Mahaux Photography/Image Bank/Getty Images

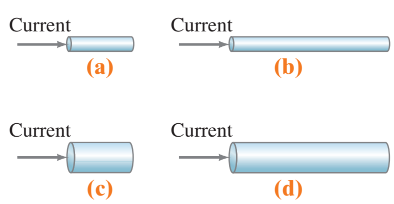
The glow of the thin wire filament of a lightbulb is caused by the electric current passing through it. Electric energy is transformed to thermal energy (via collisions between moving electrons and atoms of the wire), which causes the wire's temperature to become so high that it glows. Electric current and electric power in electric circuits are of basic importance in everyday life. We examine both dc and ac in this Chapter, and include the microscopic analysis of electric current.

Electric Currents and Resistance

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.*]

The conductors shown are all made of copper and are at the same temperature. Which conductor would have the greatest resistance to the flow of charge entering from the left? Which would offer the least resistance?



In this Chapter we introduce a study of charges in motion, and we call a flow of charge an electric current.

In everyday life we are familiar with electric currents in wires and other conductors. Indeed, most practical electrical devices depend on electric current: current through a lightbulb, current in the heating element of a stove or electric heater, and currents in electronic devices. Electric currents can exist in conductors such as wires, and also in other devices such as the CRT of a television or computer monitor whose charged electrons flow through space.

CONTENTS

1 The Electric Battery
2 Electric Current
3 Ohm's Law: Resistance and Resistors
4 Resistivity
5 Electric Power
6 Power in Household Circuits
7 Alternating Current
8 Microscopic View of Electric Current: Current Density and Drift Velocity
*9 Superconductivity
*10 Electrical Conduction in the Nervous System

Note: Sections marked with an asterisk (*) may be considered optional by the instructor.

From Chapter 25 of *Physics for Scientists & Engineers with Modern Physics*, Fourth Edition, Douglas C. Giancoli.

J.-L. Charmet/Photo Researchers, Inc.

FIGURE 1 Alessandro Volta. In this portrait, Volta exhibits his battery to Napoleon in 1801.

In electrostatic situations, the electric field must be zero inside a conductor (if it weren't, the charges would move). But when charges are *moving* in a conductor, there usually *is* an electric field in the conductor. Indeed, an electric field is needed to set charges into motion, and to keep them in motion in any normal conductor. We can control the flow of charge using electric fields and electric potential (voltage), concepts we have just been discussing. In order to have a current in a wire, a potential difference is needed, which can be provided by a battery.

We first look at electric current from a macroscopic point of view: that is, current as measured in a laboratory. Later in the Chapter we look at currents from a microscopic (theoretical) point of view as a flow of electrons in a wire.

Until the year 1800, the technical development of electricity consisted mainly of producing a static charge by friction. It all changed in 1800 when Alessandro Volta (1745–1827; Fig. 1) invented the electric battery, and with it produced the first steady flow of electric charge—that is, a steady electric current.

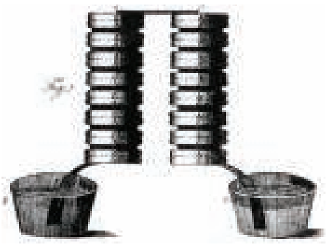
1 The Electric Battery

The events that led to the discovery of the battery are interesting. For not only was this an important discovery, but it also gave rise to a famous scientific debate.

In the 1780s, Luigi Galvani (1737–1798), professor at the University of Bologna, carried out a series of experiments on the contraction of a frog's leg muscle through electricity produced by static electricity. Galvani found that the muscle also contracted when dissimilar metals were inserted into the frog. Galvani believed that the source of the electric charge was in the frog muscle or nerve itself, and that the metal merely transmitted the charge to the proper points. When he published his work in 1791, he termed this charge "animal electricity." Many wondered, including Galvani himself, if he had discovered the long-sought "life-force."

Volta, at the University of Pavia 200 km away, was skeptical of Galvani's results, and came to believe that the source of the electricity was not in the animal itself, but rather in the *contact between the dissimilar metals*. Volta realized that a moist conductor, such as a frog muscle or moisture at the contact point of two dissimilar metals, was necessary in the circuit if it was to be effective. He also saw that the contracting frog muscle was a sensitive instrument for detecting electric "tension" or "electromotive force" (his words for what we now call potential), in fact more sensitive than the best available electroscopes that he and others had developed.[†]

FIGURE 2 A voltaic battery, from Volta's original publication.



Volta's research found that certain combinations of metals produced a greater effect than others, and, using his measurements, he listed them in order of effectiveness. (This "electrochemical series" is still used by chemists today.) He also found that carbon could be used in place of one of the metals.

Volta then conceived his greatest contribution to science. Between a disc of zinc and one of silver, he placed a piece of cloth or paper soaked in salt solution or dilute acid and piled a "battery" of such couplings, one on top of another, as shown in Fig. 2. This "pile" or "battery" produced a much increased potential difference. Indeed, when strips of metal connected to the two ends of the pile were brought close, a spark was produced. Volta had designed and built the first electric battery; he published his discovery in 1800.

[†]Volta's most sensitive electroscope measured about 40 V per degree (angle of leaf separation). Nonetheless, he was able to estimate the potential differences produced by dissimilar metals in contact: for a silver–zinc contact he got about 0.7 V, remarkably close to today's value of 0.78 V.

Electric Cells and Batteries

A battery produces electricity by transforming chemical energy into electrical energy. Today a great variety of electric cells and batteries are available, from flashlight batteries to the storage battery of a car. The simplest batteries contain two plates or rods made of dissimilar metals (one can be carbon) called **electrodes**. The electrodes are immersed in a solution, such as a dilute acid, called the **electrolyte**. Such a device is properly called an **electric cell**, and several cells connected together is a **battery**, although today even a single cell is called a battery. The chemical reactions involved in most electric cells are quite complicated. Here we describe how one very simple cell works, emphasizing the physical aspects.

The cell shown in Fig. 3 uses dilute sulfuric acid as the electrolyte. One of the electrodes is made of carbon, the other of zinc. That part of each electrode outside the solution is called the **terminal**, and connections to wires and circuits are made here. The acid tends to dissolve the zinc electrode. Each zinc atom leaves two electrons behind on the electrode and enters the solution as a positive ion. The zinc electrode thus acquires a negative charge. As the electrolyte becomes positively charged, electrons are pulled off the carbon electrode by the electrolyte. Thus the carbon electrode becomes positively charged. Because there is an opposite charge on the two electrodes, there is a potential difference between the two terminals.

In a cell whose terminals are not connected, only a small amount of the zinc is dissolved, for as the zinc electrode becomes increasingly negative, any new positive zinc ions produced are attracted back to the electrode. Thus, a particular potential difference (or voltage) is maintained between the two terminals. If charge is allowed to flow between the terminals, say, through a wire (or a lightbulb), then more zinc can be dissolved. After a time, one or the other electrode is used up and the cell becomes "dead."

The voltage that exists between the terminals of a battery depends on what the electrodes are made of and their relative ability to be dissolved or give up electrons.

When two or more cells are connected so that the positive terminal of one is connected to the negative terminal of the next, they are said to be connected in *series* and their voltages add up. Thus, the voltage between the ends of two 1.5-V flashlight batteries connected in series is 3.0 V, whereas the six 2-V cells of an automobile storage battery give 12 V. Figure 4a shows a diagram of a common "dry cell" or "flashlight battery" used in portable radios and CD players, flashlights, etc., and Fig. 4b shows two smaller ones in series, connected to a flashlight bulb. A lightbulb consists of a thin, coiled wire (filament) inside an evacuated glass bulb, as shown in Fig. 5 and in the large photo opening this Chapter. The filament gets very hot (3000 K) and glows when charge passes through it.

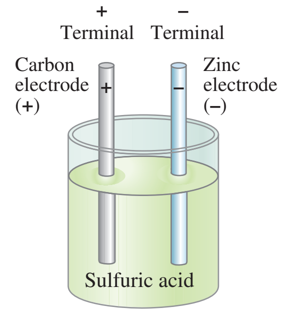


FIGURE 3 Simple electric cell.

FIGURE 4 (a) Diagram of an ordinary dry cell (like a D-cell or AA). The cylindrical zinc cup is covered on the sides; its flat bottom is the negative terminal. (b) Two dry cells (AA type) connected in series. Note that the positive terminal of one cell pushes against the negative terminal of the other.

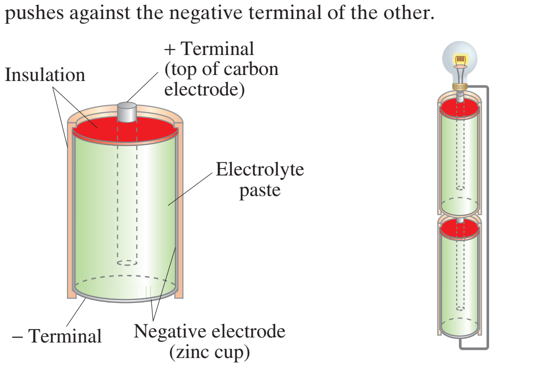
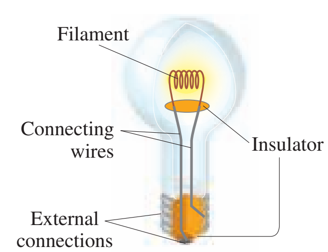


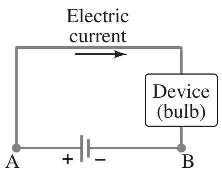
FIGURE 5 A lightbulb: the fine wire of the filament becomes so hot that it glows. This type of lightbulb is called an incandescent bulb (as compared, say, to a fluorescent bulb).



2 Electric Current

Dave King/Dorling Kindersley Media Library

(a)



(b)

FIGURE 6 (a) A simple electric circuit. (b) Schematic drawing of the same circuit, consisting of a battery, connecting wires (thick gray lines), and a lightbulb or other device.

The purpose of a battery is to produce a potential difference, which can then make charges move. When a continuous conducting path is connected between the terminals of a battery, we have an electric **circuit**, Fig. 6a. On any diagram of a circuit, as in Fig. 6b, we use the symbol

 [battery symbol]

to represent a battery. The device connected to the battery could be a lightbulb, a heater, a radio, or whatever. When such a circuit is formed, charge can flow through the wires of the circuit, from one terminal of the battery to the other, as long as the conducting path is continuous. Any flow of charge such as this is called an **electric current**.

