For office use only	Team Control Number	For office use only
T1 ________________	57136	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	C	F4 ________________

2017
MCM/ICM
Summary Sheet

(Your team's summary should be included as the first page of your electronic submission.)

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

In this paper, we explored the properties of self-driving "autonomous" vehicles and how their presence affects traffic flow on several major roads in the Seattle area. We expanded on the multiple-lane Nagel-Schreckenberg Model in a Matlab program, making adjustments to include a greater number of lanes and to account for behavioral differences in autonomous versus human-driven vehicles, specifically differences in velocity variability. We ran our model on several stretches of road data provided to us by COMAP and the state of Washington, running multiple trials with varying percentages of autonomous vehicles compared to total traffic. Our results showed that with increasing rates of congestion, higher percentages of autonomous vehicles greatly improved traffic flow at an exponential rate up until the traffic density was estimated to be equal to 1, at which point autonomous vehicles were shown to have no impact on traffic flow. Although our model makes many major simplifying assumptions, we believe these results to be indicative of a true trend in traffic, that greater numbers of autonomous vehicles compared to human-driven vehicles improve traffic flow.

Governor Jay Inslee
Governor's Office-State of Washington
416 14th Ave SW
Olympia, WA 98504

Dear Governor Inslee,

After careful analysis we have determined the impact of autonomous vehicles on roads in the State of Washington. Autonomous vehicles will be extremely beneficial to the state, especially in generally crowded sections of roads. Their impact is limited, however. Autonomous vehicles only have the ability to relieve traffic if there is sufficient space for the vehicles to navigate-it will not relieve gridlocked traffic. Many other sections of roads with less congestion will see little to no impact on traffic flow regardless of the proportion of autonomous vehicles that exist. However, autonomous vehicles navigate traffic more efficiently, and as a result will help relieve congestion in the most crowded parts of roads in the State of Washington.

We recommend that you only consider implementing lanes dedicated to autonomous vehicles in areas with already heavy traffic, but do not consider this until autonomous vehicles become a significant proportion of the vehicles on the road in Washington, and run additional tests based off information that the Department of Transportation, automakers, and any other sources accumulate regarding the behavior of autonomous vehicles over the next few years. We also recommend examining the cost and benefits of expanding existing highways, particularly I-405 and SR520, which may do to more to relieve traffic than a lane dedicated to autonomous vehicles ever could. There is a dearth of information as to how autonomous vehicles will actually behave in traffic, and as a result, further testing will give you and your constituents a far clear picture as to the effects of the changes autonomous vehicles will make to the State's road system.

Specifically, we analyzed the sections of I-5, I-90, I-405, and SR520 in Thurston, Pierce, King, and Snohomish counties at peak hours where the average traffic per lane appears to be highest. Autonomous vehicles will drastically improve traffic flow on all four of these roads, particularly in their busiest sections. Traffic will especially improve on and near intersections between major roadways, in particular the intersection of I-5/I-90 and I-405/SR520. In conclusion, automated vehicles will significantly improve traffic flow on some, but not of Washington's busiest roadways.

Sincerely,
MCM Team # 57136

Abstract

In this paper, we explored the properties of self-driving "autonomous" vehicles and how their presence affects traffic flow on several major roads in the Seattle area. We expanded on the multiple-lane Nagel-Schreckenberg Model in a Matlab program, making adjustments to include a greater number of lanes and to account for behavioral differences in autonomous versus human-driven vehicles, specifically differences in velocity variability. We ran our model on several stretches of road data provided to us by COMAP and the state of Washington, running multiple trials with varying percentages of autonomous vehicles compared to total traffic. Our results showed that with increasing rates of congestion, higher percentages of autonomous vehicles greatly improved traffic flow at an exponential rate up until the traffic density was estimated to be equal to 1, at which point autonomous vehicles were shown to have no impact on traffic flow. Although our model makes many major simplifying assumptions, we believe these results to be indicative of a true trend in traffic, that greater numbers of autonomous vehicles compared to human-driven vehicles improve traffic flow.

1 Introduction

Increasing traffic capacity on major roads in the United States is an important area of research due to the large number of cars travelling on the roads each day. In many areas, drivers experience long delays during peak travel hours because the volume of cars exceeds the designed capacity of the road. Undertaking highway expansion projects can be very expensive and time consuming, so it would be advantageous to consider alternative methods of improving traffic flow.

The advent of self-driving, cooperating cars, or autonomous vehicles, can potentially provide a solution to the issue of traffic flow without having to increase the number of lanes on the road. Although self-driving vehicles are continuously learning and improving, the standardization of their actions may help to improve traffic flow and increase highway capacity. Their ability to maintain a constant speed and a set distance behind other vehicles, as well as their quicker reaction time for breaking and responding to other road events would potentially allow self-driving cars to keep the flow of traffic moving more consistently.[1] Since there is little research on how self-driving cars interact with human-driven vehicles or how an entire fleet of self-driving cars would behave on a major highway, we decided to delve deeper into these relationships.

[1] Gates, G., Granville, K., Singhvi, A., & Russell, K. (2016, December 14). When Cars Drive Themselves. *The New York Times*. Retrieved from https://www.nytimes.com/interactive/2016/12/14/technology/how-self-driving-cars-work.html?_r=0

We were provided with information regarding several major roads in the Seattle area and asked to analyze the effect of different numbers of autonomous cars on the traffic flow of these roads. Building on previous traffic flow models, we studied the relationship between self-driving and human-driven cars, the cooperation between self-driving cars themselves, and different percentages of self-driving cars with respect to overall traffic on traffic flow.

2 Previous Work

The Nagel-Schreckenberg Model is a single-lane model for traffic simulations that divides the road up into a grid of cells[2]. Each cell is either empty or occupied by a car, and all cars on the road have a current velocity, defined by the number of cells that the car will advance in one time step, and an overall maximum velocity. In this model, cars decrease their velocity either to avoid a future collision or occasionally due to randomness, but otherwise increase their speed where possible until travelling at the maximum velocity. Random deceleration is an underlying assumption of the Nagel-Schreckenberg model, and is based off the idea that vehicles will not always accelerate.

In the paper “Two Lane Traffic Simulations using Cellular Automata” by the authors of the Nagel-Schreckenberg Model[3], they generalized their single-lane model to two lanes and included a set of rules for lane changing. This two-lane variation is especially valuable because it provides the groundwork for generalization to three or more lanes. Rather than rules only for braking and accelerating, the two-lane model also includes rules governing the conditions for a car to switch to an alternate lane. These conditions are dependant on the vehicle’s lead gap in its current lane as well as its lead and lag gaps in its target lane. It also includes an element of randomness for whether or not the car will change lanes even if the conditions are ideal to do so.

For our model, we started with Alexander Farley’s Matlab implementation of the single-lane Nagel-Schreckenberg Model[4] and heavily modified it to fit the multiple-lane model and account for differing characteristics of autonomous vehicles. We were unable to find comparable traffic models which specifically addressed autonomous vehicles, so we instead modified the properties of vehicle behavior in our existing models to account for the differences between human-driven vehicles and autonomous vehicles.

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[2] Rickert, M., Nagel, K., Schreckenberg, M., & Latour, A. (2008, February 1). Two Lane Traffic Simulations using Cellular Automata. *Los Alamos National Laboratory*, *95*(4367). Retrieved from https://arxiv.org/pdf/cond-mat/9512119.pdf

[3] Rickert, M., Nagel, K., Schreckenberg, M., & Latour, A. (2008, February 1). Two Lane Traffic Simulations using Cellular Automata. *Los Alamos National Laboratory*, *95*(4367). Retrieved from https://arxiv.org/pdf/cond-mat/9512119.pdf

[4] Farley, A. (2015, February 10) Nagel-Schreckenberg Model Implementation source code [Source code]. https://www.mathworks.com/matlabcentral/fileexchange/34956-nagel-schreckenberg-model-implementation/content/NaSchrm

3 Data

3.1 Data Provided by COMAP and by the State of Washington

The data we used was provided to us in the form of an Excel spreadsheet by COMAP and the State of Washington[5]. It consisted of the road, a the location of a start and end mile marker, the average daily traffic in 2015, and the number of lanes in the increasing and decreasing direction. We were also told that 8% of traffic occurs at peak hours, when the road is busiest, but received no information as to the duration of peak hours or how traffic was distributed outside of peak hours.

3.2 Selecting Which Data to Use

Since most congestion would occur at peak hours and is the only timeframe where we have an estimate as to traffic volumes, our modeling focused on that time period. Peak hours are assumed to be approximately one hour, since that gives the other 23 hours of the day an average traffic volume of 4%, although that would likely not be distributed evenly throughout each hour of the day. Each section of road we assumed to be a discrete independent section from all others, in order to best model the flow of those particular sections. Focus was emphasized on the busiest sections of roads, determined by the average daily traffic divided by the number of lanes. We modeled the three busiest sections of each road.

4 Model

4.1 Simplifying Assumptions

- A collision is defined as occurring when two vehicles occupy the same location. When this happens, one vehicle overwrites the others and traffic flow is not affected.
- Car location is grid-based, rather than continuous
- Velocities are on a scale of 1 - 6 car lengths per time step and increase in increments of one, rather than a continuous scale of miles per hour
- All cars have the same maximum desired velocity of 6, whereas in reality, cars have their own individual desired maximums.
- Segments of road are treated as independent of one another.
- New cars are not generated at the beginning of the selected stretch of road, but wrap back when they reach the end.

[5] 2017 MCM/ICM problems. (2017). Retrieved January 22, 2017, from COMAP Math, http://www.comap-math.com/mcm/index.html

- Only observing peak traffic, assumed to last one hour long and be uniformly distributed
- Once a car is in a lane, it will remain in that lane until it finds itself in a reasonably close physical proximity to a car ahead of it in that lane. Cars do not change lanes according to the "keep right except to pass" rule observed by a large percentage of individuals.

4.2 Variables

num_lanes:	Number of lanes in the selected segment of road
start_mile:	Mile value at the start of the selected road
end_mile:	Mile value at the end of the selected road
daily_traffic:	Total average daily traffic along the road segment
vmax:	Maximum velocity of a vehicle on the highway
av_rand:	Probability an autonomous vehicle will randomly decelerate
hv_rand:	Probability of a human-driven vehicle will randomly decelerate
av_percent:	Percentage of autonomous vehicles on the road
road_length:	Length of road in units of car lengths
total_time_steps:	Total time in simulation. 1 time step given by the time it takes a car traveling at velocity 1 to move 1 car length.
road:	Array that is num_lanes wide and road_length tall, where each cell represents a location a car can be.
road_next:	Array identical to road, but containing the future positions of cars at the next time step.
velocities:	Parallel array to road, where each cell contains the velocity of the car in the respective cell in road
velocities_next:	Future velocity data for the upcoming time step
history:	Collection of past road arrays, documenting car location data at every time step through the simulation

velocity_history: Velocity data from every step in the simulation

density: Density of vehicles on the road

4.3 The Model

The provided segment of highway is represented by a *road* array, each element of which represents a position a vehicle can occupy. The width of the *road* array is given by the number of lanes of that highway segment, designated *num_lanes*. Each cell in *road* is representative of one car length, which we determined to be an average of 14 feet, so the height of the *road* array is given by *road_length* = round(| *end_mile* - *start_mile* | * 5280 / 14). Each cell is populated with a number 0 - 2. A zero represents an empty space which no vehicle occupies, a 1 represents a human-driven vehicle (HV), and a 2 represents an autonomous vehicle (AV).

A simplified example of a road segment 70 feet long with 2 lanes, populated with both HV and AV, is represented abstractly as follows:

0	1
0	0
2	0
0	1
0	0

The *velocities* array is a parallel array to *road*, containing the velocity of each vehicle in its equivalently indexed cell. The *road_next* and *velocities_next* arrays are populated during the computation for each time step, containing the future locations and velocities of vehicles on the road for the upcoming time time step. After a step is complete, the *road_next* and *velocities_next* arrays are copied over into their counterparts. Finally, the *history* and *velocity_history* parallel arrays are the records of the *road* and *velocities* arrays at each time step, arranged horizontally with a width of *simulation_steps* * *num_lanes* and a height of *road_length*.

