Richard Megna/Fundamental Photographs, NYC

This comb has acquired a static electric charge, either from passing through hair, or being rubbed by a cloth or paper towel. The electrical charge on the comb induces a polarization (separation of charge) in scraps of paper, and thus attracts them.

Our introduction to electricity in this Chapter covers conductors and insulators, and Coulomb's law which relates the force between two point charges as a function of their distance apart. We also introduce the powerful concept of electric field.

Electric Charge and Electric Field

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.*]

Two identical tiny spheres have the same electric charge. If the electric charge on each of them is doubled, and their separation is also doubled, the force each exerts on the other will be

(a) half.
(b) double.
(c) four times larger.
(d) one-quarter as large.
(e) unchanged.

The word "electricity" may evoke an image of complex modern technology: lights, motors, electronics, and computers. But the electric force plays an even deeper role in our lives. According to atomic theory, electric forces between atoms and molecules hold them together to form liquids and solids, and electric forces are also involved in the metabolic processes that occur within our bodies. Many of the forces we have dealt with so far, such as elastic forces, the normal force, and friction and other contact forces (pushes and pulls), are now considered to result from electric forces acting at the atomic level. Gravity, on the other hand, is a separate force.†

†Physicists in the twentieth century came to recognize four different fundamental forces in nature: (1) gravitational force, (2) electromagnetic force (we will see later that electric and magnetic forces are intimately related), (3) strong nuclear force, and (4) weak nuclear force. The last two forces operate at the level of the nucleus of an atom. Recent theory has combined the electromagnetic and weak nuclear forces so they are now considered to have a common origin known as the electroweak force.

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Note: Sections marked with an asterisk (*) may be considered optional by the instructor.

From Chapter 21 of *Physics for Scientists & Engineers with Modern Physics*, Fourth Edition, Douglas C. Giancoli.

The earliest studies on electricity date back to the ancients, but only in the past two centuries has electricity been studied in detail.

1 Static Electricity; Electric Charge and Its Conservation

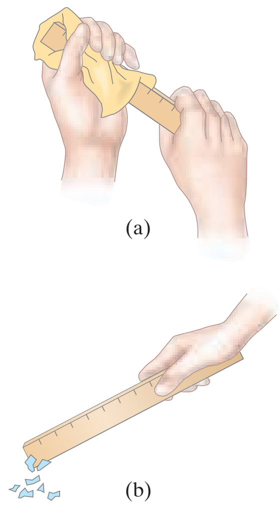
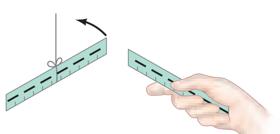


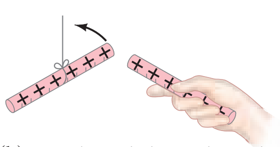
FIGURE 1 (a) Rub a plastic ruler and (b) bring it close to some tiny pieces of paper.

The word *electricity* comes from the Greek word *elektron*, which means "amber." Amber is petrified tree resin, and the ancients knew that if you rub a piece of amber with a cloth, the amber attracts small pieces of leaves or dust. A piece of hard rubber, a glass rod, or a plastic ruler rubbed with a cloth will also display this "amber effect," or **static electricity** as we call it today. You can readily pick up small pieces of paper with a plastic comb or ruler that you have just vigorously rubbed with even a paper towel. See the photo on the previous page and Fig. 1. You have probably experienced static electricity when combing your hair or when taking a synthetic blouse or shirt from a clothes dryer. And you may have felt a shock when you touched a metal doorknob after sliding across a car seat or walking across a nylon carpet. In each case, an object becomes "charged" as a result of rubbing, and is said to possess a net **electric charge**.

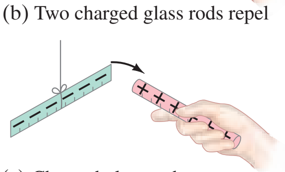
FIGURE 2 Like charges repel one another; unlike charges attract. (Note color coding: positive and negative charged objects are often colored pink and blue-green, respectively, when we want to emphasize them. We use these colors especially for point charges, but not often for real objects.)



(a) Two charged plastic rulers repel



(b) Two charged glass rods repel



(c) Charged glass rod attracts charged plastic ruler

Is all electric charge the same, or is there more than one type? In fact, there are *two* types of electric charge, as the following simple experiments show. A plastic ruler suspended by a thread is vigorously rubbed with a cloth to charge it. When a second plastic ruler, which has been charged in the same way, is brought close to the first, it is found that one ruler *repels* the other. This is shown in Fig. 2a. Similarly, if a rubbed glass rod is brought close to a second charged glass rod, again a repulsive force is seen to act, Fig. 2b. However, if the charged glass rod is brought close to the charged plastic ruler, it is found that they *attract* each other, Fig. 2c. The charge on the glass must therefore be different from that on the plastic. Indeed, it is found experimentally that all charged objects fall into one of two categories. Either they are attracted to the plastic and repelled by the glass; or they are repelled by the plastic and attracted to the glass. Thus there seem to be two, and only two, types of electric charge. Each type of charge repels the same type but attracts the opposite type. That is: **unlike charges attract; like charges repel**.

The two types of electric charge were referred to as *positive* and *negative* by the American statesman, philosopher, and scientist Benjamin Franklin (1706–1790). The choice of which name went with which type of charge was arbitrary. Franklin's choice set the charge on the rubbed glass rod to be positive charge, so the charge on a rubbed plastic ruler (or amber) is called negative charge. We still follow this convention today.

Franklin argued that whenever a certain amount of charge is produced on one object, an equal amount of the opposite type of charge is produced on another object. The positive and negative are to be treated *algebraically*, so during any process, the net change in the amount of charge produced is zero. For example, when a plastic ruler is rubbed with a paper towel, the plastic acquires a negative charge and the towel acquires an equal amount of positive charge. The charges are separated, but the sum of the two is zero.

This is an example of a law that is now well established: the **law of conservation of electric charge**, which states that

LAW OF CONSERVATION OF ELECTRIC CHARGE

> **the net amount of electric charge produced in any process is zero;**

or, said another way,

> **no net electric charge can be created or destroyed.**

If one object (or a region of space) acquires a positive charge, then an equal amount of negative charge will be found in neighboring areas or objects. No violations have ever been found, and this conservation law is as firmly established as those for energy and momentum.

2 Electric Charge in the Atom

Only within the past century has it become clear that an understanding of electricity originates inside the atom itself. It will help our understanding of electricity if we discuss it briefly now.

A simplified model of an atom shows it as having a tiny but heavy, positively charged nucleus surrounded by one or more negatively charged electrons (Fig. 3). The nucleus contains protons, which are positively charged, and neutrons, which have no net electric charge. All protons and all electrons have exactly the same magnitude of electric charge; but their signs are opposite. Hence neutral atoms, having no net charge, contain equal numbers of protons and electrons. Sometimes an atom may lose one or more of its electrons, or may gain extra electrons, in which case it will have a net positive or negative charge and is called an **ion**.

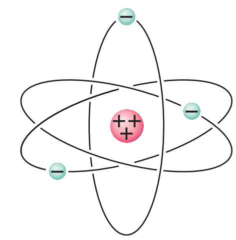


FIGURE 3 Simple model of the atom.

In solid materials the nuclei tend to remain close to fixed positions, whereas some of the electrons may move quite freely. When an object is *neutral*, it contains equal amounts of positive and negative charge. The charging of a solid object by rubbing can be explained by the transfer of electrons from one object to the other. When a plastic ruler becomes negatively charged by rubbing with a paper towel, the transfer of electrons from the towel to the plastic leaves the towel with a positive charge equal in magnitude to the negative charge acquired by the plastic. In liquids and gases, nuclei or ions can move as well as electrons.

Normally when objects are charged by rubbing, they hold their charge only for a limited time and eventually return to the neutral state. Where does the charge go? Usually the charge "leaks off" onto water molecules in the air. This is because water molecules are **polar**—that is, even though they are neutral, their charge is not distributed uniformly, Fig. 4. Thus the extra electrons on, say, a charged plastic ruler can "leak off" into the air because they are attracted to the positive end of water molecules. A positively charged object, on the other hand, can be neutralized by transfer of loosely held electrons from water molecules in the air. On dry days, static electricity is much more noticeable since the air contains fewer water molecules to allow leakage. On humid or rainy days, it is difficult to make any object hold a net charge for long.

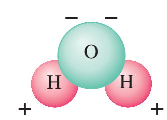


FIGURE 4 Diagram of a water molecule. Because it has opposite charges on different ends, it is called a "polar" molecule.

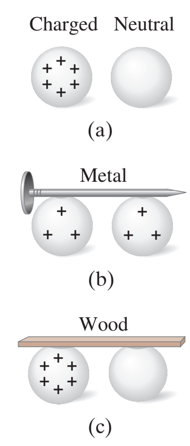
3 Insulators and Conductors

Suppose we have two metal spheres, one highly charged and the other electrically neutral (Fig. 5a). If we now place a metal object, such as a nail, so that it touches both spheres (Fig. 5b), the previously uncharged sphere quickly becomes charged. If, instead, we had connected the two spheres by a wooden rod or a piece of rubber (Fig. 5c), the uncharged ball would not become noticeably charged. Materials like the iron nail are said to be **conductors** of electricity, whereas wood and rubber are **nonconductors** or **insulators**.

Metals are generally good conductors, whereas most other materials are insulators (although even insulators conduct electricity very slightly). Nearly all natural materials fall into one or the other of these two very distinct categories. However, a few materials (notably silicon and germanium) fall into an intermediate category known as **semiconductors**.

From the atomic point of view, the electrons in an insulating material are bound very tightly to the nuclei. In a good conductor, on the other hand, some of the electrons are bound very loosely and can move about freely within the material (although they cannot *leave* the object easily) and are often referred to as *free electrons* or *conduction electrons*. When a positively charged object is brought close to or touches a conductor, the free electrons in the conductor are attracted by this positively charged object and move quickly toward it. On the other hand, the free electrons move swiftly away from a negatively charged object that is brought close to the conductor. In a semiconductor, there are many fewer free electrons, and in an insulator, almost none.

FIGURE 5 (a) A charged metal sphere and a neutral metal sphere. (b) The two spheres connected by a conductor (a metal nail), which conducts charge from one sphere to the other. (c) The original two spheres connected by an insulator (wood); almost no charge is conducted.



4 Induced Charge; the Electroscope

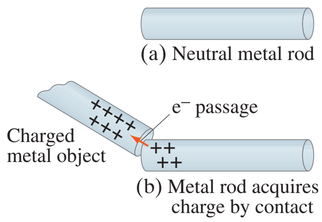


FIGURE 6 A neutral metal rod in (a) will acquire a positive charge if placed in contact (b) with a positively charged metal object. (Electrons move as shown by the orange arrow.) This is called charging by conduction.

Suppose a positively charged metal object is brought close to an uncharged metal object. If the two touch, the free electrons in the neutral one are attracted to the positively charged object and some will pass over to it, Fig. 6. Since the second object, originally neutral, is now missing some of its negative electrons, it will have a net positive charge. This process is called "charging by conduction," or "by contact," and the two objects end up with the same sign of charge.

Now suppose a positively charged object is brought close to a neutral metal rod, but does not touch it. Although the free electrons of the metal rod do not leave the rod, they still move within the metal toward the external positive charge, leaving a positive charge at the opposite end of the rod (Fig. 7). A charge is said to have been *induced* at the two ends of the metal rod. No net charge has been created in the rod: charges have merely been *separated*. The net charge on the metal rod is still zero. However, if the metal is separated into two pieces, we would have two charged objects: one charged positively and one charged negatively.

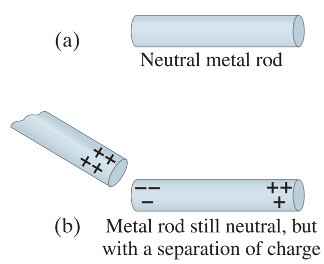


FIGURE 7 Charging by induction.

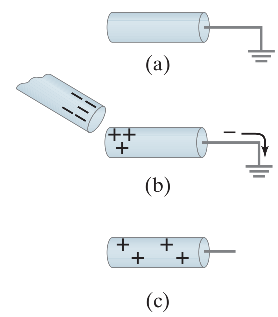


FIGURE 8 Inducing a charge on an object connected to ground.

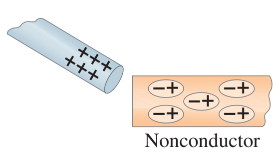
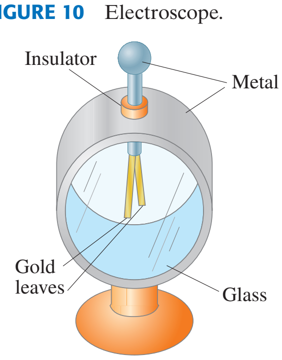


FIGURE 9 A charged object brought near an insulator causes a charge separation within the insulator's molecules.

FIGURE 10 Electroscope.



