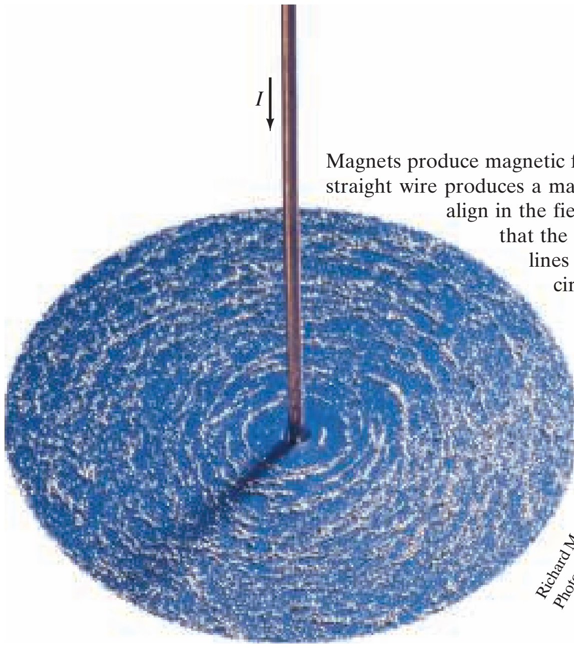


Richard Megna/Fundamental Photographs, NYC

Magnets produce magnetic fields, but so do electric currents. An electric current flowing in this straight wire produces a magnetic field which causes the tiny pieces of iron (iron "filings") to align in the field. We shall see in this Chapter how magnetic field is defined, and that the magnetic field direction is along the iron filings. The magnetic field lines due to the electric current in this long wire are in the shape of circles around the wire.

We also discuss how magnetic fields exert forces on electric currents and on charged particles, as well as useful applications of the interaction between magnetic fields and electric currents and moving electric charges.

Magnetism

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.*]

Which of the following can experience a force when placed in the magnetic field of a magnet?

(a) An electric charge at rest.
(b) An electric charge moving.
(c) An electric current in a wire.
(d) Another magnet.

The history of magnetism begins thousands of years ago. In a region of Asia Minor known as Magnesia, rocks were found that could attract each other. These rocks were called "magnets" after their place of discovery.

Not until the nineteenth century, however, was it seen that magnetism and electricity are closely related. A crucial discovery was that electric currents produce magnetic effects (we will say "magnetic fields") like magnets do. All kinds of practical devices depend on magnetism, from compasses to motors, loudspeakers, computer memory, and electric generators.

CONTENTS

1 Magnets and Magnetic Fields

We have all observed a magnet attract paper clips, nails, and other objects made of iron, Fig. 1. Any magnet, whether it is in the shape of a bar or a horseshoe, has two ends or faces, called **poles**, which is where the magnetic effect is strongest. If a bar magnet is suspended from a fine thread, it is found that one pole of the magnet will always point toward the north. It is not known for sure when this fact was discovered, but it is known that the Chinese were making use of it as an aid to navigation by the eleventh century and perhaps earlier. This is the principle of a compass.

Michael Newman/PhotoEdit, Inc.

FIGURE 1 A horseshoe magnet attracts iron tacks and paper clips.

Note: Sections marked with an asterisk (*) may be considered optional by the instructor.

From Chapter 27 of *Physics for Scientists & Engineers with Modern Physics*, Fourth Edition, Douglas C. Giancoli.

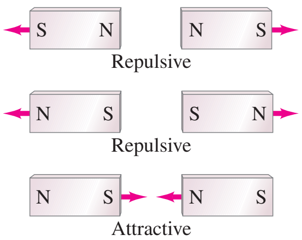
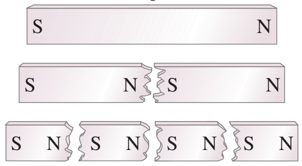


FIGURE 2 Like poles of a magnet repel; unlike poles attract. Red arrows indicate force direction.

FIGURE 3 If you split a magnet, you won't get isolated north and south poles; instead, two new magnets are produced, each with a north and a south pole.



CAUTION
Magnets do not attract all metals

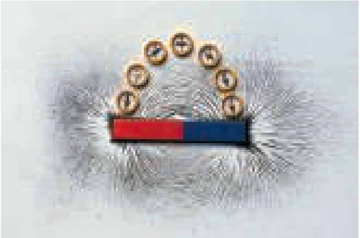
A compass needle is simply a bar magnet which is supported at its center of gravity so that it can rotate freely. The pole of a freely suspended magnet that points toward geographic north is called the **north pole** of the magnet. The other pole points toward the south and is called the **south pole**.

It is a familiar observation that when two magnets are brought near one another, each exerts a force on the other. The force can be either attractive or repulsive and can be felt even when the magnets don't touch. If the north pole of one bar magnet is brought near the north pole of a second magnet, the force is repulsive. Similarly, if the south poles of two magnets are brought close, the force is repulsive. But when a north pole is brought near the south pole of another magnet, the force is attractive. These results are shown in Fig. 2, and are reminiscent of the forces between electric charges: like poles repel, and unlike poles attract. *But do not confuse magnetic poles with electric charge*. They are very different. One important difference is that a positive or negative electric charge can easily be isolated. But an isolated single magnetic pole has never been observed. If a bar magnet is cut in half, you do not obtain isolated north and south poles. Instead, two new magnets are produced, Fig. 3, each with north (N) and south (S) poles. If the cutting operation is repeated, more magnets are produced, each with a north and a south pole. Physicists have searched for isolated single magnetic poles (monopoles), but no **magnetic monopole** has ever been observed.

Only iron and a few other materials, such as cobalt, nickel, gadolinium, and some of their oxides and alloys, show strong magnetic effects. They are said to be **ferromagnetic** (from the Latin word *ferrum* for iron). Other materials show some slight magnetic effect, but it is very weak and can be detected only with delicate instruments.

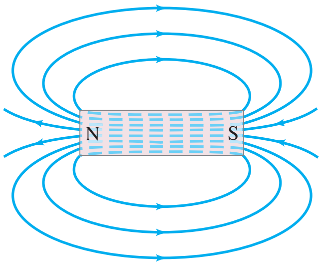
As when picturing an electric field surrounding an electric charge, in a similar way, we can picture a **magnetic field** surrounding a magnet. The force one magnet exerts on another can then be described as the interaction between one magnet and the magnetic field of the other. Just as we drew electric field lines, we can also draw **magnetic field lines**. They can be drawn, as for electric field lines, so that (1) the direction of the magnetic field is tangent to a field line at any point, and (2) the number of lines per unit area is proportional to the strength of the magnetic field.

FIGURE 4 (a) Visualizing magnetic field lines around a bar magnet, using iron filings and compass needles. The red end of the bar magnet is its north pole. The N pole of a nearby compass needle points away from the north pole of the magnet. (b) Magnetic field lines for a bar magnet.



Stephen Oliver/Dorling Kindersley Media Library

(a)



(b)

The *direction* of the magnetic field at a given point can be defined as the direction that the north pole of a compass needle would point if placed at that point. (A more precise definition will be given in Section 3.) Figure 4a shows how thin iron filings (acting like tiny magnets) reveal the magnetic field lines by lining up like the compass needles. The magnetic field determined in this way for the field surrounding a bar magnet is shown in Fig. 4b. Notice that because of our definition, the lines always point out from the north pole and in toward the south pole of a magnet (the north pole of a magnetic compass needle is attracted to the south pole of the magnet).

CAUTION
Magnetic field lines form closed loops, unlike electric field lines

Magnetic field lines continue inside a magnet, as indicated in Fig. 4b. Indeed, given the lack of single magnetic poles, magnetic field lines always form closed loops, unlike electric field lines that begin on positive charges and end on negative charges.

Earth's Magnetic Field

The Earth's magnetic field is shown in Fig. 5. The pattern of field lines is as if there were an imaginary bar magnet inside the Earth. Since the north pole (N) of a compass needle points north, the Earth's magnetic pole which is in the geographic north is magnetically a south pole, as indicated in Fig. 5 by the S on the schematic bar magnet inside the Earth. Remember that the north pole of one magnet is attracted to the south pole of another magnet. Nonetheless, Earth's pole in the north is still often called the "north magnetic pole," or "geomagnetic north," simply because it is in the north. Similarly, the Earth's southern magnetic pole, which is near the geographic south pole, is magnetically a north pole (N). The Earth's magnetic poles do not coincide with the *geographic* poles, which are on the Earth's axis of rotation. The north magnetic pole, for example, is in the Canadian Arctic,† about 900 km from the geographic north pole, or "true north." This difference must be taken into account when you use a compass (Fig. 6). The angular difference between magnetic north and true (geographical) north is called the **magnetic declination**. In the U.S. it varies from 0° to about 20°, depending on location.

Notice in Fig. 5 that the Earth's magnetic field at most locations is not tangent to the Earth's surface. The angle that the Earth's magnetic field makes with the horizontal at any point is referred to as the **angle of dip**.

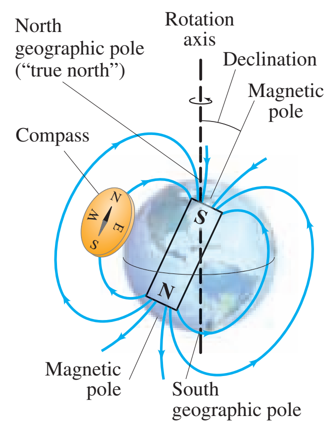


FIGURE 5 The Earth acts like a huge magnet; but its magnetic poles are not at the geographic poles, which are on the Earth's rotation axis.

EXERCISE A Does the Earth's magnetic field have a greater magnitude near the poles or near the equator? [*Hint*: Note the field lines in Fig. 5.]

