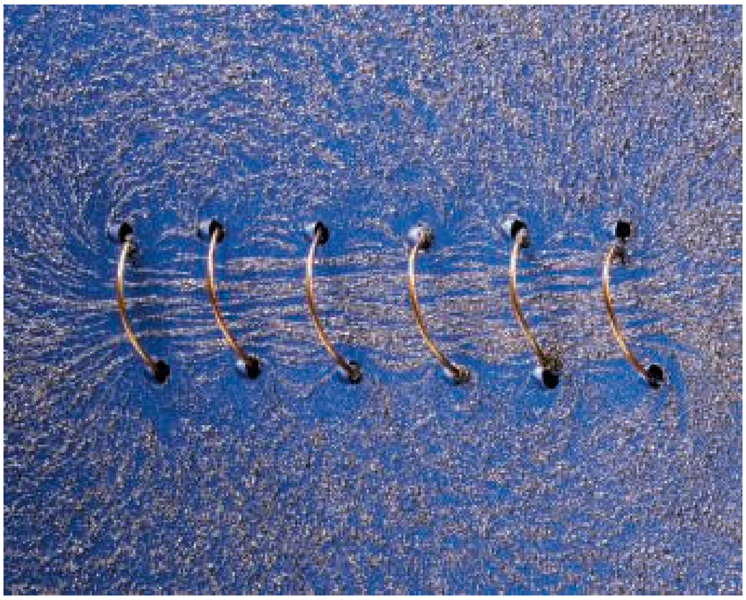


A long coil of wire with many closely spaced loops is called a solenoid. When a long solenoid carries an electric current, a nearly uniform magnetic field is produced within the loops as suggested by the alignment of the iron filings in this photo. The magnitude of the field inside a solenoid is readily found using Ampère's law, one of the great general laws of electromagnetism, relating magnetic fields and electric currents. We examine these connections in detail in this Chapter, as well as other means for producing magnetic fields.

Richard Megna/Fundamental Photographs, NYC

Sources of Magnetic Field

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.*]

Which of the following will produce a magnetic field?

(a) An electric charge at rest.
(b) A moving electric charge.
(c) An electric current.
(d) The voltage of a battery not connected to anything.
(e) Any piece of iron.
(f) A piece of any metal.

There are effects (forces and torques) that a magnetic field has on electric currents and on moving electric charges. Magnetic fields are produced not only by magnets but also by electric currents (Oersted's great discovery). It is this aspect of magnetism, the production of magnetic fields, that we discuss in this Chapter. We will see how magnetic field strengths are determined for some simple situations, and discuss some general relations between magnetic fields and their sources, electric current. Most elegant is Ampère's law. We also study the Biot-Savart Law, which can be very helpful for solving practical problems.

CONTENTS

1 Magnetic Field Due to a Straight Wire
2 Force between Two Parallel Wires
3 Definitions of the Ampere and the Coulomb
4 Ampère's Law
5 Magnetic Field of a Solenoid and a Toroid
6 Biot-Savart Law
7 Magnetic Materials—Ferromagnetism
*8 Electromagnets and Solenoids—Applications
*9 Magnetic Fields in Magnetic Materials; Hysteresis
*10 Paramagnetism and Diamagnetism

Note: Sections marked with an asterisk (*) may be considered optional by the instructor.

From Chapter 28 of *Physics for Scientists & Engineers with Modern Physics*, Fourth Edition, Douglas C. Giancoli.

1 Magnetic Field Due to a Straight Wire

The magnetic field due to the electric current in a long straight wire is such that the field lines are circles with the wire at the center (Fig. 1). You might expect that the field strength at a given point would be greater if the current flowing in the wire were greater; and that the field would be less at points farther from the wire. This is indeed the case. Careful experiments show that the magnetic field B due to a long straight wire at a point near it is directly proportional to the current I in the wire and inversely proportional to the distance r from the wire:

$$B \propto \frac{I}{r}.$$

This relation $B \propto I/r$ is valid as long as r, the perpendicular distance to the wire, is much less than the distance to the ends of the wire (i.e., the wire is long).

The proportionality constant is written† as $\mu_0/2\pi$; thus,

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}. \qquad \text{[near a long straight wire]} \quad \mathbf{(1)}$$

The value of the constant μ_0, which is called the **permeability of free space**, is $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T\cdot m/A}$.

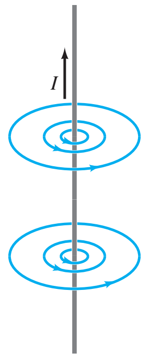
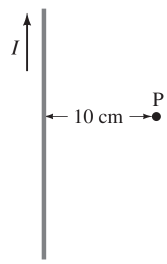


FIGURE 1 Magnetic field lines around a long straight wire carrying an electric current I.

FIGURE 2 Example 1.



CAUTION
A compass, near a current, may not point north

EXAMPLE 1 **Calculation of $\vec{B}$ near a wire.** An electric wire in the wall of a building carries a dc current of 25 A vertically upward. What is the magnetic field due to this current at a point P, 10 cm due north of the wire (Fig. 2)?

APPROACH We assume the wire is much longer than the 10-cm distance to the point P so we can apply Eq. 1.

SOLUTION According to Eq. 1:

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(25\,\mathrm{A})}{(2\pi)(0.10\,\mathrm{m})} = 5.0 \times 10^{-5}\,\mathrm{T},$$

or 0.50 G. By the right-hand rule, the field due to the current points to the west (into the page in Fig. 2) at point P.

NOTE The wire's field has about the same magnitude as Earth's magnetic field, so a compass at P would not point north but in a northwesterly direction.

NOTE Most electrical wiring in buildings consists of cables with two wires in each cable. Since the two wires carry current in opposite directions, their magnetic fields cancel to a large extent, but may still affect sensitive electronic devices.

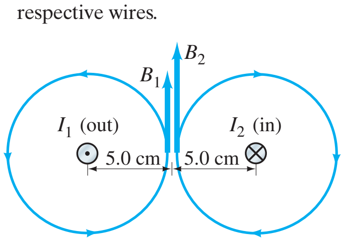
EXERCISE A We saw that a typical lightning bolt produces a 100-A current for 0.2 s. Estimate the magnetic field 10 m from a lightning bolt. Would it have a significant effect on a compass?

EXAMPLE 2 **Magnetic field midway between two currents.** Two parallel straight wires 10.0 cm apart carry currents in opposite directions (Fig. 3). Current $I_1 = 5.0\,\mathrm{A}$ is out of the page, and $I_2 = 7.0\,\mathrm{A}$ is into the page. Determine the magnitude and direction of the magnetic field halfway between the two wires.

APPROACH The magnitude of the field produced by each wire is calculated from Eq. 1. The direction of *each* wire's field is determined with the right-hand rule. The total field is the vector sum of the two fields at the midway point.

SOLUTION The magnetic field lines due to current I_1 form circles around the wire of I_1, and right-hand-rule-1 tells us they point counterclockwise around the wire. The field lines due to I_2 form circles around the wire of I_2 and point clockwise, Fig. 3. At the midpoint, both fields point upward as shown, and so add together.

FIGURE 3 Example 2. Wire 1 carrying current I_1 out towards us, and wire 2 carrying current I_2 into the page, produce magnetic fields whose lines are circles around their respective wires.



†The constant is chosen in this complicated way so that Ampère's law (Section 4), which is considered more fundamental, will have a simple and elegant form.

The midpoint is 0.050 m from each wire, and from Eq. 1 the magnitudes of B_1 and B_2 are

$$B_1 = \frac{\mu_0 I_1}{2\pi r} = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(5.0\,\mathrm{A})}{2\pi(0.050\,\mathrm{m})} = 2.0 \times 10^{-5}\,\mathrm{T};$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r} = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(7.0\,\mathrm{A})}{2\pi(0.050\,\mathrm{m})} = 2.8 \times 10^{-5}\,\mathrm{T}.$$

The total field is *up* with a magnitude of

$$B = B_1 + B_2 = 4.8 \times 10^{-5}\,\mathrm{T}.$$

EXERCISE B Suppose both I_1 and I_2 point into the page in Fig. 3. What then is the field midway between the two wires?

CONCEPTUAL EXAMPLE 3 **Magnetic field due to four wires.** Figure 4 shows four long parallel wires which carry equal currents into or out of the page as shown. In which configuration, (a) or (b), is the magnetic field greater at the center of the square?

RESPONSE It is greater in (a). The arrows illustrate the directions of the field produced by each wire; check it out, using the right-hand rule to confirm these results. The net field at the center is the superposition of the four fields, which will point to the left in (a) and is zero in (b).

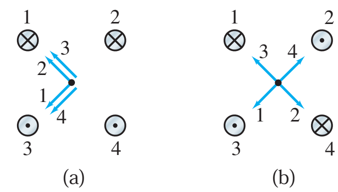


FIGURE 4 Example 3.

2 Force between Two Parallel Wires

We have seen that a wire carrying a current produces a magnetic field (magnitude given by Eq. 1 for a long straight wire). Also, a current-carrying wire feels a force when placed in a magnetic field. Thus, we expect that two current-carrying wires will exert a force on each other.

