CSE-170 Computer Graphics

Lecture 22

Implicit Curves and Metaballs

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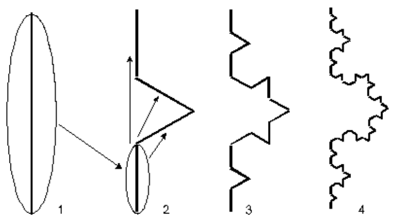
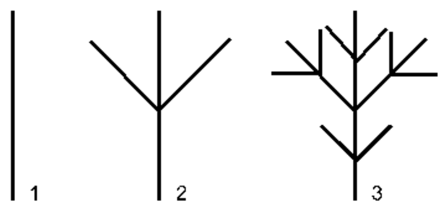
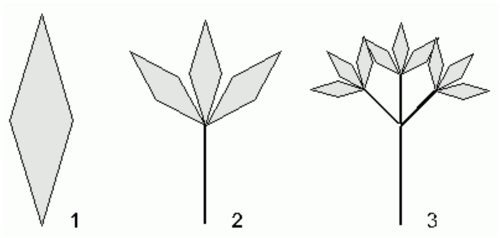
Curves

- Main approaches to define curves:
  - Procedural curves: fractals, subdivision rules







  - Parametric curves: are mappings
    - Ex.: Continuous map from 1D space to n-D space

      $f(t)=(x,y)$, ex: $f(t)=(\cos t, \sin t)$

  - Implicit curves: defined by an equation
    - Described by all points satisfying an equation

      $f(x,y)=0$ ex: $x^2+y^2-1=0$

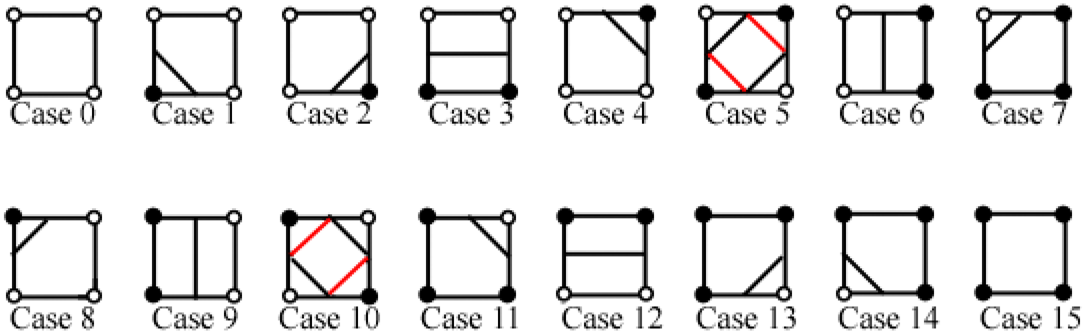
Drawing Implicit Curves with Marching Cubes and Marching Squares

Implicit Curves

- How to draw an arbitrary implicit curve?
  - Main algorithm: Marching Cubes

Implicit Curves

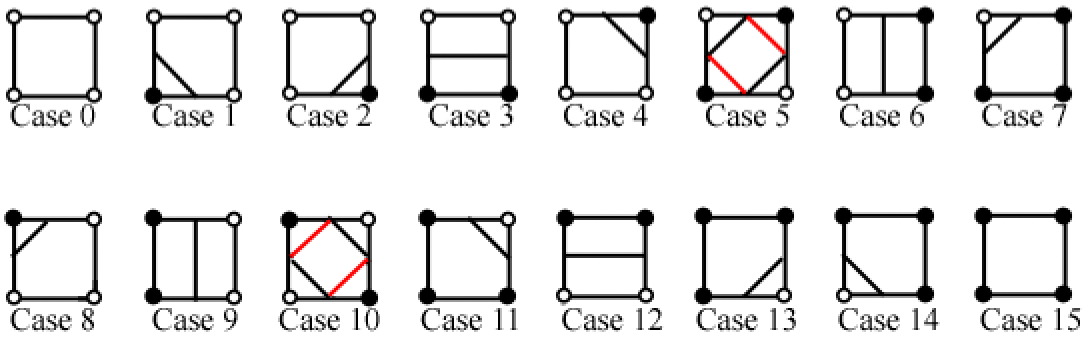
- How to draw an arbitrary implicit curve?
  - Marching squares (marching cubes for 3D)
    - Variations: instead of cubes, use tetrahedra/triangles
    - Extensions: adaptive, etc.



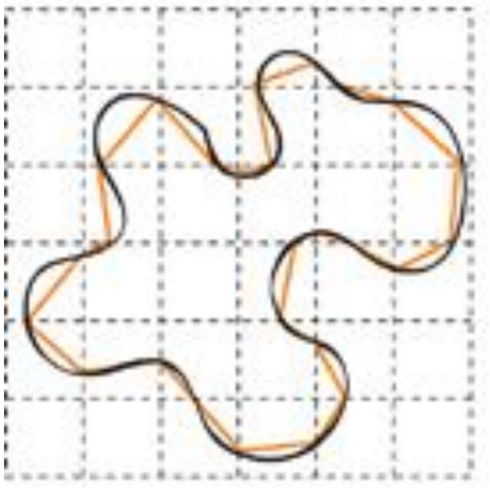
http://users.polytech.unice.fr/~lingrand/MarchingCubes/algo.html

Implicit Curves

- Cases

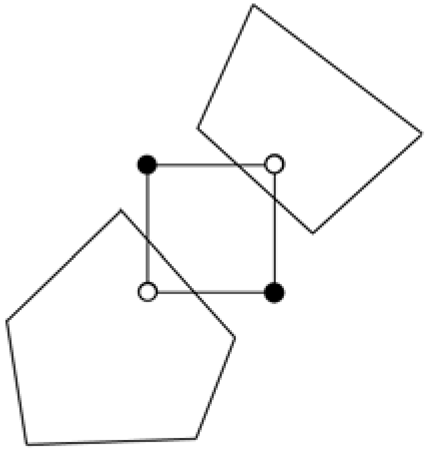


- How to apply cases:
  1) just use midpoint of each cell edge (simpler)
  2) interpolate along cell edges according to weights from the function evaluations at vertices (better)

