Wireless technology is all around us: in this photo we see a Bluetooth earpiece for wireless telephone communication and a wi-fi computer. The wi-fi antenna is just visible at the lower left. All these devices work by electromagnetic waves traveling through space, based on the great work of Maxwell which we investigate in this Chapter. Modern wireless devices are applications of Marconi's development of long distance transmission of information a century ago.

We will see in this Chapter that Maxwell predicted the existence of EM waves from his famous equations. Maxwell's equations themselves are a magnificent summary of electromagnetism. We will also examine how EM waves carry energy and momentum.

Douglas C. Giancoli

Maxwell's Equations and Electromagnetic Waves

CONTENTS

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.*]

Which of the following best describes the difference between radio waves and X-rays?

(a) X-rays are radiation while radio waves are electromagnetic waves.
(b) Both can be thought of as electromagnetic waves. They differ only in wavelength and frequency.
(c) X-rays are pure energy. Radio waves are made of fields, not energy.
(d) Radio waves come from electric currents in an antenna. X-rays are not related to electric charge.
(e) The fact that X-rays can expose film, and radio waves cannot, means they are fundamentally different.

The culmination of electromagnetic theory in the nineteenth century was the prediction, and the experimental verification, that waves of electromagnetic fields could travel through space. This achievement opened a whole new world of communication: first the wireless telegraph, then radio and television, and more recently cell phones, remote-control devices, wi-fi, and Bluetooth. Most important was the spectacular prediction that visible light is an electromagnetic wave.

The theoretical prediction of electromagnetic waves was the work of the Scottish physicist James Clerk Maxwell (1831–1879; Fig. 1), who unified, in one magnificent theory, all the phenomena of electricity and magnetism.

The development of electromagnetic theory in the early part of the nineteenth century by Oersted, Ampère, and others was not actually done in terms of electric and magnetic fields. The idea of the field was introduced somewhat later by Faraday, and was not generally used until Maxwell showed that all electric and magnetic phenomena could be described using only four equations involving electric and magnetic fields. These equations, known as **Maxwell's equations**, are the basic equations for all electromagnetism. They are fundamental in the same sense that Newton's three laws of motion and the law of universal gravitation are for mechanics. In a sense, they are even more fundamental, since they are consistent with the theory of relativity, whereas Newton's laws are not. Because all of electromagnetism is contained in this set of four equations, Maxwell's equations are considered one of the great triumphs of human intellect.

Before we discuss Maxwell's equations and electromagnetic waves, we first need to discuss a major new prediction of Maxwell's, and, in addition, Gauss's law for magnetism.

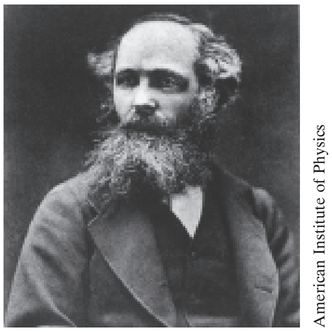


FIGURE 1 James Clerk Maxwell (1831–1879).

1 Changing Electric Fields Produce Magnetic Fields; Ampère's Law and Displacement Current

Ampère's Law

That a magnetic field is produced by an electric current was discovered by Oersted, and the mathematic relation is given by Ampère's law:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 I_{\text{encl}}.$$

Is it possible that magnetic fields could be produced in another way as well? For if a changing magnetic field produces an electric field, then perhaps the reverse might be true as well: that *a changing electric field will produce a magnetic field*. If this were true, it would signify a beautiful *symmetry* in nature.

To back up this idea that a changing electric field might produce a magnetic field, we use an indirect argument that goes something like this. According to Ampère's law, we divide any chosen closed path into short segments $d\vec{\boldsymbol{\ell}}$, take the dot product of each $d\vec{\boldsymbol{\ell}}$ with the magnetic field $\vec{\mathbf{B}}$ at that segment, and sum (integrate) all these products over the chosen closed path. That sum will equal μ_0 times the total current I that passes through a surface bounded by the path of the line integral. When we applied Ampère's law to the field around a straight wire, we imagined the current as passing through the circular area enclosed by our circular loop, and that area is the flat surface 1 shown in Fig. 2. However, we could just as well use the sackshaped surface 2 in Fig. 2 as the surface for Ampère's law, since the same current I passes through it.

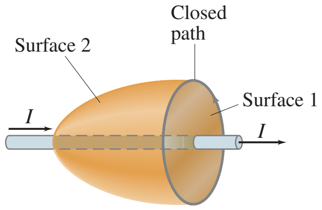
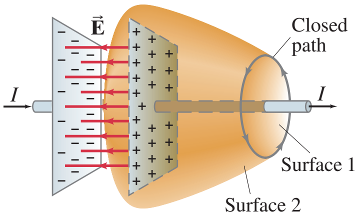


FIGURE 2 Ampère's law applied to two different surfaces bounded by the same closed path.

Now consider the closed circular path for the situation of Fig. 3, where a capacitor is being discharged. Ampère's law works for surface 1 (current I passes through surface 1), but it does not work for surface 2, since no current passes through surface 2. There is a magnetic field around the wire, so the left side of Ampère's law ($\int \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}}$) is not zero; yet no current flows through surface 2, so the right side of Ampère's law *is* zero. We seem to have a contradiction of Ampère's law.

FIGURE 3 A capacitor discharging. A conduction current passes through surface 1, but no conduction current passes through surface 2. An extra term is needed in Ampère's law.



There is a magnetic field present in Fig. 3, however, only if charge is flowing to or away from the capacitor plates. The changing charge on the plates means that the electric field between the plates is changing in time. Maxwell resolved the problem of no current through surface 2 in Fig. 3 by proposing that there needs to be an extra term on the right in Ampère's law involving the changing electric field.

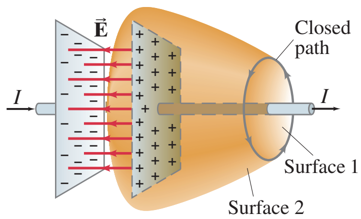


FIGURE 3 (repeated) See text.

Let us see what this term should be by determining it for the changing electric field between the capacitor plates in Fig. 3. The charge Q on a capacitor of capacitance C is $Q = CV$ where V is the potential difference between the plates. Also recall that $V = Ed$ where d is the (small) separation of the plates and E is the (uniform) electric field strength between them, if we ignore any fringing of the field. Also, for a parallel-plate capacitor, $C = \epsilon_0 A/d,$ where A is the area of each plate. We combine these to obtain:

$$Q = CV = \left(\epsilon_0 \frac{A}{d}\right)(Ed) = \epsilon_0 AE.$$

If the charge on each plate changes at a rate dQ/dt, the electric field changes at a proportional rate. That is, by differentiating this expression for Q, we have:

$$\frac{dQ}{dt} = \epsilon_0 A \frac{dE}{dt}.$$

Now dQ/dt is also the current I flowing into or out of the capacitor:

$$I = \frac{dQ}{dt} = \epsilon_0 A \frac{dE}{dt} = \epsilon_0 \frac{d\Phi_E}{dt}$$

where $\Phi_E = EA$ is the **electric flux** through the closed path (surface 2 in Fig. 3). In order to make Ampère's law work for surface 2 in Fig. 3, as well as for surface 1 (where current I flows), we therefore write:

Ampère's law (general form)

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 I_{\text{encl}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}. \qquad \textbf{(1)}$$

This equation represents the general form of **Ampère's law**,† and embodies Maxwell's idea that a magnetic field can be caused not only by an ordinary electric current, but also by a changing electric field or changing electric flux. Although we arrived at it for a special case, Eq. 1 has proved valid in general. The last term on the right in Eq. 1 is usually very small, and not easy to measure experimentally.

EXAMPLE 1 **Charging capacitor.** A 30-pF air-gap capacitor has circular plates of area $A = 100\ \text{cm}^2$. It is charged by a 70-V battery through a 2.0-Ω resistor. At the instant the battery is connected, the electric field between the plates is changing most rapidly. At this instant, calculate (*a*) the current into the plates, and (*b*) the rate of change of electric field between the plates. (*c*) Determine the magnetic field induced between the plates. Assume $\vec{\mathbf{E}}$ is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

APPROACH In RC circuits, the charge on a capacitor being charged, as a function of time, is

$$Q = CV_0\left(1 - e^{-t/RC}\right),$$

where V_0 is the voltage of the battery. To find the current at $t = 0,$ we differentiate this and substitute the values $V_0 = 70\ \text{V}, C = 30\ \text{pF},\ R = 2.0\ \Omega.$

SOLUTION (*a*) We take the derivative of Q and evaluate it at $t = 0$:

$$\left.\frac{dQ}{dt}\right|_{t=0} = \left.\frac{CV_0}{RC}e^{-t/RC}\right|_{t=0} = \frac{V_0}{R} = \frac{70\ \text{V}}{2.0\ \Omega} = 35\ \text{A}.$$

This is the rate at which charge accumulates on the capacitor and equals the current flowing in the circuit at $t = 0$.

(*b*) The electric field between two closely spaced conductors is given by

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon_0}.$$

†Actually, there is a third term on the right for the case when a magnetic field is produced by magnetized materials. This can be accounted for by changing μ_0 to μ, but we will mainly be interested in cases where no magnetic material is present. In the presence of a dielectric, ϵ_0 is replaced by $\epsilon = K\epsilon_0$.

