

The beautiful colors from the surface of this soap bubble can be nicely explained by the wave theory of light. A soap bubble is a very thin spherical film filled with air. Light reflected from the outer and inner surfaces of this thin film of soapy water interferes constructively to produce the bright colors. Which color we see at any point depends on the thickness of the soapy water film at that point and also on the viewing angle. Near the top of the bubble, we see a small black area surrounded by a silver or white area. The bubble's thickness is smallest at that black spot, perhaps only about 30 nm thick, and is fully transparent (we see the black background).

We cover fundamental aspects of the wave nature of light, including two-slit interference and interference in thin films.



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The Wave Nature of Light; Interference

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CHAPTER-OPENING QUESTION—Guess now!

[Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.]

When a thin layer of oil lies on top of water or wet pavement, you can often see swirls of color. We also see swirls of color on the soap bubble shown above. What causes these colors?

- (a) Additives in the oil or soap reflect various colors.
- (b) Chemicals in the oil or soap absorb various colors.
- (c) Dispersion due to differences in index of refraction in the oil or soap.
- (d) The interactions of the light with a thin boundary layer where the oil (or soap) and the water have mixed irregularly.
- (e) Light waves reflected from the top and bottom surfaces of the thin oil or soap film can add up constructively for particular wavelengths.

That light carries energy is obvious to anyone who has focused the Sun's rays with a magnifying glass on a piece of paper and burned a hole in it. But how does light travel, and in what form is this energy carried? In the context of waves, energy can be carried from place to place in basically two ways: by particles or by waves. In the first case, material objects or particles can carry energy, such as an avalanche of rocks or rushing water. In the second case, water waves and sound waves, for example, can carry energy over long distances even though the oscillating particles of the medium do not travel these distances. In view of this, what can we say about the nature of light: does light travel as a stream of particles away from its source, or does light travel in the form of waves that spread outward from the source?

Note: Sections marked with an asterisk (*) may be considered optional by the instructor.

Historically, this question has turned out to be a difficult one. For one thing, light does not reveal itself in any obvious way as being made up of tiny particles; nor do we see tiny light waves passing by as we do water waves. The evidence seemed to favor first one side and then the other until about 1830, when most physicists had accepted the wave theory. By the end of the nineteenth century, light was considered to be an *electromagnetic wave*. In the early twentieth century, light was shown to have a particle nature as well. We now speak of the wave-particle duality of light. The wave theory of light remains valid and has proved very successful. We now investigate the evidence for the wave theory and how it has been used to explain a wide range of phenomena.

1 Waves versus Particles; Huygens' Principle and Diffraction

The Dutch scientist Christian Huygens (1629–1695), a contemporary of Newton, proposed a wave theory of light that had much merit. Still useful today is a technique Huygens developed for predicting the future position of a wave front when an earlier position is known. By a wave front, we mean all the points along a two- or three-dimensional wave that form a wave crest—what we simply call a “wave” as seen on the ocean. Wave fronts are perpendicular to rays. **Huygens' principle** can be stated as follows: *Every point on a wave front can be considered as a source of tiny wavelets that spread out in the forward direction at the speed of the wave itself. The new wave front is the envelope of all the wavelets—that is, the tangent to all of them.*

As a simple example of the use of Huygens' principle, consider the wave front AB in Fig. 1, which is traveling away from a source S. We assume the medium is *isotropic*—that is, the speed v of the waves is the same in all directions. To find the wave front a short time t after it is at AB, tiny circles are drawn with radius $r = vt$. The centers of these tiny circles are blue dots on the original wave front AB, and the circles represent Huygens' (imaginary) wavelets. The tangent to all these wavelets, the curved line CD, is the new position of the wave front.

Huygens' principle is particularly useful for analyzing what happens when waves impinge on an obstacle and the wave fronts are partially interrupted. Huygens' principle predicts that waves bend in behind an obstacle, as shown in Fig. 2. This is just what water waves do. The bending of waves behind obstacles into the “shadow region” is known as **diffraction**. Since diffraction occurs for waves, but not for particles, it can serve as one means for distinguishing the nature of light.

Note, as shown in Fig. 2, that diffraction is most prominent when the size of the opening is on the order of the wavelength of the wave. If the opening is much larger than the wavelength, diffraction goes unnoticed.

Does light exhibit diffraction? In the mid-seventeenth century, the Jesuit priest Francesco Grimaldi (1618–1663) had observed that when sunlight entered a darkened room through a tiny hole in a screen, the spot on the opposite wall was larger than would be expected from geometric rays. He also observed that the border of the image was not clear but was surrounded by colored fringes. Grimaldi attributed this to the diffraction of light.

The wave model of light nicely accounts for diffraction. But the ray model cannot account for diffraction, and it is important to be aware of such limitations to the ray model. Geometric optics using rays is successful in a wide range of situations only because normal openings and obstacles are much larger than the wavelength of the light, and so relatively little diffraction or bending occurs.

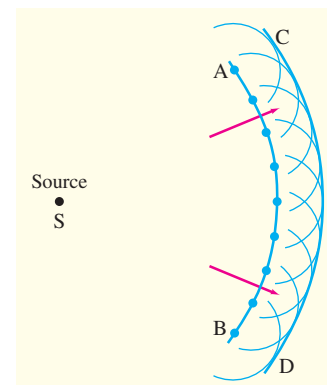
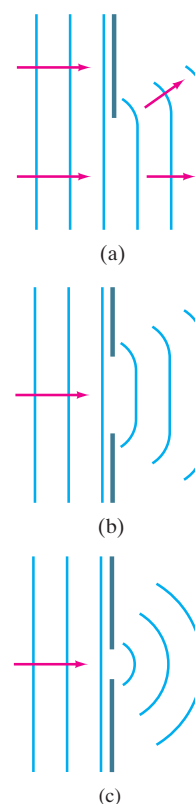


FIGURE 1 Huygens' principle, used to determine wave front CD when wave front AB is given.

FIGURE 2 Huygens' principle is consistent with diffraction (a) around the edge of an obstacle, (b) through a large hole, (c) through a small hole whose size is on the order of the wavelength of the wave.



2 Huygens' Principle and the Law of Refraction

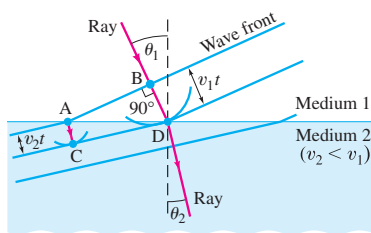


FIGURE 3 Refraction explained, using Huygens' principle. Wave fronts are perpendicular to the rays.

The laws of reflection and refraction were well known in Newton's time. The law of reflection could not distinguish between the two theories we just discussed: waves versus particles. For when waves reflect from an obstacle, the angle of incidence equals the angle of reflection. The same is true of particles—think of a tennis ball without spin striking a flat surface.

The law of refraction is another matter. Consider a ray of light entering a medium where it is bent toward the normal, as when traveling from air into water. As shown in Fig. 3, this bending can be constructed using Huygens' principle if we assume the speed of light is less in the second medium ($v_2 < v_1$). In time t , point B on wave front AB (perpendicular to the incoming ray) travels a distance $v_1 t$ to reach point D. Point A on the wave front, traveling in the second medium, goes a distance $v_2 t$ to reach point C, and $v_2 t < v_1 t$. Huygens' principle is applied to points A and B to obtain the curved wavelets shown at C and D. The wave front is tangent to these two wavelets, so the new wave front is the line CD. Hence the rays, which are perpendicular to the wave fronts, bend toward the normal if $v_2 < v_1$, as drawn.

Newton favored a particle theory of light which predicted the opposite result, that the speed of light would be greater in the second medium ($v_2 > v_1$). Thus the wave theory predicts that the speed of light in water, for example, is less than in air; and Newton's particle theory predicts the reverse. An experiment to actually measure the speed of light in water was performed in 1850 by the French physicist Jean Foucault, and it confirmed the wave-theory prediction. By then, however, the wave theory was already fully accepted, as we shall see in the next Section.

Snell's law of refraction follows directly from Huygens' principle, given that the speed of light v in any medium is related to the speed in a vacuum, c , and the index of refraction, n : that is, $v = c/n$. From the Huygens' construction of Fig. 3, angle ADC is equal to θ_2 and angle BAD is equal to θ_1 . Then for the two triangles that have the common side AD, we have

$$\sin \theta_1 = \frac{v_1 t}{AD}, \quad \sin \theta_2 = \frac{v_2 t}{AD}.$$

We divide these two equations and obtain

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2}.$$

Then, by $v_1 = c/n_1$ and $v_2 = c/n_2$, so we have

$$n_1 \sin \theta_1 = n_2 \sin \theta_2,$$

which is Snell's law of refraction. (The law of reflection can be derived from Huygens' principle in a similar way.)