More precisely, the electric current in a wire is defined as the net amount of charge that passes through the wire's full cross section at any point per unit time. Thus, the average current $\bar{I}$ is defined as

$$\bar{I} = \frac{\Delta Q}{\Delta t}, \tag{1a}$$

where ΔQ is the amount of charge that passes through the conductor at any location during the time interval Δt. The instantaneous current is defined by the derivative limit

$$I = \frac{dQ}{dt}. \tag{1b}$$

Electric current is measured in coulombs per second; this is given a special name, the **ampere** (abbreviated amp or A), after the French physicist André Ampère (1775–1836). Thus, $1\text{ A} = 1\text{ C/s}$. Smaller units of current are often used, such as the milliampere $(1\text{ mA} = 10^{-3}\text{ A})$ and microampere $(1\,\mu\text{A} = 10^{-6}\text{ A})$.

A current can flow in a circuit only if there is a *continuous* conducting path. We then have a **complete circuit**. If there is a break in the circuit, say, a cut wire, we call it an **open circuit** and no current flows. In any single circuit, with only a single path for current to follow such as in Fig. 6b, a steady current at any instant is the same at one point (say, point A) as at any other point (such as B). This follows from the conservation of electric charge: charge doesn't disappear. A battery does not create (or destroy) any net charge, nor does a lightbulb absorb or destroy charge.

CAUTION
A battery does not create charge; a lightbulb does not destroy charge

EXAMPLE 1 **Current is flow of charge.** A steady current of 2.5 A exists in a wire for 4.0 min. (*a*) How much total charge passed by a given point in the circuit during those 4.0 min? (*b*) How many electrons would this be?

APPROACH Current is flow of charge per unit time, Eqs. 1, so the amount of charge passing a point is the product of the current and the time interval. To get the number of electrons (*b*), we divide the total charge by the charge on one electron.

SOLUTION (*a*) Since the current was 2.5 A, or 2.5 C/s, then in 4.0 min (= 240 s) the total charge that flowed past a given point in the wire was, from Eq. 1a,

$$\Delta Q = I\,\Delta t = (2.5\text{ C/s})(240\text{ s}) = 600\text{ C}.$$

(*b*) The charge on one electron is 1.60×10^{-19} C, so 600 C would consist of

$$\frac{600\text{ C}}{1.60 \times 10^{-19}\text{ C/electron}} = 3.8 \times 10^{21}\text{ electrons.}$$

EXERCISE A If 1 million electrons per second pass a point in a wire, what is the current in amps?

CONCEPTUAL EXAMPLE 2 **How to connect a battery.** What is wrong with each of the schemes shown in Fig. 7 for lighting a flashlight bulb with a flashlight battery and a single wire?

RESPONSE (*a*) There is no closed path for charge to flow around. Charges might briefly start to flow from the battery toward the lightbulb, but there they run into a "dead end," and the flow would immediately come to a stop.

(*b*) Now there is a closed path passing to and from the lightbulb; but the wire touches only one battery terminal, so there is no potential difference in the circuit to make the charge move.

(*c*) Nothing is wrong here. This is a complete circuit: charge can flow out from one terminal of the battery, through the wire and the bulb, and into the other terminal. This scheme will light the bulb.

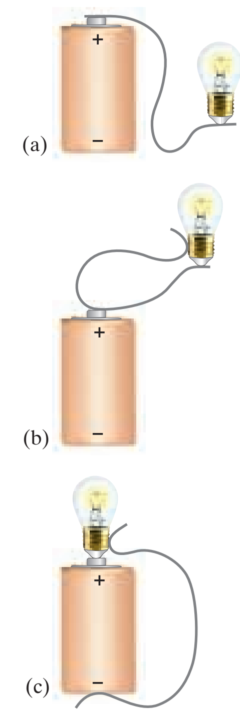
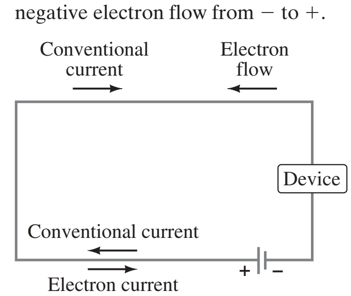


FIGURE 7 Example 2.

In many real circuits, wires are connected to a common conductor that provides continuity. This common conductor is called **ground**, usually represented as ⏚ or ⏚, and really is connected to the ground in a building or house. In a car, one terminal of the battery is called "ground," but is not connected to the ground—it is connected to the frame of the car, as is one connection to each lightbulb and other devices. Thus the car frame is a conductor in each circuit, ensuring a continuous path for charge flow.

Conductors contain many free electrons. Thus, if a continuous conducting wire is connected to the terminals of a battery, negatively charged electrons flow in the wire. When the wire is first connected, the potential difference between the terminals of the battery sets up an electric field inside the wire† and parallel to it. Free electrons at one end of the wire are attracted into the positive terminal, and at the same time other electrons leave the negative terminal of the battery and enter the wire at the other end. There is a continuous flow of electrons throughout the wire that begins as soon as the wire is connected to *both* terminals. However, when the conventions of positive and negative charge were invented two centuries ago, it was assumed that positive charge flowed in a wire. For nearly all purposes, positive charge flowing in one direction is exactly equivalent to negative charge flowing in the opposite direction,‡ as shown in Fig. 8. Today, we still use the historical convention of positive charge flow when discussing the direction of a current. So when we speak of the current direction in a circuit, we mean the direction positive charge would flow. This is sometimes referred to as **conventional current**. When we want to speak of the direction of electron flow, we will specifically state it is the electron current. In liquids and gases, both positive and negative charges (ions) can move.

FIGURE 8 Conventional current from + to − is equivalent to a negative electron flow from − to +.



3 Ohm's Law: Resistance and Resistors

To produce an electric current in a circuit, a difference in potential is required. One way of producing a potential difference along a wire is to connect its ends to the opposite terminals of a battery. It was Georg Simon Ohm (1787–1854) who established experimentally that the current in a metal wire is proportional to the potential difference V applied to its two ends:

$$I \propto V.$$

If, for example, we connect a wire to the two terminals of a 6-V battery, the current flow will be twice what it would be if the wire were connected to a 3-V battery. It is also found that reversing the sign of the voltage does not affect the magnitude of the current.

†This does not contradict that in the *static* case, there can be no electric field within a conductor since otherwise the charges would move. Indeed, when there is an electric field in a conductor, charges do move, and we get an electric current.

‡An exception exists, but is not discussed in this Chapter.

A useful analogy compares the flow of electric charge in a wire to the flow of water in a river, or in a pipe, acted on by gravity. If the river or pipe is nearly level, the flow rate is small. But if one end is somewhat higher than the other, the flow rate—or current—is greater. The greater the difference in height, the swifter the current. Electric potential is analogous, in the gravitational case, to the height of a cliff. This applies in the present case to the height through which the fluid flows. Just as an increase in height can cause a greater flow of water, so a greater electric potential difference, or voltage, causes a greater electric current.

Exactly how large the current is in a wire depends not only on the voltage but also on the resistance the wire offers to the flow of electrons. The walls of a pipe, or the banks of a river and rocks in the middle, offer resistance to the water current. Similarly, electron flow is impeded because of interactions with the atoms of the wire. The higher this resistance, the less the current for a given voltage V. We then define electrical *resistance* so that the current is inversely proportional to the resistance: that is,

$$I = \frac{V}{R} \tag{2a}$$

where R is the **resistance** of a wire or other device, V is the potential difference applied across the wire or device, and I is the current through it. Equation 2a is often written as

OHM'S "LAW"

$$\boxed{V = IR.} \tag{2b}$$

As mentioned above, Ohm found experimentally that in metal conductors R is a constant independent of V, a result known as **Ohm's law**. Equation 2b, $V = IR$, is itself sometimes called Ohm's law, but only when referring to materials or devices for which R is a constant independent of V. But R is not a constant for many substances other than metals, nor for devices such as diodes, vacuum tubes, transistors, and so on. Even for metals, R is not constant if the temperature changes much: for a lightbulb filament the measured resistance is low for small currents, but is much higher at its normal large operating current that puts it at the high temperature needed to make it glow (3000 K). Thus Ohm's "law" is not a fundamental law, but rather a description of a certain class of materials: metal conductors, whose temperature does not change much. Materials or devices that do not follow Ohm's law are said to be *nonohmic*. See Fig. 9.

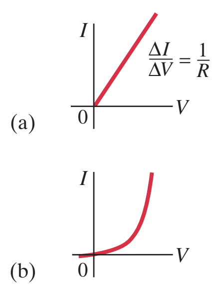
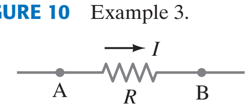


FIGURE 9 Graphs of current vs. voltage for (a) a metal conductor which obeys Ohm's law, and (b) for a nonohmic device, in this case a semiconductor diode.

The unit for resistance is called the **ohm** and is abbreviated Ω (Greek capital letter omega). Because $R = V/I$, we see that $1.0\,\Omega$ is equivalent to $1.0\,\mathrm{V/A}$.

CONCEPTUAL EXAMPLE 3 **Current and potential.** Current I enters a resistor R as shown in Fig. 10. (*a*) Is the potential higher at point A or at point B? (*b*) Is the current greater at point A or at point B?

FIGURE 10 Example 3.



RESPONSE (*a*) Positive charge always flows from + to −, from high potential to low potential. Think again of the gravitational analogy: a mass will fall down from high gravitational potential to low. So for positive current I, point A is at a higher potential than point B.

(*b*) Conservation of charge requires that whatever charge flows into the resistor at point A, an equal amount of charge emerges at point B. Charge or current does not get "used up" by a resistor, just as an object that falls through a gravitational potential difference does not gain or lose mass. So the current is the same at A and B.

An electric potential decrease, as from point A to point B in Example 3, is often called a **potential drop** or a **voltage drop**.

EXAMPLE 4 Flashlight bulb resistance. A small flashlight bulb (Fig. 11) draws 300 mA from its 1.5-V battery. (*a*) What is the resistance of the bulb? (*b*) If the battery becomes weak and the voltage drops to 1.2 V, how would the current change?

APPROACH We can apply Ohm's law to the bulb, where the voltage applied across it is the battery voltage.

