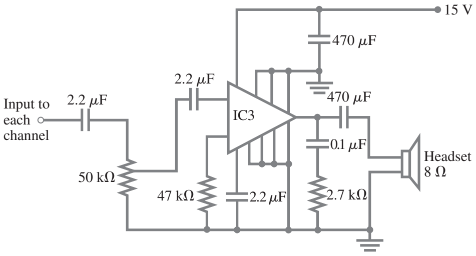
Dino Vournas/Reuters Ltd.

These MP3 players contain circuits that are dc, at least in part. (The audio signal is ac.) The circuit diagram below shows a possible amplifier circuit for each stereo channel. We have already met two of the circuit elements shown: resistors and capacitors, and we discuss them in circuits in this Chapter. (The large triangle is an amplifier chip containing transistors.) We also discuss voltmeters and ammeters, and how they are built and used to make measurements.



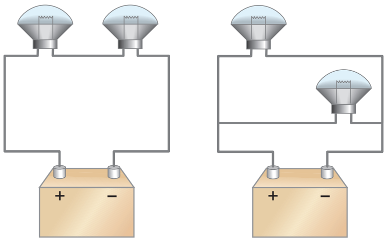
DC Circuits

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.*]

The automobile headlight bulbs shown in the circuits here are identical. The connection which produces more light is

(a) circuit 1.
(b) circuit 2.
(c) both the same.
(d) not enough information.



Circuit 1 Circuit 2

Electric circuits are basic parts of all electronic devices from radio and TV sets to computers and automobiles. Scientific measurements, from physics to biology and medicine, make use of electric circuits. We will apply the basic principles of electric current to analyze dc circuits involving combinations of batteries, resistors, and capacitors. We also study the operation of some useful instruments.†

†AC circuits that contain only a voltage source and resistors can be analyzed like the dc circuits in this Chapter. However, ac circuits that contain capacitors and other circuit elements are more complicated.

CONTENTS

Note: Sections marked with an asterisk (*) may be considered optional by the instructor .

From Chapter 26 of *Physics for Scientists & Engineers with Modern Physics*, Fourth Edition, Douglas C. Giancoli.

TABLE 1 Symbols for Circuit Elements

Symbol	Device
	Battery
or	Capacitor
	Resistor
	Wire with negligible resistance
	Switch
⏚ or ⏚	Ground

When we draw a diagram for a circuit, we represent batteries, capacitors, and resistors by the symbols shown in Table 1. Wires whose resistance is negligible compared with other resistance in the circuit are drawn simply as straight lines. Some circuit diagrams show a ground symbol (⏚ or ⏚) which may mean a real connection to the ground, perhaps via a metal pipe, or it may simply mean a common connection, such as the frame of a car.

For the most part in this Chapter, except in Section 5 on *RC* circuits, we will be interested in circuits operating in their steady state. That is, we won't be looking at a circuit at the moment a change is made in it, such as when a battery or resistor is connected or disconnected, but rather later when the currents have reached their steady values.

1 EMF and Terminal Voltage

To have current in an electric circuit, we need a device such as a battery or an electric generator that transforms one type of energy (chemical, mechanical, or light, for example) into electric energy. Such a device is called a **source** of **electromotive force** or of **emf**. (The term "electromotive force" is a misnomer since it does not refer to a "force" that is measured in newtons. Hence, to avoid confusion, we prefer to use the abbreviation, emf.) The *potential difference* between the terminals of such a source, when no current flows to an external circuit, is called the **emf** of the source. The symbol $\mathscr{E}$ is usually used for emf (don't confuse it with E for electric field), and its unit is volts.

CAUTION
Why battery voltage isn't perfectly constant

A battery is not a source of constant current—the current out of a battery varies according to the resistance in the circuit. A battery *is*, however, a nearly constant voltage source, but not perfectly constant as we now discuss. You may have noticed in your own experience that when a current is drawn from a battery, the potential difference (voltage) across its terminals drops below its rated emf. For example, if you start a car with the headlights on, you may notice the headlights dim. This happens because the starter draws a large current, and the battery voltage drops as a result. The voltage drop occurs because the chemical reactions in a battery cannot supply charge fast enough to maintain the full emf. For one thing, charge must move (within the electrolyte) between the electrodes of the battery, and there is always some hindrance to completely free flow. Thus, a battery itself has some resistance, which is called its **internal resistance**; it is usually designated r.

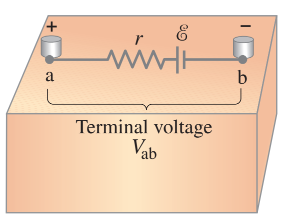


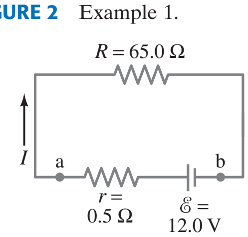
FIGURE 1 Diagram for an electric cell or battery.

A real battery is modeled as if it were a perfect emf $\mathscr{E}$ in series with a resistor r, as shown in Fig. 1. Since this resistance r is inside the battery, we can never separate it from the battery. The two points a and b in the diagram represent the two terminals of the battery. What we measure is the **terminal voltage** $V_{ab} = V_a - V_b$. When no current is drawn from the battery, the terminal voltage equals the emf, which is determined by the chemical reactions in the battery: $V_{ab} = \mathscr{E}$. However, when a current I flows naturally from the battery there is an internal drop in voltage equal to Ir. Thus the terminal voltage (the actual voltage) is[†]

$$V_{ab} = \mathscr{E} - Ir. \tag{1}$$

For example, if a 12-V battery has an internal resistance of 0.1 Ω, then when 10 A flows from the battery, the terminal voltage is $12\text{ V} - (10\text{ A})(0.1\ \Omega) = 11\text{ V}$. The internal resistance of a battery is usually small. For example, an ordinary flashlight battery when fresh may have an internal resistance of perhaps 0.05 Ω. (However, as it ages and the electrolyte dries out, the internal resistance increases to many ohms.) Car batteries have lower internal resistance.

FIGURE 2 Example 1.



EXAMPLE 1 **Battery with internal resistance.** A 65.0-Ω resistor is connected to the terminals of a battery whose emf is 12.0 V and whose internal resistance is 0.5 Ω, Fig. 2. Calculate (*a*) the current in the circuit, (*b*) the terminal voltage of the battery, V_{ab}, and (*c*) the power dissipated in the resistor R and in the battery's internal resistance r.

APPROACH We first consider the battery as a whole, which is shown in Fig. 2 as an emf $\mathscr{E}$ and internal resistance r between points a and b. Then we apply $V = IR$ to the circuit itself.

[†]When a battery is being charged, a current is forced to pass through it; we then have to write

$$V_{ab} = \mathscr{E} + Ir.$$

See Section 4 and Fig. 46.

SOLUTION (a) From Eq. 1, we have

$$V_{ab} = \mathscr{E} - Ir.$$

We apply Ohm's law to this battery and the resistance R of the circuit: $V_{ab} = IR$. Hence $IR = \mathscr{E} - Ir$ or $\mathscr{E} = I(R + r)$, and so

$$I = \frac{\mathscr{E}}{R + r} = \frac{12.0\text{ V}}{65.0\,\Omega + 0.5\,\Omega} = \frac{12.0\text{ V}}{65.5\,\Omega} = 0.183\text{ A}.$$

(b) The terminal voltage is

$$V_{ab} = \mathscr{E} - Ir = 12.0\text{ V} - (0.183\text{ A})(0.5\,\Omega) = 11.9\text{ V}.$$

(c) The power dissipated in R is

$$P_R = I^2R = (0.183\text{ A})^2(65.0\,\Omega) = 2.18\text{ W},$$

and in r is

$$P_r = I^2r = (0.183\text{ A})^2(0.5\,\Omega) = 0.02\text{ W}.$$

EXERCISE A Repeat Example 1 assuming now that the resistance $R = 10.0\,\Omega$, whereas $\mathscr{E}$ and r remain as before.

In much of what follows, unless stated otherwise, we assume that the battery's internal resistance is negligible, and that the battery voltage given is its terminal voltage, which we will usually write simply as V rather than V_{ab}. Be careful not to confuse V (italic) for voltage and V (not italic) for the volt unit.

2 Resistors in Series and in Parallel

When two or more resistors are connected end to end along a single path as shown in Fig. 3a, they are said to be connected in **series**. The resistors could be simple resistors, or they could be lightbulbs (Fig. 3b), or heating elements, or other resistive devices. Any charge that passes through R_1 in Fig. 3a will also pass through R_2 and then R_3. Hence the same current I passes through each resistor. (If it did not, this would imply that either charge was not conserved, or that charge was accumulating at some point in the circuit, which does not happen in the steady state.)

We let V represent the potential difference (voltage) across all three resistors in Fig. 3a. We assume all other resistance in the circuit can be ignored, so V equals the terminal voltage supplied by the battery. We let V_1, V_2, and V_3 be the potential differences across each of the resistors, R_1, R_2, and R_3, respectively. From Ohm's law, $V = IR$, we can write $V_1 = IR_1$, $V_2 = IR_2$, and $V_3 = IR_3$. Because the resistors are connected end to end, energy conservation tells us that the total voltage V is equal to the sum of the voltages[†] across each resistor:

$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3. \qquad \text{[series]} \quad \textbf{(2)}$$

Now let us determine the equivalent single resistance R_{eq} that would draw the same current I as our combination of three resistors in series; see Fig. 3c. Such a single resistance R_{eq} would be related to V by

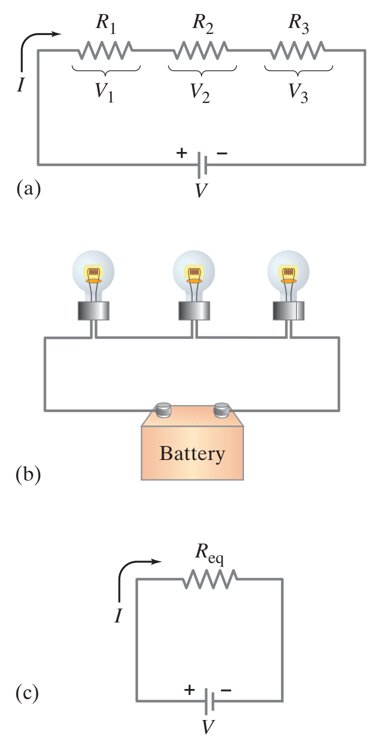
$$V = IR_{eq}.$$

We equate this expression with Eq. 2, $V = I(R_1 + R_2 + R_3)$, and find

$$R_{eq} = R_1 + R_2 + R_3. \qquad \text{[series]} \quad \textbf{(3)}$$

This is, in fact, what we expect. When we put several resistances in series, the total or equivalent resistance is the sum of the separate resistances. (Sometimes we may also call it the "net resistance.") This sum applies to any number of resistances in series. Note that when you add more resistance to the circuit, the current through the circuit will decrease. For example, if a 12-V battery is connected to a 4-Ω resistor, the current will be 3 A. But if the 12-V battery is connected to three 4-Ω resistors in series, the total resistance is 12 Ω and the current through the entire circuit will be only 1 A.

FIGURE 3 (a) Resistances connected in series. (b) Resistances could be lightbulbs, or any other type of resistance. (c) Equivalent single resistance R_{eq} that draws the same current: $R_{eq} = R_1 + R_2 + R_3$.



[†]To see in more detail why this is true, note that an electric charge q passing through R_1 loses an amount of potential energy equal to qV_1. In passing through R_2 and R_3, the potential energy U decreases by qV_2 and qV_3, for a total $\Delta U = qV_1 + qV_2 + qV_3$; this sum must equal the energy given to q by the battery, qV, so that energy is conserved. Hence $qV = q(V_1 + V_2 + V_3)$, and so $V = V_1 + V_2 + V_3$, which is Eq. 2.

