimated from a large corpus.

Counting bigrams

. . . $\boxed{\text{I would}}$ like to know your plans for . . .

I would: 1

Counting bigrams

. . . I $\boxed{\text{would like}}$ to know your plans for . . .

I would: 1   would like: 1

Counting bigrams

. . . I would $\boxed{\text{like to}}$ know your plans for . . .

I would: 1 would like: 1 like to: 1

Bigram probabilities

The word *I* occurred 1000 times...

- ... 20 times, the next word was *like*
- ... 200 times, the next word was *am*
- ... 100 times, the next word was *have*
- etc.

From the counts, we can estimate *bigram probabilities*:

- $P(\mathit{like}|I) = 0.02$
- $P(\mathit{am}|I) = 0.2$
- $P(\mathit{have}|I) = 0.1$
- etc.

Trigram probabilities

The sequence *I like* occurred 20 times...

- ... 5 times, the next word was *to*
- ... 4 times, the next word was *that*
- ... 1 time, the next word was *apples*
- etc.

From the counts, we can estimate *trigram probabilities*:

- $P(to|I\ like) = 0.25$
- $P(that|I\ like) = 0.2$
- $P(apples|I\ like) = 0.05$
- etc.

Example

- Corpus with 19 tokens (including punctuation):
  I live in Boston.
  I like ants.
  Ants like honey.
  Therefore I like honey too.
- What is P(I like Boston) using a unigram model based on the above corpus?
- What is P(I like honey) using a bigram model?
- What is P(I like Boston) using a bigram model?

Example

- Corpus with 19 tokens (including punctuation):

    I live in Boston.
    I like ants.
    Ants like honey.
    Therefore I like honey too.

- What is P(I like Boston) using a unigram model based on the above corpus? (3/19)(3/19)(1/19)
- What is P(I like honey) using a bigram model? (3/19)(2/3)(2/3)
- What is P(I like Boston) using a bigram model? (3/19)(2/3)(0/3) = 0

Zero-probabilities, and what to do about them

A problem with *n*-gram models is that many sensible word sequences will have zero-probabilities.

3 techniques to solve this problems:

- Smoothing
- Backoff
- Linear interpolation

Laplace smoothing

Smoothing: Transfer some of the probability mass from the seen sequences to the unseen sequences.

Easiest variant: *Laplace (add-one)* smoothing.

- Previously (Maximum Likelihood Estimation):

$$P_{MLE}(w_i|w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Now: (MLE with Laplace smoothing):

$$P_{Laplace}(w_i|w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$

  where V is the size of the vocabulary (number of unique words).

Laplace smoothing (example)

- Corpus with 19 tokens (including punctuation):

  I live in Boston.
  I like ants.
  Ants like honey.
  Therefore I like honey too.

- What is P(I like honey) using a bigram model with Laplace smoothing?
- What is P(I like Boston) using a bigram model with Laplace smoothing?

Laplace smoothing (example)

- Corpus with 19 tokens (including punctuation):

  I live in Boston.
  I like ants.
  Ants like honey.
  Therefore I like honey too.

- What is P(I like honey) using a bigram model with Laplace smoothing? $(\frac{3+1}{19+10})(\frac{2+1}{3+10})(\frac{2+1}{3+10})$
- What is P(I like Boston) using a bigram model with Laplace smoothing? $(\frac{3+1}{19+10})(\frac{2+1}{3+10})(\frac{0+1}{3+10})$

Advanced smoothing

Laplace smoothing turns out to be too crude for many applications

- ...but it is useful in some machine learning contexts (more on this later in the course)

More sophisticated smoothing methods exist, e.g. the *interpolated Kneser-Ney* method.

We won’t cover these in the course.

Backoff

If a particular n-gram is not present in the training corpus, then use $(n-1)$-grams instead.

If the $(n-1)$-grams do not exist either, then use $(n-2)$-grams, etc.

Example: In a bigram model built from *Moby Dick*, $P(\text{I like Boston}) = 0$, since "like Boston" does not appear in the book. Then compute the probability as:

$$P(\text{I like Boston}) = P(\text{I})P(\text{like}|\text{I})P(\text{Boston})$$

Note that this computation will most likely underestimate the actual probability (why?).

Linear interpolation

Estimate $\hat{P}(w_i|w_{i-1})$ as

$$\lambda_1 P(w_i|w_{i-1}) + \lambda_2 P(w_i) + \lambda_3$$

where the λs sum to 1.

$$\sum_i \lambda_i = 1$$

Typically $\lambda_1 = 0.99$, $\lambda_2 = 0.01 - \lambda_3$, $\lambda_3 = 10^{-6}$

This idea naturally extends to 3-grams, etc.

Assignment 2, problem 1

Write a program that computes all bigram probabilites from a given (training) corpus, and stores it in a file. Practical issue:

We will use log-probabilities rather than probabilities

- ... using the natural logarithm (because it's simpler)
- -11.99225 rather than 0.0000061919939907.