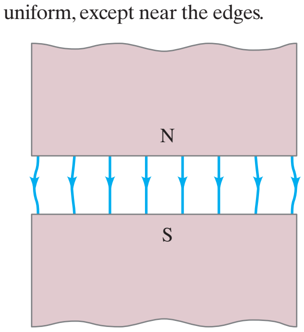
PHYSICS APPLIED
Use of a compass

FIGURE 6 Using a map and compass in the wilderness. First you align the compass case so the needle points away from true north (N) exactly the number of degrees of declination as stated on the map (15° for the place shown on this topographic map of a part of California). Then align the map with true north, as shown, *not* with the compass needle.

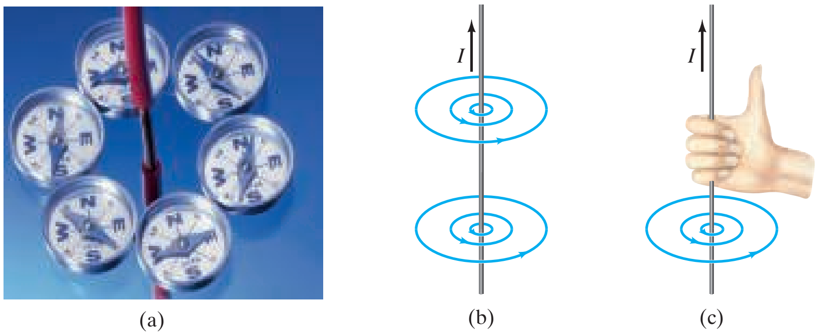
Uniform Magnetic Field

The simplest magnetic field is one that is uniform—it doesn't change in magnitude or direction from one point to another. A perfectly uniform field over a large area is not easy to produce. But the field between two flat parallel pole pieces of a magnet is nearly uniform if the area of the pole faces is large compared to their separation, as shown in Fig. 7. At the edges, the field "fringes" out somewhat: the magnetic field lines are no longer quite parallel and uniform. The parallel evenly spaced field lines in the central region of the gap indicate that the field is uniform at points not too near the edges, much like the electric field between two parallel plates.

FIGURE 7 Magnetic field between two wide poles of a magnet is nearly uniform, except near the edges.



†Magnetic north is moving many kilometers a year at present. Magnetism in rocks suggests that the Earth's poles have not only moved significantly over geologic time, but have also reversed direction 400 times over the last 330 million years.



(a) (b) (c)

Richard Megna/Fundamental Photographs, NYC

FIGURE 8 (a) Deflection of compass needles near a current-carrying wire, showing the presence and direction of the magnetic field. (b) Magnetic field lines around an electric current in a straight wire. See also the Chapter-Opening photo. (c) Right-hand rule for remembering the direction of the magnetic field: when the thumb points in the direction of the conventional current, the fingers wrapped around the wire point in the direction of the magnetic field.

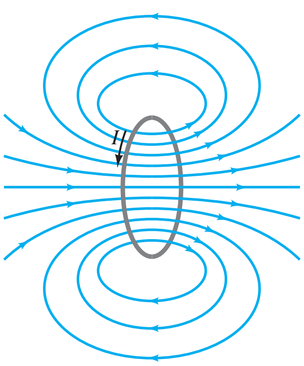


FIGURE 9 Magnetic field lines due to a circular loop of wire.

2 Electric Currents Produce Magnetic Fields

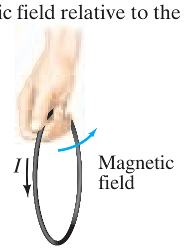
During the eighteenth century, many scientists sought to find a connection between electricity and magnetism. A stationary electric charge and a magnet were shown to have no influence on each other. But in 1820, Hans Christian Oersted (1777–1851) found that when a compass needle is placed near a wire, the needle deflects as soon as the two ends of the wire are connected to the terminals of a battery and the wire carries an electric current. As we have seen, a compass needle is deflected by a magnetic field. So Oersted's experiment showed that **an electric current produces a magnetic field**. He had found a connection between electricity and magnetism.

A compass needle placed near a straight section of current-carrying wire experiences a force, causing the needle to align tangent to a circle around the wire, Fig. 8a. Thus, the magnetic field lines produced by a current in a straight wire are in the form of circles with the wire at their center, Fig. 8b. The direction of these lines is indicated by the north pole of the compasses in Fig. 8a. There is a simple way to remember the direction of the magnetic field lines in this case. It is called a **right-hand rule**: grasp the wire with your right hand so that your thumb points in the direction of the conventional (positive) current; then your fingers will encircle the wire in the direction of the magnetic field, Fig. 8c.

Right-hand-rule 1: magnetic field direction produced by electric current

The magnetic field lines due to a circular loop of current-carrying wire can be determined in a similar way using a compass. The result is shown in Fig. 9. Again the right-hand rule can be used, as shown in Fig. 10. Unlike the uniform field shown in Fig. 7, the magnetic fields shown in Figs. 8 and 9 are *not* uniform—the fields are different in magnitude and direction at different points.

FIGURE 10 Right-hand rule for determining the direction of the magnetic field relative to the current.



EXERCISE B A straight wire carries a current directly toward you. In what direction are the magnetic field lines surrounding the wire?

3 Force on an Electric Current in a Magnetic Field; Definition of $\vec{\mathbf{B}}$

In Section 2 we saw that an electric current exerts a force on a magnet, such as a compass needle. By Newton's third law, we might expect the reverse to be true as well: we should expect that *a magnet exerts a force on a current-carrying wire*. Experiments indeed confirm this effect, and it too was first observed by Oersted.

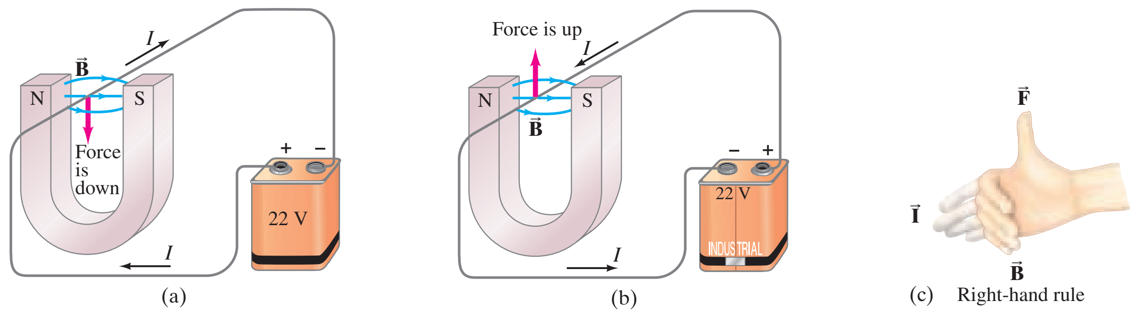


FIGURE 11 (a) Force on a current-carrying wire placed in a magnetic field $\vec{\mathbf{B}}$; (b) same, but current reversed; (c) right-hand rule for setup in (b).

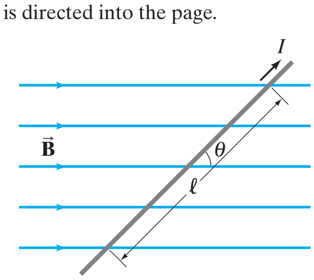
Suppose a straight wire is placed in the magnetic field between the poles of a horseshoe magnet as shown in Fig. 11. When a current flows in the wire, experiment shows that a force is exerted on the wire. But this force is *not* toward one or the other pole of the magnet. Instead, the force is directed at right angles to the magnetic field direction, downward in Fig. 11a. If the current is reversed in direction, the force is in the opposite direction, upward as shown in Fig. 11b. Experiments show that *the direction of the force is always perpendicular to the direction of the current and also perpendicular to the direction of the magnetic field*, $\vec{\mathbf{B}}$.

The direction of the force is given by another **right-hand rule**, as illustrated in Fig. 11c. Orient your right hand until your outstretched fingers can point in the direction of the conventional current I, and when you bend your fingers they point in the direction of the magnetic field lines, $\vec{\mathbf{B}}$. Then your outstretched thumb will point in the direction of the force $\vec{\mathbf{F}}$ on the wire.

Right-hand-rule 2: force on current exerted by $\vec{\mathbf{B}}$

This right-hand rule describes the direction of the force. What about the magnitude of the force on the wire? It is found experimentally that the magnitude of the force is directly proportional to the current I in the wire, and to the length ℓ of wire exposed to the magnetic field (assumed uniform). Furthermore, if the magnetic field B is made stronger, the force is found to be proportionally greater. The force also depends on the angle θ between the current direction and the magnetic field (Fig. 12), being proportional to $\sin\theta$. Thus, the force on a wire carrying a current I with length ℓ in a uniform magnetic field B is given by

FIGURE 12 Current-carrying wire in a magnetic field. Force on the wire is directed into the page.



$$F \propto I\ell B \sin\theta.$$

When the current is perpendicular to the field lines $(\theta = 90°)$, the force is strongest. When the wire is parallel to the magnetic field lines $(\theta = 0°)$, there is no force at all.

Up to now we have not defined the magnetic field strength precisely. In fact, the magnetic field B can be conveniently defined in terms of the above proportion so that the proportionality constant is precisely 1. Thus we have

$$\boxed{F = I\ell B \sin\theta.} \qquad \textbf{(1)}$$

If the direction of the current is perpendicular to the field $\vec{\mathbf{B}}$ $(\theta = 90°)$, then the force is

$$F_{\text{max}} = I\ell B. \qquad \left[\text{current} \perp \vec{\mathbf{B}}\right] \quad \textbf{(2)}$$

If the current is parallel to the field $(\theta = 0°)$, the force is zero. The magnitude of $\vec{\mathbf{B}}$ can be defined using Eq. 2 as $B = F_{\text{max}}/I\ell$, where F_{max} is the magnitude of the force on a straight length ℓ of wire carrying a current I when the wire is perpendicular to $\vec{\mathbf{B}}$.