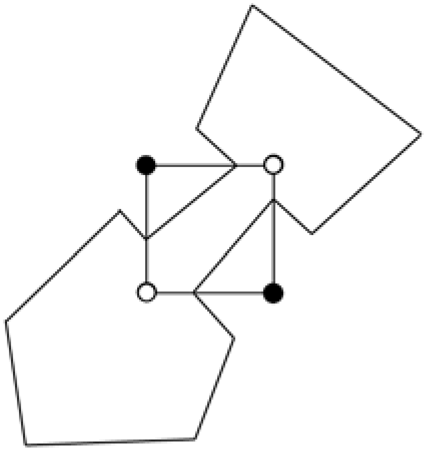


Implicit Curves

- Marching Squares
  - Ambiguous cases are possible
  - Increasing the grid evaluation resolution may solve ambiguities
  - For ex., the two options below are both possible:



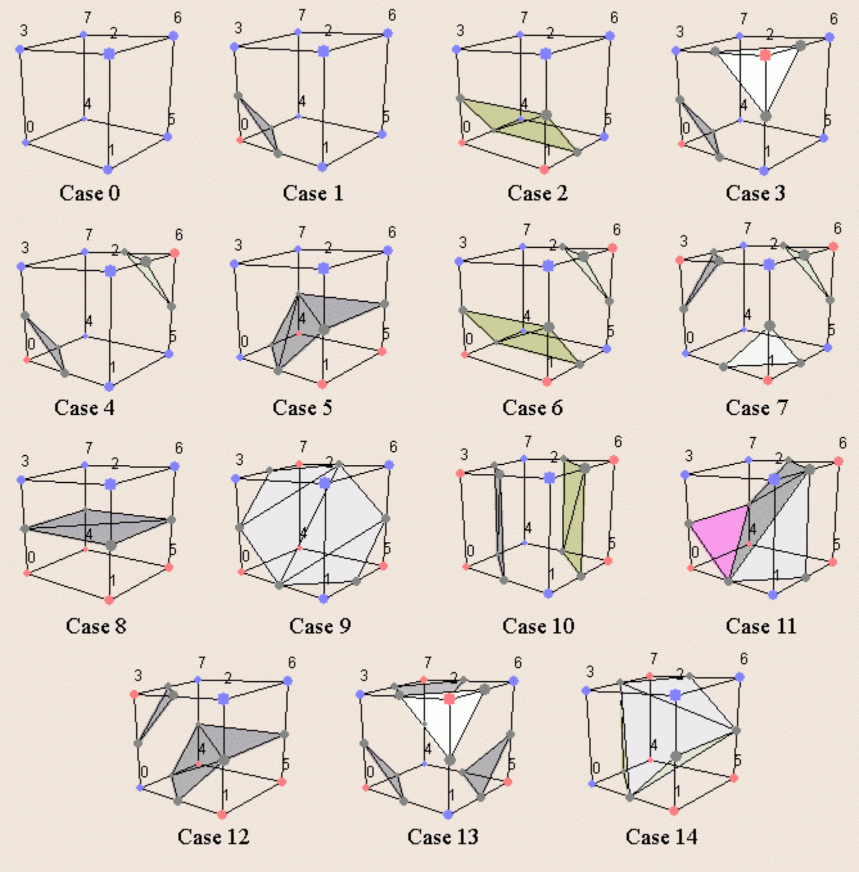
Break contour



Join contour

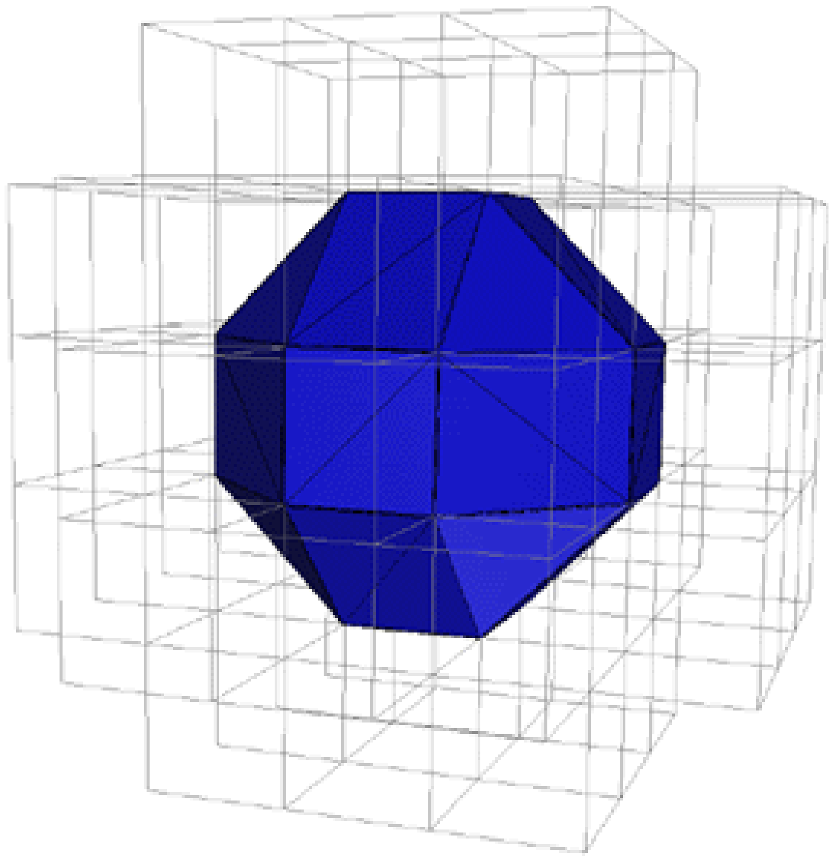
Implicit Surfaces

- Marching Cubes in 3D
  - More cases to consider
  - More ambiguities possible



Implicit Surfaces

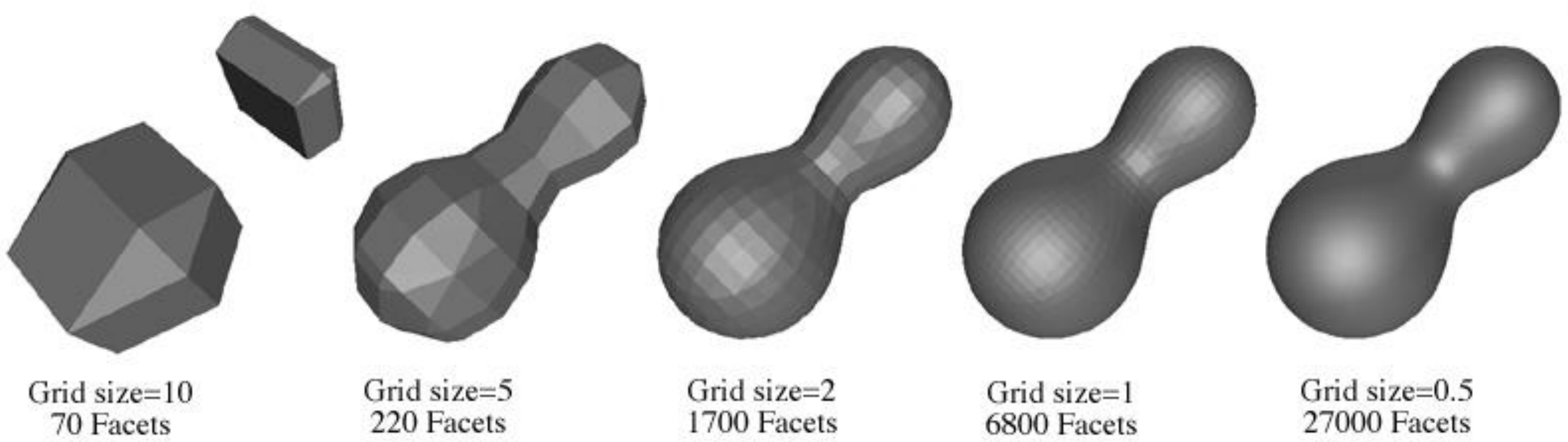
- Example:



http://paulbourke.net/geometry/polygonise/