When we begin our simulation, our first step is to populate our road with vehicles. Based on the provided traffic data, we calculated the density of vehicles on our road as follows:

The number of cars that pass through the given stretch of highway per minute during rush hour is given by:

$$cars\_per\_min = daily\_traffic * 0.08 \div 60$$

Since cars are travelling at an average rate of 60 mph, this equates to the cars traveling at a rate of 1 mile per minute. Therefore, the number of cars on our stretch of road at any given moment is given by:

$$cars\_on\_road = \frac{cars\_per\_min}{end\_mile - start\_mile}$$

To get the final density, we divide the number of cars on the road at any given point by the number of available locations a car can occupy. Therefore we have:

$$density = \frac{carsonroad}{(road\_length * num\_lanes)}$$

To populate our cars randomly on the road according to density, we step through each cell in our *road* array. At each cell we generate a random number and check if it is less than our density. If so, we generate another random number and see if this value is less than our pre-determined percentage of AVs, given by *av_percent*. If so, we place a 2, otherwise a 1. In this way, we continue until our initial road state is fully populated with 0s, 1s, and 2s.

At each step of the simulation, in accordance with the expanded Nagel - Schreckenberg model, we follow a series of steps: (1) Lane changes (2) Velocity update (3) Position update. For each step, we loop through each car in the road and follow a series of sub-steps until every car in the system has been addressed.

Lane changes are the first step in the model. A car shifts its position horizontally one lane to the left or to the right if the conditions are considered 'favorable' to do so. In our model, this is achieved through the following steps for each vehicle:

1. Examine lead gap in the current lane. In the next time step, the vehicle will want to be traveling at either the maximum velocity or one plus its current velocity, whichever is smaller. We call that value *desired_velocity*, which is between 1 and 6. We then seek ahead that many cells in front of our vehicle to see if another vehicle occupies some space in that range. If this space is clear, then we skip to the next vehicle as there is no incentive to change lanes.
2. Examine lead gap in target lane. We examine the same distance ahead in our desired target lane, to see if the the lead gap is better. If the lead gap is better in our current lane, there is no incentive to change lanes, so we skip to the next vehicle.

3. Examine lag gap in the target lane. We seek backward in the target lane up to the maximum velocity to ensure it is clear of cars. If not, we cannot change lanes.
4. If all conditions are met, there is a random chance that the car will decide not to change lanes. This is to prevent a ping-pong effect where a car repeatedly switches back and forth between two or more lanes. For our model, we have set this chance that a lane change will not occur to be 15%.

After lane changes occur, the velocities of the vehicles are updated. Similar to the lane-change model, we look ahead to the positions in front of our current car to see if its path is clear. If it is, the car will increase its velocity by one unless it has reached the maximum velocity. If our car has a current velocity v and there is a car v or fewer cells in front of it, then our car will decrease its velocity be one in order to prevent a collision. Finally, there is a chance our car will randomly decelerate by one unit, in accordance with the velocity variability that occurs in reality. For HVs, we set this likelihood to be 60%, as suggested by the writer of the original single-lane Matlab file, but since AVs have a much lower velocity variability, we set their likelihood of random deceleration to be 5%. We determined whether a given vehicle would randomly decelerate by generating a random number and checking if it was lower than our randomness value, 0.6 for HVs and 0.05 for AVs.

Our final repeating step of the simulation was updating the position of the vehicles on the road. Each vehicle's velocity value was added to its current vertical location on the road, and its data was copied over to that location in the *road_next* and *velocities_next* arrays. If two cars are found to occupy the same future location, a collision message is printed.

At the end of every time step, the road and velocity data are copied over into the *history* arrays. After the simulation runs through its entirety, we calculate the normalized flow rate for the entirety of the simulation using the following Matlab code:

$$sum(velocity\_history(:)) \div nnz(history) * nnz(history) \div numel(history)$$

4.4 Dedicated lanes

The question was also postulated as to how a dedicated lane for autonomous vehicles might affect traffic flow. We chose to first implement a model without dedicated lanes, with two options for implementing dedicated lanes. If a clear correlation emerges between autonomous vehicles and traffic flow, we simply extrapolate from our original tests, otherwise, modify our model to include dedicated lanes.

5 Results

5.1 Sections of Road Examined

As has been discussed already, we found the 3 sections of each road that had the highest volume of cars in each lane. We denoted these sections road_a, road_b, and road_c, i.e. the first section we examined of I-405 was denoted 405_a

Road	Start Mile	End Mile	Daily Traffic	# Lanes
I-5/5_a	162.79	163.36	186000	3
I-5/5_b	163.36	163.48	238000	3
I-5/5_c	163.48	164.22	242000	3
I-90/90_a	6.85	7.64	124000	3
I-90/90_b	7.64	8.7	151000	3
I-90/90_c	8.7	9.61	162000	3
I-405/405_a	7.69	8.98	152000	2
I-405/405_b	8.98	9.59	151000	2
I-405/405_c	9.59	9.96	161000	2
SR520/520_a	6.52	6.93	77000	2
SR520/520_b	6.93	9.6	109000	2
SR520/520_c	9.6	9.72	95000	2

5.2 Traffic Flow Results

We ran our traffic flow model for two-hundred discrete percentages of autonomous vehicles on the road, from zero autonomous cars to 100 percent autonomous cars. Provided below are the traffic flow rates for our selected roads for a few values of autonomous car percentages. The flow rate represents the number of cars that can pass through the section of road in one second. While the flow rates for each road fluctuated somewhat between each of these percentage points, most of the road sections we analyzed demonstrated an overall increase in flow rate as the percentage of autonomous vehicles increased.

The roads with an asterisk were ones where the density calculated was higher than one, suggesting that there were supposedly more cars on the road than available spots during the peak hour and that there was essentially bumper to bumper traffic throughout the entire simulation. Our model was not able to deal with this, so the results display that zero cars passed through this section during the peak hour.

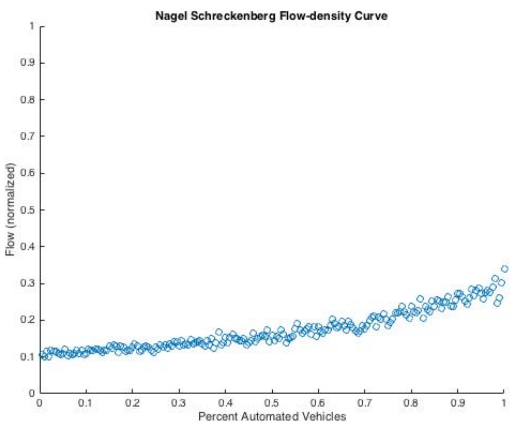
Traffic Flow Rates as Percentage of Autonomous Vehicles (AV) Increases

Road	10% AV	25% AV	50% AV	75% AV	90% AV	100% AV
5_a	0.109	0.128	0.159	0.185	0.271	0.339
5_b*	0	0	0	0	0	0
5_c	0.160	0.175	0.202	0.280	0.350	0.438
90_a	0.228	0.266	0.290	0.427	0.506	0.680
90_b	0.262	0.276	0.325	0.372	0.461	0.730
90_c	0.236	0.257	0.274	0.401	0.498	0.690
405_a	0.267	0.286	0.265	0.381	0.369	0.506
405_b	0.216	0.241	0.296	0.337	0.520	0.587
405_c*	0	0	0	0	0	0
SR520_a	0.198	0.212	0.264	0.374	0.466	0.543
SR520_b	0.091	0.067	0.057	0.075	0.080	0.068
SR520_c*	0	0	0	0	0	0

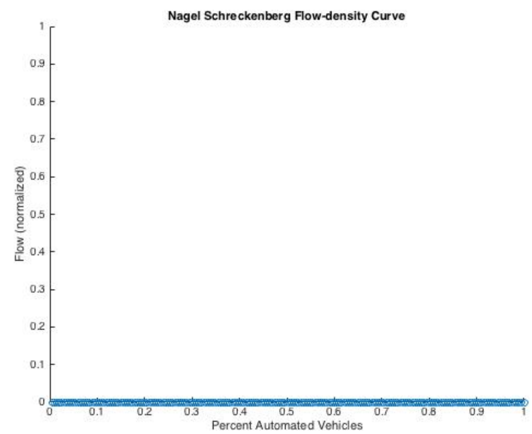
* Refers to roads where the density was greater than 1, implying the road is over capacity