SOLUTION (*a*) We change 300 mA to 0.30 A and use Eq. 2a or b:

$$R = \frac{V}{I} = \frac{1.5\text{ V}}{0.30\text{ A}} = 5.0\ \Omega.$$

(*b*) If the resistance stays the same, the current would be

$$I = \frac{V}{R} = \frac{1.2\text{ V}}{5.0\ \Omega} = 0.24\text{ A} = 240\text{ mA},$$

or a decrease of 60 mA.

NOTE With the smaller current in part (*b*), the bulb filament's temperature would be lower and the bulb less bright. Also, resistance does depend on temperature (Section 4), so our calculation is only a rough approximation.

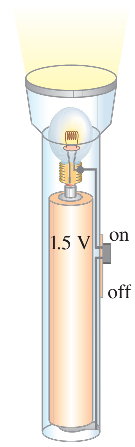


FIGURE 11 Flashlight (Example 4). Note how the circuit is completed along the side strip.

EXERCISE B What resistance should be connected across a 9.0-V battery to make a 10-mA current? (*a*) 9 Ω, (*b*) 0.9 Ω, (*c*) 900 Ω, (*d*) 1.1 Ω, (*e*) 0.11 Ω.

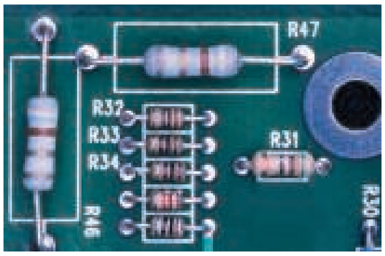
All electric devices, from heaters to lightbulbs to stereo amplifiers, offer resistance to the flow of current. The filaments of lightbulbs (Fig. 5) and electric heaters are special types of wires whose resistance results in their becoming very hot. Generally, the connecting wires have very low resistance in comparison to the resistance of the wire filaments or coils, so the connecting wires usually have a minimal effect on the magnitude of the current. In many circuits, particularly in electronic devices, **resistors** are used to control the amount of current. Resistors have resistances ranging from less than an ohm to millions of ohms (see Figs. 12 and 13). The main types are "wire-wound" resistors which consist of a coil of fine wire, "composition" resistors which are usually made of carbon, and thin carbon or metal films.

When we draw a diagram of a circuit, we use the symbol


[resistor symbol]

to indicate a resistance. Wires whose resistance is negligible, however, are shown simply as straight lines.

FIGURE 12 Photo of resistors (striped), plus other devices on a circuit board.



Tom Pantages

Resistor Color Code

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	1%
Red	2	10^2	2%
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
No color			20%

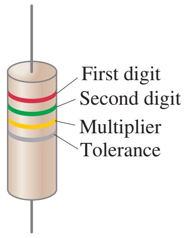


FIGURE 13 The resistance value of a given resistor is written on the exterior, or may be given as a color code as shown above and in the Table: the first two colors represent the first two digits in the value of the resistance, the third color represents the power of ten that it must be multiplied by, and the fourth is the manufactured tolerance. For example, a resistor whose four colors are red, green, yellow, and silver has a resistance of $25 \times 10^4\ \Omega = 250{,}000\ \Omega = 250\text{ k}\Omega$, plus or minus 10%. An alternate example of a simple code is a number such as 104, which means $R = 1.0 \times 10^4\ \Omega$.

Some Helpful Clarifications

CAUTION
Voltage is applied across *a device; current passes* through *a device*

Here we briefly summarize some possible misunderstandings and clarifications. Batteries do not put out a constant current. Instead, batteries are intended to maintain a constant potential difference, or very nearly so. Thus a battery should be considered a source of voltage. The voltage is applied *across* a wire or device.

Electric current passes *through* a wire or device (connected to a battery), and its magnitude depends on that device's resistance. The resistance is a *property* of the wire or device. The voltage, on the other hand, is external to the wire or device, and is applied across the two ends of the wire or device. The current through the device might be called the "response": the current increases if the voltage increases or the resistance decreases, as $I = V/R$.

In a wire, the direction of the current is always parallel to the wire, no matter how the wire curves, just like water in a pipe. The direction of conventional (positive) current is from high potential $(+)$ toward lower potential $(-)$.

CAUTION
Current is not consumed

Current and charge do not increase or decrease or get "used up" when going through a wire or other device. The amount of charge that goes in at one end comes out at the other end.

4 Resistivity

It is found experimentally that the resistance R of any wire is directly proportional to its length ℓ and inversely proportional to its cross-sectional area A. That is,

$$R = \rho \frac{\ell}{A}, \tag{3}$$

where ρ, the constant of proportionality, is called the **resistivity** and depends on the material used. Typical values of ρ, whose units are $\Omega \cdot \mathrm{m}$ (see Eq. 3), are given for various materials in the middle column of Table 1, which is divided into the categories *conductors*, *insulators*, and *semiconductors*. The values depend somewhat on purity, heat treatment, temperature, and other factors. Notice that silver has the lowest resistivity and is thus the best conductor (although it is expensive). Copper is close, and much less expensive, which is why most wires are made of copper. Aluminum, although it has a higher resistivity, is much less dense than copper; it is thus preferable to copper in some situations, such as for transmission lines, because its resistance for the same weight is less than that for copper.

TABLE 1 Resistivity and Temperature Coefficients (at 20°C)

Material	Resistivity, ρ ($\Omega \cdot$ m)	Temperature Coefficient, α $(\mathrm{C}^\circ)^{-1}$
Conductors		
Silver	1.59×10^{-8}	0.0061
Copper	1.68×10^{-8}	0.0068
Gold	2.44×10^{-8}	0.0034
Aluminum	2.65×10^{-8}	0.00429
Tungsten	5.6×10^{-8}	0.0045
Iron	9.71×10^{-8}	0.00651
Platinum	10.6×10^{-8}	0.003927
Mercury	98×10^{-8}	0.0009
Nichrome (Ni, Fe, Cr alloy)	100×10^{-8}	0.0004
Semiconductors†		
Carbon (graphite)	$(3-60) \times 10^{-5}$	-0.0005
Germanium	$(1-500) \times 10^{-3}$	-0.05
Silicon	0.1 – 60	-0.07
Insulators		
Glass	$10^{9} - 10^{12}$	
Hard rubber	$10^{13} - 10^{15}$	

† Values depend strongly on the presence of even slight amounts of impurities.

The reciprocal of the resistivity, called the **conductivity** σ, is

$$\sigma = \frac{1}{\rho} \tag{4}$$

and has units of $(\Omega \cdot \mathrm{m})^{-1}$.

EXERCISE C Return to the Chapter-Opening Question and answer it again now. Try to explain why you may have answered differently the first time.

EXERCISE D A copper wire has a resistance of $10\,\Omega$. What will its resistance be if it is only half as long? (*a*) $20\,\Omega$, (*b*) $10\,\Omega$, (*c*) $5\,\Omega$, (*d*) $1\,\Omega$, (*e*) none of these.

EXAMPLE 5 **Speaker wires.** Suppose you want to connect your stereo to remote speakers (Fig. 14). (*a*) If each wire must be 20 m long, what diameter copper wire should you use to keep the resistance less than $0.10\,\Omega$ per wire? (*b*) If the current to each speaker is 4.0 A, what is the potential difference, or voltage drop, across each wire?

APPROACH We solve Eq. 3 to get the area A, from which we can calculate the wire's radius using $A = \pi r^2$. The diameter is $2r$. In (*b*) we can use Ohm's law, $V = IR$.

SOLUTION (*a*) We solve Eq. 3 for the area A and find ρ for copper in Table 1:

$$A = \rho \frac{\ell}{R} = \frac{(1.68 \times 10^{-8}\,\Omega \cdot \mathrm{m})(20\,\mathrm{m})}{(0.10\,\Omega)} = 3.4 \times 10^{-6}\,\mathrm{m}^2.$$

The cross-sectional area A of a circular wire is $A = \pi r^2$. The radius must then be at least

$$r = \sqrt{\frac{A}{\pi}} = 1.04 \times 10^{-3}\,\mathrm{m} = 1.04\,\mathrm{mm}.$$

The diameter is twice the radius and so must be at least $2r = 2.1\,\mathrm{mm}$.

(*b*) From $V = IR$ we find that the voltage drop across each wire is

$$V = IR = (4.0\,\mathrm{A})(0.10\,\Omega) = 0.40\,\mathrm{V}.$$

NOTE The voltage drop across the wires reduces the voltage that reaches the speakers from the stereo amplifier, thus reducing the sound level a bit.

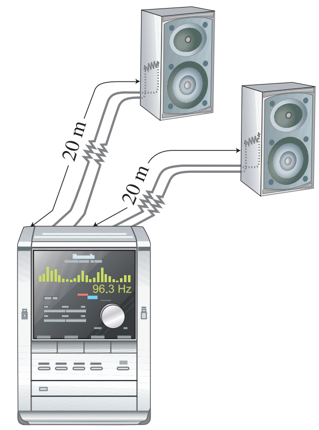


FIGURE 14 Example 5.

CONCEPTUAL EXAMPLE 6 **Stretching changes resistance.** Suppose a wire of resistance R could be stretched uniformly until it was twice its original length. What would happen to its resistance?

RESPONSE If the length ℓ doubles, then the cross-sectional area A is halved, because the volume $(V = A\ell)$ of the wire remains the same. From Eq. 3 we see that the resistance would increase by a factor of four $\left(2/\tfrac{1}{2} = 4\right)$.

EXERCISE E Copper wires in houses typically have a diameter of about 1.5 mm. How long a wire would have a $1.0\text{-}\Omega$ resistance?

Temperature Dependence of Resistivity

The resistivity of a material depends somewhat on temperature. The resistance of metals generally increases with temperature. This is not surprising, for at higher temperatures, the atoms are moving more rapidly and are arranged in a less orderly fashion. So they might be expected to interfere more with the flow of electrons. If the temperature change is not too great, the resistivity of metals usually increases nearly linearly with temperature. That is,

$$\rho_T = \rho_0\left[1 + \alpha\left(T - T_0\right)\right] \tag{5}$$

where ρ_0 is the resistivity at some reference temperature T_0 (such as 0°C or 20°C), ρ_T is the resistivity at a temperature T, and α is the *temperature coefficient of resistivity*. Values for α are given in Table 1. Note that the temperature coefficient for semiconductors can be negative. Why? It seems that at higher temperatures, some of the electrons that are normally not free in a semiconductor become free and can contribute to the current. Thus, the resistance of a semiconductor can decrease with an increase in temperature.