Implicit Surfaces

- Example:

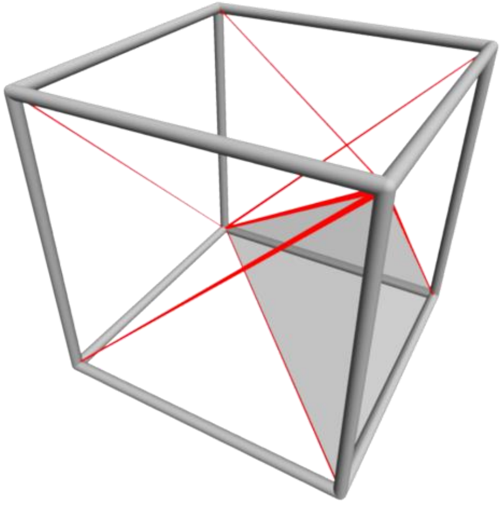


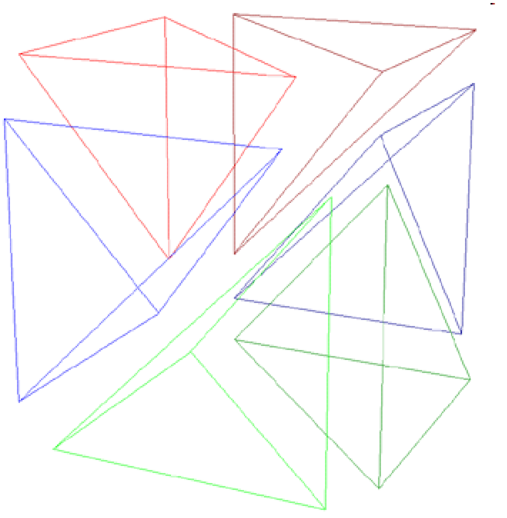
Implicit Surfaces

- Marching Tetrahedra
  - Subdivide cube in tetrahedra
    - How?
    - Why?
      - When we use tetrahedra as cells we have less vertices per cell (4 and not 8): thus, less cases to consider per cell

Implicit Surfaces

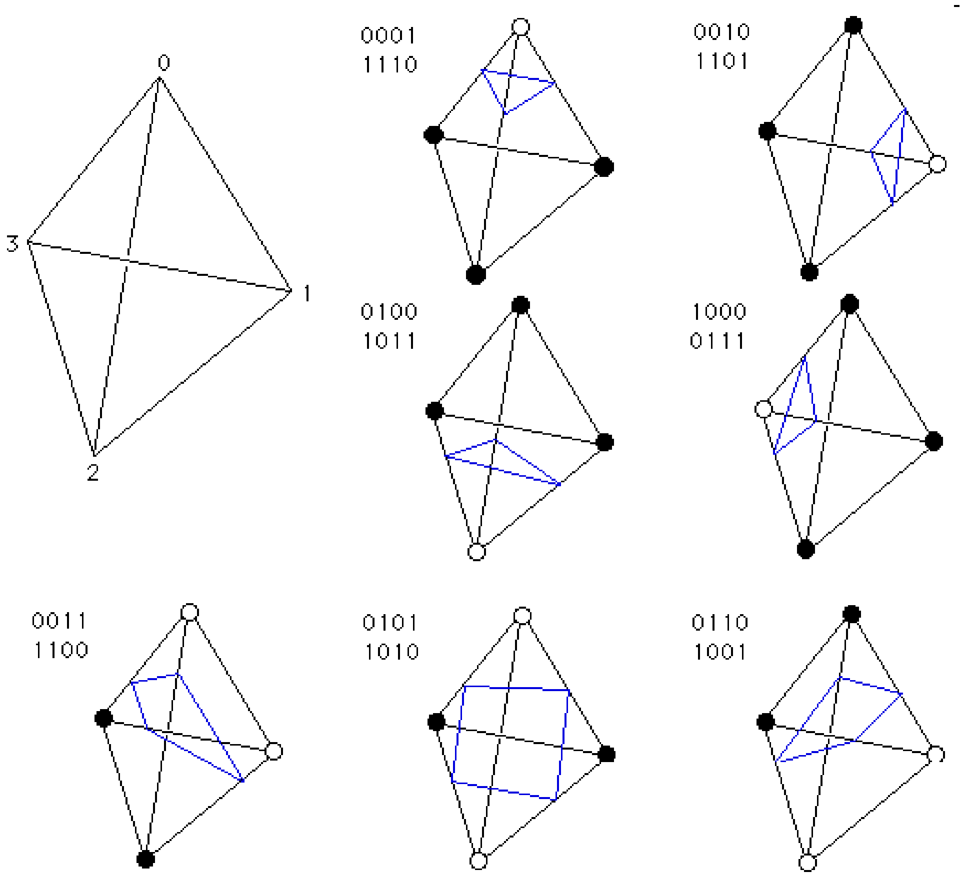
- Marching Tetrahedra
  - Subdivide cube in tetrahedra
    - How?
  - A cube can be divided in 5 or 6 tetrahedra
    - Example with 6 is shown
      - All share a diagonal