5.3 Figures

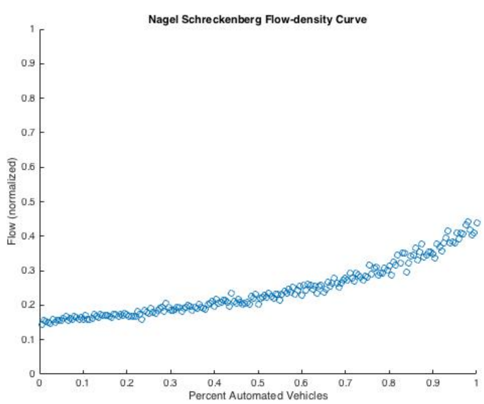
5_a



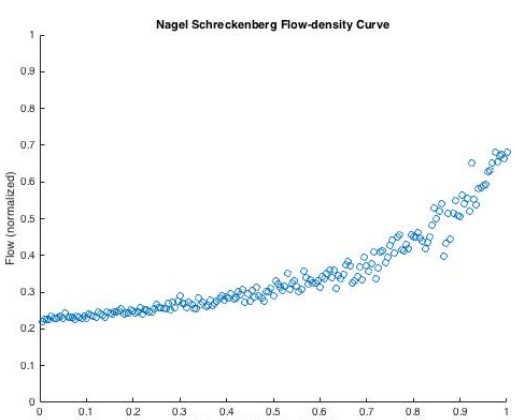
5_b



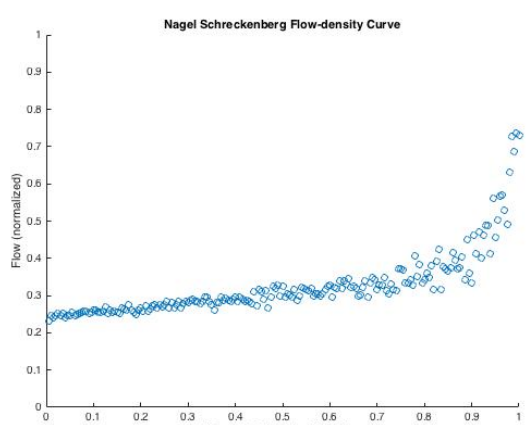
5_c



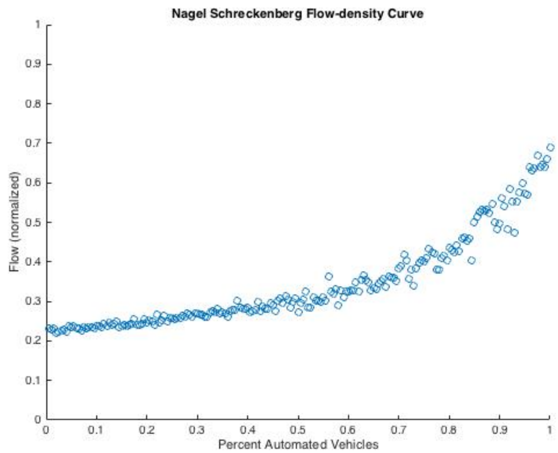
90_a



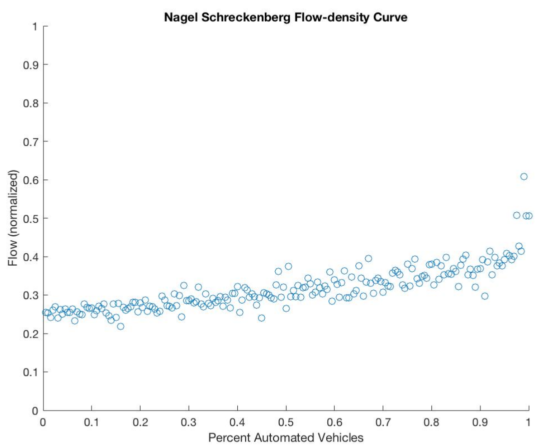
90_b



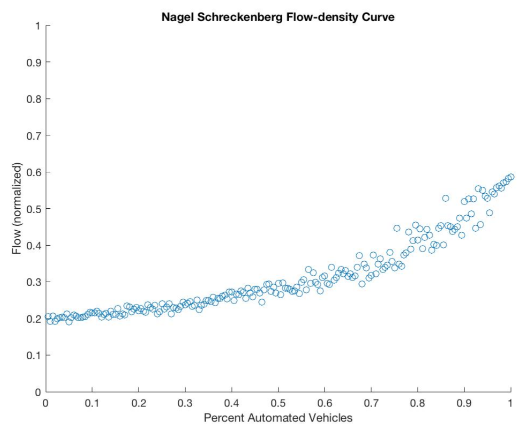
90_c



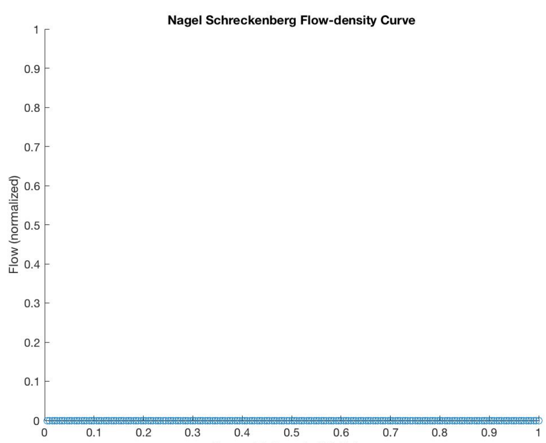
405_a



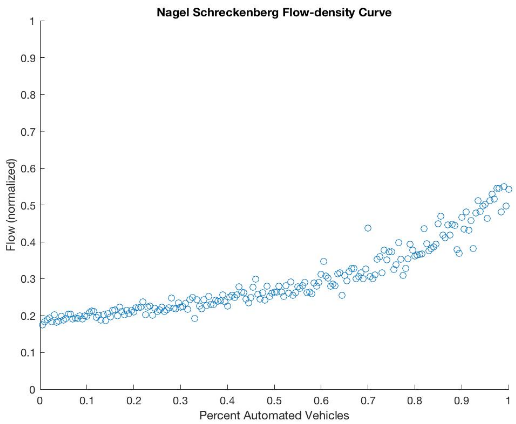
405_b



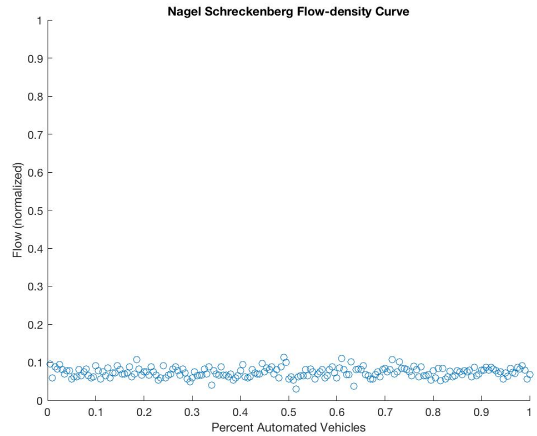
405_c



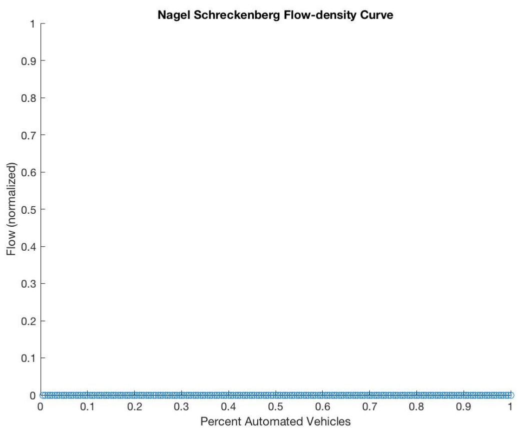
520_a



520_b



520_c



5.3 Analysis of Results

In general, There is a clear correlation between higher percentages of autonomous vehicles and higher rates of flow. However, some of our testing indicated that flow was constantly zero regardless of the percentage of automated vehicles. This only happened when our density was greater than one, implying complete and total gridlock of traffic. In general, flow improves with autonomous vehicles when there is a high density, and appears to increase exponentially as the percentage of autonomous cars increases. There does not appear to be any equilibria, although the flow improves markedly when around 70% of the vehicles on a road are autonomous vehicles.

5.4 Extrapolation to Dedicated Lanes

After examining our data without dedicated lanes to autonomous vehicles, it is apparent that autonomous vehicles flow through traffic faster than non-autonomous vehicles. This division is especially pronounced in sections of road that have a high density of vehicles. This seems to indicate that a dedicated lane for autonomous vehicles would likely improve the flow of traffic in dense areas, but only if the proportion of vehicles is greater than or equal to the proportion of the road dedicated to autonomous vehicles-it only makes sense to dedicated one lane of a 2 lane road to autonomous vehicles if at least half the vehicles can use that lane. Conceptually, this would be similar to the idea of an HOV lane, where a dedicated lane reduces traffic in congested areas.

6 Discussion

6.1 Limitations of Model

Our model has several factors that were not taken into account due to our simplifying assumptions and a number of other conditions. Firstly, we assume that every vehicle on the road is of the same size, and that every vehicle, particularly non-autonomous vehicles, obeys the rules of the road properly. When a collision occurs, our model has the following car overwrite the preceding car, so the remaining car continues at its current velocity and does not affect the flow of traffic, a humorously unrealistic simplification. Furthermore, we assume that each section of road operates as a discrete unit.

We assumed that no vehicles enter or exit the road at any point and that traffic in one section of road has no affect on any sections surrounding it. This limits the real life application of the model by suggesting that a traffic backup in one section of the road would not affect the next section of road ahead. Although we implement the ability of vehicles to change lanes, all vehicles are placed on the road with equal probability of being in any one lane, which is not realistic. In addition, we assume that peak hours last for exactly one hour and that traffic is evenly distributed throughout the peak hour.

Another limitation of our model is that we assumed the total daily traffic during peak hours was only moving in one direction, so we only took into account the two or three lanes in the Northbound direction. We did not divide up the total daily traffic during peak hours between the two directions of the road, so this simplification is a significant limitation to our model, but it allows us to produce more heavily weighted results. We also only observed peak hour traffic during our testing, while more thorough examination would compare traffic at non-peak hours as well.

We assumed that autonomous vehicles act almost identically to a non-autonomous vehicles, with the exception of a much lower chance of decelerating randomly for autonomous vehicles, 5%, than for human vehicles, 60%. Information we found regarding autonomous vehicles indicated that they had a much lower velocity variability than a human-operated car, so this factor was an implementation of that characteristic, but should be modeled at different probabilities of randomly decelerating based on empirical data. Our research also showed that autonomous vehicles could follow one another at much closer distances and have reaction times / braking speeds much faster than human-operated vehicles. We attempted to implement the faster reaction times of autonomous vehicles, but to disastrous results. We altered the model such that an autonomous vehicle would only seek half as far in front of itself and decelerate at twice the rate of a human-driven vehicle, but this caused our simulations to balloon with hundreds to thousands of collisions. We believe this was due to autonomous vehicles in our simulation braking faster than human-operated vehicles behind them could account for, causing these vehicles to be rear-ended. Although this is actually a common problem with autonomous vehicles in the real world, the degree to which these collisions appeared in our simulations was severely overblown to the point that the results regarding traffic flow were unusable. In the interest of time, we abandoned the faster reaction time and closer flanking distance characteristics, though a better model would scale down the degrees of these effects to produce a more realistic representation of the driving habits of autonomous vehicles.

We closely examined and implemented only one traffic model, the multi-lane Nagel-Schreckenberg Model. A more thorough analysis would compare and contrast different traffic models, taking into account the precise conditions observed on the roads in question. We also did not account for the effects of more general driving conditions, such as the weather or the specific time of day during which the traveling occurred. Weather conditions and the direction and intensity or lack of sunlight could have an impact on traffic flow data as well -- the sensors of autonomous cars may or may not be impaired under poor weather conditions or low light, and their performance may or may not be more reliable than impaired human sight and senses. This is a good area for further research and analysis, as autonomous versus human performance under these conditions could affect rates of vehicle collisions and traffic-related deaths.

Furthermore, our model begins with a randomized placement of cars, based off our derived density equations, but in general only one test was run for each percentage of autonomous vehicles. Since our varying percentages of autonomous vehicles were close in value, differing by 0.5% with each simulation, we can say with reasonable confidence that our trends are reliable, but further testing would enforce our results and normalize our data.

Finally, our individual determination of which sections of road from the provided data should be tested did not account for the length of road in question. We analyzed and chose road sections based on total traffic volumes but failed to account for distance -- a longer stretch of road can obviously handle more vehicles than a shorter stretch. More holistic testing would compare relative road segment capacities and choose representative segments of highway from different degrees of capacity.

6.2 Sensitivity Analysis

The model is highly sensitive to the density of vehicles. Our data shows that the advantages of autonomous vehicles become increasingly pronounced with greater density up until the density reaches a value of 1, a value implying that there is more traffic than the road can handle due to its capacity. However, at this point, flow drops to zero, which implies that no vehicles move at all. A more realistic model with a continuous representation of car location on the road could produce more realistic flow values that may be close to zero, but are not exactly zero. As with all random sampling and placement, our model is sensitive to the random values that happen to appear on any particular iteration, although this effect can be normalized across multiple trials.

6.3 Future Improvements

6.3.1 Additional Trials

As mentioned above, additional trials at steady percentages of autonomous vehicles will better display the precise effects of autonomous vehicles on traffic flow. Furthermore, a number of different traffic models exist[6], but we only examined the Nagel-Schreckenberg model, so retaining our determined characteristics of autonomous versus human-operated vehicles under a different traffic model would be valuable to explore.

6.3.2 Additional Sections of Road

The primary reason we only focused on particular sections of road was due to time constraints. Our trials only utilized the particular sections of road that have the highest overall number of cars per lane, which is not necessarily the sections of road with the highest density, as the lengths of each section differs. As a result, further testing of every section of road inevitably will provide more accurate data, whereas we simply focused on the sections of road that might see the greatest effect on traffic flow due to autonomous vehicles. Given more time, an area for further investigation is running the model on each stretch of road that was provided to observe how the traffic in each section affects other sections.

6.3.3 Improvements to Our Model

Specific areas for improvement in this model includes accounting for varying weather and light conditions, accounting for more specific behavior of autonomous cars (especially in the area of flanking distance and reaction times), implementing a realistic reaction to

[6] Seibold, B. (2009). Traffic Modeling - Phantom Traffic Jams and Traveling Jamitons. In *MIT Mathematics*. Retrieved January 20, 2017, from http://math.mitedu/projects/traffic/

collisions, accounting for cars exiting and entering the highway via exit- and on-ramps, implementing a more continuous representation of car location, taking into account increases or decreases in the number of lanes along a road segment, observing behavior connected between multiple road segments, and dynamically generating new cars at the entrance of the specified road segment instead of wrapping them around from the end of the road.

7 Conclusion

It is clear from our analysis that autonomous vehicles improve traffic congestion under certain conditions. The improvements of autonomous vehicles are most pronounced on roads that have a significant amount of moving traffic, that is, roads that are congested but not at an absolute standstill with respect to traffic. Autonomous vehicles also improve traffic congestion on less crowded roads, but to a lesser degree, since vehicles on less congested roads are spread farther apart and therefore less likely to interact with each other. In conclusion, autonomous vehicles are reasonably effective in clearing congestion on relatively crowded roads, and are increasingly more effective as they reach higher penetrations.