Another way to induce a net charge on a metal object is to first connect it with a conducting wire to the ground (or a conducting pipe leading into the ground) as shown in Fig. 8a (the symbol ⏚ means connected to "ground"). The object is then said to be "grounded" or "earthed." The Earth, because it is so large and can conduct, easily accepts or gives up electrons; hence it acts like a reservoir for charge. If a charged object—say negative this time—is brought up close to the metal object, free electrons in the metal are repelled and many of them move down the wire into the Earth, Fig. 8b. This leaves the metal positively charged. If the wire is now cut, the metal object will have a positive induced charge on it (Fig. 8c). If the wire were cut after the negative object was moved away, the electrons would all have moved back into the metal object and it would be neutral.

Charge separation can also be done in nonconductors. If you bring a positively charged object close to a neutral nonconductor as shown in Fig. 9, almost no electrons can move about freely within the nonconductor. But they can move slightly within their own atoms and molecules. Each oval in Fig. 9 represents a molecule (not to scale); the negatively charged electrons, attracted to the external positive charge, tend to move in its direction within their molecules. Because the negative charges in the nonconductor are nearer to the external positive charge, the nonconductor as a whole is attracted to the external positive charge (see the Chapter-Opening Photo).

An **electroscope** is a device that can be used for detecting charge. As shown in Fig. 10, inside of a case are two movable metal leaves, often made of gold, connected to a metal knob on the outside. (Sometimes only one leaf is movable.)

If a positively charged object is brought close to the knob, a separation of charge is induced: electrons are attracted up into the knob, leaving the leaves positively charged, Fig 11a. The two leaves repel each other as shown, because they are both positively charged. If, instead, the knob is charged by conduction, the whole apparatus acquires a net charge as shown in Fig 11b. In either case, the greater the amount of charge, the greater the separation of the leaves.

Note that you cannot tell the sign of the charge in this way, since negative charge will cause the leaves to separate just as much as an equal amount of positive charge; in either case, the two leaves repel each other. An electroscope can, however, be used to determine the sign of the charge if it is first charged by conduction, say, negatively, as in Fig. 12a. Now if a negative object is brought close, as in Fig. 12b, more electrons are induced to move down into the leaves and they separate further. If a positive charge is brought close instead, the electrons are induced to flow upward, leaving the leaves less negative and their separation is reduced, Fig. 12c.

The electroscope was used in the early studies of electricity. The same principle, aided by some electronics, is used in much more sensitive modern **electrometers**.

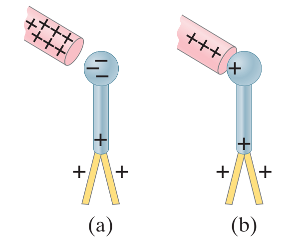
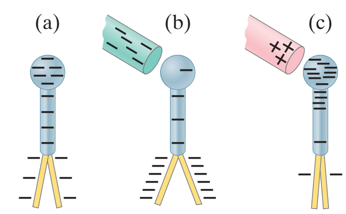


FIGURE 11 Electroscope charged (a) by induction, (b) by conduction.

FIGURE 12 A previously charged electroscope can be used to determine the sign of a charged object.



5 Coulomb's Law

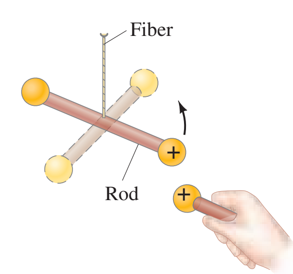
We have seen that an electric charge exerts a force of attraction or repulsion on other electric charges. What factors affect the magnitude of this force? To find an answer, the French physicist Charles Coulomb (1736–1806) investigated electric forces in the 1780s using a torsion balance (Fig. 13) much like that used by Cavendish for his studies of the gravitational force.

Precise instruments for the measurement of electric charge were not available in Coulomb's time. Nonetheless, Coulomb was able to prepare small spheres with different magnitudes of charge in which the *ratio* of the charges was known.[†] Although he had some difficulty with induced charges, Coulomb was able to argue that the force one tiny charged object exerts on a second tiny charged object is directly proportional to the charge on each of them. That is, if the charge on either one of the objects is doubled, the force is doubled; and if the charge on both of the objects is doubled, the force increases to four times the original value. This was the case when the distance between the two charges remained the same. If the distance between them was allowed to increase, he found that the force decreased with the *square of the distance* between them. That is, if the distance was doubled, the force fell to one-fourth of its original value. Thus, Coulomb concluded, the force one small charged object exerts on a second one is proportional to the product of the magnitude of the charge on one, Q_1, times the magnitude of the charge on the other, Q_2, and inversely proportional to the square of the distance r between them (Fig. 14). As an equation, we can write **Coulomb's law** as

$$F = k\frac{Q_1 Q_2}{r^2}, \qquad \text{[magnitudes]} \quad \textbf{(1)}$$

where k is a proportionality constant.[‡]

FIGURE 13 (below) Coulomb used a torsion balance to investigate how the electric force varies as a function of the magnitude of the charges and of the distance between them. When an external charged sphere is placed close to the charged one on the suspended bar, the bar rotates slightly. The suspending fiber resists the twisting motion, and the angle of twist is proportional to the electric force.



COULOMB'S LAW

FIGURE 14 Coulomb's law, Eq. 1, gives the force between two point charges, Q_1 and Q_2, a distance r apart.

[†] Coulomb reasoned that if a charged conducting sphere is placed in contact with an identical uncharged sphere, the charge on the first would be shared equally by the two of them because of symmetry. He thus had a way to produce charges equal to $\frac{1}{2}, \frac{1}{4}$, and so on, of the original charge.

[‡] The validity of Coulomb's law today rests on precision measurements that are much more sophisticated than Coulomb's original experiment. The exponent 2 in Coulomb's law has been shown to be accurate to 1 part in 10^{16} [that is, $2 \pm (1 \times 10^{-16})$].

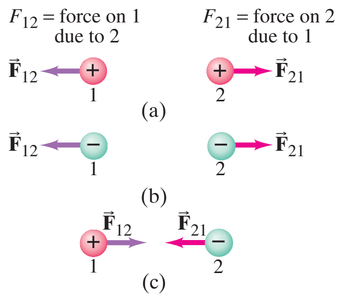


FIGURE 15 The direction of the static electric force one point charge exerts on another is always along the line joining the two charges, and depends on whether the charges have the same sign as in (a) and (b), or opposite signs (c).

As we just saw, Coulomb's law,

$$F = k\frac{Q_1 Q_2}{r^2}, \qquad \text{[magnitudes]} \quad \mathbf{(1)}$$

gives the *magnitude* of the electric force that either charge exerts on the other. The *direction* of the electric force *is always along the line joining the two charges*. If the two charges have the same sign, the force on either charge is directed away from the other (they repel each other). If the two charges have opposite signs, the force on one is directed toward the other (they attract). See Fig. 15. Notice that the force one charge exerts on the second is equal but opposite to that exerted by the second on the first, in accord with Newton's third law.

The SI unit of charge is the **coulomb** (C).† The precise definition of the coulomb today is in terms of electric current and magnetic field, but not discussed in detail here. In SI units, the constant k in Coulomb's law has the value

$$k = 8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2}$$

or, when we only need two significant figures,

$$k \approx 9.0 \times 10^9\,\mathrm{N\cdot m^2/C^2}.$$

Thus, 1 C is that amount of charge which, if placed on each of two point objects that are 1.0 m apart, will result in each object exerting a force of $(9.0 \times 10^9\,\mathrm{N\cdot m^2/C^2})(1.0\,\mathrm{C})(1.0\,\mathrm{C})/(1.0\,\mathrm{m})^2 = 9.0 \times 10^9\,\mathrm{N}$ on the other. This would be an enormous force, equal to the weight of almost a million tons. We rarely encounter charges as large as a coulomb.

Charges produced by rubbing ordinary objects (such as a comb or plastic ruler) are typically around a microcoulomb $(1\,\mu\mathrm{C} = 10^{-6}\,\mathrm{C})$ or less. Objects that carry a positive charge have a deficit of electrons, whereas negatively charged objects have an excess of electrons. The charge on one electron has been determined to have a magnitude of about 1.602×10^{-19} C, and is negative. This is the smallest charge found in nature,‡ and because it is fundamental, it is given the symbol e and is often referred to as the *elementary charge*:

$$e = 1.602 \times 10^{-19}\,\mathrm{C}.$$

Note that e is defined as a positive number, so the charge on the electron is $-e$. (The charge on a proton, on the other hand, is $+e$.) Since an object cannot gain or lose a fraction of an electron, the net charge on any object must be an integral multiple of this charge. Electric charge is thus said to be **quantized** (existing only in discrete amounts: $1e$, $2e$, $3e$, etc.). Because e is so small, however, we normally do not notice this discreteness in macroscopic charges ($1\,\mu$C requires about 10^{13} electrons), which thus seem continuous.

Coulomb's law looks a lot like the *law of universal gravitation*, $F = Gm_1 m_2/r^2$, which expresses the gravitational force a mass m_1 exerts on a mass m_2. Both are inverse square laws $(F \propto 1/r^2)$. Both also have a proportionality to a property of each object—mass for gravity, electric charge for electricity. And both act over a distance (that is, there is no need for contact). A major difference between the two laws is that gravity is always an attractive force, whereas the electric force can be either attractive or repulsive. Electric charge comes in two types, positive and negative; gravitational mass is only positive.

†In the once common cgs system of units, k is set equal to 1, and the unit of electric charge is called the *electrostatic unit* (esu) or the statcoulomb. One esu is defined as that charge, on each of two point objects 1 cm apart, that gives rise to a force of 1 dyne.

‡According to the standard model of elementary particle physics, subnuclear particles called quarks have a smaller charge than that on the electron, equal to $\frac{1}{3}e$ or $\frac{2}{3}e$. Quarks have not been detected directly as isolated objects, and theory indicates that free quarks may not be detectable.

The constant k in Eq. 1 is often written in terms of another constant, ϵ_0, called the **permittivity of free space**. It is related to k by $k = 1/4\pi\epsilon_0$. Coulomb's law can then be written

$$F = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2}, \tag{2}$$

COULOMB'S LAW (in terms of ϵ_0)

where

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2}.$$

Equation 2 looks more complicated than Eq. 1, but other fundamental equations we haven't seen yet are simpler in terms of ϵ_0 rather than k. It doesn't matter which form we use since Eqs. 1 and 2 are equivalent.

[Our convention for units, such as $\mathrm{C^2/N\cdot m^2}$ for ϵ_0, means $\mathrm{m^2}$ is in the denominator. That is, $\mathrm{C^2/N\cdot m^2}$ does *not* mean $(\mathrm{C^2/N})\cdot\mathrm{m^2} = \mathrm{C^2\cdot m^2/N}$.]

Equations 1 and 2 apply to objects whose size is much smaller than the distance between them. Ideally, it is precise for **point charges** (spatial size negligible compared to other distances). For finite-sized objects, it is not always clear what value to use for r, particularly since the charge may not be distributed uniformly on the objects. If the two objects are spheres and the charge is known to be distributed uniformly on each, then r is the distance between their centers.

Coulomb's law describes the force between two charges when they are at rest. Additional forces come into play when charges are in motion. In this Chapter we discuss only charges at rest, the study of which is called **electrostatics**, and Coulomb's law gives the **electrostatic force**.

When calculating with Coulomb's law, we usually ignore the signs of the charges and determine the direction of a force separately based on whether the force is attractive or repulsive.

PROBLEM SOLVING
Use magnitudes in Coulomb's law; find force direction from signs of charges

EXERCISE A Return to the Chapter-Opening Question and answer it again now. Try to explain why you may have answered differently the first time.

CONCEPTUAL EXAMPLE 1 **Which charge exerts the greater force?** Two positive point charges, $Q_1 = 50\,\mu\mathrm{C}$ and $Q_2 = 1\,\mu\mathrm{C}$, are separated by a distance ℓ, Fig. 16. Which is larger in magnitude, the force that Q_1 exerts on Q_2, or the force that Q_2 exerts on Q_1?

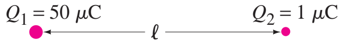


FIGURE 16 Example 1.

RESPONSE From Coulomb's law, the force on Q_1 exerted by Q_2 is

$$F_{12} = k\frac{Q_1 Q_2}{\ell^2}.$$

The force on Q_2 exerted by Q_1 is

$$F_{21} = k\frac{Q_2 Q_1}{\ell^2}$$

which is the same magnitude. The equation is symmetric with respect to the two charges, so $F_{21} = F_{12}$.

NOTE Newton's third law also tells us that these two forces must have equal magnitude.

EXERCISE B What is the magnitude of F_{12} (and F_{21}) in Example 1 if $\ell = 30\,\mathrm{cm}$?

Keep in mind that Eq. 2 (or 1) gives the force on a charge due to only *one* other charge. If several (or many) charges are present, the *net force on any one of them will be the vector sum of the forces due to each of the others*. This **principle of superposition** is based on experiment, and tells us that electric force vectors add like any other vector. For continuous distributions of charge, the sum becomes an integral.

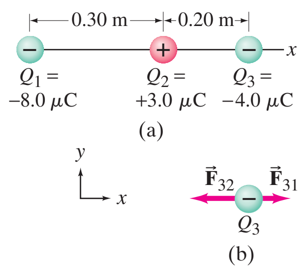


FIGURE 17 Example 2.

CAUTION
Each charge exerts its own force. No charge blocks the effect of the others

EXAMPLE 2 **Three charges in a line.** Three charged particles are arranged in a line, as shown in Fig. 17. Calculate the net electrostatic force on particle 3 (the $-4.0\ \mu C$ on the right) due to the other two charges.