Hence

$$\frac{dE}{dt} = \frac{dQ/dt}{\epsilon_0 A} = \frac{35\text{ A}}{\left(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.0 \times 10^{-2}\,\text{m}^2\right)} = 4.0 \times 10^{14}\,\text{V/m}\cdot\text{s}.$$

(*c*) Although we will not prove it, we might expect the lines of $\vec{\mathbf{B}}$, because of *symmetry*, to be circles, and to be perpendicular to $\vec{\mathbf{E}}$, as shown in Fig. 4; this is the same symmetry we saw for the inverse situation of a changing magnetic field producing an electric field. To determine the magnitude of B between the plates we apply Ampère's law, Eq. 1, with the current $I_{\text{encl}} = 0$:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}.$$

We choose our path to be a circle of radius r, centered at the center of the plate, and thus following a magnetic field line such as the one shown in Fig. 4. For $r \leq r_0$ (the radius of plate) the flux through a circle of radius r is $E(\pi r^2)$ since E is assumed uniform between the plates at any moment. So from Ampère's law we have

$$B(2\pi r) = \mu_0 \epsilon_0 \frac{d}{dt}\left(\pi r^2 E\right) = \mu_0 \epsilon_0 \pi r^2 \frac{dE}{dt}.$$

Hence

$$B = \frac{\mu_0 \epsilon_0}{2} r \frac{dE}{dt}. \qquad \left[r \leq r_0\right]$$

We assume $\vec{\mathbf{E}} = 0$ for $r > r_0$, so for points beyond the edge of the plates all the flux is contained within the plates $\left(\text{area} = \pi r_0^2\right)$ and $\Phi_E = E\pi r_0^2$. Thus Ampère's law gives

$$B(2\pi r) = \mu_0 \epsilon_0 \frac{d}{dt}\left(\pi r_0^2 E\right) = \mu_0 \epsilon_0 \pi r_0^2 \frac{dE}{dt}$$

or

$$B = \frac{\mu_0 \epsilon_0 r_0^2}{2r} \frac{dE}{dt}. \qquad \left[r \geq r_0\right]$$

B has its maximum value at $r = r_0$ which, from either relation above (using $r_0 = \sqrt{A/\pi} = 5.6\,\text{cm}$), is

$$\begin{aligned} B_{\max} &= \frac{\mu_0 \epsilon_0 r_0}{2} \frac{dE}{dt} \\ &= \tfrac{1}{2}\left(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(5.6 \times 10^{-2}\,\text{m}\right)\left(4.0 \times 10^{14}\,\text{V/m}\cdot\text{s}\right) \\ &= 1.2 \times 10^{-4}\,\text{T}. \end{aligned}$$

This is a very small field and lasts only briefly (the time constant $RC = 6.0 \times 10^{-11}\,\text{s}$) and so would be very difficult to measure.

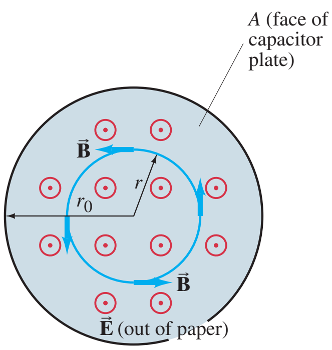


FIGURE 4 Frontal view of a circular plate of a parallel-plate capacitor. $\vec{\mathbf{E}}$ between plates points out toward viewer; lines of $\vec{\mathbf{B}}$ are circles. (Example 1.)

Let us write the magnetic field B outside the capacitor plates of Example 1 in terms of the current I that leaves the plates. The electric field between the plates is $E = \sigma/\epsilon_0 = Q/(\epsilon_0 A)$, as we saw in part b, so $dE/dt = I/(\epsilon_0 A)$. Hence B for $r > r_0$ is,

$$B = \frac{\mu_0 \epsilon_0 r_0^2}{2r} \frac{dE}{dt} = \frac{\mu_0 \epsilon_0 r_0^2}{2r} \frac{I}{\epsilon_0 \pi r_0^2} = \frac{\mu_0 I}{2\pi r}.$$

This is the same formula for the field that surrounds a wire. Thus the B field outside the capacitor is the same as that outside the wire. In other words, the magnetic field produced by the changing electric field between the plates is the same as that produced by the current in the wire.

Displacement Current

Maxwell interpreted the second term on the right in Eq. 1 as being *equivalent* to an electric current. He called it a **displacement current**, I_D. An ordinary current I is then called a **conduction current**. Ampère's law can then be written

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 (I + I_D)_{\text{encl}} \tag{2}$$

where

Displacement current

$$I_D = \epsilon_0 \frac{d\Phi_E}{dt}. \tag{3}$$

The term "displacement current" was based on an old discarded theory. Don't let it confuse you: I_D does not represent a flow of electric charge[†], nor is there a displacement.

2 Gauss's Law for Magnetism

We are almost in a position to state Maxwell's equations, but first we need to discuss the magnetic equivalent of Gauss's law. For a magnetic field $\vec{\mathbf{B}}$ the *magnetic flux* Φ_B through a surface is defined as

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

where the integral is over the area of either an open or a closed surface. The magnetic flux through a closed surface—that is, a surface which completely encloses a volume—is written

$$\Phi_B = \oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}.$$

In the electric case, the electric flux Φ_E through a closed surface is equal to the total net charge Q enclosed by the surface, divided by ϵ_0:

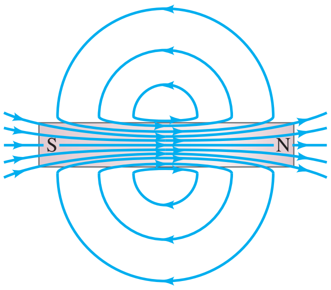
$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q}{\epsilon_0}.$$

This relation is Gauss's law for electricity.

We can write a similar relation for the magnetic flux. We have seen, however, that in spite of intense searches, no isolated magnetic poles (monopoles)—the magnetic equivalent of single electric charges—have ever been observed. Hence, **Gauss's law for magnetism** is

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0. \tag{4}$$

FIGURE 5 Magnetic field lines for a bar magnet.



In terms of magnetic field lines, this relation tells us that as many lines enter the enclosed volume as leave it. If, indeed, magnetic monopoles do not exist, then there are no "sources" or "sinks" for magnetic field lines to start or stop on, corresponding to electric field lines starting on positive charges and ending on negative charges. Magnetic field lines must then be continuous. Even for a bar magnet, a magnetic field $\vec{\mathbf{B}}$ exists inside as well as outside the magnetic material, and the lines of $\vec{\mathbf{B}}$ are closed loops as shown in Fig. 5.

[†]The interpretation of the changing electric field as a current does fit in well with the concept that an alternating current can be said to pass through a capacitor (although charge doesn't). It also means that Kirchhoff's junction rule will be valid even at a capacitor plate: conduction current flows into the plate, but no conduction current flows out of the plate—instead a "displacement current" flows out of one plate (toward the other plate).

3 Maxwell's Equations

With the extension of Ampère's law given by Eq. 1, plus Gauss's law for magnetism (Eq. 4), we are now ready to state all four of Maxwell's equations. In the absence of dielectric or magnetic materials, **Maxwell's equations** are:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q}{\epsilon_0} \tag{5a}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0 \tag{5b}$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{d\Phi_B}{dt} \tag{5c}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}. \tag{5d}$$

MAXWELL'S EQUATIONS

The first two of Maxwell's equations are the same as Gauss's law for electricity and Gauss's law for magnetism (Section 2, Eq. 4). The third is Faraday's law and the fourth is Ampère's law as modified by Maxwell (Eq. 1). (We dropped the subscripts on Q_{encl} and I_{encl} for simplicity.)

They can be summarized in words: (1) a generalized form of Coulomb's law relating electric field to its sources, electric charges; (2) the same for the magnetic field, except that if there are no magnetic monopoles, magnetic field lines are continuous—they do not begin or end (as electric field lines do on charges); (3) an electric field is produced by a changing magnetic field; (4) a magnetic field is produced by an electric current or by a changing electric field.

Maxwell's equations are the basic equations for all electromagnetism, and are as fundamental as Newton's three laws of motion and the law of universal gravitation. Maxwell's equations can also be written in differential form.

We can treat electric and magnetic fields separately if they do not vary in time. But we cannot treat them independently if they do change in time. For a changing magnetic field produces an electric field; and a changing electric field produces a magnetic field. An important outcome of these relations is the production of electromagnetic waves.

4 Production of Electromagnetic Waves

A magnetic field will be produced in empty space if there is a changing electric field. A changing magnetic field produces an electric field that is itself changing. This changing electric field will, in turn, produce a magnetic field, which will be changing, and so it too will produce a changing electric field; and so on. Maxwell found that the net result of these interacting changing fields was a *wave* of electric and magnetic fields that can propagate (travel) through space! We now examine, in a simplified way, how such **electromagnetic waves** can be produced.

Consider two conducting rods that will serve as an "antenna" (Fig. 6a). Suppose these two rods are connected by a switch to the opposite terminals of a battery. When the switch is closed, the upper rod quickly becomes positively charged and the lower one negatively charged. Electric field lines are formed as indicated by the lines in Fig. 6b. While the charges are flowing, a current exists whose direction is indicated by the black arrows. A magnetic field is therefore produced near the antenna. The magnetic field lines encircle the rod-like antenna and therefore, in Fig. 6, $\vec{\mathbf{B}}$ points into the page ($\otimes$) on the right and out of the page ($\odot$) on the left. How far out do these electric and magnetic fields extend? In the static case, the fields extend outward indefinitely far. However, when the switch in Fig. 6 is closed, the fields quickly appear nearby, but it takes time for them to reach distant points. Both electric and magnetic fields store energy, and this energy cannot be transferred to distant points at infinite speed.

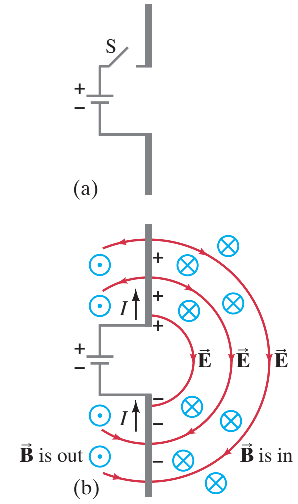


FIGURE 6 Fields produced by charge flowing into conductors. It takes time for the $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields to travel outward to distant points. The fields are shown to the right of the antenna, but they move out in all directions, symmetrically about the (vertical) antenna.

