Lester V. Bergman/Corbis

We are used to voltage in our lives—a 12-volt car battery, 110 V or 220 V at home, 1.5 volt flashlight batteries, and so on. Here we see a Van de Graaff generator, whose voltage may reach 50,000 V or more. Voltage is the same as electric potential difference between two points. Electric potential is defined as the potential energy per unit charge.

The children here, whose hair stands on end because each hair has received the same sign of charge, are not harmed by the voltage because the Van de Graaff cannot provide much current before the voltage drops. (It is current through the body that is harmful, as we will see later.)

Electric Potential

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.*]

Consider a pair of parallel plates with equal and opposite charge densities, σ. Which of the following actions will increase the voltage between the plates (assuming fixed charge density)?

(a) Moving the plates closer together.
(b) Moving the plates apart.
(c) Doubling the area of the plates.
(d) Halving the area of the plates.

CONTENTS

The energy point of view is especially useful for electricity. It not only extends the law of conservation of energy, but it gives us another way to view electrical phenomena. Energy is also a powerful tool for solving Problems more easily in many cases than by using forces and electric fields.

1 Electric Potential Energy and Potential Difference

Electric Potential Energy

To apply conservation of energy, we need to define electric potential energy as we do for other types of potential energy. Potential energy can be defined only for a conservative force. The work done by a conservative force in moving an object between any two positions is independent of the path taken. The electrostatic force between any two charges ($F = kQ_1Q_2/r^2$) is conservative since the dependence on position is just like the gravitational force, $1/r^2$, which is conservative. Hence we can define potential energy U for the electrostatic force.

Note: Sections marked with an asterisk (*) may be considered optional by the instructor.

From Chapter 23 of *Physics for Scientists & Engineers with Modern Physics*, Fourth Edition, Douglas C. Giancoli.

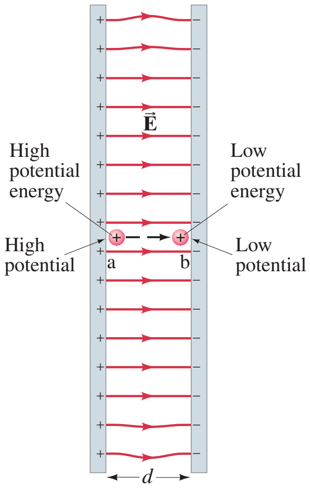


FIGURE 1 Work is done by the electric field in moving the positive charge q from position a to position b.

The change in potential energy between two points, a and b, equals the negative of the work done by the conservative force as an object moves from a to b: $\Delta U = -W$.

Thus we define the change in electric potential energy, $U_b - U_a$, when a point charge q moves from some point a to another point b, as the negative of the work done by the electric force as the charge moves from a to b. For example, consider the electric field between two equally but oppositely charged parallel plates; we assume their separation is small compared to their width and height, so the field $\vec{\mathbf{E}}$ will be uniform over most of the region, Fig. 1. Now consider a tiny positive point charge q placed at point a very near the positive plate as shown. This charge q is so small it has no effect on $\vec{\mathbf{E}}$. If this charge q at point a is released, the electric force will do work on the charge and accelerate it toward the negative plate. The work W done by the electric field E to move the charge a distance d is

$$W = Fd = qEd$$

where $F = qE$. The change in electric potential energy equals the negative of the work done by the electric force:

$$U_b - U_a = -W = -qEd \qquad [\text{uniform } \vec{\mathbf{E}}] \quad \textbf{(1)}$$

for this case of uniform electric field $\vec{\mathbf{E}}$. In the case illustrated, the potential energy decreases (ΔU is negative); and as the charged particle accelerates from point a to point b in Fig. 1, the particle's kinetic energy K increases—by an equal amount. In accord with the conservation of energy, electric potential energy is transformed into kinetic energy, and the total energy is conserved. Note that the positive charge q has its greatest potential energy at point a, near the positive plate.[†] The reverse is true for a negative charge: its potential energy is greatest near the negative plate.

Electric Potential and Potential Difference

It is useful to define the **electric potential** (or simply the **potential** when "electric" is understood) as the *electric potential energy per unit charge*. Electric potential is given the symbol V. If a positive test charge q in an electric field has electric potential energy U_a at some point a (relative to some zero potential energy), the electric potential V_a at this point is

$$V_a = \frac{U_a}{q}. \qquad \textbf{(2a)}$$

Only differences in potential energy are physically meaningful. Hence only the **difference in potential**, or the **potential difference**, between two points a and b (such as between a and b in Fig. 1) is measurable. When the electric force does positive work on a charge, the kinetic energy increases and the potential energy decreases. The difference in potential energy, $U_b - U_a$, is equal to the negative of the work, W_{ba}, done by the electric field as the charge moves from a to b; so the potential difference V_{ba} is

$$V_{ba} = \Delta V = V_b - V_a = \frac{U_b - U_a}{q} = -\frac{W_{ba}}{q}. \qquad \textbf{(2b)}$$

Note that electric potential, like electric field, does not depend on our test charge q. V depends on the other charges that create the field, not on q; q acquires potential energy by being in the potential V due to the other charges.

We can see from our definition that the positive plate in Fig. 1 is at a higher potential than the negative plate. Thus a positively charged object moves naturally from a high potential to a low potential. A negative charge does the reverse.

The unit of electric potential, and of potential difference, is joules/coulomb and is given a special name, the **volt**, in honor of Alessandro Volta (1745–1827) who is best known for inventing the electric battery. The volt is abbreviated V, so $1\text{ V} = 1\text{ J/C}$. Potential difference, since it is measured in volts, is often referred to as **voltage**.

[†]At this point the charge has its greatest ability to do work (on some other object or system).

If we wish to speak of the potential V_a at some point a, we must be aware that V_a depends on where the potential is chosen to be zero. The zero for electric potential in a given situation can be chosen arbitrarily, just as for potential energy, because only differences in potential energy can be measured. Often the ground, or a conductor connected directly to the ground (the Earth), is taken as zero potential, and other potentials are given with respect to ground. (Thus, a point where the voltage is 50 V is one where the difference of potential between it and ground is 50 V.) In other cases, as we shall see, we may choose the potential to be zero at an infinite distance $(r = \infty)$.

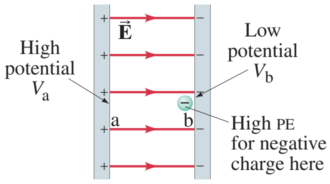


FIGURE 2 Central part of Fig. 1, showing a negative point charge near the negative plate, where its potential energy (PE) is high. Example 1.

CONCEPTUAL EXAMPLE 1 **A negative charge.** Suppose a negative charge, such as an electron, is placed near the negative plate in Fig. 1, at point b, shown here in Fig. 2. If the electron is free to move, will its electric potential energy increase or decrease? How will the electric potential change?

RESPONSE An electron released at point b will move toward the positive plate. As the electron moves toward the positive plate, its potential energy *decreases* as its kinetic energy gets larger, so $U_a < U_b$ and $\Delta U = U_a - U_b < 0$. But note that the electron moves from point b at low potential to point a at higher potential: $V_{ab} = V_a - V_b > 0$. (Potentials V_a and V_b are due to the charges on the plates, not due to the electron.) The sign of ΔU and ΔV are opposite because of the negative charge.