Consider two long parallel wires separated by a distance d, as in Fig. 5a. They carry currents I_1 and I_2, respectively. Each current produces a magnetic field that is "felt" by the other, so each must exert a force on the other. For example, the magnetic field B_1 produced by I_1 in Fig. 5 is given by Eq. 1, which at the location of wire 2 is

$$B_1 = \frac{\mu_0}{2\pi}\frac{I_1}{d}.$$

See Fig. 5b, where the field due *only* to I_1 is shown. According to $F_{\max} = I\ell B$, the force F_2 exerted by B_1 on a length ℓ_2 of wire 2, carrying current I_2, is

$$F_2 = I_2 B_1 \ell_2.$$

Note that the force on I_2 is due only to the field produced by I_1. Of course, I_2 also produces a field, but it does not exert a force on itself. We substitute B_1 into the formula for F_2 and find that the force on a length ℓ_2 of wire 2 is

$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2. \qquad \text{[parallel wires]} \qquad \textbf{(2)}$$

If we use right-hand-rule-1, we see that the lines of B_1 are as shown in Fig. 5b. Then using right-hand-rule-2, we see that the force exerted on I_2 will be to the left in Fig. 5b. That is, I_1 exerts an attractive force on I_2 (Fig. 6a). This is true as long as the currents are in the same direction. If I_2 is in the opposite direction, the right-hand rule indicates that the force is in the opposite direction. That is, I_1 exerts a repulsive force on I_2 (Fig. 6b).

Reasoning similar to that above shows that the magnetic field produced by I_2 exerts an equal but opposite force on I_1. We expect this to be true also from Newton's third law. Thus, as shown in Fig. 6, parallel currents in the same direction attract each other, whereas parallel currents in opposite directions repel.

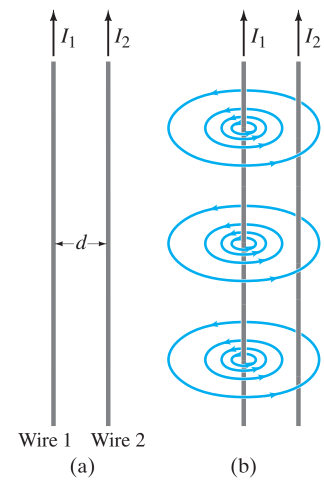
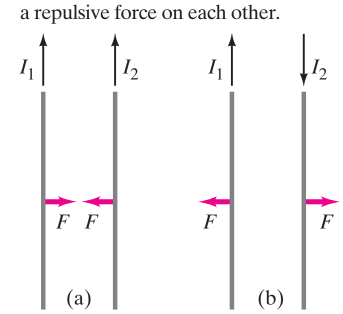


FIGURE 5 (a) Two parallel conductors carrying currents I_1 and I_2. (b) Magnetic field $\vec{\mathbf{B}}_1$ produced by I_1. (Field produced by I_2 is not shown.) $\vec{\mathbf{B}}_1$ points into page at position of I_2.

FIGURE 6 (a) Parallel currents in the same direction exert an attractive force on each other. (b) Antiparallel currents (in opposite directions) exert a repulsive force on each other.



EXAMPLE 4 **Force between two current-carrying wires.** The two wires of a 2.0-m-long appliance cord are 3.0 mm apart and carry a current of 8.0 A dc. Calculate the force one wire exerts on the other.

APPROACH Each wire is in the magnetic field of the other when the current is on, so we can apply Eq. 2.

SOLUTION Equation 2 gives

$$F = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(8.0\,\mathrm{A})^2(2.0\,\mathrm{m})}{(2\pi)(3.0 \times 10^{-3}\,\mathrm{m})} = 8.5 \times 10^{-3}\,\mathrm{N}.$$

The currents are in opposite directions (one toward the appliance, the other away from it), so the force would be repulsive and tend to spread the wires apart.

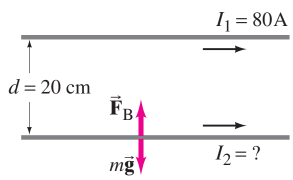


FIGURE 7 Example 5.

EXAMPLE 5 **Suspending a wire with a current.** A horizontal wire carries a current $I_1 = 80\,\mathrm{A}$ dc. A second parallel wire 20 cm below it (Fig. 7) must carry how much current I_2 so that it doesn't fall due to gravity? The lower wire has a mass of 0.12 g per meter of length.

APPROACH If wire 2 is not to fall under gravity, which acts downward, the magnetic force on it must be upward. This means that the current in the two wires must be in the same direction (Fig. 6). We can find the current I_2 by equating the magnitudes of the magnetic force and the gravitational force on the wire.

SOLUTION The force of gravity on wire 2 is downward. For each 1.0 m of wire length, the gravitational force has magnitude

$$F = mg = (0.12 \times 10^{-3}\,\mathrm{kg/m})(1.0\,\mathrm{m})(9.8\,\mathrm{m/s^2}) = 1.18 \times 10^{-3}\,\mathrm{N}.$$

The magnetic force on wire 2 must be upward, and Eq. 2 gives

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell$$

where $d = 0.20\,\mathrm{m}$ and $I_1 = 80\,\mathrm{A}$. We solve this for I_2 and set the two force magnitudes equal (letting $\ell = 1.0\,\mathrm{m}$):

$$I_2 = \frac{2\pi d}{\mu_0 I_1}\left(\frac{F}{\ell}\right) = \frac{2\pi(0.20\,\mathrm{m})}{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(80\,\mathrm{A})}\frac{(1.18 \times 10^{-3}\,\mathrm{N/m})}{(1.0\,\mathrm{m})} = 15\,\mathrm{A}.$$

3 Definitions of the Ampere and the Coulomb

You may have wondered how the constant μ_0 in Eq. 1 could be exactly $4\pi \times 10^{-7}\,\mathrm{T\cdot m/A}$. Here is how it happened. With an older definition of the ampere, μ_0 was measured experimentally to be very close to this value. Today, μ_0 is *defined* to be exactly $4\pi \times 10^{-7}\,\mathrm{T\cdot m/A}$. This could not be done if the ampere were defined independently. The ampere, the unit of current, is now defined in terms of the magnetic field B it produces using the defined value of μ_0.

In particular, we use the force between two parallel current-carrying wires, Eq. 2, to define the ampere precisely. If $I_1 = I_2 = 1\,\mathrm{A}$ exactly, and the two wires are exactly 1 m apart, then

$$\frac{F}{\ell} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})}{(2\pi)}\frac{(1\,\mathrm{A})(1\,\mathrm{A})}{(1\,\mathrm{m})} = 2 \times 10^{-7}\,\mathrm{N/m}.$$

Thus, *one* **ampere** *is defined as that current flowing in each of two long parallel wires 1 m apart, which results in a force of exactly 2×10^{-7} N per meter of length of each wire.*

This is the precise definition of the ampere. The **coulomb** is then defined as being *exactly* one ampere-second: $1\ \mathrm{C} = 1\ \mathrm{A\cdot s}$. The value of k or ϵ_0 in Coulomb's law is obtained from experiment.

This may seem a rather roundabout way of defining quantities. The reason behind it is the desire for **operational definitions** of quantities—that is, definitions of quantities that can actually be measured given a definite set of operations to carry out. For example, the unit of charge, the coulomb, could be defined in terms of the force between two equal charges after defining a value for ϵ_0 or k in Coulomb's law. However, to carry out an actual experiment to measure the force between two charges is very difficult. For one thing, any desired amount of charge is not easily obtained precisely; and charge tends to leak from objects into the air. The amount of current in a wire, on the other hand, can be varied accurately and continuously (by putting a variable resistor in a circuit). Thus the force between two current-carrying conductors is far easier to measure precisely. This is why we first define the ampere, and then define the coulomb in terms of the ampere. At the National Institute of Standards and Technology in Maryland, precise measurement of current is made using circular coils of wire rather than straight lengths because it is more convenient and accurate.

Electric and magnetic field strengths are also defined operationally: the electric field in terms of the measurable force on a charge, via $\vec{\mathbf{E}} = \dfrac{\vec{\mathbf{F}}}{q}$; and the magnetic field in terms of the force per unit length on a current-carrying wire, via $F_{\max} = I\ell B$.

4 Ampère's Law

In Section 1 we saw that Eq. 1 gives the relation between the current in a long straight wire and the magnetic field it produces. This equation is valid *only* for a long straight wire. Is there a general relation between a current in a wire of any shape and the magnetic field around it? The answer is yes: the French scientist André Marie Ampère (1775–1836) proposed such a relation shortly after Oersted's discovery. Consider an arbitrary closed path around a current as shown in Fig. 8, and imagine this path as being made up of short segments each of length $\Delta\ell$. First, we take the product of the length of each segment times the component of $\vec{\mathbf{B}}$ parallel to that segment (call this component $B_{\parallel}$). If we now sum all these terms, according to Ampère, the result will be equal to μ_0 times the net current I_{encl} that passes through the surface enclosed by the path:

$$\sum B_{\parallel}\,\Delta\ell = \mu_0 I_{\mathrm{encl}}.$$

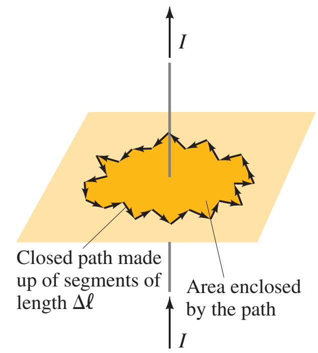
The lengths $\Delta\ell$ are chosen so that $B_{\parallel}$ is essentially constant along each length. The sum must be made over a *closed path*; and I_{encl} is the net current passing through the surface bounded by this closed path (orange in Fig. 8). In the limit $\Delta\ell \to 0$, this relation becomes

AMPÈRE'S LAW

$$\oint \vec{\mathbf{B}}\cdot d\vec{\boldsymbol{\ell}} = \mu_0 I_{\mathrm{encl}}, \qquad (3)$$

where $d\vec{\boldsymbol{\ell}}$ is an infinitesimal length vector and the vector dot product assures that the parallel component of $\vec{\mathbf{B}}$ is taken. Equation 3 is known as **Ampère's law**. The integrand in Eq. 3 is taken around a closed path, and I_{encl} is the current passing through the space enclosed by the chosen path or loop.