5_a	**Density**	0.6746						
percentage AV	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04
flow (normalized)	0.1138157563	0.1149212187	0.1089273481	0.1044297819	0.1152449973	0.1059080584	0.1110596719	0.11595457
5_b	**Density**	19.5885						
percentage AV	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04
flow (normalized)	0	0	0	0	0	0	0	0
5_c	**Density**	0.521						
percentage AV	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04
flow (normalized)	0.1555110203	0.1570162254	0.149917567	0.15488924	0.1603508006	0.1575386579	0.1594990643	0.164023244
90_a	**Density**	0.2341						
percentage AV	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04
flow (normalized)	0.2257634791	0.2173329279	0.2262585769	0.2225838175	0.2338269297	0.2303188295	0.2218965362	0.2286158281
90_b	**Density**	0.1583						
percentage AV	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04
flow (normalized)	0.24471125	0.2393479167	0.2483633333	0.2492410417	0.2441654167	0.2516933333	0.252535625	0.244204375
90_c	**Density**	0.2307						
percentage AV	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04
flow (normalized)	0.2273256325	0.2247728412	0.2293978983	0.2265255123	0.2333361666	0.2262764664	0.2318535078	0.2296256945
405_a	**Density**	0.0806						
percentage AV	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04
flow (normalized)	0.2549926424	0.2539606778	0.2410028292	0.2602251981	0.26953628	0.2405095101	0.2623557463	0.2501108914
405_b	**Density**	0.3588						
percentage AV	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04
flow (normalized)	0.2048034026	0.1925321361	0.2060170132	0.1922901701	0.1998752363	0.2023232514	0.2039924386	0.2021389414
405_c	**Density**	1.036						
percentage AV	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04

0.045	0.05	0.055	0.06	0.065	0.07	0.075	0.08	0.085
0.1141719849	0.1086648639	0.1110452497	0.1146940689	0.1082653687	0.1236920858	0.1144250946	0.1144784568	0.1119798089
0.045	0.05	0.055	0.06	0.065	0.07	0.075	0.08	0.085
0	0	0	0	0	0	0	0	0
0.045	0.05	0.055	0.06	0.065	0.07	0.075	0.08	0.085
0.1553140376	0.1548986609	0.1607220702	0.1540255135	0.151637312	0.1681752975	0.1626841896	0.1601139931	0.16005447
0.045	0.05	0.055	0.06	0.065	0.07	0.075	0.08	0.085
0.2352270168	0.2281327418	0.2322691921	0.2335949582	0.2289667883	0.2320169512	0.2318033722	0.2343531823	0.2368162846
0.045	0.05	0.055	0.06	0.065	0.07	0.075	0.08	0.085
0.2404572917	0.2417566667	0.2444670833	0.2486629167	0.262056875	0.2477997917	0.2535810417	0.2624297917	0.254733125
0.045	0.05	0.055	0.06	0.065	0.07	0.075	0.08	0.085
0.2294267978	0.2335636795	0.2322895506	0.2348752079	0.2327012271	0.2326230284	0.2312469011	0.2343943992	0.2334477981
0.045	0.05	0.055	0.06	0.065	0.07	0.075	0.08	0.085
0.2635626916	0.255192289	0.2553320628	0.2636761128	0.232845355	0.2569028836	0.2501298652	0.2496028149	0.2760173125
0.045	0.05	0.055	0.06	0.065	0.07	0.075	0.08	0.085
0.2123459357	0.1901446125	0.2027334594	0.2091606805	0.2066729679	0.2018988658	0.2023383743	0.2037741021	0.2055170132
0.045	0.05	0.055	0.06	0.065	0.07	0.075	0.08	0.085

0.09	0.095	0.1	0.105	0.11	0.115	0.12	0.125	0.13
0.111040923	0.1166994772	0.1193106184	0.120607175	0.1111231296	0.1138496485	0.1223508203	0.1200742744	0.1160209122
0.09	0.095	0.1	0.105	0.11	0.115	0.12	0.125	0.13
0	0	0	0	0	0	0	0	0
0.09	0.095	0.1	0.105	0.11	0.115	0.12	0.125	0.13
0.1630353327	0.168011288	0.1636832346	0.1631650844	0.1478038566	0.1700402102	0.1579351927	0.1609571648	0.1564346981
0.09	0.095	0.1	0.105	0.11	0.115	0.12	0.125	0.13
0.2360344129	0.2351504437	0.2289795505	0.2349931685	0.2352720598	0.2389558278	0.2313435581	0.2484107322	0.2482519556
0.09	0.095	0.1	0.105	0.11	0.115	0.12	0.125	0.13
0.270859375	0.2546752083	0.2609179167	0.2583833333	0.2616722917	0.252066875	0.254745625	0.2558783333	0.2679608333
0.09	0.095	0.1	0.105	0.11	0.115	0.12	0.125	0.13
0.2364777148	0.2372761349	0.2455116491	0.2381918532	0.2343102505	0.2380686052	0.2350823211	0.2333695994	0.2365301306
0.09	0.095	0.1	0.105	0.11	0.115	0.12	0.125	0.13
0.2675185205	0.2645489082	0.2666476226	0.24902833	0.2592039853	0.2710364761	0.2644226269	0.2763143581	0.2528854952
0.09	0.095	0.1	0.105	0.11	0.115	0.12	0.125	0.13
0.211620983	0.216173913	0.21568431	0.2149253308	0.2190529301	0.2142504726	0.2031266541	0.2102533081	0.2141143667
0.09	0.095	0.1	0.105	0.11	0.115	0.12	0.125	0.13

0.135	0.14	0.145	0.15	0.155	0.16	0.165	0.17	0.175
0.1182404904	0.1229529475	0.1259224806	0.1255446187	0.1221085271	0.116443483	0.1225195601	0.1176845142	0.1224748513
0.135	0.14	0.145	0.15	0.155	0.16	0.165	0.17	0.175
0	0	0	0	0	0	0	0	0
0.135	0.14	0.145	0.15	0.155	0.16	0.165	0.17	0.175
0.1645032823	0.1660311832	0.1701361322	0.1614915876	0.175331766	0.1755403108	0.166067154	0.167785186	0.1653875635
0.135	0.14	0.145	0.15	0.155	0.16	0.165	0.17	0.175
0.239536132	0.2458410282	0.2394528024	0.2371818837	0.2433914388	0.2405199465	0.2463511403	0.251453388	0.2599672687
0.135	0.14	0.145	0.15	0.155	0.16	0.165	0.17	0.175
0.2607560417	0.2487345833	0.2566527083	0.2671925	0.2560958333	0.26239	0.2508589583	0.27128375	0.2656791667
0.135	0.14	0.145	0.15	0.155	0.16	0.165	0.17	0.175
0.2472824532	0.2390514723	0.2392090031	0.2452144373	0.2430787059	0.2469897747	0.2442366701	0.2413328347	0.2461182557
0.135	0.14	0.145	0.15	0.155	0.16	0.165	0.17	0.175
0.2466270887	0.2343837095	0.2763938373	0.2410648103	0.2785891073	0.2175256884	0.2675549924	0.2610849226	0.2647211482
0.135	0.14	0.145	0.15	0.155	0.16	0.165	0.17	0.175
0.2038591682	0.2197391304	0.2106124764	0.2102797732	0.2265567108	0.2059461248	0.2121587902	0.2095179584	0.2346275992
0.135	0.14	0.145	0.15	0.155	0.16	0.165	0.17	0.175

0.18	0.185	0.19	0.195	0.2	0.205	0.21	0.215	0.22
0.1176513431	0.134309717	0.1313805661	0.125898684	0.1242733009	0.1294624121	0.1307322877	0.1233582116	0.1172028123
0.18	0.185	0.19	0.195	0.2	0.205	0.21	0.215	0.22
0	0	0	0	0	0	0	0	0
0.18	0.185	0.19	0.195	0.2	0.205	0.21	0.215	0.22
0.1710161312	0.1777752084	0.1615823709	0.1844512104	0.1783999863	0.1692976709	0.1653160502	0.1731238465	0.1690775641
0.18	0.185	0.19	0.195	0.2	0.205	0.21	0.215	0.22
0.2464877708	0.2493179737	0.2450265003	0.2455212228	0.2643338138	0.2543271324	0.2453898473	0.2617513475	0.2528399622
0.18	0.185	0.19	0.195	0.2	0.205	0.21	0.215	0.22
0.2600670833	0.2600620833	0.2728985417	0.2597035417	0.2602258333	0.2641179167	0.2791675	0.2705289583	0.2873910417
0.18	0.185	0.19	0.195	0.2	0.205	0.21	0.215	0.22
0.2474360173	0.2449492417	0.2490552406	0.2483803517	0.2499828586	0.2490563739	0.2500862736	0.2547351302	0.2455116491
0.18	0.185	0.19	0.195	0.2	0.205	0.21	0.215	0.22
0.2694508979	0.2809087613	0.2812218292	0.2558304416	0.27979795	0.2682928629	0.2871399719	0.2582812256	0.2705625524
0.18	0.185	0.19	0.195	0.2	0.205	0.21	0.215	0.22
0.2314357278	0.2184678639	0.2254621928	0.2296550095	0.221415879	0.2270396975	0.2195	0.2167778828	0.2369451796
0.18	0.185	0.19	0.195	0.2	0.205	0.21	0.215	0.22

0.225	0.23	0.235	0.24	0.245	0.25	0.255	0.26	0.265
0.138088336	0.117637642	0.130088336	0.133696773	0.1256160087	0.1244867496	0.1433300883	0.1277894357	0.1460573283
0.225	0.23	0.235	0.24	0.245	0.25	0.255	0.26	0.265
0	0	0	0	0	0	0	0	0
0.225	0.23	0.235	0.24	0.245	0.25	0.255	0.26	0.265
0.1833956398	0.1704196161	0.1781905851	0.1823109501	0.184690159	0.1717261255	0.1829442924	0.1860977291	0.1904090818
0.225	0.23	0.235	0.24	0.245	0.25	0.255	0.26	0.265
0.2606744441	0.2542441782	0.2471589118	0.2466078855	0.258766497	0.2540369803	0.2606721919	0.260553954	0.2436012642
0.225	0.23	0.235	0.24	0.245	0.25	0.255	0.26	0.265
0.266724375	0.2791827083	0.2720960417	0.2531352083	0.2892425	0.2635202083	0.2735358333	0.2735158333	0.2849547917
0.225	0.23	0.235	0.24	0.245	0.25	0.255	0.26	0.265
0.2479712251	0.2615486178	0.2607178982	0.2585787668	0.2650066441	0.2504424177	0.2607530309	0.2567512969	0.2570476587
0.225	0.23	0.235	0.24	0.245	0.25	0.255	0.26	0.265
0.2690741623	0.2639685203	0.252954855	0.2579308426	0.2969922292	0.2862829881	0.2724717396	0.270391999	0.266953101
0.225	0.23	0.235	0.24	0.245	0.25	0.255	0.26	0.265
0.2297769376	0.2249508507	0.2364177694	0.2124385633	0.2186786389	0.2404886578	0.2253705104	0.2307221172	0.2403506616
0.225	0.23	0.235	0.24	0.245	0.25	0.255	0.26	0.265

0.27	0.275	0.28	0.285	0.29	0.295	0.3	0.305	0.31
0.1278406346	0.1219397873	0.133983775	0.1447312061	0.12927925	0.1361153777	0.1201492699	0.1504034613	0.1451653146
0.27	0.275	0.28	0.285	0.29	0.295	0.3	0.305	0.31
0	0	0	0	0	0	0	0	0
0.27	0.275	0.28	0.285	0.29	0.295	0.3	0.305	0.31
0.2087156297	0.1805098427	0.1739528869	0.1750654111	0.1867006676	0.177914381	0.2116776506	0.202208776	0.1952518596
0.27	0.275	0.28	0.285	0.29	0.295	0.3	0.305	0.31
0.2655724967	0.2702742369	0.2485616264	0.2629307238	0.266383271	0.263635647	0.26558113	0.2584785971	0.2647264387
0.27	0.275	0.28	0.285	0.29	0.295	0.3	0.305	0.31
0.2709041667	0.2887977083	0.2676902083	0.2771139583	0.27675125	0.2737277083	0.2976452083	0.293300625	0.2875297917
0.27	0.275	0.28	0.285	0.29	0.295	0.3	0.305	0.31
0.2549637764	0.2527611794	0.2540072589	0.2734855375	0.2701890085	0.258769447	0.2708899636	0.2614440695	0.2684128212
0.27	0.275	0.28	0.285	0.29	0.295	0.3	0.305	0.31
0.3029565415	0.272150028	0.2988837074	0.2426824332	0.324195405	0.2854413941	0.2847659264	0.2896601158	0.279848336
0.27	0.275	0.28	0.285	0.29	0.295	0.3	0.305	0.31
0.2127438563	0.2302750473	0.2288402647	0.2236294896	0.2321701323	0.2443478261	0.2372863894	0.2410217391	0.2452863894
0.27	0.275	0.28	0.285	0.29	0.295	0.3	0.305	0.31

0.315	0.32	0.325	0.33	0.335	0.34	0.345	0.35	0.355
0.1337212908	0.1319704345	0.1433560483	0.1424330269	0.134538309	0.1422426537	0.1352911484	0.134453218	0.1348772309
0.315	0.32	0.325	0.33	0.335	0.34	0.345	0.35	0.355
0	0	0	0	0	0	0	0	0
0.315	0.32	0.325	0.33	0.335	0.34	0.345	0.35	0.355
0.1901864056	0.1814202455	0.1861996463	0.1991067261	0.1882970842	0.1935432484	0.1981539292	0.1928118429	0.1970439743
0.315	0.32	0.325	0.33	0.335	0.34	0.345	0.35	0.355
0.2640080026	0.2679916821	0.2583817546	0.2579249433	0.2754782067	0.2719333213	0.261648124	0.2593077639	0.2689345825
0.315	0.32	0.325	0.33	0.335	0.34	0.345	0.35	0.355
0.2865208333	0.2903702083	0.2816370833	0.2780814583	0.277981875	0.2954839583	0.280621875	0.295103125	0.2904435417
0.315	0.32	0.325	0.33	0.335	0.34	0.345	0.35	0.355
0.2688293143	0.2698286145	0.2570062927	0.2649644281	0.2599922935	0.270948046	0.2853949743	0.27111096	0.2648612965
0.315	0.32	0.325	0.33	0.335	0.34	0.345	0.35	0.355
0.2824007353	0.3205631849	0.2769974153	0.269969937	0.3023607638	0.2779125855	0.272730205	0.2913355034	0.280750857
0.315	0.32	0.325	0.33	0.335	0.34	0.345	0.35	0.355
0.2323875236	0.2359943289	0.2497873346	0.2242627599	0.235110586	0.2385529301	0.2495056711	0.2491540643	0.2462485822
0.315	0.32	0.325	0.33	0.335	0.34	0.345	0.35	0.355