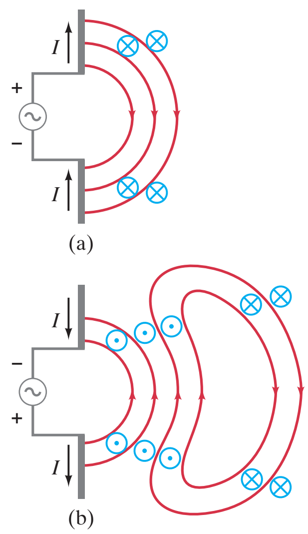


FIGURE 7 Sequence showing electric and magnetic fields that spread outward from oscillating charges on two conductors (the antenna) connected to an ac source (see the text).

Now we look at the situation of Fig. 7 where our antenna is connected to an ac generator. In Fig. 7a, the connection has just been completed. Charge starts building up and fields form just as in Fig. 6. The + and − signs in Fig. 7a indicate the net charge on each rod at a given instant. The black arrows indicate the direction of the current. The electric field is represented by the red lines in the plane of the page; and the magnetic field, according to the right-hand rule, is into (⊗) or out of (⊙) the page, in blue. In Fig. 7b, the voltage of the ac generator has reversed in direction; the current is reversed and the new magnetic field is in the opposite direction. Because the new fields have changed direction, the old lines fold back to connect up to some of the new lines and form closed loops as shown.† The old fields, however, don't suddenly disappear; they are on their way to distant points. Indeed, because a changing magnetic field produces an electric field, and a changing electric field produces a magnetic field, this combination of changing electric and magnetic fields moving outward is self-supporting, no longer depending on the antenna charges.

The fields not far from the antenna, referred to as the *near field*, become quite complicated, but we are not so interested in them. We are instead mainly interested in the fields far from the antenna (they are generally what we detect), which we refer to as the **radiation field**, or *far field*. The electric field lines form loops, as shown in Fig. 8, and continue moving outward. The magnetic field lines also form closed loops, but are not shown since they are perpendicular to the page. Although the lines are shown only on the right of the source, fields also travel in other directions. The field strengths are greatest in directions perpendicular to the oscillating charges; and they drop to zero along the direction of oscillation—above and below the antenna in Fig. 8.

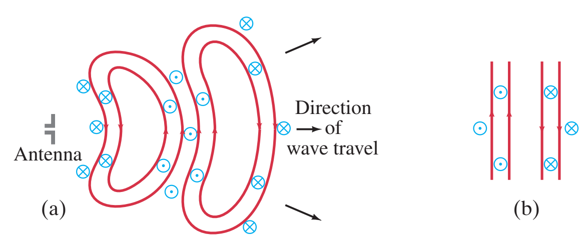


FIGURE 8 (a) The radiation fields (far from the antenna) produced by a sinusoidal signal on the antenna. The red closed loops represent electric field lines. The magnetic field lines, perpendicular to the page and represented by blue ⊗ and ⊙, also form closed loops. (b) Very far from the antenna the wave fronts (field lines) are essentially flat over a fairly large area, and are referred to as *plane waves*.

The magnitudes of both $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ in the radiation field are found to decrease with distance as $1/r$. (Compare this to the static electric field given by Coulomb's law where $\vec{\mathbf{E}}$ decreases as $1/r^2$.) The energy carried by the electromagnetic wave is proportional (as for any wave) to the square of the amplitude, E^2 or B^2, as will be discussed further in Section 8, so the intensity of the wave decreases as $1/r^2$.

Several things about the radiation field can be noted from Fig. 8. First, *the electric and magnetic fields at any point are perpendicular to each other, and to the direction of wave travel*. Second, we can see that the fields alternate in direction ($\vec{\mathbf{B}}$ is into the page at some points and out of the page at others; $\vec{\mathbf{E}}$ points up at some points and down at others). Thus, the field strengths vary from a maximum in one direction, to zero, to a maximum in the other direction. The electric and magnetic fields are "in phase": that is, they each are zero at the same points and reach their maxima at the same points in space. Finally, very far from the antenna (Fig. 8b) the field lines are quite flat over a reasonably large area, and the waves are referred to as **plane waves**.

If the source voltage varies sinusoidally, then the electric and magnetic field strengths in the radiation field will also vary sinusoidally. The sinusoidal character of the waves is diagrammed in Fig. 9, which shows the field directions and magnitudes plotted as a function of position. Notice that $\vec{\mathbf{B}}$ and $\vec{\mathbf{E}}$ are perpendicular to each other and to the direction of travel (= the direction of the wave velocity $\vec{\mathbf{v}}$). The direction of $\vec{\mathbf{v}}$ can be had from a right-hand rule using $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$: fingers along $\vec{\mathbf{E}}$, then along $\vec{\mathbf{B}}$, gives $\vec{\mathbf{v}}$ along thumb.

†We are considering waves traveling through empty space. There are no charges for lines of $\vec{\mathbf{E}}$ to start or stop on, so they form closed loops. Magnetic field lines always form closed loops.

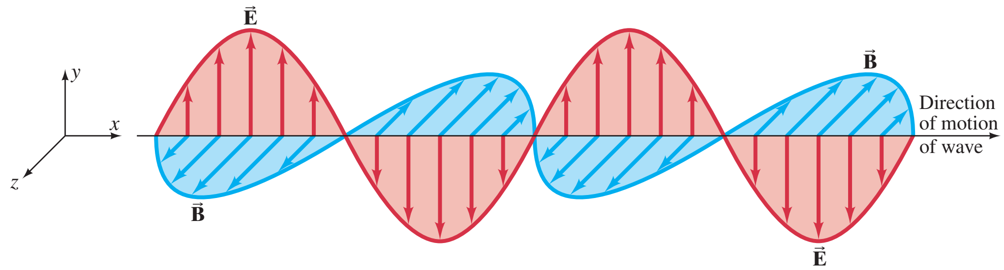


FIGURE 9 Electric and magnetic field strengths in an electromagnetic wave. $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are at right angles to each other. The entire pattern moves in a direction perpendicular to both $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$.

We call these waves electromagnetic (EM) waves. They are *transverse* waves because the amplitude is perpendicular to the direction of wave travel. However, EM waves are always waves of *fields*, not of matter (like waves on water or a rope). Because they are fields, EM waves can propagate in empty space.

As we have seen, EM waves are produced by electric charges that are oscillating, and hence are undergoing acceleration. In fact, we can say in general that

accelerating electric charges give rise to electromagnetic waves.

Electromagnetic waves can be produced in other ways as well, requiring description at the atomic and nuclear levels, as we will discuss later.

EXERCISE A At a particular instant in time, a wave has its electric field pointing north and its magnetic field pointing up. In which direction is the wave traveling? (*a*) South, (*b*) west, (*c*) east, (*d*) down, (*e*) not enough information.

5 Electromagnetic Waves, and Their Speed, Derived from Maxwell's Equations

Let us now examine how the existence of EM waves follows from Maxwell's equations. We will see that Maxwell's prediction of the existence of EM waves was startling. Equally startling was the speed at which they were predicted to travel.

We begin by considering a region of free space, where there are *no charges or conduction currents*—that is, far from the source so that the wave fronts (the field lines in Fig. 8) are essentially flat over a reasonable area. We call them **plane waves**, as we saw, because at any instant $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are uniform over a reasonably large plane perpendicular to the direction of propagation. We choose a coordinate system, so that the wave is traveling in the x direction with velocity $\vec{\mathbf{v}} = v\hat{\mathbf{i}}$, with $\vec{\mathbf{E}}$ parallel to the y axis and $\vec{\mathbf{B}}$ parallel to the z axis, as in Fig. 9.

Maxwell's equations, with $Q = I = 0$, become

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 0 \qquad \textbf{(6a)}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0 \qquad \textbf{(6b)}$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{d\Phi_B}{dt} \qquad \textbf{(6c)}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}. \qquad \textbf{(6d)}$$

Notice the beautiful *symmetry* of these equations. The term on the right in the last equation, conceived by Maxwell, is essential for this symmetry. It is also essential if electromagnetic waves are to be produced, as we will now see.

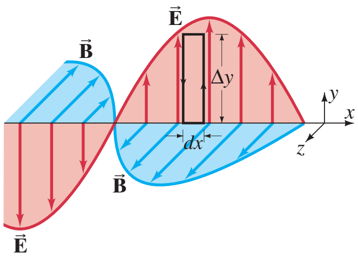


FIGURE 10 Applying Faraday's law to the rectangle $(\Delta y)(dx)$.

If the wave is sinusoidal with wavelength λ and frequency f, then such a traveling wave can be written as

$$\begin{aligned} E &= E_y = E_0 \sin(kx - \omega t) \\ B &= B_z = B_0 \sin(kx - \omega t) \end{aligned} \tag{7}$$

where

$$k = \frac{2\pi}{\lambda}, \qquad \omega = 2\pi f, \qquad \text{and} \qquad f\lambda = \frac{\omega}{k} = v, \tag{8}$$

with v being the speed of the wave. Although visualizing the wave as sinusoidal is helpful, we will not have to assume so in most of what follows.