PHYSICS APPLIED
Resistance thermometer

EXAMPLE 7 **Resistance thermometer.** The variation in electrical resistance with temperature can be used to make precise temperature measurements. Platinum is commonly used since it is relatively free from corrosive effects and has a high melting point. Suppose at 20.0°C the resistance of a platinum resistance thermometer is 164.2 Ω. When placed in a particular solution, the resistance is 187.4 Ω. What is the temperature of this solution?

APPROACH Since the resistance R is directly proportional to the resistivity ρ, we can combine Eq. 3 with Eq. 5 to find R as a function of temperature T, and then solve that equation for T.

SOLUTION We multiply Eq. 5 by (ℓ/A) to obtain (see also Eq. 3)

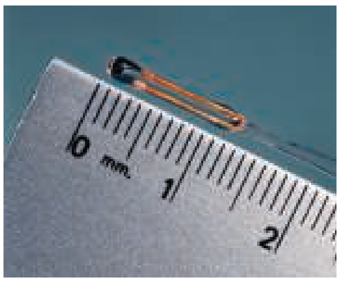
$$R = R_0[1 + \alpha(T - T_0)].$$

Here $R_0 = \rho_0\ell/A$ is the resistance of the wire at $T_0 = 20.0°\mathrm{C}$. We solve this equation for T and find (see Table 1 for α)

$$T = T_0 + \frac{R - R_0}{\alpha R_0} = 20.0°\mathrm{C} + \frac{187.4\,\Omega - 164.2\,\Omega}{(3.927 \times 10^{-3}(\mathrm{C°})^{-1})(164.2\,\Omega)} = 56.0°\mathrm{C}.$$

NOTE Resistance thermometers have the advantage that they can be used at very high or low temperatures where gas or liquid thermometers would be useless.

NOTE More convenient for some applications is a *thermistor* (Fig. 15), which consists of a metal oxide or semiconductor whose resistance also varies in a repeatable way with temperature. Thermistors can be made quite small and respond very quickly to temperature changes.



Richard Megna/Fundamental Photographs, NYC

FIGURE 15 A thermistor shown next to a millimeter ruler for scale.

EXERCISE F The resistance of the tungsten filament of a common incandescent lightbulb is how many times greater at its operating temperature of 3000 K than its resistance at room temperature? (*a*) Less than 1% greater; (*b*) roughly 10% greater; (*c*) about 2 times greater; (*d*) roughly 10 times greater; (*e*) more than 100 times greater.

The value of α in Eq. 5 itself can depend on temperature, so it is important to check the temperature range of validity of any value (say, in a handbook of physical data). If the temperature range is wide, Eq. 5 is not adequate and terms proportional to the square and cube of the temperature are needed, but they are generally very small except when $T - T_0$ is large.

5 Electric Power

Electric energy is useful to us because it can be easily transformed into other forms of energy. Motors transform electric energy into mechanical energy.

In other devices such as electric heaters, stoves, toasters, and hair dryers, electric energy is transformed into thermal energy in a wire resistance known as a "heating element." And in an ordinary lightbulb, the tiny wire filament (Fig. 5 and Chapter-opening photo) becomes so hot it glows; only a few percent of the energy is transformed into visible light, and the rest, over 90%, into thermal energy. Lightbulb filaments and heating elements (Fig. 16) in household appliances have resistances typically of a few ohms to a few hundred ohms.

FIGURE 16 Hot electric stove burner glows because of energy transformed by electric current.


Mark C. Burnett/Photo Researchers, Inc.

Electric energy is transformed into thermal energy or light in such devices, and there are many collisions between the moving electrons and the atoms of the wire. In each collision, part of the electron's kinetic energy is transferred to the atom with which it collides. As a result, the kinetic energy of the wire's atoms increases and hence the temperature of the wire element increases. The increased thermal energy can be transferred as heat by conduction and convection to the air in a heater or to food in a pan, by radiation to bread in a toaster, or radiated as light.

To find the power transformed by an electric device, recall that the energy transformed when an infinitesimal charge dq moves through a potential difference V is $dU = V\,dq$. Let dt be the time required for an amount of charge dq

to move through a potential difference V. Then the power P, which is the rate energy is transformed, is

$$P = \frac{dU}{dt} = \frac{dq}{dt}V.$$

The charge that flows per second, dq/dt, is the electric current I. Thus we have

$$P = IV. \tag{6}$$

This general relation gives us the power transformed by any device, where I is the current passing through it and V is the potential difference across it. It also gives the power delivered by a source such as a battery. The SI unit of electric power is the same as for any kind of power, the **watt** $(1\text{ W} = 1\text{ J/s})$.

The rate of energy transformation in a resistance R can be written in two other ways, starting with the general relation $P = IV$ and substituting in $V = IR$:

$$P = IV = I(IR) = I^2R \tag{7a}$$

$$P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R}. \tag{7b}$$

Equations 7a and b apply only to resistors, whereas Eq. 6, $P = IV$, is more general and applies to any device, including a resistor.

EXAMPLE 8 **Headlights.** Calculate the resistance of a 40-W automobile headlight designed for 12 V (Fig. 17).

APPROACH We solve Eq. 7b for R.

SOLUTION From Eq. 7b,

$$R = \frac{V^2}{P} = \frac{(12\text{ V})^2}{(40\text{ W})} = 3.6\ \Omega.$$

NOTE This is the resistance when the bulb is burning brightly at 40 W. When the bulb is cold, the resistance is much lower, as we saw in Eq. 5. Since the current is high when the resistance is low, lightbulbs burn out most often when first turned on.

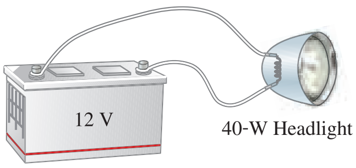


FIGURE 17 Example 8.

PHYSICS APPLIED
Why lightbulbs burn out when first turned on

CAUTION
You pay for energy, which is power × time, not for power

It is energy, not power, that you pay for on your electric bill. Since power is the *rate* energy is transformed, the total energy used by any device is simply its power consumption multiplied by the time it is on. If the power is in watts and the time is in seconds, the energy will be in joules since $1\text{ W} = 1\text{ J/s}$. Electric companies usually specify the energy with a much larger unit, the **kilowatt-hour** (kWh). One $\text{kWh} = (1000\text{ W})(3600\text{ s}) = 3.60 \times 10^6\text{ J}$.

EXAMPLE 9 **Electric heater.** An electric heater draws a steady 15.0 A on a 120-V line. How much power does it require and how much does it cost per month (30 days) if it operates 3.0 h per day and the electric company charges 9.2 cents per kWh?

APPROACH We use Eq. 6, $P = IV$, to find the power. We multiply the power (in kW) by the time (h) used in a month and by the cost per energy unit, \$0.092 per kWh, to get the cost per month.

SOLUTION The power is

$$P = IV = (15.0\text{ A})(120\text{ V}) = 1800\text{ W}$$

or 1.80 kW. The time (in hours) the heater is used per month is $(3.0\text{ h/d})(30\text{ d}) = 90\text{ h}$, which at 9.2¢/kWh would cost $(1.80\text{ kW})(90\text{ h})(\$0.092/\text{kWh}) = \$15$.

NOTE Household current is actually alternating (ac), but our solution is still valid assuming the given values for V and I are the proper averages (rms) as we discuss in Section 7.

PHYSICS APPLIED
Lightning

A&J Verkaik/Bettmann/Corbis

FIGURE 18 Example 10. A lightning bolt.

EXAMPLE 10 **ESTIMATE** **Lightning bolt.** Lightning is a spectacular example of electric current in a natural phenomenon (Fig. 18). There is much variability to lightning bolts, but a typical event can transfer 10^9 J of energy across a potential difference of perhaps 5×10^7 V during a time interval of about 0.2 s. Use this information to estimate (*a*) the total amount of charge transferred between cloud and ground, (*b*) the current in the lightning bolt, and (*c*) the average power delivered over the 0.2 s.

APPROACH We estimate the charge Q, recalling that potential energy change equals the potential difference ΔV times the charge Q. We equate ΔU with the energy transferred, $\Delta U \approx 10^9$ J. Next, the current I is Q/t (Eq. 1a) and the power P is energy/time.

SOLUTION (*a*) From $\Delta U = U_b - U_a = q(V_b - V_a) = qV_{ba}$, the energy transformed is $\Delta U = Q\,\Delta V$. We solve for Q:

$$Q = \frac{\Delta U}{\Delta V} \approx \frac{10^9\ \text{J}}{5 \times 10^7\ \text{V}} = 20\ \text{coulombs}.$$

(*b*) The current during the 0.2 s is about

$$I = \frac{Q}{t} \approx \frac{20\ \text{C}}{0.2\ \text{s}} = 100\ \text{A}.$$

(*c*) The average power delivered is

$$P = \frac{\text{energy}}{\text{time}} = \frac{10^9\ \text{J}}{0.2\ \text{s}} = 5 \times 10^9\ \text{W} = 5\ \text{GW}.$$

We can also use Eq. 6:

$$P = IV = (100\ \text{A})(5 \times 10^7\ \text{V}) = 5\ \text{GW}.$$

NOTE Since most lightning bolts consist of several stages, it is possible that individual parts could carry currents much higher than the 100 A calculated above.