The relation between the force $\vec{\mathbf{F}}$ on a wire carrying current I, and the magnetic field $\vec{\mathbf{B}}$ that causes the force, can be written as a vector equation. To do so, we recall that the direction of $\vec{\mathbf{F}}$ is given by the right-hand rule (Fig. 11c), and the magnitude by Eq. 1. This is consistent with the definition of the vector cross product, so we can write

$$\vec{\mathbf{F}} = I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}; \qquad \textbf{(3)}$$

here, $\vec{\boldsymbol{\ell}}$ is a vector whose magnitude is the length of the wire and its direction is along the wire (assumed straight) in the direction of the conventional (positive) current.

Equation 3 applies if the magnetic field is uniform and the wire is straight. If $\vec{\mathbf{B}}$ is not uniform, or if the wire does not everywhere make the same angle θ with $\vec{\mathbf{B}}$, then Eq. 3 can be written more generally as

$$d\vec{\mathbf{F}} = I\, d\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}, \tag{4}$$

where $d\vec{\mathbf{F}}$ is the infinitesimal force acting on a differential length $d\vec{\boldsymbol{\ell}}$ of the wire. The total force on the wire is then found by integrating.

Equation 4 can serve (just as well as Eq. 2 or 3) as a practical definition of $\vec{\mathbf{B}}$. An equivalent way to define $\vec{\mathbf{B}}$, in terms of the force on a moving electric charge, is discussed in the next Section.

EXERCISE C A wire carrying current I is perpendicular to a magnetic field of strength B. Assuming a fixed length of wire, which of the following changes will result in decreasing the force on the wire by a factor of 2? (*a*) Decrease the angle from 90° to 45°; (*b*) decrease the angle from 90° to 30°; (*c*) decrease the current in the wire to $I/2$; (*d*) decrease the magnetic field strength to $B/2$; (*e*) none of these will do it.

The SI unit for magnetic field B is the **tesla** (T). From Eqs. 1, 2, 3, or 4, we see that $1\,\mathrm{T} = 1\,\mathrm{N/A\cdot m}$. An older name for the tesla is the "weber per meter squared" $\left(1\,\mathrm{Wb/m^2} = 1\,\mathrm{T}\right)$. Another unit sometimes used to specify magnetic field is a cgs unit, the **gauss** (G): $1\,\mathrm{G} = 10^{-4}\,\mathrm{T}$. A field given in gauss should always be changed to teslas before using with other SI units. To get a "feel" for these units, we note that the magnetic field of the Earth at its surface is about $\frac{1}{2}$ G or 0.5×10^{-4} T. On the other hand, the field near a small magnet attached to your refrigerator may be 100 G (0.01 T) whereas strong electromagnets can produce fields on the order of 2 T and superconducting magnets can produce over 10 T.

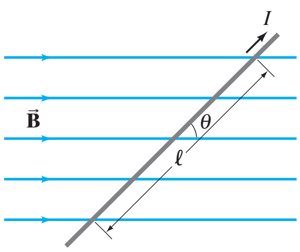


FIGURE 12 (Repeated for Example 1.) Current-carrying wire in a magnetic field. Force on the wire is directed into the page.

EXAMPLE 1 **Magnetic force on a current-carrying wire.** A wire carrying a 30-A current has a length $\ell = 12$ cm between the pole faces of a magnet at an angle $\theta = 60°$ (Fig. 12). The magnetic field is approximately uniform at 0.90 T. We ignore the field beyond the pole pieces. What is the magnitude of the force on the wire?

APPROACH We use Eq. 1, $F = I\ell B \sin\theta$.

SOLUTION The force F on the 12-cm length of wire within the uniform field B is

$$F = I\ell B \sin\theta = (30\,\mathrm{A})(0.12\,\mathrm{m})(0.90\,\mathrm{T})(0.866) = 2.8\,\mathrm{N}.$$

EXERCISE D A straight power line carries 30 A and is perpendicular to the Earth's magnetic field of 0.50×10^{-4} T. What magnitude force is exerted on 100 m of this power line?

On a diagram, when we want to represent an electric current or a magnetic field that is pointing out of the page (toward us) or into the page, we use $\odot$ or $\times$, respectively. The $\odot$ is meant to resemble the tip of an arrow pointing directly toward the reader, whereas the $\times$ or $\otimes$ resembles the tail of an arrow moving away. (See Figs. 13 and 14.)

EXAMPLE 2 **Measuring a magnetic field.** A rectangular loop of wire hangs vertically as shown in Fig. 13. A magnetic field $\vec{\mathbf{B}}$ is directed horizontally, perpendicular to the wire, and points out of the page at all points as represented by the symbol $\odot$. The magnetic field $\vec{\mathbf{B}}$ is very nearly uniform along the horizontal portion of wire ab (length $\ell = 10.0$ cm) which is near the center of the gap of a large magnet producing the field. The top portion of the wire loop is free of the field. The loop hangs from a balance which measures a downward magnetic force (in addition to the gravitational force) of $F = 3.48 \times 10^{-2}$ N when the wire carries a current $I = 0.245$ A. What is the magnitude of the magnetic field B?

APPROACH Three straight sections of the wire loop are in the magnetic field: a horizontal section and two vertical sections. We apply Eq. 1 to each section and use the right-hand rule.

SOLUTION The magnetic force on the left vertical section of wire points to the left; the force on the vertical section on the right points to the right. These two forces are equal and in opposite directions and so add up to zero. Hence, the net magnetic force on the loop is that on the horizontal section ab, whose length is $\ell = 0.100\,\text{m}$. The angle θ between $\vec{\mathbf{B}}$ and the wire is $\theta = 90°$, so $\sin\theta = 1$. Thus Eq. 1 gives

$$B = \frac{F}{I\ell} = \frac{3.48 \times 10^{-2}\,\text{N}}{(0.245\,\text{A})(0.100\,\text{m})} = 1.42\,\text{T}.$$

NOTE This technique can be a precise means of determining magnetic field strength.

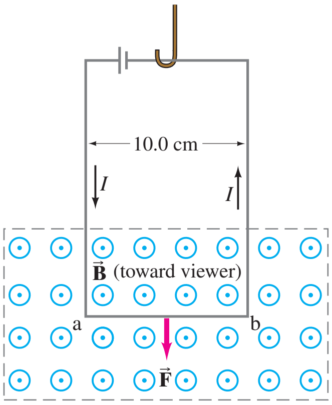


FIGURE 13 Measuring a magnetic field $\vec{\mathbf{B}}$. Example 2.

EXAMPLE 3 **Magnetic force on a semicircular wire.** A rigid wire, carrying a current I, consists of a semicircle of radius R and two straight portions as shown in Fig. 14. The wire lies in a plane perpendicular to a uniform magnetic field $\vec{\mathbf{B}}_0$. Note choice of x and y axis. The straight portions each have length ℓ within the field. Determine the net force on the wire due to the magnetic field $\vec{\mathbf{B}}_0$.

APPROACH The forces on the two straight sections are equal $(= I\ell B_0)$ and in opposite directions, so they cancel. Hence the net force is that on the semicircular portion.

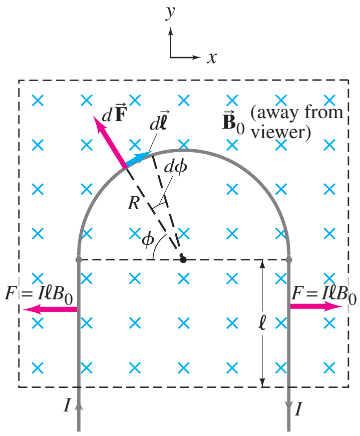


FIGURE 14 Example 3.

SOLUTION We divide the semicircle into short lengths $d\ell = R\,d\phi$ as indicated in Fig. 14, and use Eq. 4, $d\vec{\mathbf{F}} = I\,d\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}$, to find

$$dF = IB_0 R\,d\phi,$$

where dF is the force on the length $d\ell = R\,d\phi$, and the angle between $d\vec{\boldsymbol{\ell}}$ and $\vec{\mathbf{B}}_0$ is 90° (so $\sin\theta = 1$ in the cross product). The x component of the force $d\vec{\mathbf{F}}$ on the segment $d\vec{\boldsymbol{\ell}}$ shown, and the x component of $d\vec{\mathbf{F}}$ for a symmetrically located $d\vec{\boldsymbol{\ell}}$ on the other side of the semicircle, will cancel each other. Thus for the entire semicircle there will be no x component of force. Hence we need be concerned only with the y components, each equal to $dF\sin\phi$, and the total force will have magnitude

$$F = \int_0^\pi dF\sin\phi = IB_0R\int_0^\pi \sin\phi\,d\phi = -IB_0R\cos\phi\Big|_0^\pi = 2IB_0R,$$

with direction vertically upward along the y axis in Fig. 14.

4 Force on an Electric Charge Moving in a Magnetic Field

We have seen that a current-carrying wire experiences a force when placed in a magnetic field. Since a current in a wire consists of moving electric charges, we might expect that freely moving charged particles (not in a wire) would also experience a force when passing through a magnetic field. Indeed, this is the case.

From what we already know we can predict the force on a single moving electric charge. If N such particles of charge q pass by a given point in time t, they constitute a current $I = Nq/t$. We let t be the time for a charge q to travel a distance ℓ in a magnetic field $\vec{\mathbf{B}}$; then $\vec{\boldsymbol{\ell}} = \vec{\mathbf{v}}t$ where $\vec{\mathbf{v}}$ is the velocity of the particle. Thus, the force on these N particles is, by Eq. 3, $\vec{\mathbf{F}} = I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}} = (Nq/t)(\vec{\mathbf{v}}t) \times \vec{\mathbf{B}} = Nq\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. The force on *one* of the N particles is then

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}. \tag{5a}$$

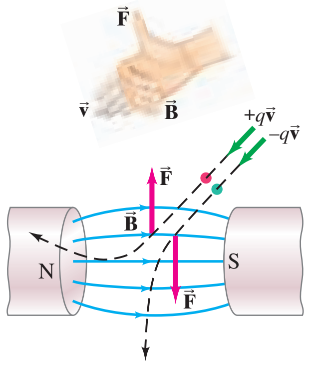


FIGURE 15 Force on charged particles due to a magnetic field is perpendicular to the magnetic field direction. If $\vec{\mathbf{v}}$ is horizontal, then $\vec{\mathbf{F}}$ is vertical.