Consider now a small rectangle in the plane of the electric field as shown in Fig. 10. This rectangle has a finite height Δy, and a very thin width which we take to be the infinitesimal distance dx. First we show that $\vec{\mathbf{E}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{v}}$ are in the orientation shown by applying Lenz's law to this rectangular loop. The changing magnetic flux through this loop is related to the electric field around the loop by Faraday's law (Maxwell's third equation, Eq. 6c). For the case shown, B through the loop is decreasing in time (the wave is moving to the right). So the electric field must be in a direction to oppose this change, meaning E must be greater on the right side of the loop than on the left, as shown (so it could produce a counterclockwise current whose magnetic field would act to oppose the change in Φ_B—but of course there is no current). This brief argument shows that the orientation of $\vec{\mathbf{E}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{v}}$ are in the correct relation as shown. That is, $\vec{\mathbf{v}}$ is in the direction of $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$. Now let us apply Faraday's law, which is Maxwell's third equation (Eq. 6c),

$$\oint \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{d\Phi_B}{dt}$$

to the rectangle of height Δy and width dx shown in Fig. 10. First we consider $\oint \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}$. Along the short top and bottom sections of length dx, $\vec{\mathbf{E}}$ is perpendicular to $d\vec{\boldsymbol{\ell}}$, so $\vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = 0$. Along the vertical sides, we let E be the electric field along the left side, and on the right side where it will be slightly larger, it is $E + dE$. Thus, if we take our loop counterclockwise,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = (E + dE)\,\Delta y - E\,\Delta y = dE\,\Delta y.$$

For the right side of Faraday's law, the magnetic flux through the loop changes as

$$\frac{d\Phi_B}{dt} = \frac{dB}{dt}\,dx\,\Delta y,$$

since the area of the loop, $(dx)(\Delta y)$, is not changing. Thus, Faraday's law gives us

$$dE\,\Delta y = -\frac{dB}{dt}\,dx\,\Delta y$$

or

$$\frac{dE}{dx} = -\frac{dB}{dt}.$$

Actually, both E and B are functions of position x and time t. We should therefore use partial derivatives:

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \tag{9}$$

where $\partial E/\partial x$ means the derivative of E with respect to x while t is held fixed, and $\partial B/\partial t$ is the derivative of B with respect to t while x is kept fixed.

We can obtain another important relation between E and B in addition to Eq. 9. To do so, we consider now a small rectangle in the plane of $\vec{\mathbf{B}}$, whose length and width are Δz and dx as shown in Fig. 11. To this rectangular loop we apply Maxwell's fourth equation (the extension of Ampère's law), Eq. 6d:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

where we have taken $I = 0$ since we assume the absence of conduction currents.

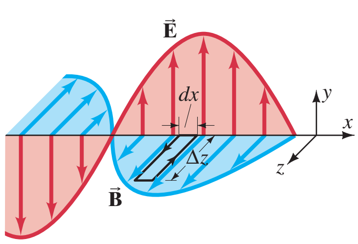


FIGURE 11 Applying Maxwell's fourth equation to the rectangle $(\Delta z)(dx)$.

Along the short sides (dx), $\vec{\mathbf{B}}\cdot d\vec{\boldsymbol{\ell}}$ is zero since $\vec{\mathbf{B}}$ is perpendicular to $d\vec{\boldsymbol{\ell}}$. Along the longer sides (Δz), we let B be the magnetic field along the left side of length Δz, and $B + dB$ be the field along the right side. We again integrate counterclockwise, so

$$\oint \vec{\mathbf{B}}\cdot d\vec{\boldsymbol{\ell}} = B\,\Delta z - (B + dB)\,\Delta z = -dB\,\Delta z.$$

The right side of Maxwell's fourth equation is

$$\mu_0\epsilon_0 \frac{d\Phi_E}{dt} = \mu_0\epsilon_0 \frac{dE}{dt}\,dx\,\Delta z.$$

Equating the two expressions, we obtain

$$-dB\,\Delta z = \mu_0\epsilon_0 \frac{dE}{dt}\,dx\,\Delta z$$

or

$$\frac{\partial B}{\partial x} = -\mu_0\epsilon_0 \frac{\partial E}{\partial t} \tag{10}$$

where we have replaced dB/dx and dE/dt by the proper partial derivatives as before.

We can use Eqs. 9 and 10 to obtain a relation between the magnitudes of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$, and the speed v. Let E and B be given by Eqs. 7 as a function of x and t. When we apply Eq. 9, taking the derivatives of E and B as given by Eqs. 7, we obtain

$$kE_0 \cos(kx - \omega t) = \omega B_0 \cos(kx - \omega t)$$

or

$$\frac{E_0}{B_0} = \frac{\omega}{k} = v,$$

since $v = \omega/k$ (see Eq. 8). Since E and B are in phase, we see that E and B are related by

$$\frac{E}{B} = v \tag{11}$$

at any point in space, where v is the velocity of the wave.

Now we apply Eq. 10 to the sinusoidal fields (Eqs. 7) and we obtain

$$kB_0 \cos(kx - \omega t) = \mu_0\epsilon_0\omega E_0 \cos(kx - \omega t)$$

or

$$\frac{B_0}{E_0} = \frac{\mu_0\epsilon_0\omega}{k} = \mu_0\epsilon_0 v.$$

We just saw that $B_0/E_0 = 1/v$, so

$$\mu_0\epsilon_0 v = \frac{1}{v}\cdot$$

Solving for v we find

$$v = c = \frac{1}{\sqrt{\epsilon_0\mu_0}}, \tag{12}$$

where c is the special symbol for the speed of electromagnetic waves in free space. We see that c is a constant, independent of the wavelength or frequency. If we put in values for ϵ_0 and μ_0 we find

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}} = \frac{1}{\sqrt{\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)}}$$
$$= 3.00\times10^{8}\,\mathrm{m/s}.$$

This is a remarkable result. For this is precisely equal to the measured speed of light!

EXAMPLE 2 **Determining $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ in EM waves.** Assume a 60.0-Hz EM wave is a sinusoidal wave propagating in the z direction with $\vec{\mathbf{E}}$ pointing in the x direction, and $E_0 = 2.00\,\mathrm{V/m}$. Write vector expressions for $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ as functions of position and time.

APPROACH We find λ from $\lambda f = v = c$. Then we use Fig. 9 and Eqs. 7 and 8 for the mathematical form of traveling electric and magnetic fields of an EM wave.

SOLUTION The wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8\,\mathrm{m/s}}{60.0\,\mathrm{s^{-1}}} = 5.00 \times 10^6\,\mathrm{m}.$$

From Eq. 8 we have

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{5.00 \times 10^6\,\mathrm{m}} = 1.26 \times 10^{-6}\,\mathrm{m^{-1}}$$
$$\omega = 2\pi f = 2\pi(60.0\,\mathrm{Hz}) = 377\,\mathrm{rad/s}.$$

From Eq. 11 with $v = c$, we find that

$$B_0 = \frac{E_0}{c} = \frac{2.00\,\mathrm{V/m}}{3.00 \times 10^8\,\mathrm{m/s}} = 6.67 \times 10^{-9}\,\mathrm{T}.$$

The direction of propagation is that of $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$, as in Fig. 9. With $\vec{\mathbf{E}}$ pointing in the x direction, and the wave propagating in the z direction, $\vec{\mathbf{B}}$ must point in the y direction. Using Eqs. 7 we find:

$$\vec{\mathbf{E}} = \hat{\mathbf{i}}(2.00\,\mathrm{V/m})\sin\left[\left(1.26 \times 10^{-6}\,\mathrm{m^{-1}}\right)z - \left(377\,\mathrm{rad/s}\right)t\right]$$
$$\vec{\mathbf{B}} = \hat{\mathbf{j}}\left(6.67 \times 10^{-9}\,\mathrm{T}\right)\sin\left[\left(1.26 \times 10^{-6}\,\mathrm{m^{-1}}\right)z - \left(377\,\mathrm{rad/s}\right)t\right]$$

*Derivation of Speed of Light (General)

We can derive the speed of EM waves without having to assume sinusoidal waves by combining Eqs. 9 and 10 as follows. We take the derivative, with respect to t of Eq. 10

$$\frac{\partial^2 B}{\partial t\, \partial x} = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}.$$

We next take the derivative of Eq. 9 with respect to x:

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial^2 B}{\partial t\, \partial x}.$$

Since $\partial^2 B/\partial t\, \partial x$ appears in both relations, we obtain

$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\mu_0 \epsilon_0}\frac{\partial^2 E}{\partial x^2}. \tag{13a}$$

By taking other derivatives of Eqs. 9 and 10 we obtain the same relation for B:

$$\frac{\partial^2 B}{\partial t^2} = \frac{1}{\mu_0 \epsilon_0}\frac{\partial^2 B}{\partial x^2}. \tag{13b}$$

Both of Eqs. 13 have the form of the **wave equation** for a plane wave traveling in the x direction:

$$\frac{\partial^2 D}{\partial t^2} = v^2 \frac{\partial^2 D}{\partial x^2},$$

where D stands for any type of displacement. We see that the velocity v for Eqs. 13 is given by

$$v^2 = \frac{1}{\mu_0 \epsilon_0}$$

in agreement with Eq. 12. Thus we see that a natural outcome of Maxwell's equations is that E and B obey the wave equation for waves traveling with speed $v = 1/\sqrt{\mu_0 \epsilon_0}$. It was on this basis that Maxwell predicted the existence of electromagnetic waves and predicted their speed.

6 Light as an Electromagnetic Wave and the Electromagnetic Spectrum

The calculations in Section 5 gave the result that Maxwell himself determined: that the speed of EM waves in empty space is given by

$$c = \frac{E}{B} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8\,\mathrm{m/s},$$

the same as the measured speed of light in vacuum.

Light had been shown some 60 years previously to behave like a wave. But nobody knew what kind of wave it was. What is it that is oscillating in a light wave? Maxwell, on the basis of the calculated speed of EM waves, argued that light must be an electromagnetic wave. This idea soon came to be generally accepted by scientists, but not fully until after EM waves were experimentally detected. EM waves were first generated and detected experimentally by Heinrich Hertz (1857–1894) in 1887, eight years after Maxwell’s death. Hertz used a spark-gap apparatus in which charge was made to rush back and forth for a short time, generating waves whose frequency was about 10^9 Hz. He detected them some distance away using a loop of wire in which an emf was produced when a changing magnetic field passed through. These waves were later shown to travel at the speed of light, $3.00 \times 10^8\,\mathrm{m/s}$, and to exhibit all the characteristics of light such as reflection, refraction, and interference. The only difference was that they were not visible. Hertz’s experiment was a strong confirmation of Maxwell’s theory.

