One of the great laws of physics is Faraday's law of induction, which says that a changing magnetic flux produces an induced emf. This photo shows a bar magnet moving inside a coil of wire, and the galvanometer registers an induced current. This phenomenon of electromagnetic induction is the basis for many practical devices, including generators, alternators, transformers, tape recording, and computer memory.

Richard Megna/Fundamental Photographs, NYC

Electromagnetic Induction and Faraday's Law

CONTENTS

CHAPTER-OPENING QUESTION — Guess now!

[*Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.*]

In the photograph above, the bar magnet is inserted into the coil of wire, and is left there for 1 minute; then it is removed from the coil. What would an observer watching the galvanometer see?

(a) No change; without a battery there is no current to detect.
(b) A small current flows while the magnet is inside the coil of wire.
(c) A current spike as the magnet enters the coil, and then nothing.
(d) A current spike as the magnet enters the coil, and then a steady small current.
(e) A current spike as the magnet enters the coil, then nothing, and then a current spike in the opposite direction as the magnet leaves the coil.

It was discovered that there are two ways in which electricity and magnetism are related: (1) an electric current produces a magnetic field; and (2) a magnetic field exerts a force on an electric current or moving electric charge. These discoveries were made in 1820–1821. Scientists then began to wonder: if electric currents produce a magnetic field, is it possible that a magnetic field can produce an electric current? Ten years later the American Joseph Henry (1797–1878) and the Englishman Michael Faraday (1791–1867) independently found that it was possible. Henry actually made the discovery first. But Faraday published his results earlier and investigated the subject in more detail. We now discuss this phenomenon and some of its world-changing applications including the electric generator.

Note: Sections marked with an asterisk (*) may be considered optional by the instructor.

1 Induced EMF

In his attempt to produce an electric current from a magnetic field, Faraday used an apparatus like that shown in Fig. 1. A coil of wire, X, was connected to a battery. The current that flowed through X produced a magnetic field that was intensified by the ring-shaped iron core around which the wire was wrapped. Faraday hoped that a strong steady current in X would produce a great enough magnetic field to produce a current in a second coil Y wrapped on the same iron ring. This second circuit, Y, contained a galvanometer to detect any current but contained no battery.

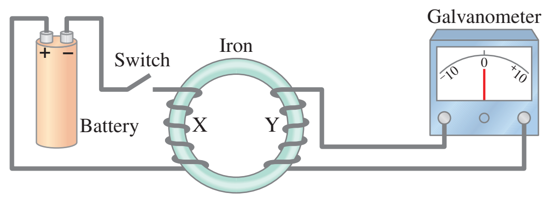


FIGURE 1 Faraday's experiment to induce an emf.

He met no success with constant currents. But the long-sought effect was finally observed when Faraday noticed the galvanometer in circuit Y deflect strongly at the moment he closed the switch in circuit X. And the galvanometer deflected strongly in the opposite direction when he opened the switch in X. A constant current in X produced a constant magnetic field which produced *no* current in Y. Only when the current in X was starting or stopping was a current produced in Y.

Faraday concluded that although a constant magnetic field produces no current in a conductor, a *changing* magnetic field can produce an electric current. Such a current is called an **induced current**. When the magnetic field through coil Y changes, a current occurs in Y as if there were a source of emf in circuit Y. We therefore say that

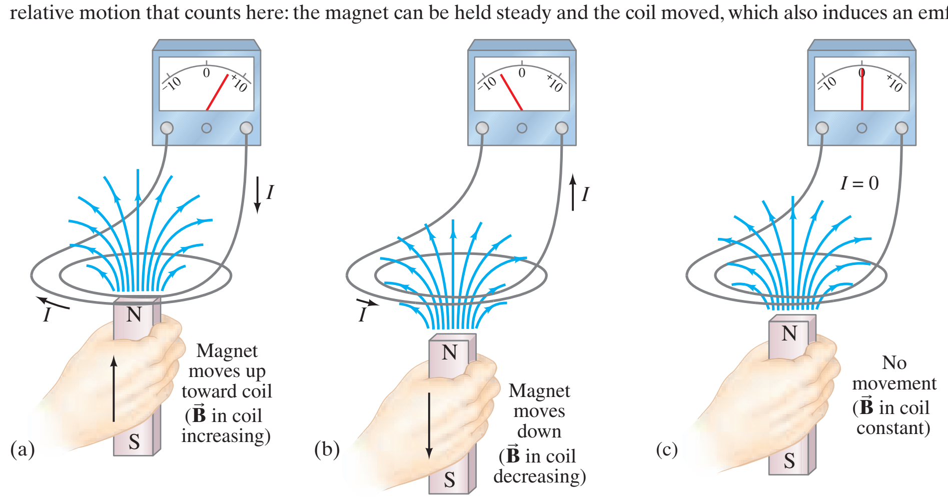
a changing magnetic field induces an emf.

CAUTION
Changing $\vec{\mathbf{B}}$, *not* $\vec{\mathbf{B}}$ *itself, induces current*

Faraday did further experiments on **electromagnetic induction**, as this phenomenon is called. For example, Fig. 2 shows that if a magnet is moved quickly into a coil of wire, a current is induced in the wire. If the magnet is quickly removed, a current is induced in the opposite direction ($\vec{\mathbf{B}}$ through the coil decreases). Furthermore, if the magnet is held steady and the coil of wire is moved toward or away from the magnet, again an emf is induced and a current flows. Motion or change is required to induce an emf. It doesn't matter whether the magnet or the coil moves. It is their *relative motion* that counts.

CAUTION
Relative motion—magnet or coil moving induces current

FIGURE 2 (a) A current is induced when a magnet is moved toward a coil, momentarily increasing the magnetic field through the coil. (b) The induced current is opposite when the magnet is moved away from the coil ($\vec{\mathbf{B}}$ decreases). Note that the galvanometer zero is at the center of the scale and the needle deflects left or right, depending on the direction of the current. In (c), no current is induced if the magnet does not move relative to the coil. It is the relative motion that counts here: the magnet can be held steady and the coil moved, which also induces an emf.



EXERCISE A Return to the Chapter-Opening Question and answer it again now. Try to explain why you may have answered differently the first time.

2 Faraday's Law of Induction; Lenz's Law

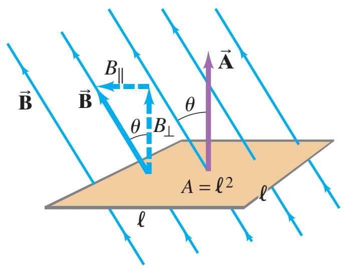


FIGURE 3 Determining the flux through a flat loop of wire. This loop is square, of side ℓ and area $A = \ell^2$.

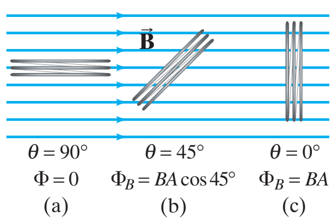
Faraday investigated quantitatively what factors influence the magnitude of the emf induced. He found first of all that the more rapidly the magnetic field changes, the greater the induced emf. He also found that the induced emf depends on the area of the circuit loop. Thus we say that the emf is proportional to the rate of change of the **magnetic flux**, Φ_B, passing through the circuit or loop of area A. Magnetic flux for a uniform magnetic field is defined in the same way as for electric flux, namely as

$$\Phi_B = B_{\perp} A = BA\cos\theta = \vec{\mathbf{B}}\cdot\vec{\mathbf{A}}. \qquad [\vec{\mathbf{B}} \text{ uniform}] \quad \textbf{(1a)}$$

Here $B_{\perp}$ is the component of the magnetic field $\vec{\mathbf{B}}$ perpendicular to the face of the loop, and θ is the angle between $\vec{\mathbf{B}}$ and the vector $\vec{\mathbf{A}}$ (representing the area) whose direction is perpendicular to the face of the loop. These quantities are shown in Fig. 3 for a square loop of side ℓ whose area is $A = \ell^2$. If the area is of some other shape, or $\vec{\mathbf{B}}$ is not uniform, the magnetic flux can be written†

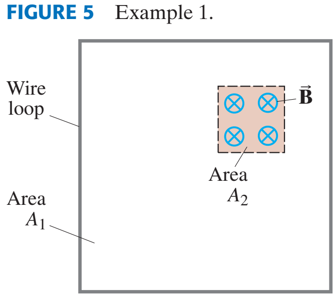
$$\Phi_B = \int \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}}. \qquad \textbf{(1b)}$$

FIGURE 4 Magnetic flux Φ_B is proportional to the number of lines of $\vec{\mathbf{B}}$ that pass through the loop.



The lines of $\vec{\mathbf{B}}$ (like lines of $\vec{\mathbf{E}}$) can be drawn such that the number of lines per unit area is proportional to the field strength. Then the flux Φ_B can be thought of as being proportional to the *total number of lines passing through the area enclosed by the loop*. This is illustrated in Fig. 4, where the loop is viewed from the side (on edge). For $\theta = 90°$, no magnetic field lines pass through the loop and $\Phi_B = 0$, whereas Φ_B is a maximum when $\theta = 0°$. The unit of magnetic flux is the tesla-meter^2; this is called a **weber**: $1\text{ Wb} = 1\text{ T}\cdot\text{m}^2$.

FIGURE 5 Example 1.



CONCEPTUAL EXAMPLE 1 **Determining flux.** A square loop of wire encloses area A_1 as shown in Fig. 5. A uniform magnetic field $\vec{\mathbf{B}}$ perpendicular to the loop extends over the area A_2. What is the magnetic flux through the loop A_1?

RESPONSE We assume that the magnetic field is zero outside the area A_2. The total magnetic flux through area A_1 is the flux through area A_2, which by Eq. 1a for a uniform field is BA_2, plus the flux through the remaining area $(= A_1 - A_2)$, which is zero because $B = 0$. So the total flux is $\Phi_B = BA_2 + 0(A_1 - A_2) = BA_2$. It is *not* equal to BA_1 because $\vec{\mathbf{B}}$ is not uniform over A_1.