6 Power in Household Circuits

PHYSICS APPLIED
Safety—wires getting hot

The electric wires that carry electricity to lights and other electric appliances have some resistance, although usually it is quite small. Nonetheless, if the current is large enough, the wires will heat up and produce thermal energy at a rate equal to I^2R, where R is the wire's resistance. One possible hazard is that the current-carrying wires in the wall of a building may become so hot as to start a fire. Thicker wires have less resistance (see Eq. 3) and thus can carry more current without becoming too hot. When a wire carries more current than is safe, it is said to be "overloaded." To prevent overloading, *fuses* or *circuit breakers* are installed in circuits. They are basically switches (Fig. 19)

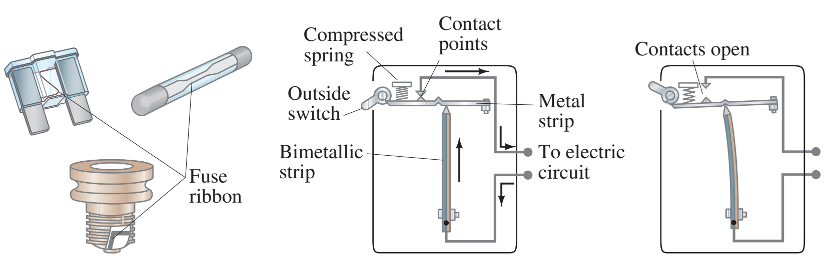


FIGURE 19 (a) Fuses. When the current exceeds a certain value, the metallic ribbon melts and the circuit opens. Then the fuse must be replaced. (b) One type of circuit breaker. The electric current passes through a bimetallic strip. When the current exceeds a safe level, the heating of the bimetallic strip causes the strip to bend so far to the left that the notch in the spring-loaded metal strip drops down over the end of the bimetallic strip; (c) the circuit then opens at the contact points (one is attached to the metal strip) and the outside switch is also flipped. As soon as the bimetallic strip cools down, it can be reset using the outside switch.

that open the circuit when the current exceeds some particular value. A 20-A fuse or circuit breaker, for example, opens when the current passing through it exceeds 20 A. If a circuit repeatedly burns out a fuse or opens a circuit breaker, there are two possibilities: there may be too many devices drawing current in that circuit; or there is a fault somewhere, such as a "short." A short, or "short circuit," means that two wires have touched that should not have (perhaps because the insulation has worn through) so the resistance is much reduced and the current becomes very large. Short circuits should be remedied immediately.

PHYSICS APPLIED
Fuses and circuit breakers

Household circuits are designed with the various devices connected so that each receives the standard voltage (usually 120 V in the United States) from the electric company (Fig. 20). Circuits with the devices arranged as in Fig. 20 are called *parallel circuits*. When a fuse blows or circuit breaker opens, it is important to check the total current being drawn on that circuit, which is the sum of the currents in each device.

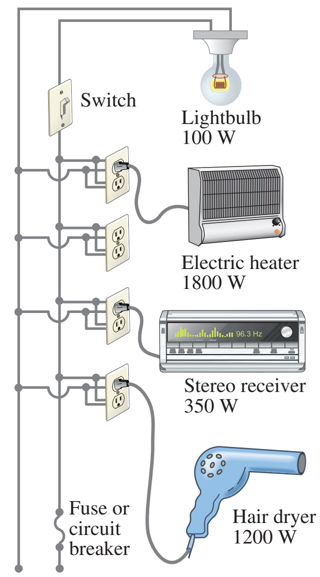


FIGURE 20 Connection of household appliances.

EXAMPLE 11 **Will a fuse blow?** Determine the total current drawn by all the devices in the circuit of Fig. 20.

APPROACH Each device has the same 120-V voltage across it. The current each draws from the source is found from $I = P/V$, Eq. 6.

SOLUTION The circuit in Fig. 20 draws the following currents: the lightbulb draws $I = P/V = 100\,\mathrm{W}/120\,\mathrm{V} = 0.8\,\mathrm{A}$; the heater draws $1800\,\mathrm{W}/120\,\mathrm{V} = 15.0\,\mathrm{A}$; the stereo draws a maximum of $350\,\mathrm{W}/120\,\mathrm{V} = 2.9\,\mathrm{A}$; and the hair dryer draws $1200\,\mathrm{W}/120\,\mathrm{V} = 10.0\,\mathrm{A}$. The total current drawn, if all devices are used at the same time, is

$$0.8\,\mathrm{A} + 15.0\,\mathrm{A} + 2.9\,\mathrm{A} + 10.0\,\mathrm{A} = 28.7\,\mathrm{A}.$$

NOTE The heater draws as much current as 18 100-W lightbulbs. For safety, the heater should probably be on a circuit by itself.

If the circuit in Fig. 20 is designed for a 20-A fuse, the fuse should blow, and we hope it will, to prevent overloaded wires from getting hot enough to start a fire. Something will have to be turned off to get this circuit below 20 A. (Houses and apartments usually have several circuits, each with its own fuse or circuit breaker; try moving one of the devices to another circuit.) If the circuit is designed with heavier wire and a 30-A fuse, the fuse shouldn't blow—if it does, a short may be the problem. (The most likely place for a short is in the cord of one of the devices.) Proper fuse size is selected according to the wire used to supply the current. A properly rated fuse should *never* be replaced by a higher-rated one. A fuse blowing or a circuit breaker opening is acting like a switch, making an "open circuit." By an open circuit, we mean that there is no longer a complete conducting path, so no current can flow; it is as if $R = \infty$.

PHYSICS APPLIED
Proper fuses and shorts

CONCEPTUAL EXAMPLE 12 **A dangerous extension cord.** Your 1800-W portable electric heater is too far from your desk to warm your feet. Its cord is too short, so you plug it into an extension cord rated at 11 A. Why is this dangerous?

PHYSICS APPLIED
Extension cords and possible danger

RESPONSE 1800 W at 120 V draws a 15-A current. The wires in the extension cord rated at 11 A could become hot enough to melt the insulation and cause a fire.

EXERCISE G How many 60-W 120-V lightbulbs can operate on a 20-A line? (*a*) 2; (*b*) 3; (*c*) 6; (*d*) 20; (*e*) 40.

7 Alternating Current

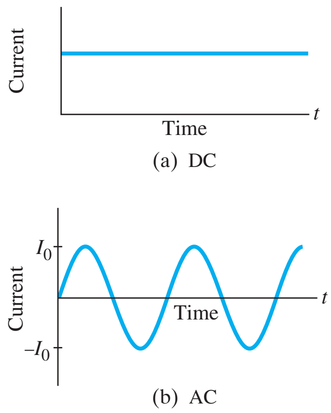


FIGURE 21 (a) Direct current. (b) Alternating current.

When a battery is connected to a circuit, the current moves steadily in one direction. This is called a **direct current**, or **dc**. Electric generators at electric power plants, however, produce **alternating current**, or **ac**. (Sometimes capital letters are used, DC and AC.) An alternating current reverses direction many times per second and is commonly sinusoidal, as shown in Fig. 21. The electrons in a wire first move in one direction and then in the other. The current supplied to homes and businesses by electric companies is ac throughout virtually the entire world. Because ac circuits are so common in real life, we will discuss some of their basic aspects here.

The voltage produced by an ac electric generator is sinusoidal, as we shall see later. The current it produces is thus sinusoidal (Fig. 21b). We can write the voltage as a function of time as

$$V = V_0 \sin 2\pi f t = V_0 \sin \omega t.$$

The potential V oscillates between $+V_0$ and $-V_0$, and V_0 is referred to as the **peak voltage**. The frequency f is the number of complete oscillations made per second, and $\omega = 2\pi f$. In most areas of the United States and Canada, f is 60 Hz (the unit "hertz" means cycles per second). In many other countries, 50 Hz is used.

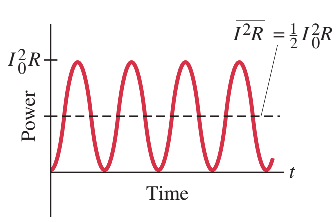
Equation 2, $V = IR$, works also for ac: if a voltage V exists across a resistance R, then the current I through the resistance is

$$I = \frac{V}{R} = \frac{V_0}{R} \sin \omega t = I_0 \sin \omega t. \qquad \textbf{(8)}$$

The quantity $I_0 = V_0/R$ is the **peak current**. The current is considered positive when the electrons flow in one direction and negative when they flow in the opposite direction. It is clear from Fig. 21b that an alternating current is as often positive as it is negative. Thus, the average current is zero. This does not mean, however, that no power is needed or that no heat is produced in a resistor. Electrons do move back and forth, and do produce heat. Indeed, the power transformed in a resistance R at any instant is

$$P = I^2 R = I_0^2 R \sin^2 \omega t.$$

FIGURE 22 Power transformed in a resistor in an ac circuit.



Because the current is squared, we see that the power is always positive, as graphed in Fig. 22. The quantity $\sin^2 \omega t$ varies between 0 and 1; and it is not too difficult to show† that its average value is $\frac{1}{2}$, as indicated in Fig. 22. Thus, the *average power* transformed, $\overline{P}$, is

$$\overline{P} = \tfrac{1}{2} I_0^2 R.$$

Since power can also be written $P = V^2/R = \left(V_0^2/R\right) \sin^2 \omega t$, we also have that the average power is

$$\overline{P} = \frac{1}{2} \frac{V_0^2}{R}.$$

The average or mean value of the *square* of the current or voltage is thus what is important for calculating average power: $\overline{I^2} = \frac{1}{2} I_0^2$ and $\overline{V^2} = \frac{1}{2} V_0^2$. The square root of each of these is the **rms** (root-mean-square) value of the current or voltage:

$$I_{\text{rms}} = \sqrt{\overline{I^2}} = \frac{I_0}{\sqrt{2}} = 0.707 I_0, \qquad \textbf{(9a)}$$

$$V_{\text{rms}} = \sqrt{\overline{V^2}} = \frac{V_0}{\sqrt{2}} = 0.707 V_0. \qquad \textbf{(9b)}$$

The rms values of V and I are sometimes called the *effective values*.

†A graph of $\cos^2 \omega t$ versus t is identical to that for $\sin^2 \omega t$ in Fig. 22, except that the points are shifted (by $\frac{1}{4}$ cycle) on the time axis. Hence the average value of $\sin^2$ and $\cos^2$, averaged over one or more full cycles, will be the same: $\overline{\sin^2 \omega t} = \overline{\cos^2 \omega t}$. From the trigonometric identity $\sin^2 \theta + \cos^2 \theta = 1$, we can write

$$\overline{\left(\sin^2 \omega t\right)} + \overline{\left(\cos^2 \omega t\right)} = 2\overline{\left(\sin^2 \omega t\right)} = 1.$$

Hence the average value of $\sin^2 \omega t$ is $\frac{1}{2}$.

They are useful because they can be substituted directly into the power formulas, Eqs. 6 and 7, to get the average power:

$$\overline{P} = I_{\text{rms}} V_{\text{rms}} \tag{10a}$$

$$\overline{P} = \tfrac{1}{2} I_0^2 R = I_{\text{rms}}^2 R \tag{10b}$$

$$\overline{P} = \tfrac{1}{2} \frac{V_0^2}{R} = \frac{V_{\text{rms}}^2}{R}. \tag{10c}$$

Thus, a direct current whose values of I and V equal the rms values of I and V for an alternating current will produce the same power. Hence it is usually the rms value of current and voltage that is specified or measured. For example, in the United States and Canada, standard line voltage[†] is 120-V ac. The 120 V is V_{rms}; the peak voltage V_0 is

$$V_0 = \sqrt{2}\, V_{\text{rms}} = 170\text{ V}.$$

In much of the world (Europe, Australia, Asia) the rms voltage is 240 V, so the peak voltage is 340 V.