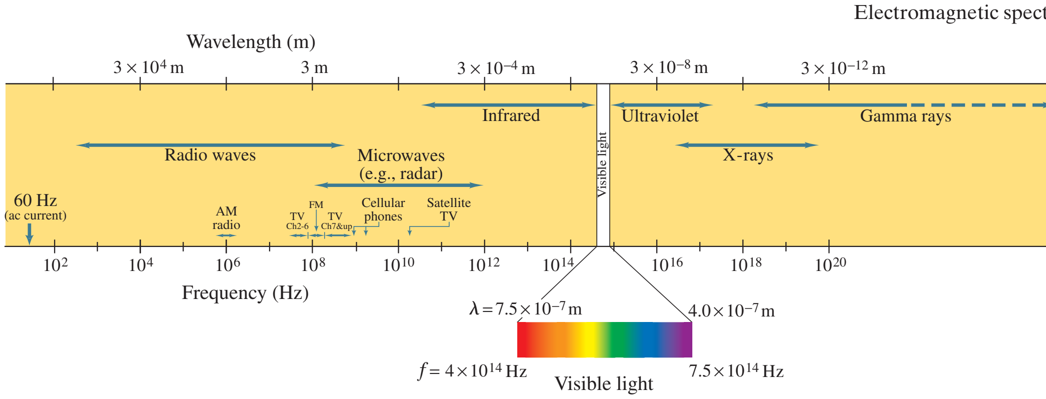
The wavelengths of visible light were measured in the first decade of the nineteenth century, long before anyone imagined that light was an electromagnetic wave. The wavelengths were found to lie between 4.0×10^{-7} m and 7.5×10^{-7} m, or 400 nm to 750 nm $(1\,\mathrm{nm} = 10^9\,\mathrm{m})$. The frequencies of visible light can be found using Eq. 8, which we rewrite here:

$$c = \lambda f, \tag{14}$$

where f and λ are the frequency and wavelength, respectively, of the wave. Here, c is the speed of light, $3.00 \times 10^8\,\mathrm{m/s}$; it gets the special symbol c because of its universality for all EM waves in free space. Equation 14 tells us that the frequencies of visible light are between 4.0×10^{14} Hz and 7.5×10^{14} Hz. (Recall that $1\,\mathrm{Hz} = 1$ cycle per second $= 1\,\mathrm{s}^{-1}$.)

But visible light is only one kind of EM wave. As we have seen, Hertz produced EM waves of much lower frequency, about 10^9 Hz. These are now called **radio waves**, because frequencies in this range are used to transmit radio and TV signals. Electromagnetic waves, or EM radiation as we sometimes call it, have been produced or detected over a wide range of frequencies. They are usually categorized as shown in Fig. 12, which is known as the **electromagnetic spectrum**.

FIGURE 12 Electromagnetic spectrum.



Radio waves and microwaves can be produced in the laboratory using electronic equipment (Fig. 7). Higher-frequency waves are very difficult to produce electronically. These and other types of EM waves are produced in natural processes, as emission from atoms, molecules, and nuclei (more on this later). EM waves can be produced by the acceleration of electrons or other charged particles, such as electrons in the antenna of Fig. 7. Another example is X-rays, which are produced when fast-moving electrons are rapidly decelerated upon striking a metal target. Even the visible light emitted by an ordinary incandescent light is due to electrons undergoing acceleration within the hot filament.

We will meet various types of EM waves later. However, it is worth mentioning here that infrared (IR) radiation (EM waves whose frequency is just less than that of visible light) is mainly responsible for the heating effect of the Sun. The Sun emits not only visible light but substantial amounts of IR and UV (ultraviolet) as well. The molecules of our skin tend to "resonate" at infrared frequencies, so it is these that are preferentially absorbed and thus warm us up. We humans experience EM waves differently, depending on their wavelengths: Our eyes detect wavelengths between about 4×10^{-7} m and 7.5×10^{-7} m (visible light), whereas our skin detects longer wavelengths (IR). Many EM wavelengths we don't detect directly at all.

EXERCISE B Return to the Chapter-Opening Question and answer it again now. Try to explain why you may have answered differently the first time.

CAUTION
Sound and EM waves are different

Light and other electromagnetic waves travel at a speed of 3×10^8 m/s. Compare this to sound, which travels at a speed of about 300 m/s in air, a million times slower; or to typical freeway speeds of a car, 30 m/s (100 km/h, or 60 mi/h), 10 million times slower than light. EM waves differ from sound waves in another big way: sound waves travel in a medium such as air, and involve motion of air molecules; EM waves do not involve any material—only fields, and they can travel in empty space.

EXAMPLE 3 Wavelengths of EM waves. Calculate the wavelength (a) of a 60-Hz EM wave, (b) of a 93.3-MHz FM radio wave, and (c) of a beam of visible red light from a laser at frequency 4.74×10^{14} Hz.

APPROACH All of these waves are electromagnetic waves, so their speed is $c = 3.00 \times 10^8$ m/s. We solve for λ in Eq. 14: $\lambda = c/f$.

SOLUTION (a) $$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8\ \mathrm{m/s}}{60\ \mathrm{s^{-1}}} = 5.0 \times 10^6\ \mathrm{m},$$
or 5000 km. 60 Hz is the frequency of ac current in the United States, and, as we see here, one wavelength stretches all the way across the continental USA.

(b) $$\lambda = \frac{3.00 \times 10^8\ \mathrm{m/s}}{93.3 \times 10^6\ \mathrm{s^{-1}}} = 3.22\ \mathrm{m}.$$
The length of an FM antenna is about half this $\left(\frac{1}{2}\lambda\right)$, or $1\frac{1}{2}$ m.

(c) $$\lambda = \frac{3.00 \times 10^8\ \mathrm{m/s}}{4.74 \times 10^{14}\ \mathrm{s^{-1}}} = 6.33 \times 10^{-7}\ \mathrm{m}\ (= 633\ \mathrm{nm}).$$

EXERCISE C What are the frequencies of (a) an 80-m-wavelength radio wave, and (b) an X-ray of wavelength 5.5×10^{-11} m?

EXAMPLE 4 ESTIMATE Cell phone antenna. The antenna of a cell phone is often $\frac{1}{4}$ wavelength long. A particular cell phone has an 8.5-cm-long straight rod for its antenna. Estimate the operating frequency of this phone.

APPROACH The basic equation relating wave speed, wavelength, and frequency is $c = \lambda f$; the wavelength λ equals four times the antenna's length.

SOLUTION The antenna is $\frac{1}{4}\lambda$ long, so $\lambda = 4(8.5\ \mathrm{cm}) = 34\ \mathrm{cm} = 0.34\ \mathrm{m}$. Then $f = c/\lambda = (3.0 \times 10^8\ \mathrm{m/s})/(0.34\ \mathrm{m}) = 8.8 \times 10^8\ \mathrm{Hz} = 880\ \mathrm{MHz}$.

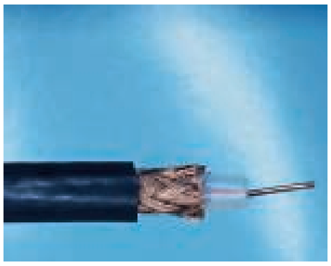
NOTE Radio antennas are not always straight conductors. The conductor may be a round loop to save space. See Fig. 21b.

EXERCISE D How long should a $\frac{1}{4}\lambda$ antenna be for an aircraft radio operating at 165 MHz?

Electromagnetic waves can travel along transmission lines as well as in empty space. When a source of emf is connected to a transmission line—be it two parallel wires or a coaxial cable (Fig. 13)—the electric field within the wire is not set up immediately at all points along the wires. This is based on the same argument we used in Section 4 with reference to Fig. 7. Indeed, it can be shown that if the wires are separated by empty space or air, the electrical signal travels along the wires at the speed $c = 3.0 \times 10^8\,\mathrm{m/s}$. For example, when you flip a light switch, the light actually goes on a tiny fraction of a second later. If the wires are in a medium whose electric permittivity is ϵ and magnetic permeability is μ; the speed is not given by Eq. 12, but by

$$v = \frac{1}{\sqrt{\epsilon\mu}}.$$

FIGURE 13 Coaxial cable.



The Image Works

EXAMPLE 5 ESTIMATE Phone call time lag. You make a telephone call from New York to a friend in London. Estimate how long it will take the electrical signal generated by your voice to reach London, assuming the signal is (*a*) carried on a telephone cable under the Atlantic Ocean, and (*b*) sent via satellite 36,000 km above the ocean. Would this cause a noticeable delay in either case?

APPROACH The signal is carried on a telephone wire or in the air via satellite. In either case it is an electromagnetic wave. Electronics as well as the wire or cable slow things down, but as a rough estimate we take the speed to be $c = 3.0 \times 10^8\,\mathrm{m/s}$.

SOLUTION The distance from New York to London is about 5000 km.
(*a*) The time delay via the cable is $t = d/c \approx (5 \times 10^6\,\mathrm{m})/(3.0 \times 10^8\,\mathrm{m/s}) = 0.017\,\mathrm{s}$.
(*b*) Via satellite the time would be longer because communications satellites, which are usually geosynchronous, move at a height of 36,000 km. The signal would have to go up to the satellite and back down, or about 72,000 km. The actual distance the signal would travel would be a little more than this as the signal would go up and down on a diagonal. Thus $t = d/c \approx 7.2 \times 10^7\,\mathrm{m}/(3 \times 10^8\,\mathrm{m/s}) = 0.24\,\mathrm{s}$.

NOTE When the signal travels via the underwater cable, there is only a hint of a delay and conversations are fairly normal. When the signal is sent via satellite, the delay *is* noticeable. The length of time between the end of when you speak and your friend receives it and replies, and then you hear the reply, is about a half second beyond the normal time in a conversation. This is enough to be noticeable, and you have to adjust for it so you don't start talking again while your friend's reply is on the way back to you.