APPROACH The net force on particle 3 is the vector sum of the force $\vec{F}_{31}$ exerted on 3 by particle 1 and the force $\vec{F}_{32}$ exerted on 3 by particle 2: $\vec{F} = \vec{F}_{31} + \vec{F}_{32}$.

SOLUTION The magnitudes of these two forces are obtained using Coulomb's law, Eq. 1:

$$F_{31} = k\frac{Q_3 Q_1}{r_{31}^2}$$
$$= \frac{(9.0\times10^9\,\mathrm{N\cdot m^2/C^2})(4.0\times10^{-6}\,\mathrm{C})(8.0\times10^{-6}\,\mathrm{C})}{(0.50\,\mathrm{m})^2} = 1.2\,\mathrm{N},$$

where $r_{31} = 0.50\,\mathrm{m}$ is the distance from Q_3 to Q_1. Similarly,

$$F_{32} = k\frac{Q_3 Q_2}{r_{32}^2}$$
$$= \frac{(9.0\times10^9\,\mathrm{N\cdot m^2/C^2})(4.0\times10^{-6}\,\mathrm{C})(3.0\times10^{-6}\,\mathrm{C})}{(0.20\,\mathrm{m})^2} = 2.7\,\mathrm{N}.$$

Since we were calculating the magnitudes of the forces, we omitted the signs of the charges. But we must be aware of them to get the direction of each force. Let the line joining the particles be the x axis, and we take it positive to the right. Then, because $\vec{F}_{31}$ is repulsive and $\vec{F}_{32}$ is attractive, the directions of the forces are as shown in Fig. 17b: F_{31} points in the positive x direction and F_{32} points in the negative x direction. The net force on particle 3 is then

$$F = -F_{32} + F_{31} = -2.7\,\mathrm{N} + 1.2\,\mathrm{N} = -1.5\,\mathrm{N}.$$

The magnitude of the net force is 1.5 N, and it points to the left.

NOTE Charge Q_1 acts on charge Q_3 just as if Q_2 were not there (this is the principle of superposition). That is, the charge in the middle, Q_2, in no way blocks the effect of charge Q_1 acting on Q_3. Naturally, Q_2 exerts its own force on Q_3.

EXERCISE C Determine the magnitude and direction of the net force on Q_1 in Fig. 17a.

EXAMPLE 3 **Electric force using vector components.** Calculate the net electrostatic force on charge Q_3 shown in Fig. 18a due to the charges Q_1 and Q_2.

APPROACH We use Coulomb's law to find the magnitudes of the individual forces. The direction of each force will be along the line connecting Q_3 to Q_1 or Q_2. The forces $\vec{F}_{31}$ and $\vec{F}_{32}$ have the directions shown in Fig. 18a, since Q_1 exerts an attractive force on Q_3, and Q_2 exerts a repulsive force. The forces $\vec{F}_{31}$ and $\vec{F}_{32}$ are *not* along the same line, so to find the resultant force on Q_3 we resolve $\vec{F}_{31}$ and $\vec{F}_{32}$ into x and y components and perform the vector addition.

SOLUTION The magnitudes of $\vec{F}_{31}$ and $\vec{F}_{32}$ are (ignoring signs of the charges since we know the directions)

$$F_{31} = k\frac{Q_3 Q_1}{r_{31}^2} = \frac{(9.0\times10^9\,\mathrm{N\cdot m^2/C^2})(6.5\times10^{-5}\,\mathrm{C})(8.6\times10^{-5}\,\mathrm{C})}{(0.60\,\mathrm{m})^2} = 140\,\mathrm{N},$$

$$F_{32} = k\frac{Q_3 Q_2}{r_{32}^2} = \frac{(9.0\times10^9\,\mathrm{N\cdot m^2/C^2})(6.5\times10^{-5}\,\mathrm{C})(5.0\times10^{-5}\,\mathrm{C})}{(0.30\,\mathrm{m})^2} = 330\,\mathrm{N}.$$

We resolve $\vec{F}_{31}$ into its components along the x and y axes, as shown in Fig. 18a:

$$F_{31x} = F_{31}\cos 30° = (140\,\mathrm{N})\cos 30° = 120\,\mathrm{N},$$
$$F_{31y} = -F_{31}\sin 30° = -(140\,\mathrm{N})\sin 30° = -70\,\mathrm{N}.$$

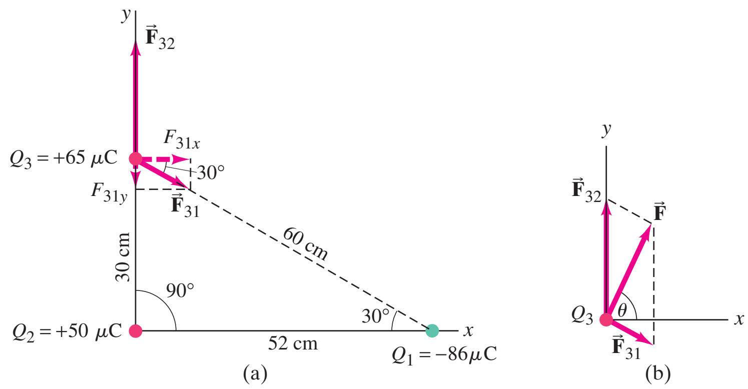


FIGURE 18 Determining the forces for Example 3. (a) The directions of the individual forces are as shown because $\vec{\mathbf{F}}_{32}$ is repulsive (the force on Q_3 is in the direction away from Q_2 because Q_3 and Q_2 are both positive) whereas $\vec{\mathbf{F}}_{31}$ is attractive (Q_3 and Q_1 have opposite signs), so $\vec{\mathbf{F}}_{31}$ points toward Q_1. (b) Adding $\vec{\mathbf{F}}_{32}$ to $\vec{\mathbf{F}}_{31}$ to obtain the net force $\vec{\mathbf{F}}$.

The force $\vec{\mathbf{F}}_{32}$ has only a y component. So the net force $\vec{\mathbf{F}}$ on Q_3 has components

$$F_x = F_{31x} = 120\,\text{N},$$
$$F_y = F_{32} + F_{31y} = 330\,\text{N} - 70\,\text{N} = 260\,\text{N}.$$

The magnitude of the net force is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(120\,\text{N})^2 + (260\,\text{N})^2} = 290\,\text{N};$$

and it acts at an angle θ (see Fig. 18b) given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{260\,\text{N}}{120\,\text{N}} = 2.2,$$

so $\theta = \tan^{-1}(2.2) = 65°$.

NOTE Because $\vec{\mathbf{F}}_{31}$ and $\vec{\mathbf{F}}_{32}$ are not along the same line, the magnitude of $\vec{\mathbf{F}}_3$ is not equal to the sum (or difference as in Example 2) of the separate magnitudes.

CONCEPTUAL EXAMPLE 4 **Make the force on Q_3 zero.** In Fig. 18, where could you place a fourth charge, $Q_4 = -50\ \mu$C, so that the net force on Q_3 would be zero?

RESPONSE By the principle of superposition, we need a force in exactly the opposite direction to the resultant $\vec{\mathbf{F}}$ due to Q_2 and Q_1 that we calculated in Example 3, Fig. 18b. Our force must have magnitude 290 N, and must point down and to the left of Q_3 in Fig. 18b. So Q_4 must be along this line. See Fig. 19.

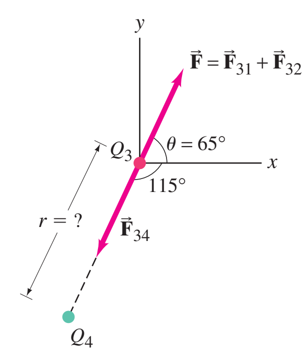


FIGURE 19 Example 4: Q_4 exerts force $(\vec{\mathbf{F}}_{34})$ that makes the net force on Q_3 zero.

EXERCISE D (*a*) Consider two point charges of the same magnitude but opposite sign ($+Q$ and $-Q$), which are fixed a distance d apart. Can you find a location where a third positive charge Q could be placed so that the net electric force on this third charge is zero? (*b*) What if the first two charges were both $+Q$?

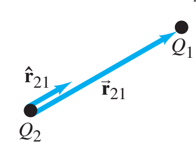
*Vector Form of Coulomb's Law

Coulomb's law can be written in vector form (as with Newton's law of universal gravitation) as

$$\vec{\mathbf{F}}_{12} = k\frac{Q_1 Q_2}{r_{21}^2}\hat{\mathbf{r}}_{21},$$

where $\vec{\mathbf{F}}_{12}$ is the vector force on charge Q_1 due to Q_2 and $\hat{\mathbf{r}}_{21}$ is the unit vector pointing from Q_2 toward Q_1. That is, $\vec{\mathbf{r}}_{21}$ points from the "source" charge (Q_2) toward the charge on which we want to know the force (Q_1). See Fig. 20. The charges Q_1 and Q_2 can be either positive or negative, and this will affect the direction of the electric force. If Q_1 and Q_2 have the same sign, the product $Q_1Q_2 > 0$ and the force on Q_1 points away from Q_2—that is, it is repulsive. If Q_1 and Q_2 have opposite signs, $Q_1Q_2 < 0$ and $\vec{\mathbf{F}}_{12}$ points toward Q_2—that is, it is attractive.

FIGURE 20 Determining the force on Q_1 due to Q_2, showing the direction of the unit vector $\hat{\mathbf{r}}_{21}$.



6 The Electric Field

Many common forces might be referred to as "contact forces," such as your hands pushing or pulling a cart, or a tennis racket hitting a tennis ball.

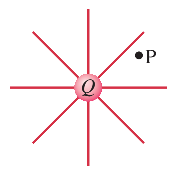
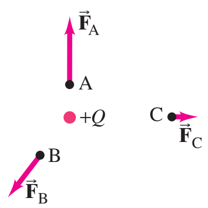


FIGURE 21 An electric field surrounds every charge. P is an arbitrary point.

In contrast, both the gravitational force and the electrical force act over a distance: there is a force between two objects even when the objects are not touching. The idea of a force *acting at a distance* was a difficult one for early thinkers. Newton himself felt uneasy with this idea when he published his law of universal gravitation. A helpful way to look at the situation uses the idea of the **field**, developed by the British scientist Michael Faraday (1791–1867). In the electrical case, according to Faraday, an *electric field* extends outward from every charge and permeates all of space (Fig. 21). If a second charge (call it Q_2) is placed near the first charge, it feels a force exerted by the electric field that is there (say, at point P in Fig. 21). The electric field at point P is considered to interact directly with charge Q_2 to produce the force on Q_2.

We can in principle investigate the electric field surrounding a charge or group of charges by measuring the force on a small positive **test charge** at rest. By a test charge we mean a charge so small that the force it exerts does not significantly affect the charges that create the field. If a tiny positive test charge q is placed at various locations in the vicinity of a single positive charge Q as shown in Fig. 22 (points A, B, C), the force exerted on q is as shown. The force at B is less than at A because B's distance from Q is greater (Coulomb's law); and the force at C is smaller still. In each case, the force on q is directed radially away from Q. The electric field is defined in terms of the force on such a positive test charge. In particular, the **electric field**, $\vec{\mathbf{E}}$, at any point in space is defined as the force $\vec{\mathbf{F}}$ exerted on a tiny positive test charge placed at that point divided by the magnitude of the test charge q:

FIGURE 22 Force exerted by charge $+Q$ on a small test charge, q, placed at points A, B, and C.



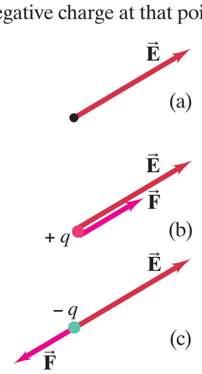
$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q}. \tag{3}$$

More precisely, $\vec{\mathbf{E}}$ is defined as the limit of $\vec{\mathbf{F}}/q$ as q is taken smaller and smaller, approaching zero. That is, q is so tiny that it exerts essentially no force on the other charges which created the field. From this definition (Eq. 3), we see that the electric field at any point in space is a vector whose direction is the direction of the force on a tiny positive test charge at that point, and whose magnitude is the *force per unit charge*. Thus $\vec{\mathbf{E}}$ has SI units of newtons per coulomb (N/C).

The reason for defining $\vec{\mathbf{E}}$ as $\vec{\mathbf{F}}/q$ (with $q \to 0$) is so that $\vec{\mathbf{E}}$ does not depend on the magnitude of the test charge q. This means that $\vec{\mathbf{E}}$ describes only the effect of the charges creating the electric field at that point.

The electric field at any point in space can be measured, based on the definition, Eq. 3. For simple situations involving one or several point charges, we can calculate $\vec{\mathbf{E}}$. For example, the electric field at a distance r from a single point charge Q would have magnitude

FIGURE 23 (a) Electric field at a given point in space. (b) Force on a positive charge at that point. (c) Force on a negative charge at that point.



$$E = \frac{F}{q} = \frac{kqQ/r^2}{q}$$

$$E = k\frac{Q}{r^2}; \qquad \text{[single point charge]} \tag{4a}$$

or, in terms of ϵ_0 as in Eq. 2 $(k = 1/4\pi\epsilon_0)$:

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}. \qquad \text{[single point charge]} \tag{4b}$$

Notice that E is independent of the test charge q—that is, E depends only on the charge Q which produces the field, and not on the value of the test charge q. Equations 4 are referred to as the electric field form of Coulomb's law.