CAUTION
A negative charge has high potential energy when potential V is low

Because the electric potential difference is defined as the potential energy difference per unit charge, then the change in potential energy of a charge q when moved between two points a and b is

$$\Delta U = U_b - U_a = q(V_b - V_a) = qV_{ba}. \tag{3}$$

That is, if an object with charge q moves through a potential difference V_{ba}, its potential energy changes by an amount qV_{ba}. For example, if the potential difference between the two plates in Fig. 1 is 6 V, then a $+1$ C charge moved (say by an external force) from point b to point a will gain $(1\text{ C})(6\text{ V}) = 6\text{ J}$ of electric potential energy. (And it will lose 6 J of electric potential energy if it moves from a to b.) Similarly, a $+2$ C charge will gain 12 J, and so on. Thus, electric potential difference is a measure of how much energy an electric charge can acquire in a given situation. And, since energy is the ability to do work, the electric potential difference is also a measure of how much work a given charge can do. The exact amount depends both on the potential difference and on the charge.

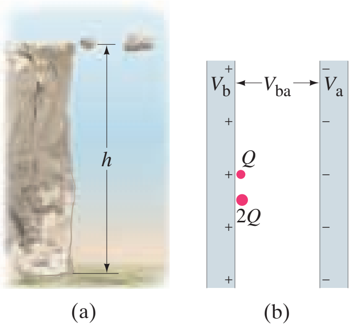


FIGURE 3 (a) Two rocks are at the same height. The larger rock has more potential energy. (b) Two charges have the same electric potential. The $2Q$ charge has more potential energy.

To better understand electric potential, let's make a comparison to the gravitational case when a rock falls from the top of a cliff. The greater the height, h, of a cliff, the more potential energy $(= mgh)$ the rock has at the top of the cliff, relative to the bottom, and the more kinetic energy it will have when it reaches the bottom. The actual amount of kinetic energy it will acquire, and the amount of work it can do, depends both on the height of the cliff and the mass m of the rock. A large rock and a small rock can be at the same height h (Fig. 3a) and thus have the same "gravitational potential," but the larger rock has the greater potential energy (it has more mass). The electrical case is similar (Fig. 3b): the potential energy change, or the work that can be done, depends both on the potential difference (corresponding to the height of the cliff) and on the charge (corresponding to mass), Eq. 3. But note a significant difference: electric charge comes in two types, $+$ and $-$, whereas gravitational mass is always $+$.

Sources of electrical energy such as batteries and electric generators are meant to maintain a potential difference. The actual amount of energy transformed by such a device depends on how much charge flows, as well as the potential difference (Eq. 3). For example, consider an automobile headlight connected to a 12.0-V battery. The amount of energy transformed (into light and thermal energy) is proportional to how much charge flows, which depends on how long the light is on. If over a given period of time 5.0 C of charge flows through the light, the total energy transformed is $(5.0\text{ C})(12.0\text{ V}) = 60\text{ J}$. If the headlight is left on twice as long, 10.0 C of charge will flow and the energy transformed is $(10.0\text{ C})(12.0\text{ V}) = 120\text{ J}$. Table 1 presents some typical voltages.

TABLE 1 Some Typical Potential Differences (Voltages)

Source	Voltage (approx.)
Thundercloud to ground	10^8 V
High-voltage power line	10^5–10^6 V
Power supply for TV tube	10^4 V
Automobile ignition	10^4 V
Household outlet	10^2 V
Automobile battery	12 V
Flashlight battery	1.5 V
Resting potential across nerve membrane	10^{-1} V
Potential changes on skin (EKG and EEG)	10^{-4} V

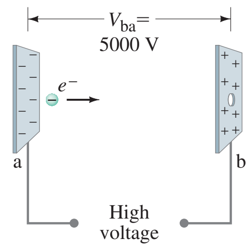


FIGURE 4 Electron accelerated in CRT. Example 2.

EXAMPLE 2 **Electron in CRT.** Suppose an electron in a cathode ray tube (Section 9) is accelerated from rest through a potential difference $V_b - V_a = V_{ba} = +5000\text{ V}$ (Fig. 4). (*a*) What is the change in electric potential energy of the electron? (*b*) What is the speed of the electron $(m = 9.1 \times 10^{-31}\text{ kg})$ as a result of this acceleration?

APPROACH The electron, accelerated toward the positive plate, will decrease in potential energy by an amount $\Delta U = qV_{ba}$ (Eq. 3). The loss in potential energy will equal its gain in kinetic energy (energy conservation).

SOLUTION (*a*) The charge on an electron is $q = -e = -1.6 \times 10^{-19}\text{ C}$. Therefore its change in potential energy is

$$\Delta U = qV_{ba} = (-1.6 \times 10^{-19}\text{ C})(+5000\text{ V}) = -8.0 \times 10^{-16}\text{ J}.$$

The minus sign indicates that the potential energy decreases. The potential difference V_{ba} has a positive sign since the final potential V_b is higher than the initial potential V_a. Negative electrons are attracted toward a positive electrode and repelled away from a negative electrode.

(*b*) The potential energy lost by the electron becomes kinetic energy K. From conservation of energy, $\Delta K + \Delta U = 0$, so

$$\Delta K = -\Delta U$$
$$\tfrac{1}{2}mv^2 - 0 = -q(V_b - V_a) = -qV_{ba},$$

where the initial kinetic energy is zero since we are given that the electron started from rest. We solve for v:

$$v = \sqrt{-\frac{2qV_{ba}}{m}} = \sqrt{-\frac{2(-1.6 \times 10^{-19}\text{ C})(5000\text{ V})}{9.1 \times 10^{-31}\text{ kg}}} = 4.2 \times 10^{7}\text{ m/s}.$$

NOTE The electric potential energy does not depend on the mass, only on the charge and voltage. The speed *does* depend on m.

2 Relation between Electric Potential and Electric Field

The effects of any charge distribution can be described either in terms of electric field or in terms of electric potential. Electric potential is often easier to use because it is a scalar, as compared to electric field which is a vector. There is a crucial connection between the electric potential produced by a given arrangement of charges and the electric field due to those charges, which we now examine.

We start by recalling the relation between a conservative force $\vec{\mathbf{F}}$ and the potential energy U associated with that force. The difference in potential energy between any two points in space, a and b, is given by:

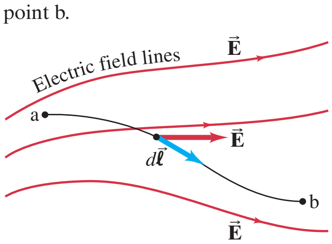
$$U_b - U_a = -\int_a^b \vec{\mathbf{F}} \cdot d\vec{\boldsymbol{\ell}},$$

where $d\vec{\boldsymbol{\ell}}$ is an infinitesimal increment of displacement, and the integral is taken along any path in space from point a to point b. For the electrical case, we are more interested in the potential difference, given by Eq. 2b, $V_{ba} = V_b - V_a = (U_b - U_a)/q$, rather than in the potential energy itself. Also, the electric field $\vec{\mathbf{E}}$ at any point in space is defined as the force per unit charge: $\vec{\mathbf{E}} = \vec{\mathbf{F}}/q$. Putting these two relations in the above equation gives us

$$V_{ba} = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}. \tag{4a}$$

This is the general relation between electric field and potential difference. See Fig. 5. If we are given the electric field due to some arrangement of electric charge, we can use Eq. 4a to determine V_{ba}.

FIGURE 5 To find V_{ba} in a nonuniform electric field $\vec{\mathbf{E}}$, we integrate $\vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}$ from point a to point b.



A simple special case is a uniform field. In Fig. 1, for example, a path parallel to the electric field lines from point a at the positive plate to point b at the negative plate gives (since $\vec{\mathbf{E}}$ and $d\vec{\boldsymbol{\ell}}$ are in the same direction at each point),

$$V_{\mathrm{ba}} = V_{\mathrm{b}} - V_{\mathrm{a}} = -\int_{\mathrm{a}}^{\mathrm{b}} \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -E\int_{\mathrm{a}}^{\mathrm{b}} d\ell = -Ed$$

or

$$V_{\mathrm{ba}} = -Ed \qquad \text{[only if } E \text{ is uniform]} \qquad \textbf{(4b)}$$

where d is the distance, parallel to the field lines, between points a and b. Be careful not to use Eq. 4b unless you are sure the electric field is uniform.