Right-hand-rule 3: force on moving charge exerted by $\vec{\mathbf{B}}$

This basic and important result can be considered as an alternative way of defining the magnetic field $\vec{\mathbf{B}}$, in place of Eq. 4 or 3. The magnitude of the force in Eq. 5a is

$$F = qvB\sin\theta. \tag{5b}$$

This gives the magnitude of the force on a particle of charge q moving with velocity $\vec{\mathbf{v}}$ at a point where the magnetic field has magnitude B. The angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$ is θ. The force is greatest when the particle moves perpendicular to $\vec{\mathbf{B}}$ ($\theta = 90°$):

$$F_{\max} = qvB. \qquad [\vec{\mathbf{v}} \perp \vec{\mathbf{B}}]$$

The force is *zero* if the particle moves *parallel* to the field lines ($\theta = 0°$). The *direction* of the force is perpendicular to the magnetic field $\vec{\mathbf{B}}$ and to the velocity $\vec{\mathbf{v}}$ of the particle. It is given again by a **right-hand rule** (as for any cross product): you orient your right hand so that your outstretched fingers point along the direction of the particle's velocity ($\vec{\mathbf{v}}$), and when you bend your fingers they must point along the direction of $\vec{\mathbf{B}}$. Then your thumb will point in the direction of the force. This is true only for *positively* charged particles, and will be "up" for the positive particle shown in Fig. 15. For negatively charged particles, the force is in exactly the opposite direction, "down" in Fig. 15.

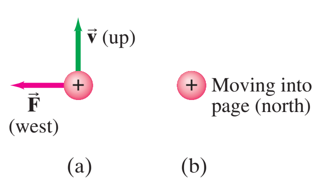
CONCEPTUAL EXAMPLE 4 **Negative charge near a magnet.** A negative charge $-Q$ is placed at rest near a magnet. Will the charge begin to move? Will it feel a force? What if the charge were positive, $+Q$?

RESPONSE No to all questions. A charge at rest has velocity equal to zero. Magnetic fields exert a force only on moving electric charges (Eqs. 5).

EXERCISE E Return to the Chapter-Opening Question and answer it again now. Try to explain why you may have answered differently the first time.

EXAMPLE 5 **Magnetic force on a proton.** A magnetic field exerts a force of 8.0×10^{-14} N toward the west on a proton moving vertically upward at a speed of 5.0×10^{6} m/s (Fig. 16a). When moving horizontally in a northerly direction, the force on the proton is zero (Fig. 16b). Determine the magnitude and direction of the magnetic field in this region. (The charge on a proton is $q = +e = 1.6 \times 10^{-19}$ C.)

FIGURE 16 Example 5.



APPROACH Since the force on the proton is zero when moving north, the field must be in a north–south direction. In order to produce a force to the west when the proton moves upward, the right-hand rule tells us that $\vec{\mathbf{B}}$ must point toward the north. (Your thumb points west and the outstretched fingers of your right hand point upward only when your bent fingers point north.) The magnitude of $\vec{\mathbf{B}}$ is found using Eq. 5b.

SOLUTION Equation 5b with $\theta = 90°$ gives

$$B = \frac{F}{qv} = \frac{8.0 \times 10^{-14}\,\mathrm{N}}{(1.6 \times 10^{-19}\,\mathrm{C})(5.0 \times 10^{6}\,\mathrm{m/s})} = 0.10\,\mathrm{T}.$$

EXAMPLE 6 ESTIMATE Magnetic force on ions during a nerve pulse. Estimate the magnetic force due to the Earth's magnetic field on ions crossing a cell membrane during an action potential. Assume the speed of the ions is 10^{-2} m/s.

APPROACH Using $F = qvB$, set the magnetic field of the Earth to be roughly $B \approx 10^{-4}$ T, and the charge $q \approx e \approx 10^{-19}$ C.

SOLUTION $F \approx (10^{-19}\,\text{C})(10^{-2}\,\text{m/s})(10^{-4}\,\text{T}) = 10^{-25}\,\text{N}.$

NOTE This is an extremely small force. Yet it is thought migrating animals do somehow detect the Earth's magnetic field, and this is an area of active research.

The path of a charged particle moving in a plane perpendicular to a uniform magnetic field is a circle as we shall now show. In Fig. 17 the magnetic field is directed *into* the paper, as represented by ×'s. An electron at point P is moving to the right, and the force on it at this point is downward as shown (use the right-hand rule and reverse the direction for negative charge). The electron is thus deflected toward the page bottom. A moment later, say, when it reaches point Q, the force is still perpendicular to the velocity and is in the direction shown. Because the force is always perpendicular to $\vec{\mathbf{v}}$, the magnitude of $\vec{\mathbf{v}}$ does not change—the electron moves at constant speed. If the force on a particle is always perpendicular to its velocity $\vec{\mathbf{v}}$, the particle moves in a circle and has a centripetal acceleration $a = v^2/r$. Thus a charged particle moves in a circular path with constant centripetal acceleration in a uniform magnetic field (see Example 7). The electron moves clockwise in Fig. 17. A positive particle in this field would feel a force in the opposite direction and would thus move counterclockwise.

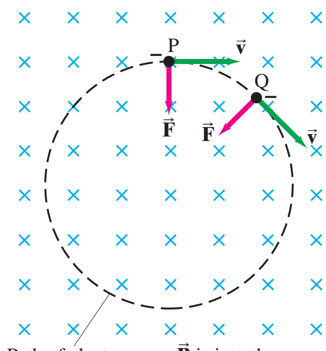


FIGURE 17 Force exerted by a uniform magnetic field on a moving charged particle (in this case, an electron) produces a circular path.

EXAMPLE 7 Electron's path in a uniform magnetic field. An electron travels at 2.0×10^7 m/s in a plane perpendicular to a uniform 0.010-T magnetic field. Describe its path quantitatively.

APPROACH The electron moves at speed v in a curved path and so must have a centripetal acceleration $a = v^2/r$. We find the radius of curvature using Newton's second law. The force is given by Eq. 5b with $\sin\theta = 1$: $F = qvB$.

SOLUTION We insert F and a into Newton's second law:

$$\Sigma F = ma$$
$$qvB = \frac{mv^2}{r}.$$

We solve for r and find

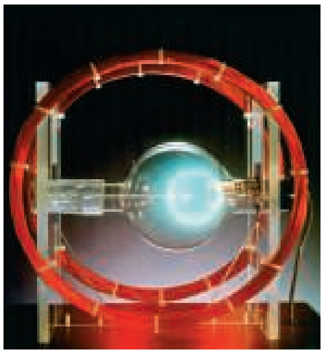
$$r = \frac{mv}{qB}.$$

Since $\vec{\mathbf{F}}$ is perpendicular to $\vec{\mathbf{v}}$, the magnitude of $\vec{\mathbf{v}}$ doesn't change. From this equation we see that if $\vec{\mathbf{B}} = \text{constant}$, then $r = \text{constant}$, and the curve must be a circle as we claimed above. To get r we put in the numbers:

$$r = \frac{(9.1 \times 10^{-31}\,\text{kg})(2.0 \times 10^7\,\text{m/s})}{(1.6 \times 10^{-19}\,\text{C})(0.010\,\text{T})} = 1.1 \times 10^{-2}\,\text{m} = 1.1\,\text{cm}.$$

NOTE See Fig. 18.

FIGURE 18 The blue ring inside the glass tube is the glow of a beam of electrons that ionize the gas molecules. The red coils of current-carrying wire produce a nearly uniform magnetic field, illustrating the circular path of charged particles in a uniform magnetic field.



Richard Megna/Fundamental Photographs, NYC

The time T required for a particle of charge q moving with constant speed v to make one circular revolution in a uniform magnetic field $\vec{\mathbf{B}}\ (\perp \vec{\mathbf{v}})$ is $T = 2\pi r/v$, where $2\pi r$ is the circumference of its circular path. From Example 7, $r = mv/qB$, so

$$T = \frac{2\pi m}{qB}.$$

Since T is the period of rotation, the frequency of rotation is

$$f = \frac{1}{T} = \frac{qB}{2\pi m}. \qquad (6)$$

This is often called the **cyclotron frequency** of a particle in a field because this is the frequency at which particles revolve in a cyclotron.

CONCEPTUAL EXAMPLE 8 **Stopping charged particles.** Can a magnetic field be used to stop a single charged particle, as an electric field can?

RESPONSE No, because the force is always *perpendicular* to the velocity of the particle and thus cannot change the magnitude of its velocity. It also means the magnetic force cannot do work on the particle and so cannot change the kinetic energy of the particle.

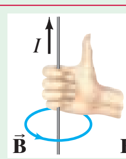
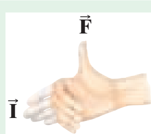
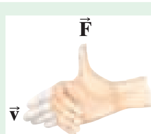
PROBLEM SOLVING

Magnetic Fields

Magnetic fields are somewhat analogous to electric fields, but there are several important differences to recall:

1. The force experienced by a charged particle moving in a magnetic field is *perpendicular* to the direction of the magnetic field (and to the direction of the velocity of the particle), whereas the force exerted by an electric field is *parallel* to the direction of the field (and unaffected by the velocity of the particle).
2. The *right-hand rule*, in its different forms, is intended to help you determine the directions of magnetic field, and the forces they exert, and/or the directions of electric current or charged particle velocity. The right-hand rules (Table 1) are designed to deal with the "perpendicular" nature of these quantities.