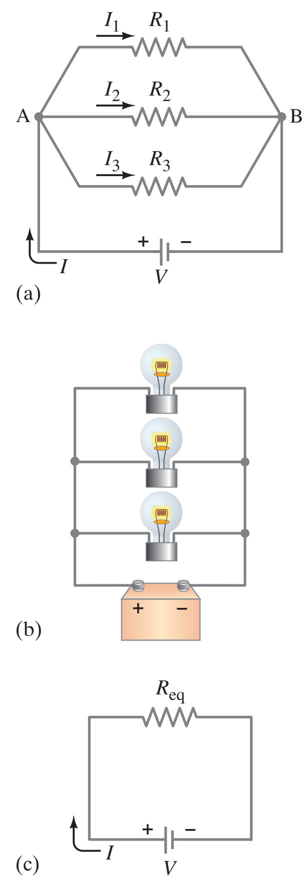
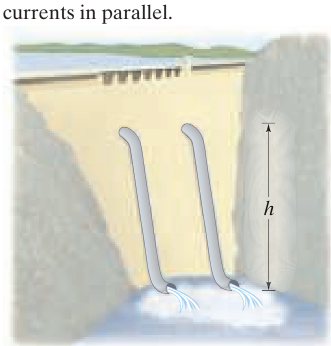


FIGURE 4 (a) Resistances connected in parallel. (b) The resistances could be lightbulbs. (c) The equivalent circuit with R_{eq} obtained from Eq. 4: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$.

FIGURE 5 Water pipes in parallel—analogy to electric currents in parallel.



Another simple way to connect resistors is in **parallel** so that the current from the source splits into separate branches or paths, as shown in Fig. 4a and b. The wiring in houses and buildings is arranged so all electric devices are in parallel. With parallel wiring, if you disconnect one device (say, R_1 in Fig. 4a), the current to the other devices is not interrupted. Compare to a series circuit, where if one device (say, R_1 in Fig. 3a) is disconnected, the current *is* stopped to all the others.

In a parallel circuit, Fig. 4a, the total current I that leaves the battery splits into three separate paths. We let I_1, I_2, and I_3 be the currents through each of the resistors, R_1, R_2, and R_3, respectively. Because *electric charge is conserved*, the current I flowing into junction A (where the different wires or conductors meet, Fig. 4a) must equal the current flowing out of the junction. Thus

$$I = I_1 + I_2 + I_3. \qquad \text{[parallel]}$$

When resistors are connected in parallel, each has the same voltage across it. (Indeed, any two points in a circuit connected by a wire of negligible resistance are at the same potential.) Hence the full voltage of the battery is applied to each resistor in Fig. 4a. Applying Ohm's law to each resistor, we have

$$I_1 = \frac{V}{R_1}, \qquad I_2 = \frac{V}{R_2}, \qquad \text{and} \qquad I_3 = \frac{V}{R_3}.$$

Let us now determine what single resistor R_{eq} (Fig. 4c) will draw the same current I as these three resistances in parallel. This equivalent resistance R_{eq} must satisfy Ohm's law too:

$$I = \frac{V}{R_{eq}}.$$

We now combine the equations above:

$$I = I_1 + I_2 + I_3,$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}.$$

When we divide out the V from each term, we have

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \qquad \text{[parallel]} \quad \textbf{(4)}$$

For example, suppose you connect two 4-Ω loudspeakers to a single set of output terminals of your stereo amplifier or receiver. (Ignore the other channel for a moment—our two speakers are both connected to the left channel, say.) The equivalent resistance of the two 4-Ω "resistors" in parallel is

$$\frac{1}{R_{eq}} = \frac{1}{4\,\Omega} + \frac{1}{4\,\Omega} = \frac{2}{4\,\Omega} = \frac{1}{2\,\Omega},$$

and so $R_{eq} = 2\,\Omega$. Thus the net (or equivalent) resistance is *less* than each single resistance. This may at first seem surprising. But remember that when you connect resistors in parallel, you are giving the current additional paths to follow. Hence the net resistance will be less.

Equations 3 and 4 make good sense. Recalling the equation for resistivity, $R = \rho\ell/A$, we see that placing resistors in series increases the length and therefore the resistance; putting resistors in parallel increases the area through which current flows, thus reducing the overall resistance.

An analogy may help here. Consider two identical pipes taking in water near the top of a dam and releasing it below as shown in Fig. 5. The gravitational potential difference, proportional to the height h, is the same for both pipes, just as the voltage is the same for parallel resistors. If both pipes are open, rather than only one, twice as much water will flow through. That is, with two equal pipes open, the net resistance to the flow of water will be reduced, by half, just as for electrical resistors in parallel. Note that if both pipes are closed, the dam offers infinite resistance to the flow of water. This corresponds in the electrical case to an open circuit—when the path is not continuous and no current flows—so the electrical resistance is infinite.

EXERCISE B You have a 10-Ω and a 15-Ω resistor. What is the smallest and largest equivalent resistance that you can make with these two resistors?

CONCEPTUAL EXAMPLE 2 **Series or parallel?** (*a*) The lightbulbs in Fig. 6 are identical. Which configuration produces more light? (*b*) Which way do you think the headlights of a car are wired? Ignore change of filament resistance R with current.

RESPONSE (*a*) The equivalent resistance of the parallel circuit is found from Eq. 4, $1/R_{eq} = 1/R + 1/R = 2/R$. Thus $R_{eq} = R/2$. The parallel combination then has lower resistance $(= R/2)$ than the series combination $(R_{eq} = R + R = 2R)$. There will be more total current in the parallel configuration (2), since $I = V/R_{eq}$ and V is the same for both circuits. The total power transformed, which is related to the light produced, is $P = IV$, so the greater current in (2) means more light produced. (*b*) Headlights are wired in parallel (2), because if one bulb goes out, the other bulb can stay lit. If they were in series (1), when one bulb burned out (the filament broke), the circuit would be open and no current would flow, so neither bulb would light.

NOTE When you answered the Chapter-Opening Question, was your answer circuit 2? Can you express any misconceptions you might have had?

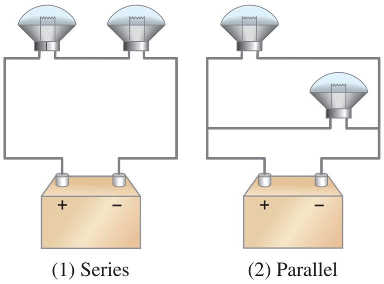


FIGURE 6 Example 2.

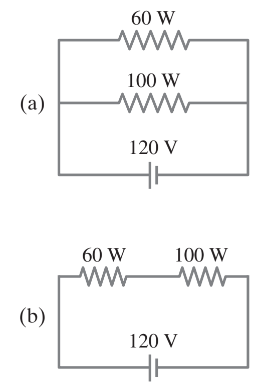
CONCEPTUAL EXAMPLE 3 **An illuminating surprise.** A 100-W, 120-V lightbulb and a 60-W, 120-V lightbulb are connected in two different ways as shown in Fig. 7. In each case, which bulb glows more brightly? Ignore change of filament resistance with current (and temperature).

RESPONSE (*a*) These are normal lightbulbs with their power rating given for 120 V. They both receive 120 V, so the 100-W bulb is naturally brighter.

(*b*) The resistance of the 100-W bulb is less than that of the 60-W bulb (calculated from $P = V^2/R$ at constant 120 V). Here they are connected in series and receive the same current. Hence, from $P = I^2R$ (I = constant) the higher-resistance "60-W" bulb will transform more power and thus be brighter.

NOTE When connected in series as in (b), the two bulbs do *not* dissipate 60 W and 100 W because neither bulb receives 120 V.

FIGURE 7 Example 3.



Note that whenever a group of resistors is replaced by the equivalent resistance, current and voltage and power in the rest of the circuit are unaffected.

EXAMPLE 4 **Circuit with series and parallel resistors.** How much current is drawn from the battery shown in Fig. 8a?

APPROACH The current I that flows out of the battery all passes through the 400-Ω resistor, but then it splits into I_1 and I_2 passing through the 500-Ω and 700-Ω resistors. The latter two resistors are in parallel with each other. We look for something that we already know how to treat. So let's start by finding the equivalent resistance, R_p, of the parallel resistors, 500 Ω and 700 Ω. Then we can consider this R_p to be in series with the 400-Ω resistor.

SOLUTION The equivalent resistance, R_p, of the 500-Ω and 700-Ω resistors in parallel is given by

$$\frac{1}{R_P} = \frac{1}{500\,\Omega} + \frac{1}{700\,\Omega} = 0.0020\,\Omega^{-1} + 0.0014\,\Omega^{-1} = 0.0034\,\Omega^{-1}.$$

This is $1/R_P$, so we take the reciprocal to find R_P. It is a common mistake to forget to take this reciprocal. Notice that the units of reciprocal ohms, Ω^{-1}, are a reminder. Thus

$$R_P = \frac{1}{0.0034\,\Omega^{-1}} = 290\,\Omega.$$

This 290 Ω is the equivalent resistance of the two parallel resistors, and is in series with the 400-Ω resistor as shown in the equivalent circuit of Fig. 8b. To find the total equivalent resistance R_{eq}, we add the 400-Ω and 290-Ω resistances together, since they are in series, and find

$$R_{eq} = 400\,\Omega + 290\,\Omega = 690\,\Omega.$$

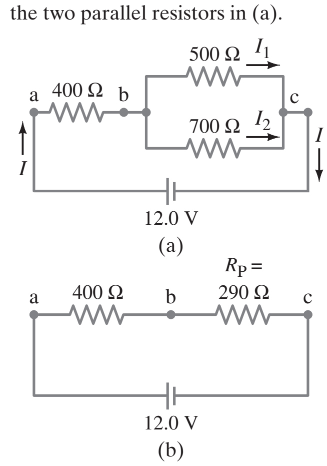
The total current flowing from the battery is then

$$I = \frac{V}{R_{eq}} = \frac{12.0\text{ V}}{690\,\Omega} = 0.0174\text{ A} \approx 17\text{ mA}.$$

NOTE This I is also the current flowing through the 400-Ω resistor, but not through the 500-Ω and 700-Ω resistors (both currents are less—see the next Example).

NOTE Complex resistor circuits can often be analyzed in this way, considering the circuit as a combination of series and parallel resistances.

FIGURE 8 (a) Circuit for Examples 4 and 5. (b) Equivalent circuit, showing the equivalent resistance of 290 Ω for the two parallel resistors in (a).



CAUTION
Remember to take the reciprocal

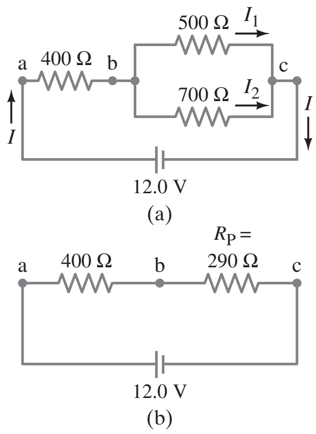


FIGURE 8 (repeated)
(a) Circuit for Examples 4 and 5.
(b) Equivalent circuit, showing the equivalent resistance of 290 Ω for the two parallel resistors in (a).

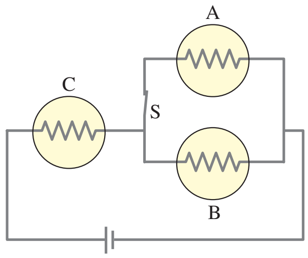


FIGURE 9 Example 6, three identical lightbulbs. Each yellow circle with -\/\/\/- inside represents a lightbulb and its resistance.

EXAMPLE 5 **Current in one branch.** What is the current through the 500-Ω resistor in Fig. 8a?

APPROACH We need to find the voltage across the 500-Ω resistor, which is the voltage between points b and c in Fig. 8a, and we call it V_{bc}. Once V_{bc} is known, we can apply Ohm's law, $V = IR$, to get the current. First we find the voltage across the 400-Ω resistor, V_{ab}, since we know that 17.4 mA passes through it (Example 4).

SOLUTION V_{ab} can be found using $V = IR$:

$$V_{ab} = (0.0174\text{ A})(400\ \Omega) = 7.0\text{ V}.$$

Since the total voltage across the network of resistors is $V_{ac} = 12.0\text{ V}$, then V_{bc} must be $12.0\text{ V} - 7.0\text{ V} = 5.0\text{ V}$. Then Ohm's law applied to the 500-Ω resistor tells us that the current I_1 through that resistor is

$$I_1 = \frac{5.0\text{ V}}{500\ \Omega} = 1.0 \times 10^{-2}\text{ A} = 10\text{ mA}.$$

This is the answer we wanted. We can also calculate the current I_2 through the 700-Ω resistor since the voltage across it is also 5.0 V:

$$I_2 = \frac{5.0\text{ V}}{700\ \Omega} = 7\text{ mA}.$$

NOTE When I_1 combines with I_2 to form the total current I (at point c in Fig. 8a), their sum is $10\text{ mA} + 7\text{ mA} = 17\text{ mA}$. This equals the total current I as calculated in Example 4, as it should.