When a light wave travels from one medium to another, its frequency does not change, but its wavelength does. This can be seen from Fig. 3, where each of the blue lines representing a wave front corresponds to a crest (peak) of the wave. Then

$$\frac{\lambda_2}{\lambda_1} = \frac{v_2 t}{v_1 t} = \frac{v_2}{v_1} = \frac{n_1}{n_2},$$

where, in the last step, we used $v = c/n$. If medium 1 is a vacuum (or air), so $n_1 = 1$, $v_1 = c$, and we call λ_1 simply λ , then the wavelength in another medium of index of refraction n ($= n_2$) will be

$$\lambda_n = \frac{\lambda}{n}. \quad (1)$$

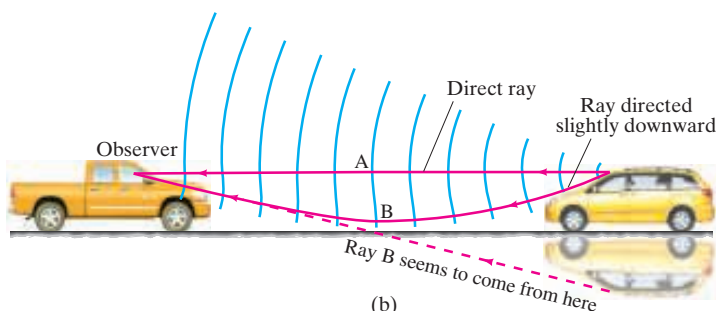
This result is consistent with the frequency f being unchanged no matter what medium the wave is traveling in, since $c = f\lambda$.

EXERCISE A A light beam in air with wavelength = 500 nm, frequency = 6.0×10^{14} Hz, and speed = 3.0×10^8 m/s goes into glass which has an index of refraction = 1.5. What are the wavelength, frequency, and speed of the light in the glass?

John M. Duany IV/Fundamental Photographs, NYC



(a)



(b)

Wave fronts can be used to explain how mirages are produced by refraction of light. For example, on a hot day motorists sometimes see a mirage of water on the highway ahead of them, with distant vehicles seemingly reflected in it (Fig. 4a). On a hot day, there can be a layer of very hot air next to the roadway (made hot by the Sun beating on the road). Hot air is less dense than cooler air, so the index of refraction is slightly lower in the hot air. In Fig. 4b, we see a diagram of light coming from one point on a distant car (on the right) heading left toward the observer. Wave fronts and two rays (perpendicular to the wave fronts) are shown. Ray A heads directly at the observer and follows a straight-line path, and represents the normal view of the distant car. Ray B is a ray initially directed slightly downward, but it bends slightly as it moves through layers of air of different index of refraction. The wave fronts, shown in blue in Fig. 4b, move slightly faster in the layers of air nearer the ground. Thus ray B is bent as shown, and seems to the observer to be coming from below (dashed line) as if reflected off the road. Hence the mirage.

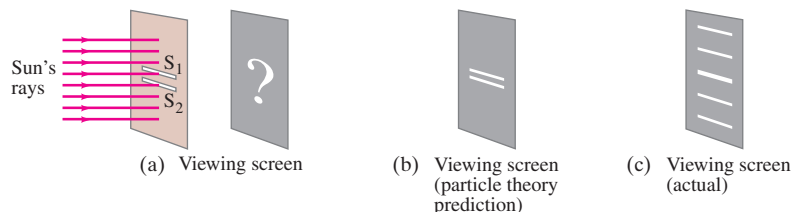
FIGURE 4 (a) A highway mirage. (b) Drawing (greatly exaggerated) showing wave fronts and rays to explain highway mirages. Note how sections of the wave fronts near the ground move faster and so are farther apart.

PHYSICS APPLIED
Highway mirages

3 Interference—Young's Double-Slit Experiment

In 1801, the Englishman Thomas Young (1773–1829) obtained convincing evidence for the wave nature of light and was even able to measure wavelengths for visible light. Figure 5a shows a schematic diagram of Young's famous double-slit experiment.

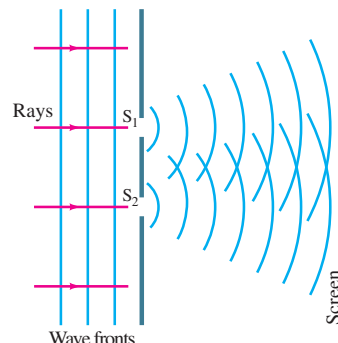
FIGURE 5 (a) Young's double-slit experiment. (b) If light consists of particles, we would expect to see two bright lines on the screen behind the slits. (c) In fact, many lines are observed. The slits and their separation need to be very thin.



To have light from a single source, Young used the Sun passing through a very narrow slit in a window covering. This beam of parallel rays falls on a screen containing two closely spaced slits, S_1 and S_2 . (The slits and their separation are very narrow, not much larger than the wavelength of the light.) If light consists of tiny particles, we might expect to see two bright lines on a screen placed behind the slits as in (b). But instead a series of bright lines are seen, as in (c). Young was able to explain this result as a **wave-interference** phenomenon.

To understand why, we consider the simple situation of plane waves of light of a single wavelength—called **monochromatic**, meaning “one color”—falling on the two slits as shown in Fig. 6. Because of diffraction, the waves leaving the two small slits spread out as shown. This is equivalent to the interference pattern produced when two rocks are thrown into a lake, or when sound from two loudspeakers interferes. Recall wave interference.

FIGURE 6 If light is a wave, light passing through one of two slits should interfere with light passing through the other slit.



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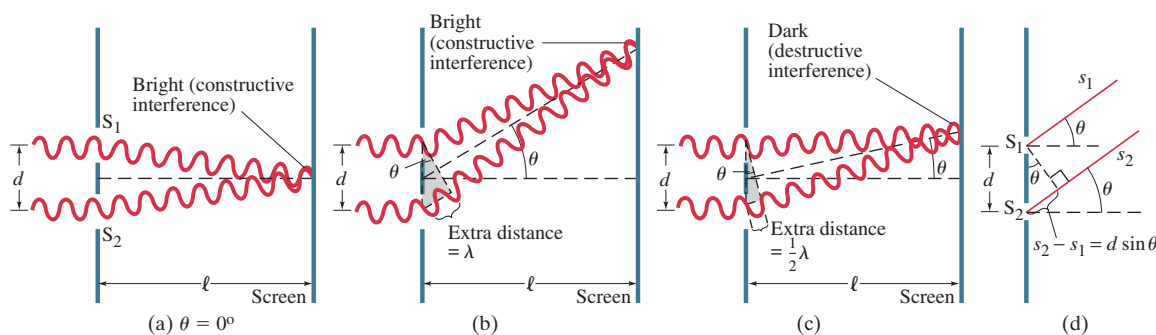
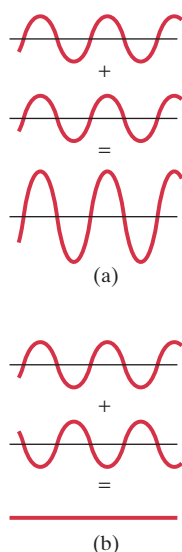


FIGURE 7 How the wave theory explains the pattern of lines seen in the double-slit experiment. (a) At the center of the screen the waves from each slit travel the same distance and are in phase. (b) At this angle θ , the lower wave travels an extra distance of one whole wavelength, and the waves are in phase; note from the shaded triangle that the path difference equals $d \sin \theta$. (c) For this angle θ , the lower wave travels an extra distance equal to one-half wavelength, so the two waves arrive at the screen fully out of phase. (d) A more detailed diagram showing the geometry for parts (b) and (c).

FIGURE 8 Two traveling waves are shown undergoing (a) constructive interference, (b) destructive interference.