0.36	0.365	0.37	0.375	0.38	0.385	0.39	0.395	0.4
0.1315283937	0.1380919416	0.1311894718	0.1512796106	0.145422751	0.1351923562	0.1335100054	0.1451321435	0.1457991707
0.36	0.365	0.37	0.375	0.38	0.385	0.39	0.395	0.4
0	0	0	0	0	0	0	0	0
0.36	0.365	0.37	0.375	0.38	0.385	0.39	0.395	0.4
0.1944348094	0.1977098616	0.1977334138	0.1998004479	0.1947885219	0.1988562154	0.2020117933	0.1963601016	0.2044963451
0.36	0.365	0.37	0.375	0.38	0.385	0.39	0.395	0.4
0.2653600438	0.2650852814	0.2789908112	0.2945734426	0.2729933336	0.2799825083	0.2668457127	0.2920499077	0.2732328123
0.36	0.365	0.37	0.375	0.38	0.385	0.39	0.395	0.4
0.2777889583	0.2752497917	0.2855775	0.2925452083	0.2744258333	0.2825491667	0.2961875	0.28654875	0.2940979167
0.36	0.365	0.37	0.375	0.38	0.385	0.39	0.395	0.4
0.291330993	0.2604357028	0.2673370223	0.285396391	0.2750279787	0.2701017433	0.2824656393	0.2740725945	0.2798218996
0.36	0.365	0.37	0.375	0.38	0.385	0.39	0.395	0.4
0.2856713989	0.2948968879	0.2714650734	0.2941001986	0.286231548	0.2668476909	0.3039301089	0.3038624356	0.3215154594
0.36	0.365	0.37	0.375	0.38	0.385	0.39	0.395	0.4
0.2576606805	0.2424480151	0.2542741021	0.2554338374	0.2609404537	0.2635869565	0.2527533081	0.2721852552	0.2727844991
0.36	0.365	0.37	0.375	0.38	0.385	0.39	0.395	0.4

0.405	0.41	0.415	0.42	0.425	0.43	0.435	0.44	0.445
0.1447362538	0.1354101316	0.1328732648	0.1370037858	0.1531638724	0.13738958	0.1564333874	0.1569972958	0.1645588606
0.405	0.41	0.415	0.42	0.425	0.43	0.435	0.44	0.445
0	0	0	0	0	0	0	0	0
0.405	0.41	0.415	0.42	0.425	0.43	0.435	0.44	0.445
0.2051836436	0.2009262471	0.2130578144	0.1978742993	0.2083053918	0.2046162477	0.2113513444	0.2107518317	0.2015608741
0.405	0.41	0.415	0.42	0.425	0.43	0.435	0.44	0.445
0.2807373542	0.2841076979	0.2861916881	0.2782175728	0.2918063	0.2778493461	0.2900331066	0.2832034593	0.2842356951
0.405	0.41	0.415	0.42	0.425	0.43	0.435	0.44	0.445
0.2967929167	0.3042735417	0.2995133333	0.2931514583	0.3045210417	0.2944229167	0.308625	0.289675	0.294980625
0.405	0.41	0.415	0.42	0.425	0.43	0.435	0.44	0.445
0.2843795244	0.2658679632	0.2675500854	0.2836590196	0.2810472961	0.2949204838	0.2754832879	0.3023190451	0.2984688919
0.405	0.41	0.415	0.42	0.425	0.43	0.435	0.44	0.445
0.2553752809	0.2862260666	0.3193459095	0.3131229208	0.2940770084	0.302552821	0.2955936484	0.273806231	0.2925394972
0.405	0.41	0.415	0.42	0.425	0.43	0.435	0.44	0.445
0.2487325142	0.2661833648	0.2655179584	0.2757533081	0.2709773157	0.2545699433	0.2819555766	0.2688128544	0.2585311909
0.405	0.41	0.415	0.42	0.425	0.43	0.435	0.44	0.445

0.45	0.455	0.46	0.465	0.47	0.475	0.48	0.485	0.49
0.1475392104	0.1468166577	0.1596690103	0.1547568055	0.1527874527	0.1533217956	0.1474216694	0.1569021092	0.1444261763
0.45	0.455	0.46	0.465	0.47	0.475	0.48	0.485	0.49
0	0	0	0	0	0	0	0	0
0.45	0.455	0.46	0.465	0.47	0.475	0.48	0.485	0.49
0.2164163701	0.2251683988	0.2194768824	0.2268684455	0.2142636914	0.2076493536	0.2230439828	0.233048993	0.2233167611
0.45	0.455	0.46	0.465	0.47	0.475	0.48	0.485	0.49
0.2959941744	0.3118718376	0.2914579674	0.2925611459	0.2945100071	0.2878173656	0.2960962719	0.296605258	0.2983146405
0.45	0.455	0.46	0.465	0.47	0.475	0.48	0.485	0.49
0.299735625	0.306280625	0.308930625	0.3079608333	0.3009327083	0.3087233333	0.3038360417	0.2933854167	0.3087425
0.45	0.455	0.46	0.465	0.47	0.475	0.48	0.485	0.49
0.2881282459	0.2957067208	0.285818834	0.3105191431	0.2984085429	0.3005714739	0.3000141664	0.3335370466	0.3147721896
0.45	0.455	0.46	0.465	0.47	0.475	0.48	0.485	0.49
0.2407374067	0.3059744317	0.3032710852	0.300489946	0.292259528	0.2898547028	0.3257801399	0.3615809823	0.2938128508
0.45	0.455	0.46	0.465	0.47	0.475	0.48	0.485	0.49
0.2794243856	0.2795954631	0.2695557656	0.2437986767	0.277405482	0.292258034	0.2936880907	0.2732022684	0.2852844991
0.45	0.455	0.46	0.465	0.47	0.475	0.48	0.485	0.49

0.495	0.5	0.505	0.51	0.515	0.52	0.525	0.53	0.535
0.1491768524	0.1498756084	0.1569021092	0.1593207139	0.1594562827	0.1613737155	0.1529597981	0.1650160447	0.1596863169
0.495	0.5	0.505	0.51	0.515	0.52	0.525	0.53	0.535
0	0	0	0	0	0	0	0	0
0.495	0.5	0.505	0.51	0.515	0.52	0.525	0.53	0.535
0.2147218903	0.2224414726	0.2296467586	0.2318662402	0.2313647906	0.2197830621	0.2238293444	0.2213713424	0.2271956081
0.495	0.5	0.505	0.51	0.515	0.52	0.525	0.53	0.535
0.3054821104	0.2916587841	0.302015675	0.3106673123	0.2988247526	0.313241896	0.330943426	0.3173922346	0.3135647043
0.495	0.5	0.505	0.51	0.515	0.52	0.525	0.53	0.535
0.32219375	0.29815375	0.3155285417	0.3040072917	0.3147870833	0.320625625	0.3295091667	0.2940716667	0.3327022917
0.495	0.5	0.505	0.51	0.515	0.52	0.525	0.53	0.535
0.2951080474	0.2884127079	0.2989885167	0.335697711	0.3053228955	0.3169033878	0.3014934254	0.3420196233	0.3241883342
0.495	0.5	0.505	0.51	0.515	0.52	0.525	0.53	0.535
0.3211043602	0.2646117326	0.3739181765	0.2953381344	0.3116210382	0.2959741788	0.3245259709	0.2942041329	0.3195622109
0.495	0.5	0.505	0.51	0.515	0.52	0.525	0.53	0.535
0.2696767486	0.296305293	0.2647930057	0.2973185255	0.2831909263	0.2830907372	0.2795652174	0.2739678639	0.2747088847
0.495	0.5	0.505	0.51	0.515	0.52	0.525	0.53	0.535

0.54	0.545	0.55	0.555	0.56	0.565	0.57	0.575	0.58
0.1563389219	0.1557815035	0.1542275104	0.1622765459	0.1552507662	0.1596480981	0.1680079322	0.1649972958	0.1698842618
0.54	0.545	0.55	0.555	0.56	0.565	0.57	0.575	0.58
0	0	0	0	0	0	0	0	0
0.54	0.545	0.55	0.555	0.56	0.565	0.57	0.575	0.58
0.2334832115	0.2440483379	0.2328164678	0.2566548049	0.2447210767	0.2321338797	0.2326850032	0.2309220077	0.2433177888
0.54	0.545	0.55	0.555	0.56	0.565	0.57	0.575	0.58
0.2795647343	0.3033834812	0.2985773914	0.3201575004	0.3199833341	0.3372839812	0.3229377806	0.3168596009	0.339909989
0.54	0.545	0.55	0.555	0.56	0.565	0.57	0.575	0.58
0.2836529167	0.3145227083	0.3366160417	0.2872689583	0.315976875	0.2821964583	0.2661066667	0.320794375	0.3110579167
0.54	0.545	0.55	0.555	0.56	0.565	0.57	0.575	0.58
0.3184869116	0.308547459	0.3201914168	0.3373971163	0.3000220996	0.3162163158	0.3179267709	0.3377206776	0.3176085928
0.54	0.545	0.55	0.555	0.56	0.565	0.57	0.575	0.58
0.3202494845	0.3431696385	0.3285461	0.3004195321	0.3074107493	0.3340695875	0.3189603194	0.3023196539	0.3228811523
0.54	0.545	0.55	0.555	0.56	0.565	0.57	0.575	0.58
0.285	0.2676701323	0.29868431	0.3057221172	0.2786153119	0.3336663516	0.2951852552	0.3250311909	0.2978865784
0.54	0.545	0.55	0.555	0.56	0.565	0.57	0.575	0.58

0.585	0.59	0.595	0.6	0.605	0.61	0.615	0.62	0.625
0.1754202271	0.1560670633	0.1709406887	0.1609980169	0.1802531098	0.1903508203	0.172933117	0.1680648999	0.1995024337
0.585	0.59	0.595	0.6	0.605	0.61	0.615	0.62	0.625
0	0	0	0	0	0	0	0	0
0.585	0.59	0.595	0.6	0.605	0.61	0.615	0.62	0.625
0.239142183	0.2484187853	0.251436047	0.2575545021	0.2552296776	0.2494747841	0.2620551295	0.2568406538	0.2452803364
0.585	0.59	0.595	0.6	0.605	0.61	0.615	0.62	0.625
0.3383743975	0.307875396	0.3449060102	0.3402448088	0.3361098599	0.3177807306	0.351169617	0.3301003709	0.3441432818
0.585	0.59	0.595	0.6	0.605	0.61	0.615	0.62	0.625
0.3105297917	0.3113691667	0.3167804167	0.3234316667	0.3398385417	0.30363375	0.3371875	0.2894172917	0.3166795833
0.585	0.59	0.595	0.6	0.605	0.61	0.615	0.62	0.625
0.3350947876	0.3374393323	0.3125803591	0.3476575803	0.3247303419	0.3215100284	0.3253315654	0.3441403384	0.3668839797
0.585	0.59	0.595	0.6	0.605	0.61	0.615	0.62	0.625
0.3145478541	0.3593951992	0.2843474484	0.3401388883	0.3275369041	0.2934582513	0.3325447255	0.3633562987	0.292703726
0.585	0.59	0.595	0.6	0.605	0.61	0.615	0.62	0.625
0.2920510397	0.2745775047	0.3122325142	0.3160198488	0.2951153119	0.2932287335	0.3389640832	0.3048591682	0.3111795841
0.585	0.59	0.595	0.6	0.605	0.61	0.615	0.62	0.625