EXAMPLE 13 **Hair dryer.** (*a*) Calculate the resistance and the peak current in a 1000-W hair dryer (Fig. 23) connected to a 120-V line. (*b*) What happens if it is connected to a 240-V line in Britain?

APPROACH We are given $\overline{P}$ and V_{rms}, so $I_{\text{rms}} = \overline{P}/V_{\text{rms}}$ (Eq. 10a or 6), and $I_0 = \sqrt{2}\, I_{\text{rms}}$. Then we find R from $V = IR$.

SOLUTION (*a*) We solve Eq. 10a for the rms current:

$$I_{\text{rms}} = \frac{\overline{P}}{V_{\text{rms}}} = \frac{1000\text{ W}}{120\text{ V}} = 8.33\text{ A}.$$

Then

$$I_0 = \sqrt{2}\, I_{\text{rms}} = 11.8\text{ A}.$$

The resistance is

$$R = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{120\text{ V}}{8.33\text{ A}} = 14.4\,\Omega.$$

The resistance could equally well be calculated using peak values:

$$R = \frac{V_0}{I_0} = \frac{170\text{ V}}{11.8\text{ A}} = 14.4\,\Omega.$$

(*b*) When connected to a 240-V line, more current would flow and the resistance would change with the increased temperature (Section 4). But let us make an estimate of the power transformed based on the same 14.4-Ω resistance. The average power would be

$$\overline{P} = \frac{V_{\text{rms}}^2}{R} = \frac{(240\text{ V})^2}{(14.4\,\Omega)} = 4000\text{ W}.$$

This is four times the dryer's power rating and would undoubtedly melt the heating element or the wire coils of the motor.

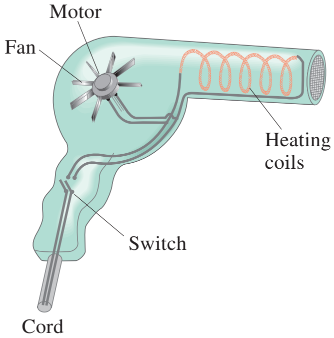


FIGURE 23 A hair dryer. Most of the current goes through the heating coils, a pure resistance; a small part goes to the motor to turn the fan. Example 13.

EXERCISE H Each channel of a stereo receiver is capable of an average power output of 100 W into an 8-Ω loudspeaker (see Fig. 14). What are the rms voltage and the rms current fed to the speaker (*a*) at the maximum power of 100 W, and (*b*) at 1.0 W when the volume is turned down?

[†]The line voltage can vary, depending on the total load; the frequency of 60 Hz or 50 Hz, however, remains extremely steady.

8 Microscopic View of Electric Current: Current Density and Drift Velocity

Up to now in this Chapter we have dealt mainly with a macroscopic view of electric current. We saw, however, that according to atomic theory, the electric current in metal wires is carried by negatively charged electrons, and that in liquid solutions current can also be carried by positive and/or negative ions. Let us now look at this microscopic picture in more detail.

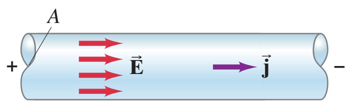


FIGURE 24 Electric field $\vec{\mathbf{E}}$ in a uniform wire of cross-sectional area A carrying a current I. The current density $j = I/A$.

When a potential difference is applied to the two ends of a wire of uniform cross section, the direction of the electric field $\vec{\mathbf{E}}$ is parallel to the walls of the wire (Fig. 24). The existence of $\vec{\mathbf{E}}$ within the conducting wire does not contradict our earlier result that $\vec{\mathbf{E}} = 0$ inside a conductor in the electrostatic case, as we are no longer dealing with the static case. Charges are free to move in a conductor, and hence can move under the action of the electric field. If all the charges are at rest, then $\vec{\mathbf{E}}$ must be zero (electrostatics).

We now define a new microscopic quantity, the **current density**, $\vec{\mathbf{j}}$. It is defined as the *electric current per unit cross-sectional area* at any point in space. If the current density $\vec{\mathbf{j}}$ in a wire of cross-sectional area A is uniform over the cross section, then j is related to the electric current by

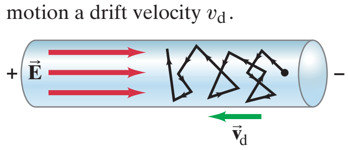
$$j = \frac{I}{A} \qquad \text{or} \qquad I = jA. \tag{11}$$

If the current density is not uniform, then the general relation is

$$I = \int \vec{\mathbf{j}} \cdot d\vec{\mathbf{A}}, \tag{12}$$

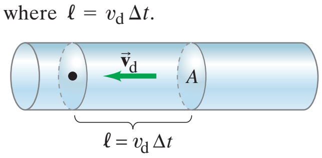
where $d\vec{\mathbf{A}}$ is an element of surface and I is the current through the surface over which the integration is taken. The direction of the current density at any point is the direction that a positive charge would move when placed at that point—that is, the direction of $\vec{\mathbf{j}}$ at any point is generally the same as the direction of $\vec{\mathbf{E}}$, Fig. 24. The current density exists for any *point* in space. The current I, on the other hand, refers to a conductor as a whole, and hence is a macroscopic quantity.

FIGURE 25 Electric field $\vec{\mathbf{E}}$ in a wire gives electrons in random motion a drift velocity v_d.



The direction of $\vec{\mathbf{j}}$ is chosen to represent the direction of net flow of positive charge. In a conductor, it is negatively charged electrons that move, so they move in the direction of $-\vec{\mathbf{j}}$, or $-\vec{\mathbf{E}}$ (to the left in Fig. 24). We can imagine the free electrons as moving about randomly at high speeds, bouncing off the atoms of the wire (somewhat like the molecules of a gas). When an electric field exists in the wire, Fig. 25, the electrons feel a force and initially begin to accelerate. But they soon reach a more or less steady average velocity in the direction of $\vec{\mathbf{E}}$, known as their **drift velocity**, $\vec{v}_d$ (collisions with atoms in the wire keep them from accelerating further). The drift velocity is normally very much smaller than the electrons' average random speed.

FIGURE 26 Electrons in the volume $A\ell$ will all pass through the cross section indicated in a time Δt, where $\ell = v_d \Delta t$.



We can relate the drift velocity v_d to the macroscopic current I in the wire. In a time Δt, the electrons will travel a distance $\ell = v_d \Delta t$ on average. Suppose the wire has cross-sectional area A. Then in time Δt, electrons in a volume $V = A\ell = Av_d \Delta t$ will pass through the cross section A of wire, as shown in Fig. 26. If there are n free electrons (each of charge $-e$) per unit volume $(n = N/V)$, then the total charge ΔQ that passes through the area A in a time Δt is

$$\begin{aligned} \Delta Q &= (\text{no. of charges}, N) \times (\text{charge per particle}) \\ &= (nV)(-e) = -(nAv_d \Delta t)(e). \end{aligned}$$

The current I in the wire is thus

$$I = \frac{\Delta Q}{\Delta t} = -neAv_d. \tag{13}$$

The current density, $j = I/A$, is

$$j = -nev_d. \tag{14}$$

In vector form, this is written

$$\vec{\mathbf{j}} = -ne\vec{\mathbf{v}}_d, \tag{15}$$

where the minus sign indicates that the direction of (positive) current flow is opposite to the drift velocity of electrons.

We can generalize Eq. 15 to any type of charge flow, such as flow of ions in an electrolyte. If there are several types of ions (which can include free electrons), each of density n_i (number per unit volume), charge q_i ($q_i = -e$ for electrons) and drift velocity $\vec{\mathbf{v}}_{\mathrm{d}i}$, then the net current density at any point is

$$\vec{\mathbf{j}} = \sum_i n_i q_i \vec{\mathbf{v}}_{\mathrm{d}i}. \tag{16}$$

The total current I passing through an area A perpendicular to a uniform $\vec{\mathbf{j}}$ is then

$$I = \sum_i n_i q_i v_{\mathrm{d}i} A.$$

EXAMPLE 14 **Electron speeds in a wire.** A copper wire 3.2 mm in diameter carries a 5.0-A current. Determine (*a*) the current density in the wire, and (*b*) the drift velocity of the free electrons. (*c*) Estimate the rms speed of electrons assuming they behave like an ideal gas at 20°C. Assume that one electron per Cu atom is free to move (the others remain bound to the atom).

APPROACH For (*a*) $j = I/A = I/\pi r^2$. For (*b*) we can apply Eq. 14 to find v_d if we can determine the number n of free electrons per unit volume. Since we assume there is one free electron per atom, the density of free electrons, n, is the same as the density of Cu atoms. The atomic mass of Cu is 63.5 u, so 63.5 g of Cu contains one mole or 6.02×10^{23} free electrons. The mass density of copper is $\rho_\mathrm{D} = 8.9 \times 10^3\,\mathrm{kg/m^3}$, where $\rho_\mathrm{D} = m/V$. (We use ρ_D to distinguish it here from ρ for resistivity.) In (*c*) we use $K = \frac{3}{2}kT$. (Do not confuse V for volume with V for voltage.)

SOLUTION (*a*) The current density is (with $r = \frac{1}{2}(3.2\,\mathrm{mm}) = 1.6 \times 10^{-3}\,\mathrm{m}$)

$$j = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{5.0\,\mathrm{A}}{\pi\left(1.6 \times 10^{-3}\,\mathrm{m}\right)^2} = 6.2 \times 10^5\,\mathrm{A/m^2}.$$

(*b*) The number of free electrons per unit volume, $n = N/V$ (where $V = m/\rho_\mathrm{D}$), is

$$n = \frac{N}{V} = \frac{N}{m/\rho_\mathrm{D}} = \frac{N(1\text{ mole})}{m(1\text{ mole})}\rho_\mathrm{D}$$

$$n = \left(\frac{6.02 \times 10^{23}\text{ electrons}}{63.5 \times 10^{-3}\,\mathrm{kg}}\right)\left(8.9 \times 10^3\,\mathrm{kg/m^3}\right) = 8.4 \times 10^{28}\,\mathrm{m^{-3}}.$$

Then, by Eq. 14, the drift velocity has magnitude

$$v_\mathrm{d} = \frac{j}{ne} = \frac{6.2 \times 10^5\,\mathrm{A/m^2}}{\left(8.4 \times 10^{28}\,\mathrm{m^{-3}}\right)\left(1.6 \times 10^{-19}\,\mathrm{C}\right)} = 4.6 \times 10^{-5}\,\mathrm{m/s} \approx 0.05\,\mathrm{mm/s}.$$

(*c*) If we model the free electrons as an ideal gas (a rather rough approximation), we use to estimate the random rms speed of an electron as it darts around:

$$v_\mathrm{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38 \times 10^{-23}\,\mathrm{J/K}\right)(293\,\mathrm{K})}{9.11 \times 10^{-31}\,\mathrm{kg}}} = 1.2 \times 10^5\,\mathrm{m/s}.$$

The drift velocity (average speed in the direction of the current) is very much less than the rms thermal speed of the electrons, by a factor of about 10^9.