To see how an interference pattern is produced on the screen, we make use of Fig. 7. Waves of wavelength λ are shown entering the slits S_1 and S_2 , which are a distance d apart. The waves spread out in all directions after passing through the slits (Fig. 6), but they are shown only for three different angles θ . In Fig. 7a, the waves reaching the center of the screen are shown ($\theta = 0^\circ$). The waves from the two slits travel the same distance, so they are in phase: a crest of one wave arrives at the same time as a crest of the other wave. Hence the amplitudes of the two waves add to form a larger amplitude as shown in Fig. 8a. This is **constructive interference**, and there is a bright area at the center of the screen. Constructive interference also occurs when the paths of the two rays differ by one wavelength (or any whole number of wavelengths), as shown in Fig. 7b; also here there will be brightness on the screen. But if one ray travels an extra distance of one-half wavelength (or $\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$, and so on), the two waves are exactly out of phase when they reach the screen: the crests of one wave arrive at the same time as the troughs of the other wave, and so they add to produce zero amplitude (Fig. 8b). This is **destructive interference**, and the screen is dark, Fig. 7c. Thus, there will be a series of bright and dark lines (or **fringes**) on the viewing screen.

To determine exactly where the bright lines fall, first note that Fig. 7 is somewhat exaggerated; in real situations, the distance d between the slits is very small compared to the distance ℓ to the screen. The rays from each slit for each case will therefore be essentially parallel, and θ is the angle they make with the horizontal as shown in Fig. 7d. From the shaded right triangles shown in Figs. 7b and c, we can see that the extra distance traveled by the lower ray is $d \sin \theta$ (seen more clearly in Fig. 7d). Constructive interference will occur, and a bright fringe will appear on the screen, when the *path difference*, $d \sin \theta$, equals a whole number of wavelengths:

$$d \sin \theta = m\lambda, \quad m = 0, 1, 2, \dots \quad \left[\begin{array}{l} \text{constructive} \\ \text{interference} \\ \text{(bright)} \end{array} \right] \quad (2a)$$

The value of m is called the **order** of the interference fringe. The first order ($m = 1$), for example, is the first fringe on each side of the central fringe (which is at $\theta = 0$, $m = 0$). Destructive interference occurs when the path difference $d \sin \theta$ is $\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, and so on:

$$d \sin \theta = (m + \frac{1}{2})\lambda, \quad m = 0, 1, 2, \dots \quad \left[\begin{array}{l} \text{destructive} \\ \text{interference} \\ \text{(dark)} \end{array} \right] \quad (2b)$$

The bright fringes are peaks or maxima of light intensity, the dark fringes are minima.

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The intensity of the bright fringes is greatest for the central fringe ($m = 0$) and decreases for higher orders, as shown in Fig. 9. How much the intensity decreases with increasing order depends on the width of the two slits.

CONCEPTUAL EXAMPLE 1 Interference pattern lines. (a) Will there be an infinite number of points on the viewing screen where constructive and destructive interference occur, or only a finite number of points? (b) Are neighboring points of constructive interference uniformly spaced, or is the spacing between neighboring points of constructive interference not uniform?

RESPONSE (a) When you look at Eqs. 2a and b you might be tempted to say, given the statement $m = 0, 1, 2, \dots$ beside the equations, that there are an infinite number of points of constructive and destructive interference. However, recall that $\sin \theta$ cannot exceed 1. Thus, there is an upper limit to the values of m that can be used in these equations. For Eq. 2a, the maximum value of m is the integer closest in value but smaller than d/λ . So there are a *finite* number of points of constructive and destructive interference no matter how large the screen. (b) The spacing between neighboring points of constructive or destructive interference is not uniform: The spacing gets larger as θ gets larger, and you can verify this statement mathematically. For small values of θ the spacing is nearly uniform as you will see in Example 2.

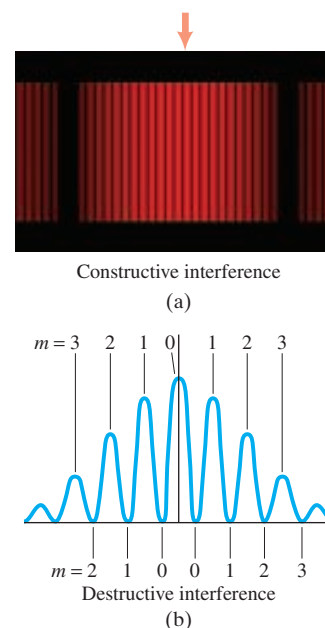


FIGURE 9 (a) Interference fringes produced by a double-slit experiment and detected by photographic film placed on the viewing screen. The arrow marks the central fringe. (b) Graph of the intensity of light in the interference pattern. Also shown are values of m for Eq. 2a (constructive interference) and Eq. 2b (destructive interference).

EXAMPLE 2 Line spacing for double-slit interference. A screen containing two slits 0.100 mm apart is 1.20 m from the viewing screen. Light of wavelength $\lambda = 500$ nm falls on the slits from a distant source. Approximately how far apart will adjacent bright interference fringes be on the screen?

APPROACH The angular position of bright (constructive interference) fringes is found using Eq. 2a. The distance between the first two fringes (say) can be found using right triangles as shown in Fig. 10.

SOLUTION Given $d = 0.100$ mm $= 1.00 \times 10^{-4}$ m, $\lambda = 500 \times 10^{-9}$ m, and $\ell = 1.20$ m, the first-order fringe ($m = 1$) occurs at an angle θ given by

$$\sin \theta_1 = \frac{m\lambda}{d} = \frac{(1)(500 \times 10^{-9} \text{ m})}{1.00 \times 10^{-4} \text{ m}} = 5.00 \times 10^{-3}.$$

This is a very small angle, so we can take $\sin \theta \approx \theta$, with θ in radians. The first-order fringe will occur a distance x_1 above the center of the screen (see Fig. 10), given by $x_1/\ell = \tan \theta_1 \approx \theta_1$, so

$$x_1 \approx \ell \theta_1 = (1.20 \text{ m})(5.00 \times 10^{-3}) = 6.00 \text{ mm}.$$

The second-order fringe ($m = 2$) will occur at

$$x_2 \approx \ell \theta_2 = \ell \frac{2\lambda}{d} = 12.0 \text{ mm}$$

above the center, and so on. Thus the lower order fringes are 6.00 mm apart.

NOTE The spacing between fringes is essentially uniform until the approximation $\sin \theta \approx \theta$ is no longer valid.

CONCEPTUAL EXAMPLE 3 Changing the wavelength. (a) What happens to the interference pattern shown in Fig. 10, Example 2, if the incident light (500 nm) is replaced by light of wavelength 700 nm? (b) What happens instead if the wavelength stays at 500 nm but the slits are moved farther apart?

RESPONSE (a) When λ increases in Eq. 2a but d stays the same, then the angle θ for bright fringes increases and the interference pattern spreads out. (b) Increasing the slit spacing d reduces θ for each order, so the lines are closer together.

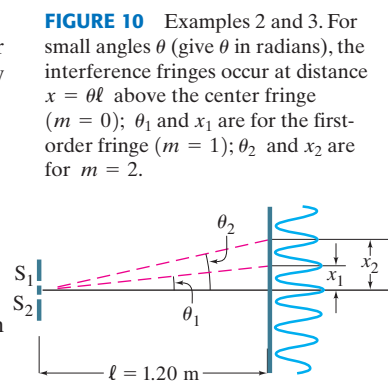


FIGURE 10 Examples 2 and 3. For small angles θ (give θ in radians), the interference fringes occur at distance $x = \theta \ell$ above the center fringe ($m = 0$); θ_1 and x_1 are for the first-order fringe ($m = 1$); θ_2 and x_2 are for $m = 2$.

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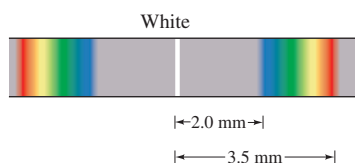
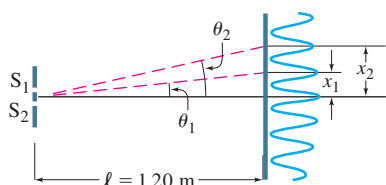


FIGURE 11 First-order fringes are a full spectrum, like a rainbow. Also Example 4.

FIGURE 10 (Repeated.)

For small angles θ (give θ in radians), the interference fringes occur at distance $x = \theta \ell$ above the center fringe ($m = 0$); θ_1 and x_1 are for the first-order fringe ($m = 1$), θ_2 and x_2 are for $m = 2$.