From either of Eqs. 4 we can see that the units for electric field intensity can be written as volts per meter (V/m) as well as newtons per coulomb (N/C). These are equivalent in general, since $1\,\mathrm{N/C} = 1\,\mathrm{N\cdot m/C\cdot m} = 1\,\mathrm{J/C\cdot m} = 1\,\mathrm{V/m}$.

EXERCISE A Return to the Chapter-Opening Question and answer it again now. Try to explain why you may have answered differently the first time.

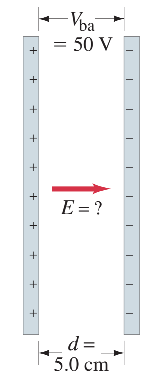


FIGURE 6 Example 3.

EXAMPLE 3 **Electric field obtained from voltage.** Two parallel plates are charged to produce a potential difference of 50 V. If the separation between the plates is 0.050 m, calculate the magnitude of the electric field in the space between the plates (Fig. 6).

APPROACH We apply Eq. 4b to obtain the magnitude of E, assumed uniform.

SOLUTION The electric field magnitude is $E = V_{\mathrm{ba}}/d = (50\,\mathrm{V}/0.050\,\mathrm{m}) = 1000\,\mathrm{V/m}$.

EXAMPLE 4 **Charged conducting sphere.** Determine the potential at a distance r from the center of a charged conducting sphere of radius r_0 for (*a*) $r > r_0$, (*b*) $r = r_0$, (*c*) $r < r_0$. The total charge on the sphere is Q.

APPROACH The charge Q is distributed over the surface of the sphere since it is a conductor. The electric field outside a conducting sphere is

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \qquad [r > r_0]$$

and points radially outward (inward if $Q < 0$). Since we know $\vec{\mathbf{E}}$, we can start by using Eq. 4a.

SOLUTION (*a*) We use Eq. 4a and integrate along a radial line with $d\vec{\boldsymbol{\ell}}$ parallel to $\vec{\mathbf{E}}$ (Fig. 7) between two points which are distances r_{a} and r_{b} from the sphere's center:

$$V_{\mathrm{b}} - V_{\mathrm{a}} = -\int_{r_{\mathrm{a}}}^{r_{\mathrm{b}}} \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{Q}{4\pi\epsilon_0}\int_{r_{\mathrm{a}}}^{r_{\mathrm{b}}} \frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_{\mathrm{b}}} - \frac{1}{r_{\mathrm{a}}}\right)$$

and we set $d\ell = dr$. If we let $V = 0$ for $r = \infty$ (let's choose $V_{\mathrm{b}} = 0$ at $r_{\mathrm{b}} = \infty$), then at any other point r (for $r > r_0$) we have

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}. \qquad [r > r_0]$$

We will see in the next Section that this same equation applies for the potential a distance r from a single point charge. Thus the electric potential outside a spherical conductor with a uniformly distributed charge is the same as if all the charge were at its center.

(*b*) As r approaches r_0, we see that

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0} \qquad [r = r_0]$$

at the surface of the conductor.

(*c*) For points within the conductor, $E = 0$. Thus the integral, $\int \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}$, between $r = r_0$ and any point within the conductor gives zero change in V. Hence V is constant within the conductor:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0}. \qquad [r \le r_0]$$

The whole conductor, not just its surface, is at this same potential. Plots of both E and V as a function of r are shown in Fig. 8 for a positively charged conducting sphere.

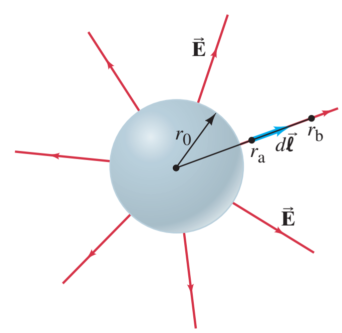
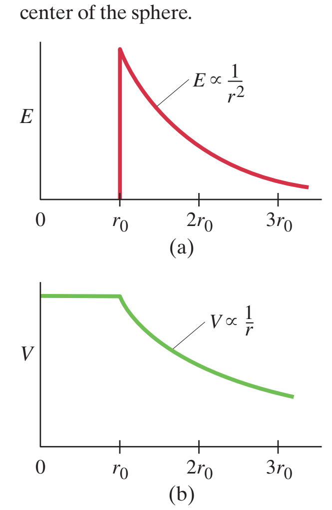


FIGURE 7 Example 4. Integrating $\vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}$ for the field outside a spherical conductor.

FIGURE 8 (a) E versus r, and (b) V versus r, for a positively charged solid conducting sphere of radius r_0 (the charge distributes itself evenly on the surface); r is the distance from the center of the sphere.



EXAMPLE 5 **Breakdown voltage.** In many kinds of equipment, very high voltages are used. A problem with high voltage is that the air can become ionized due to the high electric fields: free electrons in the air (produced by cosmic rays, for example) can be accelerated by such high fields to speeds sufficient to ionize O_2 and N_2 molecules by collision, knocking out one or more of their electrons. The air then becomes conducting and the high voltage cannot be maintained as charge flows. The breakdown of air occurs for electric fields of about $3 \times 10^6\,\mathrm{V/m}$. (*a*) Show that the breakdown voltage for a spherical conductor in air is proportional to the radius of the sphere, and (*b*) estimate the breakdown voltage in air for a sphere of diameter 1.0 cm.

APPROACH The electric potential at the surface of a spherical conductor of radius r_0 (Example 4), and the electric field just outside its surface, are

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0} \qquad \text{and} \qquad E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0^2}.$$

SOLUTION (*a*) We combine these two equations and obtain

$$V = r_0 E. \qquad \text{[at surface of spherical conductor]}$$

(*b*) For $r_0 = 5 \times 10^{-3}\,\mathrm{m}$, the breakdown voltage in air is

$$V = \left(5 \times 10^{-3}\,\mathrm{m}\right)\left(3 \times 10^6\,\mathrm{V/m}\right) \approx 15{,}000\,\mathrm{V}.$$

When high voltages are present, a glow may be seen around sharp points, known as a **corona discharge**, due to the high electric fields at these points which ionize air molecules. The light we see is due to electrons jumping down to empty lower states. **Lightning rods**, with their sharp tips, are intended to ionize the surrounding air when a storm cloud is near, and to provide a conduction path to discharge a dangerous high-voltage cloud slowly, over a period of time. Thus lightning rods, connected to the ground, are intended to draw electric charge off threatening clouds before a large buildup of charge results in a swift destructive lightning bolt.

EXERCISE B On a dry day, a person can become electrically charged by rubbing against rugs and other ordinary objects. Suppose you notice a small shock as you reach for a metal doorknob, noting that the shock occurs along with a tiny spark when your hand is about 3.0 mm from the doorknob. As a rough estimate, use Eq. 4b to estimate the potential difference between your hand and the doorknob. (*a*) 9 V; (*b*) 90 V; (*c*) 900 V; (*d*) 9000 V; (*e*) none of these.