FIGURE 8 Arbitrary path enclosing a current, for Ampère's law. The path is broken down into segments of equal length $\Delta\ell$.



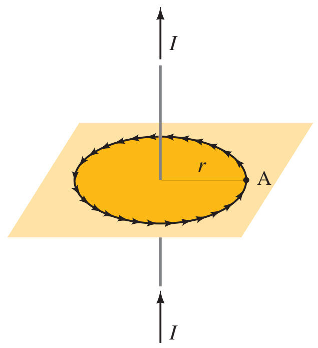


FIGURE 9 Circular path of radius r.

To understand Ampère's law better, let us apply it to the simple case of a single long straight wire carrying a current I which we've already examined, and which served as an inspiration for Ampère himself. Suppose we want to find the magnitude of $\vec{\mathbf{B}}$ at some point A which is a distance r from the wire (Fig. 9). We know the magnetic field lines are circles with the wire at their center. So to apply Eq. 3 we choose as our path of integration a circle of radius r. The choice of path is ours, so we choose one that will be convenient: at any point on this circular path, $\vec{\mathbf{B}}$ will be tangent to the circle. Furthermore, since all points on the path are the same distance from the wire, by symmetry we expect B to have the same magnitude at each point. Thus for any short segment of the circle (Fig. 9), $\vec{\mathbf{B}}$ will be parallel to that segment, and (setting $I_{\text{encl}} = I$)

$$\begin{aligned}\mu_0 I &= \oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} \\ &= \oint B\, d\ell = B \oint d\ell = B(2\pi r),\end{aligned}$$

where $\oint d\ell = 2\pi r$, the circumference of the circle. We solve for B and obtain

$$B = \frac{\mu_0 I}{2\pi r}.$$

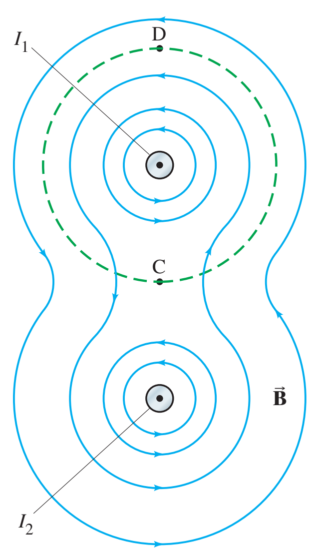
This is just Eq. 1 for the field near a long straight wire as discussed earlier.

Ampère's law thus works for this simple case. A great many experiments indicate that Ampère's law is valid in general. However, as with Gauss's law for the electric field, its practical value as a means to calculate the magnetic field is limited mainly to simple or symmetric situations. Its importance is that it relates the magnetic field to the current in a direct and mathematically elegant way. Ampère's law is thus considered one of the basic laws of electricity and magnetism. It is valid for any situation where the currents and fields are steady and not changing in time, and no magnetic materials are present.

We now can see why the constant in Eq. 1 is written $\mu_0/2\pi$. This is done so that only μ_0 appears in Eq. 3, rather than, say, $2\pi k$ if we had used k in Eq. 1. In this way, the more fundamental equation, Ampère's law, has the simpler form.

It should be noted that the $\vec{\mathbf{B}}$ in Ampère's law is not necessarily due only to the current I_{encl}. Ampère's law, like Gauss's law for the electric field, is valid in general. $\vec{\mathbf{B}}$ is the field at each point in space along the chosen path due to all sources—including the current I enclosed by the path, but also due to any other sources. For example, the field surrounding two parallel current-carrying wires is the vector sum of the fields produced by each, and the field lines are shown in Fig. 10. If the path chosen for the integral (Eq. 3) is a circle centered on one of the wires with radius less than the distance between the wires (the dashed line in Fig. 10), only the current (I_1) in the encircled wire is included on the right side of Eq. 3. $\vec{\mathbf{B}}$ on the left side of the equation must be the total $\vec{\mathbf{B}}$ at each point due to both wires. Note also that $\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}}$ for the path shown in Fig. 10 is the same whether the second wire is present or not (in both cases, it equals $\mu_0 I_1$ according to Ampère's law). How can this be? It can be so because the fields due to the two wires partially cancel one another at some points between them, such as point C in the diagram ($\vec{\mathbf{B}} = 0$ at a point midway between the wires if $I_1 = I_2$); at other points, such as D in Fig. 10, the fields add together to produce a larger field. In the *sum*, $\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}}$, these effects just balance so that $\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 I_1$, whether the second wire is there or not. The integral $\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}}$ will be the same in each case, even though $\vec{\mathbf{B}}$ will not be the same at every point for each of the two cases.

FIGURE 10 Magnetic field lines around two long parallel wires whose equal currents, I_1 and I_2, are coming out of the paper toward the viewer.



EXAMPLE 6 **Field inside and outside a wire.** A long straight cylindrical wire conductor of radius R carries a current I of uniform current density in the conductor. Determine the magnetic field due to this current at (*a*) points outside the conductor ($r > R$), and (*b*) points inside the conductor ($r < R$). See Fig. 11. Assume that r, the radial distance from the axis, is much less than the length of the wire. (*c*) If $R = 2.0\,\text{mm}$ and $I = 60\,\text{A}$, what is B at $r = 1.0\,\text{mm}$, $r = 2.0\,\text{mm}$, and $r = 3.0\,\text{mm}$?

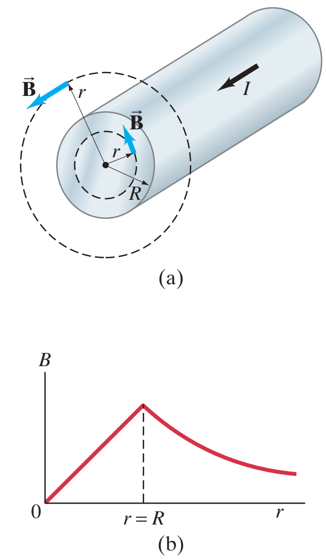


FIGURE 11 Magnetic field inside and outside a cylindrical conductor (Example 6).

APPROACH We can use *symmetry*: Because the wire is long, straight, and cylindrical, we expect from symmetry that the magnetic field must be the same at all points that are the same distance from the center of the conductor. There is no reason why any such point should have preference over others at the same distance from the wire (they are physically equivalent). So B must have the same value at all points the same distance from the center. We also expect $\vec{\mathbf{B}}$ to be tangent to circles around the wire (Fig. 1), so we choose a circular path of integration as we did in Fig. 9.

SOLUTION (*a*) We apply Ampère's law, integrating around a circle ($r > R$) centered on the wire (Fig. 11a), and then $I_{\text{encl}} = I$:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = B(2\pi r) = \mu_0 I_{\text{encl}}$$

or

$$B = \frac{\mu_0 I}{2\pi r}, \qquad [r > R]$$

which is the same result as for a thin wire.

(*b*) Inside the wire ($r < R$), we again choose a circular path concentric with the cylinder; we expect $\vec{\mathbf{B}}$ to be tangential to this path, and again, because of the symmetry, it will have the same magnitude at all points on the circle. The current enclosed in this case is less than I by a factor of the ratio of the areas:

$$I_{\text{encl}} = I\,\frac{\pi r^2}{\pi R^2}.$$

So Ampère's law gives

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 I_{\text{encl}}$$

$$B(2\pi r) = \mu_0 I\left(\frac{\pi r^2}{\pi R^2}\right)$$

so

$$B = \frac{\mu_0 I r}{2\pi R^2}. \qquad [r < R]$$

The field is zero at the center of the conductor and increases linearly with r until $r = R$; beyond $r = R$, B decreases as $1/r$. This is shown in Fig. 11b. Note that these results are valid only for points close to the center of the conductor as compared to its length. For a current to flow, there must be connecting wires (to a battery, say), and the field due to these conducting wires, if not very far away, will destroy the assumed symmetry.

CAUTION
Connecting wires can destroy assumed symmetry

(*c*) At $r = 2.0\,\text{mm}$, the surface of the wire, $r = R$, so

$$B = \frac{\mu_0 I}{2\pi R} = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(60\,\text{A})}{(2\pi)(2.0 \times 10^{-3}\,\text{m})} = 6.0 \times 10^{-3}\,\text{T}.$$

We saw in (*b*) that inside the wire B is linear in r. So at $r = 1.0\,\text{mm}$, B will be half what it is at $r = 2.0\,\text{mm}$ or $3.0 \times 10^{-3}\,\text{T}$. Outside the wire, B falls off as $1/r$, so at $r = 3.0\,\text{mm}$ it will be two-thirds as great as at $r = 2.0\,\text{mm}$, or $B = 4.0 \times 10^{-3}\,\text{T}$. To check, we use our result in (*a*), $B = \mu_0 I/2\pi r$, which gives the same result.

PHYSICS APPLIED
Coaxial cable (shielding)

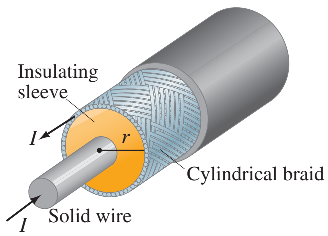


FIGURE 12 Coaxial cable. Example 7.

FIGURE 13 Exercise C.