From Eqs. 2 we can see that, except for the zeroth-order fringe at the center, the position of the fringes depends on wavelength. Consequently, when white light falls on the two slits, as Young found in his experiments, the central fringe is white, but the first- (and higher-) order fringes contain a spectrum of colors like a rainbow; θ was found to be smallest for violet light and largest for red (Fig. 11). By measuring the position of these fringes, Young was the first to determine the wavelengths of visible light (using Eqs. 2). In doing so, he showed that what distinguishes different colors physically is their wavelength (or frequency), an idea put forward earlier by Grimaldi in 1665.

EXAMPLE 4 Wavelengths from double-slit interference. White light passes through two slits 0.50 mm apart, and an interference pattern is observed on a screen 2.5 m away. The first-order fringe resembles a rainbow with violet and red light at opposite ends. The violet light is about 2.0 mm and the red 3.5 mm from the center of the central white fringe (Fig. 11). Estimate the wavelengths for the violet and red light.

APPROACH We find the angles for violet and red light from the distances given and the diagram of Fig. 10. Then we use Eq. 2a to obtain the wavelengths. Because 3.5 mm is much less than 2.5 m, we can use the small-angle approximation.

SOLUTION We use Eq. 2a with $m = 1$ and $\sin \theta \approx \tan \theta \approx \theta$. Then for violet light, $x = 2.0$ mm, so (see also Fig. 10)

$$\lambda = \frac{d \sin \theta}{m} \approx \frac{d \theta}{m} \approx \frac{d x}{m \ell} = \left(\frac{5.0 \times 10^{-4} \text{ m}}{1} \right) \left(\frac{2.0 \times 10^{-3} \text{ m}}{2.5 \text{ m}} \right) = 4.0 \times 10^{-7} \text{ m},$$

or 400 nm. For red light, $x = 3.5$ mm, so

$$\lambda = \frac{d x}{m \ell} = \left(\frac{5.0 \times 10^{-4} \text{ m}}{1} \right) \left(\frac{3.5 \times 10^{-3} \text{ m}}{2.5 \text{ m}} \right) = 7.0 \times 10^{-7} \text{ m} = 700 \text{ nm}.$$

Coherence

The two slits in Fig. 7 act as if they were two sources of radiation. They are called **coherent sources** because the waves leaving them have the same wavelength and frequency, and bear the same phase relationship to each other at all times. This happens because the waves come from a single source to the left of the two slits in Fig. 7, splitting the original beam into two. An interference pattern is observed only when the sources are coherent. If two tiny lightbulbs replaced the two slits, an interference pattern would not be seen. The light emitted by one lightbulb would have a random phase with respect to the second bulb, and the screen would be more or less uniformly illuminated. Two such sources, whose output waves have phases that bear no fixed relationship to each other over time, are called **incoherent sources**.

*4 Intensity in the Double-Slit Interference Pattern

We saw in Section 3 that the interference pattern produced by the coherent light from two slits, S_1 and S_2 (Figs. 7 and 9), produces a series of bright and dark fringes. If the two monochromatic waves of wavelength λ are in phase at the slits, the maxima (brightest points) occur at angles θ given by (Eqs. 2)

$$d \sin \theta = m \lambda,$$

and the minima (darkest points) when

$$d \sin \theta = \left(m + \frac{1}{2} \right) \lambda,$$

where m is an integer ($m = 0, 1, 2, \dots$).

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We now determine the intensity of the light at all points in the pattern, assuming that if either slit were covered, the light passing through the other would diffract sufficiently to illuminate a large portion of the screen uniformly. The intensity I of the light at any point is proportional to the square of its wave amplitude. Treating light as an electromagnetic wave, I is proportional to the square of the electric field E (or to the magnetic field B): $I \propto E^2$. The electric field \vec{E} at any point P (see Fig. 12) will be the sum of the electric field vectors of the waves coming from each of the two slits, \vec{E}_1 and \vec{E}_2 . Since \vec{E}_1 and \vec{E}_2 are essentially parallel (on a screen far away compared to the slit separation), the magnitude of the electric field at angle θ (that is, at point P) will be

$$E_\theta = E_1 + E_2.$$

Both E_1 and E_2 vary sinusoidally with frequency $f = c/\lambda$, but they differ in phase, depending on their different travel distances from the slits. The electric field at P can then be written for the light from each of the two slits, using $\omega = 2\pi f$, as

$$\begin{aligned} E_1 &= E_{10} \sin \omega t \\ E_2 &= E_{20} \sin(\omega t + \delta) \end{aligned} \quad (3)$$

where E_{10} and E_{20} are their respective amplitudes and δ is the phase difference. The value of δ depends on the angle θ , so let us now determine δ as a function of θ .

At the center of the screen (point 0), $\delta = 0$. If the difference in path length from P to S_1 and S_2 is $d \sin \theta = \lambda/2$, the two waves are exactly out of phase so $\delta = \pi$ (or 180°). If $d \sin \theta = \lambda$, the two waves differ in phase by $\delta = 2\pi$. In general, then, δ is related to θ by

$$\frac{\delta}{2\pi} = \frac{d \sin \theta}{\lambda}$$

or

$$\delta = \frac{2\pi}{\lambda} d \sin \theta. \quad (4)$$

To determine $E_\theta = E_1 + E_2$, we add the two scalars E_1 and E_2 which are sine functions differing by the phase δ . One way to determine the sum of E_1 and E_2 is to use a **phasor diagram**. As shown in Fig. 13, we draw an arrow of length E_{10} to represent the amplitude of E_1 (Eq. 3); and the arrow of length E_{20} , which we draw to make a fixed angle δ with E_{10} , represents the amplitude of E_2 . When the diagram rotates at angular frequency ω about the origin, the projections of E_{10} and E_{20} on the vertical axis represent E_1 and E_2 as a function of time (see Eq. 3). We let $E_{\theta 0}$ be the “vector” sum[†] of E_{10} and E_{20} ; $E_{\theta 0}$ is the amplitude of the sum $E_\theta = E_1 + E_2$, and the projection of $E_{\theta 0}$ on the vertical axis is just E_θ . If the two slits provide equal illumination, so that $E_{10} = E_{20} = E_0$, then from *symmetry* in Fig. 13, the angle $\phi = \delta/2$, and we can write

$$E_\theta = E_{\theta 0} \sin \left(\omega t + \frac{\delta}{2} \right). \quad (5a)$$

From Fig. 13 we can also see that

$$E_{\theta 0} = 2E_0 \cos \phi = 2E_0 \cos \frac{\delta}{2}. \quad (5b)$$

Combining Eqs. 5a and b, we obtain

$$E_\theta = 2E_0 \cos \frac{\delta}{2} \sin \left(\omega t + \frac{\delta}{2} \right), \quad (5c)$$

where δ is given by Eq. 4.

[†]We are not adding the actual electric field vectors; instead we are using the “phasor” technique to add the amplitudes, taking into account the phase difference of the two waves.

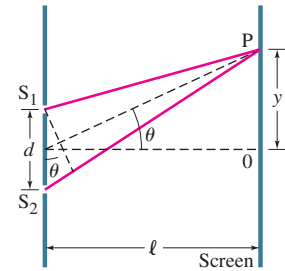
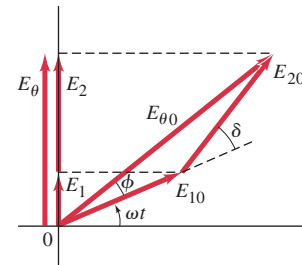


FIGURE 12 Determining the intensity in a double-slit interference pattern. Not to scale: in fact $\ell \gg d$, and the two rays become essentially parallel.

FIGURE 13 Phasor diagram for double-slit interference pattern.



The Wave Nature of Light; Interference

We are not really interested in E_θ as a function of time, since for visible light the frequency (10^{14} to 10^{15} Hz) is much too high to be noticeable. We are interested in the average intensity, which is proportional to the amplitude squared, $E_{\theta 0}^2$. We now drop the word “average,” and we let I_θ ($I_\theta \propto E_\theta^2$) be the intensity at any point P at an angle θ to the horizontal. We let I_0 be the intensity at point O, the center of the screen, where $\theta = \delta = 0$, so $I_0 \propto (E_{10} + E_{20})^2 = (2E_0)^2$. Then the ratio I_θ/I_0 is equal to the ratio of the squares of the electric-field amplitudes at these two points, so

$$\frac{I_\theta}{I_0} = \frac{E_{\theta 0}^2}{(2E_0)^2} = \cos^2 \frac{\delta}{2}$$

where we used Eq. 5b. Thus the intensity I_θ at any point is related to the maximum intensity at the center of the screen by

$$\begin{aligned} I_\theta &= I_0 \cos^2 \frac{\delta}{2} \\ &= I_0 \cos^2 \left(\frac{\pi d \sin \theta}{\lambda} \right) \end{aligned} \quad (6)$$

where δ was given by Eq. 4. This is the relation we sought.