CONCEPTUAL EXAMPLE 6 **Bulb brightness in a circuit.** The circuit shown in Fig. 9 has three identical lightbulbs, each of resistance R. (*a*) When switch S is closed, how will the brightness of bulbs A and B compare with that of bulb C? (*b*) What happens when switch S is opened? Use a minimum of mathematics in your answers.

RESPONSE (*a*) With switch S closed, the current that passes through bulb C must split into two equal parts when it reaches the junction leading to bulbs A and B. It splits into equal parts because the resistance of bulb A equals that of B. Thus, bulbs A and B each receive half of C's current; A and B will be equally bright, but they will be less bright than bulb C ($P = I^2R$). (*b*) When the switch S is open, no current can flow through bulb A, so it will be dark. We now have a simple one-loop series circuit, and we expect bulbs B and C to be equally bright. However, the equivalent resistance of this circuit ($= R + R$) is greater than that of the circuit with the switch closed. When we open the switch, we increase the resistance and reduce the current leaving the battery. Thus, bulb C will be dimmer when we open the switch. Bulb B gets more current when the switch is open (you may have to use some mathematics here), and so it will be brighter than with the switch closed; and B will be as bright as C.

EXAMPLE 7 **ESTIMATE** **A two-speed fan.** One way a multiple-speed ventilation fan for a car can be designed is to put resistors in series with the fan motor. The resistors reduce the current through the motor and make it run more slowly. Suppose the current in the motor is 5.0 A when it is connected directly across a 12-V battery. (*a*) What series resistor should be used to reduce the current to 2.0 A for low-speed operation? (*b*) What power rating should the resistor have?

APPROACH An electric motor in series with a resistor can be treated as two resistors in series. The power comes from $P = IV$.

SOLUTION (*a*) When the motor is connected to 12 V and drawing 5.0 A, its resistance is $R = V/I = (12\text{ V})/(5.0\text{ A}) = 2.4\ \Omega$. We will assume that this is the motor's resistance for all speeds. (This is an approximation because the current through the motor depends on its speed.) Then, when a current of 2.0 A is flowing, the voltage across the motor is $(2.0\text{ A})(2.4\ \Omega) = 4.8\text{ V}$. The remaining $12.0\text{ V} - 4.8\text{ V} = 7.2\text{ V}$ must appear across the series resistor. When 2.0 A flows through the resistor, its resistance must be $R = (7.2\text{ V})/(2.0\text{ A}) = 3.6\ \Omega$. (*b*) The power dissipated by the resistor is $P = (7.2\text{ V})(2.0\text{ A}) = 14.4\text{ W}$. To be safe, a power rating of 20 W would be appropriate.

EXAMPLE 8 **Analyzing a circuit.** A 9.0-V battery whose internal resistance r is $0.50\ \Omega$ is connected in the circuit shown in Fig. 10a. (*a*) How much current is drawn from the battery? (*b*) What is the terminal voltage of the battery? (*c*) What is the current in the $6.0\text{-}\Omega$ resistor?

APPROACH To find the current out of the battery, we first need to determine the equivalent resistance R_{eq} of the entire circuit, including r, which we do by identifying and isolating simple series or parallel combinations of resistors. Once we find I from Ohm's law, $I = \mathscr{E}/R_{\text{eq}}$, we get the terminal voltage using $V_{\text{ab}} = \mathscr{E} - Ir$. For (*c*) we apply Ohm's law to the $6.0\text{-}\Omega$ resistor.

SOLUTION (*a*) We want to determine the equivalent resistance of the circuit. But where do we start? We note that the $4.0\text{-}\Omega$ and $8.0\text{-}\Omega$ resistors are in parallel, and so have an equivalent resistance R_{eq1} given by

$$\frac{1}{R_{\text{eq1}}} = \frac{1}{8.0\ \Omega} + \frac{1}{4.0\ \Omega} = \frac{3}{8.0\ \Omega};$$

so $R_{\text{eq1}} = 2.7\ \Omega$. This $2.7\ \Omega$ is in series with the $6.0\text{-}\Omega$ resistor, as shown in the equivalent circuit of Fig. 10b. The net resistance of the lower arm of the circuit is then

$$R_{\text{eq2}} = 6.0\ \Omega + 2.7\ \Omega = 8.7\ \Omega,$$

as shown in Fig. 10c. The equivalent resistance R_{eq3} of the $8.7\text{-}\Omega$ and $10.0\text{-}\Omega$ resistances in parallel is given by

$$\frac{1}{R_{\text{eq3}}} = \frac{1}{10.0\ \Omega} + \frac{1}{8.7\ \Omega} = 0.21\ \Omega^{-1},$$

so $R_{\text{eq3}} = \left(1/0.21\ \Omega^{-1}\right) = 4.8\ \Omega$. This $4.8\ \Omega$ is in series with the $5.0\text{-}\Omega$ resistor and the $0.50\text{-}\Omega$ internal resistance of the battery (Fig. 10d), so the total equivalent resistance R_{eq} of the circuit is $R_{\text{eq}} = 4.8\ \Omega + 5.0\ \Omega + 0.50\ \Omega = 10.3\ \Omega$. Hence the current drawn is

$$I = \frac{\mathscr{E}}{R_{\text{eq}}} = \frac{9.0\ \text{V}}{10.3\ \Omega} = 0.87\ \text{A}.$$

(*b*) The terminal voltage of the battery is

$$V_{\text{ab}} = \mathscr{E} - Ir = 9.0\ \text{V} - (0.87\ \text{A})(0.50\ \Omega) = 8.6\ \text{V}.$$

(*c*) Now we can work back and get the current in the $6.0\text{-}\Omega$ resistor. It must be the same as the current through the $8.7\ \Omega$ shown in Fig. 10c (why?). The voltage across that $8.7\ \Omega$ will be the emf of the battery minus the voltage drops across r and the $5.0\text{-}\Omega$ resistor: $V_{8.7} = 9.0\ \text{V} - (0.87\ \text{A})(0.50\ \Omega + 5.0\ \Omega)$. Applying Ohm's law, we get the current (call it I')

$$I' = \frac{9.0\ \text{V} - (0.87\ \text{A})(0.50\ \Omega + 5.0\ \Omega)}{8.7\ \Omega} = 0.48\ \text{A}.$$

This is the current through the $6.0\text{-}\Omega$ resistor.

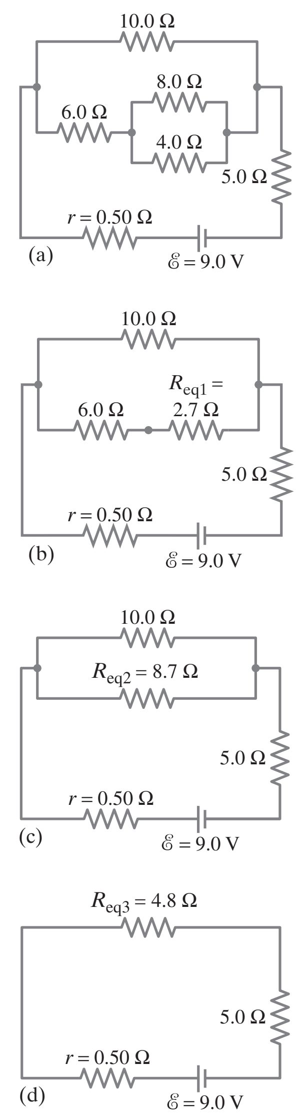


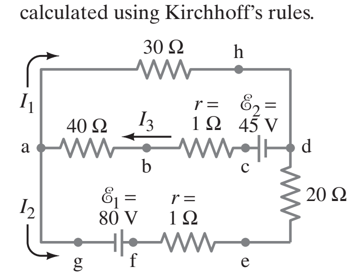
FIGURE 10 Circuit for Example 8, where r is the internal resistance of the battery.

3 Kirchhoff's Rules

In the last few Examples we have been able to find the currents in circuits by combining resistances in series and parallel, and using Ohm's law. This technique can be used for many circuits. However, some circuits are too complicated for that analysis. For example, we cannot find the currents in each part of the circuit shown in Fig. 11 simply by combining resistances as we did before.

To deal with such complicated circuits, we use Kirchhoff's rules, devised by G. R. Kirchhoff (1824–1887) in the mid-nineteenth century. There are two rules, and they are simply convenient applications of the laws of conservation of charge and energy.

FIGURE 11 Currents can be calculated using Kirchhoff's rules.



Kirchhoff's first rule or **junction rule** is based on the conservation of electric charge that we already used to derive the rule for parallel resistors. It states that

Junction rule (conservation of charge)

at any junction point, the sum of all currents entering the junction must equal the sum of all currents leaving the junction.

That is, whatever charge goes in must come out. We already saw an instance of this in the NOTE at the end of Example 5.

Kirchhoff's second rule or **loop rule** is based on the conservation of energy. It states that

Loop rule (conservation of energy)

the sum of the changes in potential around any closed loop of a circuit must be zero.

To see why this rule should hold, consider a rough analogy with the potential energy of a roller coaster on its track. When the roller coaster starts from the station, it has a particular potential energy. As it climbs the first hill, its potential energy increases and reaches a maximum at the top. As it descends the other side, its potential energy decreases and reaches a local minimum at the bottom of the hill. As the roller coaster continues on its path, its potential energy goes through more changes. But when it arrives back at the starting point, it has exactly as much potential energy as it had when it started at this point. Another way of saying this is that there was as much uphill as there was downhill.

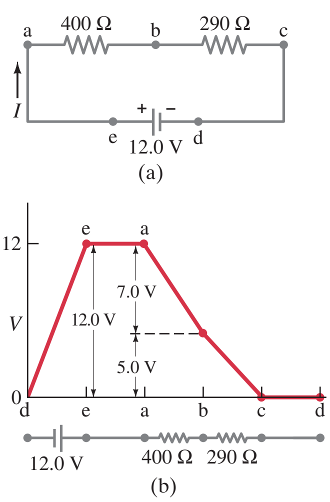


FIGURE 12 Changes in potential around the circuit in (a) are plotted in (b).

Similar reasoning can be applied to an electric circuit. We will analyze the circuit of Fig. 11 shortly but first we consider the simpler circuit in Fig. 12. We have chosen it to be the same as the equivalent circuit of Fig. 8b already discussed. The current in this circuit is $I = (12.0\text{ V})/(690\,\Omega) = 0.0174\text{ A}$, as we calculated in Example 4. (We keep an extra digit in I to reduce rounding errors.) The positive side of the battery, point e in Fig. 12a, is at a high potential compared to point d at the negative side of the battery. That is, point e is like the top of a hill for a roller coaster. We follow the current around the circuit starting at any point. We choose to start at point d and follow a positive test charge completely around this circuit. As we go, we note all changes in potential. When the test charge returns to point d, the potential will be the same as when we started (total change in potential around the circuit is zero). We plot the changes in potential around the circuit in Fig. 12b; point d is arbitrarily taken as zero.

As our positive test charge goes from point d, which is the negative or low potential side of the battery, to point e, which is the positive terminal (high potential side) of the battery, the potential increases by 12.0 V. (This is like the roller coaster being pulled up the first hill.) That is,

$$V_{\text{ed}} = +12.0\text{ V}.$$

When our test charge moves from point e to point a, there is no change in potential since there is no source of emf and we assume negligible resistance in the connecting wires. Next, as the charge passes through the 400-Ω resistor to get to point b, there is a decrease in potential of $V = IR = (0.0174\text{ A})(400\,\Omega) = 7.0\text{ V}$. The positive test charge is flowing "downhill" since it is heading toward the negative terminal of the battery, as indicated in the graph of Fig. 12b. Because this is a *decrease* in potential, we use a *negative* sign:

PROBLEM SOLVING
Be consistent with signs when applying the loop rule

$$V_{\text{ba}} = V_{\text{b}} - V_{\text{a}} = -7.0\text{ V}.$$

As the charge proceeds from b to c there is another potential decrease (a "voltage drop") of $(0.0174\text{ A}) \times (290\,\Omega) = 5.0\text{ V}$, and this too is a decrease in potential:

$$V_{\text{cb}} = -5.0\text{ V}.$$

There is no change in potential as our test charge moves from c to d as we assume negligible resistance in the wires.