This is to avoid underflow when computing with very small probabilities.

... and we can add rather than multiply

- $\log(p_1 \times \ldots \times p_n) = \log(p_1) + \ldots + \log(p_n)$

Probability of a sentence

When calculating the probability of a sentence, it is useful to include punctuation or boundary symbols, e.g.

$$P(\langle b \rangle \text{ I like Boston } \langle b \rangle) =$$
$$P(\text{I}|\langle b \rangle) \times P(\text{like}|\text{I}) \times P(\text{Boston}|\text{like}) \times P(\langle b \rangle|\text{Boston})$$

- $P(\text{I}|\langle b \rangle)$: Probability that "I" will be the first word of a sentence.
- $P(\langle b \rangle|\text{Boston})$: Probability that "Boston" will be the last word of a sentence.

n-gram models and linguistic structure

How much linguistic structure is captured by *n*-gram-models?

- Higher $n \Rightarrow$ we capture more language structure, BUT
- Higher $n \Rightarrow$ we need more training data to get accurate probabilities.
- 4-grams and above require Google quantities of data, OR a restricted domain!

Long-distance dependencies will always be a problem (regardless of the choice of *n*):

> **The struggle** *between conservatives and socialists* **is** *being fought on many fronts.*

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3b: Text generation from language models

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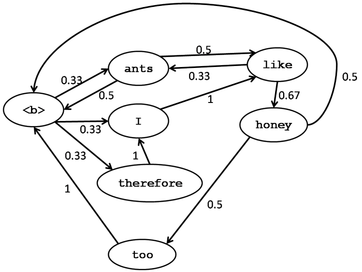
Markov chains

n-gram models are examples of *Markov chains*. In particular, a bigram model is a *first order Markov chain*.

First-order Markov chains are often depicted as a finite-state automaton with probabilities on the edges.



I like ants. Ants like honey. Therefore I like honey too.

Text generation from bigram models

Kafka:

```
the middle of year when her eyes with them appeared with how he called again
after him and mother to our breakfast ? that '' , called gregor 's spokesman to
this way they disappeared ; and it was still falling out again , only did not want
to make them , walls and cheaper than a brother any revulsion for more than he
really happened , that gregor hardly even with the time was already ? it had poured
some secret prohibition on the living room before . all his chin on his parents
into a painful silence .
```

Austen:

```
the course may i hope you , `` my junior , which her uncle and she would be very
 likely to see the british court , that my worked on the hands with an instrument
till summoned from making any of their behaviour while the solitude . what ,
however , for better worth speaking of bingley would be married to be done , she
, when dinner was again ; `` what news ; but look at last question , nothing . '
' i should take one readily engaged to hesitate . '' cried elizabeth , and at her
```

Text generation from bigram models

Guardian:

the south of président butter and cafe , arts) , which is that it ’ d nurtured . < > facebook twitter pinterest villa o ’ s emotions thinking about sex ” over with the page , and executive poached artichoke , paindepices.fr christmas day . q & tech news brands such as a mist and sports correspondent at sheena ’s too salty and her polycystic ovary syndrome on the fruit , 55 % of conversation with soldeu is intended targets for ways to make a deeply entrenched scepticism about the shoreline , uae ahas already marginalised – proving

small:

i live in boston . i like honey too ! live in boston . i like ants . therefore i like honey too ! therefore i like ants . i like ants like honey too ! therefore i like honey too ! like honey too ! honey too ! ants . therefore i live in boston . i like honey too ! i like honey . i like honey . ants . ants like honey too ! honey too ! in boston . i like honey too ! honey too ! live in boston . i live in boston .

Assignment 2, problem 2

- Write a program that generates words from a language model.
- Generate 100 words each from the models you have created.

Text generation (Sherlock Holmes)

Unigram model

1. your something
2. you she to offices the possible his of of his said sight , was laughing had .
3. white was not full meet old be to made , you no I . described that power he the, man , And ,
4. was Captain That she point labyrinth now must be far from . door had the from again what almost result fill , for coming as . a with made
5. his then a country-town by you ’ ago Men ?

Text generation (Sherlock Holmes)

Bigram model

1. Then here is the mud-bank what you , and instantly , and two officers waiting at once more valuable as I asked .
2. I may place is her husband and illegal constraint and outstanding , not recognised shape of finding that your heart , for communication between this man .
3. Mrs. Toller knows I mean that I have done very heartily at the 11 : That is a lad, his neighbour .
4. Then there has offered to its centre one left this case , upon me to violin-land , though the corner and hurried across the very large staples .
5. Holmes ran up by old-fashioned shutters of treachery to attend to which I thought I have a foreigner , too late Ezekiah Hopkins , with this rather cumbrous .