3 Electric Potential Due to Point Charges

The electric potential at a distance r from a single point charge Q can be derived directly from Eq. 4a, $V_b - V_a = -\int \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}$. The electric field due to a single point charge has magnitude

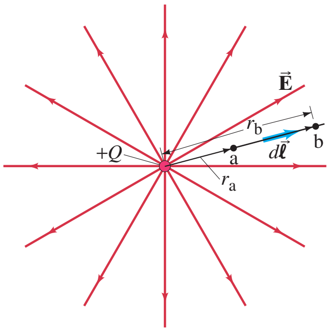
$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \qquad \text{or} \qquad E = k\frac{Q}{r^2}$$

(where $k = 1/4\pi\epsilon_0 = 8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2}$), and is directed radially outward from a positive charge (inward if $Q < 0$). We take the integral in Eq. 4a along a (straight) field line (Fig. 9) from point a, a distance r_a from Q, to point b, a distance r_b from Q. Then $d\vec{\boldsymbol{\ell}}$ will be parallel to $\vec{\mathbf{E}}$ and $d\ell = dr$. Thus

$$V_b - V_a = -\int_{r_a}^{r_b} \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{Q}{4\pi\epsilon_0}\int_{r_a}^{r_b}\frac{1}{r^2}\,dr = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r_b} - \frac{Q}{r_a}\right).$$

As mentioned earlier, only differences in potential have physical meaning. We are free, therefore, to choose the value of the potential at some one point to

FIGURE 9 We integrate Eq. 4a along the straight line (shown in black) from point a to point b. The line ab is parallel to a field line.



be whatever we please. It is common to choose the potential to be zero at infinity (let $V_b = 0$ at $r_b = \infty$). Then the electric potential V at a distance r from a single point charge is

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}. \qquad \begin{bmatrix}\text{single point charge;} \\ V = 0 \text{ at } r = \infty\end{bmatrix} \qquad \textbf{(5)}$$

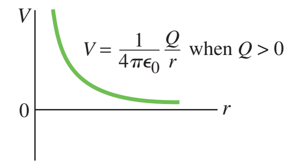
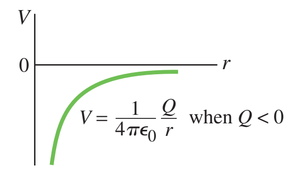


FIGURE 10 Potential V as a function of distance r from a single point charge Q when the charge is positive.

FIGURE 11 Potential V as a function of distance r from a single point charge Q when the charge is negative.



We can think of V here as representing the absolute potential, where $V = 0$ at $r = \infty$, or we can think of V as the potential difference between r and infinity. Notice that the potential V decreases with the first power of the distance, whereas the electric field decreases as the *square* of the distance. The potential near a positive charge is large, and it decreases toward zero at very large distances (Fig. 10). For a negative charge, the potential is negative and increases toward zero at large distances (Fig. 11). Equation 5 is sometimes called the **Coulomb potential** (it has its origin in Coulomb's law).

In Example 4 we found that the potential due to a uniformly charged sphere is given by the same relation, Eq. 5, for points outside the sphere. Thus we see that the potential outside a uniformly charged sphere is the same as if all the charge were concentrated at its center.

EXERCISE C What is the potential at a distance of 3.0 cm from a point charge $Q = -2.0 \times 10^{-9}$ C? (*a*) 600 V; (*b*) 60 V; (*c*) 6 V; (*d*) −600 V; (*e*) −60 V; (*f*) −6 V.

EXAMPLE 6 Work required to bring two positive charges close together. What minimum work must be done by an external force to bring a charge $q = 3.00\,\mu\text{C}$ from a great distance away (take $r = \infty$) to a point 0.500 m from a charge $Q = 20.0\,\mu\text{C}$?

APPROACH To find the work we cannot simply multiply the force times distance because the force is not constant. Instead we can set the change in potential energy equal to the (positive of the) work required of an *external* force, and Eq. 3: $W = \Delta U = q(V_b - V_a)$. We get the potentials V_b and V_a using Eq. 5.

CAUTION
We cannot use $W = Fd$ when F is not constant

SOLUTION The work required is equal to the change in potential energy:

$$\begin{aligned} W &= q(V_b - V_a) \\ &= q\left(\frac{kQ}{r_b} - \frac{kQ}{r_a}\right), \end{aligned}$$

where $r_b = 0.500$ m and $r_a = \infty$. The right-hand term within the parentheses is zero $(1/\infty = 0)$ so

$$W = (3.00 \times 10^{-6}\,\text{C})\frac{(8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(2.00 \times 10^{-5}\,\text{C})}{(0.500\,\text{m})} = 1.08\text{ J}.$$

NOTE We could not use Eq. 4b here because it applies *only* to uniform fields. But we did use Eq. 3 because it is always valid.

To determine the electric field at points near a collection of two or more point charges requires adding up the electric fields due to each charge. Since the electric field is a vector, this can be time consuming or complicated. To find the electric potential at a point due to a collection of point charges is far easier, since the electric potential is a scalar, and hence you only need to add numbers (with appropriate signs) without concern for direction. This is a major advantage in using electric potential for solving Problems.

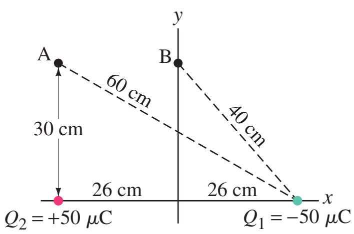


FIGURE 12 Example 7.

EXAMPLE 7 **Potential above two charges.** Calculate the electric potential (*a*) at point A in Fig. 12 due to the two charges shown, and (*b*) at point B.

APPROACH The total potential at point A (or at point B) is the sum of the potentials at that point due to each of the two charges Q_1 and Q_2. The potential due to each single charge is given by Eq. 5. We do not have to worry about directions because electric potential is a scalar quantity. But we do have to keep track of the signs of charges.

SOLUTION (*a*) We add the potentials at point A due to each charge Q_1 and Q_2, and we use Eq. 5 for each:

$$V_A = V_{A2} + V_{A1} = k\frac{Q_2}{r_{2A}} + k\frac{Q_1}{r_{1A}}$$

where $r_{1A} = 60\text{ cm}$ and $r_{2A} = 30\text{ cm}$. Then

$$\begin{aligned} V_A &= \frac{(9.0 \times 10^9\,\mathrm{N\cdot m^2/C^2})(5.0 \times 10^{-5}\,\mathrm{C})}{0.30\,\mathrm{m}} \\ &\quad + \frac{(9.0 \times 10^9\,\mathrm{N\cdot m^2/C^2})(-5.0 \times 10^{-5}\,\mathrm{C})}{0.60\,\mathrm{m}} \\ &= 1.50 \times 10^6\,\mathrm{V} - 0.75 \times 10^6\,\mathrm{V} \\ &= 7.5 \times 10^5\,\mathrm{V}. \end{aligned}$$

(*b*) At point B, $r_{1B} = r_{2B} = 0.40\text{ m}$, so

$$\begin{aligned} V_B &= V_{B2} + V_{B1} \\ &= \frac{(9.0 \times 10^9\,\mathrm{N\cdot m^2/C^2})(5.0 \times 10^{-5}\,\mathrm{C})}{0.40\,\mathrm{m}} \\ &\quad + \frac{(9.0 \times 10^9\,\mathrm{N\cdot m^2/C^2})(-5.0 \times 10^{-5}\,\mathrm{C})}{0.40\,\mathrm{m}} \\ &= 0\,\mathrm{V}. \end{aligned}$$

NOTE The two terms in the sum in (*b*) cancel for any point equidistant from Q_1 and Q_2 $(r_{1B} = r_{2B})$. Thus the potential will be zero everywhere on the plane equidistant between the two opposite charges. This plane where V is constant is called an equipotential surface.

CAUTION
Potential is a scalar and has no components

Simple summations like these can easily be performed for any number of point charges.

EXERCISE D Consider the three pairs of charges, Q_1 and Q_2, in Fig. 13. (*a*) Which set has a positive potential energy? (*b*) Which set has the most negative potential energy? (*c*) Which set requires the most work to separate the charges to infinity? Assume the charges all have the same magnitude.