The sum of all the changes in potential around the circuit of Fig. 12 is

$$+12.0\text{ V} - 7.0\text{ V} - 5.0\text{ V} = 0.$$

This is exactly what Kirchhoff's loop rule said it would be.

PROBLEM SOLVING

Kirchhoff's Rules

1. **Label the current** in each separate branch of the given circuit with a different subscript, such as I_1, I_2, I_3 (see Fig. 11 or 13). Each current refers to a segment between two junctions. Choose the direction of each current, using an arrow. The direction can be chosen arbitrarily: if the current is actually in the opposite direction, it will come out with a minus sign in the solution.
2. **Identify the unknowns.** You will need as many independent equations as there are unknowns. You may write down more equations than this, but you will find that some of the equations will be redundant (that is, not be independent in the sense of providing new information). You may use $V = IR$ for each resistor, which sometimes will reduce the number of unknowns.
3. **Apply Kirchhoff's junction rule** at one or more junctions.
4. **Apply Kirchhoff's loop rule** for one or more loops: follow each loop in one direction only. Pay careful attention to subscripts, and to signs:
   (a) For a resistor, apply Ohm's law; the potential difference is negative (a decrease) if your chosen loop direction is the same as the chosen current direction through that resistor; the potential difference is positive (an increase) if your chosen loop direction is opposite to the chosen current direction.
   (b) For a battery, the potential difference is positive if your chosen loop direction is from the negative terminal toward the positive terminal; the potential difference is negative if the loop direction is from the positive terminal toward the negative terminal.
5. **Solve the equations** algebraically for the unknowns. Be careful when manipulating equations not to err with signs. At the end, check your answers by plugging them into the original equations, or even by using any additional loop or junction rule equations not used previously.

EXAMPLE 9 **Using Kirchhoff's rules.** Calculate the currents I_1, I_2, and I_3 in the three branches of the circuit in Fig. 13 (which is the same as Fig. 11).

APPROACH AND SOLUTION

1. **Label the currents** and their directions. Figure 13 uses the labels I_1, I_2, and I_3 for the current in the three separate branches. Since (positive) current tends to move away from the positive terminal of a battery, we choose I_2 and I_3 to have the directions shown in Fig. 13. The direction of I_1 is not obvious in advance, so we arbitrarily chose the direction indicated. If the current actually flows in the opposite direction, our answer will have a negative sign.
2. **Identify the unknowns.** We have three unknowns and therefore we need three equations, which we get by applying Kirchhoff's junction and loop rules.
3. **Junction rule:** We apply Kirchhoff's junction rule to the currents at point a, where I_3 enters and I_2 and I_1 leave:
$$I_3 = I_1 + I_2. \qquad (a)$$
This same equation holds at point d, so we get no new information by writing an equation for point d.
4. **Loop rule:** We apply Kirchhoff's loop rule to two different closed loops. First we apply it to the upper loop ahdcba. We start (and end) at point a. From a to h we have a potential decrease $V_{ha} = -(I_1)(30\,\Omega)$. From h to d there is no change, but from d to c the potential increases by 45 V: that is, $V_{cd} = +45\,\mathrm{V}$. From c to a the potential decreases through the two resistances by an amount $V_{ac} = -(I_3)(40\,\Omega + 1\,\Omega) = -(41\,\Omega)I_3$. Thus we have $V_{ha} + V_{cd} + V_{ac} = 0$, or
$$-30I_1 + 45 - 41I_3 = 0, \qquad (b)$$
where we have omitted the units (volts and amps) so we can more easily do the algebra. For our second loop, we take the outer loop ahdefga. (We could have chosen the lower loop abcdefga instead.) Again we start at point a and have $V_{ha} = -(I_1)(30\,\Omega)$, and $V_{dh} = 0$. But when we take our positive test charge from d to e, it actually is going uphill, against the current—or at least against the *assumed* direction of the current, which is what counts in this calculation. Thus $V_{ed} = I_2(20\,\Omega)$ has a *positive* sign. Similarly, $V_{fe} = I_2(1\,\Omega)$. From f to g there is a decrease in potential of 80 V since we go from the high potential terminal of the battery to the low. Thus $V_{gf} = -80\,\mathrm{V}$. Finally, $V_{ag} = 0$, and the sum of the potential changes around this loop is
$$-30I_1 + (20 + 1)I_2 - 80 = 0. \qquad (c)$$
Our major work is done. The rest is algebra.

PROBLEM SOLVING
Choose current directions arbitrarily

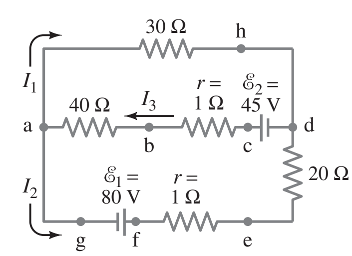


FIGURE 13 Currents can be calculated using Kirchhoff's rules. See Example 9.

5. **Solve the equations.** We have three equations—labeled (a), (b), and (c)—and three unknowns. From Eq. (c) we have

$$I_2 = \frac{80 + 30I_1}{21} = 3.8 + 1.4I_1. \qquad (d)$$

From Eq. (b) we have

$$I_3 = \frac{45 - 30I_1}{41} = 1.1 - 0.73I_1. \qquad (e)$$

We substitute Eqs. (d) and (e) into Eq. (a):

$$I_1 = I_3 - I_2 = 1.1 - 0.73I_1 - 3.8 - 1.4I_1.$$

We solve for I_1, collecting terms:

$$3.1I_1 = -2.7$$
$$I_1 = -0.87\text{ A}.$$

The negative sign indicates that the direction of I_1 is actually opposite to that initially assumed and shown in Fig. 13. The answer automatically comes out in amperes because all values were in volts and ohms. From Eq. (d) we have

$$I_2 = 3.8 + 1.4I_1 = 3.8 + 1.4(-0.87) = 2.6\text{ A},$$

and from Eq. (e)

$$I_3 = 1.1 - 0.73I_1 = 1.1 - 0.73(-0.87) = 1.7\text{ A}.$$

This completes the solution.

NOTE The unknowns in different situations are not necessarily currents. It might be that the currents are given and we have to solve for unknown resistance or voltage. The variables are then different, but the technique is the same.

EXERCISE C Write the equation for the lower loop abcdefga of Example 9 and show, assuming the currents calculated in this Example, that the potentials add to zero for this lower loop.

4 Series and Parallel EMFs; Battery Charging

When two or more sources of emf, such as batteries, are arranged in series as in Fig. 14a, the total voltage is the algebraic sum of their respective voltages. On the other hand, when a 20-V and a 12-V battery are connected oppositely, as shown in Fig. 14b, the net voltage V_{ca} is 8 V (ignoring voltage drop across internal resistances). That is, a positive test charge moved from a to b gains in potential by 20 V, but when it passes from b to c it drops by 12 V. So the net change is $20\text{ V} - 12\text{ V} = 8\text{ V}$. You might think that connecting batteries in reverse like this would be wasteful. For most purposes that would be true. But such a reverse arrangement is precisely how a battery charger works. In Fig. 14b, the 20-V source is charging up the 12-V battery. Because of its greater voltage, the 20-V source is forcing charge back into the 12-V battery: electrons are being forced into its negative terminal and removed from its positive terminal.

An automobile alternator keeps the car battery charged in the same way. A voltmeter placed across the terminals of a (12-V) car battery with the engine running fairly fast can tell you whether or not the alternator is charging the battery. If it is, the voltmeter reads 13 or 14 V. If the battery is not being charged, the voltage will be 12 V, or less if the battery is discharging. Car batteries can be recharged, but other batteries may not be rechargeable, since the chemical reactions in many cannot be reversed. In such cases, the arrangement of Fig. 14b would simply waste energy.

Sources of emf can also be arranged in parallel, Fig. 14c. With equal emfs, a parallel arrangement can provide more energy when large currents are needed. Each of the cells in parallel has to produce only a fraction of the total current, so the energy loss due to internal resistance is less than for a single cell; and the batteries will go dead less quickly.

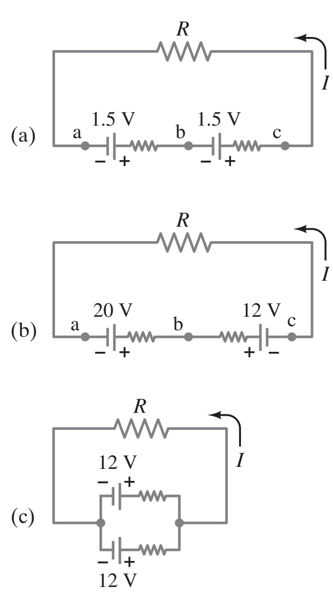


FIGURE 14 Batteries in series (a) and (b), and in parallel (c).

EXAMPLE 10 **Jump starting a car.** A good car battery is being used to jump start a car with a weak battery. The good battery has an emf of 12.5 V and internal resistance 0.020 Ω. Suppose the weak battery has an emf of 10.1 V and internal resistance 0.10 Ω. Each copper jumper cable is 3.0 m long and 0.50 cm in diameter, and can be attached as shown in Fig. 15. Assume the starter motor can be represented as a resistor $R_s = 0.15\,\Omega$. Determine the current through the starter motor (*a*) if only the weak battery is connected to it, and (*b*) if the good battery is also connected, as shown in Fig. 15.

APPROACH We apply Kirchhoff's rules, but in (*b*) we will first need to determine the resistance of the jumper cables using their dimensions and the resistivity ($\rho = 1.68 \times 10^{-8}\,\Omega\cdot\mathrm{m}$ for copper).

SOLUTION (*a*) The circuit with only the weak battery and no jumper cables is simple: an emf of 10.1 V connected to two resistances in series, $0.10\,\Omega + 0.15\,\Omega = 0.25\,\Omega$. Hence the current is $I = V/R = (10.1\,\mathrm{V})/(0.25\,\Omega) = 40\,\mathrm{A}$.

(*b*) We need to find the resistance of the jumper cables that connect the good battery. Each has resistance $R_J = \rho\ell/A = (1.68 \times 10^{-8}\,\Omega\cdot\mathrm{m})(3.0\,\mathrm{m})/(\pi)(0.25 \times 10^{-2}\,\mathrm{m})^2 = 0.0026\,\Omega$. Kirchhoff's loop rule for the full outside loop gives

$$12.5\,\mathrm{V} - I_1(2R_J + r_1) - I_3 R_S = 0$$

$$12.5\,\mathrm{V} - I_1(0.025\,\Omega) - I_3(0.15\,\Omega) = 0 \qquad (a)$$

since $(2R_J + r) = (0.0052\,\Omega + 0.020\,\Omega) = 0.025\,\Omega$.

The loop rule for the lower loop, including the weak battery and the starter, gives

$$10.1\,\mathrm{V} - I_3(0.15\,\Omega) - I_2(0.10\,\Omega) = 0. \qquad (b)$$

The junction rule at point B gives

$$I_1 + I_2 = I_3. \qquad (c)$$

We have three equations in three unknowns. From Eq. (*c*), $I_1 = I_3 - I_2$ and we substitute this into Eq. (*a*):

$$12.5\,\mathrm{V} - (I_3 - I_2)(0.025\,\Omega) - I_3(0.15\,\Omega) = 0,$$

$$12.5\,\mathrm{V} - I_3(0.175\,\Omega) + I_2(0.025\,\Omega) = 0.$$

Combining this last equation with (*b*) gives $I_3 = 71\,\mathrm{A}$, quite a bit better than in (*a*). The other currents are $I_2 = -5\,\mathrm{A}$ and $I_1 = 76\,\mathrm{A}$. Note that $I_2 = -5\,\mathrm{A}$ is in the opposite direction from that assumed in Fig. 15. The terminal voltage of the weak 10.1-V battery is now $V_{BA} = 10.1\,\mathrm{V} - (-5\,\mathrm{A})(0.10\,\Omega) = 10.6\,\mathrm{V}$.

NOTE The circuit shown in Fig. 15, without the starter motor, is how a battery can be charged. The stronger battery pushes charge back into the weaker battery.