If we are given the electric field $\vec{\mathbf{E}}$ at a given point in space, then we can calculate the force $\vec{\mathbf{F}}$ on any charge q placed at that point by writing (see Eq. 3):

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}. \tag{5}$$

This is valid even if q is not small as long as q does not cause the charges creating $\vec{\mathbf{E}}$ to move. If q is positive, $\vec{\mathbf{F}}$ and $\vec{\mathbf{E}}$ point in the same direction. If q is negative, $\vec{\mathbf{F}}$ and $\vec{\mathbf{E}}$ point in opposite directions. See Fig. 23.

EXAMPLE 5 **Photocopy machine.** A photocopy machine works by arranging positive charges (in the pattern to be copied) on the surface of a drum, then gently sprinkling negatively charged dry toner (ink) particles onto the drum. The toner particles temporarily stick to the pattern on the drum (Fig. 24) and are later transferred to paper and "melted" to produce the copy. Suppose each toner particle has a mass of 9.0×10^{-16} kg and carries an average of 20 extra electrons to provide an electric charge. Assuming that the electric force on a toner particle must exceed twice its weight in order to ensure sufficient attraction, compute the required electric field strength near the surface of the drum.

APPROACH The electric force on a toner particle of charge $q = 20e$ is $F = qE$, where E is the needed electric field. This force needs to be at least as great as twice the weight (mg) of the particle.

SOLUTION The minimum value of electric field satisfies the relation

$$qE = 2mg$$

where $q = 20e$. Hence

$$E = \frac{2mg}{q} = \frac{2(9.0 \times 10^{-16}\,\text{kg})(9.8\,\text{m/s}^2)}{20(1.6 \times 10^{-19}\,\text{C})} = 5.5 \times 10^3\,\text{N/C}.$$

PHYSICS APPLIED
Photocopier

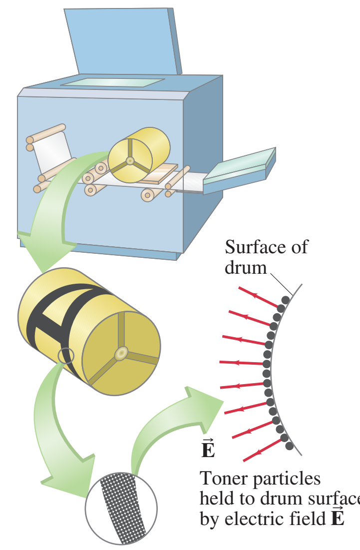


FIGURE 24 Example 5.

EXAMPLE 6 **Electric field of a single point charge.** Calculate the magnitude and direction of the electric field at a point P which is 30 cm to the right of a point charge $Q = -3.0 \times 10^{-6}$ C.

APPROACH The magnitude of the electric field due to a single point charge is given by Eq. 4. The direction is found using the sign of the charge Q.

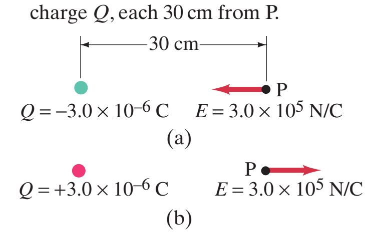
SOLUTION The magnitude of the electric field is:

$$E = k\frac{Q}{r^2} = \frac{(9.0 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(3.0 \times 10^{-6}\,\text{C})}{(0.30\,\text{m})^2} = 3.0 \times 10^5\,\text{N/C}.$$

The direction of the electric field is *toward* the charge Q, to the left as shown in Fig. 25a, since we defined the direction as that of the force on a positive test charge which here would be attractive. If Q had been positive, the electric field would have pointed away, as in Fig. 25b.

NOTE There is no electric charge at point P. But there is an electric field there. The only real charge is Q.

FIGURE 25 Example 6. Electric field at point P (a) due to a negative charge Q, and (b) due to a positive charge Q, each 30 cm from P.



This Example illustrates a general result: The electric field $\vec{\mathbf{E}}$ due to a positive charge points away from the charge, whereas $\vec{\mathbf{E}}$ due to a negative charge points toward that charge.

EXERCISE E Four charges of equal magnitude, but possibly different sign, are placed on the corners of a square. What arrangement of charges will produce an electric field with the greatest magnitude at the center of the square? (*a*) All four positive charges; (*b*) all four negative charges; (*c*) three positive and one negative; (*d*) two positive and two negative; (*e*) three negative and one positive.

If the electric field at a given point in space is due to more than one charge, the individual fields (call them $\vec{\mathbf{E}}_1$, $\vec{\mathbf{E}}_2$, etc.) due to each charge are added vectorially to get the total field at that point:

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \cdots.$$

The validity of this **superposition principle** for electric fields is fully confirmed by experiment.

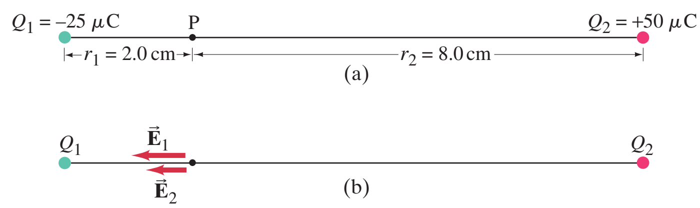


FIGURE 26 Example 7. In (b), we don't know the relative lengths of $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ until we do the calculation.

EXAMPLE 7 ***E* at a point between two charges.** Two point charges are separated by a distance of 10.0 cm. One has a charge of $-25\,\mu$C and the other $+50\,\mu$C. (*a*) Determine the direction and magnitude of the electric field at a point P between the two charges that is 2.0 cm from the negative charge (Fig. 26a). (*b*) If an electron $\left(\text{mass} = 9.11 \times 10^{-31}\,\text{kg}\right)$ is placed at rest at P and then released, what will be its initial acceleration (direction and magnitude)?

APPROACH The electric field at P will be the vector sum of the fields created separately by Q_1 and Q_2. The field due to the negative charge Q_1 points toward Q_1, and the field due to the positive charge Q_2 points away from Q_2. Thus both fields point to the left as shown in Fig. 26b and we can add the magnitudes of the two fields together algebraically, ignoring the signs of the charges. In (*b*) we use Newton's second law $(F = ma)$ to determine the acceleration, where $F = qE$ (Eq. 5).

SOLUTION (*a*) Each field is due to a point charge as given by Eq. 4, $E = kQ/r^2$. The total field is

$$\begin{aligned} E &= k\frac{Q_1}{r_1^2} + k\frac{Q_2}{r_2^2} = k\left(\frac{Q_1}{r_1^2} + \frac{Q_2}{r_2^2}\right) \\ &= \left(9.0 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{25 \times 10^{-6}\,\text{C}}{\left(2.0 \times 10^{-2}\,\text{m}\right)^2} + \frac{50 \times 10^{-6}\,\text{C}}{\left(8.0 \times 10^{-2}\,\text{m}\right)^2}\right) \\ &= 6.3 \times 10^8\,\text{N/C}. \end{aligned}$$

(*b*) The electric field points to the left, so the electron will feel a force to the *right* since it is negatively charged. Therefore the acceleration $a = F/m$ (Newton's second law) will be to the right. The force on a charge q in an electric field E is $F = qE$ (Eq. 5). Hence the magnitude of the acceleration is

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{\left(1.60 \times 10^{-19}\,\text{C}\right)\left(6.3 \times 10^8\,\text{N/C}\right)}{9.11 \times 10^{-31}\,\text{kg}} = 1.1 \times 10^{20}\,\text{m/s}^2.$$

NOTE By carefully considering the directions of *each* field ($\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$) before doing any calculations, we made sure our calculation could be done simply and correctly.

EXAMPLE 8 **$\vec{\mathbf{E}}$ above two point charges.** Calculate the total electric field (*a*) at point A and (*b*) at point B in Fig. 27 due to both charges, Q_1 and Q_2.

APPROACH The calculation is much like that of Example 3, except now we are dealing with electric fields instead of force. The electric field at point A is the vector sum of the fields $\vec{\mathbf{E}}_{A1}$ due to Q_1, and $\vec{\mathbf{E}}_{A2}$ due to Q_2. We find the magnitude of the field produced by each point charge, then we add their components to find the total field at point A. We do the same for point B.

SOLUTION (*a*) The magnitude of the electric field produced at point A by each of the charges Q_1 and Q_2 is given by $E = kQ/r^2$, so

PROBLEM SOLVING
Ignore signs of charges and determine direction physically, showing directions on diagram

$$E_{A1} = \frac{\left(9.0 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(50 \times 10^{-6}\,\text{C}\right)}{(0.60\,\text{m})^2} = 1.25 \times 10^6\,\text{N/C},$$

$$E_{A2} = \frac{\left(9.0 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(50 \times 10^{-6}\,\text{C}\right)}{(0.30\,\text{m})^2} = 5.0 \times 10^6\,\text{N/C}.$$

The direction of E_{A1} points from A toward Q_1 (negative charge), whereas E_{A2} points

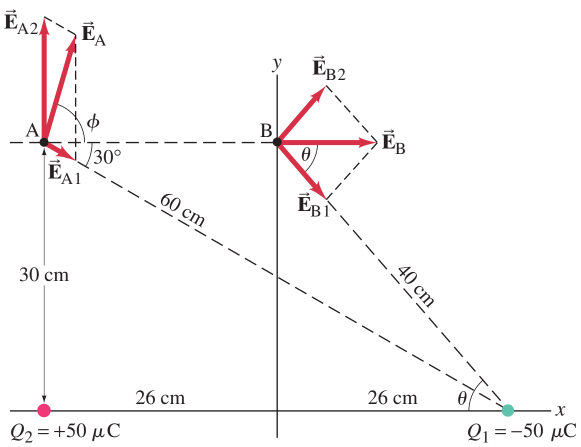


FIGURE 27 Calculation of the electric field at points A and B for Example 8.

from A away from Q_2, as shown; so the total electric field at A, $\vec{E}_A$, has components

$$E_{Ax} = E_{A1}\cos 30° = 1.1 \times 10^6\,\text{N/C},$$
$$E_{Ay} = E_{A2} - E_{A1}\sin 30° = 4.4 \times 10^6\,\text{N/C}.$$

Thus the magnitude of $\vec{E}_A$ is

$$E_A = \sqrt{(1.1)^2 + (4.4)^2} \times 10^6\,\text{N/C} = 4.5 \times 10^6\,\text{N/C},$$

and its direction is ϕ given by $\tan\phi = E_{Ay}/E_{Ax} = 4.4/1.1 = 4.0$, so $\phi = 76°$.

(b) Because B is equidistant from the two equal charges (40 cm by the Pythagorean theorem), the magnitudes of E_{B1} and E_{B2} are the same; that is,

$$E_{B1} = E_{B2} = \frac{kQ}{r^2} = \frac{(9.0 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(50 \times 10^{-6}\,\text{C})}{(0.40\,\text{m})^2}$$
$$= 2.8 \times 10^6\,\text{N/C}.$$

PROBLEM SOLVING

Use symmetry to save work, when possible

Also, because of the symmetry, the y components are equal and opposite, and so cancel out. Hence the total field E_B is horizontal and equals $E_{B1}\cos\theta + E_{B2}\cos\theta = 2E_{B1}\cos\theta$. From the diagram, $\cos\theta = 26\,\text{cm}/40\,\text{cm} = 0.65$. Then

$$E_B = 2E_{B1}\cos\theta = 2(2.8 \times 10^6\,\text{N/C})(0.65)$$
$$= 3.6 \times 10^6\,\text{N/C},$$

and the direction of $\vec{E}_B$ is along the $+x$ direction.

NOTE We could have done part (b) in the same way we did part (a). But symmetry allowed us to solve the problem with less effort.

PROBLEM SOLVING

Electrostatics: Electric Forces and Electric Fields

Follows, to a large extent, the general problem-solving procedure discussed. Whether you use electric field or electrostatic forces, the procedure in solving electrostatics problems is similar:

1. **Draw** a careful **diagram**—namely, a free-body diagram for each object, showing all the forces acting on that object, or showing the electric field at a point due to all significant charges present. Determine the **direction** of each force or electric field physically: like charges repel each other, unlike charges attract; fields point away from a + charge, and toward a − charge. Show and label each vector force or field on your diagram.
2. **Apply Coulomb's law** to calculate the magnitude of the force that each contributing charge exerts on a charged object, or the magnitude of the electric field each charge produces at a given point. Deal only with magnitudes of charges (leaving out minus signs), and obtain the magnitude of each force or electric field.
3. **Add vectorially** all the forces on an object, or the contributing fields at a point, to get the resultant. Use **symmetry** (say, in the geometry) whenever possible.
4. **Check** your answer. Is it **reasonable**? If a function of distance, does it give reasonable results in limiting cases?