TABLE 1 Summary of Right-hand Rules (= RHR)

Physical Situation	Example	How to Orient Right Hand	Result
1. Magnetic field produced by current (RHR-1)	 **Fig. 8c**	Wrap fingers around wire with thumb pointing in direction of current I	Fingers point in direction of $\vec{\mathbf{B}}$
2. Force on electric current I due to magnetic field (RHR-2)	 **Fig. 11c**	Fingers point straight along current I, then bend along magnetic field $\vec{\mathbf{B}}$	Thumb points in direction of the force $\vec{\mathbf{F}}$
3. Force on electric charge $+q$ due to magnetic field (RHR-3)	 **Fig. 15**	Fingers point along particle's velocity $\vec{\mathbf{v}}$, then along $\vec{\mathbf{B}}$	Thumb points in direction of the force $\vec{\mathbf{F}}$

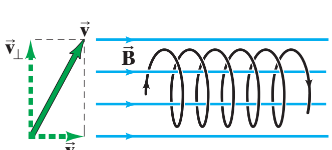


FIGURE 19 Example 9.

CONCEPTUAL EXAMPLE 9 **A helical path.** What is the path of a charged particle in a uniform magnetic field if its velocity is *not* perpendicular to the magnetic field?

RESPONSE The velocity vector can be broken down into components parallel and perpendicular to the field. The velocity component parallel to the field lines experiences no force $(\theta = 0)$, so this component remains constant. The velocity component perpendicular to the field results in circular motion about the field lines. Putting these two motions together produces a helical (spiral) motion around the field lines as shown in Fig. 19.

EXERCISE F What is the sign of the charge in Fig. 19? How would you modify the drawing if the sign were reversed?

*Aurora Borealis

Charged ions approach the Earth from the Sun (the "solar wind") and enter the atmosphere mainly near the poles, sometimes causing a phenomenon called the **aurora borealis** or "northern lights" in northern latitudes. To see why, consider Example 9 and Fig. 20 (see also Fig. 19). In Fig. 20 we imagine a stream of charged particles approaching the Earth. The velocity component *perpendicular* to the field for each particle becomes a circular orbit around the field lines, whereas the velocity component *parallel* to the field carries the particle along the field lines toward the poles. As a particle approaches the N pole, the magnetic field is stronger and the radius of the helical path becomes smaller.

A high concentration of charged particles ionizes the air, and as the electrons recombine with atoms, light is emitted which is the aurora. Auroras are especially spectacular during periods of high sunspot activity when the solar wind brings more charged particles toward Earth.

PHYSICS APPLIED
The aurora borealis

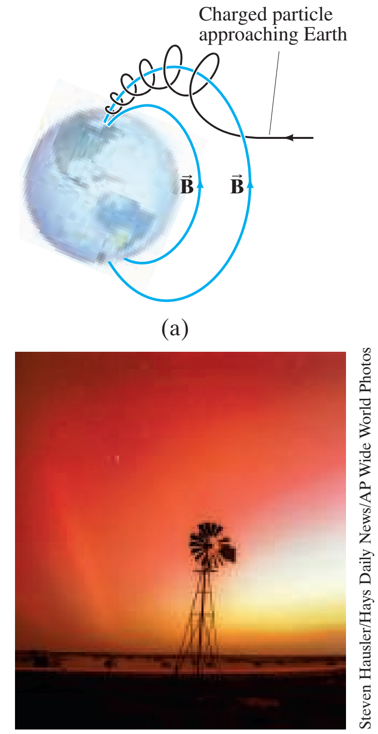


FIGURE 20 (a) Diagram showing a negatively charged particle that approaches the Earth and is "captured" by the magnetic field of the Earth. Such particles follow the field lines toward the poles as shown. (b) Photo of aurora borealis (here, in Kansas, where it is a rare sight).

Lorentz Equation

If a particle of charge q moves with velocity $\vec{\mathbf{v}}$ in the presence of both a magnetic field $\vec{\mathbf{B}}$ and an electric field $\vec{\mathbf{E}}$, it will feel a force

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \qquad \textbf{(7)}$$

where we have made use of Eq. 5a. Equation 7 is often called the **Lorentz equation** and is considered one of the basic equations in physics.

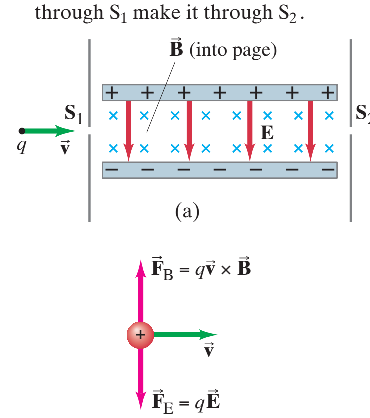
CONCEPTUAL EXAMPLE 10 **Velocity selector, or filter: Crossed $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields.** Some electronic devices and experiments need a beam of charged particles all moving at nearly the same velocity. This can be achieved using both a uniform electric field and a uniform magnetic field, arranged so they are at right angles to each other. As shown in Fig. 21a, particles of charge q pass through slit S_1 and enter the region where $\vec{\mathbf{B}}$ points into the page and $\vec{\mathbf{E}}$ points down from the positive plate toward the negative plate. If the particles enter with different velocities, show how this device "selects" a particular velocity, and determine what this velocity is.

RESPONSE After passing through slit S_1, each particle is subject to two forces as shown in Fig. 21b. If q is positive, the magnetic force is upwards and the electric force downwards. (Vice versa if q is negative.) The exit slit, S_2, is assumed to be directly in line with S_1 and the particles' velocity $\vec{\mathbf{v}}$. Depending on the magnitude of $\vec{\mathbf{v}}$, some particles will be bent upwards and some downwards. The only ones to make it through the slit S_2 will be those for which the net force is zero: $\Sigma F = qvB - qE = 0$. Hence this device selects particles whose velocity is

$$v = \frac{E}{B}. \qquad \textbf{(8)}$$

This result does not depend on the sign of the charge q.

FIGURE 21 A velocity selector: if $v = E/B$, the particles passing through S_1 make it through S_2.



EXERCISE G A particle in a velocity selector as diagrammed in Fig. 21 hits below the exit hole, S_2. This means that the particle (*a*) is going faster than the selected speed; (*b*) is going slower than the selected speed; (*c*) answer *a* is true if $q > 0$, *b* is true if $q < 0$; (*d*) answer *a* is true if $q < 0$, *b* is true if $q > 0$.

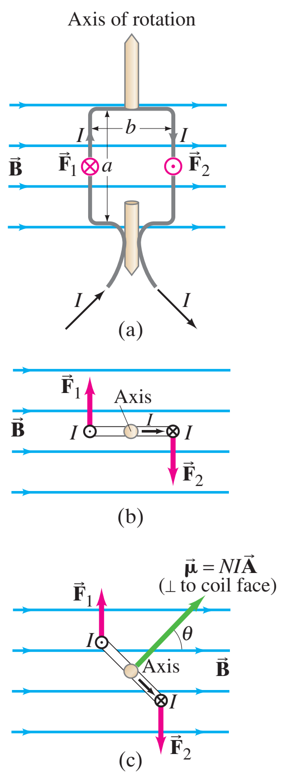


FIGURE 22 Calculating the torque on a current loop in a magnetic field $\vec{\mathbf{B}}$. (a) Loop face parallel to $\vec{\mathbf{B}}$ field lines; (b) top view; (c) loop makes an angle to $\vec{\mathbf{B}}$, reducing the torque since the lever arm is reduced.

5 Torque on a Current Loop; Magnetic Dipole Moment

When an electric current flows in a closed loop of wire placed in an external magnetic field, as shown in Fig. 22, the magnetic force on the current can produce a torque. This is the principle behind a number of important practical devices, including motors and analog voltmeters and ammeters, which we discuss in the next Section. The interaction between a current and a magnetic field is important in other areas as well, including atomic physics.

Current flows through the rectangular loop in Fig. 22a, whose face we assume is parallel to $\vec{\mathbf{B}}$. $\vec{\mathbf{B}}$ exerts no force and no torque on the horizontal segments of wire because they are parallel to the field and $\sin\theta = 0$ in Eq. 1. But the magnetic field does exert a force on each of the vertical sections of wire as shown, $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ (see also top view, Fig. 22b). By right-hand-rule 2 (Fig. 11c or Table 1) the direction of the force on the upward current on the left is in the opposite direction from the equal magnitude force $\vec{\mathbf{F}}_2$ on the downward current on the right. These forces give rise to a net torque that acts to rotate the coil about its vertical axis.

Let us calculate the magnitude of this torque. From Eq. 2 $\left(\text{current} \perp \vec{\mathbf{B}}\right)$, the force $F = IaB$, where a is the length of the vertical arm of the coil. The lever arm for each force is $b/2$, where b is the width of the coil and the "axis" is at the midpoint. The torques produced by $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ act in the same direction, so the total torque is the sum of the two torques:

$$\tau = IaB\frac{b}{2} + IaB\frac{b}{2} = IabB = IAB,$$

where $A = ab$ is the area of the coil. If the coil consists of N loops of wire, the current is then NI, so the torque becomes

$$\tau = NIAB.$$

If the coil makes an angle θ with the magnetic field, as shown in Fig. 22c, the forces are unchanged, but each lever arm is reduced from $\frac{1}{2}b$ to $\frac{1}{2}b\sin\theta$. Note that the angle θ is taken to be the angle between $\vec{\mathbf{B}}$ and the perpendicular to the face of the coil, Fig. 22c. So the torque becomes

$$\tau = NIAB\sin\theta. \tag{9}$$

This formula, derived here for a rectangular coil, is valid for any shape of flat coil.

The quantity NIA is called the **magnetic dipole moment** of the coil and is considered a vector:

$$\vec{\boldsymbol{\mu}} = NI\vec{\mathbf{A}}, \tag{10}$$

where the direction of $\vec{\mathbf{A}}$ (and therefore of $\vec{\boldsymbol{\mu}}$) is *perpendicular* to the plane of the coil (the green arrow in Fig. 22c) consistent with the right-hand rule (cup your right hand so your fingers wrap around the loop in the direction of current flow, then your thumb points in the direction of $\vec{\boldsymbol{\mu}}$ and $\vec{\mathbf{A}}$). With this definition of $\vec{\boldsymbol{\mu}}$, we can rewrite Eq. 9 in vector form:

$$\vec{\boldsymbol{\tau}} = NI\vec{\mathbf{A}} \times \vec{\mathbf{B}}$$

or

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}, \tag{11}$$

which gives the correct magnitude and direction for the torque $\vec{\boldsymbol{\tau}}$.