Alamy Images

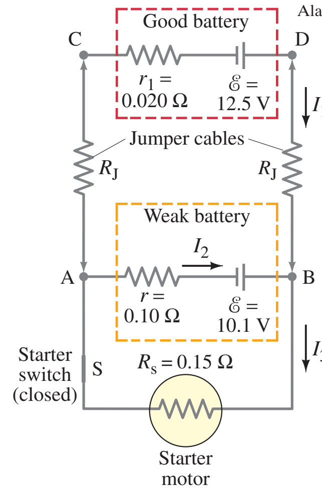
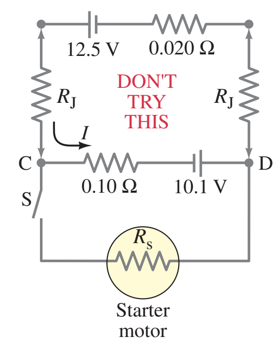


FIGURE 15 Example 10, a jump start.

EXERCISE D If the jumper cables of Example 10 were mistakenly connected in reverse, the positive terminal of each battery would be connected to the negative terminal of the other battery (Fig. 16). What would be the current I even before the starter motor is engaged (the switch S in Fig. 16 is open)? Why could this cause the batteries to explode?

FIGURE 16 Exercise D.



5 Circuits Containing Resistor and Capacitor (*RC* Circuits)

Our study of circuits in this Chapter has, until now, dealt with steady currents that do not change in time. Now we examine circuits that contain both resistance and capacitance. Such a circuit is called an ***RC* circuit**. *RC* circuits are common in everyday life: they are used to control the speed of a car's windshield wiper, and the timing of the change of traffic lights. They are used in camera flashes, in heart pacemakers, and in many other electronic devices. In *RC* circuits, we are not so interested in the final "steady state" voltage and charge on the capacitor, but rather in how these variables change in time.

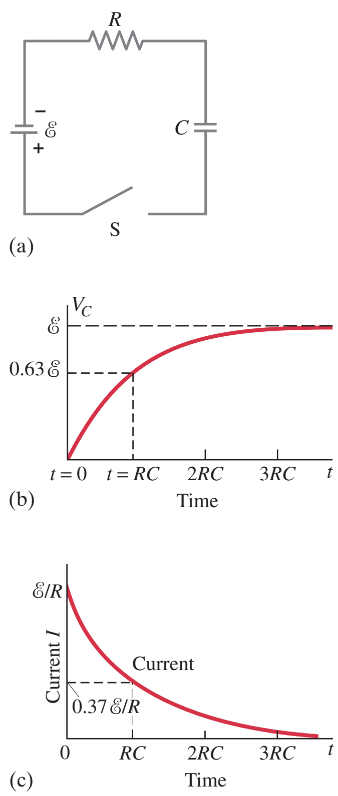


FIGURE 17 After the switch S closes in the RC circuit shown in (a), the voltage across the capacitor increases with time as shown in (b), and the current through the resistor decreases with time as shown in (c).

Let us now examine the RC circuit shown in Fig. 17a. When the switch S is closed, current immediately begins to flow through the circuit. Electrons will flow out from the negative terminal of the battery, through the resistor R, and accumulate on the upper plate of the capacitor. And electrons will flow into the positive terminal of the battery, leaving a positive charge on the other plate of the capacitor. As charge accumulates on the capacitor, the potential difference across it increases $(V_C = Q/C)$, and the current is reduced until eventually the voltage across the capacitor equals the emf of the battery, $\mathscr{E}$. There is then no potential difference across the resistor, and no further current flows. Potential difference V_C across the capacitor thus increases in time as shown in Fig. 17b. The mathematical form of this curve—that is, V_C as a function of time—can be derived using conservation of energy (or Kirchhoff's loop rule). The emf $\mathscr{E}$ of the battery will equal the sum of the voltage drops across the resistor (IR) and the capacitor (Q/C):

$$\mathscr{E} = IR + \frac{Q}{C}. \tag{5}$$

The resistance R includes all resistance in the circuit, including the internal resistance of the battery; I is the current in the circuit at any instant, and Q is the charge on the capacitor at that same instant. Although $\mathscr{E}$, R, and C are constants, both Q and I are functions of time. The rate at which charge flows through the resistor $(I = dQ/dt)$ is equal to the rate at which charge accumulates on the capacitor. Thus we can write

$$\mathscr{E} = R\frac{dQ}{dt} + \frac{1}{C}Q.$$

This equation can be solved by rearranging it:

$$\frac{dQ}{C\mathscr{E} - Q} = \frac{dt}{RC}.$$

We now integrate from $t = 0$, when $Q = 0$, to time t when a charge Q is on the capacitor:

$$\int_0^Q \frac{dQ}{C\mathscr{E} - Q} = \frac{1}{RC}\int_0^t dt$$

$$-\ln(C\mathscr{E} - Q)\Big|_0^Q = \frac{t}{RC}\Big|_0^t$$

$$-\ln(C\mathscr{E} - Q) - (-\ln C\mathscr{E}) = \frac{t}{RC}$$

$$\ln(C\mathscr{E} - Q) - \ln(C\mathscr{E}) = -\frac{t}{RC}$$

$$\ln\left(1 - \frac{Q}{C\mathscr{E}}\right) = -\frac{t}{RC}.$$

We take the exponential† of both sides

$$1 - \frac{Q}{C\mathscr{E}} = e^{-t/RC}$$

or

$$Q = C\mathscr{E}\left(1 - e^{-t/RC}\right). \tag{6a}$$

The potential difference across the capacitor is $V_C = Q/C$, so

$$V_C = \mathscr{E}\left(1 - e^{-t/RC}\right). \tag{6b}$$

From Eqs. 6 we see that the charge Q on the capacitor, and the voltage V_C across it, increase from zero at $t = 0$ to maximum values $Q_{\max} = C\mathscr{E}$ and $V_C = \mathscr{E}$ after a very long time. The quantity RC that appears in the exponent is called the **time constant** τ of the circuit:

$$\tau = RC. \tag{7}$$

It represents the time‡ required for the capacitor to reach $\left(1 - e^{-1}\right) = 0.63$ or 63% of its full charge and voltage. Thus the product RC is a measure of how quickly the

†The constant e, known as the base for natural logarithms, has the value $e = 2.718\cdots$. Do not confuse this e with e for the charge on the electron.

‡The units of RC are $\Omega\cdot\mathrm{F} = (\mathrm{V/A})(\mathrm{C/V}) = \mathrm{C/(C/s)} = \mathrm{s}$.

capacitor gets charged. In a circuit, for example, where $R = 200\,\text{k}\Omega$ and $C = 3.0\,\mu\text{F}$, the time constant is $(2.0 \times 10^5\,\Omega)(3.0 \times 10^{-6}\,\text{F}) = 0.60\,\text{s}$. If the resistance is much lower, the time constant is much smaller. This makes sense, since a lower resistance will retard the flow of charge less. All circuits contain some resistance (if only in the connecting wires), so a capacitor never can be charged instantaneously when connected to a battery.

From Eqs. 6, it appears that Q and V_C never quite reach their maximum values within a finite time. However, they reach 86% of maximum in $2RC$, 95% in $3RC$, 98% in $4RC$, and so on. Q and V_C approach their maximum values asymptotically. For example, if $R = 20\,\text{k}\Omega$ and $C = 0.30\,\mu\text{F}$, the time constant is $(2.0 \times 10^4\,\Omega)(3.0 \times 10^{-7}\,\text{F}) = 6.0 \times 10^{-3}\,\text{s}$. So the capacitor is more than 98% charged in less than $\frac{1}{40}$ of a second.

The current I through the circuit of Fig. 17a at any time t can be obtained by differentiating Eq. 6a:

$$I = \frac{dQ}{dt} = \frac{\mathscr{E}}{R}e^{-t/RC}. \tag{8}$$

Thus, at $t = 0$, the current is $I = \mathscr{E}/R$, as expected for a circuit containing only a resistor (there is not yet a potential difference across the capacitor). The current then drops exponentially in time with a time constant equal to RC, as the voltage across the capacitor increases. This is shown in Fig. 17c. The time constant RC represents the time required for the current to drop to $1/e \approx 0.37$ of its initial value.

EXAMPLE 11 ***RC* circuit, with emf.** The capacitance in the circuit of Fig. 17a is $C = 0.30\,\mu\text{F}$, the total resistance is $20\,\text{k}\Omega$, and the battery emf is 12 V. Determine (*a*) the time constant, (*b*) the maximum charge the capacitor could acquire, (*c*) the time it takes for the charge to reach 99% of this value, (*d*) the current I when the charge Q is half its maximum value, (*e*) the maximum current, and (*f*) the charge Q when the current I is 0.20 its maximum value.

APPROACH We use Fig. 17 and Eqs. 5, 6, 7, and 8.

SOLUTION (*a*) The time constant is $RC = (2.0 \times 10^4\,\Omega)(3.0 \times 10^{-7}\,\text{F}) = 6.0 \times 10^{-3}\,\text{s}$.
(*b*) The maximum charge would be $Q = C\mathscr{E} = (3.0 \times 10^{-7}\,\text{F})(12\,\text{V}) = 3.6\,\mu\text{C}$.
(*c*) In Eq. 6a, we set $Q = 0.99C\mathscr{E}$:

$$0.99C\mathscr{E} = C\mathscr{E}\left(1 - e^{-t/RC}\right),$$

or

$$e^{-t/RC} = 1 - 0.99 = 0.01.$$

Then

$$\frac{t}{RC} = -\ln(0.01) = 4.6$$

so

$$t = 4.6RC = 28 \times 10^{-3}\,\text{s}$$

or 28 ms (less than $\frac{1}{30}$ s).
(*d*) From part (*b*) the maximum charge is $3.6\,\mu\text{C}$. When the charge is half this value, $1.8\,\mu\text{C}$, the current I in the circuit can be found using the original differential equation, or Eq. 5:

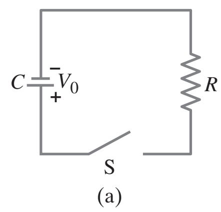
$$I = \frac{1}{R}\left(\mathscr{E} - \frac{Q}{C}\right) = \frac{1}{2.0 \times 10^4\,\Omega}\left(12\,\text{V} - \frac{1.8 \times 10^{-6}\,\text{C}}{0.30 \times 10^{-6}\,\text{F}}\right) = 300\,\mu\text{A}.$$

(*e*) The current is a maximum when there is no charge on the capacitor $(Q = 0)$:

$$I_{\max} = \frac{\mathscr{E}}{R} = \frac{12\,\text{V}}{2.0 \times 10^4\,\Omega} = 600\,\mu\text{A}.$$

(*f*) Again using Eq. 5, now with $I = 0.20I_{\max} = 120\,\mu\text{A}$, we have

$$\begin{aligned} Q &= C(\mathscr{E} - IR) \\ &= (3.0 \times 10^{-7}\,\text{F})\left[12\,\text{V} - (1.2 \times 10^{-4}\,\text{A})(2.0 \times 10^4\,\Omega)\right] = 2.9\,\mu\text{C}. \end{aligned}$$



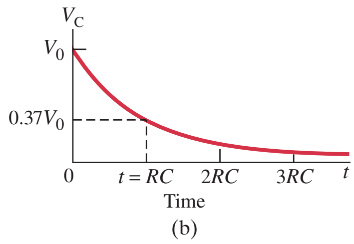


FIGURE 18 For the RC circuit shown in (a), the voltage V_C across the capacitor decreases with time, as shown in (b), after the switch S is closed at $t = 0$. The charge on the capacitor follows the same curve since $V_C \propto Q$.

The circuit just discussed involved the *charging* of a capacitor by a battery through a resistance. Now let us look at another situation: when a capacitor is already charged (say to a voltage V_0), and it is then allowed to *discharge* through a resistance R as shown in Fig. 18a. (In this case there is no battery.) When the switch S is closed, charge begins to flow through resistor R from one side of the capacitor toward the other side, until the capacitor is fully discharged. The voltage across the resistor at any instant equals that across the capacitor:

$$IR = \frac{Q}{C}.$$

The rate at which charge leaves the capacitor equals the negative of the current in the resistor, $I = -dQ/dt$, because the capacitor is discharging (Q is decreasing). So we write the above equation as

$$-\frac{dQ}{dt}R = \frac{Q}{C}.$$

We rearrange this to

$$\frac{dQ}{Q} = -\frac{dt}{RC}$$

and integrate it from $t = 0$ when the charge on the capacitor is Q_0, to some time t later when the charge is Q:

$$\ln\frac{Q}{Q_0} = -\frac{t}{RC}$$

or

$$Q = Q_0 e^{-t/RC}. \tag{9a}$$

The voltage across the capacitor $(V_C = Q/C)$ as a function of time is

$$V_C = V_0 e^{-t/RC}, \tag{9b}$$

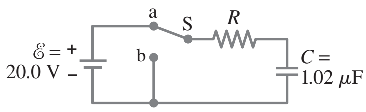
where the initial voltage $V_0 = Q_0/C$. Thus the charge on the capacitor, and the voltage across it, decrease exponentially in time with a time constant RC. This is shown in Fig. 18b. The current is

$$I = -\frac{dQ}{dt} = \frac{Q_0}{RC}e^{-t/RC} = I_0 e^{-t/RC}, \tag{10}$$

and it too is seen to decrease exponentially in time with the same time constant RC. The charge on the capacitor, the voltage across it, and the current in the resistor all decrease to 37% of their original value in one time constant $t = \tau = RC$.