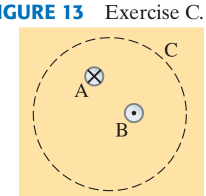
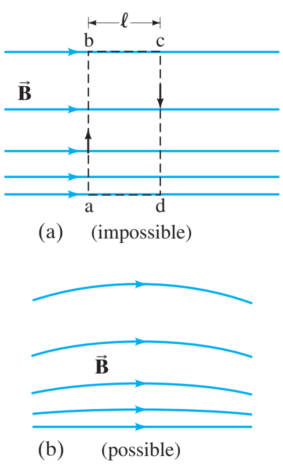


FIGURE 14 Example 8.



CONCEPTUAL EXAMPLE 7 **Coaxial cable.** A *coaxial cable* is a single wire surrounded by a cylindrical metallic braid, as shown in Fig. 12. The two conductors are separated by an insulator. The central wire carries current to the other end of the cable, and the outer braid carries the return current and is usually considered ground. Describe the magnetic field (*a*) in the space between the conductors, and (*b*) outside the cable.

RESPONSE (*a*) In the space between the conductors, we can apply Ampère's law for a circular path around the center wire, just as we did for the case shown in Figs. 9 and 11. The magnetic field lines will be concentric circles centered on the center of the wire, and the magnitude is given by Eq. 1. The current in the outer conductor has no bearing on this result. (Ampère's law uses only the current enclosed *inside* the path; as long as the currents outside the path don't affect the *symmetry* of the field, they do not contribute to the field along the path at all). (*b*) Outside the cable, we can draw a similar circular path, for we expect the field to have the same cylindrical symmetry. Now, however, there are two currents enclosed by the path, and they add up to zero. The field outside the cable is zero.

The nice feature of coaxial cables is that they are self-shielding: no stray magnetic fields exist outside the cable. The outer cylindrical conductor also shields external electric fields from coming in. This makes them ideal for carrying signals near sensitive equipment. Audiophiles use coaxial cables between stereo equipment components and even to the loudspeakers.

EXERCISE C In Fig. 13, A and B are wires each carrying a 3.0-A current but in opposite directions. On the circle C, which statement is true? (*a*) $B = 0$; (*b*) $\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = 0$; (*c*) $B = 3\mu_0$; (*d*) $B = -3\mu_0$; (*e*) $\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = 6\mu_0$.

EXAMPLE 8 **A nice use for Ampère's law.** Use Ampère's law to show that in any region of space where there are no currents the magnetic field cannot be both unidirectional and nonuniform as shown in Fig. 14a.

APPROACH The wider spacing of lines near the top of Fig. 14a indicates the field $\vec{\mathbf{B}}$ has a smaller magnitude at the top than it does lower down. We apply Ampère's law to the rectangular path abcd shown dashed in Fig. 14a.

SOLUTION Because no current is enclosed by the chosen path, Ampère's law gives

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = 0.$$

The integral along sections ab and cd is zero, since $\vec{\mathbf{B}} \perp d\vec{\boldsymbol{\ell}}$. Thus

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = B_{\mathrm{bc}}\ell - B_{\mathrm{da}}\ell = (B_{\mathrm{bc}} - B_{\mathrm{da}})\ell,$$

which is not zero since the field B_{bc} along the path bc is less than the field B_{da} along path da. Hence we have a contradiction: $\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}}$ cannot be both zero (since $I = 0$) and nonzero. Thus we have shown that a nonuniform unidirectional field is not consistent with Ampère's law. A nonuniform field whose direction also changes, as in Fig. 14b, is consistent with Ampère's law (convince yourself this is so), and possible. The fringing of a permanent magnet's field has this shape.

PROBLEM SOLVING

Ampère's Law

1. Ampère's law, like Gauss's law, is always a valid statement. But as a calculation tool it is limited mainly to systems with a high degree of symmetry. The first step in applying Ampère's law is to identify useful **symmetry**.
2. Choose an integration path that reflects the symmetry (see the Examples). Search for paths where B has constant magnitude along the entire path or along segments of the path. Make sure your integration path passes through the point where you wish to evaluate the magnetic field.
3. Use symmetry to determine the direction of $\vec{\mathbf{B}}$ along the integration path. With a smart choice of path, $\vec{\mathbf{B}}$ will be either parallel or perpendicular to the path.
4. Determine the enclosed current, I_{encl}. Be careful with signs. Let the fingers of your right hand curl along the direction of $\vec{\mathbf{B}}$ so that your thumb shows the direction of positive current. If you have a solid conductor and your integration path does not enclose the full current, you can use the current density (current per unit area) multiplied by the enclosed area (as in Example 6).

5 Magnetic Field of a Solenoid and a Toroid

A long coil of wire consisting of many loops is called a **solenoid**. Each loop produces a magnetic field as was shown. In Fig. 15a, we see the field due to a solenoid when the coils are far apart. Near each wire, the field lines are very nearly circles as for a straight wire (that is, at distances that are small compared to the curvature of the wire). Between any two wires, the fields due to each loop tend to cancel. Toward the center of the solenoid, the fields add up to give a field that can be fairly large and fairly uniform. For a long solenoid with closely packed coils, the field is nearly uniform and parallel to the solenoid axis within the entire cross section, as shown in Fig. 15b. The field outside the solenoid is very small compared to the field inside, except near the ends. Note that the same number of field lines that are concentrated inside the solenoid, spread out into the vast open space outside.

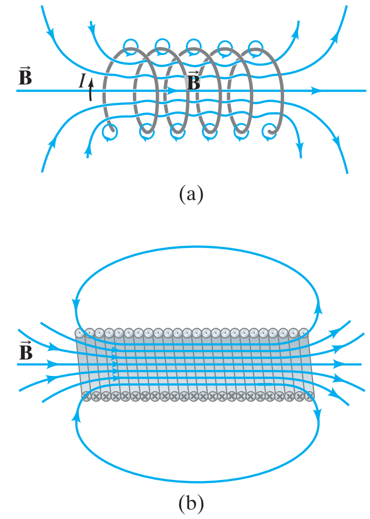


FIGURE 15 Magnetic field due to a solenoid: (a) loosely spaced turns, (b) closely spaced turns.

We now use Ampère's law to determine the magnetic field inside a very long (ideally, infinitely long) closely packed solenoid. We choose the path abcd shown in Fig. 16, far from either end, for applying Ampère's law. We will consider this path as made up of four segments, the sides of the rectangle: ab, bc, cd, da. Then the left side of Eq. 3, Ampère's law, becomes

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \int_a^b \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} + \int_b^c \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} + \int_c^d \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} + \int_d^a \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}}.$$

The field outside the solenoid is so small as to be negligible compared to the field inside. Thus the first term in this sum will be zero. Furthermore, $\vec{\mathbf{B}}$ is perpendicular to the segments bc and da inside the solenoid, and is nearly zero between and outside the coils,

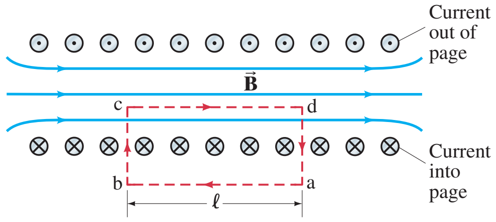


FIGURE 16 Cross-sectional view into a solenoid. The magnetic field inside is straight except at the ends. Red dashed lines indicate the path chosen for use in Ampère's law. $\odot$ and $\otimes$ are electric current direction (in the wire loops) out of the page and into the page.

so these terms too are zero. Therefore we have reduced the integral to the segment cd where $\vec{\mathbf{B}}$ is the nearly uniform field inside the solenoid, and is parallel to $d\vec{\boldsymbol{\ell}}$, so

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \int_c^d \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = B\ell,$$

where ℓ is the length cd. Now we determine the current enclosed by this loop for the right side of Ampère's law, Eq. 3. If a current I flows in the wire of the solenoid, the total current enclosed by our path abcd is NI where N is the number of loops our path encircles (five in Fig. 16). Thus Ampère's law gives us

$$B\ell = \mu_0 NI.$$

If we let $n = N/\ell$ be the *number of loops per unit length*, then

$$B = \mu_0 nI. \qquad \text{[solenoid]} \quad \textbf{(4)}$$

This is the magnitude of the magnetic field within a solenoid. Note that B depends only on the number of loops per unit length, n, and the current I. The field does not depend on position within the solenoid, so B is uniform. This is strictly true only for an infinite solenoid, but it is a good approximation for real ones for points not close to the ends.

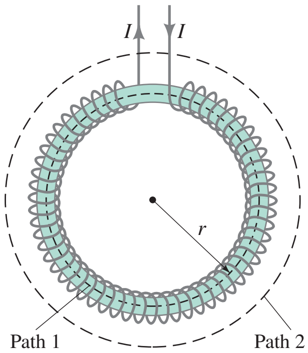
EXAMPLE 9 **Field inside a solenoid.** A thin 10-cm-long solenoid used for fast electromechanical switching has a total of 400 turns of wire and carries a current of 2.0 A. Calculate the field inside near the center.

APPROACH We use Eq. 4, where the number of turns per unit length is $n = 400/0.10\,\text{m} = 4.0 \times 10^3\,\text{m}^{-1}$.

SOLUTION $B = \mu_0 nI = (4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(4.0 \times 10^3\,\text{m}^{-1})(2.0\,\text{A}) = 1.0 \times 10^{-2}\,\text{T}.$

A close look at Fig. 15 shows that the field outside of a solenoid is much like that of a bar magnet. Indeed, a solenoid acts like a magnet, with one end acting as a north pole and the other as south pole, depending on the direction of the current in the loops. Since magnetic field lines leave the north pole of a magnet, the north poles of the solenoids in Fig. 15 are on the right.