From Eq. 6 we see that maxima occur where $\cos \delta/2 = \pm 1$, which corresponds to $\delta = 0, 2\pi, 4\pi, \dots$. From Eq. 4, δ has these values when

$$d \sin \theta = m\lambda, \quad m = 0, 1, 2, \dots$$

Minima occur where $\delta = \pi, 3\pi, 5\pi, \dots$, which corresponds to

$$d \sin \theta = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, 1, 2, \dots$$

These are the same results we obtained in Section 3. But now we know not only the position of maxima and minima, but from Eq. 6 we can determine the intensity at all points.

In the usual situation where the distance ℓ to the screen from the slits is large compared to the slit separation d ($\ell \gg d$), if we consider only points P whose distance y from the center (point O) is small compared to ℓ ($y \ll \ell$)—see Fig. 12—then

$$\sin \theta = \frac{y}{\ell}.$$

From this it follows (see Eq. 4) that

$$\delta = \frac{2\pi d}{\lambda \ell} y.$$

Equation 6 then becomes

$$I_\theta = I_0 \left[\cos \left(\frac{\pi d}{\lambda \ell} y \right) \right]^2. \quad [y \ll \ell, d \ll \ell] \quad (7)$$

EXERCISE B What are the values for the intensity I_θ when (a) $y = 0$, (b) $y = \lambda\ell/4d$, and (c) $y = \lambda\ell/2d$?

The intensity I_θ as a function of the phase difference δ is plotted in Fig. 14. In the approximation of Eq. 7, the horizontal axis could as well be y , the position on the screen.

The Wave Nature of Light; Interference

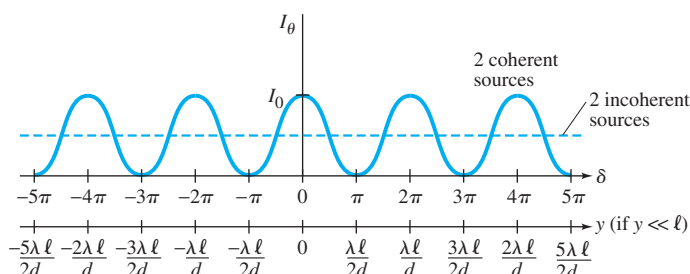


FIGURE 14 Intensity I as a function of phase difference δ and position on screen y (assuming $y \ll \ell$).

The intensity pattern expressed in Eqs. 6 and 7, and plotted in Fig. 14, shows a series of maxima of equal height, and is based on the assumption that each slit (alone) would illuminate the screen uniformly. This is never quite true, as you shall see when you study diffraction. The center maximum is strongest and each succeeding maximum to each side is a little less strong.

EXAMPLE 5 Antenna intensity. Two radio antennas are located close to each other as shown in Fig. 15, separated by a distance d . The antennas radiate in phase with each other, emitting waves of intensity I_0 at wavelength λ . (a) Calculate the net intensity as a function of θ for points very far from the antennas. (b) For $d = \lambda$, determine I and find in which directions I is a maximum and a minimum. (c) Repeat part (b) when $d = \lambda/2$.

APPROACH This setup is similar to Young's double-slit experiment.

SOLUTION (a) Points of constructive and destructive interference are still given by Eqs. 2a and b, and the net intensity as a function of θ is given by Eq. 6. (b) We let $d = \lambda$ in Eq. 6, and find for the intensity,

$$I = I_0 \cos^2(\pi \sin \theta).$$

I is a maximum, equal to I_0 , when $\sin \theta = 0, 1$, or -1 , meaning $\theta = 0, 90^\circ, 180^\circ$, and 270° . I is zero when $\sin \theta = \frac{1}{2}$ and $-\frac{1}{2}$, for which $\theta = 30^\circ, 150^\circ, 210^\circ$, and 330° . (c) For $d = \lambda/2$, I is maximized for $\theta = 0$ and 180° , and minimized for 90° and 270° .

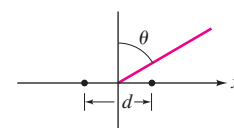


FIGURE 15 Example 5. The two dots represent the antennas.

5 Interference in Thin Films

Interference of light gives rise to many everyday phenomena such as the bright colors reflected from soap bubbles and from thin oil or gasoline films on water, Fig. 16. In these and other cases, the colors are a result of constructive interference between light reflected from the two surfaces of the thin film.

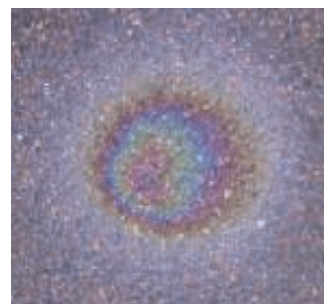
FIGURE 16 Thin film interference patterns seen in (a) a soap bubble, (b) a thin film of soapy water, and (c) a thin layer of oil on wet pavement.



(a)



(b)



(c)

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The Wave Nature of Light; Interference

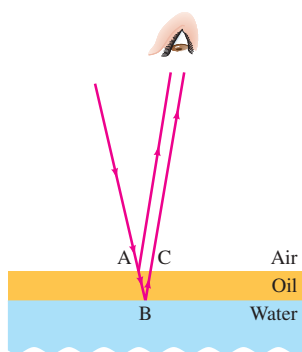


FIGURE 17 Light reflected from the upper and lower surfaces of a thin film of oil lying on water. This analysis assumes the light strikes the surface nearly perpendicularly, but is shown here at an angle so we can display each ray.

To see how this **thin-film interference** happens, consider a smooth surface of water on top of which is a thin uniform layer of another substance, say an oil whose index of refraction is less than that of water (we'll see why we assume this in a moment); see Fig. 17. Assume for the moment that the incident light is of a single wavelength. Part of the incident light is reflected at A on the top surface, and part of the light transmitted is reflected at B on the lower surface. The part reflected at the lower surface must travel the extra distance ABC. If this *path difference* ABC equals one or a whole number of wavelengths in the film (λ_n), the two waves will reach the eye in phase and interfere constructively. Hence the region AC on the surface film will appear bright. But if ABC equals $\frac{1}{2}\lambda_n$, $\frac{3}{2}\lambda_n$, and so on, the two waves will be exactly out of phase and destructive interference occurs: the area AC on the film will show no reflection—it will be dark (or better, transparent to the dark material below). The wavelength λ_n is the *wavelength in the film*: $\lambda_n = \lambda/n$, where n is the index of refraction in the film and λ is the wavelength in vacuum. See Eq. 1.

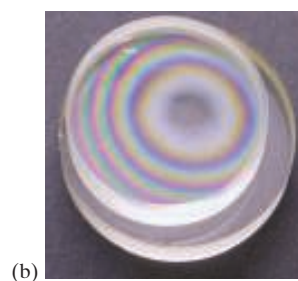
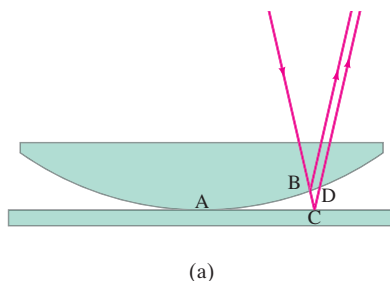
When white light falls on such a film, the path difference ABC will equal λ_n (or $m\lambda_n$, with $m = \text{an integer}$) for only one wavelength at a given viewing angle. The color corresponding to λ (λ in air) will be seen as very bright. For light viewed at a slightly different angle, the path difference ABC will be longer or shorter and a different color will undergo constructive interference. Thus, for an extended (nonpoint) source emitting white light, a series of bright colors will be seen next to one another. Variations in thickness of the film will also alter the path difference ABC and therefore affect the color of light that is most strongly reflected.

EXERCISE C Return to the Chapter-Opening Question and answer it again now. Try to explain why you may have answered differently the first time.