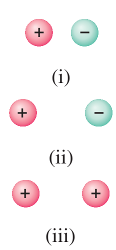


FIGURE 13 Exercise D.

4 Potential Due to Any Charge Distribution

If we know the electric field in a region of space due to any distribution of electric charge, we can determine the difference in potential between two points in the region using Eq. 4a, $V_{ba} = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}$. In many cases we don't know $\vec{\mathbf{E}}$ as a function of position, and it may be difficult to calculate. We can calculate the potential V due to a given charge distribution in another way, using the potential due to a single point charge, Eq. 5:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r},$$

where we choose $V = 0$ at $r = \infty$. Then we can sum over all the charges.

If we have n individual point charges, the potential at some point a (relative to $V = 0$ at $r = \infty$) is

$$V_a = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{Q_i}{r_{ia}}, \tag{6a}$$

where r_{ia} is the distance from the i^{th} charge (Q_i) to the point a. (We already used this approach in Example 7.) If the charge distribution can be considered continuous, then

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}, \tag{6b}$$

where r is the distance from a tiny element of charge, dq, to the point where V is being determined.

EXAMPLE 8 **Potential due to a ring of charge.** A thin circular ring of radius R has a uniformly distributed charge Q. Determine the electric potential at a point P on the axis of the ring a distance x from its center, Fig. 14.

APPROACH We integrate over the ring using Eq. 6b.

SOLUTION Each point on the ring is equidistant from point P, and this distance is $(x^2 + R^2)^{\frac{1}{2}}$. So the potential at P is:

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \frac{1}{(x^2 + R^2)^{\frac{1}{2}}} \int dq = \frac{1}{4\pi\epsilon_0} \frac{Q}{(x^2 + R^2)^{\frac{1}{2}}}.$$

NOTE For points very far away from the ring, $x \gg R$, this result reduces to $(1/4\pi\epsilon_0)(Q/x)$, the potential of a point charge, as we should expect.

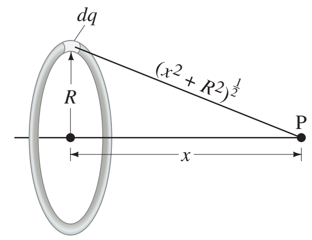


FIGURE 14 Example 8. Calculating the potential at point P, a distance x from the center of a uniform ring of charge.

EXAMPLE 9 **Potential due to a charged disk.** A thin flat disk, of radius R_0, has a uniformly distributed charge Q, Fig. 15. Determine the potential at a point P on the axis of the disk, a distance x from its center.

APPROACH We divide the disk into thin rings of radius R and thickness dR and use the result of Example 8 to sum over the disk.

SOLUTION The charge Q is distributed uniformly, so the charge contained in each ring is proportional to its area. The disk has area πR_0^2 and each thin ring has area $dA = (2\pi R)(dR)$. Hence

$$\frac{dq}{Q} = \frac{2\pi R\, dR}{\pi R_0^2}$$

so

$$dq = Q\frac{(2\pi R)(dR)}{\pi R_0^2} = \frac{2QR\, dR}{R_0^2}.$$

Then the potential at P, using Eq. 6b in which r is replaced by $(x^2 + R^2)^{\frac{1}{2}}$, is

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{(x^2 + R^2)^{\frac{1}{2}}} = \frac{2Q}{4\pi\epsilon_0 R_0^2} \int_0^{R_0} \frac{R\, dR}{(x^2 + R^2)^{\frac{1}{2}}} = \frac{Q}{2\pi\epsilon_0 R_0^2} (x^2 + R^2)^{\frac{1}{2}} \Big|_{R=0}^{R=R_0}$$

$$= \frac{Q}{2\pi\epsilon_0 R_0^2} \left[(x^2 + R_0^2)^{\frac{1}{2}} - x\right].$$

NOTE For $x \gg R_0$, this formula reduces to

$$V \approx \frac{Q}{2\pi\epsilon_0 R_0^2} \left[x\left(1 + \frac{1}{2}\frac{R_0^2}{x^2}\right) - x\right] = \frac{Q}{4\pi\epsilon_0 x}.$$

This is the formula for a point charge, as we expect.

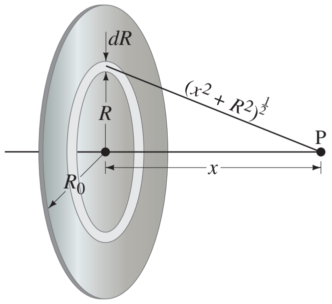


FIGURE 15 Example 9. Calculating the electric potential at point P on the axis of a uniformly charged thin disk.

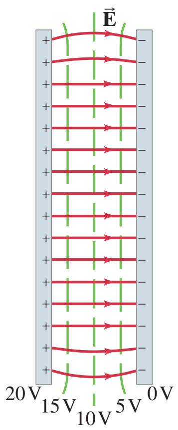


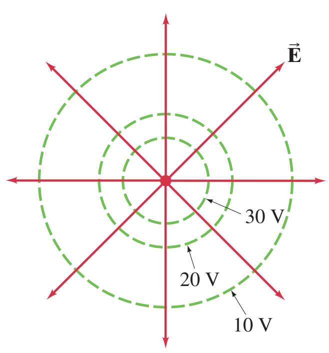
FIGURE 16 Equipotential lines (the green dashed lines) between two oppositely charged parallel plates. Note that they are perpendicular to the electric field lines (solid red lines).

5 Equipotential Surfaces

The electric potential can be represented graphically by drawing **equipotential lines** or, in three dimensions, **equipotential surfaces**. An equipotential surface is one on which all points are at the same potential. That is, the potential difference between any two points on the surface is zero, and no work is required to move a charge from one point to the other. An *equipotential surface must be perpendicular to the electric field* at any point. If this were not so—that is, if there were a component of $\vec{\mathbf{E}}$ parallel to the surface—it would require work to move the charge along the surface against this component of $\vec{\mathbf{E}}$; and this would contradict the idea that it is an equipotential surface. This can also be seen from Eq. 4a, $\Delta V = -\int \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}$. On a surface where V is constant, $\Delta V = 0$, so we must have either $\vec{\mathbf{E}} = 0$, $d\vec{\boldsymbol{\ell}} = 0$, or $\cos\theta = 0$ where θ is the angle between $\vec{\mathbf{E}}$ and $d\vec{\boldsymbol{\ell}}$. Thus in a region where $\vec{\mathbf{E}}$ is not zero, the path $d\vec{\boldsymbol{\ell}}$ along an equipotential must have $\cos\theta = 0$, meaning $\theta = 90°$ and $\vec{\mathbf{E}}$ is perpendicular to the equipotential.

The fact that the electric field lines and equipotential surfaces are mutually perpendicular helps us locate the equipotentials when the electric field lines are known. In a normal two-dimensional drawing, we show equipotential *lines*, which are the intersections of equipotential surfaces with the plane of the drawing. In Fig. 16, a few of the equipotential lines are drawn (dashed green lines) for the electric field (red lines) between two parallel plates at a potential difference of 20 V. The negative plate is arbitrarily chosen to be zero volts and the potential of each equipotential line is indicated. Note that $\vec{\mathbf{E}}$ points toward lower values of V.

FIGURE 17 Example 10. Electric field lines and equipotential surfaces for a point charge.



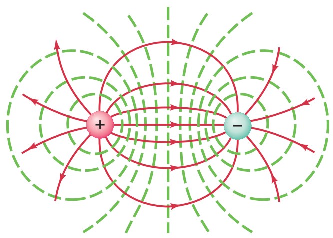
EXAMPLE 10 **Point charge equipotential surfaces.** For a single point charge with $Q = 4.0 \times 10^{-9}\,\text{C}$, sketch the equipotential surfaces (or lines in a plane containing the charge) corresponding to $V_1 = 10\,\text{V}$, $V_2 = 20\,\text{V}$, and $V_3 = 30\,\text{V}$.