7 Electric Field Calculations for Continuous Charge Distributions

In many cases we can treat charge as being distributed continuously.† We can divide up a charge distribution into infinitesimal charges dQ, each of which will act as a tiny point charge. The contribution to the electric field at a distance r from each dQ is

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2}. \tag{6a}$$

Then the electric field, $\vec{\mathbf{E}}$, at any point is obtained by summing over all the infinitesimal contributions, which is the integral

$$\vec{\mathbf{E}} = \int d\vec{\mathbf{E}}. \tag{6b}$$

Note that $d\vec{\mathbf{E}}$ is a vector (Eq. 6a gives its magnitude). [In situations where Eq. 6b is difficult to evaluate, other techniques not covered in this Chapter can often be used instead to determine $\vec{\mathbf{E}}$. Numerical integration can also be used in many cases.]

EXAMPLE 9 **A ring of charge.** A thin, ring-shaped object of radius a holds a total charge $+Q$ distributed uniformly around it. Determine the electric field at a point P on its axis, a distance x from the center. See Fig. 28. Let λ be the charge per unit length (C/m).

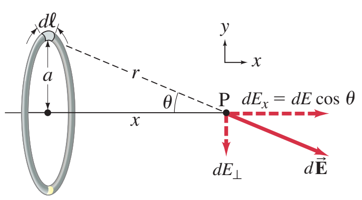


FIGURE 28 Example 9.

APPROACH AND SOLUTION We explicitly follow the steps of the Problem Solving Strategy on the previous page.

1. **Draw** a careful **diagram**. The **direction** of the electric field due to one infinitesimal length $d\ell$ of the charged ring is shown in Fig. 28.
2. **Apply Coulomb's law**. The electric field, $d\vec{\mathbf{E}}$, due to this particular segment of the ring of length $d\ell$ has magnitude
$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2}.$$
The whole ring has length (circumference) of $2\pi a$, so the charge on a length $d\ell$ is
$$dQ = Q\left(\frac{d\ell}{2\pi a}\right) = \lambda\, d\ell$$
where $\lambda = Q/2\pi a$ is the charge per unit length. Now we write dE as
$$dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, d\ell}{r^2}.$$
3. Add **vectorially** and use **symmetry**: The vector $d\vec{\mathbf{E}}$ has components dE_x along the x axis and $dE_\perp$ perpendicular to the x axis (Fig. 28). We are going to sum (integrate) around the entire ring. We note that an equal-length segment diametrically opposite the $d\ell$ shown will produce a $d\vec{\mathbf{E}}$ whose component perpendicular to the x axis will just cancel the $dE_\perp$ shown. This is true for all segments of the ring, so by symmetry $\vec{\mathbf{E}}$ will have zero y component, and so we need only sum the x components, dE_x. The total field is then
$$E = E_x = \int dE_x = \int dE\cos\theta = \frac{1}{4\pi\epsilon_0}\lambda\int\frac{d\ell}{r^2}\cos\theta.$$
Since $\cos\theta = x/r$, where $r = (x^2 + a^2)^{\frac{1}{2}}$, we have
$$E = \frac{\lambda}{(4\pi\epsilon_0)}\frac{x}{(x^2+a^2)^{\frac{3}{2}}}\int_0^{2\pi a} d\ell = \frac{1}{4\pi\epsilon_0}\frac{\lambda x(2\pi a)}{(x^2+a^2)^{\frac{3}{2}}} = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{\frac{3}{2}}}.$$
4. To **check reasonableness**, note that at great distances, $x \gg a$, this result reduces to $E = Q/(4\pi\epsilon_0 x^2)$. We would expect this result because at great distances the ring would appear to be a point charge ($1/r^2$ dependence). Also note that our result gives $E = 0$ at $x = 0$, as we might expect because all components will cancel at the center of the circle.

PROBLEM SOLVING
Use symmetry when possible

PROBLEM SOLVING
Check result by noting that at a great distance the ring looks like a point charge

†Because we believe there is a minimum charge (e), the treatment here is only for convenience; it is nonetheless useful and accurate since e is usually very much smaller than macroscopic charges.

Note in this Example three important problem-solving techniques that can be used elsewhere: (1) using symmetry to reduce the complexity of the problem; (2) expressing the charge dQ in terms of a charge density (here linear, $\lambda = Q/2\pi a$); and (3) checking the answer at the limit of large r, which serves as an indication (but not proof) of the correctness of the answer—if the result does not check at large r, your result has to be wrong.

PROBLEM SOLVING
Use symmetry, charge density, and values at $r = 0$ and ∞

CONCEPTUAL EXAMPLE 10 **Charge at the center of a ring.** Imagine a small positive charge placed at the center of a nonconducting ring carrying a uniformly distributed negative charge. Is the positive charge in equilibrium if it is displaced slightly from the center along the axis of the ring, and if so is it stable? What if the small charge is negative? Neglect gravity, as it is much smaller than the electrostatic forces.

RESPONSE The positive charge is in equilibrium because there is no net force on it, by *symmetry*. If the positive charge moves away from the center of the ring along the axis in either direction, the net force will be back towards the center of the ring and so the charge is in *stable* equilibrium. A negative charge at the center of the ring would feel no net force, but is in *unstable* equilibrium because if it moved along the ring's axis, the net force would be away from the ring and the charge would be pushed farther away.

EXAMPLE 11 **Long line of charge.** Determine the magnitude of the electric field at any point P a distance x from the midpoint 0 of a very long line (a wire, say) of uniformly distributed positive charge, Fig. 29. Assume x is much smaller than the length of the wire, and let λ be the charge per unit length (C/m).

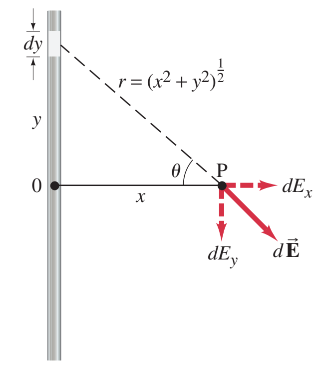


FIGURE 29 Example 11.

APPROACH We set up a coordinate system so the wire is on the y axis with origin 0 as shown. A segment of wire dy has charge $dQ = \lambda\, dy$. The field $d\vec{\mathbf{E}}$ at point P due to this length dy of wire (at y) has magnitude

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dy}{\left(x^2 + y^2\right)},$$

where $r = \left(x^2 + y^2\right)^{\frac{1}{2}}$ as shown in Fig. 29. The vector $d\vec{\mathbf{E}}$ has components dE_x and dE_y as shown where $dE_x = dE\cos\theta$ and $dE_y = dE\sin\theta$.

SOLUTION Because 0 is at the midpoint of the wire, the y component of $\vec{\mathbf{E}}$ will be zero since there will be equal contributions to $E_y = \int dE_y$ from above and below point 0:

$$E_y = \int dE\sin\theta = 0.$$

Thus we have

$$E = E_x = \int dE\cos\theta = \frac{\lambda}{4\pi\epsilon_0}\int\frac{\cos\theta\, dy}{x^2 + y^2}.$$

The integration here is over y, along the wire, with x treated as constant. We must now write θ as a function of y, or y as a function of θ. We do the latter: since $y = x\tan\theta$, then $dy = x\, d\theta/\cos^2\theta$. Furthermore, because $\cos\theta = x/\sqrt{x^2 + y^2}$, then $1/\left(x^2 + y^2\right) = \cos^2\theta/x^2$ and our integrand above is $(\cos\theta)\left(x\, d\theta/\cos^2\theta\right)\left(\cos^2\theta/x^2\right) = \cos\theta\, d\theta/x$. Hence

$$E = \frac{\lambda}{4\pi\epsilon_0}\frac{1}{x}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta = \frac{\lambda}{4\pi\epsilon_0 x}(\sin\theta)\Big|_{-\pi/2}^{\pi/2} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x},$$

where we have assumed the wire is extremely long in both directions $(y \rightarrow \pm\infty)$ which corresponds to the limits $\theta = \pm\pi/2$. Thus the field near a long straight wire of uniform charge decreases inversely as the first power of the distance from the wire.

NOTE This result, obtained for an infinite wire, is a good approximation for a wire of finite length as long as x is small compared to the distance of P from the ends of the wire.

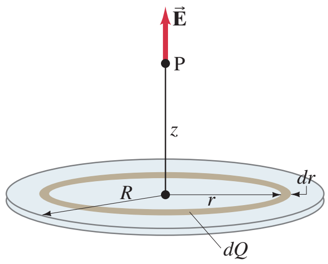


FIGURE 30 Example 12; a uniformly charged flat disk of radius R.

EXAMPLE 12 **Uniformly charged disk.** Charge is distributed uniformly over a thin circular disk of radius R. The charge per unit area $(\mathrm{C/m^2})$ is σ. Calculate the electric field at a point P on the axis of the disk, a distance z above its center, Fig. 30.

APPROACH We can think of the disk as a set of concentric rings. We can then apply the result of Example 9 to each of these rings, and then sum over all the rings.

SOLUTION For the ring of radius r shown in Fig. 30, the electric field has magnitude (see result of Example 9)

$$dE = \frac{1}{4\pi\epsilon_0}\frac{z\,dQ}{\left(z^2 + r^2\right)^{\frac{3}{2}}}$$

where we have written dE (instead of E) for this thin ring of total charge dQ. The ring has area $(dr)(2\pi r)$ and charge per unit area $\sigma = dQ/(2\pi r\,dr)$. We solve this for dQ $(= \sigma\, 2\pi r\,dr)$ and insert it in the equation above for dE:

$$dE = \frac{1}{4\pi\epsilon_0}\frac{z\sigma 2\pi r\,dr}{\left(z^2 + r^2\right)^{\frac{3}{2}}} = \frac{z\sigma r\,dr}{2\epsilon_0\left(z^2 + r^2\right)^{\frac{3}{2}}}.$$

Now we sum over all the rings, starting at $r = 0$ out to the largest with $r = R$:

$$E = \frac{z\sigma}{2\epsilon_0}\int_0^R \frac{r\,dr}{\left(z^2 + r^2\right)^{\frac{3}{2}}} = \frac{z\sigma}{2\epsilon_0}\left[-\frac{1}{\left(z^2 + r^2\right)^{\frac{1}{2}}}\right]_0^R$$

$$= \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\left(z^2 + R^2\right)^{\frac{1}{2}}}\right].$$

This gives the magnitude of $\vec{\mathbf{E}}$ at any point z along the axis of the disk. The direction of each $d\vec{\mathbf{E}}$ due to each ring is along the z axis (as in Example 9), and therefore the direction of $\vec{\mathbf{E}}$ is along z. If Q (and σ) are positive, $\vec{\mathbf{E}}$ points away from the disk; if Q (and σ) are negative, $\vec{\mathbf{E}}$ points toward the disk.

If the radius of the disk in Example 12 is much greater than the distance of our point P from the disk (i.e., $z \ll R$) then we can obtain a very useful result: the second term in the solution above becomes very small, so

$$E = \frac{\sigma}{2\epsilon_0}. \qquad \text{[infinite plane]} \quad \textbf{(7)}$$

This result is valid for any point above (or below) an infinite plane of any shape holding a uniform charge density σ. It is also valid for points close to a finite plane, as long as the point is close to the plane compared to the distance to the edges of the plane. Thus the field near a large uniformly charged plane is uniform, and directed outward if the plane is positively charged.

It is interesting to compare here the distance dependence of the electric field due to a point charge $\left(E \sim 1/r^2\right)$, due to a very long uniform line of charge $(E \sim 1/r)$, and due to a very large uniform plane of charge (E does not depend on r).

EXAMPLE 13 **Two parallel plates.** Determine the electric field between two large parallel plates or sheets, which are very thin and are separated by a distance d which is small compared to their height and width. One plate carries a uniform surface charge density σ and the other carries a uniform surface charge density $-\sigma$, as shown in Fig. 31 (the plates extend upward and downward beyond the part shown).

APPROACH From Eq. 7, each plate sets up an electric field of magnitude $\sigma/2\epsilon_0$. The field due to the positive plate points away from that plate whereas the field due to the negative plate points toward that plate.

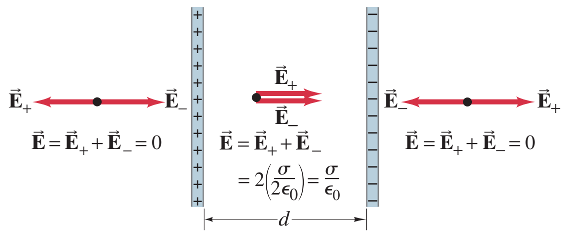


FIGURE 31 Example 13. (Only the center portion of these large plates is shown: their dimensions are large compared to their separation d.)

SOLUTION In the region between the plates, the fields add together as shown:

$$E = E_+ + E_- = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}.$$

The field is uniform, since the plates are very large compared to their separation, so this result is valid for any point, whether near one or the other of the plates, or midway between them as long as the point is far from the ends. Outside the plates, the fields cancel,

$$E = E_+ + E_- = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0,$$

as shown in Fig. 31. These results are valid ideally for infinitely large plates; they are a good approximation for finite plates if the separation is much less than the dimensions of the plate and for points not too close to the edge.

NOTE: These useful and extraordinary results illustrate the principle of superposition and its great power.