When a curved glass surface is placed in contact with a flat glass surface, Fig. 18, a series of concentric rings is seen when illuminated from above by either white light (as shown) or by monochromatic light. These are called **Newton's rings**[†] and they are due to interference between waves reflected by the top and bottom surfaces of the very thin *air gap* between the two pieces of glass. Because this gap (which is equivalent to a thin film) increases in width from the central contact point out to the edges, the extra path length for the lower ray (equal to BCD) varies; where it equals $0, \frac{1}{2}\lambda, \lambda, \frac{3}{2}\lambda, 2\lambda$, and so on, it corresponds to constructive and destructive interference; and this gives rise to the series of bright colored circles seen in Fig. 18b. The color you see at a given radius corresponds to constructive interference; at that radius, other colors partially or fully destructively interfere. (If monochromatic light is used, the rings are alternately bright and dark.)

[†] Although Newton gave an elaborate description of them, they had been first observed and described by his contemporary, Robert Hooke.

FIGURE 18 Newton's rings. (a) Light rays reflected from upper and lower surfaces of the thin air gap can interfere. (b) Photograph of interference patterns using white light.



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The Wave Nature of Light; Interference

The point of contact of the two glass surfaces (A in Fig. 18a) is dark in Fig. 18b. Since the path difference is zero here, our previous analysis would suggest that the waves reflected from each surface are in phase and so this central area ought to be bright. But it is dark, which tells us something else is happening here: the two waves must be completely out of phase. This can happen if one of the waves, upon reflection, flips over—a crest becomes a trough—see Fig. 19. We say that the reflected wave has undergone a phase change of 180° , or of half a wave cycle. Indeed, this and other experiments reveal that, at normal incidence,

a beam of light reflected by a material with index of refraction greater than that of the material in which it is traveling, changes phase by 180° or $\frac{1}{2}$ cycle;

see Fig. 19. This phase change acts just like a path difference of $\frac{1}{2}\lambda$. If the index of refraction of the reflecting material is less than that of the material in which the light is traveling, no phase change occurs.[†]

Thus the wave reflected at the curved surface above the air gap in Fig. 18a undergoes no change in phase. But the wave reflected at the lower surface, where the beam in air strikes the glass, undergoes a $\frac{1}{2}$ -cycle phase change, equivalent to a $\frac{1}{2}\lambda$ path difference. Thus the two waves reflected near the point of contact A of the two glass surfaces (where the air gap approaches zero thickness) will be a half cycle (or 180°) out of phase, and a dark spot occurs. Bright colored rings will occur when the path difference is $\frac{1}{2}\lambda, \frac{3}{2}\lambda$, and so on, because the phase change at one surface effectively adds a path difference of $\frac{1}{2}\lambda$ ($=\frac{1}{2}$ cycle). (If monochromatic light is used, the bright Newton's rings will be separated by dark bands which occur when the path difference BCD in Fig. 18a is equal to an integral number of wavelengths.)

Returning for a moment to Fig. 17, the light reflecting at both interfaces, air–oil and oil–water, underwent a phase change of 180° equivalent to a path difference of $\frac{1}{2}\lambda$, since we assumed $n_{\text{water}} > n_{\text{oil}} > n_{\text{air}}$; since the phase changes were equal, they didn't affect our analysis.

EXAMPLE 6 Thin film of air, wedge-shaped. A very fine wire 7.35×10^{-3} mm in diameter is placed between two flat glass plates as in Fig. 20a. Light whose wavelength in air is 600 nm falls (and is viewed) perpendicular to the plates and a series of bright and dark bands is seen, Fig. 20b. How many light and dark bands will there be in this case? Will the area next to the wire be bright or dark?

APPROACH We need to consider two effects: (1) path differences for rays reflecting from the two close surfaces (thin wedge of air between the two glass plates), and (2) the $\frac{1}{2}$ -cycle phase change at the lower surface (point E in Fig. 20a), where rays in air can enter glass. Because of the phase change at the lower surface, there will be a dark band (no reflection) when the path difference is $0, \lambda, 2\lambda, 3\lambda$, and so on. Since the light rays are perpendicular to the plates, the extra path length equals $2t$, where t is the thickness of the air gap at any point.

SOLUTION Dark bands will occur where

$$2t = m\lambda, \quad m = 0, 1, 2, \dots$$

Bright bands occur when $2t = (m + \frac{1}{2})\lambda$, where m is an integer. At the position of the wire, $t = 7.35 \times 10^{-6}$ m. At this point there will be $2t/\lambda = (2)(7.35 \times 10^{-6} \text{ m})/(6.00 \times 10^{-7} \text{ m}) = 24.5$ wavelengths. This is a “half integer,” so the area next to the wire will be bright. There will be a total of 25 dark lines along the plates, corresponding to path lengths of $0\lambda, 1\lambda, 2\lambda, 3\lambda, \dots, 24\lambda$, including the one at the point of contact A ($m = 0$). Between them, there will be 24 bright lines plus the one at the end, or 25.

NOTE The bright and dark bands will be straight only if the glass plates are extremely flat. If they are not, the pattern is uneven, as in Fig. 20c. Thus we see a very precise way of testing a glass surface for flatness. Spherical lens surfaces can be tested for precision by placing the lens on a flat glass surface and observing Newton's rings (Fig. 18b) for perfect circularity.

[†]This result can be derived from Maxwell's equations. It corresponds to the reflection of a wave traveling along a cord when it reaches the end; if the end is tied down, the wave changes phase and the pulse flips over, but if the end is free, no phase change occurs.

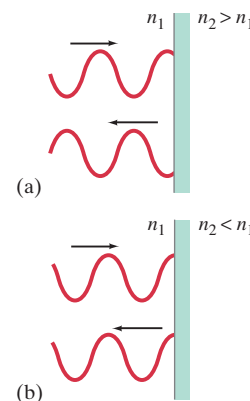
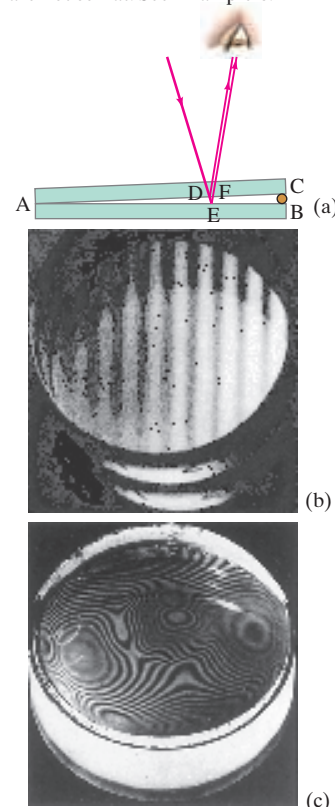


FIGURE 19 (a) Reflected ray changes phase by 180° or $\frac{1}{2}$ cycle if $n_2 > n_1$, but (b) does not if $n_2 < n_1$.

FIGURE 20 (a) Light rays reflected from the upper and lower surfaces of a thin wedge of air interfere to produce bright and dark bands. (b) Pattern observed when glass plates are optically flat; (c) pattern when plates are not so flat. See Example 6.



PHYSICS APPLIED
Testing glass for flatness

Bausch & Lomb Inc.

The Wave Nature of Light; Interference

If the wedge between the two glass plates of Example 6 is filled with some transparent substance other than air—say, water—the pattern shifts because the wavelength of the light changes. In a material where the index of refraction is n , the wavelength is $\lambda_n = \lambda/n$, where λ is the wavelength in vacuum (Eq. 1). For instance, if the thin wedge of Example 6 were filled with water, then $\lambda_n = 600 \text{ nm}/1.33 = 450 \text{ nm}$; instead of 25 dark lines, there would be 33.

When white light (rather than monochromatic light) is incident on the thin wedge of air in Figs. 18a or 20a, a colorful series of fringes is seen because constructive interference occurs for different wavelengths in the reflected light at different thicknesses along the wedge.

A soap bubble (Fig. 16a and Chapter-Opening Photo) is a thin spherical shell (or film) with air inside. The variations in thickness of a soap bubble film gives rise to bright colors reflected from the soap bubble. (There is air on both sides of the bubble film.) Similar variations in film thickness produce the bright colors seen reflecting from a thin layer of oil or gasoline on a puddle or lake (Fig. 16c). Which wavelengths appear brightest also depends on the viewing angle.

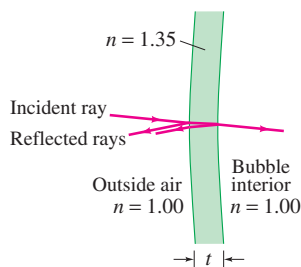


FIGURE 21 Example 7. The incident and reflected rays are assumed to be perpendicular to the bubble's surface. They are shown at a slight angle so we can distinguish them.