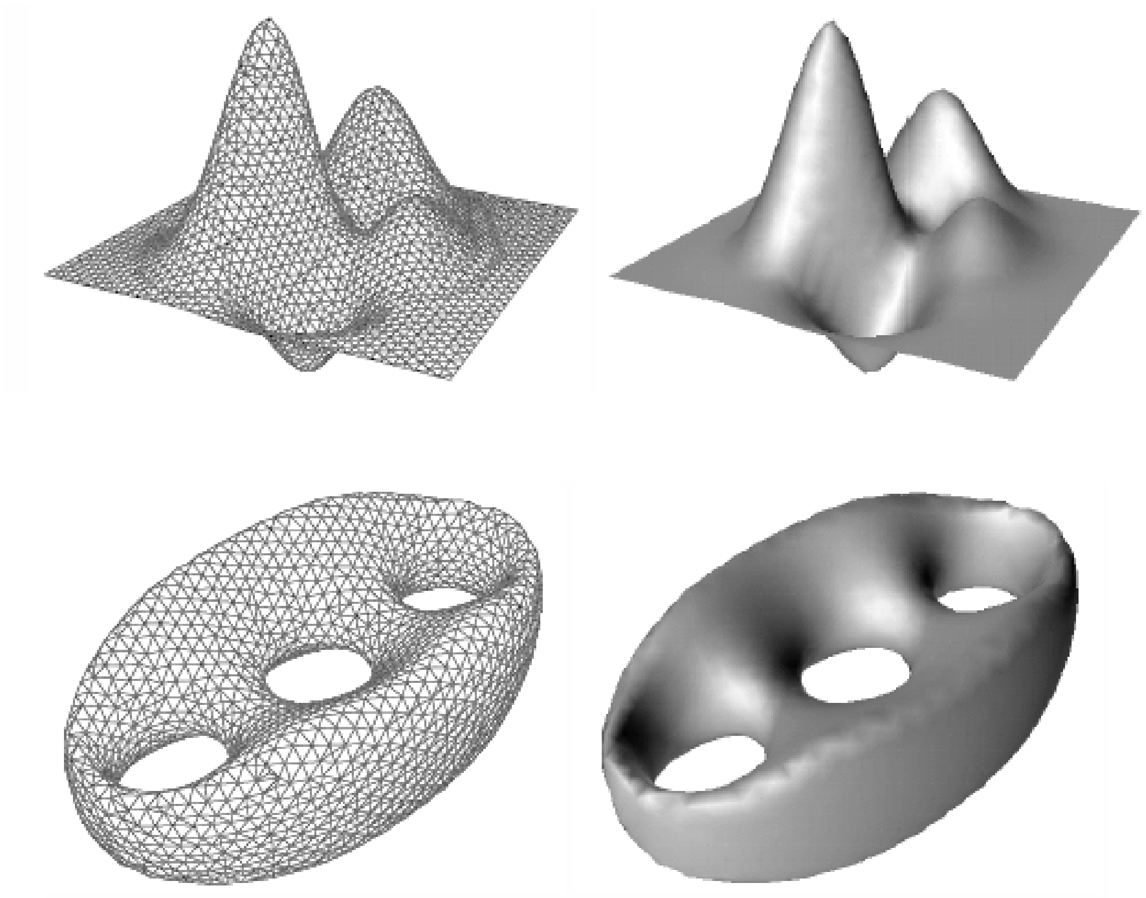
Implicit Surfaces

- Marching Tetrahedra
  - Computing intersections



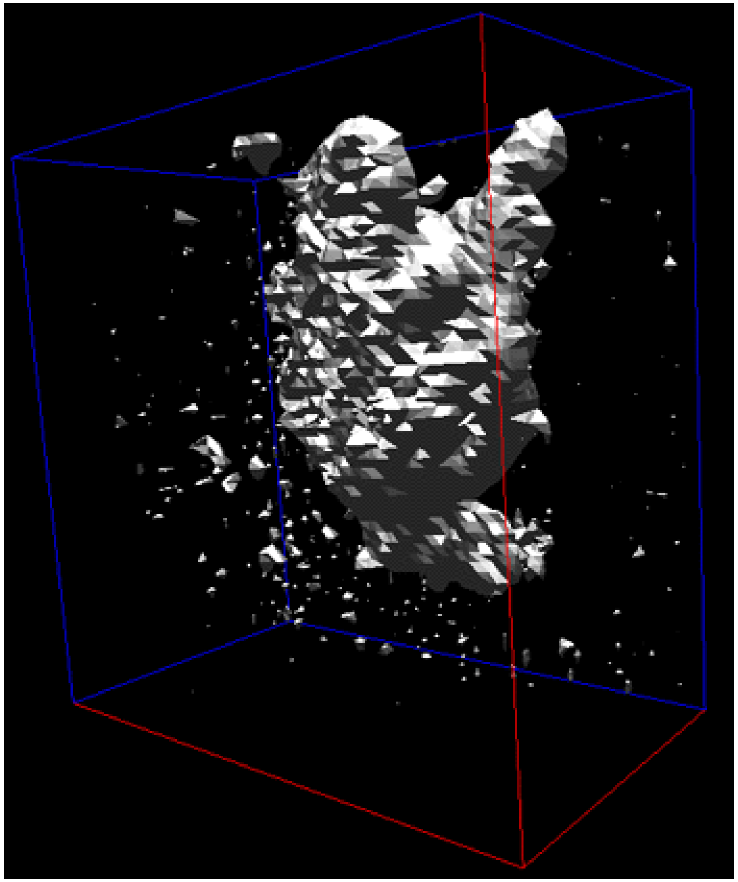
Implicit Surfaces

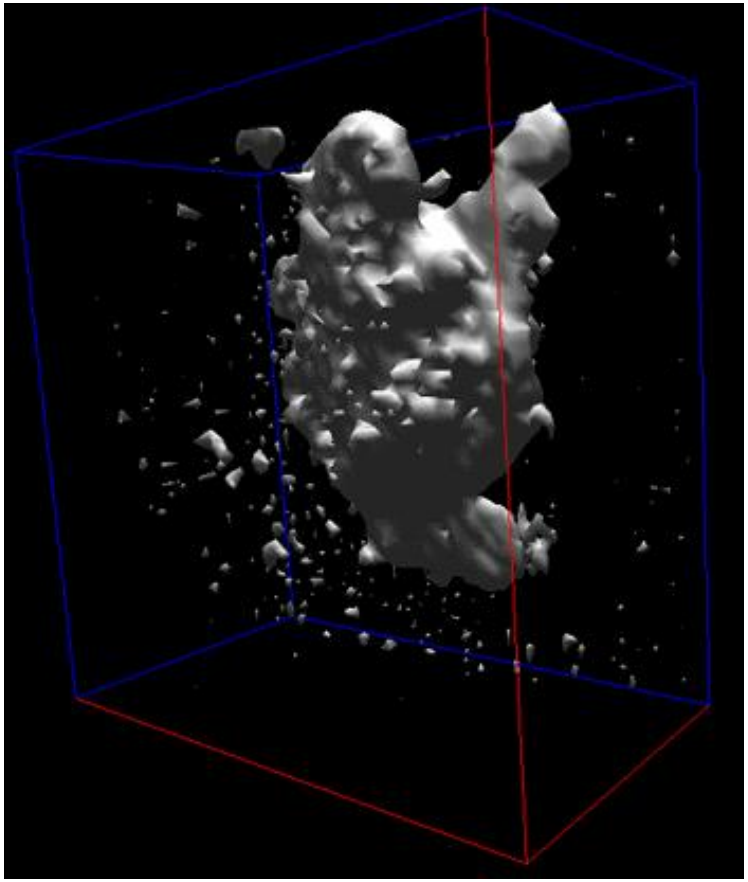