8 Field Lines

Since the electric field is a vector, it is sometimes referred to as a **vector field**. We could indicate the electric field with arrows at various points in a given situation, such as at A, B, and C in Fig. 32. The directions of $\vec{\mathbf{E}}_A$, $\vec{\mathbf{E}}_B$, and $\vec{\mathbf{E}}_C$ are the same as that of the forces shown earlier in Fig. 22, but the magnitudes (arrow lengths) are different since we divide $\vec{\mathbf{F}}$ in Fig. 22 by q to get $\vec{\mathbf{E}}$. However, the relative lengths of $\vec{\mathbf{E}}_A$, $\vec{\mathbf{E}}_B$, and $\vec{\mathbf{E}}_C$ are the same as for the forces since we divide by the same q each time. To indicate the electric field in such a way at *many* points, however, would result in many arrows, which might appear complicated or confusing. To avoid this, we use another technique, that of field lines.

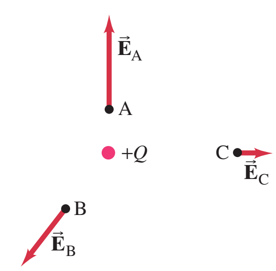


FIGURE 32 Electric field vector shown at three points, due to a single point charge Q. (Compare to Fig. 22.)

To visualize the electric field, we draw a series of lines to indicate the direction of the electric field at various points in space. These **electric field lines** (sometimes called **lines of force**) are drawn so that they indicate the direction of the force due to the given field on a positive test charge. The lines of force due to a single isolated positive charge are shown in Fig. 33a, and for a single isolated negative charge in Fig. 33b. In part (a) the lines point radially outward from the charge, and in part (b) they point radially inward toward the charge because that is the direction the force would be on a positive test charge in each case (as in Fig. 25). Only a few representative lines are shown. We could just as well draw lines in between those shown since the electric field exists there as well. We can draw the lines so that the *number of lines starting on a positive charge, or ending on a negative charge, is proportional to the magnitude of the charge*. Notice that nearer the charge, where the electric field is greater $(F \propto 1/r^2)$, the lines are closer together. This is a general property of electric field lines: *the closer together the lines are, the stronger the electric field in that region*. In fact, field lines can be drawn so that the number of lines crossing unit area perpendicular to $\vec{\mathbf{E}}$ is proportional to the magnitude of the electric field.

FIGURE 33 Electric field lines (a) near a single positive point charge, (b) near a single negative point charge.

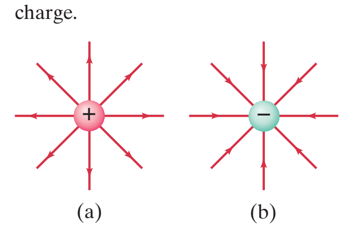


Figure 34a shows the electric field lines due to two equal charges of opposite sign, a combination known as an **electric dipole**. The electric field lines are curved in this case and are directed from the positive charge to the negative charge. The direction of the electric field at any point is tangent to the field line at that point as shown by the vector arrow $\vec{\mathbf{E}}$ at point P. To satisfy yourself that this is the correct pattern for the electric field lines, you can make a few calculations such as those done in Example 8 for just this case (see Fig. 27). Figure 34b shows the electric field lines for two equal positive charges, and Fig. 34c for unequal charges, $-Q$ and $+2Q$. Note that twice as many lines leave $+2Q$, as enter $-Q$ (number of lines is proportional to magnitude of Q). Finally, in Fig. 34d, we see the field lines between two parallel plates carrying equal but opposite charges. Notice that the electric field lines between the two plates start out perpendicular to the surface of the metal plates (we will see why this is true in the next Section) and go directly from one plate to the other, as we expect because a positive test charge placed between the plates would feel a strong repulsion from the positive plate and a strong attraction to the negative plate. The field lines between two close plates are parallel and equally spaced in the central region, but fringe outward near the edges. Thus, in the central region, the electric field has the same magnitude at all points, and we can write (see Example 13)

$$E = \text{constant} = \frac{\sigma}{\epsilon_0}. \qquad \begin{bmatrix}\text{between two closely spaced,} \\ \text{oppositely charged, parallel plates}\end{bmatrix} \qquad \textbf{(8)}$$

The fringing of the field near the edges can often be ignored, particularly if the separation of the plates is small compared to their height and width.

We summarize the properties of field lines as follows:

1. Electric field lines indicate the direction of the electric field; the field points in the direction tangent to the field line at any point.
2. The lines are drawn so that the magnitude of the electric field, E, is proportional to the number of lines crossing unit area perpendicular to the lines. The closer together the lines, the stronger the field.
3. Electric field lines start on positive charges and end on negative charges; and the number starting or ending is proportional to the magnitude of the charge.

Also note that field lines never cross. Why not? Because the electric field can not have two directions at the same point, nor exert more than one force on a test charge.

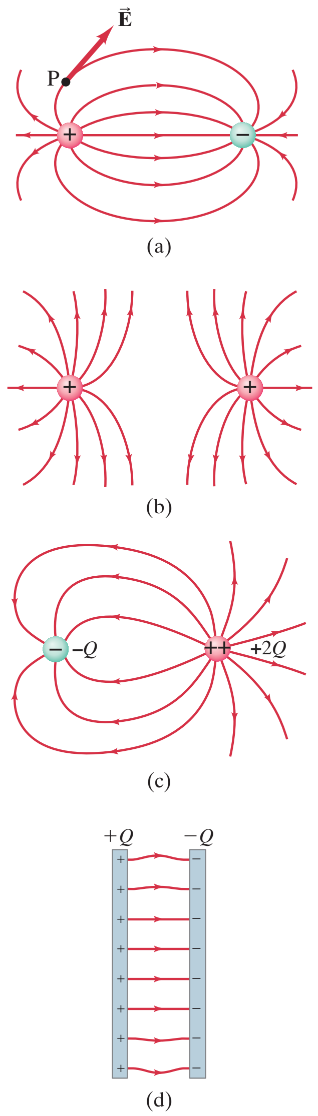


FIGURE 34 Electric field lines for four arrangements of charges.

Gravitational Field

The field concept can also be applied to the gravitational force. Thus we can say that a **gravitational field** exists for every object that has mass. One object attracts another by means of the gravitational field. The Earth, for example, can be said to possess a gravitational field (Fig. 35) which is responsible for the gravitational force on objects. The *gravitational field* is defined as the *force per unit mass*. The magnitude of the Earth's gravitational field at any point above the Earth's surface is thus (GM_E/r^2), where M_E is the mass of the Earth, r is the distance of the point from the Earth's center, and G is the gravitational constant. At the Earth's surface, r is the radius of the Earth and the gravitational field is equal to g, the acceleration due to gravity. Beyond the Earth, the gravitational field can be calculated at any point as a sum of terms due to Earth, Sun, Moon, and other bodies that contribute significantly.

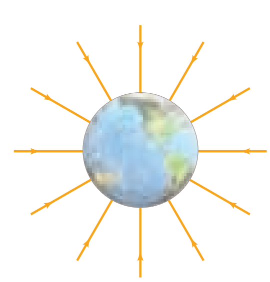


FIGURE 35 The Earth's gravitational field.

9 Electric Fields and Conductors

We now discuss some properties of conductors. First, *the electric field inside a conductor is zero in the static situation*—that is, when the charges are at rest. If there were an electric field within a conductor, there would be a force on the free electrons. The electrons would move until they reached positions where the electric field, and therefore the electric force on them, did become zero.

This reasoning has some interesting consequences. For one, *any net charge on a conductor distributes itself on the surface*. (If there *were* charges inside, there would be an electric field.) For a negatively charged conductor, you can imagine that the negative charges repel one another and race to the surface to get as far from one another as possible. Another consequence is the following. Suppose that a positive charge Q is surrounded by an isolated uncharged metal conductor whose shape is a spherical shell, Fig. 36. Because there can be no field within the metal, the lines leaving the central positive charge must end on negative charges on the inner surface of the metal. Thus an equal amount of negative charge, $-Q$, is induced on the inner surface of the spherical shell. Then, since the shell is neutral, a positive charge of the same magnitude, $+Q$, must exist on the outer surface of the shell. Thus, although no field exists in the metal itself, an electric field exists outside of it, as shown in Fig. 36, as if the metal were not even there.

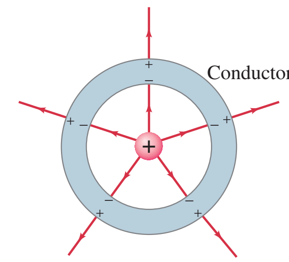
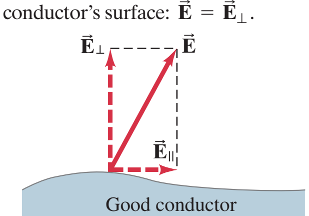


FIGURE 36 A charge inside a neutral spherical metal shell induces charge on its surfaces. The electric field exists even beyond the shell, but not within the conductor itself.

A related property of static electric fields and conductors is that *the electric field is always perpendicular to the surface outside of a conductor*. If there were a component of $\vec{\mathbf{E}}$ parallel to the surface (Fig. 37), it would exert a force on free electrons at the surface, causing the electrons to move along the surface until they reached positions where no net force was exerted on them parallel to the surface—that is, until the electric field was perpendicular to the surface.

FIGURE 37 If the electric field $\vec{\mathbf{E}}$ at the surface of a conductor had a component parallel to the surface, $\vec{\mathbf{E}}_{||}$, the latter would accelerate electrons into motion. In the static case, $\vec{\mathbf{E}}_{||}$ must be zero, and the electric field must be perpendicular to the conductor's surface: $\vec{\mathbf{E}} = \vec{\mathbf{E}}_{\perp}$.



These properties apply only to conductors. Inside a nonconductor, which does not have free electrons, a static electric field can exist as we will see in the next Chapter. Also, the electric field outside a nonconductor does not necessarily make an angle of 90° to the surface.

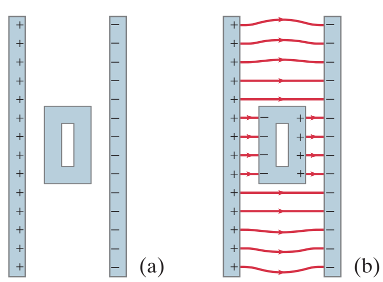
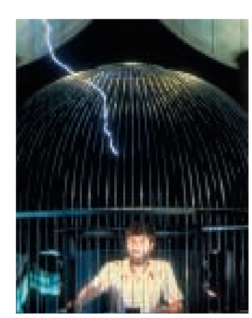


FIGURE 38 Example 14.

CONCEPTUAL EXAMPLE 14 **Shielding, and safety in a storm.** A neutral hollow metal box is placed between two parallel charged plates as shown in Fig. 38a. What is the field like inside the box?

RESPONSE If our metal box had been solid, and not hollow, free electrons in the box would have redistributed themselves along the surface until all their individual fields would have canceled each other inside the box. The net field inside the box would have been zero. For a hollow box, the external field is not changed since the electrons in the metal can move just as freely as before to the surface. Hence the field inside the hollow metal box is also zero, and the field lines are shown in Fig. 38b. A conducting box used in this way is an effective device for shielding delicate instruments and electronic circuits from unwanted external electric fields. We also can see that a relatively safe place to be during a lightning storm is inside a parked car, surrounded by metal. See also Fig. 39, where a person inside a porous "cage" is protected from a strong electric discharge.

FIGURE 39 A strong electric field exists in the vicinity of this "Faraday cage," so strong that stray electrons in the atmosphere are accelerated to the kinetic energy needed to knock electrons out of air atoms, causing an avalanche of charge which flows to (or from) the metal cage. Yet the person inside the cage is not affected.



Michael J. Lutch/Boston Museum of Science

PHYSICS APPLIED
Electrical shielding

10 Motion of a Charged Particle in an Electric Field

If an object having an electric charge q is at a point in space where the electric field is $\vec{\mathbf{E}}$, the force on the object is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

(see Eq. 5). In the past few Sections we have seen how to determine $\vec{\mathbf{E}}$ for some particular situations. Now let us suppose we know $\vec{\mathbf{E}}$ and we want to find the force on a charged object and the object's subsequent motion. (We assume no other forces act.)

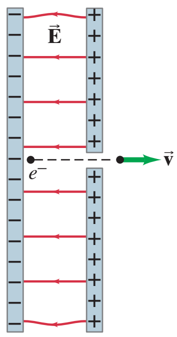


FIGURE 40 Example 15.

EXAMPLE 15 **Electron accelerated by electric field.** An electron (mass $m = 9.1 \times 10^{-31}$ kg) is accelerated in the uniform field $\vec{\mathbf{E}}\,(E = 2.0 \times 10^4\,\text{N/C})$ between two parallel charged plates. The separation of the plates is 1.5 cm. The electron is accelerated from rest near the negative plate and passes through a tiny hole in the positive plate, Fig. 40. (*a*) With what speed does it leave the hole? (*b*) Show that the gravitational force can be ignored. Assume the hole is so small that it does not affect the uniform field between the plates.

APPROACH We can obtain the electron's velocity using kinematic equations, after first finding its acceleration from Newton's second law, $F = ma$. The magnitude of the force on the electron is $F = qE$ and is directed to the right.