APPROACH The electric potential V depends on the distance r from the charge (Eq. 5).

SOLUTION The electric field for a positive point charge is directed radially outward. Since the equipotential surfaces must be perpendicular to the lines of electric field, they will be spherical in shape, centered on the point charge, Fig. 17. From Eq. 5 we have $r = (1/4\pi\epsilon_0)(Q/V)$, so that for $V_1 = 10\,\text{V}$, $r_1 = (9.0 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(4.0 \times 10^{-9}\,\text{C})/(10\,\text{V}) = 3.6\,\text{m}$, for $V_2 = 20\,\text{V}$, $r_2 = 1.8\,\text{m}$, and for $V_3 = 30\,\text{V}$, $r_3 = 1.2\,\text{m}$, as shown.

NOTE The equipotential surface with the largest potential is closest to the positive charge. How would this change if Q were negative?

FIGURE 18 Equipotential lines (green, dashed) are always perpendicular to the electric field lines (solid red) shown here for two equal but oppositely charged particles.



The equipotential lines for the case of two equal but oppositely charged particles are shown in Fig. 18 as green dashed lines. Equipotential lines and surfaces, unlike field lines, are always continuous and never end, and so continue beyond the borders of Figs. 16 and 18.

There can be no electric field within a conductor in the static case, for otherwise the free electrons would feel a force and would move. Indeed, the entire volume of *a conductor must be entirely at the same potential in the static case*, and the surface of a conductor is then an equipotential surface. (If it weren't, the free electrons at the surface would move, since whenever there is a potential difference between two points, free charges will move.) This is fully consistent with our result, discussed earlier, that the electric field at the surface of a conductor must be perpendicular to the surface.

A useful analogy for equipotential lines is a topographic map: the contour lines are essentially gravitational equipotential lines (Fig. 19).

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FIGURE 19 A topographic map (here, a portion of the Sierra Nevada in California) shows continuous contour lines, each of which is at a fixed height above sea level. Here they are at 80 ft (25 m) intervals. If you walk along one contour line, you neither climb nor descend. If you cross lines, and especially if you climb perpendicular to the lines, you will be changing your gravitational potential (rapidly, if the lines are close together).

6 Electric Dipole Potential

Two equal point charges Q, of opposite sign, separated by a distance ℓ, are called an **electric dipole**. Also, the two charges we saw in Figs. 12 and 18 constitute an electric dipole, and the latter shows the electric field lines and equipotential surfaces for a dipole. Because electric dipoles occur often in physics, as well as in other fields, it is useful to examine them more closely.

The electric potential at an arbitrary point P due to a dipole, Fig. 20, is the sum of the potentials due to each of the two charges (we take $V = 0$ at $r = \infty$):

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} + \frac{1}{4\pi\epsilon_0}\frac{(-Q)}{(r+\Delta r)} = \frac{1}{4\pi\epsilon_0}Q\left(\frac{1}{r} - \frac{1}{r+\Delta r}\right) = \frac{Q}{4\pi\epsilon_0}\frac{\Delta r}{r(r+\Delta r)},$$

where r is the distance from P to the positive charge and $r + \Delta r$ is the distance to the negative charge. This equation becomes simpler if we consider points P whose distance from the dipole is much larger than the separation of the two charges—that is, for $r \gg \ell$. From Fig. 20 we see that $\Delta r \approx \ell\cos\theta$; since $r \gg \Delta r = \ell\cos\theta$, we can neglect Δr in the denominator as compared to r. Therefore, we obtain

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q\ell\cos\theta}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2} \qquad [\text{dipole; } r \gg \ell] \quad \textbf{(7)}$$

where $p = Q\ell$ is called the **dipole moment**. We see that the potential decreases as the *square* of the distance from the dipole, whereas for a single point charge the potential decreases with the first power of the distance (Eq. 5). It is not surprising that the potential should fall off faster for a dipole; for when you are far from a dipole, the two equal but opposite charges appear so close together as to tend to neutralize each other.

Table 2 gives the dipole moments for several molecules. The + and − signs indicate on which atoms these charges lie. The last two entries are a part of many organic molecules and play an important role in molecular biology. A dipole moment has units of coulomb-meters (C·m), although for molecules a smaller unit called a *debye* is sometimes used: 1 debye $= 3.33 \times 10^{-30}$ C·m.

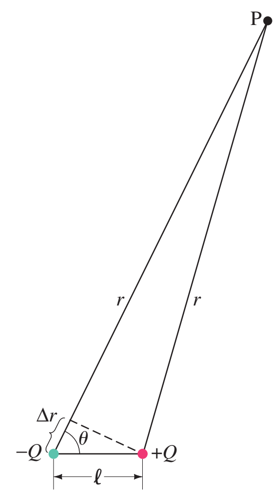

FIGURE 20 Electric dipole. Calculation of potential V at point P.

TABLE 2 Dipole Moments of Selected Molecules

Molecule	Dipole Moment (C · m)
$H_2^{(+)}O^{(-)}$	6.1×10^{-30}
$H^{(+)}Cl^{(-)}$	3.4×10^{-30}
$N^{(-)}H_3^{(+)}$	5.0×10^{-30}
$>N^{(-)}-H^{(+)}$	$\approx 3.0^{\dagger} \times 10^{-30}$
$>C^{(+)}=O^{(-)}$	$\approx 8.0^{\dagger} \times 10^{-30}$

† These groups often appear on larger molecules; hence the value for the dipole moment will vary somewhat, depending on the rest of the molecule.

7 $\vec{\mathbf{E}}$ Determined from V

We can use Eq. 4a, $V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}$, to determine the difference in potential between two points if the electric field is known in the region between those two points. By inverting Eq. 4a, we can write the electric field in terms of the potential. Then the electric field can be determined from a knowledge of V. Let us see how to do this.

We write Eq. 4a in differential form as

$$dV = -\vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -E_\ell \, d\ell,$$

where dV is the infinitesimal difference in potential between two points a distance $d\ell$ apart, and E_ℓ is the component of the electric field in the direction of the infinitesimal displacement $d\vec{\boldsymbol{\ell}}$. We can then write

$$E_\ell = -\frac{dV}{d\ell}. \tag{8}$$

Thus *the component of the electric field in any direction is equal to the negative of the rate of change of the electric potential with distance in that direction*. The quantity $dV/d\ell$ is called the gradient of V in a particular direction. If the direction is not specified, the term *gradient* refers to that direction in which V changes most rapidly; this would be the direction of $\vec{\mathbf{E}}$ at that point, so we can write

$$E = -\frac{dV}{d\ell}. \qquad [\text{if } d\vec{\boldsymbol{\ell}} \| \vec{\mathbf{E}}]$$

If $\vec{\mathbf{E}}$ is written as a function of x, y, and z, and we let ℓ refer to the x, y, and z axes, then Eq. 8 becomes

$$E_x = -\frac{\partial V}{\partial x}, \qquad E_y = -\frac{\partial V}{\partial y}, \qquad E_z = -\frac{\partial V}{\partial z}. \tag{9}$$

Here, $\partial V/\partial x$ is the "partial derivative" of V with respect to x, with y and z held constant.† For example, if $V(x, y, z) = (2\,\mathrm{V/m^2})x^2 + (8\,\mathrm{V/m^3})y^2 z + (2\,\mathrm{V/m^2})z^2$, then

$$E_x = -\partial V/\partial x = -(4\,\mathrm{V/m^2})x,$$

$$E_y = -\partial V/\partial y = -(16\,\mathrm{V/m^3})yz,$$

and

$$E_z = -\partial V/\partial z = -(8\,\mathrm{V/m^3})y^2 - (4\,\mathrm{V/m^2})z.$$

EXAMPLE 11 **$\vec{\mathbf{E}}$ for ring and disk.** Use electric potential to determine the electric field at point P on the axis of (*a*) a circular ring of charge (Fig. 14) and (*b*) a uniformly charged disk (Fig. 15).