CAUTION

A formula is not enough: you must also check for phase changes at surfaces

EXAMPLE 7 Thickness of soap bubble skin. A soap bubble appears green ($\lambda = 540 \text{ nm}$) at the point on its front surface nearest the viewer. What is the smallest thickness the soap bubble film could have? Assume $n = 1.35$.

APPROACH Assume the light is reflected perpendicularly from the point on a spherical surface nearest the viewer, Fig. 21. The light rays also reflect from the inner surface of the soap bubble film as shown. The path difference of these two reflected rays is $2t$, where t is the thickness of the soap film. Light reflected from the first (outer) surface undergoes a 180° phase change (index of refraction of soap is greater than that of air), whereas reflection at the second (inner) surface does not. To determine the thickness t for an interference maximum, we must use the wavelength of light in the soap ($n = 1.35$).

SOLUTION The 180° phase change at only one surface is equivalent to a $\frac{1}{2}\lambda$ path difference. Therefore, green light is bright when the minimum path difference equals $\frac{1}{2}\lambda_n$. Thus, $2t = \lambda/2n$, so

$$t = \frac{\lambda}{4n} = \frac{(540 \text{ nm})}{(4)(1.35)} = 100 \text{ nm}.$$

This is the smallest thickness; but the green color is more likely to be seen at the *next* thickness that gives constructive interference, $2t = 3\lambda/2n$, because other colors would be more fully cancelled by destructive interference. The more likely thickness is $3\lambda/4n = 300 \text{ nm}$, or even $5\lambda/4n = 500 \text{ nm}$. Note that green is seen in air, so $\lambda = 540 \text{ nm}$ (not λ/n).

*Colors in a Thin Soap Film

The thin film of soapy water shown in Fig. 16b (repeated here) has stood vertically for a long time so that gravity has pulled much of the soapy water toward the bottom. The top section is so thin (perhaps 30 nm thick) that light reflected from the front and back surfaces have almost no path difference. Thus the 180° phase change at the front surface assures that the two reflected waves are 180° out of phase for all wavelengths of visible light. The white light incident on this thin film does not reflect at the top part of the film. Thus the top is transparent, and we see the background which is black.

Below the black area at the top, there is a thin blue line, and then a white band. The film thickness is perhaps 75 to 100 nm , so the shortest wavelength (blue) light begins to partially interfere constructively; but just below, where the thickness is slightly greater (100 nm), the path length is reasonably close to $\lambda/2$ for much of the spectrum and we see white or silver. (Why? Recall that red starts at 600 nm in air; so most colors in the spectrum lie between 450 nm and 600 nm in air; but in water the wavelengths are $n = 1.33$ times smaller, 340 nm to 450 nm , so a 100 nm thickness is a 200 nm path length, not far from $\lambda/2$ for most colors.) Immediately below the white band we see a brown band (around 200 nm in thickness) where selected wavelengths (not all) are close to exactly λ and those colors destructively interfere, leaving only a few colors to partially interfere constructively, giving us murky brown.

FIGURE 16b (Repeated.)

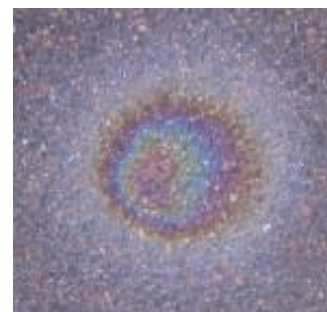


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The Wave Nature of Light; Interference

Farther down, with increasing thickness t , a path length $2t = 510 \text{ nm}$ corresponds nicely to $\frac{3}{2}\lambda$ for blue, but not for other colors, so we see blue ($\frac{3}{2}\lambda$ path difference plus $\frac{1}{2}\lambda$ phase change = constructive interference). Other colors experience constructive interference (at $\frac{3}{2}\lambda$ and then at $\frac{5}{2}\lambda$) at still greater thicknesses, so we see a series of separated colors something like a rainbow.

In the soap bubble of our Chapter-Opening Photo, similar things happen: at the top (where the film is thinnest) we see black and then silver-white, just as within the loop shown in Fig. 16b. And examine the oil film on wet pavement shown in Fig. 16c (repeated here); the oil film is thickest at the center and thins out toward the edges. Notice the whitish outer ring where most colors constructively interfere, which would suggest a thickness on the order of 100 nm as discussed above for the white band in the soap film. Beyond the outer white band of the oil film, Fig. 16c, there is still some oil, but the film is so thin that reflected light from upper and lower surfaces destructively interfere and you can see right through this very thin oil film.



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FIGURE 16c (Repeated.)

Lens Coatings

An important application of thin-film interference is in the coating of glass to make it “nonreflecting,” particularly for lenses. A glass surface reflects about 4% of the light incident upon it. Good-quality cameras, microscopes, and other optical devices may contain six to ten thin lenses. Reflection from all these surfaces can reduce the light level considerably, and multiple reflections produce a background haze that reduces the quality of the image. By reducing reflection, transmission is increased. A very thin coating on the lens surfaces can reduce reflections considerably. The thickness of the film is chosen so that light (at least for one wavelength) reflecting from the front and rear surfaces of the film destructively interferes. The amount of reflection at a boundary depends on the difference in index of refraction between the two materials. Ideally, the coating material should have an index of refraction which is the geometric mean ($= \sqrt{n_1 n_2}$) of those for air and glass, so that the amount of reflection at each surface is about equal. Then destructive interference can occur nearly completely for one particular wavelength depending on the thickness of the coating. Nearby wavelengths will at least partially destructively interfere, but a single coating cannot eliminate reflections for all wavelengths. Nonetheless, a single coating can reduce total reflection from 4% to 1% of the incident light. Often the coating is designed to eliminate the center of the reflected spectrum (around 550 nm). The extremes of the spectrum—red and violet—will not be reduced as much. Since a mixture of red and violet produces purple, the light seen reflected from such coated lenses is purple (Fig. 22). Lenses containing two or three separate coatings can more effectively reduce a wider range of reflecting wavelengths.

PHYSICS APPLIED

Lens coatings

FIGURE 22 A coated lens. Note color of light reflected from the front lens surface.



Kristen Brochmann/Fundamental Photographs, NYC

PROBLEM SOLVING

Interference

- Interference effects** depend on the simultaneous arrival of two or more waves at the same point in space.
- Constructive interference** occurs when the waves arrive in phase with each other: a crest of one wave arrives at the same time as a crest of the other wave. The amplitudes of the waves then add to form a larger amplitude. Constructive interference also occurs when the path difference is exactly one full wavelength or any integer multiple of a full wavelength: 1λ , 2λ , 3λ , \dots .
- Destructive interference** occurs when a crest of one wave arrives at the same time as a trough of the

other wave. The amplitudes add, but they are of opposite sign, so the total amplitude is reduced to zero if the two amplitudes are equal. Destructive interference occurs whenever the phase difference is half a wave cycle, or the path difference is a half-integral number of wavelengths. Thus, the total amplitude will be zero if two identical waves arrive one-half wavelength out of phase, or $(m + \frac{1}{2})\lambda$ out of phase, where m is an integer.

- For thin-film interference, an extra half-wavelength **phase shift** occurs when light **reflects** from an optically more dense medium (going from a material of lesser toward greater index of refraction).

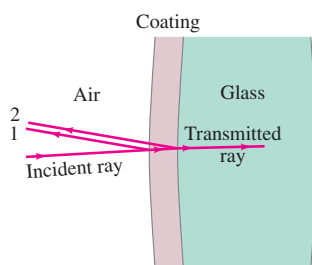


FIGURE 23 Example 8. Incident ray of light is partially reflected at the front surface of a lens coating (ray 1) and again partially reflected at the rear surface of the coating (ray 2), with most of the energy passing as the transmitted ray into the glass.

EXAMPLE 8 Nonreflective coating. What is the thickness of an optical coating of MgF_2 whose index of refraction is $n = 1.38$ and which is designed to eliminate reflected light at wavelengths (in air) around 550 nm when incident normally on glass for which $n = 1.50$?

APPROACH We explicitly follow the procedure outlined in the Problem Solving Strategy on the previous page.