SOLUTION (*a*) The magnitude of the electron's acceleration is

$$a = \frac{F}{m} = \frac{qE}{m}.$$

Between the plates $\vec{\mathbf{E}}$ is uniform so the electron undergoes uniformly accelerated motion with acceleration

$$a = \frac{(1.6 \times 10^{-19}\,\text{C})(2.0 \times 10^4\,\text{N/C})}{(9.1 \times 10^{-31}\,\text{kg})} = 3.5 \times 10^{15}\,\text{m/s}^2.$$

It travels a distance $x = 1.5 \times 10^{-2}$ m before reaching the hole, and since its initial speed was zero, we can use the kinematic equation, $v^2 = v_0^2 + 2ax$, with $v_0 = 0$:

$$v = \sqrt{2ax} = \sqrt{2(3.5 \times 10^{15}\,\text{m/s}^2)(1.5 \times 10^{-2}\,\text{m})} = 1.0 \times 10^7\,\text{m/s}.$$

There is no electric field outside the plates, so after passing through the hole, the electron moves with this speed, which is now constant.

(*b*) The magnitude of the electric force on the electron is

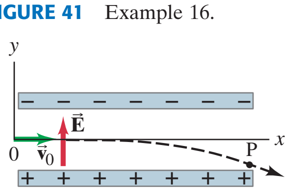
$$qE = (1.6 \times 10^{-19}\,\text{C})(2.0 \times 10^4\,\text{N/C}) = 3.2 \times 10^{-15}\,\text{N}.$$

The gravitational force is

$$mg = (9.1 \times 10^{-31}\,\text{kg})(9.8\,\text{m/s}^2) = 8.9 \times 10^{-30}\,\text{N},$$

which is 10^{14} times smaller! Note that the electric field due to the electron does not enter the problem (since a particle cannot exert a force on itself).

FIGURE 41 Example 16.



EXAMPLE 16 **Electron moving perpendicular to $\vec{\mathbf{E}}$.** Suppose an electron traveling with speed v_0 enters a uniform electric field $\vec{\mathbf{E}}$, which is at right angles to $\vec{\mathbf{v}}_0$ as shown in Fig. 41. Describe its motion by giving the equation of its path while in the electric field. Ignore gravity.

APPROACH Again we use Newton's second law, with $F = qE$, and kinematic equations.

SOLUTION When the electron enters the electric field (at $x = y = 0$) it has velocity $\vec{\mathbf{v}}_0 = v_0\hat{\mathbf{i}}$ in the x direction. The electric field $\vec{\mathbf{E}}$, pointing vertically upward, imparts a uniform vertical acceleration to the electron of

$$a_y = \frac{F}{m} = \frac{qE}{m} = -\frac{eE}{m},$$

where we set $q = -e$ for the electron.

The electron's vertical position is given by

$$y = \frac{1}{2} a_y t^2 = -\frac{eE}{2m} t^2$$

since the motion is at constant acceleration. The horizontal position is given by

$$x = v_0 t$$

since $a_x = 0$. We eliminate t between these two equations and obtain

$$y = -\frac{eE}{2mv_0^2} x^2,$$

which is the equation of a parabola (just as in projectile motion).

11 Electric Dipoles

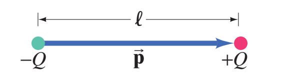
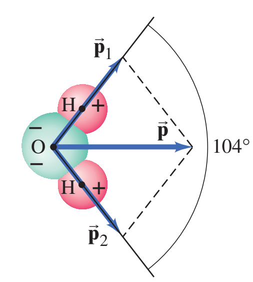


FIGURE 42 A dipole consists of equal but opposite charges, $+Q$ and $-Q$, separated by a distance ℓ. The dipole moment is $\vec{\mathbf{p}} = Q\vec{\ell}$ and points from the negative to the positive charge.

The combination of two equal charges of opposite sign, $+Q$ and $-Q$, separated by a distance ℓ, is referred to as an **electric dipole**. The quantity $Q\ell$ is called the **dipole moment** and is represented† by the symbol p. The dipole moment can be considered to be a vector $\vec{\mathbf{p}}$, of magnitude $Q\ell$, that points from the negative to the positive charge as shown in Fig. 42. Many molecules, such as the diatomic molecule CO, have a dipole moment (C has a small positive charge and O a small negative charge of equal magnitude), and are referred to as **polar molecules**. Even though the molecule as a whole is neutral, there is a separation of charge that results from an uneven sharing of electrons by the two atoms.‡ (Symmetric diatomic molecules, like O_2, have no dipole moment.) The water molecule, with its uneven sharing of electrons (O is negative, the two H are positive), also has a dipole moment—see Fig. 43.

FIGURE 43 In the water molecule (H_2O), the electrons spend more time around the oxygen atom than around the two hydrogen atoms. The net dipole moment $\vec{\mathbf{p}}$ can be considered as the vector sum of two dipole moments $\vec{\mathbf{p}}_1$ and $\vec{\mathbf{p}}_2$ that point from the O toward each H as shown: $\vec{\mathbf{p}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2$.



Dipole in an External Field

First let us consider a dipole, of dipole moment $p = Q\ell$, that is placed in a uniform electric field $\vec{\mathbf{E}}$, as shown in Fig. 44. If the field is uniform, the force $Q\vec{\mathbf{E}}$ on the positive charge and the force $-Q\vec{\mathbf{E}}$ on the negative charge result in no net force on the dipole. There will, however, be a *torque* on the dipole (Fig. 44) which has magnitude (calculated about the center, 0, of the dipole)

$$\tau = QE\frac{\ell}{2}\sin\theta + QE\frac{\ell}{2}\sin\theta = pE\sin\theta. \tag{9a}$$

This can be written in vector notation as

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}. \tag{9b}$$

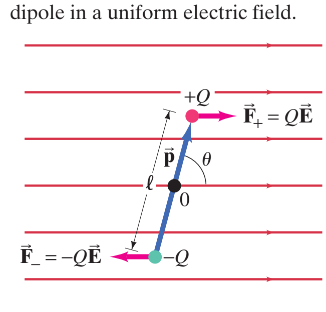
The effect of the torque is to try to turn the dipole so $\vec{\mathbf{p}}$ is parallel to $\vec{\mathbf{E}}$. The work done on the dipole by the electric field to change the angle θ from θ_1 to θ_2 is

$$W = \int_{\theta_1}^{\theta_2} \tau\, d\theta.$$

We need to write the torque as $\tau = -pE\sin\theta$ because its direction is opposite to the direction of increasing θ (right-hand rule). Then

$$W = \int_{\theta_1}^{\theta_2} \tau\, d\theta = -pE\int_{\theta_1}^{\theta_2} \sin\theta\, d\theta = pE\cos\theta\Big|_{\theta_1}^{\theta_2} = pE(\cos\theta_2 - \cos\theta_1).$$

FIGURE 44 (below) An electric dipole in a uniform electric field.



Positive work done by the field decreases the potential energy, U, of the dipole in this field. (Recall the relation between work and potential energy, $\Delta U = -W$.) If we choose $U = 0$ when $\vec{\mathbf{p}}$ is perpendicular to $\vec{\mathbf{E}}$ (that is, choosing $\theta_1 = 90°$ so $\cos\theta_1 = 0$), and setting $\theta_2 = \theta$, then

$$U = -W = -pE\cos\theta = -\vec{\mathbf{p}}\cdot\vec{\mathbf{E}}. \tag{10}$$

If the electric field is *not* uniform, the force on the $+Q$ of the dipole may not have the same magnitude as on the $-Q$, so there may be a net force as well as a torque.

†Be careful not to confuse this p for dipole moment with p for momentum.

‡The value of the separated charges may be a fraction of e (say $\pm 0.2e$ or $\pm 0.4e$) but note that such charges do not violate what we said about e being the smallest charge. These charges less than e cannot be isolated and merely represent how much time electrons spend around one atom or the other.

EXAMPLE 17 **Dipole in a field.** The dipole moment of a water molecule is $6.1 \times 10^{-30}\,\mathrm{C\cdot m}$. A water molecule is placed in a uniform electric field with magnitude $2.0 \times 10^5\,\mathrm{N/C}$. (*a*) What is the magnitude of the maximum torque that the field can exert on the molecule? (*b*) What is the potential energy when the torque is at its maximum? (*c*) In what position will the potential energy take on its greatest value? Why is this different than the position where the torque is maximum?

APPROACH The torque is given by Eq. 9 and the potential energy by Eq. 10.

SOLUTION (*a*) From Eq. 9 we see that τ is maximized when θ is 90°. Then $\tau = pE = (6.1 \times 10^{-30}\,\mathrm{C\cdot m})(2.0 \times 10^5\,\mathrm{N/C}) = 1.2 \times 10^{-24}\,\mathrm{N\cdot m}$.

(*b*) The potential energy for $\theta = 90°$ is zero (Eq. 10). Note that the potential energy is negative for smaller values of θ, so U is not a minimum for $\theta = 90°$.

(*c*) The potential energy U will be a maximum when $\cos\theta = -1$ in Eq. 10, so $\theta = 180°$, meaning $\vec{\mathbf{E}}$ and $\vec{\mathbf{p}}$ are antiparallel. The potential energy is maximized when the dipole is oriented so that it has to rotate through the largest angle, 180°, to reach the equilibrium position at $\theta = 0°$. The torque on the other hand is maximized when the electric forces are perpendicular to $\vec{\mathbf{p}}$.

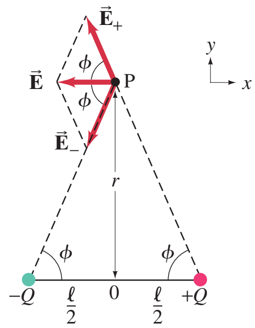


FIGURE 45 Electric field due to an electric dipole.

Electric Field Produced by a Dipole

We have just seen how an external electric field affects an electric dipole. Now let us suppose that there is no external field, and we want to determine the electric field produced *by* the dipole. For brevity, we restrict ourselves to points that are on the perpendicular bisector of the dipole, such as point P in Fig. 45 which is a distance r above the midpoint of the dipole. Note that r in Fig. 45 is not the distance from either charge to point P; the latter distance is $(r^2 + \ell^2/4)^{\frac{1}{2}}$ and this is what must be used in Eq. 4. The total field at P is

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_+ + \vec{\mathbf{E}}_-,$$

where $\vec{\mathbf{E}}_+$ and $\vec{\mathbf{E}}_-$ are the fields due to the + and − charges respectively. The magnitudes E_+ and E_- are equal:

$$E_+ = E_- = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2 + \ell^2/4}.$$

Their y components cancel at point P (*symmetry* again), so the magnitude of the total field $\vec{\mathbf{E}}$ is

$$E = 2E_+\cos\phi = \frac{1}{2\pi\epsilon_0}\left(\frac{Q}{r^2 + \ell^2/4}\right)\frac{\ell}{2(r^2 + \ell^2/4)^{\frac{1}{2}}}$$

or, setting $Q\ell = p$,

$$E = \frac{1}{4\pi\epsilon_0}\frac{p}{(r^2 + \ell^2/4)^{\frac{3}{2}}}. \qquad \left[\begin{array}{c}\text{on perpendicular bisector}\\ \text{of dipole}\end{array}\right] \qquad \textbf{(11)}$$

Far from the dipole, $r \gg \ell$, this reduces to

$$E = \frac{1}{4\pi\epsilon_0}\frac{p}{r^3}. \qquad \left[\begin{array}{c}\text{on perpendicular bisector}\\ \text{of dipole; } r \gg \ell\end{array}\right] \qquad \textbf{(12)}$$

So the field decreases more rapidly for a dipole than for a single point charge $(1/r^3$ versus $1/r^2)$, which we expect since at large distances the two opposite charges appear so close together as to neutralize each other. This $1/r^3$ dependence also applies for points not on the perpendicular bisector.

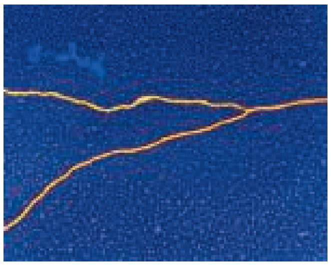
*12 Electric Forces in Molecular Biology; DNA

The interior of every biological cell is mainly water. We can imagine a cell as a vast sea of molecules continually in motion (kinetic theory), colliding with one another with various amounts of kinetic energy. These molecules interact with one another because of *electrostatic attraction* between molecules.

PHYSICS APPLIED
Inside a cell: kinetic theory plus electrostatic force

Indeed, cellular processes are now considered to be the result of *random ("thermal") molecular motion plus the ordering effect of the electrostatic force*. As an example, we look at DNA structure and replication. The picture we present has not been seen "in action." Rather, it is a model of what happens based on physical theories and experiment.

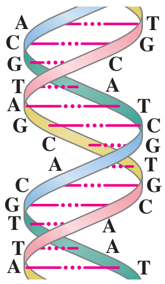
The genetic information that is passed on from generation to generation in all living cells is contained in the chromosomes, which are made up of genes. Each gene contains the information needed to produce a particular type of protein molecule, and that information is built into the principal molecule of a chromosome, DNA (deoxyribonucleic acid), Fig. 46. DNA molecules are made up of many small molecules known as nucleotide bases which are each polar due to unequal sharing of electrons. There are four types of nucleotide bases in DNA: adenine (A), cytosine (C), guanine (G), and thymine (T).