Equation 11 has the form for an electric dipole (with electric dipole moment $\vec{\mathbf{p}}$) in an electric field $\vec{\mathbf{E}}$, which is $\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$. And just as an electric dipole has potential energy given by $U = -\vec{\mathbf{p}}\cdot\vec{\mathbf{E}}$ when in an electric field, we expect a similar form for a magnetic dipole in a magnetic field. In order to rotate a current loop (Fig. 22) so as to increase θ, we must do work against the torque due to the magnetic field.

Hence the potential energy depends on angle (see the work-energy principle for rotational motion) as

$$U = \int \tau\, d\theta = \int NIAB \sin\theta\, d\theta = -\mu B \cos\theta + C.$$

If we choose $U = 0$ at $\theta = \pi/2$, then the arbitrary constant C is zero and the potential energy is

$$U = -\mu B \cos\theta = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}, \qquad \textbf{(12)}$$

as expected. Bar magnets and compass needles, as well as current loops, can be considered as magnetic dipoles. Note the striking similarities of the fields produced by a bar magnet and a current loop, Figs. 4b and 9.

EXAMPLE 11 **Torque on a coil.** A circular coil of wire has a diameter of 20.0 cm and contains 10 loops. The current in each loop is 3.00 A, and the coil is placed in a 2.00-T external magnetic field. Determine the maximum and minimum torque exerted on the coil by the field.

APPROACH Equation 9 is valid for any shape of coil, including circular loops. Maximum and minimum torque are determined by the angle θ the coil makes with the magnetic field.

SOLUTION The area of one loop of the coil is

$$A = \pi r^2 = \pi(0.100\,\mathrm{m})^2 = 3.14 \times 10^{-2}\,\mathrm{m}^2.$$

The maximum torque occurs when the coil's face is parallel to the magnetic field, so $\theta = 90°$ in Fig. 22c, and $\sin\theta = 1$ in Eq. 9:

$$\tau = NIAB \sin\theta = (10)(3.00\,\mathrm{A})\left(3.14 \times 10^{-2}\,\mathrm{m}^2\right)(2.00\,\mathrm{T})(1) = 1.88\,\mathrm{N \cdot m}.$$

The minimum torque occurs if $\sin\theta = 0$, for which $\theta = 0°$, and then $\tau = 0$ from Eq. 9.

NOTE If the coil is free to turn, it will rotate toward the orientation with $\theta = 0°$.

EXAMPLE 12 **Magnetic moment of a hydrogen atom.** Determine the magnetic dipole moment of the electron orbiting the proton of a hydrogen atom at a given instant, assuming (in the Bohr model) it is in its ground state with a circular orbit of radius 0.529×10^{-10} m. [This is a very rough picture of atomic structure, but nonetheless gives an accurate result.]

APPROACH We start by setting the electrostatic force on the electron due to the proton equal to $ma = mv^2/r$ since the electron's acceleration is centripetal.

SOLUTION The electron is held in its orbit by the coulomb force, so Newton's second law, $F = ma$, gives

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r};$$

so

$$v = \sqrt{\frac{e^2}{4\pi\epsilon_0 mr}}$$

$$= \sqrt{\frac{\left(8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2}\right)\left(1.60 \times 10^{-19}\,\mathrm{C}\right)^2}{\left(9.11 \times 10^{-31}\,\mathrm{kg}\right)\left(0.529 \times 10^{-10}\,\mathrm{m}\right)}} = 2.19 \times 10^6\,\mathrm{m/s}.$$

Since current is the electric charge that passes a given point per unit time, the revolving electron is equivalent to a current

$$I = \frac{e}{T} = \frac{ev}{2\pi r},$$

where $T = 2\pi r/v$ is the time required for one orbit. Since the area of the orbit is $A = \pi r^2$, the magnetic dipole moment is

$$\mu = IA = \frac{ev}{2\pi r}\left(\pi r^2\right) = \tfrac{1}{2} evr$$

$$= \tfrac{1}{2}\left(1.60 \times 10^{-19}\,\mathrm{C}\right)\left(2.19 \times 10^6\,\mathrm{m/s}\right)\left(0.529 \times 10^{-10}\,\mathrm{m}\right) = 9.27 \times 10^{-24}\,\mathrm{A \cdot m^2},$$

or 9.27×10^{-24} J/T.

*6 Applications: Motors, Loudspeakers, Galvanometers

*Electric Motors

PHYSICS APPLIED
DC motor

An **electric motor** changes electric energy into (rotational) mechanical energy. A motor works on the principle that a torque is exerted on a coil of current-carrying wire suspended in the magnetic field of a magnet, described in Section 5. The coil is mounted on a large cylinder called the **rotor** or **armature**, Fig. 23, so that it can rotate continuously in one direction. Actually, there are several coils, although only one is indicated in the Figure. The armature is mounted on a shaft or axle. When the armature is in the position shown in Fig. 23, the magnetic field exerts forces on the current in the loop as shown (perpendicular to $\vec{\mathbf{B}}$ and to the current direction). However, when the coil, which is rotating clockwise in Fig. 23, passes beyond the vertical position, the forces would then act to return the coil back to vertical if the current remained the same. But if the current could somehow be reversed at that critical moment, the forces would reverse, and the coil would continue rotating in the same direction. Thus, alternation of the current is necessary if a motor is to turn continuously in one direction. This can be achieved in a **dc motor** with the use of **commutators** and **brushes**: as shown in Fig. 24, input current passes through stationary brushes that rub against the conducting commutators mounted on the motor shaft. At every half revolution, each commutator changes its connection over to the other brush. Thus the current in the coil reverses every half revolution as required for continuous rotation.

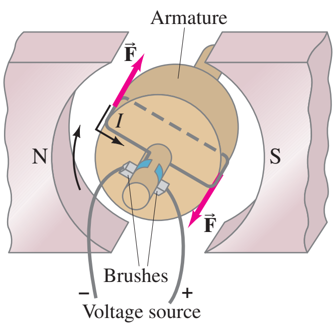


FIGURE 23 Diagram of a simple dc motor.

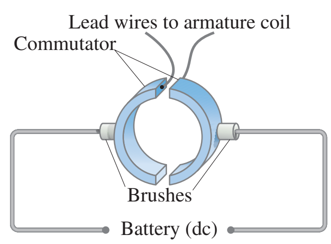


FIGURE 24 The commutator-brush arrangement in a dc motor ensures alternation of the current in the armature to keep rotation continuous. The commutators are attached to the motor shaft and turn with it, whereas the brushes remain stationary.

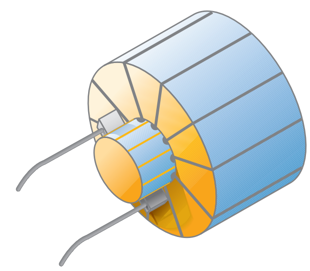


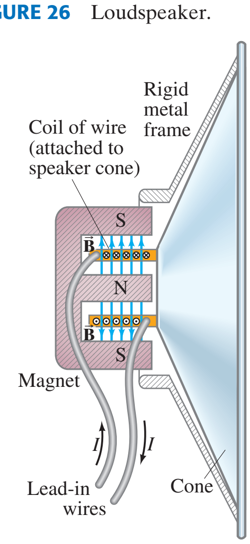
FIGURE 25 Motor with many windings.

Most motors contain several coils, called *windings*, each located in a different place on the armature, Fig. 25. Current flows through each coil only during a small part of a revolution, at the time when its orientation results in the maximum torque. In this way, a motor produces a much steadier torque than can be obtained from a single coil.

An **ac motor**, with ac current as input, can work without commutators since the current itself alternates. Many motors use wire coils to produce the magnetic field (electromagnets) instead of a permanent magnet. Indeed the design of most motors is more complex than described here, but the general principles remain the same.

*Loudspeakers

FIGURE 26 Loudspeaker.



A **loudspeaker** also works on the principle that a magnet exerts a force on a current-carrying wire. The electrical output of a stereo or TV set is connected to the wire leads of the speaker. The speaker leads are connected internally to a coil of wire, which is itself attached to the speaker cone, Fig. 26. The speaker cone is usually made of stiffened cardboard and is mounted so that it can move back and forth freely. A permanent magnet is mounted directly in line with the coil of wire. When the alternating current of an audio signal flows through the wire coil, which is free to move within the magnet, the coil experiences a force due to the magnetic field of the magnet. (The force is to the right at the instant shown in Fig. 26.)

As the current alternates at the frequency of the audio signal, the coil and attached speaker cone move back and forth at the same frequency, causing alternate compressions and rarefactions of the adjacent air, and sound waves are produced. A speaker thus changes electrical energy into sound energy, and the frequencies and intensities of the emitted sound waves can be an accurate reproduction of the electrical input.

*Galvanometer

The basic component of analog meters (those with pointer and dial), including analog ammeters, voltmeters, and ohmmeters, is a galvanometer. We have already seen how these meters are designed, and now we can examine how the crucial element, a galvanometer, works. As shown in Fig. 27, a **galvanometer** consists of a coil of wire (with attached pointer) suspended in the magnetic field of a permanent magnet. When current flows through the loop of wire, the magnetic field exerts a torque on the loop, as given by Eq. 9,

$$\tau = NIAB\sin\theta.$$

This torque is opposed by a spring which exerts a torque τ_s approximately proportional to the angle ϕ through which it is turned (Hooke's law). That is,

$$\tau_s = k\phi,$$

where k is the stiffness constant of the spring. The coil and attached pointer rotate to the angle where the torques balance. When the needle is in equilibrium at rest, the torques are equal: $k\phi = NIAB\sin\theta$, or

$$\phi = \frac{NIAB\sin\theta}{k}.$$

The deflection of the pointer, ϕ, is directly proportional to the current I flowing in the coil, but also depends on the angle θ the coil makes with $\vec{\mathbf{B}}$. For a useful meter we need ϕ to depend only on the current I, independent of θ. To solve this problem, magnets with curved pole pieces are used and the galvanometer coil is wrapped around a cylindrical iron core as shown in Fig. 28. The iron tends to concentrate the magnetic field lines so that $\vec{\mathbf{B}}$ always points parallel to the face of the coil at the wire outside the core. The force is then always perpendicular to the face of the coil, and the torque will not vary with angle. Thus ϕ will be proportional to I as required.