SOLUTION

- Interference effects.** Consider two rays reflected from the front and rear surfaces of the coating on the lens as shown in Fig. 23. The rays are drawn not quite perpendicular to the lens so we can see each of them. These two reflected rays will interfere with each other.
- Constructive interference.** We want to eliminate reflected light, so we do not consider constructive interference.
- Destructive interference.** To eliminate reflection, we want reflected rays 1 and 2 to be $\frac{1}{2}$ cycle out of phase with each other so that they destructively interfere. The phase difference is due to the path difference $2t$ traveled by ray 2, as well as any phase change in either ray due to reflection.
- Reflection phase shift.** Rays 1 and 2 *both* undergo a change of phase by $\frac{1}{2}$ cycle when they reflect from the coating's front and rear surfaces, respectively (at both surfaces the index of refraction increases). Thus there is no net change in phase due to the reflections. The net phase difference will be due to the extra path $2t$ taken by ray 2 in the coating, where $n = 1.38$. We want $2t$ to equal $\frac{1}{2}\lambda_n$ so that destructive interference occurs, where $\lambda_n = \lambda/n$ is the wavelength in the coating. With $2t = \lambda_n/2 = \lambda/2n$, then

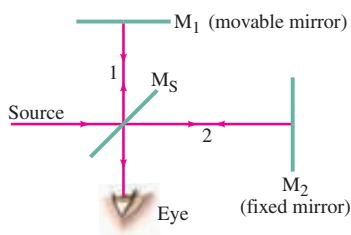
$$t = \frac{\lambda_n}{4} = \frac{\lambda}{4n} = \frac{(550 \text{ nm})}{(4)(1.38)} = 99.6 \text{ nm}.$$

NOTE We could have set $2t = (m + \frac{1}{2})\lambda_n$, where m is an integer. The smallest thickness ($m = 0$) is usually chosen because destructive interference will occur over the widest angle.

NOTE Complete destructive interference occurs only for the given wavelength of visible light. Longer and shorter wavelengths will have only partial cancellation.

*6 Michelson Interferometer

FIGURE 24 Michelson interferometer.



A useful instrument involving wave interference is the **Michelson interferometer** (Fig. 24),[†] invented by the American Albert A. Michelson. Monochromatic light from a single point on an extended source is shown striking a half-silvered mirror M_S . This **beam splitter** mirror M_S has a thin layer of silver that reflects only half the light that hits it, so that half of the beam passes through to a fixed mirror M_2 , where it is reflected back. The other half is reflected by M_S to a mirror M_1 that is movable (by a fine-thread screw), where it is also reflected back. Upon its return, part of beam 1 passes through M_S and reaches the eye; and part of beam 2, on its return, is reflected by M_S into the eye. If the two path lengths are identical, the two coherent beams entering the eye constructively interfere and brightness will be seen. If the movable mirror is moved a distance $\lambda/4$, one beam will travel an extra distance equal to $\lambda/2$ (because it travels back and forth over the distance $\lambda/4$). In this case, the two beams will destructively interfere and darkness will be seen. As M_1 is moved farther, brightness will recur (when the path difference is λ), then darkness, and so on.

Very precise length measurements can be made with an interferometer. The motion of mirror M_1 by only $\frac{1}{4}\lambda$ produces a clear difference between brightness and darkness. For $\lambda = 400 \text{ nm}$, this means a precision of 100 nm or 10^{-4} mm ! If mirror M_1 is tilted very slightly, the bright or dark spots are seen instead as a series of bright and dark lines or “fringes.” By counting the number of fringes (or fractions thereof) that pass a reference line, extremely precise length measurements can be made.

[†]There are other types of interferometer, but Michelson's is the best known.

*7 Luminous Intensity

The *intensity* of light, as for any electromagnetic wave, is measured by the Poynting vector in W/m^2 , and the total power output of a source can be measured in watts (the *radiant flux*). But for measuring the visual sensation we call brightness, we must consider *only* the visible spectrum as well as the eye's sensitivity to different wavelengths—the eye is most sensitive in the central, 550-nm (green), portion of the spectrum.

These factors are taken into account in the quantity **luminous flux**, F_ℓ , whose unit is the **lumen** (lm). One lumen is equivalent to $\frac{1}{683}$ watts of 555-nm light.

Since the luminous flux from a source may not be uniform over all directions, we define the **luminous intensity** I_ℓ as the luminous flux per unit solid angle[†] (steradian). Its unit is the **candela** (cd) where $1 \text{ cd} = 1 \text{ lm}/\text{sr}$, and it is one of the seven basic quantities in the SI.

The **illuminance**, E_ℓ , is the luminous flux incident on a surface per unit area of the surface: $E_\ell = F_\ell/A$. Its unit is the lumen per square meter (lm/m^2) and is a measure of the illumination falling on a surface.[‡]

EXAMPLE 9 Lightbulb illuminance. The brightness of a particular type of 100-W lightbulb is rated at 1700 lm. Determine (a) the luminous intensity and (b) the illuminance at a distance of 2.0 m.

APPROACH Assume the light output is uniform in all directions.

SOLUTION (a) A full sphere corresponds to $4\pi \text{ sr}$. Hence, $I_\ell = 1700 \text{ lm}/4\pi \text{ sr} = 135 \text{ cd}$. It does not depend on distance. (b) At $d = 2.0 \text{ m}$ from the source, the luminous flux per unit area is

$$E_\ell = \frac{F_\ell}{4\pi d^2} = \frac{1700 \text{ lm}}{(4\pi)(2.0 \text{ m})^2} = 34 \text{ lm}/\text{m}^2.$$

The illuminance decreases as the square of the distance.

[†]A solid angle is a sort of two-dimensional angle and is measured in steradians. Think of a solid angle starting at a point and intercepting an area ΔA on a sphere of radius r surrounding that point. The solid angle has magnitude $\Delta A/r^2$ steradians. A solid angle including all of space intercepts the full surface area of the sphere, $4\pi r^2$, and so has magnitude $4\pi r^2/r^2 = 4\pi$ steradians. (Compare to a normal angle, for which a full circle subtends 2π radians.)

[‡]The British unit is the foot-candle, or lumen per square foot.

Summary

The wave theory of light is strongly supported by the observations that light exhibits **interference** and **diffraction**. Wave theory also explains the refraction of light and the fact that light travels more slowly in transparent solids and liquids than it does in air.

An aid to predicting wave behavior is **Huygens' principle**, which states that every point on a wave front can be considered as a source of tiny wavelets that spread out in the forward direction at the speed of the wave itself. The new wave front is the envelope (the common tangent) of all the wavelets.

The wavelength of light in a medium with index of refraction n is

$$\lambda_n = \frac{\lambda}{n}, \quad (1)$$

where λ is the wavelength in vacuum; the frequency is not changed.

Young's double-slit experiment clearly demonstrated the interference of light. The observed bright spots of the interference pattern are explained as constructive interference between the beams coming through the two slits, where the beams differ in path length by an integral number of wavelengths. The dark areas in between are due to destructive interference when the path lengths differ by $\frac{1}{2}\lambda, \frac{3}{2}\lambda$, and so on. The angles θ at which **constructive interference** occurs are given by

$$\sin \theta = m \frac{\lambda}{d}, \quad (2a)$$

where λ is the wavelength of the light, d is the separation of the slits, and m is an integer ($0, 1, 2, \dots$). **Destructive interference** occurs at angles θ given by

$$\sin \theta = \left(m + \frac{1}{2}\right) \frac{\lambda}{d}, \quad (2b)$$

where m is an integer ($0, 1, 2, \dots$).

The Wave Nature of Light; Interference

The light intensity I_θ at any point in a double-slit interference pattern can be calculated using a phasor diagram, which predicts that

$$I_\theta = I_0 \cos^2 \frac{\delta}{2} \quad (6)$$

where I_0 is the intensity at $\theta = 0$ and the phase angle δ is

$$\delta = \frac{2\pi d}{\lambda} \sin \theta. \quad (4)$$

Two sources of light are perfectly **coherent** if the waves leaving them are of the same single frequency and maintain the

same phase relationship at all times. If the light waves from the two sources have a random phase with respect to each other over time (as for two incandescent lightbulbs) the two sources are **incoherent**.

Light reflected from the front and rear surfaces of a thin film of transparent material can interfere constructively or destructively, depending on the path difference. A phase change of 180° or $\frac{1}{2}\lambda$ occurs when the light reflects at a surface where the index of refraction increases. Such **thin-film interference** has many practical applications, such as lens coatings and using Newton's rings to check the uniformity of glass surfaces.

Answers to Exercises

A: 333 nm; 6.0×10^{14} Hz; 2.0×10^8 m/s.

B: (a) I_0 ; (b) $0.50I_0$; (c) 0.

C: (e).