NOTE The result in (*c*) is an underestimate. Quantum theory calculations, and experiments, give the rms speed in copper to be about $1.6 \times 10^6\,\mathrm{m/s}$.

The drift velocity of electrons in a wire is very slow, only about 0.05 mm/s (Example 14 above), which means it takes an electron 20×10^3 s, or $5\frac{1}{2}$ h, to travel only 1 m. This is not, of course, how fast "electricity travels": when you flip a light switch, the light—even if many meters away—goes on nearly instantaneously. Why? Because electric fields travel essentially at the speed of light $\left(3 \times 10^8\,\mathrm{m/s}\right)$. We can think of electrons in a wire as being like a pipe full of water: when a little water enters one end of the pipe, almost immediately some water comes out at the other end.

*Electric Field Inside a Wire

Equation 2b, $V = IR$, can be written in terms of microscopic quantities as follows. We write the resistance R in terms of the resistivity ρ:

$$R = \rho \frac{\ell}{A};$$

and we write V and I as

$$I = jA \qquad \text{and} \qquad V = E\ell.$$

We assume the electric field is uniform within the wire and ℓ is the length of the wire (or a portion of the wire) between whose ends the potential difference is V. Thus, from $V = IR$, we have

$$E\ell = (jA)\left(\rho \frac{\ell}{A}\right) = j\rho\ell$$

so

$$j = \frac{1}{\rho} E = \sigma E, \tag{17}$$

where $\sigma = 1/\rho$ is the *conductivity* (Eq. 4). For a metal conductor, ρ and σ do not depend on V (and hence not on E). Therefore the current density $\vec{\mathbf{j}}$ is proportional to the electrical field $\vec{\mathbf{E}}$ in the conductor. This is the "microscopic" statement of Ohm's law. Equation 17, which can be written in vector form as

$$\vec{\mathbf{j}} = \sigma \vec{\mathbf{E}} = \frac{1}{\rho} \vec{\mathbf{E}},$$

is sometimes taken as the definition of conductivity σ and resistivity ρ.

EXAMPLE 15 **Electric field inside a wire.** What is the electric field inside the wire of Example 14?

APPROACH We use Eq. 17 and $\rho = 1.68 \times 10^{-8}\,\Omega\cdot\text{m}$ for copper.

SOLUTION Example 14 gives $j = 6.2 \times 10^{5}\,\text{A/m}^2$, so

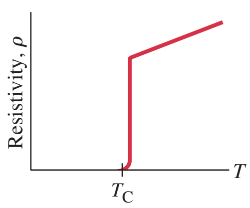
$$E = \rho j = \left(1.68 \times 10^{-8}\,\Omega\cdot\text{m}\right)\left(6.2 \times 10^{5}\,\text{A/m}^2\right) = 1.0 \times 10^{-2}\,\text{V/m}.$$

NOTE For comparison, the electric field between the plates of a capacitor is often much larger. Thus we see that only a modest electric field is needed for current flow in practical cases.

*9 Superconductivity

At very low temperatures, well below 0°C, the resistivity (Section 4) of certain metals and certain compounds or alloys becomes zero as measured by the highest-precision techniques. Materials in such a state are said to be **superconducting**. It was first observed by H. K. Onnes (1853–1926) in 1911 when he cooled mercury below 4.2 K (−269°C) and found that the resistance of mercury suddenly dropped to zero. In general, superconductors become superconducting only below a certain *transition temperature* or *critical temperature*, T_C, which is usually within a few degrees of absolute zero. Current in a ring-shaped superconducting material has been observed to flow for years in the absence of a potential difference, with no measurable decrease. Measurements show that the resistivity ρ of superconductors is less than $4 \times 10^{-25}\,\Omega\cdot\text{m}$, which is over 10^{16} times smaller than that for copper, and is considered to be zero in practice. See Fig. 27.

FIGURE 27 A superconducting material has zero resistivity when its temperature is below T_C, its "critical temperature." At T_C, the resistivity jumps to a "normal" nonzero value and increases with temperature as most materials do (Eq. 5).



Before 1986 the highest temperature at which a material was found to superconduct was 23 K, which required liquid helium to keep the material cold. In 1987, a compound of yttrium, barium, copper, and oxygen (YBCO) was developed that can be superconducting at 90 K. This was an important breakthrough since liquid nitrogen, which boils at 77 K (sufficiently cold to keep the material superconducting), is more easily and cheaply obtained than the liquid helium needed for conventional superconductors. Superconductivity at temperatures as high as 160 K has been reported, though in fragile compounds.

Most applications today use a bismuth-strontium-calcium-copper oxide, known (for short) as BSCCO. A major challenge is how to make a useable, bendable wire out of the BSCCO, which is very brittle. (One solution is to embed tiny filaments of the high-T_C superconductor in a metal alloy, which is not resistanceless, but the resistance is much less than that of a conventional copper cable.)

*10 Electrical Conduction in the Nervous System

The flow of electric charge in the human nervous system provides us the means for being aware of the world. Although the detailed functioning is not well understood, we do have a reasonable understanding of how messages are transmitted within the nervous system: they are electrical signals passing along the basic element of the nervous system, the *neuron*.

Neurons are living cells of unusual shape (Fig. 28). Attached to the main cell body are several small appendages known as *dendrites* and a long tail called the *axon*. Signals are received by the dendrites and are propagated along the axon. When a signal reaches the nerve endings, it is transmitted to the next neuron or to a muscle at a connection called a *synapse*.

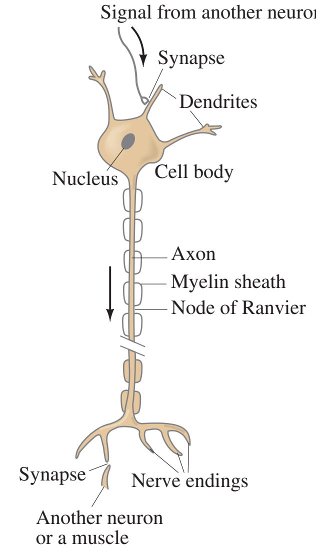


FIGURE 28 A simplified sketch of a typical neuron.

A neuron, before transmitting an electrical signal, is in the so-called "resting state." Like nearly all living cells, neurons have a net positive charge on the outer surface of the cell membrane and a negative charge on the inner surface. This difference in charge, or "dipole layer," means that a potential difference exists across the cell membrane. When a neuron is not transmitting a signal, this "resting potential," normally stated as

$$V_{\text{inside}} - V_{\text{outside}},$$

is typically $-60\,\text{mV}$ to $-90\,\text{mV}$, depending on the type of organism. The most common ions in a cell are K^+, Na^+, and Cl^-. There are large differences in the concentrations of these ions inside and outside a cell, as indicated by the typical values given in Table 2. Other ions are also present, so the fluids both inside and outside the axon are electrically neutral. Because of the differences in concentration, there is a tendency for ions to diffuse across the membrane. However, in the resting state the cell membrane prevents any net flow of Na^+ (through a mechanism of "active pumping" of Na^+ out of the cell). But it does allow the flow of Cl^- ions, and less so of K^+ ions, and it is these two ions that produce the dipole charge layer on the membrane. Because there is a greater concentration of K^+ inside the cell than outside, more K^+ ions tend to diffuse outward across the membrane than diffuse inward. A K^+ ion that passes through the membrane becomes attached to the outer surface of the membrane, and leaves behind an equal negative charge that lies on the inner surface of the membrane (Fig. 29). Independently, Cl^- ions tend to diffuse *into* the cell since their concentration outside is higher. Both K^+ and Cl^- diffusion tends to charge the interior surface of the membrane negatively and the outside positively. As charge accumulates on the membrane surface, it becomes increasingly difficult for more ions to diffuse: K^+ ions trying to move outward, for example, are repelled by the positive charge already there. Equilibrium is reached when the tendency to diffuse because of the concentration difference is just balanced by the electrical potential difference across the membrane. The greater the concentration difference, the greater the potential difference across the membrane ($-60\,\text{mV}$ to $-90\,\text{mV}$).

TABLE 2
Concentrations of Ions Inside and Outside a Typical Axon

	Concentration inside axon (mol/m^3)	Concentration outside axon (mol/m^3)
K^+	140	5
Na^+	15	140
Cl^-	9	125

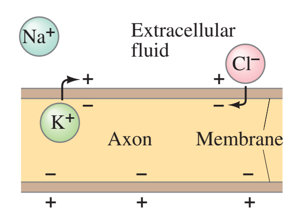
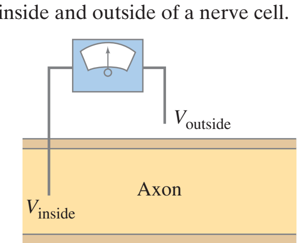


FIGURE 29 How a dipole layer of charge forms on a cell membrane.

The most important aspect of a neuron is not that it has a resting potential (most cells do), but rather that it can respond to a stimulus and conduct an electrical signal along its length. The stimulus could be thermal (when you touch a hot stove) or chemical (as in taste buds); it could be pressure (as on the skin or at the eardrum), or light (as in the eye); or it could be the electric stimulus of a signal coming from the brain or another neuron. In the laboratory, the stimulus is usually electrical and is applied by a tiny probe at some point on the neuron. If the stimulus exceeds some threshold, a voltage pulse will travel down the axon. This voltage pulse can be detected at a point on the axon using a voltmeter or an oscilloscope connected as in Fig. 30.

FIGURE 30 Measuring the potential difference between the inside and outside of a nerve cell.