With our definition of flux, Eqs. 1, we can now write down the results of Faraday's investigations: The emf induced in a circuit is equal to the rate of change of magnetic flux through the circuit:

FARADAY'S LAW OF INDUCTION

$$\mathscr{E} = -\frac{d\Phi_B}{dt}. \qquad \textbf{(2a)}$$

This fundamental result is known as **Faraday's law of induction**, and is one of the basic laws of electromagnetism.

†The integral is taken over an open surface—that is, one bounded by a closed curve such as a circle or square. In the present discussion, the area is that enclosed by the loop under discussion. The area is not an enclosed surface as we use in Gauss's law.

If the circuit contains N loops that are closely wrapped so the same flux passes through each, the emfs induced in each loop add together, so

$$\mathscr{E} = -N\frac{d\Phi_B}{dt}. \qquad [N \text{ loops}] \quad \textbf{(2b)}$$

FARADAY'S LAW OF INDUCTION

EXAMPLE 2 **A loop of wire in a magnetic field.** A square loop of wire of side $\ell = 5.0\,\text{cm}$ is in a uniform magnetic field $B = 0.16\,\text{T}$. What is the magnetic flux in the loop (*a*) when $\vec{\mathbf{B}}$ is perpendicular to the face of the loop and (*b*) when $\vec{\mathbf{B}}$ is at an angle of 30° to the area $\vec{\mathbf{A}}$ of the loop? (*c*) What is the magnitude of the average current in the loop if it has a resistance of $0.012\,\Omega$ and it is rotated from position (*b*) to position (*a*) in 0.14 s?

APPROACH We use the definition $\Phi_B = \vec{\mathbf{B}}\cdot\vec{\mathbf{A}}$ to calculate the magnetic flux. Then we use Faraday's law of induction to find the induced emf in the coil, and from that the induced current $(I = \mathscr{E}/R)$.

SOLUTION The area of the coil is $A = \ell^2 = (5.0 \times 10^{-2}\,\text{m})^2 = 2.5 \times 10^{-3}\,\text{m}^2$, and the direction of $\vec{\mathbf{A}}$ is perpendicular to the face of the loop (Fig. 3).

(*a*) $\vec{\mathbf{B}}$ is perpendicular to the coil's face, and thus parallel to $\vec{\mathbf{A}}$ (Fig. 3), so

$$\begin{aligned}\Phi_B &= \vec{\mathbf{B}}\cdot\vec{\mathbf{A}} \\ &= BA\cos 0° = (0.16\,\text{T})(2.5 \times 10^{-3}\,\text{m}^2)(1) = 4.0 \times 10^{-4}\,\text{Wb}.\end{aligned}$$

(*b*) The angle between $\vec{\mathbf{B}}$ and $\vec{\mathbf{A}}$ is 30°, so

$$\begin{aligned}\Phi_B &= \vec{\mathbf{B}}\cdot\vec{\mathbf{A}} \\ &= BA\cos\theta = (0.16\,\text{T})(2.5 \times 10^{-3}\,\text{m}^2)\cos 30° = 3.5 \times 10^{-4}\,\text{Wb}.\end{aligned}$$

(*c*) The magnitude of the induced emf is

$$\mathscr{E} = \frac{\Delta\Phi_B}{\Delta t} = \frac{(4.0 \times 10^{-4}\,\text{Wb}) - (3.5 \times 10^{-4}\,\text{Wb})}{0.14\,\text{s}} = 3.6 \times 10^{-4}\,\text{V}.$$

The current is then

$$I = \frac{\mathscr{E}}{R} = \frac{3.6 \times 10^{-4}\,\text{V}}{0.012\,\Omega} = 0.030\,\text{A} = 30\,\text{mA}.$$

The minus signs in Eqs. 2a and b are there to remind us in which direction the induced emf acts. Experiments show that

a current produced by an induced emf moves in a direction so that the magnetic field created by that current opposes the original change in flux.

This is known as **Lenz's law**. Be aware that we are now discussing two distinct magnetic fields: (1) the changing magnetic field or flux that induces the current, and (2) the magnetic field produced by the induced current (all currents produce a field). The second field opposes the change in the first.

CAUTION *Distinguish two different magnetic fields*

Lenz's law can be said another way, valid even if no current can flow (as when a circuit is not complete):

An induced emf is always in a direction that opposes the original change in flux that caused it.

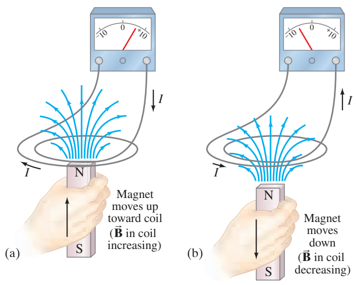


FIGURE 2 (repeated).

Let us apply Lenz's law to the relative motion between a magnet and a coil, Fig. 2. The changing flux through the coil induces an emf in the coil, producing a current. This induced current produces its own magnetic field. In Fig. 2a the distance between the coil and the magnet decreases. The magnet's magnetic field (and number of field lines) through the coil increases, and therefore the flux increases. The magnetic field of the magnet points upward. To oppose the upward increase, the magnetic field inside the coil produced by the induced current needs to point *downward*. Thus, Lenz's law tells us that the current moves as shown (use the right-hand rule). In Fig. 2b, the flux *decreases* (because the magnet is moved away and B decreases), so the induced current in the coil produces an *upward* magnetic field through the coil that is "trying" to maintain the status quo. Thus the current in Fig. 2b is in the opposite direction from Fig. 2a.

It is important to note that an emf is induced whenever there is a change in *flux* through the coil, and we now consider some more possibilities.

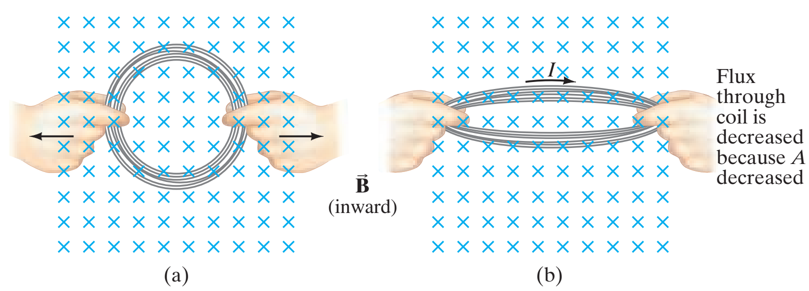


FIGURE 6 A current can be induced by changing the area of the coil, even though B doesn't change. Here the area is reduced by pulling on its sides: the *flux* through the coil is reduced as we go from (a) to (b). Here the brief induced current acts in the direction shown so as to try to maintain the original flux $(\Phi = BA)$ by producing its own magnetic field into the page. That is, as the area A decreases, the current acts to increase B in the original (inward) direction.

Since magnetic flux $\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int B \cos\theta \, dA$, we see that an emf can be induced in three ways: (1) by a changing magnetic field B; (2) by changing the area A of the loop in the field; or (3) by changing the loop's orientation θ with respect to the field. Figures 1 and 2 illustrated case 1. Examples of cases 2 and 3 are illustrated in Figs. 6 and 7, respectively.

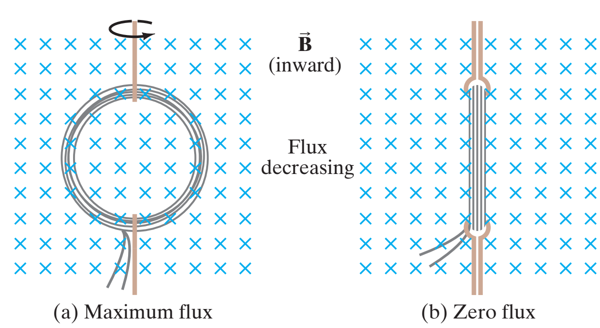


FIGURE 7 A current can be induced by rotating a coil in a magnetic field. The flux through the coil changes from (a) to (b) because θ (in Eq. 1a, $\Phi = BA\cos\theta$) went from $0°$ $(\cos\theta = 1)$ to $90°$ $(\cos\theta = 0)$.

CONCEPTUAL EXAMPLE 3 **Induction stove.** In an induction stove (Fig. 8), an ac current exists in a coil that is the "burner" (a burner that never gets hot). Why will it heat a metal pan but not a glass container?

RESPONSE The ac current sets up a changing magnetic field that passes through the pan bottom. This changing magnetic field induces a current in the pan bottom, and since the pan offers resistance, electric energy is transformed to thermal energy which heats the pot and its contents. A glass container offers such high resistance that little current is induced and little energy is transferred $(P = V^2/R)$.

FIGURE 8 Example 3: An induction stove.

Diva de Provence/ DIVA Induction froid

PROBLEM SOLVING

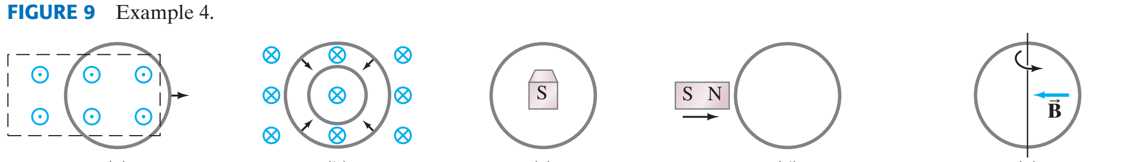
Lenz's Law

Lenz's law is used to determine the direction of the (conventional) electric current induced in a loop due to a change in magnetic flux inside the loop. To produce an induced current you need

(a) a closed conducting loop, and
(b) an external magnetic flux through the loop that is changing in time.

1. Determine whether the magnetic flux ($\Phi_B = BA\cos\theta$) inside the loop is decreasing, increasing, or unchanged.
2. The magnetic field due to the induced current: (a) points in the same direction as the external field if the flux is decreasing; (b) points in the opposite direction from the external field if the flux is increasing; or (c) is zero if the flux is not changing.
3. Once you know the direction of the induced magnetic field, use the right-hand rule to find the direction of the induced current.
4. Always keep in mind that there are two magnetic fields: (1) an external field whose flux must be changing if it is to induce an electric current, and (2) a magnetic field produced by the induced current.

FIGURE 9 Example 4.