APPROACH We obtained V as a function of x in Examples 8 and 9, so we find E by taking derivatives (Eqs. 9).

SOLUTION (*a*) From Example 8,

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{(x^2 + R^2)^{\frac{1}{2}}}.$$

Then

$$E_x = -\frac{\partial V}{\partial x} = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2 + R^2)^{\frac{3}{2}}}.$$

(*b*) From Example 9,

$$V = \frac{Q}{2\pi\epsilon_0 R_0^2}\left[(x^2 + R_0^2)^{\frac{1}{2}} - x\right],$$

so

$$E_x = -\frac{\partial V}{\partial x} = \frac{Q}{2\pi\epsilon_0 R_0^2}\left[1 - \frac{x}{(x^2 + R_0^2)^{\frac{1}{2}}}\right].$$

For points very close to the disk, $x \ll R_0$, this can be approximated by

$$E_x \approx \frac{Q}{2\pi\epsilon_0 R_0^2} = \frac{\sigma}{2\epsilon_0}$$

where $\sigma = Q/\pi R_0^2$ is the surface charge density.

†Equation 9 can be written as a vector equation,

$$\vec{\mathbf{E}} = -\mathrm{grad}\, V = -\vec{\boldsymbol{\nabla}} V = -\left(\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}\right)V$$

where the symbol $\vec{\boldsymbol{\nabla}}$ is called the *del* or *gradient operator*: $\vec{\boldsymbol{\nabla}} = \hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}.$

Here, as for many charge distributions, it is easier to calculate V first, and then $\vec{\mathbf{E}}$ from Eq. 9, rather than to calculate $\vec{\mathbf{E}}$ due to each charge from Coulomb's law. This is because V due to many charges is a scalar sum, whereas $\vec{\mathbf{E}}$ is a vector sum.

8 Electrostatic Potential Energy; the Electron Volt

Suppose a point charge q is moved between two points in space, a and b, where the electric potential due to other charges is V_a and V_b, respectively. The change in electrostatic potential energy of q in the field of these other charges is, according to Eq. 2b,

$$\Delta U = U_b - U_a = q(V_b - V_a).$$

Now suppose we have a system of several point charges. What is the electrostatic potential energy of the system? It is most convenient to choose the electric potential energy to be zero when the charges are very far (ideally infinitely far) apart. A single point charge, Q_1, in isolation, has no potential energy, because if there are no other charges around, no electric force can be exerted on it. If a second point charge Q_2 is brought close to Q_1, the potential due to Q_1 at the position of this second charge is

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{r_{12}},$$

where r_{12} is the distance between the two. The potential energy of the two charges, relative to $V = 0$ at $r = \infty$, is

$$U = Q_2 V = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r_{12}}. \tag{10}$$

This represents the work that needs to be done by an external force to bring Q_2 from infinity $(V = 0)$ to a distance r_{12} from Q_1. It is also the negative of the work needed to separate them to infinity.

If the system consists of three charges, the total potential energy will be the work needed to bring all three together. Equation 10 represents the work needed to bring Q_2 close to Q_1; to bring a third charge Q_3 so that it is a distance r_{13} from Q_1 and r_{23} from Q_2 requires work equal to

$$\frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_3}{r_{13}} + \frac{1}{4\pi\epsilon_0}\frac{Q_2 Q_3}{r_{23}}.$$

So the potential energy of a system of three point charges is

$$U = \frac{1}{4\pi\epsilon_0}\left(\frac{Q_1 Q_2}{r_{12}} + \frac{Q_1 Q_3}{r_{13}} + \frac{Q_2 Q_3}{r_{23}}\right). \qquad [V = 0 \text{ at } r = \infty]$$

For a system of four charges, the potential energy would contain six such terms, and so on. (Caution must be used when making such sums to avoid double counting of the different pairs.)

The Electron Volt Unit

The joule is a very large unit for dealing with energies of electrons, atoms, or molecules (see Example 2), and for this purpose, the unit **electron volt** (eV) is used. One electron volt is defined as the energy acquired by a particle carrying a charge whose magnitude equals that on the electron $(q = e)$ as a result of moving through a potential difference of 1 V. Since $e = 1.6 \times 10^{-19}\,\mathrm{C}$, and since the change in potential energy equals qV, 1 eV is equal to $(1.6 \times 10^{-19}\,\mathrm{C})(1.0\,\mathrm{V}) = 1.6 \times 10^{-19}\,\mathrm{J}$:

$$1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}.$$

An electron that accelerates through a potential difference of 1000 V will lose 1000 eV of potential energy and will thus gain 1000 eV or 1 keV (kiloelectron volt) of kinetic energy. On the other hand, if a particle with a charge equal to twice the magnitude of the charge on the electron $(= 2e = 3.2 \times 10^{-19}\,\mathrm{C})$ moves through a potential difference of 1000 V, its energy will change by $2000\,\mathrm{eV} = 2\,\mathrm{keV}$.

Although the electron volt is handy for *stating* the energies of molecules and elementary particles, it is not a proper SI unit. For calculations it should be converted to joules using the conversion factor given above. In Example 2, for example, the electron acquired a kinetic energy of 8.0×10^{-16} J. We normally would quote this energy as 5000 eV $\left(= 8.0 \times 10^{-16}\,\mathrm{J}/1.6 \times 10^{-19}\,\mathrm{J/eV}\right)$. But when determining the speed of a particle in SI units, we must use the kinetic energy in J.

EXERCISE E What is the kinetic energy of a He^{2+} ion released from rest and accelerated through a potential difference of 1.0 kV? (*a*) 1000 eV, (*b*) 500 eV, (*c*) 2000 eV, (*d*) 4000 eV, (*e*) 250 eV.

EXAMPLE 12 **Disassembling a hydrogen atom.** Calculate the work needed to "disassemble" a hydrogen atom. Assume that the proton and electron are initially separated by a distance equal to the "average" radius of the hydrogen atom in its ground state, 0.529×10^{-10} m, and that they end up an infinite distance apart from each other.

APPROACH The work necessary will be equal to the total energy, kinetic plus potential, of the electron and proton as an atom, compared to their total energy when infinitely far apart.

SOLUTION From Eq. 10 we have initially

$$U = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{(e)(-e)}{r} = \frac{-\left(8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2}\right)\left(1.60 \times 10^{-19}\,\mathrm{C}\right)^2}{\left(0.529 \times 10^{-10}\,\mathrm{m}\right)}$$
$$= -27.2\left(1.60 \times 10^{-19}\right)\mathrm{J} = -27.2\,\mathrm{eV}.$$

This represents the potential energy. The total energy must include also the kinetic energy of the electron moving in an orbit of radius $r = 0.529 \times 10^{-10}$ m. From $F = ma$ for centripetal acceleration, we have

$$\frac{1}{4\pi\epsilon_0}\left(\frac{e^2}{r^2}\right) = \frac{mv^2}{r}.$$

Then

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(\frac{1}{4\pi\epsilon_0}\right)\frac{e^2}{r}$$

which equals $-\tfrac{1}{2}U$ (as calculated above), so $K = +13.6$ eV. The total energy initially is $E = K + U = 13.6\,\mathrm{eV} - 27.2\,\mathrm{eV} = -13.6\,\mathrm{eV}$. To separate a stable hydrogen atom into a proton and an electron at rest very far apart ($U = 0$ at $r = \infty$, $K = 0$ because $v = 0$) requires +13.6 eV. This is, in fact, the measured ionization energy for hydrogen.