Text generation (Sherlock Holmes)

Trigram model

1. However , when last seen , but now I will leave no survivor from a solution by the Underground and hurried me into a bedroom , which boomed out every quarter of a brickish red .
2. ’ I beg that you have ever done yet , among the trees and wayside hedges were just being lighted as we stepped from her imprudence in allowing this brute to trace some geese which were new to me .
3. Holmes had sat up in my uncle’s life , and that a woman .
4. James and his hand and at the open , and has seen , but there are a thousand details which seem to have been hanged on far slighter evidence , I thought of !
5. Mr. Windibank draws my interest every quarter and pays it over to him . Natural

Text generation (Sherlock Holmes)

4-gram model

1. Seeing that his passion was becoming ungovernable , I left him and returned towards Hatherley Farm .
2. You will excuse me , said my wife , and in order to see whether the objections are fatal , or if he had been to the side from which I could see that two of them were of the war he fought in Jackson's army , and afterwards from your gesture, that Miss Rucastle was perfectly happy , and that I can .
3. I rang the bell and called for the weekly county paper , which contained a verbatim account of the matter , but you do not see the point .
4. It hadn't pulled up before she shot out of the window ?
5. Why does fate play such tricks with poor , helpless worms ?

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3c: Evaluation of language models

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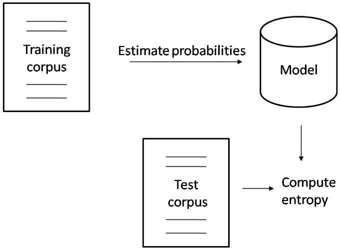
Evaluation of *n*-gram models

Extrinsic evaluation: Put your *n*-gram to use in an application, e.g. a machine translation system. Measure the performance.

Intrinsic evaluation: Compute the *entropy* of the model.



Information

The *information* of an event having probability p is

$$-\log_2 p$$

Information is measured in *bits*.

Entropy

The *entropy* of a random variable X is the expected value of the information.

$$H(X) = -\sum_{i=1}^{n} P(X = x_i) \log_2 P(X = x_i)$$

Special case: We assume that $0 \log_2 0 = 0$.

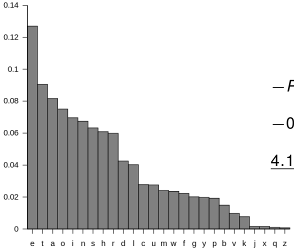
The entropy of X is a measure of the difficulty of predicting the value of X.

Entropy examples

Entropy for a–z using a uniform distribution:

$$-\sum_{i=1}^{26} \frac{1}{26} \log_2 \frac{1}{26} = -\log_2 \frac{1}{26} = \log_2 26 = \underline{4.7}$$

Entropy for a–z using probabilities for English:



$$-P(\mathtt{a}) \log_2 P(\mathtt{a}) \quad -P(\mathtt{b}) \log_2 P(\mathtt{b}) \quad - \ldots \quad -P(\mathtt{z}) \log_2 P(\mathtt{z}) =$$

$$-0.082 \log_2 0.082 \quad -0.015 \log_2 0.015 \quad - \ldots \quad -0.00074 \log_2 0.00074 =$$

$\underline{4.18}$

Entropy if $P(\mathtt{a}) = 1$: $-1 \log_2 1 - 0 \log_2 0 - \ldots 0 \log_2 0 = \underline{0}$

Basic properties of entropy

More predictability $\Leftrightarrow$ lower entropy.

The entropy of a certain event is 0.

Maximum entropy is obtained if all events are equally probable.

Cross-entropy

Suppose `a,b,c,d` is distributed according to p:

$$p_{\mathrm{a}} = P(\mathrm{a}) = 0.5,\ p_{\mathrm{b}} = 0.2,\ p_{\mathrm{c}} = 0.2,\ p_{\mathrm{d}} = 0.1$$

but we *believe* it is distributed according to q:

$$q_{\mathrm{a}} = P(\mathrm{a}) = 0.1,\ q_{\mathrm{b}} = 0.2,\ q_{\mathrm{c}} = 0.2,\ q_{\mathrm{d}} = 0.5$$

The *cross-entropy of p on q* is then computed as:

$$H(X) = -\sum_{i=\mathrm{a,b,c,d}} p_i \log_2 q_i$$

Cross-entropy measures how difficult it is to predict the symbol, and how efficiently we can encode strings, under this belief.

Entropy as an evaluation metric

Entropy (on the word level) can be used as an evaluation metric for language models.

Given a model P estimated from a *training corpus*, one can approximate the entropy as:

$$-\frac{1}{N}\log_2 P(w_1, w_2, \ldots, w_N)$$

where $w_1, w_2, \ldots, w_N$ is a very long sequence of words from a *test corpus*.

The above computation really approximates the *cross-entropy* of the test set on P, which is an upper bound of the entropy of P.

Entropy as an evaluation metric

The *lower* the entropy of the test corpus, the *better* the language model learned from the training corpus.

The tacit assumption here is that the test corpus is representative of actual data.

Assignment 2, problem 3

- Write a program that evaluates a language model (a model constructed with your program in (a)) on a given test set.
- Build a number of models and evaluate them on different test sets.
- Draw conclusions.