(a) Pulling a round loop to the right out of a magnetic field which points out of the page

(b) Shrinking a loop in a magnetic field pointing into the page

(c) S magnetic pole moving from below, up toward the loop

(d) N magnetic pole moving toward loop in the plane of the page

(e) Rotating the loop by pulling the left side toward us and pushing the right side in; the magnetic field points from right to left

CONCEPTUAL EXAMPLE 4 **Practice with Lenz's law.** In which direction is the current induced in the circular loop for each situation in Fig. 9?

RESPONSE (*a*) Initially, the magnetic field pointing out of the page passes through the loop. If you pull the loop out of the field, magnetic flux through the loop decreases; so the induced current will be in a direction to maintain the decreasing flux through the loop: the current will be counterclockwise to produce a magnetic field outward (toward the reader).

(*b*) The external field is into the page. The coil area gets smaller, so the flux will decrease; hence the induced current will be clockwise, producing its own field into the page to make up for the flux decrease.

(*c*) Magnetic field lines point into the S pole of a magnet, so as the magnet moves toward us and the loop, the magnet's field points into the page and is getting stronger. The current in the loop will be induced in the counterclockwise direction in order to produce a field $\vec{\mathbf{B}}$ *out* of the page.

(*d*) The field is in the plane of the loop, so no magnetic field lines pass through the loop and the flux through the loop is zero throughout the process; hence there is no change in external magnetic flux with time, and there will be no induced emf or current in the loop.

(*e*) Initially there is no flux through the loop. When you start to rotate the loop, the external field through the loop begins increasing to the left. To counteract this change in flux, the loop will have current induced in a counterclockwise direction so as to produce its own field to the right.

CAUTION
Magnetic field created by induced current opposes change in external flux, not necessarily opposing the external field

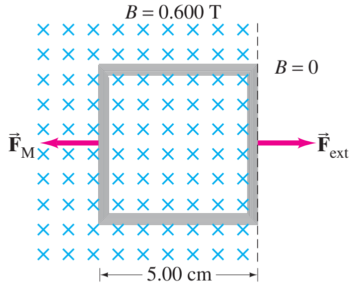


FIGURE 10 Example 5. The square coil in a magnetic field $B = 0.600\text{ T}$ is pulled abruptly to the right to a region where $B = 0$.

EXAMPLE 5 **Pulling a coil from a magnetic field.** A 100-loop square coil of wire, with side $\ell = 5.00\text{ cm}$ and total resistance $100\,\Omega$, is positioned perpendicular to a uniform 0.600-T magnetic field, as shown in Fig. 10. It is quickly pulled from the field at constant speed (moving perpendicular to $\vec{\mathbf{B}}$) to a region where B drops abruptly to zero. At $t = 0$, the right edge of the coil is at the edge of the field. It takes 0.100 s for the whole coil to reach the field-free region. Find (*a*) the rate of change in flux through the coil, and (*b*) the emf and current induced. (*c*) How much energy is dissipated in the coil? (*d*) What was the average force required (F_{ext})?

APPROACH We start by finding how the magnetic flux, $\Phi_B = BA$, changes during the time interval $\Delta t = 0.100\text{ s}$. Faraday's law then gives the induced emf and Ohm's law gives the current.

SOLUTION (*a*) The area of the coil is $A = \ell^2 = \left(5.00 \times 10^{-2}\text{ m}\right)^2 = 2.50 \times 10^{-3}\text{ m}^2$. The flux through one loop is initially $\Phi_B = BA = (0.600\text{ T})\left(2.50 \times 10^{-3}\text{ m}^2\right) = 1.50 \times 10^{-3}\text{ Wb}$. After 0.100 s, the flux is zero. The rate of change in flux is constant (because the coil is square), equal to

$$\frac{\Delta\Phi_B}{\Delta t} = \frac{0 - \left(1.50 \times 10^{-3}\text{ Wb}\right)}{0.100\text{ s}} = -1.50 \times 10^{-2}\text{ Wb/s}.$$

(*b*) The emf induced (Eq. 2) in the 100-loop coil during this 0.100-s interval is

$$\mathscr{E} = -N\frac{\Delta\Phi_B}{\Delta t} = -(100)\left(-1.50 \times 10^{-2}\text{ Wb/s}\right) = 1.50\text{ V}.$$

The current is found by applying Ohm's law to the 100-Ω coil:

$$I = \frac{\mathscr{E}}{R} = \frac{1.50\text{ V}}{100\,\Omega} = 1.50 \times 10^{-2}\text{ A} = 15.0\text{ mA}.$$

By Lenz's law, the current must be clockwise to produce more $\vec{\mathbf{B}}$ into the page and thus oppose the decreasing flux into the page.
(*c*) The total energy dissipated in the coil is the product of the power $\left(= I^2R\right)$ and the time:

$$E = Pt = I^2Rt = \left(1.50 \times 10^{-2}\text{ A}\right)^2(100\,\Omega)(0.100\text{ s}) = 2.25 \times 10^{-3}\text{ J}.$$

(*d*) We can use the result of part (*c*) and apply the work-energy principle: the energy dissipated E is equal to the work W needed to pull the coil out of the field. Because $W = \bar{F}d$ where $d = 5.00\text{ cm}$, then

$$\bar{F} = \frac{W}{d} = \frac{2.25 \times 10^{-3}\text{ J}}{5.00 \times 10^{-2}\text{ m}} = 0.0450\text{ N}.$$

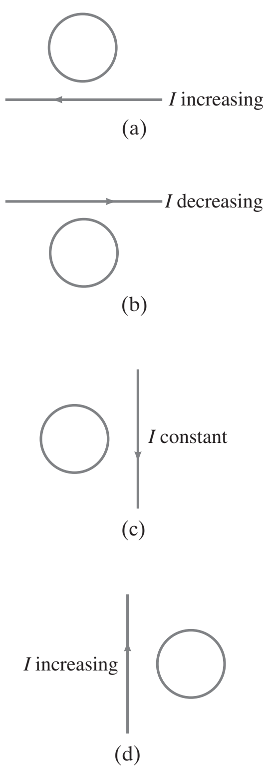
Alternate Solution (*d*) We can also calculate the force directly using $\vec{\mathbf{F}} = I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}$, which here for constant $\vec{\mathbf{B}}$ is $F = I\ell B$. The force the magnetic field exerts on the top and bottom sections of the square coil of Fig. 10 are in opposite directions and cancel each other. The magnetic force $\vec{\mathbf{F}}_M$ exerted on the left vertical section of the square coil acts to the left as shown because the current is up (clockwise). The right side of the loop is in the region where $\vec{\mathbf{B}} = 0$. Hence the external force, to the right, needed to just overcome the magnetic force to the left (on $N = 100$ loops) is

$$F_{ext} = NI\ell B = (100)(0.0150\text{ A})(0.0500\text{ m})(0.600\text{ T}) = 0.0450\text{ N},$$

which is the same answer, confirming our use of energy conservation above.

EXERCISE B What is the direction of the induced current in the circular loop due to the current shown in each part of Fig. 11?

FIGURE 11 Exercise B.



3 EMF Induced in a Moving Conductor

Another way to induce an emf is shown in Fig. 12a, and this situation helps illuminate the nature of the induced emf. Assume that a uniform magnetic field $\vec{\mathbf{B}}$ is perpendicular to the area bounded by the U-shaped conductor and the movable rod resting on it. If the rod is made to move at a speed v, it travels a distance $dx = v\,dt$ in a time dt. Therefore, the area of the loop increases by an amount $dA = \ell\,dx = \ell v\,dt$ in a time dt. By Faraday's law there is an induced emf $\mathscr{E}$ whose magnitude is given by

$$\mathscr{E} = \frac{d\Phi_B}{dt} = \frac{B\,dA}{dt} = \frac{B\ell v\,dt}{dt} = B\ell v. \qquad \textbf{(3)}$$

Equation 3 is valid as long as B, ℓ, and v are mutually perpendicular. (If they are not, we use only the components of each that are mutually perpendicular.) An emf induced on a conductor moving in a magnetic field is sometimes called *motional emf*.

We can also obtain Eq. 3 without using Faraday's law. A charged particle moving perpendicular to a magnetic field B with speed v experiences a force $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. When the rod of Fig. 12a moves to the right with speed v, the electrons in the rod also move with this speed. Therefore, since $\vec{\mathbf{v}} \perp \vec{\mathbf{B}}$, each electron feels a force $F = qvB$, which acts up the page as shown in Fig. 12b. If the rod was not in contact with the U-shaped conductor, electrons would collect at the upper end of the rod, leaving the lower end positive (see signs in Fig. 12b). There must thus be an induced emf. If the rod is in contact with the U-shaped conductor (Fig. 12a), the electrons will flow into the U. There will then be a clockwise (conventional) current in the loop. To calculate the emf, we determine the work W needed to move a charge q from one end of the rod to the other against this potential difference: $W = \text{force} \times \text{distance} = (qvB)(\ell)$. The emf equals the work done per unit charge, so $\mathscr{E} = W/q = qvB\ell/q = B\ell v$, the same result† as from Faraday's law above, Eq. 3.

EXERCISE C In what direction will the electrons flow in Fig. 12 if the rod moves to the left, decreasing the area of the current loop?

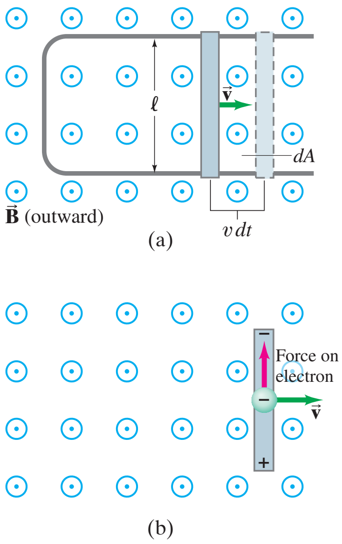


FIGURE 12 (a) A conducting rod is moved to the right on a U-shaped conductor in a uniform magnetic field $\vec{\mathbf{B}}$ that points out of the page. The induced current is clockwise. (b) Upward force on an electron in the metal rod (moving to the right) due to $\vec{\mathbf{B}}$ pointing out of the page; hence electrons can collect at top of rod, leaving + charge at bottom.