- Examples:



Implicit Surfaces

- Examples:

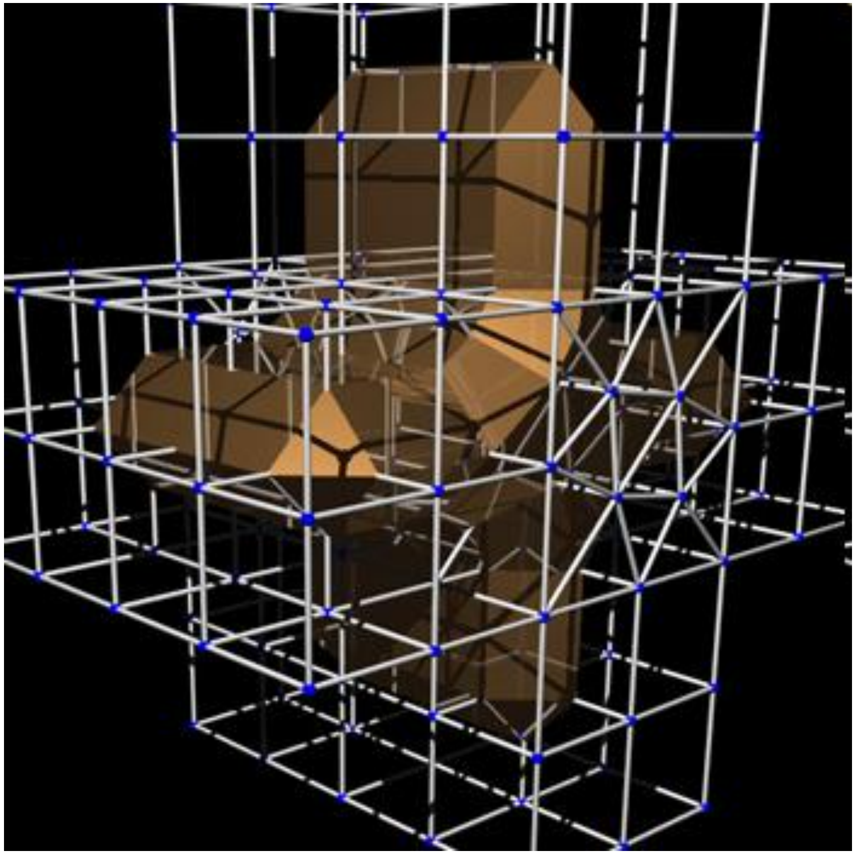




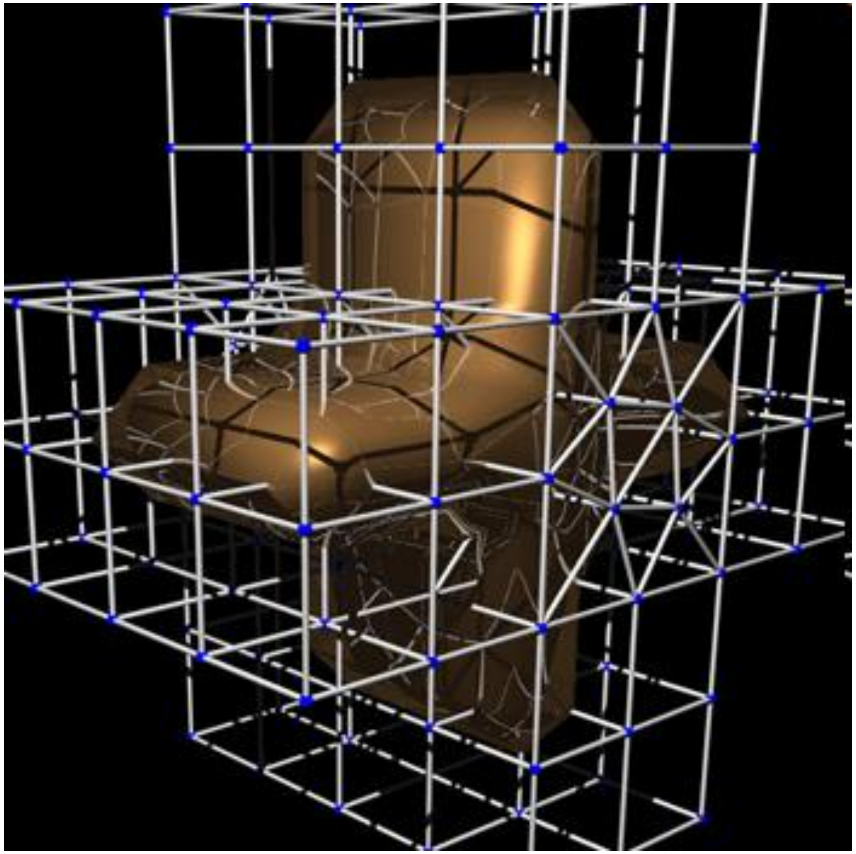
Implicit Surfaces

- Examples:

Result:

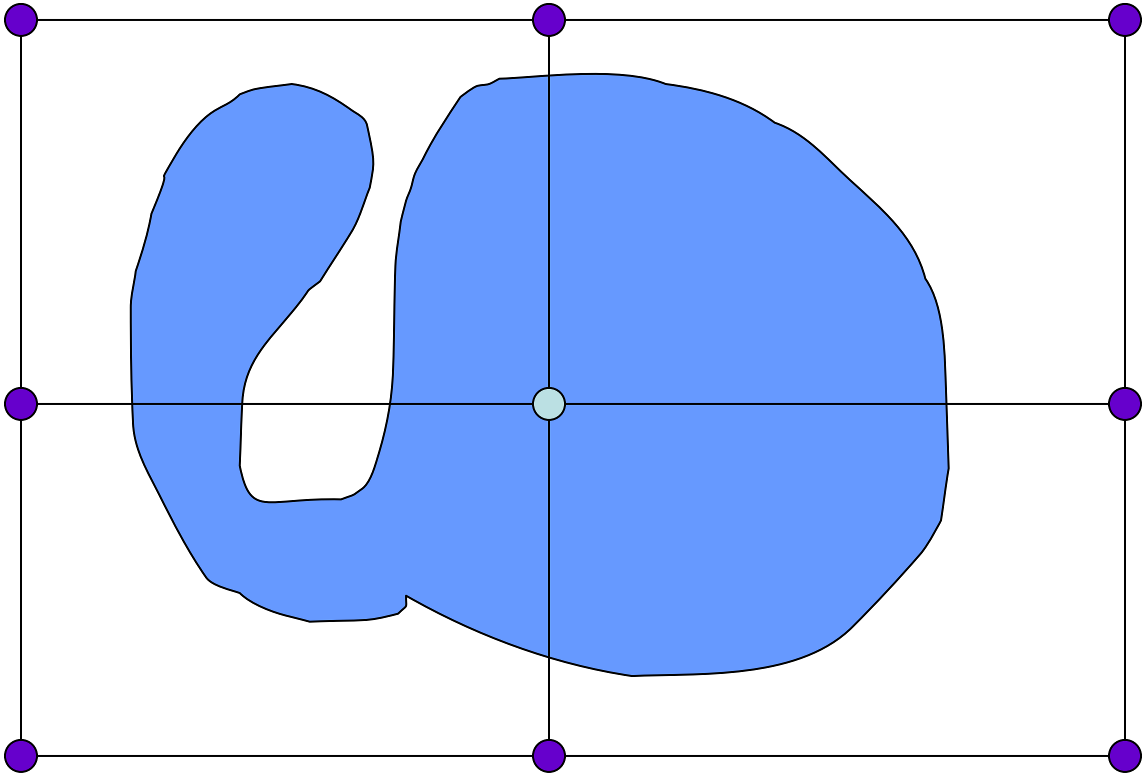


With Additional Smoothing:



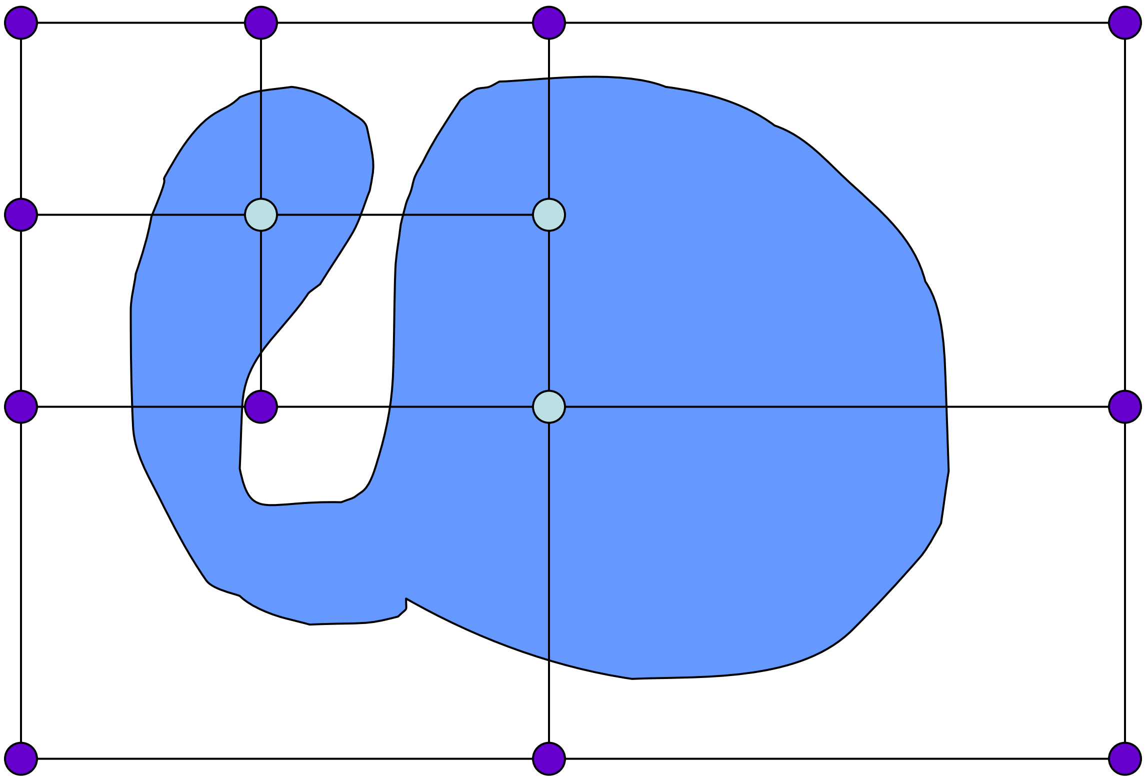
Adaptive Marching Cubes

- "Octree/quadtree" recursive subdivision:



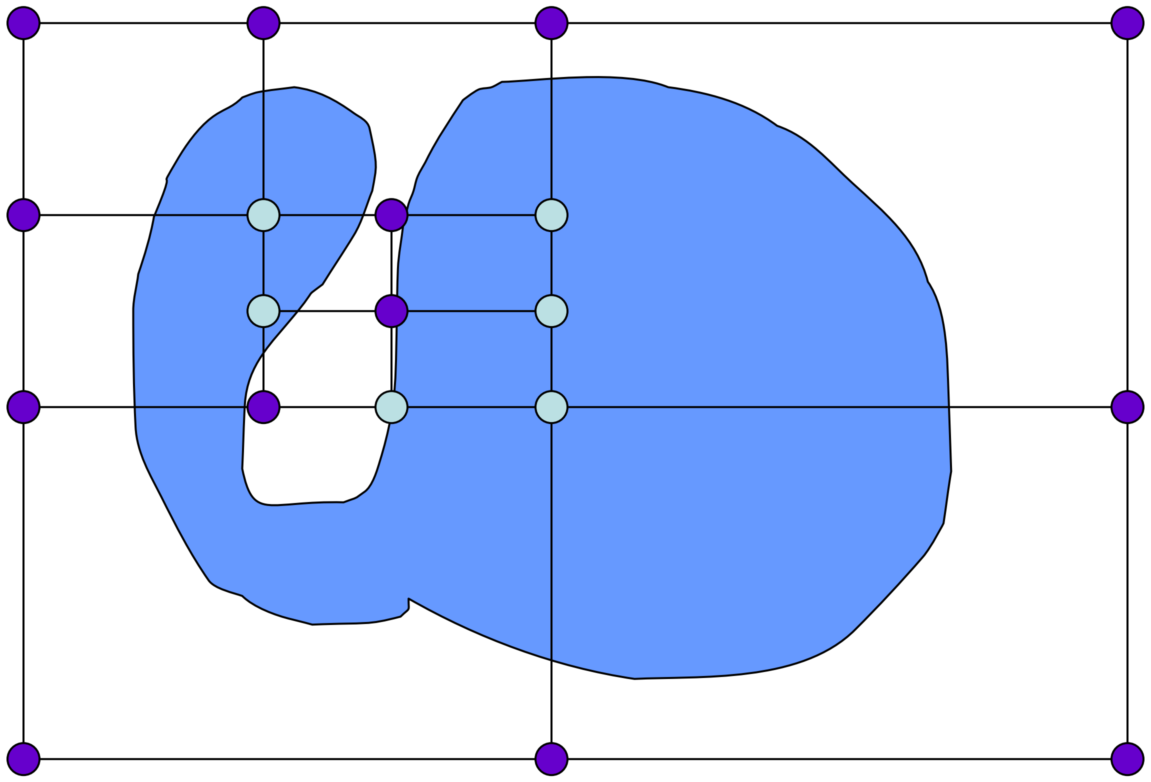
Adaptive Marching Cubes

- Each "mixed region" is further subdivided



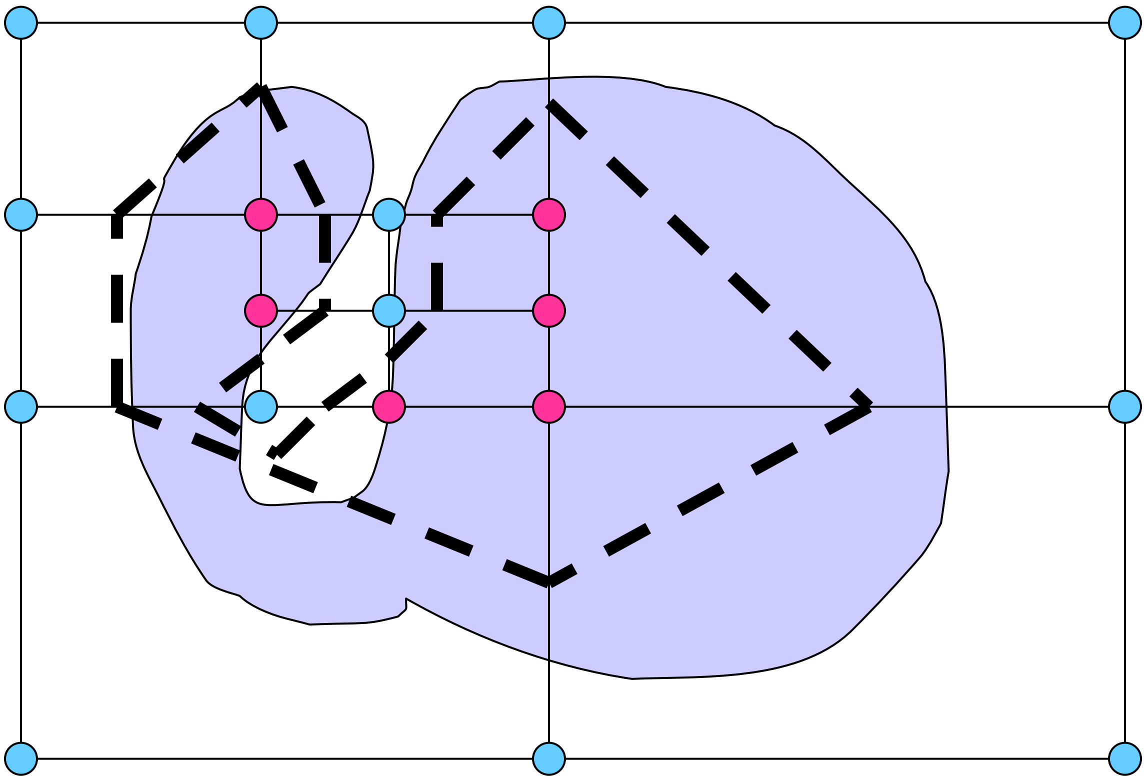
Adaptive Marching Cubes

- Until desired precision is reached



Adaptive Marching Cubes

- Boundary reconstruction is then based on cells of different sizes



Metaballs

Metaballs

- Basic idea of metaballs
  - Use implicit equation of the form:

$$\sum_{i=0}^{n} \text{metaball}_i(x,y,z) \leq \text{threshold}$$

  - ...where each metaball function f(x,y,z) is for ex:

$f(x,y,z) = 1/((x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2) = 1/r^2$,