EXERCISE E In 10 time constants, the charge on the capacitor in Fig. 18 will be about (*a*) $Q_0/20{,}000$, (*b*) $Q_0/5000$, (*c*) $Q_0/1000$, (*d*) $Q_0/10$, (*e*) $Q_0/3$?

FIGURE 19 Example 12.



EXAMPLE 12 Discharging *RC* circuit. In the RC circuit shown in Fig. 19, the battery has fully charged the capacitor, so $Q_0 = C\mathscr{E}$. Then at $t = 0$ the switch is thrown from position a to b. The battery emf is 20.0 V, and the capacitance $C = 1.02\,\mu\text{F}$. The current I is observed to decrease to 0.50 of its initial value in $40\,\mu\text{s}$. (*a*) What is the value of Q, the charge on the capacitor, at $t = 0$? (*b*) What is the value of R? (*c*) What is Q at $t = 60\,\mu\text{s}$?

APPROACH At $t = 0$, the capacitor has charge $Q_0 = C\mathscr{E}$, and then the battery is removed from the circuit and the capacitor begins discharging through the resistor, as in Fig. 18. At any time t later (Eq. 9a) we have

$$Q = Q_0 e^{-t/RC} = C\mathscr{E}e^{-t/RC}.$$

SOLUTION (*a*) At $t = 0$,

$$Q = Q_0 = C\mathscr{E} = (1.02 \times 10^{-6}\,\mathrm{F})(20.0\,\mathrm{V}) = 2.04 \times 10^{-5}\,\mathrm{C} = 20.4\,\mu\mathrm{C}.$$

(*b*) To find R, we are given that at $t = 40\,\mu\mathrm{s}$, $I = 0.50 I_0$. Hence

$$0.50 I_0 = I_0 e^{-t/RC}.$$

Taking natural logs on both sides $(\ln 0.50 = -0.693)$:

$$0.693 = \frac{t}{RC}$$

so

$$R = \frac{t}{(0.693)C} = \frac{(40 \times 10^{-6}\,\mathrm{s})}{(0.693)(1.02 \times 10^{-6}\,\mathrm{F})} = 57\,\Omega.$$

(*c*) At $t = 60\,\mu\mathrm{s}$,

$$Q = Q_0 e^{-t/RC} = (20.4 \times 10^{-6}\,\mathrm{C})e^{-\frac{60 \times 10^{-6}\,\mathrm{s}}{(57\,\Omega)(1.02 \times 10^{-6}\,\mathrm{F})}} = 7.3\,\mu\mathrm{C}.$$

CONCEPTUAL EXAMPLE 13 **Bulb in *RC* circuit.** In the circuit of Fig. 20, the capacitor is originally uncharged. Describe the behavior of the lightbulb from the instant switch S is closed until a long time later.

RESPONSE When the switch is first closed, the current in the circuit is high and the lightbulb burns brightly. As the capacitor charges, the voltage across the capacitor increases causing the current to be reduced, and the lightbulb dims. As the potential difference across the capacitor approaches the same voltage as the battery, the current decreases toward zero and the lightbulb goes out.

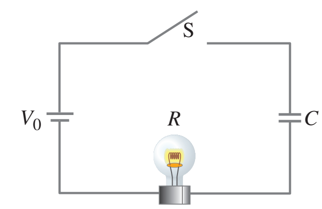


FIGURE 20 Example 13.

*Applications of *RC* Circuits

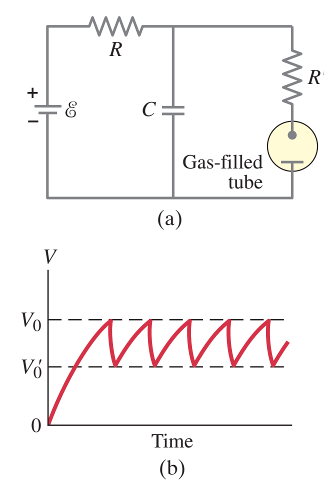
The charging and discharging in an RC circuit can be used to produce voltage pulses at a regular frequency. The charge on the capacitor increases to a particular voltage, and then discharges. One way of initiating the discharge of the capacitor is by the use of a gas-filled tube which has an electrical breakdown when the voltage across it reaches a certain value V_0. After the discharge is finished, the tube no longer conducts current and the recharging process repeats itself, starting at a lower voltage V_0'. Figure 21 shows a possible circuit, and the "sawtooth" voltage it produces.

A simple blinking light can be an application of a sawtooth oscillator circuit. Here the emf is supplied by a battery; the neon bulb flashes on at a rate of perhaps 1 cycle per second. The main component of a "flasher unit" is a moderately large capacitor.

The intermittent windshield wipers of a car can also use an RC circuit. The RC time constant, which can be changed using a multi-positioned switch for different values of R with fixed C, determines the rate at which the wipers come on.

PHYSICS APPLIED
Sawtooth, blinkers, windshield wipers

FIGURE 21 (a) An RC circuit, coupled with a gas-filled tube as a switch, can produce a repeating "sawtooth" voltage, as shown in (b).



EXAMPLE 14 **ESTIMATE** **Resistor in a turn signal.** Estimate the order of magnitude of the resistor in a turn-signal circuit.

APPROACH A typical turn signal flashes perhaps twice per second, so the time constant is on the order of 0.5 s. A moderate capacitor might have $C = 1\,\mu\mathrm{F}$.

SOLUTION Setting $\tau = RC = 0.5\,\mathrm{s}$, we find

$$R = \frac{\tau}{C} = \frac{0.5\,\mathrm{s}}{1 \times 10^{-6}\,\mathrm{F}} \approx 500\,\mathrm{k}\Omega.$$

PHYSICS APPLIED
Heart pacemaker

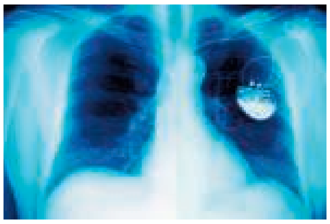


FIGURE 22 Electronic battery-powered pacemaker can be seen on the rib cage in this X-ray.

An interesting medical use of an *RC* circuit is the electronic heart pacemaker, which can make a stopped heart start beating again by applying an electric stimulus through electrodes attached to the chest. The stimulus can be repeated at the normal heartbeat rate if necessary. The heart itself contains *pacemaker* cells, which send out tiny electric pulses at a rate of 60 to 80 per minute. These signals induce the start of each heartbeat. In some forms of heart disease, the natural pacemaker fails to function properly, and the heart loses its beat. Such patients use *electronic pacemakers* which produce a regular voltage pulse that starts and controls the frequency of the heartbeat. The electrodes are implanted in or near the heart (Fig. 22), and the circuit contains a capacitor and a resistor. The charge on the capacitor increases to a certain point and then discharges a pulse to the heart. Then it starts charging again. The pulsing rate depends on the values of R and C.

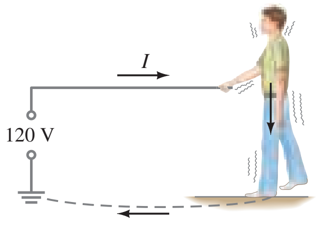
6 Electric Hazards

PHYSICS APPLIED
Dangers of electricity

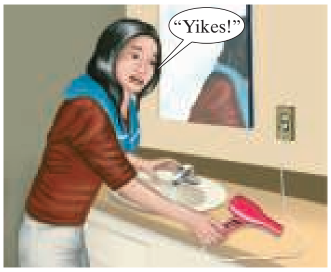
Excess electric current can heat wires in buildings and cause fires. Electric current can also damage the human body or even be fatal. Electric current through the human body can cause damage in two ways: (1) Electric current heats tissue and can cause burns; (2) electric current stimulates nerves and muscles, and we feel a "shock." The severity of a shock depends on the magnitude of the current, how long it acts, and through what part of the body it passes. A current passing through vital organs such as the heart or brain is especially serious for it can interfere with their operation.

Most people can "feel" a current of about 1 mA. Currents of a few mA cause pain but rarely cause much damage in a healthy person. Currents above 10 mA cause severe contraction of the muscles, and a person may not be able to let go of the source of the current (say, a faulty appliance or wire). Death from paralysis of the respiratory system can occur. Artificial respiration, however, can sometimes revive a victim. If a current above about 80 to 100 mA passes across the torso, so that a portion passes through the heart for more than a second or two, the heart muscles will begin to contract irregularly and blood will not be properly pumped. This condition is called **ventricular fibrillation**. If it lasts for long, death results. Strangely enough, if the current is much larger, on the order of 1 A, death by heart failure may be less likely,† but such currents can cause serious burns, especially if concentrated through a small area of the body.

FIGURE 23 A person receives an electric shock when the circuit is completed.



(a)



(b)

The seriousness of a shock depends on the applied voltage and on the effective resistance of the body. Living tissue has low resistance since the fluid of cells contains ions that can conduct quite well. However, the outer layer of skin, when dry, offers high resistance and is thus protective. The effective resistance between two points on opposite sides of the body when the skin is dry is in the range of 10^4 to $10^6\ \Omega$. But when the skin is wet, the resistance may be $10^3\ \Omega$ or less. A person who is barefoot or wearing thin-soled shoes will be in good contact with the ground, and touching a 120-V line with a wet hand can result in a current

$$I = \frac{120\text{ V}}{1000\ \Omega} = 120\text{ mA}.$$

As we saw, this could be lethal.

A person who has received a shock has become part of a complete circuit. Figure 23 shows two ways the circuit might be completed when a person

†Larger currents apparently bring the entire heart to a standstill. Upon release of the current, the heart returns to its normal rhythm. This may not happen when fibrillation occurs because, once started, it can be hard to stop. Fibrillation may also occur as a result of a heart attack or during heart surgery. A device known as a *defibrillator* can apply a brief high current to the heart, causing complete heart stoppage which is often followed by resumption of normal beating.

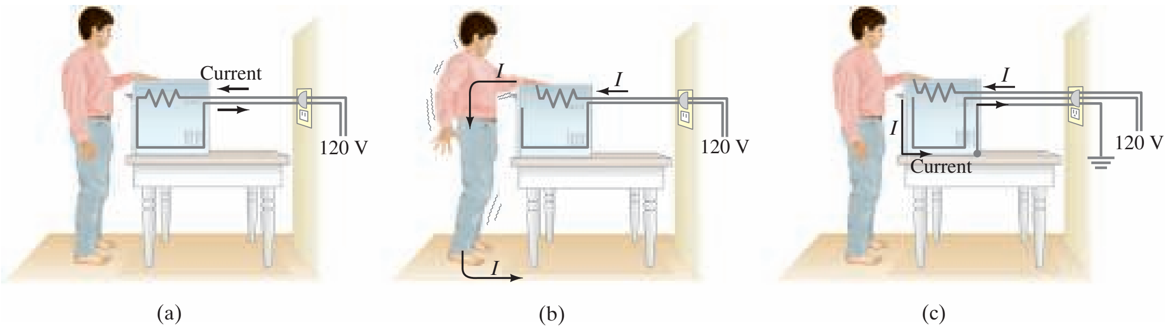


FIGURE 24 (a) An electric oven operating normally with a 2-prong plug. (b) Short to the case with ungrounded case: shock. (c) Short to the case with the case grounded by a 3-prong plug.

accidentally touches a "hot" electric wire—"hot" meaning a high potential such as 120 V (normal U.S. household voltage) relative to ground. The other wire of building wiring is connected to ground—either by a wire connected to a buried conductor, or via a metal water pipe into the ground. In Fig. 23a, the current passes from the high-voltage wire, through the person, to the ground through his bare feet, and back along the ground (a fair conductor) to the ground terminal of the source. If the person stands on a good insulator—thick rubber-soled shoes or a dry wood floor—there will be much more resistance in the circuit and consequently much less current through the person. If the person stands with bare feet on the ground, or is in a bathtub, there is lethal danger because the resistance is much less and the current greater. In a bathtub (or swimming pool), not only are you wet, which reduces your resistance, but the water is in contact with the drain pipe (typically metal) that leads to the ground. It is strongly recommended that you not touch anything electrical when wet or in bare feet. Building codes that require the use of non-metal pipes would be protective.