EXAMPLE 6 ESTIMATE Does a moving airplane develop a large emf? An airplane travels 1000 km/h in a region where the Earth's magnetic field is about 5×10^{-5} T and is nearly vertical (Fig. 13). What is the potential difference induced between the wing tips that are 70 m apart?

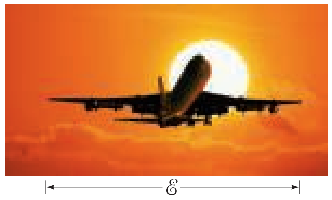
APPROACH We consider the wings to be a 70-m-long conductor moving through the Earth's magnetic field. We use Eq. 3 to get the emf.

SOLUTION Since $v = 1000\text{ km/h} = 280\text{ m/s}$, and $\vec{\mathbf{v}} \perp \vec{\mathbf{B}}$, we have

$$\mathscr{E} = B\ell v = (5 \times 10^{-5}\,\text{T})(70\,\text{m})(280\,\text{m/s}) \approx 1\,\text{V}.$$

NOTE Not much to worry about.

FIGURE 13 Example 6.



Jeff Hunter/Image Bank/Getty Images

EXAMPLE 7 Electromagnetic blood-flow measurement. The rate of blood flow in our body's vessels can be measured using the apparatus shown in Fig. 14, since blood contains charged ions. Suppose that the blood vessel is 2.0 mm in diameter, the magnetic field is 0.080 T, and the measured emf is 0.10 mV. What is the flow velocity v of the blood?

APPROACH The magnetic field $\vec{\mathbf{B}}$ points horizontally from left to right (N pole toward S pole). The induced emf acts over the width $\ell = 2.0$ mm of the blood vessel, perpendicular to $\vec{\mathbf{B}}$ and $\vec{\mathbf{v}}$ (Fig. 14), just as in Fig. 12. We can then use Eq. 3 to get v. ($\vec{\mathbf{v}}$ in Fig. 14 corresponds to $\vec{\mathbf{v}}$ in Fig. 12.)

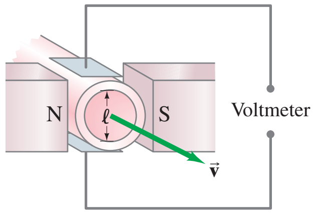
SOLUTION We solve for v in Eq. 3:

$$v = \frac{\mathscr{E}}{B\ell} = \frac{(1.0 \times 10^{-4}\,\text{V})}{(0.080\,\text{T})(2.0 \times 10^{-3}\,\text{m})} = 0.63\,\text{m/s}.$$

NOTE In actual practice, an alternating current is used to produce an alternating magnetic field. The induced emf is then alternating.

PHYSICS APPLIED
Blood-flow measurement

FIGURE 14 Measurement of blood velocity from the induced emf. Example 7.



†This force argument, which is basically the same as for the Hall effect, explains this one way of inducing an emf. It does not explain the general case of electromagnetic induction.

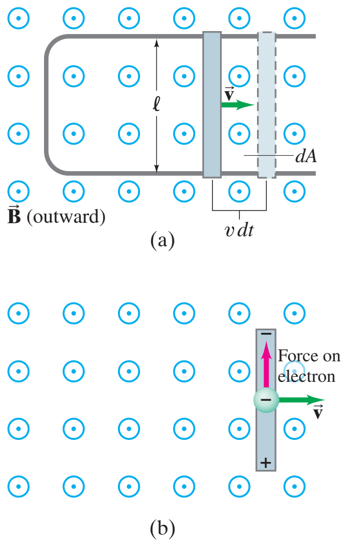


FIGURE 12 (repeated) (a) A conducting rod is moved to the right on a U-shaped conductor in a uniform magnetic field $\vec{\mathbf{B}}$ that points out of the page. The induced current is clockwise. (b) Upward force on an electron in the metal rod (moving to the right) due to $\vec{\mathbf{B}}$ pointing out of the page; hence electrons can collect at top of rod, leaving + charge at bottom.

EXAMPLE 8 **Force on the rod.** To make the rod of Fig. 12a move to the right at constant speed v, you need to apply an external force on the rod to the right. (*a*) Explain and determine the magnitude of the required force. (*b*) What external power is needed to move the rod? (Do not confuse this external force on the rod with the upward force on the electrons shown in Fig. 12b.)

APPROACH When the rod moves to the right, electrons flow upward in the rod according to the right-hand rule. So the conventional current is downward in the rod. We can see this also from Lenz's law: the outward magnetic flux through the loop is increasing, so the induced current must oppose the increase. Thus the current is clockwise so as to produce a magnetic field into the page (right-hand rule). The magnetic force on the moving rod is $\vec{\mathbf{F}} = I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}$ for a constant $\vec{\mathbf{B}}$. The right-hand rule tells us this magnetic force is to the left, and is thus a "drag force" opposing our effort to move the rod to the right.

SOLUTION (*a*) The magnitude of the external force, to the right, needs to balance the magnetic force $F = I\ell B$, to the left. The current $I = \mathscr{E}/R = B\ell v/R$ (see Eq. 3), and the resistance R is that of the whole circuit: the rod and the U-shaped conductor. The force F required to move the rod is thus

$$F = I\ell B = \left(\frac{B\ell v}{R}\right)\ell B = \frac{B^2\ell^2}{R}v.$$

If B, ℓ, and R are constant, then a constant speed v is produced by a constant external force. (Constant R implies that the parallel rails have negligible resistance.)

(*b*) The external power needed to move the rod for constant R is

$$P_{\text{ext}} = Fv = \frac{B^2\ell^2v^2}{R}.$$

The power dissipated in the resistance is $P = I^2R$. With $I = \mathscr{E}/R = B\ell v/R$,

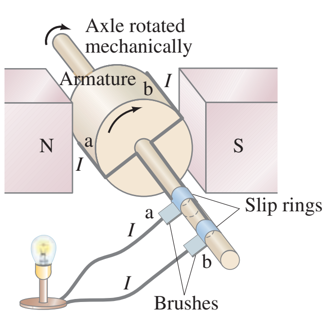
$$P_{\text{R}} = I^2R = \frac{B^2\ell^2v^2}{R},$$

so the power input equals the power dissipated in the resistance at any moment.

4 Electric Generators

Here we examine how alternating currents (ac) are generated: by an **electric generator** or **dynamo**, one of the most important practical results of Faraday's great discovery. A generator transforms mechanical energy into electric energy, just the opposite of what a motor does. A simplified diagram of an **ac generator** is shown in Fig. 15. A generator consists of many loops of wire (only one is shown) wound on an *armature* that can rotate in a magnetic field. The axle is turned by some mechanical means (falling water, steam turbine, car motor belt), and an emf is induced in the rotating coil. An electric current is thus the *output* of a generator. Suppose in Fig. 15 that the armature is rotating clockwise; then $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ applied to charged particles in the wire (or Lenz's law) tells us that the (conventional) current in the wire labeled b on the armature is outward, toward us; therefore the current is outward from brush b. (Each brush is fixed and presses against a continuous slip ring that rotates with the armature.) After one-half revolution, wire b will be where wire a is now in the drawing, and the current then at brush b will be inward. Thus the current produced is alternating.

FIGURE 15 An ac generator.



Let us assume the loop is being made to rotate in a uniform magnetic field $\vec{\mathbf{B}}$ with constant angular velocity ω. From Faraday's law (Eq. 2a), the induced emf is

$$\mathscr{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} = -\frac{d}{dt}[BA\cos\theta]$$

where A is the area of the loop and θ is the angle between $\vec{\mathbf{B}}$ and $\vec{\mathbf{A}}$. Since $\omega = d\theta/dt$, then $\theta = \theta_0 + \omega t$. We arbitrarily take $\theta_0 = 0$, so

$$\mathscr{E} = -BA\frac{d}{dt}(\cos\omega t) = BA\omega\sin\omega t.$$

If the rotating coil contains N loops,

$$\begin{aligned}\mathscr{E} &= NBA\omega\sin\omega t \\ &= \mathscr{E}_0\sin\omega t.\end{aligned} \qquad \textbf{(4)}$$

Thus the output emf is sinusoidal (Fig. 16) with amplitude $\mathscr{E}_0 = NBA\omega$. Such a rotating coil in a magnetic field is the basic operating principle of an ac generator.

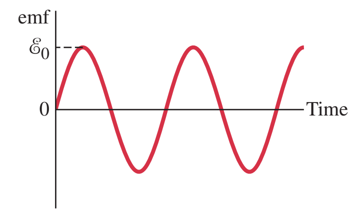


FIGURE 16 An ac generator produces an alternating current. The output emf $\mathscr{E} = \mathscr{E}_0\sin\omega t$, where $\mathscr{E}_0 = NAB\omega$ (Eq. 4).

The frequency $f\,(=\omega/2\pi)$ is 60 Hz for general use in the United States and Canada, whereas 50 Hz is used in many countries. Most of the power generated in the United States is done at steam plants, where the burning of fossil fuels (coal, oil, natural gas) boils water to produce high-pressure steam that turns a turbine connected to the generator axle. Falling water from the top of a dam (hydroelectric) is also common (Fig. 17). At nuclear power plants, the nuclear energy released is used to produce steam to turn turbines. Indeed, a heat engine connected to a generator is the principal means of generating electric power. The frequency of 60 Hz or 50 Hz is maintained very precisely by power companies, and in doing Problems, we will assume it is at least as precise as other numbers given.

PHYSICS APPLIED
Power plants

FIGURE 17 Water-driven generators at the base of Bonneville Dam, Oregon.

Rick Bowmer/AP Wide World Photos

EXAMPLE 9 **An ac generator.** The armature of a 60-Hz ac generator rotates in a 0.15-T magnetic field. If the area of the coil is $2.0 \times 10^{-2}\,\text{m}^2$, how many loops must the coil contain if the peak output is to be $\mathscr{E}_0 = 170\,\text{V}$?

APPROACH From Eq. 4 we see that the maximum emf is $\mathscr{E}_0 = NBA\omega$.