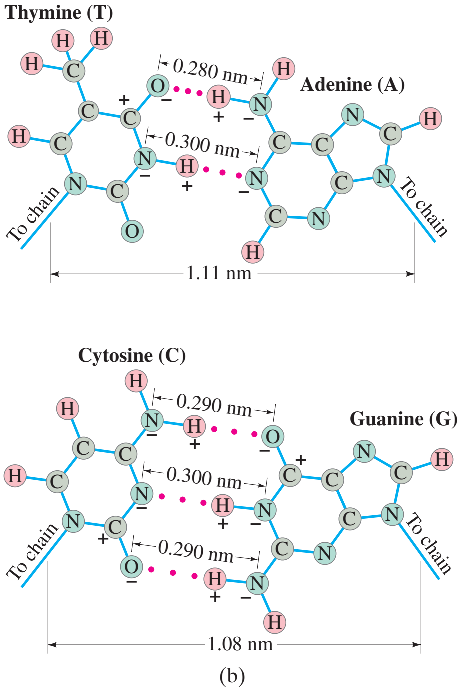
Dr. Gopal Murti/Science Photo Library/Photo Researchers, Inc.

FIGURE 46 DNA replicating in a human HeLa cancer cell. This is a false-color image made by a transmission electron microscope (TEM).

The DNA of a chromosome generally consists of two long DNA strands wrapped about one another in the shape of a "double helix." The genetic information is contained in the specific order of the four bases (A, C, G, T) along the strand. As shown in Fig. 47, the two strands are attracted by electrostatic forces—that is, by the attraction of positive charges to negative charges that exist on parts of the molecules. We see in Fig. 47a that an A (adenine) on one strand is always opposite a T on the other strand; similarly, a G is always opposite a C. This important ordering effect occurs because the shapes of A, T, C, and G are such that a T fits closely only into an A, and a G into a C; and only in the case of this close proximity of the charged portions is the electrostatic force great enough to hold them together even for a short time (Fig. 47b), forming what are referred to as "weak bonds."



(a)



(b)

FIGURE 47 (a) Section of a DNA double helix. (b) "Close-up" view of the helix, showing how A and T attract each other and how G and C attract each other through electrostatic forces. The + and − signs represent net charges, usually a fraction of e, due to uneven sharing of electrons. The red dots indicate the electrostatic attraction (often called a "weak bond" or "hydrogen bond"). Note that there are two weak bonds between A and T, and three between C and G.

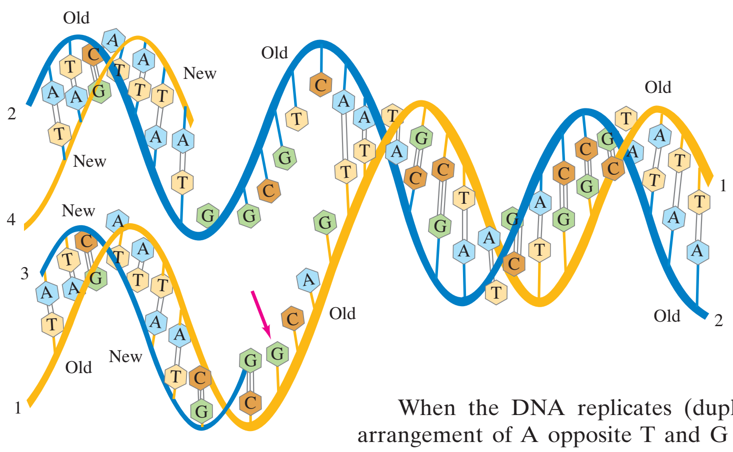


FIGURE 48 Replication of DNA.

When the DNA replicates (duplicates) itself just before cell division, the arrangement of A opposite T and G opposite C is crucial for ensuring that the genetic information is passed on accurately to the next generation, Fig. 48. The two strands of DNA separate (with the help of enzymes, which also operate via the electrostatic force), leaving the charged parts of the bases exposed. Once replication starts, let us see how the correct order of bases occurs by looking at the G molecule indicated by the red arrow in Fig. 48. Many unattached nucleotide bases of all four kinds are bouncing around in the cellular fluid, and the only type that will experience attraction to our G, if it bounces close to it, will be a C. The charges on the other three bases can not get close enough to those on the G to provide a significant attractive force—remember that the force decreases rapidly with distance $(\propto 1/r^2)$. Because the G does not attract an A, T, or G appreciably, an A, T, or G will be knocked away by collisions with other molecules before enzymes can attach it to the growing chain (number 3). But the electrostatic force will often hold a C opposite our G long enough so that an enzyme can attach the C to the growing end of the new chain. Thus we see that electrostatic forces are responsible for selecting the bases in the proper order during replication.

This process of DNA replication is often presented as if it occurred in clockwork fashion—as if each molecule knew its role and went to its assigned place. But this is not the case. The forces of attraction are rather weak, and if the molecular shapes are not just right, there is almost no electrostatic attraction, which is why there are few mistakes. Thus, out of the random motion of the molecules, the electrostatic force acts to bring order out of chaos.

The random (thermal) velocities of molecules in a cell affect *cloning*. When a bacterial cell divides, the two new bacteria have nearly identical DNA. Even if the DNA were perfectly identical, the two new bacteria would not end up behaving in the same way. Long protein, DNA, and RNA molecules get bumped into different shapes, and even the expression of genes can thus be different. Loosely held parts of large molecules such as a methyl group (CH_3) can also be knocked off by a strong collision with another molecule. Hence, cloned organisms are not identical, even if their DNA were identical. Indeed, there can not really be genetic determinism.

* 13 Photocopy Machines and Computer Printers Use Electrostatics

Photocopy machines and laser printers use electrostatic attraction to print an image. They each use a different technique to project an image onto a special cylindrical drum. The drum is typically made of aluminum, a good conductor; its surface is coated with a thin layer of selenium, which has the interesting property (called "photoconductivity") of being an electrical nonconductor in the dark, but a conductor when exposed to light.

In a *photocopier*, lenses and mirrors focus an image of the original sheet of paper onto the drum, much like a camera lens focuses an image on film. Step 1 is

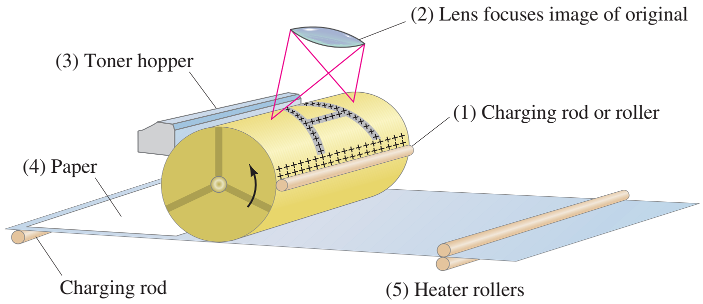


FIGURE 49 Inside a photocopy machine: (1) the selenium drum is given a + charge; (2) the lens focuses image on drum—only dark spots stay charged; (3) toner particles (negatively charged) are attracted to positive areas on drum; (4) the image is transferred to paper; (5) heat binds the image to the paper.

PHYSICS APPLIED
Photocopy machines

the placing of a uniform positive charge on the drum's selenium layer by a charged rod or roller, done in the dark. In step 2, the image to be copied or printed is projected onto the drum. For simplicity, let us assume the image is a dark letter A on a white background (as on the page of a book) as shown in Fig. 49. The letter A on the drum is dark, but all around it is light. At all these light places, the selenium becomes conducting and electrons flow in from the aluminum beneath, neutralizing those positive areas. In the dark areas of the letter A, the selenium is nonconducting and so retains a positive charge, Fig. 49. In step 3, a fine dark powder known as *toner* is given a negative charge, and brushed on the drum as it rotates. The negatively charged toner particles are attracted to the positive areas on the drum (the A in our case) and stick only there. In step 4, the rotating drum presses against a piece of paper which has been positively charged more strongly than the selenium, so the toner particles are transferred to the paper, forming the final image. Finally, step 5, the paper is heated to fix the toner particles firmly on the paper.

In a color copier (or printer), this process is repeated for each color—black, cyan (blue), magenta (red), and yellow. Combining these four colors in different proportions produces any desired color.

PHYSICS APPLIED
Laser printer

A *laser printer*, on the other hand, uses a computer output to program the intensity of a laser beam onto the selenium-coated drum. The thin beam of light from the laser is scanned (by a movable mirror) from side to side across the drum in a series of horizontal lines, each line just below the previous line. As the beam sweeps across the drum, the intensity of the beam is varied by the computer output, being strong for a point that is meant to be white or bright, and weak or zero for points that are meant to come out dark. After each sweep, the drum rotates very slightly for additional sweeps, Fig. 50, until a complete image is formed on it. The light parts of the selenium become conducting and lose their electric charge, and the toner sticks only to the dark, electrically charged areas. The drum then transfers the image to paper, as in a photocopier.

PHYSICS APPLIED
Inkjet printer

An *inkjet printer* does not use a drum. Instead nozzles spray tiny droplets of ink directly at the paper. The nozzles are swept across the paper, each sweep just above the previous one as the paper moves down. On each sweep, the ink makes dots on the paper, except for those points where no ink is desired, as directed by the computer. The image consists of a huge number of very tiny dots. The quality or resolution of a printer is usually specified in dots per inch (dpi) in each (linear) direction.

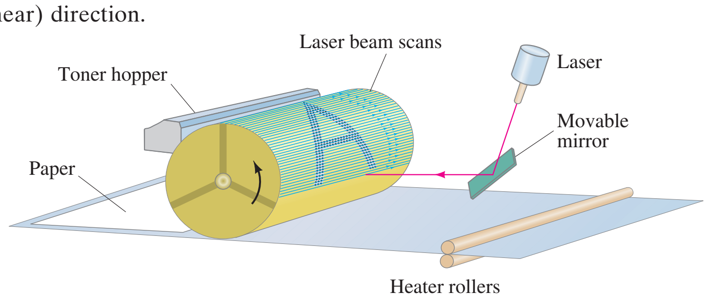


FIGURE 50 Inside a laser printer: A movable mirror sweeps the laser beam in horizontal lines across the drum.

Summary

There are two kinds of **electric charge**, positive and negative. These designations are to be taken algebraically—that is, any charge is plus or minus so many coulombs (C), in SI units.

Electric charge is **conserved**: if a certain amount of one type of charge is produced in a process, an equal amount of the opposite type is also produced; thus the *net* charge produced is zero.

According to atomic theory, electricity originates in the atom, each consisting of a positively charged nucleus surrounded by negatively charged electrons. Each electron has a charge $-e = -1.6 \times 10^{-19}\,\mathrm{C}$.

Electric **conductors** are those materials in which many electrons are relatively free to move, whereas electric **insulators** are those in which very few electrons are free to move.

An object is negatively charged when it has an excess of electrons, and positively charged when it has less than its normal amount of electrons. The charge on any object is thus a whole number times $+e$ or $-e$. That is, charge is **quantized**.

An object can become charged by rubbing (in which electrons are transferred from one material to another), by conduction (which is transfer of charge from one charged object to another by touching), or by induction (the separation of charge within an object because of the close approach of another charged object but without touching).

Electric charges exert a force on each other. If two charges are of opposite types, one positive and one negative, they each exert an attractive force on the other. If the two charges are the same type, each repels the other.

The magnitude of the force one point charge exerts on another is proportional to the product of their charges, and inversely proportional to the square of the distance between them:

$$F = k\frac{Q_1 Q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2}; \qquad \textbf{(1, 2)}$$

this is **Coulomb's law**. In SI units, k is often written as $1/4\pi\epsilon_0$.

We think of an **electric field** as existing in space around any charge or group of charges. The force on another charged object is then said to be due to the electric field present at its location.

The *electric field*, $\vec{\mathbf{E}}$, at any point in space due to one or more charges, is defined as the force per unit charge that would act on a positive test charge q placed at that point:

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q}. \qquad \textbf{(3)}$$

The magnitude of the electric field a distance r from a point charge Q is

$$E = k\frac{Q}{r^2}. \qquad \textbf{(4a)}$$

The total electric field at a point in space is equal to the vector sum of the individual fields due to each contributing charge (**principle of superposition**).

Electric fields are represented by **electric field lines** that start on positive charges and end on negative charges. Their direction indicates the direction the force would be on a tiny positive test charge placed at each point. The lines can be drawn so that the number per unit area is proportional to the magnitude of E.

The static electric field inside a conductor is zero, and the electric field lines just outside a charged conductor are perpendicular to its surface.

An **electric dipole** is a combination of two equal but opposite charges, $+Q$ and $-Q$, separated by a distance ℓ. The **dipole moment** is $p = Q\ell$. A dipole placed in a uniform electric field feels no net force but does feel a net torque (unless $\vec{\mathbf{p}}$ is parallel to $\vec{\mathbf{E}}$). The electric field produced by a dipole decreases as the third power of the distance r from the dipole $(E \propto 1/r^3)$ for r large compared to ℓ.

[*In the replication of DNA, the electrostatic force plays a crucial role in selecting the proper molecules so the genetic information is passed on accurately from generation to generation.]

Answers to Exercises

A: (*e*).
B: 5 N.
C: 1.2 N, to the right.
D: (*a*) No; (*b*) yes, midway between them.
E: (*d*), if the two + charges are not at opposite corners (use symmetry).