Solenoids have many practical applications, and we discuss some of them later in the Chapter, in Section 8.



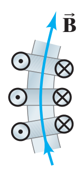


FIGURE 17 (a) A toroid. (b) A section of the toroid showing direction of the current for three loops: ⊙ means current toward you, ⊗ means current away from you.

EXAMPLE 10 Toroid. Use Ampère's law to determine the magnetic field (*a*) inside and (*b*) outside a toroid, which is like a solenoid bent into the shape of a circle as shown in Fig. 17a.

APPROACH The magnetic field lines inside the toroid will be circles concentric with the toroid. (If you think of the toroid as a solenoid bent into a circle, the field lines bend along with the solenoid.) The direction of $\vec{\mathbf{B}}$ is clockwise. We choose as our path of integration one of these field lines of radius r inside the toroid as shown by the dashed line labeled "path 1" in Fig. 17a. We make this choice to use the *symmetry* of the situation, so B will be tangent to the path and will have the same magnitude at all points along the path (although it is not necessarily the same across the whole cross section of the toroid). This chosen path encloses *all* the coils; if there are N coils, each carrying current I, then $I_{\text{encl}} = NI$.

SOLUTION (*a*) Ampère's law applied along this path gives

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 I_{\text{encl}}$$

$$B(2\pi r) = \mu_0 NI,$$

where N is the total number of coils and I is the current in each of the coils. Thus

$$B = \frac{\mu_0 NI}{2\pi r}.$$

The magnetic field B is not uniform within the toroid: it is largest along the inner edge (where r is smallest) and smallest at the outer edge. However, if the toroid is large, but thin (so that the difference between the inner and outer radii is small compared to the average radius), the field will be essentially uniform within the toroid. In this case, the formula for B reduces to that for a straight solenoid $B = \mu_0 nI$ where $n = N/(2\pi r)$ is the number of coils per unit length. (*b*) Outside the toroid, we choose as our path of integration a circle concentric with the toroid, "path 2" in Fig. 17a. This path encloses N loops carrying current I in one direction and N loops carrying the same current in the opposite direction. (Figure 17b shows the directions of the current for the parts of the loop on the inside and outside of the toroid.) Thus the net current enclosed by path 2 is zero. For a very tightly packed toroid, all points on path 2 are equidistant from the toroid and equivalent, so we expect B to be the same at all points along the path. Hence, Ampère's law gives

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 I_{\text{encl}}$$

$$B(2\pi r) = 0$$

or

$$B = 0.$$

The same is true for a path taken at a radius smaller than that of the toroid. So there is no field exterior to a very tightly wound toroid. It is all inside the loops.

6 Biot-Savart Law

The usefulness of Ampère's law for determining the magnetic field $\vec{\mathbf{B}}$ due to particular electric currents is restricted to situations where the symmetry of the given currents allows us to evaluate $\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}}$ readily. This does not, of course, invalidate Ampère's law nor does it reduce its fundamental importance. Recall the electric case, where Gauss's law is considered fundamental but is limited in its use for actually calculating $\vec{\mathbf{E}}$. We must often determine the electric field $\vec{\mathbf{E}}$ by another method summing over contributions due to infinitesimal charge elements dq via Coulomb's law: $dE = (1/4\pi\epsilon_0)(dq/r^2)$. A magnetic equivalent to this infinitesimal form of Coulomb's law would be helpful for currents that do not have great symmetry. Such a law was developed by Jean Baptiste Biot (1774–1862) and Felix Savart (1791–1841) shortly after Oersted's discovery in 1820 that a current produces a magnetic field.

According to Biot and Savart, a current I flowing in any path can be considered as many tiny (infinitesimal) current elements, such as in the wire of Fig. 18. If $d\vec{\boldsymbol{\ell}}$ represents any infinitesimal length along which the current is flowing, then the magnetic field, $d\vec{\mathbf{B}}$, at any point P in space, due to this element of current, is given by

$$d\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\boldsymbol{\ell}} \times \hat{\mathbf{r}}}{r^2}, \qquad (5)$$

Biot-Savart law

where $\vec{\mathbf{r}}$ is the displacement vector from the element $d\vec{\boldsymbol{\ell}}$ to the point P, and $\hat{\mathbf{r}} = \vec{\mathbf{r}}/r$ is the unit vector (magnitude = 1) in the direction of $\vec{\mathbf{r}}$ (see Fig. 18).

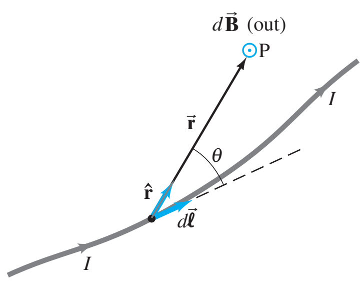


FIGURE 18 Biot-Savart law: the field at P due to current element $I\,d\vec{\boldsymbol{\ell}}$ is $d\vec{\mathbf{B}} = (\mu_0 I/4\pi)(d\vec{\boldsymbol{\ell}} \times \hat{\mathbf{r}}/r^2)$.

Equation 5 is known as the **Biot-Savart law**. The magnitude of $d\vec{\mathbf{B}}$ is

$$dB = \frac{\mu_0 I\, d\ell \sin\theta}{4\pi r^2}, \qquad (6)$$

where θ is the angle between $d\vec{\boldsymbol{\ell}}$ and $\vec{\mathbf{r}}$ (Fig. 18). The total magnetic field at point P is then found by summing (integrating) over all current elements:

$$\vec{\mathbf{B}} = \int d\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\boldsymbol{\ell}} \times \hat{\mathbf{r}}}{r^2}.$$

Note that this is a *vector* sum. The Biot-Savart law is the magnetic equivalent of Coulomb's law in its infinitesimal form. It is even an inverse square law, like Coulomb's law.

An important difference between the Biot-Savart law and Ampère's law (Eq. 3) is that in Ampère's law $\left[\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 I_{\text{encl}}\right]$, $\vec{\mathbf{B}}$ is not necessarily due only to the current enclosed by the path of integration. But in the Biot-Savart law the field $d\vec{\mathbf{B}}$ in Eq. 5 is due only, and entirely, to the current element $I\,d\vec{\boldsymbol{\ell}}$. To find the total $\vec{\mathbf{B}}$ at any point in space, it is necessary to include *all* currents.

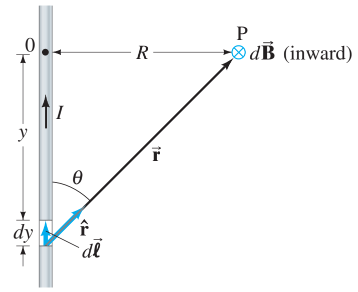


FIGURE 19 Determining $\vec{\mathbf{B}}$ due to a long straight wire using the Biot-Savart law.

EXAMPLE 11 **$\vec{\mathbf{B}}$ due to current I in straight wire.** For the field near a long straight wire carrying a current I, show that the Biot-Savart law gives the same result as Eq. 1, $B = \mu_0 I/2\pi r$.

APPROACH We calculate the magnetic field in Fig. 19 at point P, which is a perpendicular distance R from an infinitely long wire. The current is moving upwards, and both $d\vec{\boldsymbol{\ell}}$ and $\hat{\mathbf{r}}$, which appear in the cross product of Eq. 5, are in the plane of the page. Hence the direction of the field $d\vec{\mathbf{B}}$ due to each element of current must be directed into the plane of the page as shown (right-hand rule for the cross product $d\vec{\boldsymbol{\ell}} \times \hat{\mathbf{r}}$). Thus all the $d\vec{\mathbf{B}}$ have the same direction at point P, and add up to give $\vec{\mathbf{B}}$ the same direction consistent with our previous results (Figs. 1 and 11).

SOLUTION The magnitude of $\vec{\mathbf{B}}$ will be

$$B = \frac{\mu_0 I}{4\pi} \int_{y=-\infty}^{+\infty} \frac{dy \sin\theta}{r^2},$$

where $dy = d\ell$ and $r^2 = R^2 + y^2$. Note that we are integrating over y (the length of the wire) so R is considered constant. Both y and θ are variables, but they are not independent. In fact, $y = -R/\tan\theta$. Note that we measure y as positive upward from point 0, so for the current element we are considering $y < 0$. Then

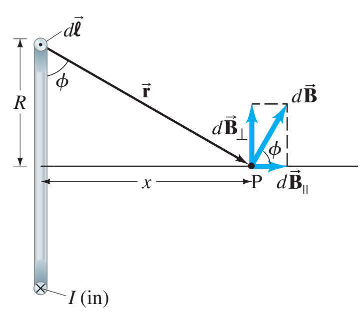
$$dy = +R\csc^2\theta\, d\theta = \frac{R\, d\theta}{\sin^2\theta} = \frac{R\, d\theta}{(R/r)^2} = \frac{r^2\, d\theta}{R}.$$

From Fig. 19 we can see that $y = -\infty$ corresponds to $\theta = 0$ and that $y = +\infty$ corresponds to $\theta = \pi$ radians. So our integral becomes

$$B = \frac{\mu_0 I}{4\pi}\frac{1}{R}\int_{\theta=0}^{\pi} \sin\theta\, d\theta = -\frac{\mu_0 I}{4\pi R}\cos\theta\Big|_0^{\pi} = \frac{\mu_0 I}{2\pi R}.$$

This is just Eq. 1 for the field near a long wire, where R has been used instead of r.