SOLUTION We solve Eq. 4 for N with $\omega = 2\pi f = (6.28)(60\,\text{s}^{-1}) = 377\,\text{s}^{-1}$:

$$N = \frac{\mathscr{E}_0}{BA\omega} = \frac{170\,\text{V}}{(0.15\,\text{T})(2.0\times10^{-2}\,\text{m}^2)(377\,\text{s}^{-1})} = 150 \text{ turns.}$$

A **dc generator** is much like an ac generator, except the slip rings are replaced by split-ring commutators, Fig. 18a, just as in a dc motor. The output of such a generator is as shown and can be smoothed out by placing a capacitor in parallel with the output. More common is the use of many armature windings, as in Fig. 18b, which produces a smoother output.

PHYSICS APPLIED
DC generator

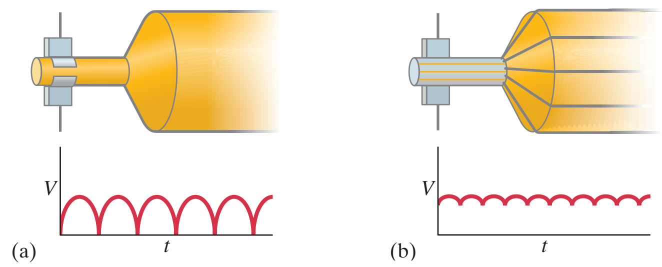
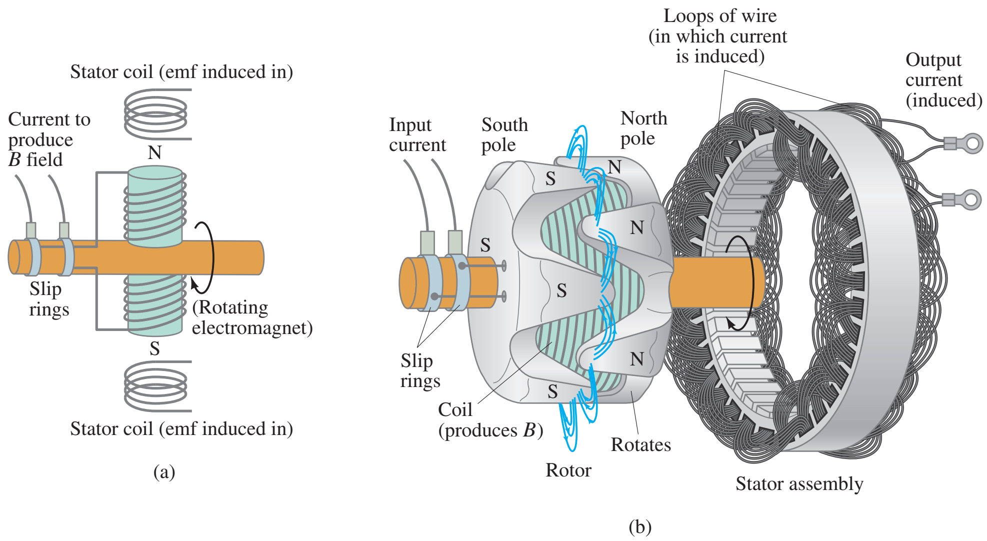


FIGURE 18 (a) A dc generator with one set of commutators, and (b) a dc generator with many sets of commutators and windings.

FIGURE 19 (a) Simplified schematic diagram of an alternator. The input current to the rotor from the battery is connected through continuous slip rings. Sometimes the rotor electromagnet is replaced by a permanent magnet. (b) Actual shape of an alternator. The rotor is made to turn by a belt from the engine. The current in the wire coil of the rotor produces a magnetic field inside it on its axis that points horizontally from left to right, thus making north and south poles of the plates attached at either end. These end plates are made with triangular fingers that are bent over the coil—hence there are alternating N and S poles quite close to one another, with magnetic field lines between them as shown by the blue lines. As the rotor turns, these field lines pass through the fixed stator coils (shown on the right for clarity, but in operation the rotor rotates within the stator), inducing a current in them, which is the output.



PHYSICS APPLIED
Alternators

Automobiles used to use dc generators. Today they mainly use **alternators**, which avoid the problems of wear and electrical arcing (sparks) across the split-ring commutators of dc generators. Alternators differ from generators in that an electromagnet, called the *rotor*, is fed by current from the battery and is made to rotate by a belt from the engine. The magnetic field of the turning rotor passes through a surrounding set of stationary coils called the *stator* (Fig. 19), inducing an alternating current in the stator coils, which is the output. This ac output is changed to dc for charging the battery by the use of semiconductor diodes, which allow current flow in one direction only.

*5 Back EMF and Counter Torque; Eddy Currents

*Back EMF, in a Motor

A motor turns and produces mechanical energy when a current is made to flow in it. From a description of a simple dc motor, you might expect that the armature would accelerate indefinitely due to the torque on it. However, as the armature of the motor turns, the magnetic flux through the coil changes and an emf is generated. This induced emf acts to oppose the motion (Lenz's law) and is called the **back emf** or **counter emf**. The greater the speed of the motor, the greater the back emf. A motor normally turns and does work on something, but if there were no load, the motor's speed would increase until the back emf equaled the input voltage. When there is a mechanical load, the speed of the motor may be limited also by the load. The back emf will then be less than the external applied voltage. The greater the mechanical load, the slower the motor rotates and the lower is the back emf ($\mathscr{E} \propto \omega$, Eq. 4).

EXAMPLE 10 **Back emf in a motor.** The armature windings of a dc motor have a resistance of $5.0\,\Omega$. The motor is connected to a 120-V line, and when the motor reaches full speed against its normal load, the back emf is 108 V. Calculate (*a*) the current into the motor when it is just starting up, and (*b*) the current when the motor reaches full speed.

APPROACH As the motor is just starting up, it is turning very slowly, so there is no induced back emf. The only voltage is the 120-V line. The current is given by Ohm's law with $R = 5.0\,\Omega$. At full speed, we must include as emfs both the 120-V applied emf and the opposing back emf.

SOLUTION (*a*) At start up, the current is controlled by the 120 V applied to the coil's 5.0-Ω resistance. By Ohm's law,

$$I = \frac{V}{R} = \frac{120\text{ V}}{5.0\,\Omega} = 24\text{ A}.$$

(*b*) When the motor is at full speed, the back emf must be included in the equivalent circuit shown in Fig. 20. In this case, Ohm's law (or Kirchhoff's rule) gives

$$120\text{ V} - 108\text{ V} = I(5.0\,\Omega).$$

Therefore

$$I = \frac{12\text{ V}}{5.0\,\Omega} = 2.4\text{ A}.$$

NOTE This result shows that the current can be very high when a motor first starts up. This is why the lights in your house may dim when the motor of the refrigerator (or other large motor) starts up. The large initial current causes the voltage to the lights and at the outlets to drop, since the house wiring has resistance and there is some voltage drop across it when large currents are drawn.

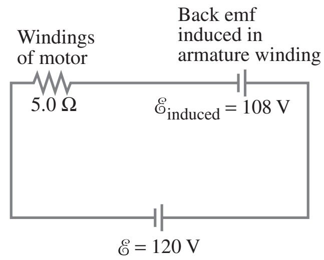


FIGURE 20 Circuit of a motor showing induced back emf. Example 10.

CONCEPTUAL EXAMPLE 11 **Motor overload.** When using an appliance such as a blender, electric drill, or sewing machine, if the appliance is overloaded or jammed so that the motor slows appreciably or stops while the power is still connected, the device can burn out and be ruined. Explain why this happens.

RESPONSE The motors are designed to run at a certain speed for a given applied voltage, and the designer must take the expected back emf into account. If the rotation speed is reduced, the back emf will not be as high as expected ($\mathscr{E} \propto \omega$, Eq. 4), and the current will increase, and may become large enough that the windings of the motor heat up to the point of ruining the motor.

PHYSICS APPLIED
Burning out a motor

*Counter Torque

In a generator, the situation is the reverse of that for a motor. As we saw, the mechanical turning of the armature induces an emf in the loops, which is the output. If the generator is not connected to an external circuit, the emf exists at the terminals but there is no current. In this case, it takes little effort to turn the armature. But if the generator *is* connected to a device that draws current, then a current flows in the coils of the armature. Because this current-carrying coil is in an external magnetic field, there will be a torque exerted on it (as in a motor), and this torque opposes the motion (use the right-hand rule for the force on a wire in Fig. 15). This is called a **counter torque**. The greater the electrical load—that is, the more current that is drawn—the greater will be the counter torque. Hence the external applied torque will have to be greater to keep the generator turning. This makes sense from the conservation of energy principle. More mechanical-energy input is needed to produce more electrical-energy output.

EXERCISE D A bicycle headlight is powered by a generator that is turned by the bicycle wheel. (*a*) If you pedal faster, how does the power to the light change? (*b*) Does the generator resist being turned as the bicycle's speed increases, and if so how?

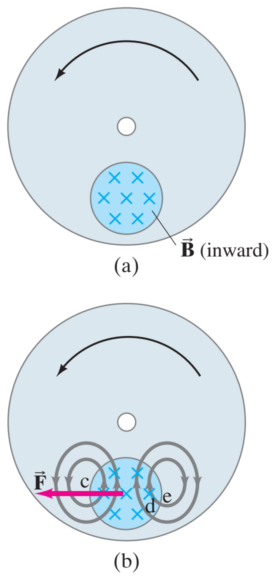


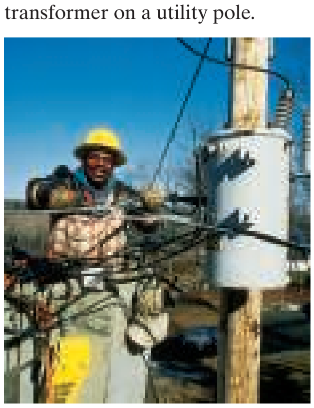
FIGURE 21 Production of eddy currents in a rotating wheel. The grey lines in (b) indicate induced current.

FIGURE 22 Airport metal detector.

Jack Hollingsworth/ Photodisc/Getty Images

PHYSICS APPLIED
Airport metal detector

FIGURE 23 Repairing a step-down transformer on a utility pole.