0.63	0.635	0.64	0.645	0.65	0.655	0.66	0.665	0.67
0.1758276546	0.1914916171	0.1824784568	0.169500631	0.2068887687	0.1914065261	0.1865361457	0.1922927709	0.1936224986
0.63	0.635	0.64	0.645	0.65	0.655	0.66	0.665	0.67
0	0	0	0	0	0	0	0	0
0.63	0.635	0.64	0.645	0.65	0.655	0.66	0.665	0.67
0.2388925288	0.2688064987	0.2474060371	0.2434188495	0.2514561735	0.2493313292	0.2745404093	0.2489296558	0.2689846396
0.63	0.635	0.64	0.645	0.65	0.655	0.66	0.665	0.67
0.3468728886	0.3629213399	0.3513302704	0.325348708	0.3277307328	0.3563668303	0.3713590229	0.3560669189	0.3575627224
0.63	0.635	0.64	0.645	0.65	0.655	0.66	0.665	0.67
0.3228766667	0.32046125	0.3131270833	0.3238570833	0.306184375	0.3224404167	0.3365208333	0.3078839583	0.3139247917
0.63	0.635	0.64	0.645	0.65	0.655	0.66	0.665	0.67
0.3444638997	0.3776799349	0.3491830218	0.3346703613	0.3762763248	0.3572955713	0.3244671296	0.3371684701	0.3223679476
0.63	0.635	0.64	0.645	0.65	0.655	0.66	0.665	0.67
0.2931709034	0.3460433278	0.3035133175	0.3123780933	0.3757029797	0.344508768	0.2963677799	0.3332914504	0.3954941413
0.63	0.635	0.64	0.645	0.65	0.655	0.66	0.665	0.67
0.3234744802	0.3331994329	0.3224385633	0.3307655955	0.3145340265	0.3219026465	0.3110482042	0.3161531191	0.3387627599
0.63	0.635	0.64	0.645	0.65	0.655	0.66	0.665	0.67

0.675	0.68	0.685	0.69	0.695	0.7	0.705	0.71	0.715
0.1980667027	0.1765502073	0.1737631152	0.1687622138	0.1820155039	0.1915586804	0.2000245178	0.1989652064	0.1943464936
0.675	0.68	0.685	0.69	0.695	0.7	0.705	0.71	0.715
0	0	0	0	0	0	0	0	0
0.675	0.68	0.685	0.69	0.695	0.7	0.705	0.71	0.715
0.2541933771	0.2595230448	0.274292468	0.2851928932	0.2616628769	0.276524368	0.2695045028	0.2927330498	0.2810228543
0.675	0.68	0.685	0.69	0.695	0.7	0.705	0.71	0.715
0.3715560861	0.3561371109	0.3852315211	0.370514091	0.3780122517	0.4214123238	0.400876462	0.3411696921	0.3549753765
0.675	0.68	0.685	0.69	0.695	0.7	0.705	0.71	0.715
0.32481625	0.3449145833	0.3307720833	0.3348797917	0.3261154167	0.328744375	0.3416870833	0.2895925	0.3778489583
0.675	0.68	0.685	0.69	0.695	0.7	0.705	0.71	0.715
0.3646728829	0.3650865427	0.3386332792	0.3654188873	0.3512649208	0.3389228411	0.3921347964	0.3670145943	0.3881755618
0.675	0.68	0.685	0.69	0.695	0.7	0.705	0.71	0.715
0.3306962546	0.3036771248	0.3382136789	0.344188743	0.3348270642	0.3070365436	0.331387323	0.3228113708	0.3214304989
0.675	0.68	0.685	0.69	0.695	0.7	0.705	0.71	0.715
0.3718553875	0.294473535	0.3481389414	0.3378610586	0.3108289225	0.318137051	0.3726635161	0.3213620038	0.3475812854
0.675	0.68	0.685	0.69	0.695	0.7	0.705	0.71	0.715

0.72	0.725	0.73	0.735	0.74	0.745	0.75	0.755	0.76
0.1987221922	0.227255814	0.2135893276	0.174897422	0.2032673517	0.2063601947	0.2089698936	0.2061582838	0.2192861006
0.72	0.725	0.73	0.735	0.74	0.745	0.75	0.755	0.76
0	0	0	0	0	0	0	0	0
0.72	0.725	0.73	0.735	0.74	0.745	0.75	0.755	0.76
0.2667780047	0.2650646831	0.2898999242	0.279641834	0.2870342536	0.28522501	0.2783181956	0.2906775778	0.3055283634
0.72	0.725	0.73	0.735	0.74	0.745	0.75	0.755	0.76
0.3648026365	0.3602840713	0.4109721034	0.3903161269	0.3749084125	0.3887809108	0.4002728856	0.3921268561	0.4223702386
0.72	0.725	0.73	0.735	0.74	0.745	0.75	0.755	0.76
0.3168925	0.3743358333	0.3549229167	0.305954375	0.3177554167	0.3418004167	0.3442789583	0.337705625	0.3601347917
0.72	0.725	0.73	0.735	0.74	0.745	0.75	0.755	0.76
0.3552564549	0.4114620609	0.3633418049	0.4050228505	0.4048936526	0.3959860829	0.3826279866	0.4114110617	0.4248731396
0.72	0.725	0.73	0.735	0.74	0.745	0.75	0.755	0.76
0.3572503152	0.3640939583	0.3596197226	0.3522549321	0.3263411323	0.3173228795	0.3806859244	0.323603422	0.3679382634
0.72	0.725	0.73	0.735	0.74	0.745	0.75	0.755	0.76
0.3626890359	0.3339555766	0.3390217391	0.3456493384	0.3808232514	0.3559064272	0.3373062382	0.4452986767	0.3481408318
0.72	0.725	0.73	0.735	0.74	0.745	0.75	0.755	0.76

0.765	0.77	0.775	0.78	0.785	0.79	0.795	0.8	0.805
0.1999430323	0.2085920317	0.2079206778	0.1908815576	0.1887095727	0.22672075	0.2204550207	0.2294854877	0.2177854696
0.765	0.77	0.775	0.78	0.785	0.79	0.795	0.8	0.805
0	0	0	0	0	0	0	0	0
0.765	0.77	0.775	0.78	0.785	0.79	0.795	0.8	0.805
0.2975698325	0.3141168108	0.3151359823	0.3055292198	0.3066760019	0.3339636781	0.303871139	0.2945949649	0.3144238469
0.765	0.77	0.775	0.78	0.785	0.79	0.795	0.8	0.805
0.3858493611	0.4201702626	0.3777048331	0.4423614552	0.4888420942	0.4600029278	0.3998183265	0.4767690645	0.4200576551
0.765	0.77	0.775	0.78	0.785	0.79	0.795	0.8	0.805
0.3505147917	0.3656254167	0.3584620833	0.3811664583	0.384394375	0.3978470833	0.3752402083	0.3467170833	0.34790125
0.765	0.77	0.775	0.78	0.785	0.79	0.795	0.8	0.805
0.3738379983	0.4194986783	0.4375192309	0.4652406735	0.4236522764	0.3819638643	0.3976747217	0.4188490057	0.4362298022
0.765	0.77	0.775	0.78	0.785	0.79	0.795	0.8	0.805
0.3932094835	0.3417044386	0.3313390452	0.3474931378	0.351352622	0.3442694872	0.3794699139	0.3807498029	0.3260438759
0.765	0.77	0.775	0.78	0.785	0.79	0.795	0.8	0.805
0.3420992439	0.3733090737	0.3783251418	0.4364886578	0.3885274102	0.4120879017	0.4541606805	0.4138979206	0.4440879017
0.765	0.77	0.775	0.78	0.785	0.79	0.795	0.8	0.805

0.81	0.815	0.82	0.825	0.83	0.835	0.84	0.845	0.85
0.2275197404	0.2261813593	0.2400540833	0.2133145845	0.2484665585	0.2244579052	0.237650622	0.2475788715	0.2172965567
0.81	0.815	0.82	0.825	0.83	0.835	0.84	0.845	0.85
0	0	0	0	0	0	0	0	0
0.81	0.815	0.82	0.825	0.83	0.835	0.84	0.845	0.85
0.3061420074	0.2918320679	0.3025487853	0.3029380404	0.3310847326	0.3371697863	0.3396072335	0.3107950823	0.3047451429
0.81	0.815	0.82	0.825	0.83	0.835	0.84	0.845	0.85
0.4470309145	0.4449397925	0.476379067	0.4652147801	0.4467092323	0.4343197754	0.4622862334	0.442961278	0.4736412023
0.81	0.815	0.82	0.825	0.83	0.835	0.84	0.845	0.85
0.3452847917	0.3779375	0.3888747917	0.376165625	0.3916052083	0.3875172917	0.382955	0.3643035417	0.3715302083
0.81	0.815	0.82	0.825	0.83	0.835	0.84	0.845	0.85
0.4301093365	0.4151215338	0.4435031322	0.4454464835	0.4022093969	0.409942286	0.446123639	0.497052532	0.4342955175
0.81	0.815	0.82	0.825	0.83	0.835	0.84	0.845	0.85
0.3843600977	0.3407783058	0.3763434513	0.3529554874	0.3982282676	0.3558772437	0.3532968052	0.3686310184	0.3606065717
0.81	0.815	0.82	0.825	0.83	0.835	0.84	0.845	0.85
0.3901587902	0.4214669187	0.4436918715	0.4272353497	0.3865557656	0.4024981096	0.3989981096	0.4458487713	0.4539234405
0.81	0.815	0.82	0.825	0.83	0.835	0.84	0.845	0.85

0.855	0.86	0.865	0.87	0.875	0.88	0.885	0.89	0.895
0.2421777537	0.2488999459	0.2378013341	0.2570795024	0.2413982333	0.2393690283	0.2638615468	0.2340876149	0.2576787453
0.855	0.86	0.865	0.87	0.875	0.88	0.885	0.89	0.895
0	0	0	0	0	0	0	0	0
0.855	0.86	0.865	0.87	0.875	0.88	0.885	0.89	0.895
0.3512480569	0.3623386133	0.3627895325	0.3306295311	0.3670713377	0.3306458036	0.3416177421	0.3541359952	0.3654008385
0.855	0.86	0.865	0.87	0.875	0.88	0.885	0.89	0.895
0.4619416543	0.4911467952	0.5119750612	0.4905548549	0.5015742534	0.5594684173	0.4939195682	0.5112476165	0.4819069712
0.855	0.86	0.865	0.87	0.875	0.88	0.885	0.89	0.895
0.3962933333	0.4061670833	0.3155770833	0.44757875	0.5347925	0.357045	0.4327208333	0.4466241667	0.42822625
0.855	0.86	0.865	0.87	0.875	0.88	0.885	0.89	0.895
0.5203478143	0.5032934123	0.4680150278	0.4979858166	0.4902597274	0.5072523637	0.4593233545	0.4738518814	0.4959591667
0.855	0.86	0.865	0.87	0.875	0.88	0.885	0.89	0.895
0.3212519343	0.3779610742	0.3940597211	0.4035240693	0.3527273801	0.3669320189	0.3505496081	0.3211610708	0.3673768494
0.855	0.86	0.865	0.87	0.875	0.88	0.885	0.89	0.895
0.4008412098	0.5274035917	0.4530680529	0.4502958412	0.4369678639	0.4425349716	0.4500529301	0.4735567108	0.4266294896
0.855	0.86	0.865	0.87	0.875	0.88	0.885	0.89	0.895

0.9	0.905	0.91	0.915	0.92	0.925	0.93	0.935	0.94
0.2518875068	0.2628433387	0.2566677483	0.2818236885	0.2599596178	0.2736261042	0.260325942	0.267216874	0.2453686677
0.9	0.905	0.91	0.915	0.92	0.925	0.93	0.935	0.94
0	0	0	0	0	0	0	0	0
0.9	0.905	0.91	0.915	0.92	0.925	0.93	0.935	0.94
0.3376224183	0.3784278208	0.3749129636	0.3646989804	0.3795382039	0.3795921601	0.3762918428	0.386666838	0.3890593218
0.9	0.905	0.91	0.915	0.92	0.925	0.93	0.935	0.94
0.4958222603	0.5459453778	0.5246464123	0.5231352191	0.5804374428	0.5701815234	0.5511437172	0.5860535561	0.6074170833
0.9	0.905	0.91	0.915	0.92	0.925	0.93	0.935	0.94
0.4554370833	0.4469372917	0.4383497917	0.448854375	0.4311464583	0.552588125	0.523575	0.5103197917	0.4268714583
0.9	0.905	0.91	0.915	0.92	0.925	0.93	0.935	0.94
0.4787370342	0.5227408081	0.5700139114	0.4934582246	0.5749333469	0.5763871063	0.5362723015	0.5805936869	0.53232355
0.9	0.905	0.91	0.915	0.92	0.925	0.93	0.935	0.94
0.3693332602	0.3924579098	0.2968967276	0.3854182039	0.413497759	0.3526955462	0.3983045845	0.3757152073	0.3834017093
0.9	0.905	0.91	0.915	0.92	0.925	0.93	0.935	0.94
0.5195151229	0.474010397	0.5264848771	0.4859669187	0.5256427221	0.4456767486	0.5538837429	0.4556379962	0.5502476371
0.9	0.905	0.91	0.915	0.92	0.925	0.93	0.935	0.94