EXERCISE E If you are on the phone via satellite to someone only 100 km away, would you hear the same effect?

EXERCISE F If your voice traveled as a sound wave, how long would it take to go from New York to London?

7 Measuring the Speed of Light

Galileo attempted to measure the speed of light by trying to measure the time required for light to travel a known distance between two hilltops. He stationed an assistant on one hilltop and himself on another, and ordered the assistant to lift the cover from a lamp the instant he saw a flash from Galileo's lamp. Galileo measured the time between the flash of his lamp and when he received the light from his assistant's lamp. The time was so short that Galileo concluded it merely represented human reaction time, and that the speed of light must be extremely high.

The first successful determination that the speed of light is finite was made by the Danish astronomer Ole Roemer (1644–1710). Roemer had noted that the carefully measured orbital period of Io, a moon of Jupiter with an average period of 42.5 h, varied slightly, depending on the relative position of Earth and Jupiter. He attributed this variation in the apparent period to the change in distance between the Earth and Jupiter during one of Io's periods, and the time it took light to travel the extra distance. Roemer concluded that the speed of light—though great—is finite.

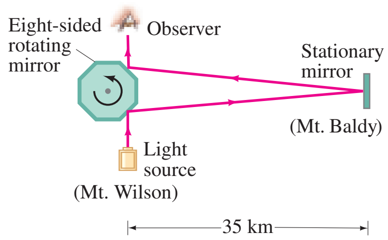


FIGURE 14 Michelson's speed-of-light apparatus (not to scale).

Since then a number of techniques have been used to measure the speed of light. Among the most important were those carried out by the American Albert A. Michelson (1852–1931). Michelson used the rotating mirror apparatus diagrammed in Fig. 14 for a series of high-precision experiments carried out from 1880 to the 1920s. Light from a source would hit one face of a rotating eight-sided mirror. The reflected light traveled to a stationary mirror a large distance away and back again as shown. If the rotating mirror was turning at just the right rate, the returning beam of light would reflect from one of the eight mirrors into a small telescope through which the observer looked. If the speed of rotation was only slightly different, the beam would be deflected to one side and would not be seen by the observer. From the required speed of the rotating mirror and the known distance to the stationary mirror, the speed of light could be calculated. In the 1920s, Michelson set up the rotating mirror on the top of Mt. Wilson in southern California and the stationary mirror on Mt. Baldy (Mt. San Antonio) 35 km away. He later measured the speed of light in vacuum using a long evacuated tube.

Today the speed of light, c, in vacuum is taken as

$$c = 2.99792458 \times 10^8 \text{ m/s},$$

and is defined to be this value. This means that the standard for length, the meter, is no longer defined separately. Instead, the meter is now formally defined as the distance light travels in vacuum in 1/299,792,458 of a second. We usually round off c to

$$c = 3.00 \times 10^8 \text{ m/s}$$

when extremely precise results are not required. In air, the speed is only slightly less.

8 Energy in EM Waves; the Poynting Vector

Electromagnetic waves carry energy from one region of space to another. This energy is associated with the moving electric and magnetic fields. The energy density u_E (J/m^3) stored in an electric field E is $u_E = \frac{1}{2}\epsilon_0 E^2$. The energy density stored in a magnetic field B is given by $u_B = \frac{1}{2}B^2/\mu_0$. Thus, the total energy stored per unit volume in a region of space where there is an electromagnetic wave is

$$u = u_E + u_B = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}. \tag{15}$$

In this equation, E and B represent the electric and magnetic field strengths of the wave at any instant in a small region of space. We can write Eq. 15 in terms of the E field alone, using Eqs. 11 $(B = E/c)$ and 12 $\left(c = 1/\sqrt{\epsilon_0\mu_0}\right)$ to obtain

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{\epsilon_0\mu_0 E^2}{\mu_0} = \epsilon_0 E^2. \tag{16a}$$

Note here that the energy density associated with the B field equals that due to the E field, and each contributes half to the total energy. We can also write the energy density in terms of the B field only:

$$u = \epsilon_0 E^2 = \epsilon_0 c^2 B^2 = \frac{B^2}{\mu_0}, \tag{16b}$$

or in one term containing both E and B,

$$u = \epsilon_0 E^2 = \epsilon_0 EcB = \frac{\epsilon_0 EB}{\sqrt{\epsilon_0\mu_0}} = \sqrt{\frac{\epsilon_0}{\mu_0}}\,EB. \tag{16c}$$

Equations 16 give the energy density in any region of space at any instant.

Now let us determine the energy the wave transports per unit time per unit area. This is given by a vector $\vec{\mathbf{S}}$, which is called the **Poynting vector**.† The units of $\vec{\mathbf{S}}$ are W/m^2. The direction of $\vec{\mathbf{S}}$ is the direction in which the energy is transported, which is the direction in which the wave is moving.

†After J. H. Poynting (1852–1914).

Let us imagine the wave is passing through an area A perpendicular to the x axis as shown in Fig. 15. In a short time dt, the wave moves to the right a distance $dx = c\,dt$ where c is the wave speed. The energy that passes through A in the time dt is the energy that occupies the volume $dV = A\,dx = Ac\,dt$. The energy density u is $u = \epsilon_0 E^2$ where E is the electric field in this volume at the given instant. So the total energy dU contained in this volume dV is the energy density u times the volume: $dU = u\ dV = (\epsilon_0 E^2)(Ac\,dt)$. Therefore the energy crossing the area A per time dt is

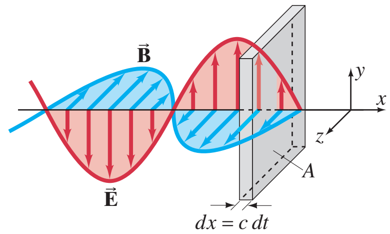


FIGURE 15 Electromagnetic wave carrying energy through area A.

$$S = \frac{1}{A}\frac{dU}{dt} = \epsilon_0 c E^2. \tag{17}$$

Since $E = cB$ and $c = 1/\sqrt{\epsilon_0 \mu_0}$, this can also be written:

$$S = \epsilon_0 c E^2 = \frac{cB^2}{\mu_0} = \frac{EB}{\mu_0}.$$

The direction of $\vec{\mathbf{S}}$ is along $\vec{\mathbf{v}}$, perpendicular to $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$, so the Poynting vector $\vec{\mathbf{S}}$ can be written

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}(\vec{\mathbf{E}} \times \vec{\mathbf{B}}). \tag{18}$$

Equation 17 or 18 gives the energy transported per unit area per unit time at any *instant*. We often want to know the *average* over an extended period of time since the frequencies are usually so high we don't detect the rapid time variation. If E and B are sinusoidal, then $\overline{E^2} = E_0^2/2$, just as for electric currents and voltages, where E_0 is the *maximum* value of E. Thus we can write for the magnitude of the Poynting vector, on the average,

$$\overline{S} = \frac{1}{2}\epsilon_0 c E_0^2 = \frac{1}{2}\frac{c}{\mu_0}B_0^2 = \frac{E_0 B_0}{2\mu_0}, \tag{19a}$$

where B_0 is the maximum value of B. This time averaged value of $\vec{\mathbf{S}}$ is the **intensity**, defined as the average power transferred across unit area. We can also write for the average value of S:

$$\overline{S} = \frac{E_{\mathrm{rms}} B_{\mathrm{rms}}}{\mu_0} \tag{19b}$$

where E_{rms} and B_{rms} are the rms values $\left(E_{\mathrm{rms}} = \sqrt{\overline{E^2}},\, B_{\mathrm{rms}} = \sqrt{\overline{B^2}}\right)$.

EXAMPLE 6 ***E* and *B* from the Sun.** Radiation from the Sun reaches the Earth (above the atmosphere) at a rate of about $1350\ \mathrm{J/s\cdot m^2}$ $(= 1350\ \mathrm{W/m^2})$. Assume that this is a single EM wave, and calculate the maximum values of E and B.

APPROACH We solve Eq. 19a $\left(\overline{S} = \frac{1}{2}\epsilon_0 c E_0^2\right)$ for E_0 in terms of $\overline{S}$ using $\overline{S} = 1350\ \mathrm{J/s\cdot m^2}$.

SOLUTION
$$E_0 = \sqrt{\frac{2\overline{S}}{\epsilon_0 c}} = \sqrt{\frac{2(1350\ \mathrm{J/s\cdot m^2})}{(8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(3.00 \times 10^8\ \mathrm{m/s})}}$$
$$= 1.01 \times 10^3\ \mathrm{V/m}.$$

From Eq. 11, $B = E/c$, so

$$B_0 = \frac{E_0}{c} = \frac{1.01 \times 10^3\ \mathrm{V/m}}{3.00 \times 10^8\ \mathrm{m/s}} = 3.37 \times 10^{-6}\ \mathrm{T}.$$

NOTE Although B has a small numerical value compared to E (because of the way the different units for E and B are defined), B contributes the same energy to the wave as E does, as we saw earlier (Eqs. 15 and 16).

9 Radiation Pressure

If electromagnetic waves carry energy, then we might expect them to also carry linear momentum. When an electromagnetic wave encounters the surface of an object, a force will be exerted on the surface as a result of the momentum transfer $(F = dp/dt)$, just as when a moving object strikes a surface. The force per unit area exerted by the waves is called **radiation pressure**, and its existence was predicted by Maxwell. He showed that if a beam of EM radiation (light, for example) is completely absorbed by an object, then the momentum transferred is

$$\Delta p = \frac{\Delta U}{c} \qquad \left[\begin{array}{c}\text{radiation}\\ \text{fully}\\ \text{absorbed}\end{array}\right] \qquad \textbf{(20a)}$$

where ΔU is the energy absorbed by the object in a time Δt, and c is the speed of light.† If instead, the radiation is fully reflected (suppose the object is a mirror), then the momentum transferred is twice as great, just as when a ball bounces elastically off a surface:

$$\Delta p = \frac{2\,\Delta U}{c}. \qquad \left[\begin{array}{c}\text{radiation}\\ \text{fully}\\ \text{reflected}\end{array}\right] \qquad \textbf{(20b)}$$

If a surface absorbs some of the energy, and reflects some of it, then $\Delta p = a\,\Delta U/c$, where a is a factor between 1 and 2.