Robert Houser

*Eddy Currents

Induced currents are not always confined to well-defined paths such as in wires. Consider, for example, the rotating metal wheel in Fig. 21a. An external magnetic field is applied to a limited area of the wheel as shown and points into the page. The section of wheel in the magnetic field has an emf induced in it because the conductor is moving, carrying electrons with it. The flow of induced (conventional) current in the wheel is upward in the region of the magnetic field (Fig. 21b), and the current follows a downward return path outside that region. Why? According to Lenz's law, the induced currents oppose the change that causes them. Consider the part of the wheel labeled c in Fig. 21b, where the magnetic field is zero but is just about to enter a region where $\vec{\mathbf{B}}$ points into the page. To oppose this inward increase in magnetic field, the induced current is counterclockwise to produce a field pointing out of the page (right-hand-rule 1). Similarly, region d is about to move to e, where $\vec{\mathbf{B}}$ is zero; hence the current is clockwise to produce an inward field opposed to this decreasing flux inward. These currents are referred to as **eddy currents**. They can be present in any conductor that is moving across a magnetic field or through which the magnetic flux is changing.

In Fig. 21b, the magnetic field exerts a force $\vec{\mathbf{F}}$ on the induced currents it has created, and that force opposes the rotational motion. Eddy currents can be used in this way as a smooth braking device on, say, a rapid-transit car. In order to stop the car, an electromagnet can be turned on that applies its field either to the wheels or to the moving steel rail below. Eddy currents can also be used to dampen (reduce) the oscillation of a vibrating system. Eddy currents, however, can be a problem. For example, eddy currents induced in the armature of a motor or generator produce heat $(P = I\mathscr{E})$ and waste energy. To reduce the eddy currents, the armatures are *laminated*; that is, they are made of very thin sheets of iron that are well insulated from one another. The total path length of the eddy currents is confined to each slab, which increases the total resistance; hence the current is less and there is less wasted energy.

Walk-through metal detectors at airports (Fig. 22) detect metal objects using electromagnetic induction and eddy currents. Several coils are situated in the walls of the walk-through at different heights. In a technique called "pulse induction," the coils are given repeated brief pulses of current (on the order of microseconds), hundreds or thousands of times a second. Each pulse in a coil produces a magnetic field for a very brief period of time. When a passenger passes through the walk-through, any metal object being carried will have eddy currents induced in it. The eddy currents persist briefly after each input pulse, and the small magnetic field produced by the persisting eddy current (before the next external pulse) can be detected, setting off an alert or alarm. Stores and libraries sometimes use similar systems to discourage theft.

6 Transformers and Transmission of Power

A transformer is a device for increasing or decreasing an ac voltage. Transformers are found everywhere: on utility poles (Fig. 23) to reduce the high voltage from the electric company to a usable voltage in houses (120 V or 240 V), in chargers for cell phones, laptops, and other electronic devices, in CRT monitors and in your car to give the needed high voltage (to the spark plugs), and in many other applications. A **transformer** consists of two coils of wire known as the **primary** and **secondary** coils. The two coils can be interwoven (with insulated wire); or they can be linked by an iron core which is laminated to minimize eddy-current losses (Section 5), as shown in Fig. 24. Transformers are designed so that (nearly) all the magnetic flux produced by the current in the primary coil also passes through the secondary coil, and we assume this is true in what follows. We also assume that energy losses (in resistance and hysteresis) can be ignored—a good approximation for real transformers, which are often better than 99% efficient.

When an ac voltage is applied to the primary coil, the changing magnetic field it produces will induce an ac voltage of the same frequency in the secondary coil. However, the voltage will be different according to the number of loops in each coil. From Faraday's law, the voltage or emf induced in the secondary coil is

$$V_S = N_S \frac{d\Phi_B}{dt},$$

where N_S is the number of turns in the secondary coil, and $d\Phi_B/dt$ is the rate at which the magnetic flux changes.

The input primary voltage, V_P, is related to the rate at which the flux changes through it,

$$V_P = N_P \frac{d\Phi_B}{dt},$$

where N_P is the number of turns in the primary coil. This follows because the changing flux produces a back emf, $N_P\, d\Phi_B/dt$, in the primary that exactly balances the applied voltage V_P if the resistance of the primary can be ignored (Kirchhoff's rules). We divide these two equations, assuming little or no flux is lost, to find

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}. \tag{5}$$

This **transformer equation** tells how the secondary (output) voltage is related to the primary (input) voltage; V_S and V_P in Eq. 5 can be the rms values for both, or peak values for both. DC voltages don't work in a transformer because there would be no changing magnetic flux.

If the secondary coil contains more loops than the primary coil $(N_S > N_P)$, we have a **step-up transformer**. The secondary voltage is greater than the primary voltage. For example, if the secondary coil has twice as many turns as the primary coil, then the secondary voltage will be twice that of the primary voltage. If N_S is less than N_P, we have a **step-down transformer**.

Although ac voltage can be increased (or decreased) with a transformer, we don't get something for nothing. Energy conservation tells us that the power output can be no greater than the power input. A well-designed transformer can be greater than 99% efficient, so little energy is lost to heat. The power output thus essentially equals the power input. Since power $P = IV$, we have

$$I_P V_P = I_S V_S,$$

or

$$\frac{I_S}{I_P} = \frac{N_P}{N_S}. \tag{6}$$

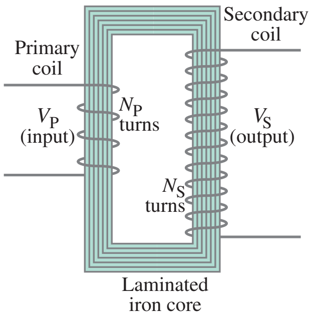


FIGURE 24 Step-up transformer $(N_P = 4, N_S = 12)$.

EXAMPLE 12 **Cell phone charger.** The charger for a cell phone contains a transformer that reduces 120-V (or 240-V) ac to 5.0-V ac to charge the 3.7-V battery. (It also contains diodes to change the 5.0-V ac to 5.0-V dc.) Suppose the secondary coil contains 30 turns and the charger supplies 700 mA. Calculate (*a*) the number of turns in the primary coil, (*b*) the current in the primary, and (*c*) the power transformed.

APPROACH We assume the transformer is ideal, with no flux loss, so we can use Eq. 5 and then Eq. 6.

SOLUTION (*a*) This is a step-down transformer, and from Eq. 5 we have

$$N_P = N_S \frac{V_P}{V_S} = \frac{(30)(120\text{ V})}{(5.0\text{ V})} = 720\text{ turns}.$$

(*b*) From Eq. 6

$$I_P = I_S \frac{N_S}{N_P} = (0.70\text{ A})\left(\frac{30}{720}\right) = 29\text{ mA}.$$

(*c*) The power transformed is

$$P = I_S V_S = (0.70\text{ A})(5.0\text{ V}) = 3.5\text{ W}.$$

NOTE The power in the primary coil, $P = (0.029\text{ A})(120\text{ V}) = 3.5\text{ W}$, is the same as the power in the secondary coil. There is 100% efficiency in power transfer for our ideal transformer.

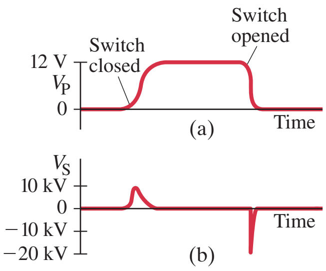


FIGURE 25 A dc voltage turned on and off as shown in (a) produces voltage pulses in the secondary (b). Voltage scales in (a) and (b) are not the same.

PHYSICS APPLIED
Transformers help power transmission

A transformer operates only on ac. A dc current in the primary coil does not produce a changing flux and therefore induces no emf in the secondary. However, if a dc voltage is applied to the primary through a switch, at the instant the switch is opened or closed there will be an induced voltage in the secondary. For example, if the dc is turned on and off as shown in Fig. 25a, the voltage induced in the secondary is as shown in Fig. 25b. Notice that the secondary voltage drops to zero when the dc voltage is steady. This is basically how, in the **ignition system** of an automobile, the high voltage is created to produce the spark across the gap of a spark plug that ignites the gas-air mixture. The transformer is referred to simply as an "ignition coil," and transforms the 12 V of the battery (when switched off in the primary) into a spike of as much as 30 kV in the secondary.

Transformers play an important role in the transmission of electricity. Power plants are often situated some distance from metropolitan areas, so electricity must then be transmitted over long distances (Fig. 26). There is always some power loss in the transmission lines, and this loss can be minimized if the power is transmitted at high voltage, using transformers, as the following Example shows.

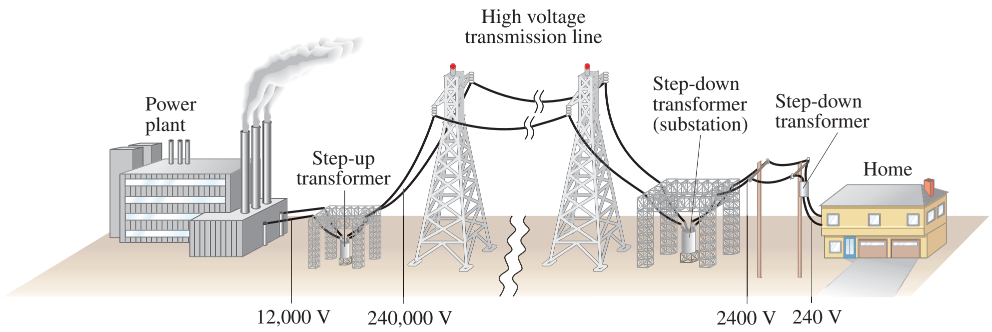


FIGURE 26 The transmission of electric power from power plants to homes makes use of transformers at various stages.

EXAMPLE 13 **Transmission lines.** An average of 120 kW of electric power is sent to a small town from a power plant 10 km away. The transmission lines have a total resistance of 0.40 Ω. Calculate the power loss if the power is transmitted at (*a*) 240 V and (*b*) 24,000 V.

APPROACH We cannot use $P = V^2/R$ because if R is the resistance of the transmission lines, we don't know the voltage drop along them; the given voltages are applied across the lines plus the load (the town). But we can determine the current I in the lines $(= P/V)$, and then find the power loss from $P_L = I^2R$, for both cases (*a*) and (*b*).