In Fig. 23b, a person touches a faulty "hot" wire with one hand, and the other hand touches a sink faucet (connected to ground via the pipe). The current is particularly dangerous because it passes across the chest, through the heart and lungs. A useful rule: if one hand is touching something electrical, keep your other hand in your pocket (don't use it!), and wear thick rubber-soled shoes. It is also a good idea to remove metal jewelry, especially rings (your finger is usually moist under a ring).

CAUTION
Keep one hand in your pocket when other touches electricity

You can come into contact with a hot wire by touching a bare wire whose insulation has worn off, or from a bare wire inside an appliance when you're tinkering with it. (Always unplug an electrical device before investigating† its insides!) Another possibility is that a wire inside a device may break or lose its insulation and come in contact with the case. If the case is metal, it will conduct electricity. A person could then suffer a severe shock merely by touching the case, as shown in Fig. 24b. To prevent an accident, metal cases are supposed to be connected directly to ground by a separate ground wire. Then if a "hot" wire touches the grounded case, a short circuit to ground immediately occurs internally, as shown in Fig. 24c, and most of the current passes through the low-resistance ground wire rather than through the person. Furthermore, the high current should open the fuse or circuit breaker. Grounding a metal case is done by a separate ground wire connected to the third (round) prong of a 3-prong plug. Never cut off the third prong of a plug—it could save your life.

PHYSICS APPLIED
Grounding and shocks

†Even then you can get a bad shock from a capacitor that hasn't been discharged until you touch it.

A three-prong plug, and an adapter, are shown in Figs. 25a and b.

Why is a third wire needed? The 120 V is carried by the other two wires—one **hot** (120 V ac), the other **neutral**, which is itself grounded. The third "dedicated" ground wire with the round prong may seem redundant. But it is protection for two reasons: (1) it protects against internal wiring that may have been done incorrectly; (2) the neutral wire carries normal current ("return" current from the 120 V) and it does have resistance; so there can be a voltage drop along it—normally small, but if connections are poor or corroded, or the plug is loose, the resistance could be large enough that you might feel that voltage if you touched the neutral wire some distance from its grounding point.

Some electrical devices come with only two wires, and the plug's two prongs are of different widths; the plug can be inserted only one way into the outlet so that the intended neutral (wider prong) in the device is connected to neutral in the wiring (Fig. 25c). For example, the screw threads of a lightbulb are meant to be connected to neutral (and the base contact to hot), to avoid shocks when changing a bulb in a possibly protruding socket. Devices with 2-prong plugs do *not* have their cases grounded; they are supposed to have double electric insulation. Take extra care anyway.

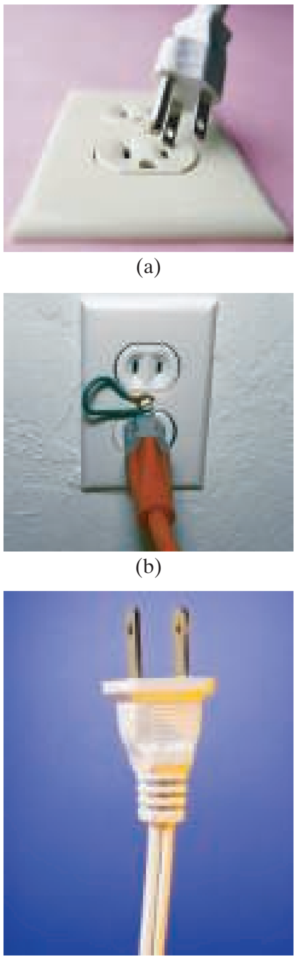
The insulation on a wire may be color coded. Hand-held meters may have red (hot) and black (ground) lead wires. But in a house, black is usually hot (or it may be red), whereas white is neutral and green is the dedicated ground, Fig. 26. But beware: these color codes cannot always be trusted. [In the U.S., three wires normally enter a house: two *hot* wires at 120 V each (which add together to 240 V for appliances or devices that run on 240 V) plus the grounded *neutral* (carrying return current for the two hots). See Fig. 26. The "dedicated" *ground* wire (non-current carrying) is a fourth wire that does not come from the electric company but enters the house from a nearby heavy stake in the ground or a buried metal pipe. The two hot wires can feed separate 120-V circuits in the house, so each 120-V circuit inside the house has only three wires, including ground.]

Normal circuit breakers protect equipment and buildings from overload and fires. They protect humans only in some circumstances, such as the very high currents that result from a short, if they respond quickly enough. *Ground fault circuit interrupters* (GFCI) are designed to protect people from the much lower currents (10 mA to 100 mA) that are lethal but would not throw a 15-A circuit breaker or blow a 20-A fuse.

It is current that harms, but it is voltage that drives the current. 30 volts is sometimes said to be the threshhold for danger. But even a 12-V car battery (which can supply large currents) can cause nasty burns and shock.

Another danger is **leakage current**, by which we mean a current along an unintended path. Leakage currents are often "capacitively coupled." For example, a wire in a lamp forms a capacitor with the metal case; charges moving in one conductor attract or repel charge in the other, so there is a current. Typical electrical codes limit leakage currents to 1 mA for any device. A 1-mA leakage current is usually harmless. It can be very dangerous, however, to a hospital patient with implanted electrodes connected to ground through the apparatus. This is due to the absence of the protective skin layer and because the current can pass directly through the heart as compared to the usual situation where the current enters at the hands and spreads out through the body. Although 100 mA may be needed to cause heart fibrillation when entering through the hands (very little of it actually passes through the heart), as little as 0.02 mA has been known to cause fibrillation when passing directly to the heart. Thus, a "wired" patient is in considerable danger from leakage current even from as simple an act as touching a lamp.

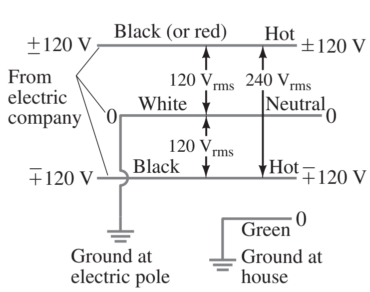
Finally, don't touch a downed power line (lethal!) or even get near it. A hot power line is at thousands of volts. A huge current can flow along the ground or pavement, from where the high-voltage wire touches the ground along its path to the grounding point of the neutral line, enough that the voltage between your two feet could be large. Tip: stand on one foot or run (only one foot touching the ground at a time).



Photodisc/Getty Images
(a)
William E. Ferguson
(b)
GreenStockCreative/Shutterstock
(c)

FIGURE 25 (a) A 3-prong plug, and (b) an adapter (gray) for old-fashioned 2-prong outlets—be sure to screw down the ground tab. (c) A polarized 2-prong plug.

FIGURE 26 Four wires entering a typical house. The color codes for wires are not always as shown here—be careful!

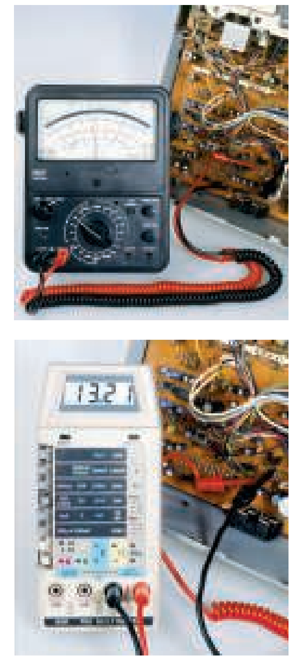


*7 Ammeters and Voltmeters

An **ammeter** is used to measure current, and a **voltmeter** measures potential difference or voltage. Measurements of current and voltage are made with meters that are of two types: (1) *analog* meters, which display numerical values by the position of a pointer that can move across a scale (Fig. 27a); and (2) *digital* meters, which display the numerical value in numbers (Fig. 27b). We now discuss the meters themselves and how they work, then how they are connected to circuits to make measurements. Finally we will discuss how using meters affects the circuit being measured, possibly causing erroneous results—and what to do about it.

PHYSICS APPLIED

DC meters



(a) Paul Silverman/Fundamental Photographs, NYC

(b) Paul Silverman/Fundamental Photographs, NYC

FIGURE 27 (a) An analog multimeter being used as a voltmeter. (b) An electronic digital meter.

*Analog Ammeters and Voltmeters

The crucial part of an analog ammeter or voltmeter, in which the reading is by a pointer on a scale (Fig. 27a), is a *galvanometer*. The galvanometer works on the principle of the force between a magnetic field and a current-carrying coil of wire. For now, we merely need to know that the deflection of the needle of a galvanometer is proportional to the current flowing through it. The *full-scale current sensitivity* of a galvanometer, I_m, is the electric current needed to make the needle deflect full scale.

A galvanometer can be used directly to measure small dc currents. For example, a galvanometer whose sensitivity I_m is 50 μA can measure currents from about 1 μA (currents smaller than this would be hard to read on the scale) up to 50 μA. To measure larger currents, a resistor is placed in parallel with the galvanometer. Thus, an analog **ammeter**, represented by the symbol •Ⓐ•, consists of a galvanometer (•Ⓖ•) in parallel with a resistor called the **shunt resistor**, as shown in Fig. 28. ("Shunt" is a synonym for "in parallel.") The shunt resistance is R_{sh}, and the resistance of the galvanometer coil, through which current passes, is r. The value of R_{sh} is chosen according to the full-scale deflection desired; R_{sh} is normally very small—giving an ammeter a very small net resistance—so most of the current passes through R_{sh} and very little ($\lesssim$ 50 μA) passes through the galvanometer to deflect the needle.

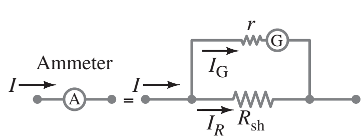


FIGURE 28 An ammeter is a galvanometer in parallel with a (shunt) resistor with low resistance, R_{sh}.

EXAMPLE 15 **Ammeter design.** Design an ammeter to read 1.0 A at full scale using a galvanometer with a full-scale sensitivity of 50 μA and a resistance $r = 30\,\Omega$. Check if the scale is linear.

APPROACH Only $50\,\mu\text{A}\;(= I_G = 0.000050\text{ A})$ of the 1.0-A current must pass through the galvanometer to give full-scale deflection. The rest of the current $(I_R = 0.999950\text{ A})$ passes through the small shunt resistor, R_{sh}, Fig. 28. The potential difference across the galvanometer equals that across the shunt resistor (they are in parallel). We apply Ohm's law to find R_{sh}.

SOLUTION Because $I = I_G + I_R$, when $I = 1.0$ A flows into the meter, we want I_R through the shunt resistor to be $I_R = 0.999950$ A. The potential difference across the shunt is the same as across the galvanometer, so Ohm's law tells us

$$I_R R_{sh} = I_G r;$$

then

$$R_{sh} = \frac{I_G r}{I_R} = \frac{(5.0 \times 10^{-5}\text{ A})(30\,\Omega)}{(0.999950\text{ A})} = 1.5 \times 10^{-3}\,\Omega,$$

or 0.0015 Ω. The shunt resistor must thus have a very low resistance and most of the current passes through it.

Because $I_G = I_R(R_{sh}/r)$ and (R_{sh}/r) is constant, we see that the scale is linear.

An analog **voltmeter** (-Ⓥ-) also consists of a galvanometer and a resistor. But the resistor R_{ser} is connected in series, Fig. 29, and it is usually large, giving a voltmeter a high internal resistance.

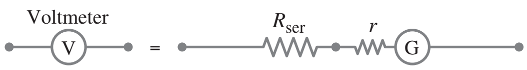


FIGURE 29 A voltmeter is a galvanometer in series with a resistor with high resistance, R_{ser}.

EXAMPLE 16 Voltmeter design. Using a galvanometer with internal resistance $r = 30\,\Omega$ and full-scale current sensitivity of $50\,\mu A$, design a voltmeter that reads from 0 to 15 V. Is the scale linear?