0.945	0.95	0.955	0.96	0.965	0.97	0.975	0.98	0.985
0.270499369	0.2833113395	0.2827048855	0.2858957995	0.2757490535	0.286427258	0.2748729043	0.310270777	0.2705210023
0.945	0.95	0.955	0.96	0.965	0.97	0.975	0.98	0.985
0	0	0	0	0	0	0	0	0
0.945	0.95	0.955	0.96	0.965	0.97	0.975	0.98	0.985
0.3873931047	0.3753184911	0.4150499094	0.396631167	0.4158661031	0.432648176	0.4582799125	0.4382497655	0.4376301264
0.945	0.95	0.955	0.96	0.965	0.97	0.975	0.98	0.985
0.5856969656	0.5871574854	0.6222478717	0.5435734877	0.6303454799	0.6294217227	0.6167950393	0.6473101812	0.6560871883
0.945	0.95	0.955	0.96	0.965	0.97	0.975	0.98	0.985
0.5727877083	0.4392347917	0.60828125	0.5167195833	0.531285625	0.5796639583	0.579865625	0.6322935417	0.5951075
0.945	0.95	0.955	0.96	0.965	0.97	0.975	0.98	0.985
0.6276154777	0.5383315342	0.5808115666	0.6680266442	0.6216293098	0.6398663255	0.6736691345	0.6228337399	0.6589241444
0.945	0.95	0.955	0.96	0.965	0.97	0.975	0.98	0.985
0.3756424744	0.3926023216	0.4077725166	0.4019205714	0.3914744338	0.4002572006	0.5076966214	0.4274032441	0.4140165873
0.945	0.95	0.955	0.96	0.965	0.97	0.975	0.98	0.985
0.5338185255	0.5283024575	0.4876928166	0.5455926276	0.5398015123	0.5568487713	0.5614584121	0.5554716446	0.5700245747
0.945	0.95	0.955	0.96	0.965	0.97	0.975	0.98	0.985

0.99	0.995	1
0.2959365423	0.3083627186	0.291079863
0.99	0.995	1
0	0	0
0.99	0.995	1
0.4303259208	0.4450349644	0.4552545145
0.99	0.995	1
0.6514717806	0.6831039893	0.6832631413
0.99	0.995	1
0.6677397917	0.7286764583	0.733085625
0.99	0.995	1
0.6813221249	0.6949493833	0.6960121491
0.99	0.995	1
0.6074527025	0.5056027558	0.5054655119
0.99	0.995	1
0.5729300567	0.5821039698	0.5868185255
0.99	0.995	1

flow (normalized)	0	0	0	0	0	0	0	0
520_a	**Density**	0.4039						
percentage AV	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04
flow (normalized)	0.175119667	0.1836899063	0.1897127992	0.1929947971	0.1829469303	0.2015483871	0.1816940687	0.1850509886
520_b	**Density**	0.0135						
percentage AV	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04
flow (normalized)	0.09634248444	0.05891702472	0.08834908372	0.08297626643	0.09368423025	0.0803312266	0.06962415031	0.0776275111
520_c	**Density**	5.8642						
percentage AV	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04
flow (normalized)	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.045	0.05	0.055	0.06	0.065	0.07	0.075	0.08	0.085
0.1978855359	0.1875691988	0.1922663892	0.2030260146	0.2030260146	0.1909032258	0.1939958377	0.1916087409	0.1987263267
0.045	0.05	0.055	0.06	0.065	0.07	0.075	0.08	0.085
0.0776404296	0.0562327856	0.06158089994	0.06159016971	0.08032279505	0.06430419043	0.07498247126	0.08303094821	0.0642646953
0.045	0.05	0.055	0.06	0.065	0.07	0.075	0.08	0.085
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.09	0.095	0.1	0.105	0.11	0.115	0.12	0.125	0.13
0.190314256	0.1994630593	0.1974942768	0.2081727367	0.2121165453	0.2108824142	0.1952237253	0.2003121748	0.1870093652
0.09	0.095	0.1	0.105	0.11	0.115	0.12	0.125	0.13
0.05889932341	0.0616050605	0.09104190231	0.07763880246	0.05628401586	0.07503207439	0.06427110524	0.08569950762	0.0589798422
0.09	0.095	0.1	0.105	0.11	0.115	0.12	0.125	0.13
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.135	0.14	0.145	0.15	0.155	0.16	0.165	0.17	0.175
0.2017419355	0.1862039542	0.2044994797	0.1961602497	0.2143704475	0.2146784599	0.1993173777	0.2231841831	0.2110988554
0.135	0.14	0.145	0.15	0.155	0.16	0.165	0.17	0.175
0.07229739391	0.07231687029	0.09109426665	0.08032861331	0.06969278605	0.06966152523	0.07229058951	0.08833749651	0.06160111592
0.135	0.14	0.145	0.15	0.155	0.16	0.165	0.17	0.175
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.18	0.185	0.19	0.195	0.2	0.205	0.21	0.215	0.22
0.201471384	0.2143204995	0.2056878252	0.2138543184	0.2091716961	0.2207200832	0.2205931322	0.2233215401	0.2375858481
0.18	0.185	0.19	0.195	0.2	0.205	0.21	0.215	0.22
0.06965225546	0.1070890065	0.08304554316	0.06697777918	0.0749935654	0.07503917464	0.06698497804	0.0884457753	0.07501106948
0.18	0.185	0.19	0.195	0.2	0.205	0.21	0.215	0.22
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.225	0.23	0.235	0.24	0.245	0.25	0.255	0.26	0.265
0.2015296566	0.2219084287	0.2253298647	0.2005390219	0.2202122789	0.2114797086	0.2156378772	0.2226264308	0.2112715921
0.225	0.23	0.235	0.24	0.245	0.25	0.255	0.26	0.265
0.06695711943	0.05365618427	0.05894996198	0.09106769002	0.05903516497	0.06700795524	0.06972079258	0.08304228888	0.08842472109
0.225	0.23	0.235	0.24	0.245	0.25	0.255	0.26	0.265
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.27	0.275	0.28	0.285	0.29	0.295	0.3	0.305	0.31
0.2149115505	0.2209510926	0.2469115505	0.2196087409	0.2181540062	0.2335962539	0.2227887617	0.2235962539	0.2330364204
0.27	0.275	0.28	0.285	0.29	0.295	0.3	0.305	0.31
0.0776580323	0.06697950494	0.08312113123	0.07233743143	0.05628736876	0.04832448925	0.05901401214	0.07502009272	0.06440748918
0.27	0.275	0.28	0.285	0.29	0.295	0.3	0.305	0.31
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.315	0.32	0.325	0.33	0.335	0.34	0.345	0.35	0.355
0.2167513007	0.2436212279	0.2486389178	0.1916212279	0.2431841831	0.2255026015	0.2184162331	0.2424994797	0.2262788762
0.315	0.32	0.325	0.33	0.335	0.34	0.345	0.35	0.355
0.06701870422	0.06700203836	0.08306654807	0.06967784594	0.08843275818	0.04027389209	0.07773312729	0.06968810186	0.06698936639
0.315	0.32	0.325	0.33	0.335	0.34	0.345	0.35	0.355
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.36	0.365	0.37	0.375	0.38	0.385	0.39	0.395	0.4
0.2514110302	0.230074922	0.2302684703	0.2421248699	0.2385619147	0.2405806452	0.2554776275	0.2369490114	0.2257502601
0.36	0.365	0.37	0.375	0.38	0.385	0.39	0.395	0.4
0.08843571662	0.06700672255	0.06711761463	0.06164687308	0.06970471841	0.05359499393	0.05897678515	0.06428757387	0.07779338079
0.36	0.365	0.37	0.375	0.38	0.385	0.39	0.395	0.4
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.405	0.41	0.415	0.42	0.425	0.43	0.435	0.44	0.445
0.2497627471	0.2544370447	0.2484016649	0.256780437	0.2787429761	0.2628012487	0.262528616	0.2442601457	0.2343537981
0.405	0.41	0.415	0.42	0.425	0.43	0.435	0.44	0.445
0.09383279309	0.06165343095	0.05893768447	0.06173439351	0.08036347356	0.07238363235	0.06982216836	0.06973415486	0.09651856074
0.405	0.41	0.415	0.42	0.425	0.43	0.435	0.44	0.445
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.45	0.455	0.46	0.465	0.47	0.475	0.48	0.485	0.49
0.2489719043	0.2767596254	0.2980770031	0.2573215401	0.244792924	0.262301769	0.242049948	0.2801768991	0.2513277836
0.45	0.455	0.46	0.465	0.47	0.475	0.48	0.485	0.49
0.0753016373	0.08576207856	0.08058032699	0.08847028102	0.06978449759	0.08039291001	0.05901884426	0.08849138454	0.1125708422
0.45	0.455	0.46	0.465	0.47	0.475	0.48	0.485	0.49
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.495	0.5	0.505	0.51	0.515	0.52	0.525	0.53	0.535
0.2606389178	0.2639209157	0.2631800208	0.2801748179	0.2641727367	0.252503642	0.2817044745	0.2607117586	0.2913798127
0.495	0.5	0.505	0.51	0.515	0.52	0.525	0.53	0.535
0.09930042828	0.05647192591	0.06182418207	0.05364050455	0.02959329382	0.06188640786	0.06448988165	0.06441380052	0.08064807519
0.495	0.5	0.505	0.51	0.515	0.52	0.525	0.53	0.535
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.54	0.545	0.55	0.555	0.56	0.565	0.57	0.575	0.58
0.2551737773	0.2620645161	0.2783621228	0.2737065557	0.2816711759	0.2895920916	0.2623912591	0.2637169615	0.2586305931
0.54	0.545	0.55	0.555	0.56	0.565	0.57	0.575	0.58
0.0645111824	0.08313681094	0.07511057158	0.05634895355	0.06979914186	0.07521707531	0.08054886894	0.05916518827	0.06436459185
0.54	0.545	0.55	0.555	0.56	0.565	0.57	0.575	0.58
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.585	0.59	0.595	0.6	0.605	0.61	0.615	0.62	0.625
0.2887304891	0.279421436	0.2894672216	0.3118584807	0.3468761707	0.3074630593	0.3017440166	0.2795150884	0.2860811655
0.585	0.59	0.595	0.6	0.605	0.61	0.615	0.62	0.625
0.08043166553	0.08849389921	0.0726074381	0.05931868184	0.085884262	0.1101631184	0.0805025694	0.06719502706	0.06715518678
0.585	0.59	0.595	0.6	0.605	0.61	0.615	0.62	0.625
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.63	0.635	0.64	0.645	0.65	0.655	0.66	0.665	0.67
0.2812549428	0.3127263267	0.3166951093	0.2543870968	0.3083537981	0.2947887617	0.3183975026	0.3277294485	0.3274297607
0.63	0.635	0.64	0.645	0.65	0.655	0.66	0.665	0.67
0.1019163275	0.0378437827	0.08059748592	0.0831642751	0.08067775818	0.09122744562	0.06736173499	0.06464337522	0.05642350616
0.63	0.635	0.64	0.645	0.65	0.655	0.66	0.665	0.67
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.675	0.68	0.685	0.69	0.695	0.7	0.705	0.71	0.715
0.2994776275	0.3062726327	0.3164308012	0.3006909469	0.3260312175	0.43778564	0.3065078044	0.299508845	0.309773153
0.675	0.68	0.685	0.69	0.695	0.7	0.705	0.71	0.715
0.05648696463	0.06728042728	0.07548091857	0.06189178235	0.0805127267	0.08339143375	0.07519375296	0.08047466148	0.1072222841
0.675	0.68	0.685	0.69	0.695	0.7	0.705	0.71	0.715
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.72	0.725	0.73	0.735	0.74	0.745	0.75	0.755	0.76
0.3520728408	0.360251821	0.3165452653	0.3768511967	0.3506347555	0.3732819979	0.3735359001	0.3241997919	0.3379542144
0.72	0.725	0.73	0.735	0.74	0.745	0.75	0.755	0.76
0.06991629596	0.07523408632	0.1020236695	0.08336456128	0.08333256085	0.08105939654	0.07520188867	0.06459401863	0.08949271682
0.72	0.725	0.73	0.735	0.74	0.745	0.75	0.755	0.76
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.765	0.77	0.775	0.78	0.785	0.79	0.795	0.8	0.805
0.3972736733	0.3526701353	0.3092216441	0.3283725286	0.3535442248	0.392905307	0.3778439126	0.3613381894	0.3647346514
0.765	0.77	0.775	0.78	0.785	0.79	0.795	0.8	0.805
0.08061282049	0.0622218946	0.08901389381	0.06479469927	0.06489425067	0.06719601321	0.05378443251	0.07803681084	0.05910074365
0.765	0.77	0.775	0.78	0.785	0.79	0.795	0.8	0.805
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.81	0.815	0.82	0.825	0.83	0.835	0.84	0.845	0.85
0.366201873	0.367132154	0.435408949	0.3954609781	0.3764724246	0.3819958377	0.3866014568	0.3928824142	0.4495317378
0.81	0.815	0.82	0.825	0.83	0.835	0.84	0.845	0.85
0.08340943091	0.05112721377	0.08327003922	0.05578911867	0.06153031067	0.07594381534	0.06186550157	0.06484632399	0.0782375901
0.81	0.815	0.82	0.825	0.83	0.835	0.84	0.845	0.85
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.855	0.86	0.865	0.87	0.875	0.88	0.885	0.89	0.895
0.4692570239	0.4183204995	0.4106742976	0.4466722164	0.4188824142	0.4469198751	0.4439021852	0.37821436	0.3693090531
0.855	0.86	0.865	0.87	0.875	0.88	0.885	0.89	0.895
0.07523413563	0.06761561818	0.07804854598	0.07569821577	0.07885442419	0.0618988826	0.08663871272	0.06463331654	0.07000682413
0.855	0.86	0.865	0.87	0.875	0.88	0.885	0.89	0.895
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.9	0.905	0.91	0.915	0.92	0.925	0.93	0.935	0.94
0.4657315297	0.4348720083	0.4811862643	0.4308116545	0.457132154	0.3811987513	0.4775192508	0.5119854318	0.4820395421
0.9	0.905	0.91	0.915	0.92	0.925	0.93	0.935	0.94
0.07973327719	0.07914070227	0.08687883919	0.07933068323	0.0868308139	0.08237900733	0.07847480743	0.07062203109	0.0753720974
0.9	0.905	0.91	0.915	0.92	0.925	0.93	0.935	0.94
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0.945	0.95	0.955	0.96	0.965	0.97	0.975	0.98	0.985
0.4964370447	0.5013673257	0.4634547347	0.5118355879	0.5299188345	0.5158792924	0.5448865765	0.5459167534	0.4809802289
0.945	0.95	0.955	0.96	0.965	0.97	0.975	0.98	0.985
0.05671363021	0.07279515092	0.06417184968	0.08352027368	0.07347825401	0.07086620075	0.08684432409	0.08267332249	0.09153043886
0.945	0.95	0.955	0.96	0.965	0.97	0.975	0.98	0.985
0	0	0	0	0	0	0	0	0