SOLUTION (*a*) If 120 kW is sent at 240 V, the total current will be

$$I = \frac{P}{V} = \frac{1.2 \times 10^5\ \mathrm{W}}{2.4 \times 10^2\ \mathrm{V}} = 500\ \mathrm{A}.$$

The power loss in the lines, P_L, is then

$$P_L = I^2R = (500\ \mathrm{A})^2(0.40\ \Omega) = 100\ \mathrm{kW}.$$

Thus, over 80% of all the power would be wasted as heat in the power lines!

(*b*) If 120 kW is sent at 24,000 V, the total current will be

$$I = \frac{P}{V} = \frac{1.2 \times 10^5\,\mathrm{W}}{2.4 \times 10^4\,\mathrm{V}} = 5.0\,\mathrm{A}.$$

The power loss in the lines is then

$$P_\mathrm{L} = I^2 R = (5.0\,\mathrm{A})^2(0.40\,\Omega) = 10\,\mathrm{W},$$

which is less than $\frac{1}{100}$ of 1%: a far better efficiency!

NOTE We see that the higher voltage results in less current, and thus less power is wasted as heat in the transmission lines. It is for this reason that power is usually transmitted at very high voltages, as high as 700 kV.

The great advantage of ac, and a major reason it is in nearly universal use, is that the voltage can easily be stepped up or down by a transformer. The output voltage of an electric generating plant is stepped up prior to transmission. Upon arrival in a city, it is stepped down in stages at electric substations prior to distribution. The voltage in lines along city streets is typically 2400 V or 7200 V (but sometimes less), and is stepped down to 240 V or 120 V for home use by transformers (Figs. 23 and 26).

PHYSICS APPLIED
Fluorescent lightbulb ballast

Fluorescent lights require a very high voltage initially to ionize the gas inside the bulb. The high voltage is obtained using a step-up transformer, called a ballast, and can be replaced independently of the bulb in many fluorescent light fixtures. When the ballast starts to fail, the tube is slow to light. Replacing the bulb will not solve the problem. In newer compact fluorescent bulbs designed to replace incandescent bulbs, the ballast (transformer) is part of the bulb, and is very small.

7 A Changing Magnetic Flux Produces an Electric Field

When an electric current flows in a wire, there is an electric field in the wire that does the work of moving the electrons in the wire. In this Chapter we have seen that a changing magnetic flux induces a current in the wire, which implies that there is an electric field in the wire induced by the changing magnetic flux. Thus we come to the important conclusion that

a changing magnetic flux produces an electric field.

This result applies not only to wires and other conductors, but is actually a general result that applies to any region in space. Indeed, an electric field will be produced at any point in space where there is a changing magnetic field.

Faraday's Law—General Form

We can put these ideas into mathematical form by generalizing our relation between an electric field and the potential difference between two points a and b: $V_\mathrm{ab} = \int_\mathrm{a}^\mathrm{b} \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}$ where $d\vec{\boldsymbol{\ell}}$ is an element of displacement along the path of integration. The emf $\mathscr{E}$ induced in a circuit is equal to the work done per unit charge by the electric field, which equals the integral of $\vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}$ around the closed path:

$$\mathscr{E} = \oint \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}. \tag{7}$$

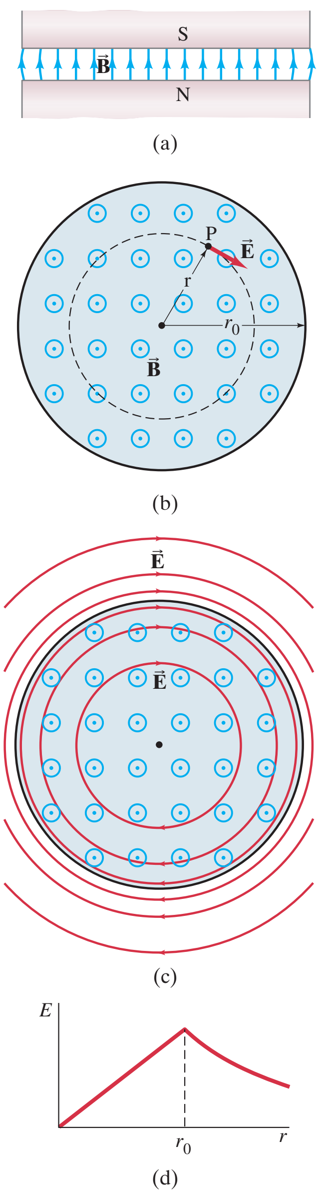
We combine this with Eq. 2a, to obtain a more elegant and general form of Faraday's law

FARADAY'S LAW (general form)

$$\oint \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{d\Phi_B}{dt} \tag{8}$$

which relates the changing magnetic flux to the electric field it produces. The integral on the left is taken around a path enclosing the area through which the magnetic flux Φ_B is changing. This more elegant statement of Faraday's law (Eq. 8) is valid not only in conductors, but in any region of space. To illustrate this, let us take an Example.

FIGURE 27 Example 14.
(a) Side view of nearly constant $\vec{\mathbf{B}}$.
(b) Top view, for determining the electric field $\vec{\mathbf{E}}$ at point P. (c) Lines of $\vec{\mathbf{E}}$ produced by increasing $\vec{\mathbf{B}}$ (pointing outward). (d) Graph of E vs. r.



EXAMPLE 14 **$\vec{\mathbf{E}}$ produced by changing $\vec{\mathbf{B}}$.** A magnetic field $\vec{\mathbf{B}}$ between the pole faces of an electromagnet is nearly uniform at any instant over a circular area of radius r_0 as shown in Figs. 27a and b. The current in the windings of the electromagnet is increasing in time so that $\vec{\mathbf{B}}$ changes in time at a constant rate $d\vec{\mathbf{B}}/dt$ at each point. Beyond the circular region $(r > r_0)$, we assume $\vec{\mathbf{B}} = 0$ at all times. Determine the electric field $\vec{\mathbf{E}}$ at any point P a distance r from the center of the circular area due to the changing $\vec{\mathbf{B}}$.

APPROACH The changing magnetic flux through a circle of radius r, shown dashed in Fig. 27b, will produce an emf around this circle. Because all points on the dashed circle are equivalent physically, the electric field too will show this symmetry and will be in the plane perpendicular to $\vec{\mathbf{B}}$. Thus we can expect $\vec{\mathbf{E}}$ to be perpendicular to $\vec{\mathbf{B}}$ and to be tangent to the circle of radius r. The direction of $\vec{\mathbf{E}}$ will be as shown in Fig. 27b and c, since by Lenz's law the induced $\vec{\mathbf{E}}$ needs to be capable of producing a current that generates a magnetic field opposing the original change in $\vec{\mathbf{B}}$. By *symmetry*, we also expect $\vec{\mathbf{E}}$ to have the same magnitude at all points on the circle of radius r.

SOLUTION We take the circle shown in Fig 27b as our path of integration in Eq. 8. We ignore the minus sign so we can concentrate on magnitude since we already found the direction of $\vec{\mathbf{E}}$ from Lenz's law, and obtain

$$E(2\pi r) = (\pi r^2)\frac{dB}{dt}, \qquad [r < r_0]$$

since $\Phi_B = BA = B(\pi r^2)$ at any instant. We solve for E and obtain

$$E = \frac{r}{2}\frac{dB}{dt}. \qquad [r < r_0]$$

This expression is valid up to the edge of the circle $(r \leq r_0)$, beyond which $\vec{\mathbf{B}} = 0$. If we now consider a point where $r > r_0$, the flux through a circle of radius r is $\Phi_B = \pi r_0^2 B$. Then Eq. 8 gives

$$E(2\pi r) = \pi r_0^2 \frac{dB}{dt} \qquad [r > r_0]$$

or

$$E = \frac{r_0^2}{2r}\frac{dB}{dt}. \qquad [r > r_0]$$

Thus the magnitude of the induced electric field increases linearly from zero at the center of the magnet to $E = (dB/dt)(r_0/2)$ at the edge, and then decreases inversely with distance in the region beyond the edge of the magnetic field. The electric field lines are circles as shown in Fig. 27c. A graph of E vs. r is shown in Fig. 27d.

EXERCISE E Consider the magnet shown in Fig. 27 with a radius $r_0 = 6.0\,\text{cm}$. If the magnetic field changes uniformly from 0.040 T to 0.090 T in 0.18 s, what is the magnitude of the resulting electric field at (*a*) $r = 3.0\,\text{cm}$ and (*b*) $r = 9.0\,\text{cm}$?

*Forces Due to Changing $\vec{\mathbf{B}}$ are Nonconservative

Example 14 illustrates an important difference between electric fields produced by changing magnetic fields and electric fields produced by electric charges at rest (electrostatic fields). Electric field lines produced in the electrostatic case start and stop on electric charges. But the electric field lines produced by a changing magnetic field are continuous; they form closed loops. This distinction goes even further and is an important one. In the electrostatic case, the potential difference between two points is given by

$$V_{\text{ba}} = V_{\text{b}} - V_{\text{a}} = -\int_{\text{a}}^{\text{b}} \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}.$$

If the integral is around a closed loop, so points a and b are the same, then $V_{\text{ba}} = 0$.

Hence the integral of $\vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}$ around a closed path is zero:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = 0. \qquad \text{[electrostatic field]}$$

This followed from the fact that the electrostatic force (Coulomb's law) is a conservative force, and so a potential energy function could be defined. Indeed, the relation above, $\oint \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = 0$, tells us that the work done per unit charge around any closed path is zero (or the work done between any two points is independent of path), which is a property only of a conservative force. But in the nonelectrostatic case, when the electric field is produced by a changing magnetic field, the integral around a closed path is *not* zero, but is given by Eq. 8:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{d\Phi_B}{dt}.$$

We thus come to the conclusion that the forces due to changing magnetic fields are *nonconservative*. We are not able therefore to define a potential energy, or potential function, at a given point in space for the nonelectrostatic case. Although static electric fields are *conservative fields*, the electric field produced by a changing magnetic field is a **nonconservative field**.