APPROACH When a potential difference of 15 V exists across the terminals of our voltmeter, we want $50\,\mu A$ to be passing through it so as to give a full-scale deflection.

SOLUTION From Ohm's law, $V = IR$, we have (see Fig. 29)

$$15\,\text{V} = (50\,\mu\text{A})(r + R_{ser}),$$

so

$$R_{ser} = \frac{15\,\text{V}}{5.0 \times 10^{-5}\,\text{A}} - r = 300\,\text{k}\Omega - 30\,\Omega = 300\,\text{k}\Omega.$$

Notice that $r = 30\,\Omega$ is so small compared to the value of R_{ser} that it doesn't influence the calculation significantly. The scale will again be linear: if the voltage to be measured is 6.0 V, the current passing through the voltmeter will be $(6.0\,\text{V})/(3.0 \times 10^5\,\Omega) = 2.0 \times 10^{-5}\,\text{A}$, or $20\,\mu A$. This will produce two-fifths of full-scale deflection, as required $(6.0\,\text{V}/15.0\,\text{V} = 2/5)$.

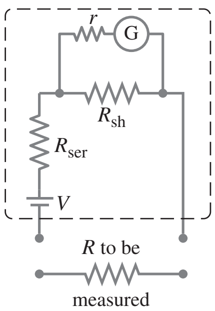


FIGURE 30 An ohmmeter.

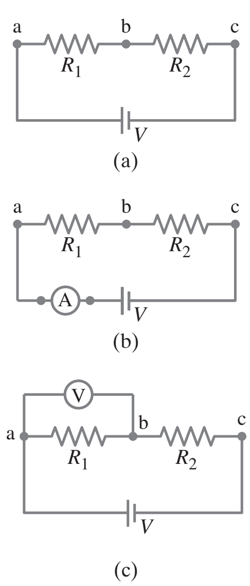
The meters just described are for direct current. A dc meter can be modified to measure ac (alternating current) with the addition of diodes, which allow current to flow in one direction only. An ac meter can be calibrated to read rms or peak values.

Voltmeters and ammeters can have several series or shunt resistors to offer a choice of range. **Multimeters** can measure voltage, current, and resistance. Sometimes a multimeter is called a VOM (Volt-Ohm-Meter or Volt-Ohm-Milliammeter).

An **ohmmeter** measures resistance, and must contain a battery of known voltage connected in series to a resistor (R_{ser}) and to an ammeter (Fig. 30). The resistor whose resistance is to be measured completes the circuit. The needle deflection is inversely proportional to the resistance. The scale calibration depends on the value of the series resistor. Because an ohmmeter sends a current through the device whose resistance is to be measured, it should not be used on very delicate devices that could be damaged by the current.

The **sensitivity** of a meter is generally specified on the face. It may be given as so many ohms per volt, which indicates how many ohms of resistance there are in the meter per volt of full-scale reading. For example, if the sensitivity is $30{,}000\,\Omega/\text{V}$, this means that on the 10-V scale the meter has a resistance of $300{,}000\,\Omega$, whereas on a 100-V scale the meter resistance is $3\,\text{M}\Omega$. The full-scale current sensitivity, I_m, discussed earlier, is just the reciprocal of the sensitivity in Ω/V.

FIGURE 31 Measuring current and voltage.



* How to Connect Meters

Suppose you wish to determine the current I in the circuit shown in Fig. 31a, and the voltage V across the resistor R_1. How exactly are ammeters and voltmeters connected to the circuit being measured?

Because an ammeter is used to measure the current flowing in the circuit, it must be inserted directly into the circuit, in series with the other elements, as shown in Fig. 31b. The smaller its internal resistance, the less it affects the circuit.

A voltmeter, on the other hand, is connected "externally," in parallel with the circuit element across which the voltage is to be measured. It is used to measure the potential difference between two points. Its two wire leads (connecting wires) are connected to the two points, as shown in Fig. 31c, where the voltage across R_1 is being measured. The larger its internal resistance, $(R_{ser} + r)$ in Fig. 29, the less it affects the circuit being measured.

*Effects of Meter Resistance

It is important to know the sensitivity of a meter, for in many cases the resistance of the meter can seriously affect your results. Take the following Example.

PHYSICS APPLIED
Correcting for meter resistance

EXAMPLE 17 **Voltage reading versus true voltage.** Suppose you are testing an electronic circuit which has two resistors, R_1 and R_2, each $15\,\mathrm{k\Omega}$, connected in series as shown in Fig. 32a. The battery maintains 8.0 V across them and has negligible internal resistance. A voltmeter whose sensitivity is $10{,}000\,\Omega/\mathrm{V}$ is put on the 5.0-V scale. What voltage does the meter read when connected across R_1, Fig. 32b, and what error is caused by the finite resistance of the meter?

APPROACH The meter acts as a resistor in parallel with R_1. We use parallel and series resistor analyses and Ohm's law to find currents and voltages.

SOLUTION On the 5.0-V scale, the voltmeter has an internal resistance of $(5.0\,\mathrm{V})(10{,}000\,\Omega/\mathrm{V}) = 50{,}000\,\Omega$. When connected across R_1, as in Fig. 32b, we have this $50\,\mathrm{k\Omega}$ in parallel with $R_1 = 15\,\mathrm{k\Omega}$. The net resistance R_{eq} of these two is given by

$$\frac{1}{R_{\mathrm{eq}}} = \frac{1}{50\,\mathrm{k\Omega}} + \frac{1}{15\,\mathrm{k\Omega}} = \frac{13}{150\,\mathrm{k\Omega}};$$

so $R_{\mathrm{eq}} = 11.5\,\mathrm{k\Omega}$. This $R_{\mathrm{eq}} = 11.5\,\mathrm{k\Omega}$ is in series with $R_2 = 15\,\mathrm{k\Omega}$, so the total resistance of the circuit is now $26.5\,\mathrm{k\Omega}$ (instead of the original $30\,\mathrm{k\Omega}$). Hence the current from the battery is

$$I = \frac{8.0\,\mathrm{V}}{26.5\,\mathrm{k\Omega}} = 3.0 \times 10^{-4}\,\mathrm{A} = 0.30\,\mathrm{mA}.$$

Then the voltage drop across R_1, which is the same as that across the voltmeter, is $(3.0 \times 10^{-4}\,\mathrm{A})(11.5 \times 10^3\,\Omega) = 3.5\,\mathrm{V}$. [The voltage drop across R_2 is $(3.0 \times 10^{-4}\,\mathrm{A})(15 \times 10^3\,\Omega) = 4.5\,\mathrm{V}$, for a total of 8.0 V.] If we assume the meter is precise, it will read 3.5 V. In the original circuit, without the meter, $R_1 = R_2$ so the voltage across R_1 is half that of the battery, or 4.0 V. Thus the voltmeter, because of its internal resistance, gives a low reading. In this case it is off by 0.5 V, or more than 10%.

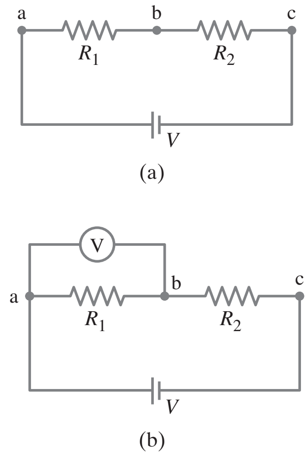


FIGURE 32 Example 17.

Example 17 illustrates how seriously a meter can affect a circuit and give a misleading reading. If the resistance of a voltmeter is much higher than the resistance of the circuit, however, it will have little effect and its readings can be trusted, at least to the manufactured precision of the meter, which for ordinary analog meters is typically 3% to 4% of full-scale deflection. An ammeter also can interfere with a circuit, but the effect is minimal if its resistance is much less than that of the circuit as a whole. For both voltmeters and ammeters, the more sensitive the galvanometer, the less effect it will have. A $50{,}000\text{-}\Omega/\mathrm{V}$ meter is far better than a $1000\text{-}\Omega/\mathrm{V}$ meter.

*Digital Meters

Digital meters (see Fig. 27b) are used in the same way as analog meters: they are inserted directly into the circuit, in series, to measure current (Fig. 31b), and connected "outside," in parallel with the circuit, to measure voltage (Fig. 31c).

The internal construction of digital meters, however, is different from that of analog meters in that digital meters do not use a galvanometer. The electronic circuitry and digital readout are more sensitive than a galvanometer, and have less effect on the circuit to be measured. When we measure dc voltages, a digital meter's resistance is very high, commonly on the order of 10 to $100\,\mathrm{M\Omega}$ $(10^7\text{–}10^8\,\Omega)$, and doesn't change significantly when different voltage scales are selected. A $100\text{-}\mathrm{M\Omega}$ digital meter draws off very little current when connected across even a $1\text{-}\mathrm{M\Omega}$ resistance.

The precision of digital meters is exceptional, often one part in 10^4 $(=0.01\%)$ or better. This precision is not the same as accuracy, however. A precise meter of internal resistance $10^8\,\Omega$ will not give accurate results if used to measure a voltage across a $10^8\text{-}\Omega$ resistor—in which case it is necessary to do a calculation like that in Example 17.

Whenever we make a measurement on a circuit, to some degree we affect that circuit (Example 17). This is true for other types of measurement as well: when we make a measurement on a system, we affect that system in some way. On a temperature measurement, for example, the thermometer can exchange heat with the system, thus altering its temperature. It is important to be able to make needed corrections, as we saw in Example 17.

Summary

A device that transforms another type of energy into electrical energy is called a **source** of **emf**. A battery behaves like a source of emf in series with an **internal resistance**. The emf is the potential difference determined by the chemical reactions in the battery and equals the terminal voltage when no current is drawn. When a current is drawn, the voltage at the battery's terminals is less than its emf by an amount equal to the potential decrease Ir across the internal resistance.

When resistances are connected in **series** (end to end in a single linear path), the equivalent resistance is the sum of the individual resistances:

$$R_{eq} = R_1 + R_2 + \cdots. \tag{3}$$

In a series combination, R_{eq} is greater than any component resistance.

When resistors are connected in **parallel**, the reciprocal of the equivalent resistance equals the sum of the reciprocals of the individual resistances:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots. \tag{4}$$

In a parallel connection, the net resistance is less than any of the individual resistances.

Kirchhoff's rules are helpful in determining the currents and voltages in circuits. Kirchhoff's **junction rule** is based on conservation of electric charge and states that the sum of all currents entering any junction equals the sum of all currents leaving that junction. The second, or **loop rule**, is based on conservation of energy and states that the algebraic sum of the changes in potential around any closed path of the circuit must be zero.

When an ***RC* circuit** containing a resistor R in series with a capacitance C is connected to a dc source of emf, the voltage across the capacitor rises gradually in time characterized by an exponential of the form $\left(1 - e^{-t/RC}\right)$, where the **time constant**,

$$\tau = RC, \tag{7}$$

is the time it takes for the voltage to reach 63 percent of its maximum value. The current through the resistor decreases as $e^{-t/RC}$.

A capacitor discharging through a resistor is characterized by the same time constant: in a time $\tau = RC$, the voltage across the capacitor drops to 37 percent of its initial value. The charge on the capacitor, and voltage across it, decreases as $e^{-t/RC}$, as does the current.

Electric shocks are caused by current passing through the body. To avoid shocks, the body must not become part of a complete circuit by allowing different parts of the body to touch objects at different potentials. Commonly, shocks are caused by one part of the body touching ground and another part touching a high electric potential.

[*An **ammeter** measures current. An analog ammeter consists of a galvanometer and a parallel **shunt resistor** that carries most of the current. An analog **voltmeter** consists of a galvanometer and a series resistor. An ammeter is inserted *into* the circuit whose current is to be measured. A voltmeter is external, being connected in parallel to the element whose voltage is to be measured. Digital voltmeters have greater internal resistance and affect the circuit to be measured less than do analog meters.]

Answers to Exercises

A: (*a*) 1.14 A; (*b*) 11.4 V; (*c*) $P_R = 13.1$ W, $P_r = 0.65$ W.

B: 6 Ω and 25 Ω.

C: $41I_3 - 45 + 21I_2 - 80 = 0$.

D: 180 A; this high current through the batteries could cause them to become very hot: the power dissipated in the weak battery would be $P = I^2r = (180\text{ A})^2(0.10\,\Omega) = 3200$ W!

E: (*a*).