0	0	0
0.99	0.995	1
0.5491758585	0.4966805411	0.5430260146
0.99	0.995	1
0.07972676863	0.05611834339	0.06791067296
0.99	0.995	1
0	0	0

```matlab
% Nagel Schreckenberg Multi-Lane Model Simulation with Autonomous Vehicles
% Modified from the original code written by Alexander Farley
% https://www.mathworks.com/matlabcentral/fileexchange/34956-nagel-schreckenberg
%  -model-implementation/content/NaSchr.m

function l = nagel_schrenberg_basic_density(num_lanes, start_mile, end_mile, daily_traffic)

% -------------------------------- Variables --------------------------------
%       av_percent: percentage of autonomous vehicles on the road
%             vmax: maximum allowed velocity of the cars
%          hv_rand: "randomness" variable: probability a human car will
%                   decelerate
%          av_rand: "randomness" variable: probability an autonomous vehicle
%                   will decelerate
%      road_length: total length of observed road in units of car lengths.
%                   total miles, converted to feet, divided by 1 car length (14
%                   feet).
% total_time_steps: total simulation time. 1 step = time it takes a car
%                   traveling at vmax to move 10 car lenghts
%             road: an array num_lanes wide and road_length tall, where each
%                   cell represents a location a car can be.
%                   0 = no car; 1 = human-driven car; 2 = autonomous car
%        road_next: an array just like road, but containing the future positions
%                   of cars at the next time step
%       velocities: a parallel array to road, where each cell contains the
%                   velocity of the car in the respective cell in road
%  velocities_next: just like velocities, but contains data for the upcoming
%                   time step.
%          history: an array of past road arrays arrange horizontally,
%                   documenting car location data at every time step through the
%                   simulation
% velocity_history: same as history, but stores velocity data

hv_rand = 0.6;
av_rand = 0.05;
lane_change_prob = 0.85;

vmax             = 6;
road_length      = round(abs(end_mile - start_mile) * 5280 / 14);
total_time_steps = 10 * road_length;
road             = zeros(num_lanes, road_length);
velocities       = zeros(num_lanes, road_length);
road_next        = road;
velocities_next  = velocities;
history          = zeros(total_time_steps * num_lanes, road_length);
velocity_history = zeros(total_time_steps * num_lanes, road_length);

density = ((daily_traffic * .08) / 60 / (end_mile - start_mile) / (road_length * num_lanes));


% Graph-related drawing variables
render_on = 0;
pause_on = 0;
delay_on = 0;
delay_length = 0.05; % 10 FPS


% Running multiple simulations
num_sims    = 200;
simulations = zeros(2, num_sims); % Contains flow rate for each sample
collisions  = 0;


figure


for curr_sim = 1:num_sims

    % Varying the percentage of autonomous vehicles with each trial
    av_percent = curr_sim / num_sims;

    % Resets road to be full of zeroes at start of new sim
    road = zeros(num_lanes, road_length);
    road_next = road;

    % Steps through road and places cars randomly based on density
    for curr_lane = 1:num_lanes
        for curr_index = 1:road_length
            if rand < density
                if rand < av_percent
                    road(curr_lane, curr_index) = 2;
                else
                    road(curr_lane, curr_index) = 1;
                end;
            end
        end
    end

```

```
    % Drawing graph
    if render_on
        imshow(road);
        drawnow
    end

    % Run through one time step
    for time = 1:total_time_steps

        % Record current locations and velocities in history
        for curr_lane = 1:num_lanes
            history((time - 1) * num_lanes + curr_lane, :) = road(curr_lane, :);
            velocity_history((time - 1) * num_lanes + curr_lane, :) = velocities(curr_lane, :);
        end

        % ------------------------- Lane Changing ------------------------- %

        % Loop through each lane
        for curr_lane = 1:num_lanes

            % Loop through each 'cell' in the lane
            for curr_car = 1:road_length

                % Checks for a car
                if road(curr_lane, curr_car) == 0
                    continue
                end


                % STEP 1: CHECK CURR LANE LEAD GAP ----------------------------

                desired_velocity = min(velocities(curr_lane, curr_car) + 1, vmax);
                lead_gap = 0;
                barrier = 0;

                % Seek ahead
                for v_curr = 1:desired_velocity
                    lead_gap = v_curr;

                    % Updates index, wraps to beginning
                    index = mod(curr_car - 1 + v_curr, road_length) + 1;

                    if road(curr_lane, index) ~= 0
                        barrier = 1;
                        break
                    end

                end

                % check if curr lane conditions are met
                if barrier ~= 1
                    break
                end


                % Sets target lane to be lane to left and lane to right
                for target_lane = (num_lanes - 1):2:(num_lanes + 1)

                    % Checks that target lane is within bounds
                    if (target_lane > num_lanes) || (target_lane < 1)
                        continue
                    end


                    % STEP 2: CHECK TARGET LANE LEAD GAP -----------------------

                    barrier = 0;

                    % Seek ahead
                    for v_curr = 0:desired_velocity
                        target_lead_gap = v_curr;

                        % Updates index, wraps to beginning
                        index = mod(curr_car - 1 + v_curr, road_length) + 1;

                        if road(target_lane, index) ~= 0
                            barrier = 1;
                            break
                        end
                    end

                    % check if target lane lead conditions are met
                    if (target_lead_gap <= lead_gap) && (barrier == 1)
                        continue
                    end


                    % STEP 3: CHECK TARGET LANE LAG GAP ------------------------
```

```
                barrier = 0;

                % Seek behind
                for v_curr = 1:vmax

                    % Updates index, wraps to beginning
                    index = mod(curr_car - 1 + v_curr, road_length) + 1;

                    if road(target_lane, index) ~= 0
                        barrier = 1;
                        break
                    end
                end

                % check if target lane lag conditions are met
                if barrier == 1
                    continue
                end


                % STEP 4: RANDOM CHANCE OF CHANGE --------------------------

                if rand < lane_change_prob
                    road(target_lane, curr_car) = road(curr_lane, curr_car);
                    road(curr_lane, curr_car) = 0;
                    velocities(target_lane, curr_car) = velocities(curr_lane, curr_car);
                end

                break
            end
        end
    end



    % ------------------------ Velocity Update ------------------------ %

    % Loop through each lane
    for curr_lane = 1:num_lanes

        % Loop through each 'cell' in the lane
        for curr_car = 1:road_length

            % Checks for a car
            if road(curr_lane, curr_car) == 0
                continue
            end

            % Distane = how much the car will move up in the next step
            distance = 0;

            barrier = 0;

            % Seek ahead
            for v_curr = 1:vmax
                distance = v_curr;

                % Updates index, wraps to beginning
                index = mod(curr_car - 1 + v_curr, road_length) + 1;

                if road(curr_lane, index) ~= 0
                    barrier = 1;
                    break
                end
            end

            % Acceleration
            if velocities(curr_lane, curr_car) < vmax
                velocities(curr_lane, curr_car) = velocities(curr_lane, curr_car) + 1;
            end

            % Braking for collision avoidance
            if (velocities(curr_lane, curr_car) >= distance) && (barrier == 1)
                velocities(curr_lane, curr_car) = distance - 1;
            end

            % Braking based on random deceleration
            if velocities(curr_lane, curr_car) > 0
                if (road(curr_lane, curr_car) == 1 && rand < hv_rand)
                    velocities(curr_lane, curr_car) = velocities(curr_lane, curr_car) - 1;
                elseif road(curr_lane, curr_car) == 2 && rand < av_rand
                    velocities(curr_lane, curr_car) = velocities(curr_lane, curr_car) - 1;
                end
            end
        end
    end

```

```
        % ------------------------- Location Update ------------------------- %

        % Loop through lanes
        for curr_lane = 1:num_lanes

            % Loop through possible car locations in road
            for curr_car = 1:road_length

                % Checks for a car
                if road(curr_lane, curr_car) == 0
                    continue
                end

                % Updates index, wraps to beginning
                loc_next = mod(curr_car - 1 + velocities(curr_lane, curr_car), road_length) + 1;

                % Collision detection
                if road_next(curr_lane, loc_next) ~= 0
                    disp('Collision detected!')
                    collisions = collisions + 1;
                end

                % Places car and its velocity in their future location
                road_next(curr_lane, loc_next) = road(curr_lane, curr_car);
                velocities_next(curr_lane, loc_next) = velocities(curr_lane, curr_car);
            end
        end

        % Copies over road and velocity data and resets them
        velocities      = velocities_next;
        road            = road_next;
        velocities_next = zeros(num_lanes,road_length);
        road_next       = zeros(num_lanes, road_length);

        % Graphing things
        if render_on
            imshow(road);
            drawnow
        end

        if pause_on
            pause
        end

        if delay_on
            pause(delay_length)
        end

    end

    % Record flow rate for this run of the simulation.
    simulations(:, curr_sim) = [av_percent sum(velocity_history(:)) / nnz(history) * nnz(history) / numel(history)];

    % Display progress
    disp('Simulation Number:')
    curr_sim

end

disp ('Total collisions:')
collisions

l = simulations;

% Draws graph
scatter(simulations(1,:), simulations(2,:));

axis([0 1 0 1]);
xlabel('Percent Automated Vehicles')
ylabel('Flow (normalized)')
title('Nagel Schreckenberg Flow-density Curve')
```