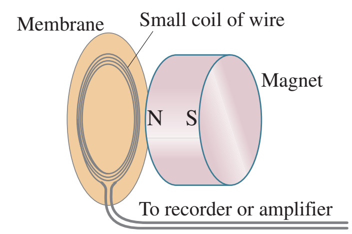


FIGURE 28 Diagram of a microphone that works by induction.

*8 Applications of Induction: Sound Systems, Computer Memory, Seismograph, GFCI

*Microphone

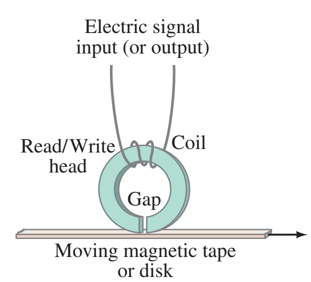
There are various types of *microphones*, and many operate on the principle of induction. In one form, a microphone is just the inverse of a loudspeaker. A small coil connected to a membrane is suspended close to a small permanent magnet, as shown in Fig. 28. The coil moves in the magnetic field when sound waves strike the membrane and this motion induces an emf. The frequency of the induced emf will be just that of the impinging sound waves, and this emf is the "signal" that can be amplified and sent to loudspeakers, or sent to a recorder.

*Read/Write on Tape and Disks

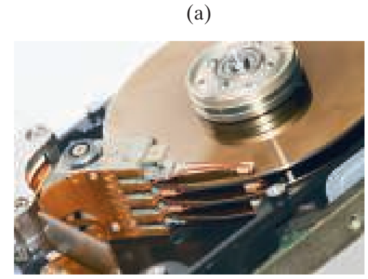
Recording and playback on tape or disks is done by magnetic *heads*. Recording tapes for use in audio and video tape recorders contain a thin layer of magnetic oxide on a thin plastic tape. During recording, the audio and/or video signal voltage is sent to the recording head, which acts as a tiny electromagnet (Fig. 29) that magnetizes the tiny section of tape passing over the narrow gap in the head at each instant. In playback, the changing magnetism of the moving tape at the gap causes corresponding changes in the magnetic field within the soft-iron head, which in turn induces an emf in the coil (Faraday's law). This induced emf is the output signal that can be amplified and sent to a loudspeaker (audio) or to the picture tube (video). In audio and video recorders, the signals may be *analog*—they vary continuously in amplitude over time. The variation in degree of magnetization of the tape at any point reflects the variation in amplitude and frequency of the audio or video signal.

Digital information, such as used on computer hard drives or on magnetic computer tape and some types of digital tape recorders, is read and written using heads that are basically the same as just described (Fig. 29). The essential difference is in the signals, which are not analog, but are digital, and in particular binary, meaning that only two values are possible for each of the extremely high number of predetermined spaces on the tape or disk. The two possible values are usually referred to as 1 and 0. The signal voltage does not vary continuously but rather takes on only two values, +5 V and 0 V, for example, corresponding to the 1 or 0. Thus, information is carried as a series of **bits**, each of which can have only one of two values, 1 or 0.

FIGURE 29 (a) Read/Write (playback/recording) head for tape or disk. In writing or recording, the electric input signal to the head, which acts as an electromagnet, magnetizes the passing tape or disk. In reading or playback, the changing magnetic field of the passing tape or disk induces a changing magnetic field in the head, which in turn induces in the coil an emf that is the output signal. (b) Photo of a hard drive showing several platters and read/write heads that can quickly move from the edge of the disk to the center.



(a)



Terence Kearey

(b)

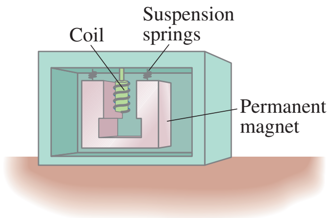


FIGURE 30 One type of seismograph, in which the coil is fixed to the case and moves with the Earth. The magnet, suspended by springs, has inertia and does not move instantaneously with the coil (and case), so there is relative motion between magnet and coil.

*Credit Card Swipe

When you swipe your credit card at a store or gas station, the magnetic stripe on the back of the card passes over a read head just as in a tape recorder or computer. The magnetic stripe contains personal information about your account and connects by telephone line for approval if your account is in order.

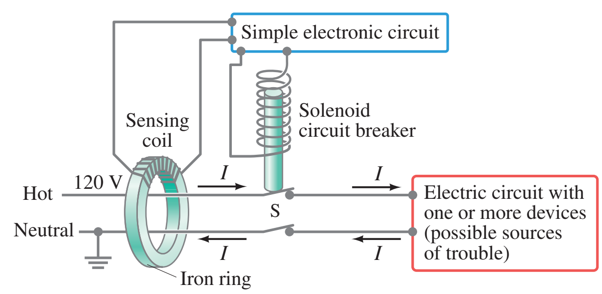
*Seismograph

In geophysics, a **seismograph** measures the intensity of earthquake waves using a magnet and a coil of wire. Either the magnet or the coil is fixed to the case, and the other is inertial (suspended by a spring; Fig. 30). The relative motion of magnet and coil when the Earth shakes induces an emf output.

*Ground Fault Circuit Interrupter (GFCI)

Fuses and circuit breakers protect buildings from fire, and apparatus from damage, due to undesired high currents. But they do not turn off the current until it is very much greater than that which causes permanent damage to humans or death (≈ 100 mA). If fast enough, they may protect in case of a short. A *ground fault circuit interrupter* (GFCI) is meant to protect humans; GFCIs can react to currents as small as 5 mA.

FIGURE 31 A ground fault circuit interrupter (GFCI).



Electromagnetic induction is the physical basis of a GFCI. As shown in Fig. 31, the two conductors of a power line leading to an electrical circuit or device (red) pass through a small iron ring. Around the ring are many loops of thin wire that serve as a sensing coil. Under normal conditions (no ground fault), the current moving in the hot wire is exactly balanced by the returning current in the neutral wire. If something goes wrong and the hot wire touches the ungrounded metal case of the device or appliance, some of the entering current can pass through a person who touches the case and then to ground (a *ground fault*). Then the return current in the neutral wire will be less than the entering current in the hot wire, so there is a *net current* passing through the GFCI's iron ring. Because the current is ac, it is changing and the current difference produces a changing magnetic field in the iron, thus inducing an emf in the sensing coil wrapped around the iron. For example, if a device draws 8.0 A, and there is a ground fault through a person of 100 mA ($=0.1$ A), then 7.9 A will appear in the neutral wire. The emf induced in the sensing coil by this 100-mA difference is amplified by a simple transistor circuit and sent to its own solenoid circuit breaker that opens the circuit at the switch S, thus protecting your life.

If the case of the faulty device is grounded, the current difference is even higher when there is a fault, and the GFCI trips immediately.

GFCIs can sense currents as low as 5 mA and react in 1 msec, saving lives. They can be small enough to fit as a wall outlet (Fig. 32a), or as a plug-in unit into which you plug a hair dryer or toaster (Fig. 32b). It is especially important to have GFCIs installed in kitchens, bathrooms, outdoors, and near swimming pools, where people are most in danger of touching ground. GFCIs always have a "test" button (to be sure it works) and a "reset" button (after it goes off).

FIGURE 32 (a) A GFCI wall outlet. GFCIs can be recognized because they have "test" and "reset" buttons. (b) Add-on GFCI that plugs into outlet.

Richard Megna/Fundamental Photographs, NYC (a)

Christian Botting (b)

Summary

The **magnetic flux** passing through a loop is equal to the product of the area of the loop times the perpendicular component of the (uniform) magnetic field: $\Phi_B = B_\perp A = BA\cos\theta$. If $\vec{\mathbf{B}}$ is not uniform, then

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}. \quad \textbf{(1b)}$$

If the magnetic flux through a coil of wire changes in time, an emf is induced in the coil. The magnitude of the induced emf equals the time rate of change of the magnetic flux through the loop times the number N of loops in the coil:

$$\mathscr{E} = -N\frac{d\Phi_B}{dt}. \quad \textbf{(2b)}$$

This is **Faraday's law of induction**.

The induced emf can produce a current whose magnetic field opposes the original change in flux (**Lenz's law**).

We can also see from Faraday's law that a straight wire of length ℓ moving with speed v perpendicular to a magnetic field of strength B has an emf induced between its ends equal to:

$$\mathscr{E} = B\ell v. \quad \textbf{(3)}$$

Faraday's law also tells us that *a changing magnetic field produces an electric field*. The mathematical relation is

$$\oint \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{d\Phi_B}{dt} \quad \textbf{(8)}$$

and is the general form of Faraday's law. The integral on the left is taken around the loop through which the magnetic flux Φ_B is changing.

An electric **generator** changes mechanical energy into electrical energy. Its operation is based on Faraday's law: a coil of wire is made to rotate uniformly by mechanical means in a magnetic field, and the changing flux through the coil induces a sinusoidal current, which is the output of the generator.

[*A motor, which operates in the reverse of a generator, acts like a generator in that a **back emf** is induced in its rotating coil; since this counter emf opposes the input voltage, it can act to limit the current in a motor coil. Similarly, a generator acts somewhat like a motor in that a **counter torque** acts on its rotating coil.]

A **transformer**, which is a device to change the magnitude of an ac voltage, consists of a primary coil and a secondary coil. The changing flux due to an ac voltage in the primary coil induces an ac voltage in the secondary coil. In a 100% efficient transformer, the ratio of output to input voltages (V_S/V_P) equals the ratio of the number of turns N_S in the secondary to the number N_P in the primary:

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}. \quad \textbf{(5)}$$

The ratio of secondary to primary current is in the inverse ratio of turns:

$$\frac{I_S}{I_P} = \frac{N_P}{N_S}. \quad \textbf{(6)}$$

[*Microphones, ground fault circuit interrupters, seismographs, and read/write heads for computer drives and tape recorders are applications of electromagnetic induction.]

Answers to Exercises

A: (*e*).

B: (*a*) Counterclockwise; (*b*) clockwise; (*c*) zero; (*d*) counterclockwise.

C: Electrons flow clockwise (conventional current counterclockwise).

D: (*a*) increases; (*b*) yes; increases (counter torque).

E: (*a*) 4.2×10^{-3} V/m; (*b*) 5.6×10^{-3} V/m.