Steven Hausler/Hays Daily News/AP Wide World Photos

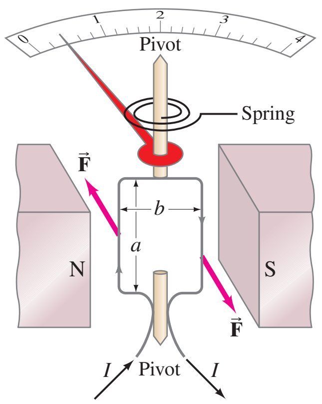
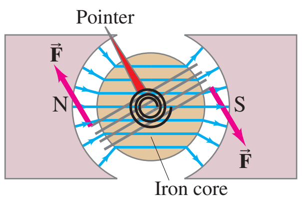


FIGURE 27 Galvanometer.

FIGURE 28 Galvanometer coil wrapped on an iron core.



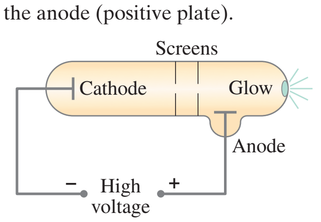
7 Discovery and Properties of the Electron

The electron plays a basic role in our understanding of electricity and magnetism today. But its existence was not suggested until the 1890s. We discuss it here because magnetic fields were crucial for measuring its properties.

Toward the end of the nineteenth century, studies were being done on the discharge of electricity through rarefied gases. One apparatus, diagrammed in Fig. 29, was a glass tube fitted with electrodes and evacuated so only a small amount of gas remained inside. When a very high voltage was applied to the electrodes, a dark space seemed to extend outward from the cathode (negative electrode) toward the opposite end of the tube; and that far end of the tube would glow. If one or more screens containing a small hole was inserted as shown, the glow was restricted to a tiny spot on the end of the tube. It seemed as though something being emitted by the cathode traveled to the opposite end of the tube. These "somethings" were named **cathode rays**.

There was much discussion at the time about what these rays might be. Some scientists thought they might resemble light. But the observation that the bright spot at the end of the tube could be deflected to one side by an electric or magnetic field suggested that cathode rays could be charged particles; and the direction of the deflection was consistent with a negative charge. Furthermore, if the tube contained certain types of rarefied gas, the path of the cathode rays was made visible by a slight glow.

FIGURE 29 Discharge tube. In some models, one of the screens is the anode (positive plate).



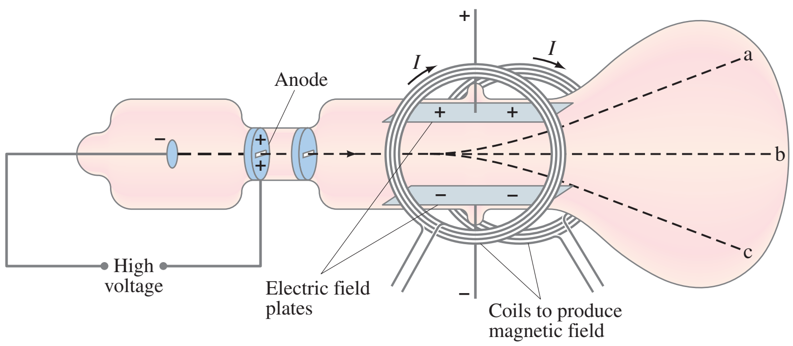


FIGURE 30 Cathode rays deflected by electric and magnetic fields.

Estimates of the charge e of the (assumed) cathode-ray particles, as well as of their charge-to-mass ratio e/m, had been made by 1897. But in that year, J. J. Thomson (1856–1940) was able to measure e/m directly, using the apparatus shown in Fig. 30. Cathode rays are accelerated by a high voltage and then pass between a pair of parallel plates built into the tube. The voltage applied to the plates produces an electric field, and a pair of coils produces a magnetic field. When only the electric field is present, say with the upper plate positive, the cathode rays are deflected upward as in path a in Fig. 30. If only a magnetic field exists, say inward, the rays are deflected downward along path c. These observations are just what is expected for a negatively charged particle. The force on the rays due to the magnetic field is $F = evB$, where e is the charge and v is the velocity of the cathode rays. In the absence of an electric field, the rays are bent into a curved path, so we have, from $F = ma$,

$$evB = m\frac{v^2}{r},$$

and thus

$$\frac{e}{m} = \frac{v}{Br}.$$

The radius of curvature r can be measured and so can B. The velocity v can be found by applying an electric field in addition to the magnetic field. The electric field E is adjusted so that the cathode rays are undeflected and follow path b in Fig. 30. This is just like the velocity selector of Example 10 where the force due to the electric field, $F = eE$, is balanced by the force due to the magnetic field, $F = evB$. Thus $eE = evB$ and $v = E/B$. Combining this with the above equation we have

$$\frac{e}{m} = \frac{E}{B^2r}. \tag{13}$$

The quantities on the right side can all be measured so that although e and m could not be determined separately, the ratio e/m could be determined. The accepted value today is $e/m = 1.76 \times 10^{11}\,\mathrm{C/kg}$. Cathode rays soon came to be called **electrons**.

It is worth noting that the "discovery" of the electron, like many others in science, is not quite so obvious as discovering gold or oil. Should the discovery of the electron be credited to the person who first saw a glow in the tube? Or to the person who first called them cathode rays? Perhaps neither one, for they had no conception of the electron as we know it today. In fact, the credit for the discovery is generally given to Thomson, but not because he was the first to see the glow in the tube. Rather it is because he believed that this phenomenon was due to tiny negatively charged particles and made careful measurements on them. Furthermore he argued that these particles were constituents of atoms, and not ions or atoms themselves as many thought, and he developed an electron theory of matter. His view is close to what we accept today, and this is why Thomson is credited with the "discovery." Note, however, that neither he nor anyone else ever actually saw an electron itself. We discuss this briefly, for it illustrates the fact that discovery in science is not always a clear-cut matter. In fact some philosophers of science think the word "discovery" is often not appropriate, such as in this case.

Thomson believed that an electron was not an atom, but rather a constituent, or part, of an atom. Convincing evidence for this came soon with the determination of the charge and the mass of the cathode rays. Thomson's student J. S. Townsend made the first direct (but rough) measurements of e in 1897. But it was the more refined **oil-drop experiment** of Robert A. Millikan (1868–1953) that yielded a precise value for the charge on the electron and showed that charge comes in discrete amounts. In this experiment, tiny droplets of mineral oil carrying an electric charge were allowed to fall under gravity between two parallel plates, Fig. 31. The electric field E between the plates was adjusted until the drop was suspended in midair. The downward pull of gravity, mg, was then just balanced by the upward force due to the electric field. Thus $qE = mg$, so the charge $q = mg/E$. The mass of the droplet was determined by measuring its terminal velocity in the absence of the electric field. Sometimes the drop was charged negatively, and sometimes positively, suggesting that the drop had acquired or lost electrons (by friction, leaving the atomizer). Millikan's painstaking observations and analysis presented convincing evidence that any charge was an integral multiple of a smallest charge, e, that was ascribed to the electron, and that the value of e was $1.6 \times 10^{-19}\,\mathrm{C}$. This value of e, combined with the measurement of e/m, gives the mass of the electron to be $(1.6 \times 10^{-19}\,\mathrm{C})/(1.76 \times 10^{11}\,\mathrm{C/kg}) = 9.1 \times 10^{-31}\,\mathrm{kg}$. This mass is less than a thousandth the mass of the smallest atom, and thus confirmed the idea that the electron is only a part of an atom. The accepted value today for the mass of the electron is $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$.

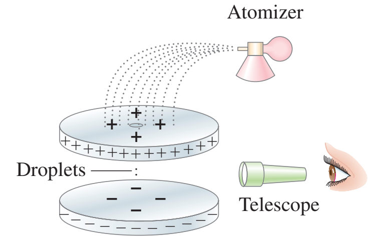


FIGURE 31 Millikan's oil-drop experiment.

CRT, Revisited

The cathode ray tube (CRT) can serve as the picture tube of TV sets, oscilloscopes, and computer monitors. There we see a design using electric deflection plates to maneuver the electron beam. Many CRTs, however, make use of the magnetic field produced by coils to maneuver the electron beam. They operate much like the coils shown in Fig. 30.

8 The Hall Effect

When a current-carrying conductor is held fixed in a magnetic field, the field exerts a sideways force on the charges moving in the conductor. For example, if electrons move to the right in the rectangular conductor shown in Fig. 32a, the inward magnetic field will exert a downward force on the electrons $\vec{\mathbf{F}}_B = -e\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}$, where $\vec{\mathbf{v}}_d$ is the drift velocity of the electrons. Thus the electrons will tend to move nearer to face D than face C. There will thus be a potential difference between faces C and D of the conductor. This potential difference builds up until the electric field $\vec{\mathbf{E}}_H$ that it produces exerts a force, $e\vec{\mathbf{E}}_H$, on the moving charges that is equal and opposite to the magnetic force. This effect is called the **Hall effect** after E. H. Hall, who discovered it in 1879. The difference of potential produced is called the **Hall emf**.