FIGURE 20 Determining $\vec{\mathbf{B}}$ due to a current loop.



EXAMPLE 12 **Current loop.** Determine $\vec{\mathbf{B}}$ for points on the axis of a circular loop of wire of radius R carrying a current I, Fig. 20.

APPROACH For an element of current at the top of the loop, the magnetic field $d\vec{\mathbf{B}}$ at point P on the axis is perpendicular to $\vec{\mathbf{r}}$ as shown, and has magnitude (Eq. 5)

$$dB = \frac{\mu_0 I\, d\ell}{4\pi r^2}$$

since $d\vec{\boldsymbol{\ell}}$ is perpendicular to $\vec{\mathbf{r}}$ so $|d\vec{\boldsymbol{\ell}} \times \hat{\mathbf{r}}| = d\ell$. We can break $d\vec{\mathbf{B}}$ down into components $dB_\parallel$ and $dB_\perp$, which are parallel and perpendicular to the axis as shown.

SOLUTION When we sum over all the elements of the loop, *symmetry* tells us that the perpendicular components will cancel on opposite sides, so $B_\perp = 0$. Hence, the total $\vec{\mathbf{B}}$ will point along the axis, and will have magnitude

$$B = B_\parallel = \int dB\cos\phi = \int dB\,\frac{R}{r} = \int dB\,\frac{R}{(R^2 + x^2)^{\frac{1}{2}}},$$

where x is the distance of P from the center of the ring, and $r^2 = R^2 + x^2$. Now we put in dB from the equation above and integrate around the current loop, noting that all segments $d\vec{\boldsymbol{\ell}}$ of current are the same distance, $(R^2 + x^2)^{\frac{1}{2}}$, from point P:

$$B = \frac{\mu_0 I}{4\pi}\frac{R}{(R^2 + x^2)^{\frac{3}{2}}}\int d\ell = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{\frac{3}{2}}}$$

since $\int d\ell = 2\pi R$, the circumference of the loop.

NOTE At the very center of the loop (where $x = 0$) the field has its maximum value

$$B = \frac{\mu_0 I}{2R}. \qquad \text{[at center of current loop]}$$

Recall that a current loop, such as that just discussed (Fig. 20), is considered a **magnetic dipole**. We saw there that a current loop has a magnetic dipole moment

$$\mu = NIA,$$

where A is the area of the loop and N is the number of coils in the loop, each carrying current I. A magnetic dipole placed in an external magnetic field experiences a torque and possesses potential energy, just like an electric dipole. In Example 12, we looked at another aspect of a magnetic dipole: the magnetic field *produced by* a magnetic dipole has magnitude, along the dipole axis, of

$$B = \frac{\mu_0 IR^2}{2\left(R^2 + x^2\right)^{\frac{3}{2}}}.$$

We can write this in terms of the magnetic dipole moment $\mu = IA = I\pi R^2$ (for a single loop $N = 1$):

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{\left(R^2 + x^2\right)^{\frac{3}{2}}}. \qquad \text{[magnetic dipole]} \quad \textbf{(7a)}$$

(Be careful to distinguish μ for dipole moment from μ_0, the magnetic permeability constant.) For distances far from the loop, $x \gg R$, this becomes

$$B \approx \frac{\mu_0}{2\pi}\frac{\mu}{x^3}. \qquad \left[\begin{matrix}\text{on axis,}\\ \text{magnetic dipole, } x \gg R\end{matrix}\right] \quad \textbf{(7b)}$$

The magnetic field on the axis of a magnetic dipole decreases with the cube of the distance, just as the electric field does for an electric dipole. B decreases as the cube of the distance also for points not on the axis, although the multiplying factor is not the same. The magnetic field due to a current loop can be determined at various points using the Biot-Savart law and the results are in accord with experiment. The field lines around a current loop are shown in Fig. 21.

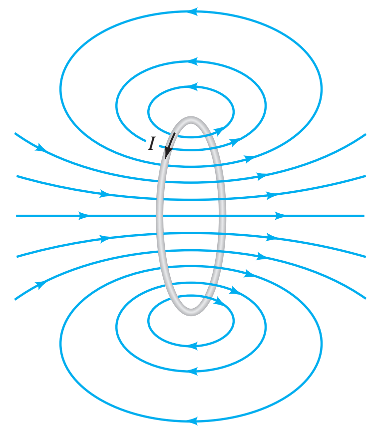
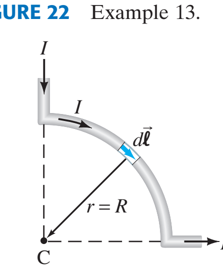


FIGURE 21 Magnetic field due to a circular loop of wire.

EXAMPLE 13 $\vec{\mathbf{B}}$ **due to a wire segment.** One quarter of a circular loop of wire carries a current I as shown in Fig. 22. The current I enters and leaves on straight segments of wire, as shown; the straight wires are along the radial direction from the center C of the circular portion. Find the magnetic field at point C.

FIGURE 22 Example 13.



APPROACH The current in the straight sections produces no magnetic field at point C because $d\vec{\boldsymbol{\ell}}$ and $\hat{\mathbf{r}}$ in the Biot-Savart law (Eq. 5) are parallel and therefore $d\vec{\boldsymbol{\ell}} \times \hat{\mathbf{r}} = 0$. Each piece $d\vec{\boldsymbol{\ell}}$ of the curved section of wire produces a field $d\vec{\mathbf{B}}$ that points into the page at C (right-hand rule).

SOLUTION The magnitude of each $d\vec{\mathbf{B}}$ due to each $d\ell$ of the circular portion of wire is (Eq. 6)

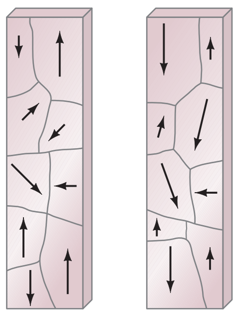
$$dB = \frac{\mu_0 I\, d\ell}{4\pi R^2}$$

where $r = R$ is the radius of the curved section, and $\sin\theta$ in Eq. 6 is $\sin 90° = 1$. With $r = R$ for all pieces $d\vec{\boldsymbol{\ell}}$, we integrate over a quarter of a circle.

$$B = \int dB = \frac{\mu_0 I}{4\pi R^2}\int d\ell = \frac{\mu_0 I}{4\pi R^2}\left(\frac{1}{4}2\pi R\right) = \frac{\mu_0 I}{8R}.$$

7 Magnetic Materials—Ferromagnetism

Magnetic fields can be produced (1) by magnetic materials (magnets) and (2) by electric currents. Common magnetic materials include ordinary magnets, iron cores in motors and electromagnets, recording tape, computer hard drives and magnetic stripes on credit cards. Iron (and a few other materials) can be made into strong magnets. These materials are said to be **ferromagnetic**. We now look into the sources of ferromagnetism.



(a) (b)

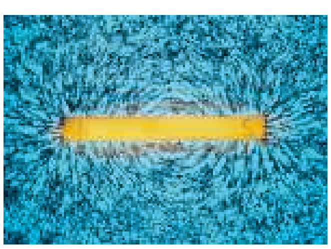
FIGURE 23 (a) An unmagnetized piece of iron is made up of domains that are randomly arranged. Each domain is like a tiny magnet; the arrows represent the magnetization direction, with the arrowhead being the N pole. (b) In a magnet, the domains are preferentially aligned in one direction (down in this case), and may be altered in size by the magnetization process.

A bar magnet, with its two opposite poles near either end, resembles an electric dipole (equal-magnitude positive and negative charges separated by a distance). Indeed, a bar magnet is sometimes referred to as a "magnetic dipole." There are opposite "poles" separated by a distance. And the magnetic field lines of a bar magnet form a pattern much like that for the electric field of an electric dipole.

Microscopic examination reveals that a piece of iron is made up of tiny regions known as **domains**, less than 1 mm in length or width. Each domain behaves like a tiny magnet with a north and a south pole. In an unmagnetized piece of iron, these domains are arranged randomly, as shown in Fig. 23a. The magnetic effects of the domains cancel each other out, so this piece of iron is not a magnet. In a magnet, the domains are preferentially aligned in one direction as shown in Fig. 23b (downward in this case). A magnet can be made from an unmagnetized piece of iron by placing it in a strong magnetic field. (You can make a needle magnetic, for example, by stroking it with one pole of a strong magnet.) The magnetization direction of domains may actually rotate slightly to be more nearly parallel to the external field, and the borders of domains may move so domains with magnetic orientation parallel to the external field grow larger (compare Figs. 23a and b).

We can now explain how a magnet can pick up unmagnetized pieces of iron like paper clips. The field of the magnet's south pole (say) causes a slight realignment of the domains in the unmagnetized object, which then becomes a temporary magnet with its north pole facing the south pole of the permanent magnet; thus, attraction results. Similarly, elongated iron filings in a magnetic field acquire aligned domains and align themselves to reveal the shape of the magnetic field, Fig. 24.

FIGURE 24 Iron filings line up along magnetic field lines due to a permanent magnet.



Richard Megna/Fundamental Photographs, NYC

An iron magnet can remain magnetized for a long time, and is referred to as a "permanent magnet." But if you drop a magnet on the floor or strike it with a hammer, you can jar the domains into randomness and the magnet loses some or all of its magnetism. Heating a permanent magnet can also cause loss of magnetism, for raising the temperature increases the random thermal motion of atoms, which tends to randomize the domains. Above a certain temperature known as the **Curie temperature** (1043 K for iron), a magnet cannot be made at all. Iron, nickel, cobalt, gadolinium, and certain alloys are ferromagnetic at room temperature; several other elements and alloys have low Curie temperature and thus are ferromagnetic only at low temperatures. Most other metals, such as aluminum and copper, do not show any noticeable magnetic effect (but see Section 10).