Using Newton's second law we can calculate the force and the pressure exerted by radiation on the object. The force F is given by

$$F = \frac{dp}{dt}.$$

The average rate that energy is delivered to the object is related to the Poynting vector by

$$\frac{dU}{dt} = \overline{S}A,$$

where A is the cross-sectional area of the object which intercepts the radiation. The radiation pressure P (assuming full absorption) is given by (see Eq. 20a)

$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} = \frac{1}{Ac}\frac{dU}{dt} = \frac{\overline{S}}{c}. \qquad \left[\begin{array}{c}\text{fully}\\ \text{absorbed}\end{array}\right] \qquad \textbf{(21a)}$$

If the light is fully reflected, the pressure is twice as great (Eq. 20b):

$$P = \frac{2\overline{S}}{c}. \qquad \left[\begin{array}{c}\text{fully}\\ \text{reflected}\end{array}\right] \qquad \textbf{(21b)}$$

EXAMPLE 7 ESTIMATE Solar pressure. Radiation from the Sun that reaches the Earth's surface (after passing through the atmosphere) transports energy at a rate of about $1000\ \mathrm{W/m^2}$. Estimate the pressure and force exerted by the Sun on your outstretched hand.

APPROACH The radiation is partially reflected and partially absorbed, so let us estimate simply $P = \overline{S}/c$.

SOLUTION $P \approx \dfrac{\overline{S}}{c} = \dfrac{1000\ \mathrm{W/m^2}}{3 \times 10^8\ \mathrm{m/s}} \approx 3 \times 10^{-6}\ \mathrm{N/m^2}.$

An estimate of the area of your outstretched hand might be about 10 cm by 20 cm, so $A = 0.02\ \mathrm{m^2}$. Then the force is

$$F = PA \approx \left(3 \times 10^{-6}\ \mathrm{N/m^2}\right)\left(0.02\ \mathrm{m^2}\right) \approx 6 \times 10^{-8}\ \mathrm{N}.$$

NOTE These numbers are tiny. The force of gravity on your hand, for comparison, is maybe a half pound, or with $m = 0.2$ kg, $mg \approx (0.2\ \mathrm{kg})\left(9.8\ \mathrm{m/s^2}\right) \approx 2$ N. The radiation pressure on your hand is imperceptible compared to gravity.

†Very roughly, if we think of light as particles (and we do), the force that would be needed to bring such a particle moving at speed c to "rest" (i.e. absorption) is $F = \Delta p/\Delta t$. But F is also related to energy by $F = \Delta U/\Delta x$, so $\Delta p = F\,\Delta t = \Delta U/(\Delta x/\Delta t) = \Delta U/c$ where we identify $(\Delta x/\Delta t)$ with the speed of light c.

EXAMPLE 8 **ESTIMATE** **A solar sail.** Proposals have been made to use the radiation pressure from the Sun to help propel spacecraft around the solar system. (*a*) About how much force would be applied on a $1\,\text{km} \times 1\,\text{km}$ highly reflective sail, and (*b*) by how much would this increase the speed of a 5000-kg spacecraft in one year? (*c*) If the spacecraft started from rest, about how far would it travel in a year?

APPROACH Pressure P is force per unit area, so $F = PA$. We use the estimate of Example 7, doubling it for a reflecting surface $P = 2\overline{S}/c$. We find the acceleration from Newton's second law, and assume it is constant, and then find the speed from $v = v_0 + at$. The distance traveled is given by $x = \frac{1}{2}at^2$.

SOLUTION (*a*) Doubling the result of Example 7, the solar pressure is $2\overline{S}/c = 6 \times 10^{-6}\,\text{N/m}^2$. Then the force is $F \approx PA = (6 \times 10^{-6}\,\text{N/m}^2)(10^6\,\text{m}^2) \approx 6\,\text{N}$.
(*b*) The acceleration is $a \approx F/m \approx (6\,\text{N})/(5000\,\text{kg}) \approx 1.2 \times 10^{-3}\,\text{m/s}^2$. The speed increase is $v - v_0 = at = (1.2 \times 10^{-3}\,\text{m/s}^2)(365\,\text{days})(24\,\text{hr/day})(3600\,\text{s/hr}) \approx 4 \times 10^4\,\text{m/s}$ ($\approx 150{,}000\,\text{km/h}$!). (*c*) Starting from rest, this acceleration would result in a distance of about $\frac{1}{2}at^2 \approx 6 \times 10^{11}\,\text{m}$ in a year, about four times the Sun-Earth distance. The starting point should be far from the Earth so the Earth's gravitational force is small compared to 6 N.

NOTE A large sail providing a small force over a long time can result in a lot of motion.

PHYSICS APPLIED
Optical tweezers

Although you cannot directly feel the effects of radiation pressure, the phenomenon is quite dramatic when applied to atoms irradiated by a finely focused laser beam. An atom has a mass on the order of $10^{-27}\,\text{kg}$, and a laser beam can deliver energy at a rate of $1000\,\text{W/m}^2$. This is the same intensity used in Example 7, but here a radiation pressure of $10^{-6}\,\text{N/m}^2$ would be very significant on a molecule whose mass might be 10^{-23} to $10^{-26}\,\text{kg}$. It is possible to move atoms and molecules around by steering them with a laser beam, in a device called "optical tweezers." Optical tweezers have some remarkable applications. They are of great interest to biologists, especially since optical tweezers can manipulate live microorganisms, and components within a cell, without damaging them. Optical tweezers have been used to measure the elastic properties of DNA by pulling each end of the molecule with such a laser "tweezers."

10 Radio and Television; Wireless Communication

PHYSICS APPLIED
Wireless transmission

Electromagnetic waves offer the possibility of transmitting information over long distances. Among the first to realize this and put it into practice was Guglielmo Marconi (1874–1937) who, in the 1890s, invented and developed wireless communication. With it, messages could be sent at the speed of light without the use of wires. The first signals were merely long and short pulses that could be translated into words by a code, such as the "dots" and "dashes" of the Morse code: they were digital wireless, believe it or not. In 1895 Marconi sent wireless signals a kilometer or two in Italy. By 1901 he had sent test signals 3000 km across the ocean from Newfoundland, Canada, to Cornwall, England. In 1903 he sent the first practical commercial messages from Cape Cod, Massachusetts, to England: the London *Times* printed news items sent from its New York correspondent. 1903 was also the year of the first powered airplane flight by the Wright brothers. The hallmarks of the modern age—wireless communication and flight—date from the same year. Our modern world of wireless communication, including radio, television, cordless phones, cell phones, Bluetooth, wi-fi, and satellite communication, are simply modern applications of Marconi's pioneering work.

The next decade saw the development of vacuum tubes. Out of this early work radio and television were born. We now discuss briefly (1) how radio and TV signals are transmitted, and (2) how they are received at home.

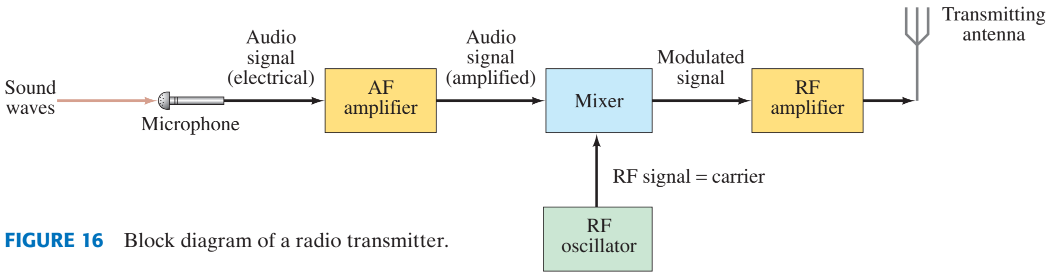


FIGURE 16 Block diagram of a radio transmitter.

The process by which a radio station transmits information (words and music) is outlined in Fig. 16. The audio (sound) information is changed into an electrical signal of the same frequencies by, say, a microphone or magnetic read/write head. This electrical signal is called an audiofrequency (AF) signal, since the frequencies are in the audio range (20 to 20,000 Hz). The signal is amplified electronically and is then mixed with a radio-frequency (RF) signal called its **carrier frequency**, which represents that station. AM radio stations have carrier frequencies from about 530 kHz to 1700 kHz. For example, "710 on your dial" means a station whose carrier frequency is 710 kHz. FM radio stations have much higher carrier frequencies, between 88 MHz and 108 MHz. The carrier frequencies for broadcast TV stations in the United States lie between 54 MHz and 72 MHz, between 76 MHz and 88 MHz, between 174 MHz and 216 MHz, and between 470 MHz and 698 MHz.

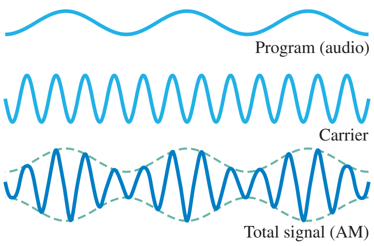
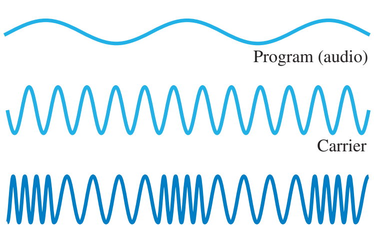


FIGURE 17 In amplitude modulation (AM), the amplitude of the carrier signal is made to vary in proportion to the audio signal's amplitude.