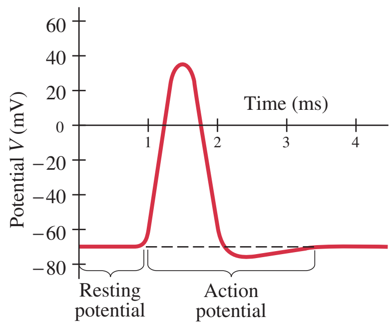
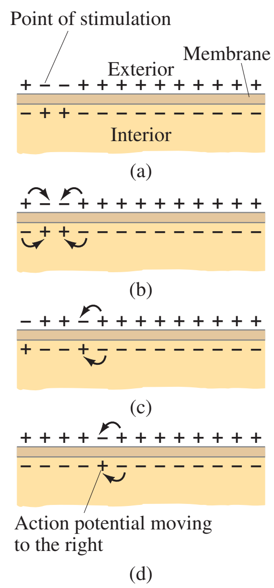


FIGURE 31 Action potential.

FIGURE 32 Propagation of an action potential along an axon membrane.



This voltage pulse has the shape shown in Fig. 31, and is called an **action potential**. As can be seen, the potential increases from a resting potential of about $-70\,\mathrm{mV}$ and becomes a positive $30\,\mathrm{mV}$ or $40\,\mathrm{mV}$. The action potential lasts for about 1 ms and travels down an axon with a speed of 30 m/s to 150 m/s. When an action potential is stimulated, the nerve is said to have "fired."

What causes the action potential? Apparently, the cell membrane has the ability to alter its permeability properties. At the point where the stimulus occurs, the membrane suddenly becomes much more permeable to Na^+ than to K^+ and Cl^- ions. Thus, Na^+ ions rush into the cell and the inner surface of the wall becomes positively charged, and the potential difference quickly swings positive ($\approx +35\,\mathrm{mV}$ in Fig. 31). Just as suddenly, the membrane returns to its original characteristics: it becomes impermeable to Na^+ and in fact pumps out Na^+ ions. The diffusion of Cl^- and K^+ ions again predominates and the original resting potential is restored ($-70\,\mathrm{mV}$ in Fig. 31).

What causes the action potential to travel along the axon? The action potential occurs at the point of stimulation, as shown in Fig. 32a. The membrane momentarily is positive on the inside and negative on the outside at this point. Nearby charges are attracted toward this region, as shown in Fig. 32b. The potential in these adjacent regions then drops, causing an action potential there. Thus, as the membrane returns to normal at the original point, nearby it experiences an action potential, so the action potential moves down the axon (Figs. 32c and d).

You may wonder if the number of ions that pass through the membrane would significantly alter the concentrations. The answer is no; and we can show why by treating the axon as a capacitor in the following Example.

EXAMPLE 16 **ESTIMATE** **Capacitance of an axon.** (*a*) Do an order-of-magnitude estimate for the capacitance of an axon 10 cm long of radius $10\,\mu\mathrm{m}$. The thickness of the membrane is about $10^{-8}\,\mathrm{m}$, and the dielectric constant is about 3. (*b*) By what factor does the concentration (number of ions per volume) of Na^+ ions in the cell change as a result of one action potential?

APPROACH We model the membrane of an axon as a cylindrically shaped parallel-plate capacitor, with opposite charges on each side. The separation of the "plates" is the thickness of the membrane, $d \approx 10^{-8}\,\mathrm{m}$. We first calculate the area of the cylinder and then can use $C = K\epsilon_0 A/d$ to find the capacitance. In (*b*), we use the voltage change during one action potential to find the amount of charge moved across the membrane.

SOLUTION (*a*) The area A is the area of a cylinder of radius r and length ℓ:

$$A = 2\pi r\ell \approx (6.28)(10^{-5}\,\mathrm{m})(0.1\,\mathrm{m}) \approx 6 \times 10^{-6}\,\mathrm{m}^2.$$

From the previous Equation, we have

$$C = K\epsilon_0 \frac{A}{d} \approx (3)(8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2})\frac{6 \times 10^{-6}\,\mathrm{m}^2}{10^{-8}\,\mathrm{m}} \approx 10^{-8}\,\mathrm{F}.$$

(*b*) Since the voltage changes from $-70\,\mathrm{mV}$ to about $+30\,\mathrm{mV}$, the total change is about 100 mV. The amount of charge that moves is then

$$Q = CV \approx (10^{-8}\,\mathrm{F})(0.1\,\mathrm{V}) = 10^{-9}\,\mathrm{C}.$$

Each ion carries a charge $e = 1.6 \times 10^{-19}\,\mathrm{C}$, so the number of ions that flow per action potential is $Q/e = (10^{-9}\,\mathrm{C})/(1.6 \times 10^{-19}\,\mathrm{C}) \approx 10^{10}$. The volume of our cylindrical axon is

$$V = \pi r^2\ell \approx (3)(10^{-5}\,\mathrm{m})^2(0.1\,\mathrm{m}) = 3 \times 10^{-11}\,\mathrm{m}^3,$$

and the concentration of Na^+ ions inside the cell (Table 2) is $15\,\mathrm{mol/m^3} = 15 \times 6.02 \times 10^{23}\,\mathrm{ions/m^3} \approx 10^{25}\,\mathrm{ions/m^3}$. Thus, the cell contains $(10^{25}\,\mathrm{ions/m^3}) \times (3 \times 10^{-11}\,\mathrm{m^3}) \approx 3 \times 10^{14}\,Na^+$ ions. One action potential, then, will change the concentration of Na^+ ions by about $10^{10}/(3 \times 10^{14}) = \frac{1}{3} \times 10^{-4}$, or 1 part in 30,000. This tiny change would not be measurable.

Thus, even 1000 action potentials will not alter the concentration significantly. The sodium pump does not, therefore, have to remove Na^+ ions quickly after an action potential, but can operate slowly over time to maintain a relatively constant concentration.

Summary

An electric **battery** serves as a source of nearly constant potential difference by transforming chemical energy into electric energy. A simple battery consists of two electrodes made of different metals immersed in a solution or paste known as an electrolyte.

Electric current, I, refers to the rate of flow of electric charge and is measured in **amperes** (A): 1 A equals a flow of 1 C/s past a given point.

The direction of **conventional current** is that of positive charge flow. In a wire, it is actually negatively charged electrons that move, so they flow in a direction opposite to the conventional current. A positive charge flow in one direction is almost always equivalent to a negative charge flow in the opposite direction. Positive conventional current always flows from a high potential to a low potential.

The **resistance** R of a device is defined by the relation

$$V = IR, \tag{2}$$

where I is the current in the device when a potential difference V is applied across it. For materials such as metals, R is a constant independent of V (thus $I \propto V$), a result known as **Ohm's law**. Thus, the current I coming from a battery of voltage V depends on the resistance R of the circuit connected to it.

Voltage is applied *across* a device or between the ends of a wire. Current passes *through* a wire or device. Resistance is a property *of* the wire or device.

The unit of resistance is the **ohm** (Ω), where $1\,\Omega = 1\,\mathrm{V/A}$. See Table 3.

TABLE 3 Summary of Units

Current	$1\,\mathrm{A} = 1\,\mathrm{C/s}$
Potential difference	$1\,\mathrm{V} = 1\,\mathrm{J/C}$
Power	$1\,\mathrm{W} = 1\,\mathrm{J/s}$
Resistance	$1\,\Omega = 1\,\mathrm{V/A}$

The resistance R of a wire is inversely proportional to its cross-sectional area A, and directly proportional to its length ℓ and to a property of the material called its resistivity:

$$R = \frac{\rho\ell}{A}. \tag{3}$$

The **resistivity**, ρ, increases with temperature for metals, but for semiconductors it may decrease.

The rate at which energy is transformed in a resistance R from electric to other forms of energy (such as heat and light) is equal to the product of current and voltage. That is, the **power** transformed, measured in watts, is given by

$$P = IV, \tag{6}$$

which for resistors can be written as

$$P = I^2R = \frac{V^2}{R}. \tag{7}$$

The SI unit of power is the **watt** $(1\,\mathrm{W} = 1\,\mathrm{J/s})$.

The total electric energy transformed in any device equals the product of the power and the time during which the device is operated. In SI units, energy is given in joules $(1\,\mathrm{J} = 1\,\mathrm{W\cdot s})$, but electric companies use a larger unit, the **kilowatt-hour** $(1\,\mathrm{kWh} = 3.6 \times 10^6\,\mathrm{J})$.

Electric current can be **direct current** (**dc**), in which the current is steady in one direction; or it can be **alternating current** (**ac**), in which the current reverses direction at a particular frequency f, typically 60 Hz. Alternating currents are typically sinusoidal in time,

$$I = I_0 \sin \omega t, \tag{8}$$

where $\omega = 2\pi f$, and are produced by an alternating voltage.

The **rms** values of sinusoidally alternating currents and voltages are given by

$$I_{\mathrm{rms}} = \frac{I_0}{\sqrt{2}} \quad \text{and} \quad V_{\mathrm{rms}} = \frac{V_0}{\sqrt{2}}, \tag{9}$$

respectively, where I_0 and V_0 are the **peak** values. The power relationship, $P = IV = I^2R = V^2/R$, is valid for the average power in alternating currents when the rms values of V and I are used.

Current density $\vec{\mathbf{j}}$ is the current per cross-sectional area. From a microscopic point of view, the current density is related to the number of charge carriers per unit volume, n, their charge, q, and their **drift velocity**, $\vec{\mathbf{v}}_{\mathrm{d}}$, by

$$\vec{\mathbf{j}} = nq\vec{\mathbf{v}}_{\mathrm{d}}. \tag{16}$$

The electric field within a wire is related to $\vec{\mathbf{j}}$ by $\vec{\mathbf{j}} = \sigma\vec{\mathbf{E}}$ where $\sigma = 1/\rho$ is the **conductivity**.

[*At very low temperatures certain materials become **superconducting**, which means their electrical resistance becomes zero.]

[*The human nervous system operates via electrical conduction: when a nerve "fires," an electrical signal travels as a voltage pulse known as an **action potential**.]

Answers to Exercises

A: 1.6×10^{-13} A.

B: (*c*).

C: (*b*), (*c*).

D: (*c*).

E: 110 m.

F: (*d*).

G: (*e*).

H: (*a*) 28 V, 3.5 A; (*b*) 2.8 V, 0.35 A.