GOAL: DEFINE VALUES THAT SUMMARIZE A PROBABILITY DISTRIBUTION

variable name
$$P(X = x)$$
↑ value
eg 3, "e", etc

expectation
$$E[X] = \sum_x x\, P(X=x)$$
think "for"

$$P(X=0) = \tfrac{1}{2}$$
$$P(X=1) = \tfrac{1}{2}$$

$$E[X] = 0 \cdot \tfrac{1}{2} + 1 \cdot \tfrac{1}{2} = \tfrac{1}{2}$$
easy to calculate

$$P(X=0) = \tfrac{1}{100}$$
$$\vdots$$
$$P(X=99) = \tfrac{1}{100}$$

hard to do by hand — use computers!

expectation of a function of X
$$E[x^2] = \sum_x x^2 P(X=x)$$
$$= 0^2 \cdot \tfrac{1}{2} + 1^2 \cdot \tfrac{1}{2} = \tfrac{1}{2}$$

shift the dist'n to mean ⓪

$$E[X - E[X]] = E[X] - E[E[X]]$$
expectation is a sum, ∴ linear
("therefore")
$$= E[X] - E[X]$$

same for all x
$$\sum_x (E[X]) P(X=x)$$
$$= E[X] \sum_x P(X=x)$$
$$= E[X] \cdot 1$$

ok, not actually hard:
$$\sum_{x=1}^{100} x\, P(X=x) =$$
$$\frac{1}{100} \sum_{x=1}^{100} x = \frac{1}{100} \cdot \frac{100 \cdot (100-1)}{2} = 49.5$$
Then shift down by 1

$$E[(X - E[x])^2] = Var(X)$$
square the value to make it positive