PHYSICS APPLIED
AM and FM

The mixing of the audio and carrier frequencies is done in two ways. In **amplitude modulation** (AM), the amplitude of the high-frequency carrier wave is made to vary in proportion to the amplitude of the audio signal, as shown in Fig. 17. It is called "amplitude modulation" because the *amplitude* of the carrier is altered ("modulate" means to change or alter). In **frequency modulation** (FM), the *frequency* of the carrier wave is made to change in proportion to the audio signal's amplitude, as shown in Fig. 18. The mixed signal is amplified further and sent to the transmitting antenna, where the complex mixture of frequencies is sent out in the form of EM waves. In digital communication, the signal is put into a digital form which modulates the carrier.

FIGURE 18 In frequency modulation (FM), the frequency of the carrier signal is made to change in proportion to the audio signal's amplitude. This method is used by FM radio and television.

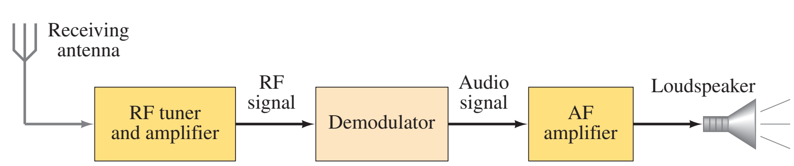


A television transmitter works in a similar way, using FM for audio and AM for video; both audio and video signals are mixed with carrier frequencies.

PHYSICS APPLIED
Radio and TV receivers

Now let us look at the other end of the process, the reception of radio and TV programs at home. A simple radio receiver is diagrammed in Fig. 19. The EM waves sent out by all stations are received by the antenna. The signals the antenna detects and sends to the receiver are very small and contain frequencies from many different stations. The receiver selects out a particular RF frequency (actually a narrow range of frequencies) corresponding to a particular station using a resonant LC circuit. A simple way of tuning

FIGURE 19 Block diagram of a simple radio receiver.



a station is shown in Fig. 20. A particular station is "tuned in" by adjusting C and/or L so that the resonant frequency of the circuit equals that of the station's carrier frequency. The signal, containing both audio and carrier frequencies, next goes to the *demodulator*, or *detector* (Fig. 19), where "demodulation" takes place—that is, the RF carrier frequency is separated from the audio signal. The audio signal is amplified and sent to a loudspeaker or headphones.

Modern receivers have more stages than those shown. Various means are used to increase the sensitivity and selectivity (ability to detect weak signals and distinguish them from other stations), and to minimize distortion of the original signal.†

A television receiver does similar things to both the audio and the video signals. The audio signal goes finally to the loudspeaker, and the video signal to the monitor, such as a *cathode ray tube* (CRT) or LCD screen.

One kind of antenna consists of one or more conducting rods; the electric field in the EM waves exerts a force on the electrons in the conductor, causing them to move back and forth at the frequencies of the waves (Fig. 21a). A second type of antenna consists of a tubular coil of wire which detects the magnetic field of the wave: the changing B field induces an emf in the coil (Fig. 21b).

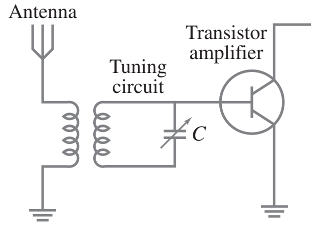


FIGURE 20 Simple tuning stage of a radio.

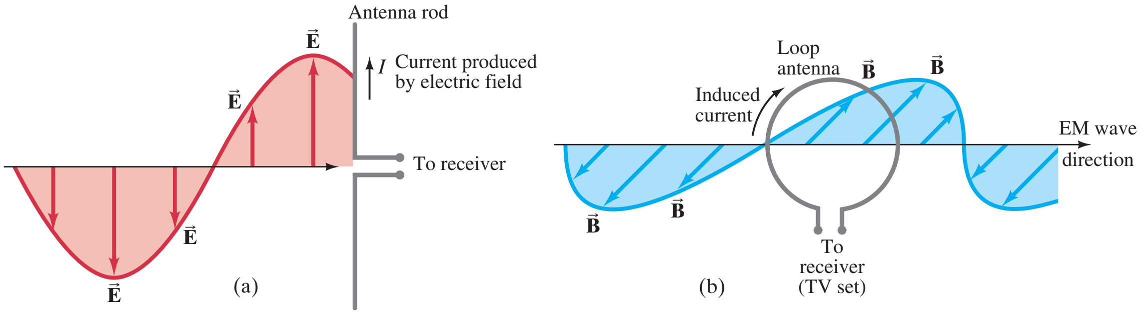


FIGURE 21 Antennas. (a) Electric field of EM wave produces a current in an antenna consisting of straight wire or rods. (b) Changing magnetic field induces an emf and current in a loop antenna.

A satellite dish (Fig. 22) consists of a parabolic reflector that focuses the EM waves onto a "horn," similar to a concave mirror telescope.

FIGURE 22 A satellite dish.

Spencer Grant/PhotoEdit, Inc.

EXAMPLE 9 **Tuning a station.** Calculate the transmitting wavelength of an FM radio station that transmits at 100 MHz.

APPROACH Radio is transmitted as an EM wave, so the speed is $c = 3.0 \times 10^8\,\mathrm{m/s}$. The wavelength is found from Eq. 14, $\lambda = c/f$.

SOLUTION The carrier frequency is $f = 100\,\mathrm{MHz} = 1.0 \times 10^8\,\mathrm{s^{-1}}$, so

$$\lambda = \frac{c}{f} = \frac{(3.0 \times 10^8\,\mathrm{m/s})}{(1.0 \times 10^8\,\mathrm{s^{-1}})} = 3.0\,\mathrm{m}.$$

NOTE The wavelengths of other FM signals (88 MHz to 108 MHz) are close to the 3.0-m wavelength of this station. FM antennas are typically 1.5 m long, or about a half wavelength. This length is chosen so that the antenna reacts in a resonant fashion and thus is more sensitive to FM frequencies. AM radio antennas would have to be very long to be either $\frac{1}{2}\lambda$ or $\frac{1}{4}\lambda$.

†For *FM stereo broadcasting*, two signals are carried by the carrier wave. One signal contains frequencies up to about 15 kHz, which includes most audio frequencies. The other signal includes the same range of frequencies, but 19 kHz is added to it. A stereo receiver subtracts this 19,000-Hz signal and distributes the two signals to the left and right channels. The first signal consists of the sum of left and right channels (L + R), so mono radios detect all the sound. The second signal is the difference between left and right (L − R). Hence the receiver must add and subtract the two signals to get pure left and right signals for each channel.

Other EM Wave Communications

PHYSICS APPLIED
Cell phones, radio control, remote control, cable TV, and satellite TV and radio

The various regions of the radio-wave spectrum are assigned by governmental agencies for various purposes. Besides those mentioned above, there are "bands" assigned for use by ships, airplanes, police, military, amateurs, satellites and space, and radar. Cell phones, for example, are complete radio transmitters and receivers. In the U.S., CDMA cell phones function on two different bands: 800 MHz and 1900 MHz (= 1.9 GHz). Europe, Asia, and much of the rest of the world use a different system: the international standard called GSM (Global System for Mobile Communication), on 900-MHz and 1800-MHz bands. The U.S. now also has the GSM option (at 850 MHz and 1.9 GHz), as does much of the rest of the Americas. A 700-MHz band is now being made available for cell phones (it used to carry TV broadcast channels 52–69, now no longer used). Radio-controlled toys (cars, sailboats, robotic animals, etc.) can use various frequencies from 27 MHz to 75 MHz. Automobile remote (keyless) entry may operate around 300 MHz or 400 MHz.

Cable TV channels are carried as electromagnetic waves along a coaxial cable (see Fig. 13) rather than being broadcast and received through the "air." The channels are in the same part of the EM spectrum, hundreds of MHz, but some are at frequencies not available for TV broadcast. Digital satellite TV and radio are carried in the microwave portion of the spectrum (12 to 14 GHz and 2.3 GHz, respectively).

Summary

James Clerk Maxwell synthesized an elegant theory in which all electric and magnetic phenomena could be described using four equations, now called **Maxwell's equations**. They are based on earlier ideas, but Maxwell added one more—that a changing electric field produces a magnetic field. Maxwell's equations are

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q}{\epsilon_0} \qquad \textbf{(5a)}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0 \qquad \textbf{(5b)}$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{d\Phi_B}{dt} \qquad \textbf{(5c)}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}. \qquad \textbf{(5d)}$$

The first two are Gauss's laws for electricity and for magnetism; the other two are Faraday's law and Ampère's law (as extended by Maxwell), respectively.

Maxwell's theory predicted that transverse **electromagnetic (EM) waves** would be produced by accelerating electric charges, and these waves would propagate through space at the speed of light c, given by

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8\,\mathrm{m/s}. \qquad \textbf{(12)}$$

The wavelength λ and frequency f of EM waves are related to their speed c by

$$c = \lambda f, \qquad \textbf{(14)}$$

just as for other waves.

The oscillating electric and magnetic fields in an EM wave are perpendicular to each other and to the direction of propagation. EM waves are waves of fields, not matter, and can propagate in empty space.

After EM waves were experimentally detected in the late 1800s, the idea that light is an EM wave (although of much higher frequency than those detected directly) became generally accepted. The **electromagnetic spectrum** includes EM waves of a wide variety of wavelengths, from microwaves and radio waves to visible light to X-rays and gamma rays, all of which travel through space at a speed $c = 3.00 \times 10^8\,\mathrm{m/s}$.

The energy carried by EM waves can be described by the **Poynting vector**

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}} \qquad \textbf{(18)}$$

which gives the rate energy is carried across unit area per unit time when the electric and magnetic fields in an EM wave in free space are $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$.

EM waves carry momentum and exert a **radiation pressure** proportional to the intensity S of the wave.

Answers to Exercises

A: (*c*).

B: (*b*).

C: (*a*) 3.8×10^6 Hz; (*b*) 5.5×10^{18} Hz.

D: 45 cm.

E: Yes; the signal still travels 72,000 km.

F: Over 4 hours.