The striking similarity between the fields produced by a bar magnet and by a loop of electric current offers a clue that perhaps magnetic fields produced by electric currents may have something to do with ferromagnetism. According to modern atomic theory, atoms can be visualized as having electrons that orbit around a central nucleus. The electrons are charged, and so constitute an electric current and therefore produce a magnetic field; but the fields due to orbiting electrons generally all add up to zero. Electrons themselves produce an additional magnetic field, as if they and their electric charge were spinning about their own axes. It is the magnetic field due to electron **spin**† that is believed to produce ferromagnetism in most ferromagnetic materials.

CAUTION

$\vec{\mathbf{B}}$ lines form closed loops, $\vec{\mathbf{E}}$ start on ⊕ and end on ⊖

It is believed today that *all* magnetic fields are caused by electric currents. This means that magnetic field lines always form closed loops, unlike electric field lines which begin on positive charges and end on negative charges.

†The name "spin" comes from an early suggestion that this intrinsic magnetic moment arises from the electron "spinning" on its axis (as well as "orbiting" the nucleus) to produce the extra field. However this view of a spinning electron is oversimplified and not valid.

EXERCISE D Return to the Chapter-Opening Question and answer it again now. Try to explain why you may have answered differently the first time.

*8 Electromagnets and Solenoids—Applications

A long coil of wire consisting of many loops of wire, as discussed in Section 5, is called a solenoid. The magnetic field within a solenoid can be fairly large since it will be the sum of the fields due to the current in each loop (see Fig. 25). The solenoid acts like a magnet; one end can be considered the north pole and the other the south pole, depending on the direction of the current in the loops (use the right-hand rule). Since the magnetic field lines leave the north pole of a magnet, the north pole of the solenoid in Fig. 25 is on the right.

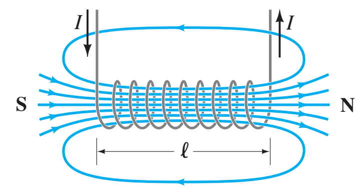


FIGURE 25 Magnetic field of a solenoid. The north pole of this solenoid, thought of as a magnet, is on the right, and the south pole is on the left.

If a piece of iron is placed inside a solenoid, the magnetic field is increased greatly because the domains of the iron are aligned by the magnetic field produced by the current. The resulting magnetic field is the sum of that due to the current and that due to the iron, and can be hundreds or thousands of times larger than that due to the current alone (see Section 9). This arrangement is called an **electromagnet**. The alloys of iron used in electromagnets acquire and lose their magnetism quite readily when the current is turned on or off, and so are referred to as "soft iron." (It is "soft" only in a magnetic sense.) Iron that holds its magnetism even when there is no externally applied field is called "hard iron." Hard iron is used in permanent magnets. Soft iron is usually used in electromagnets so that the field can be turned on and off readily. Whether iron is hard or soft depends on heat treatment, type of alloy, and other factors.

PHYSICS APPLIED
Electromagnets and solenoids

Electromagnets have many practical applications, from use in motors and generators to producing large magnetic fields for research. Sometimes an iron core is not present—the magnetic field comes only from the current in the wire coils. When the current flows continuously in a normal electromagnet, a great deal of waste heat (I^2R power) can be produced. Cooling coils, which are tubes carrying water, are needed to absorb the heat in larger installations.

For some applications, the current-carrying wires are made of superconducting material kept below the transition temperature. Very high fields can be produced with superconducting wire without an iron core. No electric power is needed to maintain large current in the superconducting coils, which means large savings of electricity; nor must huge amounts of heat be dissipated. It is not a free ride, though, because energy is needed to keep the superconducting coils at the necessary low temperature.

Another useful device consists of a solenoid into which a rod of iron is partially inserted. This combination is also referred to as a solenoid. One simple use is as a doorbell (Fig. 26). When the circuit is closed by pushing the button, the coil effectively becomes a magnet and exerts a force on the iron rod. The rod is pulled into the coil and strikes the bell. A large solenoid is used in the starters of cars; when you engage the starter, you are closing a circuit that not only turns the starter motor, but activates a solenoid that first moves the starter into direct contact with the gears on the engine's flywheel. Solenoids are used as switches in many devices. They have the advantage of moving mechanical parts quickly and accurately.

PHYSICS APPLIED
Doorbell, car starter

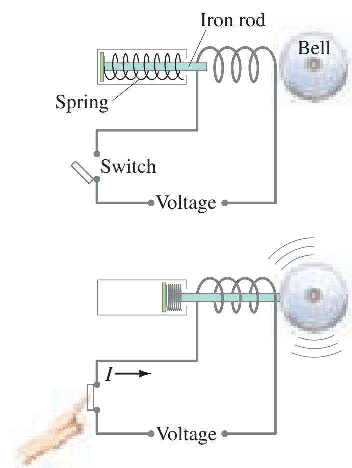


FIGURE 26 Solenoid used as a doorbell.

*Magnetic Circuit Breakers

Modern circuit breakers that protect houses and buildings from overload and fire contain not only a "thermal" part (bimetallic strip) but also a magnetic sensor. If the current is above a certain level, the magnetic field it produces pulls an iron plate that breaks contact points. In more sophisticated circuit breakers, including ground fault circuit interrupters (GFCIs), a solenoid is used. The iron rod of Fig. 26, instead of striking a bell, strikes one side of a pair of points, opening them and opening the circuit. Magnetic circuit breakers react quickly (<10 msec), and for buildings are designed to react to the high currents of shorts (but not shut off for the start-up surges of motors).

PHYSICS APPLIED
Magnetic circuit breakers

*9 Magnetic Fields in Magnetic Materials; Hysteresis

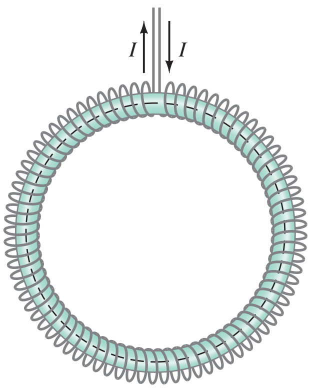


FIGURE 27 Iron-core toroid.

The field of a long solenoid is directly proportional to the current. Indeed, Eq. 4 tells us that the field B_0 inside a solenoid is given by

$$B_0 = \mu_0 nI.$$

This is valid if there is only air inside the coil. If we put a piece of iron or other ferromagnetic material inside the solenoid, the field will be greatly increased, often by hundreds or thousands of times. This occurs because the domains in the iron become preferentially aligned by the external field. The resulting magnetic field is the sum of that due to the current and that due to the iron. It is sometimes convenient to write the total field in this case as a sum of two terms:

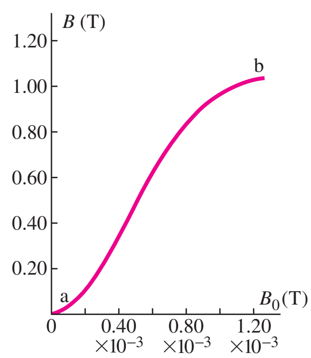
$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_0 + \vec{\mathbf{B}}_{\mathrm{M}}. \tag{8}$$

Here, $\vec{\mathbf{B}}_0$ refers to the field due only to the current in the wire (the "external field"). It is the field that would be present in the absence of a ferromagnetic material. Then $\vec{\mathbf{B}}_{\mathrm{M}}$ represents the additional field due to the ferromagnetic material itself; often $\vec{\mathbf{B}}_{\mathrm{M}} \gg \vec{\mathbf{B}}_0$.

The total field inside a solenoid in such a case can also be written by replacing the constant μ_0 in Eq. 4 by another constant, μ, characteristic of the material inside the coil:

$$B = \mu nI; \tag{9}$$

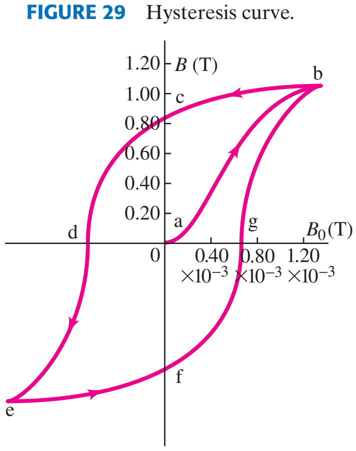
FIGURE 28 Total magnetic field B in an iron-core toroid as a function of the external field B_0 (B_0 is caused by the current I in the coil).



μ is called the **magnetic permeability** of the material (do not confuse it with $\vec{\boldsymbol{\mu}}$ for magnetic dipole moment). For ferromagnetic materials, μ is much greater than μ_0. For all other materials, its value is very close to μ_0 (Section 10). The value of μ, however, is not constant for ferromagnetic materials; it depends on the value of the external field B_0, as the following experiment shows.