NOTE To treat atoms properly, we need to use quantum theory. But our "classical" calculation does give the correct answer here.

EXERCISE F The kinetic energy of a 1000-kg automobile traveling 20 m/s (70 km/h) would be about (*a*) 100 GeV, (*b*) 1000 TeV, (*c*) 10^6 TeV, (*d*) 10^{12} TeV, (*e*) 10^{18} TeV.

*9 Cathode Ray Tube: TV and Computer Monitors, Oscilloscope

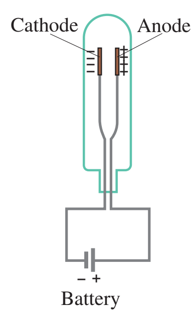

FIGURE 21 If the cathode inside the evacuated glass tube is heated to glowing, negatively charged "cathode rays" (electrons) are "boiled off" and flow across to the anode (+) to which they are attracted.

An important device that makes use of voltage, and that allows us to "visualize" how a voltage changes in time, is the *cathode ray tube* (CRT). A CRT used in this way is an *oscilloscope*. The CRT has also been used for many years as the picture tube of television sets and computer monitors, but LCD and other screens are now common.

The operation of a CRT depends on the phenomenon of **thermionic emission** discovered by Thomas Edison (1847–1931). Consider two small plates (electrodes) inside an evacuated "bulb" or "tube" as shown in Fig. 21, to which is applied a potential difference. The negative electrode is called the **cathode**, the positive one the **anode**. If the negative cathode is heated (usually by an electric current, as in a lightbulb) so that it becomes hot and glowing, it is found that negative charge leaves the cathode and flows to the positive anode. These negative charges are now called electrons, but originally they were called **cathode rays** since they seemed to come from the cathode.

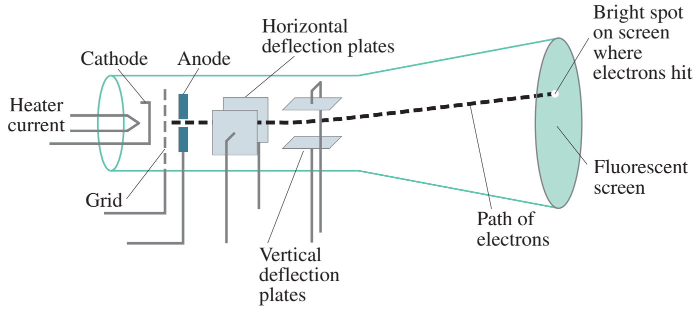

FIGURE 22 A cathode ray tube. Magnetic deflection coils are often used in place of the electric deflection plates shown here. The relative positions of the elements have been exaggerated for clarity.

PHYSICS APPLIED
CRT, TV and computer monitors

The **cathode ray tube** (CRT) derives its name from the fact that inside an evacuated glass tube, a beam of cathode rays (electrons) is directed to various parts of a screen to produce a "picture." A simple CRT is diagrammed in Fig. 22. Electrons emitted by the heated cathode are accelerated by a high voltage (5000–50,000 V) applied between the anode and cathode. The electrons pass out of this "electron gun" through a small hole in the anode. The inside of the tube face is coated with a fluorescent material that glows when struck by electrons. A tiny bright spot is thus visible where the electron beam strikes the screen. Two horizontal and two vertical plates can deflect the beam of electrons when a voltage is applied to them. The electrons are deflected toward whichever plate is positive. By varying the voltage on the deflection plates, the bright spot can be placed at any point on the screen. Many CRTs use magnetic deflection coils instead of electric plates.

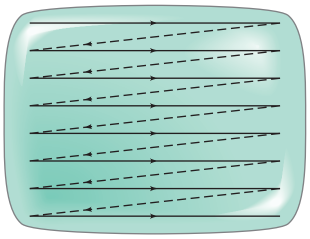

FIGURE 23 Electron beam sweeps across a television screen in a succession of horizontal lines. Each horizontal sweep is made by varying the voltage on the horizontal deflection plates. Then the electron beam is moved down a short distance by a change in voltage on the vertical deflection plates, and the process is repeated.

In the picture tube or monitor for a computer or television set, the electron beam is made to sweep over the screen in the manner shown in Fig. 23 by changing voltages applied to the deflection plates. For standard television in the United States, 525 lines constitutes a complete sweep in $\frac{1}{30}$ s, over the entire screen. High-definition TV provides more than double this number of lines (1080), giving greater picture sharpness. We see a picture because the image is retained by the fluorescent screen and by our eyes for about $\frac{1}{20}$ s. The picture we see consists of the varied brightness of the spots on the screen, controlled by the grid (a "porous" electrode, such as a wire grid, that allows passage of electrons). The grid limits the flow of electrons by means of the voltage (the "video signal") applied to it: the more negative this voltage, the more electrons are repelled and the fewer pass through. This video signal sent out by the TV station, and received by the TV set, is accompanied by signals that synchronize the grid voltage to the horizontal and vertical sweeps.

PHYSICS APPLIED
Oscilloscope

An **oscilloscope** is a device for amplifying, measuring, and visually observing an electrical signal as a function of time on the screen of a CRT (a "signal" is usually a time-varying voltage). The electron beam is swept horizontally at a uniform rate in time by the horizontal deflection plates. The signal to be displayed is applied (after amplification) to the vertical deflection plates. The visible "trace" on the screen, which could be an electrocardiogram (Fig. 24), or a signal from an experiment on nerve conduction, is a plot of the signal voltage (vertically) versus time (horizontally).

WavebreakmediaMicro/Fotolia

FIGURE 24 An electrocardiogram (ECG) trace displayed on a CRT.

Summary

Electric potential is defined as electric potential energy per unit charge. That is, the **electric potential difference** between any two points in space is defined as the difference in potential energy of a test charge q placed at those two points, divided by the charge q:

$$V_{ba} = \frac{U_b - U_a}{q}. \tag{2b}$$

Potential difference is measured in volts $(1\,V = 1\,J/C)$ and is sometimes referred to as **voltage**.

The change in potential energy of a charge q when it moves through a potential difference V_{ba} is

$$\Delta U = qV_{ba}. \tag{3}$$

The potential difference V_{ba} between two points, a and b, is given by the relation

$$V_{ba} = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}. \tag{4a}$$

Thus V_{ba} can be found in any region where $\vec{\mathbf{E}}$ is known. If the electric field is uniform, the integral is easy: $V_{ba} = -Ed$, where d is the distance (parallel to the field lines) between the two points.

An **equipotential line** or **surface** is all at the same potential, and is perpendicular to the electric field at all points.

The electric potential due to a single point charge Q, relative to zero potential at infinity, is given by

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}. \tag{5}$$

The potential due to any charge distribution can be obtained by summing (or integrating) over the potentials for all the charges.

The potential due to an **electric dipole** drops off as $1/r^2$. The **dipole moment** is $p = Q\ell$, where ℓ is the distance between the two equal but opposite charges of magnitude Q.

When V is known, the components of $\vec{\mathbf{E}}$ can be found from the inverse of Eq. 4a, namely

$$E_x = -\frac{\partial V}{\partial x}, \qquad E_y = -\frac{\partial V}{\partial y}, \qquad E_z = -\frac{\partial V}{\partial z}. \tag{9}$$

[*Television and computer monitors traditionally use a **cathode ray tube** (CRT) that accelerates electrons by high voltage, and sweeps them across the screen in a regular way using deflection plates.]

Answers to Exercises

A: (*b*).

B: (*d*).

C: (*d*).

D: (*a*) iii, (*b*) i, (*c*) i.

E: (*c*).

F: (*d*).