The electric field due to the separation of charge is called the *Hall field*, $\vec{\mathbf{E}}_H$, and points downward in Fig. 32a, as shown. In equilibrium, the force due to this electric field is balanced by the magnetic force $ev_d B$, so

$$eE_H = ev_d B.$$

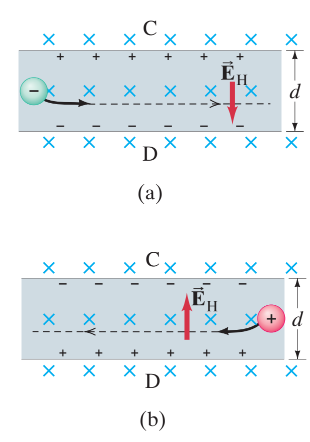
Hence $E_H = v_d B$. The Hall emf is then (assuming the conductor is long and thin so E_H is uniform)

$$\mathscr{E}_H = E_H d = v_d B d, \qquad \textbf{(14)}$$

where d is the width of the conductor.

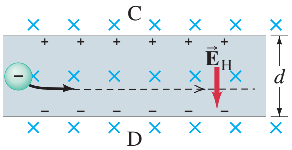
A current of negative charges moving to the right is equivalent to positive charges moving to the left, at least for most purposes. But the Hall effect can distinguish these two. As can be seen in Fig. 32b, positive particles moving to the left are deflected downward, so that the bottom surface is positive relative to the top surface. This is the reverse of part (a). Indeed, the direction of the emf in the Hall effect first revealed that it is negative particles that move in metal conductors.

FIGURE 32 The Hall effect. (a) Negative charges moving to the right as the current. (b) Positive charges moving to the left as the current.



The magnitude of the Hall emf is proportional to the strength of the magnetic field. The Hall effect can thus be used to measure magnetic field strengths. First the conductor, called a *Hall probe*, is calibrated with known magnetic fields. Then, for the same current, its emf output will be a measure of B. Hall probes can be made very small and are convenient and accurate to use.

The Hall effect can also be used to measure the drift velocity of charge carriers when the external magnetic field B is known. Such a measurement also allows us to determine the density of charge carriers in the material.



(a)

FIGURE 32a (Repeated here for Example 13.)

EXAMPLE 13 **Drift velocity using the Hall effect.** A long copper strip 1.8 cm wide and 1.0 mm thick is placed in a 1.2-T magnetic field as in Fig. 32a. When a steady current of 15 A passes through it, the Hall emf is measured to be 1.02 μV. Determine the drift velocity of the electrons and the density of free (conducting) electrons (number per unit volume) in the copper.

APPROACH We use Eq. 14 to obtain the drift velocity, and another to find the density of conducting electrons.

SOLUTION The drift velocity (Eq. 14) is

$$v_d = \frac{\mathscr{E}_H}{Bd} = \frac{1.02 \times 10^{-6}\,\text{V}}{(1.2\,\text{T})(1.8 \times 10^{-2}\,\text{m})} = 4.7 \times 10^{-5}\,\text{m/s}.$$

The density of charge carriers n is obtained from $I = nev_d A$, where A is the cross-sectional area through which the current I flows. Then

$$n = \frac{I}{ev_d A} = \frac{15\,\text{A}}{(1.6 \times 10^{-19}\,\text{C})(4.7 \times 10^{-5}\,\text{m/s})(1.8 \times 10^{-2}\,\text{m})(1.0 \times 10^{-3}\,\text{m})}$$
$$= 11 \times 10^{28}\,\text{m}^{-3}.$$

This value for the density of free electrons in copper, $n = 11 \times 10^{28}$ per m^3, is the experimentally measured value. It represents *more* than one free electron per atom, which as we see in $8.4 \times 10^{28}\,\text{m}^{-3}$.

*9 Mass Spectrometer

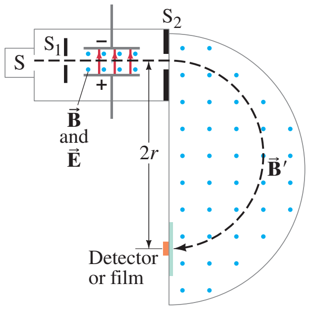
PHYSICS APPLIED
The mass spectrometer

A **mass spectrometer** is a device to measure masses of atoms. It is used today not only in physics but also in chemistry, geology, and medicine, often to identify atoms (and their concentration) in given samples. As shown in Fig. 33, ions are produced by heating, or by an electric current, in the source or sample S. The particles, of mass m and electric charge q, pass through slit S_1 and enter crossed electric and magnetic fields. Ions follow a straight-line path in this "velocity selector" (as in Example 10) if the electric force qE is balanced by the magnetic force qvB: that is, if $qE = qvB$, or $v = E/B$. Thus only those ions whose speed is $v = E/B$ will pass through undeflected and emerge through slit S_2. In the semicircular region, after S_2, there is only a magnetic field, B', so the ions follow a circular path. The radius of the circular path is found from their mark on film (or detectors) if B' is fixed; or else r is fixed by the position of a detector and B' is varied until detection occurs. Newton's second law, $\Sigma F = ma$, applied to an ion moving in a circle under the influence only of the magnetic field B' gives $qvB' = mv^2/r$. Since $v = E/B$, we have

$$m = \frac{qB'r}{v} = \frac{qBB'r}{E}.$$

All the quantities on the right side are known or can be measured, and thus m can be determined.

FIGURE 33 Bainbridge-type mass spectrometer. The magnetic fields B and B' point out of the paper (indicated by the dots), for positive ions.



Historically, the masses of many atoms were measured this way. When a pure substance was used, it was sometimes found that two or more closely spaced marks would appear on the film. For example, neon produced two marks whose radii corresponded to atoms of mass 20 and 22 atomic mass units (u). Impurities were ruled out and it was concluded that there must be two types of neon with different masses. These different forms were called **isotopes**. It was soon found that most elements are mixtures of isotopes, and the difference in mass is due to different numbers of neutrons.

EXAMPLE 14 **Mass spectrometry.** Carbon atoms of atomic mass 12.0 u are found to be mixed with another, unknown, element. In a mass spectrometer with fixed B', the carbon traverses a path of radius 22.4 cm and the unknown's path has a 26.2-cm radius. What is the unknown element? Assume the ions of both elements have the same charge.

APPROACH The carbon and unknown atoms pass through the same electric and magnetic fields. Hence their masses are proportional to the radius of their respective paths (see equation on previous page).

SOLUTION We write a ratio for the masses, using the equation at the bottom of the previous page:

$$\frac{m_x}{m_C} = \frac{qBB'r_x/E}{qBB'r_C/E} = \frac{26.2\text{ cm}}{22.4\text{ cm}} = 1.17.$$

Thus $m_x = 1.17 \times 12.0\text{ u} = 14.0\text{ u}$. The other element is probably nitrogen (see the Periodic Table, inside the back cover).

NOTE The unknown could also be an isotope such as carbon-14 $\left({}^{14}_{6}\text{C}\right)$. Further physical or chemical analysis would be needed.

Summary

A magnet has two **poles**, north and south. The north pole is that end which points toward geographic north when the magnet is freely suspended. Like poles of two magnets repel each other, whereas unlike poles attract.

We can picture that a **magnetic field** surrounds every magnet. The SI unit for magnetic field is the **tesla** (T).

Electric currents produce magnetic fields. For example, the lines of magnetic field due to a current in a straight wire form circles around the wire, and the field exerts a force on magnets (or currents) near it.

A magnetic field exerts a force on an electric current. The force on an infinitesimal length of wire $d\vec{\boldsymbol{\ell}}$ carrying a current I in a magnetic field $\vec{\mathbf{B}}$ is

$$d\vec{\mathbf{F}} = I\, d\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}. \tag{4}$$

If the field $\vec{\mathbf{B}}$ is uniform over a straight length $\vec{\boldsymbol{\ell}}$ of wire, then the force is

$$\vec{\mathbf{F}} = I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}} \tag{3}$$

which has magnitude

$$F = I\ell B \sin\theta \tag{1}$$

where θ is the angle between magnetic field $\vec{\mathbf{B}}$ and the wire. The direction of the force is perpendicular to the wire and to the magnetic field, and is given by the right-hand rule. This relation serves as the definition of magnetic field $\vec{\mathbf{B}}$.

Similarly, a magnetic field $\vec{\mathbf{B}}$ exerts a force on a charge q moving with velocity $\vec{\mathbf{v}}$ given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}. \tag{5a}$$

The magnitude of the force is

$$F = qvB \sin\theta, \tag{5b}$$

where θ is the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$.

The path of a charged particle moving perpendicular to a uniform magnetic field is a circle.

If both electric and magnetic fields ($\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$) are present, the force on a charge q moving with velocity $\vec{\mathbf{v}}$ is

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}. \tag{7}$$

The torque on a current loop in a magnetic field $\vec{\mathbf{B}}$ is

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}, \tag{11}$$

where $\vec{\boldsymbol{\mu}}$ is the **magnetic dipole moment** of the loop:

$$\vec{\boldsymbol{\mu}} = NI\vec{\mathbf{A}}. \tag{10}$$

Here N is the number of coils carrying current I in the loop and $\vec{\mathbf{A}}$ is a vector perpendicular to the plane of the loop (use right-hand rule, fingers along current in loop) and has magnitude equal to the area of the loop.

The measurement of the charge-to-mass ratio (e/m) of the electron was done using magnetic and electric fields. The charge e on the electron was first measured in the Millikan oil-drop experiment and then its mass was obtained from the measured value of the e/m ratio.

[*In the **Hall effect**, moving charges in a conductor placed in a magnetic field are forced to one side, producing an emf between the two sides of the conductor.]

[*A **mass spectrometer** uses magnetic and electric fields to measure the mass of ions.]

Answers to Exercises

A: Near the poles, where the field lines are closer together.

B: Counterclockwise.

C: (*b*), (*c*), (*d*).

D: 0.15 N.

E: (*b*), (*c*), (*d*).

F: Negative; the direction of the helical path would be reversed (still going to the right).

G: (*d*).