Measurements on magnetic materials are generally done using a toroid, which is essentially a long solenoid bent into the shape of a circle (Fig. 27), so that practically all the lines of $\vec{\mathbf{B}}$ remain within the toroid. Suppose the toroid has an iron core that is initially unmagnetized and there is no current in the windings of the toroid. Then the current I is slowly increased, and B_0 increases linearly with I. The total field B also increases, but follows the curved line shown in the graph of Fig. 28. (Note the different scales: $B \gg B_0$.) Initially, point a, the domains (Section 7) are randomly oriented. As B_0 increases, the domains become more and more aligned until at point b, nearly all are aligned. The iron is said to be approaching **saturation**. Point b is typically 70% of full saturation. (If B_0 is increased further, the curve continues to rise very slowly, and reaches 98% saturation only when B_0 reaches a value about a thousandfold above that at point b; the last few domains are very difficult to align.) Next, suppose the external field B_0 is reduced by decreasing the current in the toroid coils. As the current is reduced to zero, shown as point c in Fig. 29, the domains do not become completely random. Some permanent magnetism remains. If the current is then reversed in direction, enough domains can be turned around so $B = 0$ (point d). As the reverse current is increased further, the iron approaches saturation in the opposite direction (point e). Finally, if the current is again reduced to zero and then increased in the original direction, the total field follows the path efgb, again approaching saturation at point b.

FIGURE 29 Hysteresis curve.



Notice that the field did not pass through the origin (point a) in this cycle. The fact that the curves do not retrace themselves on the same path is called **hysteresis**. The curve bcdefgb is called a **hysteresis loop**. In such a cycle, much energy is transformed to thermal energy (friction) due to realigning of the domains. It can be shown that the energy dissipated in this way is proportional to the area of the hysteresis loop.

At points c and f, the iron core is magnetized even though there is no current in the coils. These points correspond to a permanent magnet. For a permanent magnet, it is desired that ac and af be as large as possible. Materials for which this is true are said to have high **retentivity**.

Materials with a broad hysteresis curve as in Fig. 29 are said to be magnetically "hard" and make good permanent magnets. On the other hand, a hysteresis curve such as that in Fig. 30 occurs for "soft" iron, which is preferred for electromagnets and transformers since the field can be more readily switched off, and the field can be reversed with less loss of energy.

A ferromagnetic material can be demagnetized—that is, made unmagnetized. This can be done by reversing the magnetizing current repeatedly while decreasing its magnitude. This results in the curve of Fig. 31. The heads of a tape recorder are demagnetized in this way. The alternating magnetic field acting at the heads due to a handheld demagnetizer is strong when the demagnetizer is placed near the heads and decreases as it is moved slowly away. Video and audio tapes themselves can be erased and ruined by a magnetic field, as can computer hard disks, other magnetic storage devices, and the magnetic stripes on credit cards.

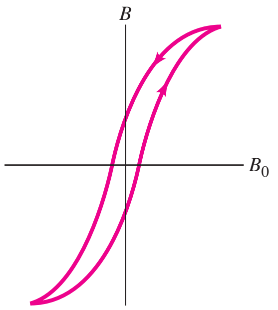
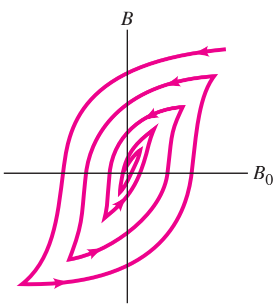


FIGURE 30 Hysteresis curve for soft iron.

FIGURE 31 Successive hysteresis loops during demagnetization.



*10 Paramagnetism and Diamagnetism

All materials are magnetic to at least a tiny extent. Nonferromagnetic materials fall into two principal classes: *paramagnetic*, in which the magnetic permeability μ is very slightly greater than μ_0; and *diamagnetic*, in which μ is very slightly less than μ_0. The ratio of μ to μ_0 for any material is called the **relative permeability** K_m:

$$K_m = \frac{\mu}{\mu_0}.$$

Another useful parameter is the **magnetic susceptibility** χ_m defined as

$$\chi_m = K_m - 1.$$

Paramagnetic substances have $K_m > 1$ and $\chi_m > 0$, whereas diamagnetic substances have $K_m < 1$ and $\chi_m < 0$. See Table 1, and note how small the effect is.

TABLE 1 Paramagnetism and Diamagnetism: Magnetic Susceptibilities

Paramagnetic substance	χ_m	Diamagnetic substance	χ_m
Aluminum	2.3×10^{-5}	Copper	-9.8×10^{-6}
Calcium	1.9×10^{-5}	Diamond	-2.2×10^{-5}
Magnesium	1.2×10^{-5}	Gold	-3.6×10^{-5}
Oxygen (STP)	2.1×10^{-6}	Lead	-1.7×10^{-5}
Platinum	2.9×10^{-4}	Nitrogen (STP)	-5.0×10^{-9}
Tungsten	6.8×10^{-5}	Silicon	-4.2×10^{-6}

The difference between paramagnetic and diamagnetic materials can be understood theoretically at the molecular level on the basis of whether or not the molecules have a permanent magnetic dipole moment. One type of **paramagnetism** occurs in materials whose molecules (or ions) have a permanent magnetic dipole moment.† In the absence of an external field, the molecules are randomly oriented and no magnetic effects are observed. However, when an external magnetic field is applied, say, by putting the material in a solenoid, the applied field exerts a torque on the magnetic dipoles, tending to align them parallel to the field. The total magnetic field (external plus that due to aligned magnetic dipoles) will be slightly greater than B_0. The thermal motion of the molecules reduces the alignment, however. A useful

†Other types of paramagnetism also occur whose origin is different from that described here, such as in metals where free electrons can contribute.

quantity is the **magnetization vector**, $\vec{\mathbf{M}}$, defined as the magnetic dipole moment per unit volume,

$$\vec{\mathbf{M}} = \frac{\vec{\boldsymbol{\mu}}}{V},$$

where $\vec{\boldsymbol{\mu}}$ is the magnetic dipole moment of the sample and V its volume. It is found experimentally that M is directly proportional to the external magnetic field (tending to align the dipoles) and inversely proportional to the kelvin temperature T (tending to randomize dipole directions). This is called *Curie's law*, after Pierre Curie (1859–1906), who first noted it:

$$M = C\frac{B}{T},$$

where C is a constant. If the ratio B/T is very large (B very large or T very small) Curie's law is no longer accurate; as B is increased (or T decreased), the magnetization approaches some maximum value, M_{max}. This makes sense, of course, since M_{max} corresponds to complete alignment of all the permanent magnetic dipoles. However, even for very large magnetic fields, $\approx 2.0\,\mathrm{T}$, deviations from Curie's law are normally noted only at very low temperatures, on the order of a few kelvins.

Ferromagnetic materials, as mentioned in Section 7, are no longer ferromagnetic above a characteristic temperature called the Curie temperature (1043 K for iron). Above this Curie temperature, they generally are paramagnetic.

Diamagnetic materials (for which μ_{m} is slightly less than μ_0) are made up of molecules that have no permanent magnetic dipole moment. When an external magnetic field is applied, magnetic dipoles are induced, but the induced magnetic dipole moment is in the direction opposite to that of the field. Hence the total field will be slightly less than the external field. The effect of the external field—in the crude model of electrons orbiting nuclei—is to increase the "orbital" speed of electrons revolving in one direction, and to decrease the speed of electrons revolving in the other direction; the net result is a net dipole moment opposing the external field. Diamagnetism is present in all materials, but is weaker even than paramagnetism and so is overwhelmed by paramagnetic and ferromagnetic effects in materials that display these other forms of magnetism.

Summary

The magnetic field B at a distance r from a long straight wire is directly proportional to the current I in the wire and inversely proportional to r:

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}. \quad \textbf{(1)}$$

The magnetic field lines are circles centered at the wire.

The force that one long current-carrying wire exerts on a second parallel current-carrying wire 1 m away serves as the definition of the ampere unit, and ultimately of the coulomb as well.

Ampère's law states that the line integral of the magnetic field $\vec{\mathbf{B}}$ around any closed loop is equal to μ_0 times the total net current I_{encl} enclosed by the loop:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 I_{\mathrm{encl}}. \quad \textbf{(3)}$$

The magnetic field inside a long tightly wound solenoid is

$$B = \mu_0 n I \quad \textbf{(4)}$$

where n is the number of coils per unit length and I is the current in each coil.

The **Biot-Savart law** is useful for determining the magnetic field due to a known arrangement of currents. It states that

$$d\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}} \times \hat{\mathbf{r}}}{r^2}, \quad \textbf{(5)}$$

where $d\vec{\mathbf{B}}$ is the contribution to the total field at some point P due to a current I along an infinitesimal length $d\vec{\boldsymbol{\ell}}$ of its path, and $\hat{\mathbf{r}}$ is the unit vector along the direction of the displacement vector $\vec{\mathbf{r}}$ from $d\vec{\boldsymbol{\ell}}$ to P. The total field $\vec{\mathbf{B}}$ will be the integral over all $d\vec{\mathbf{B}}$.

Iron and a few other materials can be made into strong permanent magnets. They are said to be **ferromagnetic**. Ferromagnetic materials are made up of tiny **domains**—each a tiny magnet—which are preferentially aligned in a permanent magnet, but randomly aligned in a nonmagnetized sample.

[*When a ferromagnetic material is placed in a magnetic field B_0 due to a current, say inside a solenoid or toroid, the material becomes magnetized. When the current is turned off, however, the material remains magnetized, and when the current is increased in the opposite direction (and then again reversed), a graph of the total field B versus B_0 is a **hysteresis loop**, and the fact that the curves do not retrace themselves is called **hysteresis**.]

[*All materials exhibit some magnetic effects. Nonferromagnetic materials have much smaller paramagnetic or diamagnetic properties.]

Answers to Exercises

A: 2×10^{-6} T; not at this distance, and then only briefly.

B: 0.8×10^{-5} T, up.

C: (*b*).

D: (*b*), (*c*).