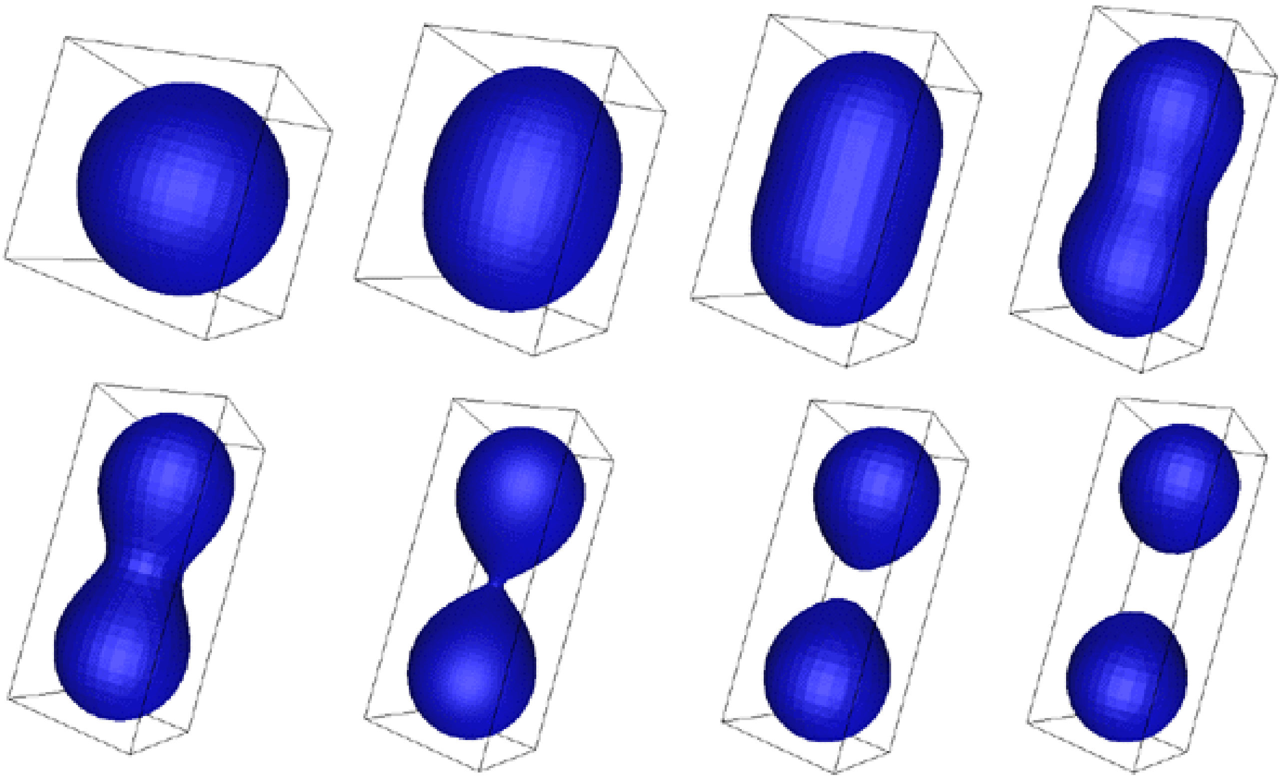
with (x_0, y_0, z_0) being the center of the metaball, and r = distance ((x,y,z), (x_0,y_0,z_0)).

So: $f(x,y,z) = 1/r^2$

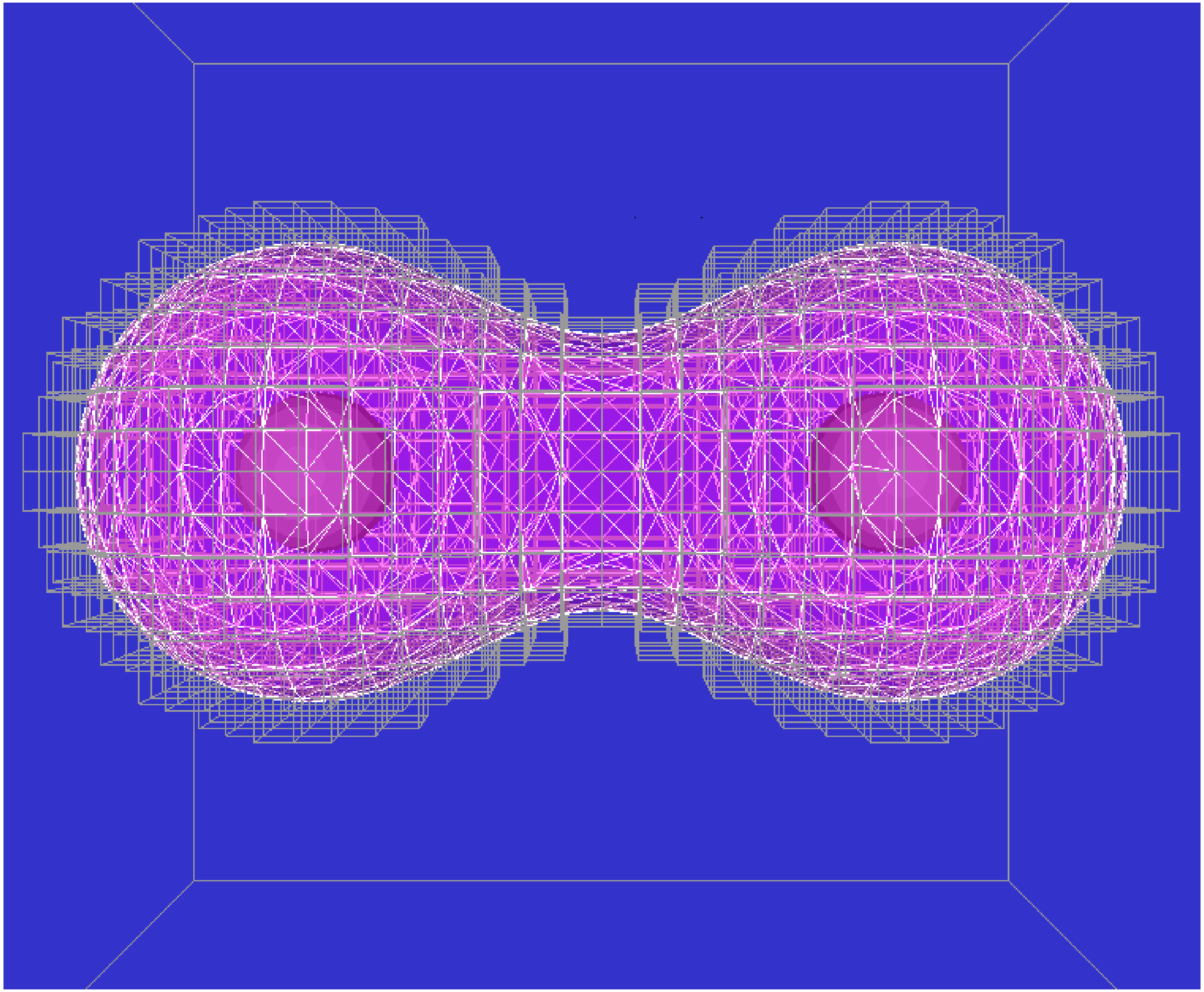
  - Other formulations exist

Metaballs: result

- Final surface depends on the proximity between the metaball centers:



Metaballs: marching cubes evaluation



Many uses