CSE-170 Computer Graphics

Lecture 7

Barycentric Coordinates

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Barycentric Coordinates

- Given point **p** and reference points **x**, **y**, **z**, the barycentric coordinates of **p** are 3 coordinates alpha, beta, gamma, such that:

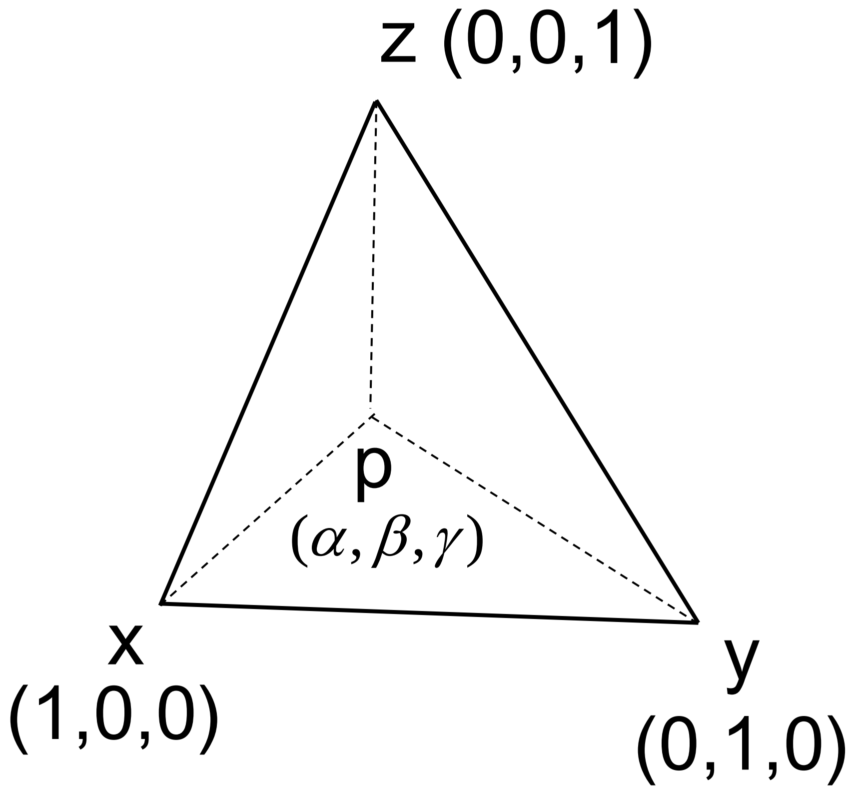
$$\mathbf{p} = \alpha\ \mathbf{x} + \beta\ \mathbf{y} + \gamma\ \mathbf{z}$$

$$\alpha + \beta + \gamma = 1$$

- Note that the above definition works in any dimension
  - while in these slides **x**, **y**, **z** and **p** are seen in 2D, barycentric coordinates work in the exact same way in 3D

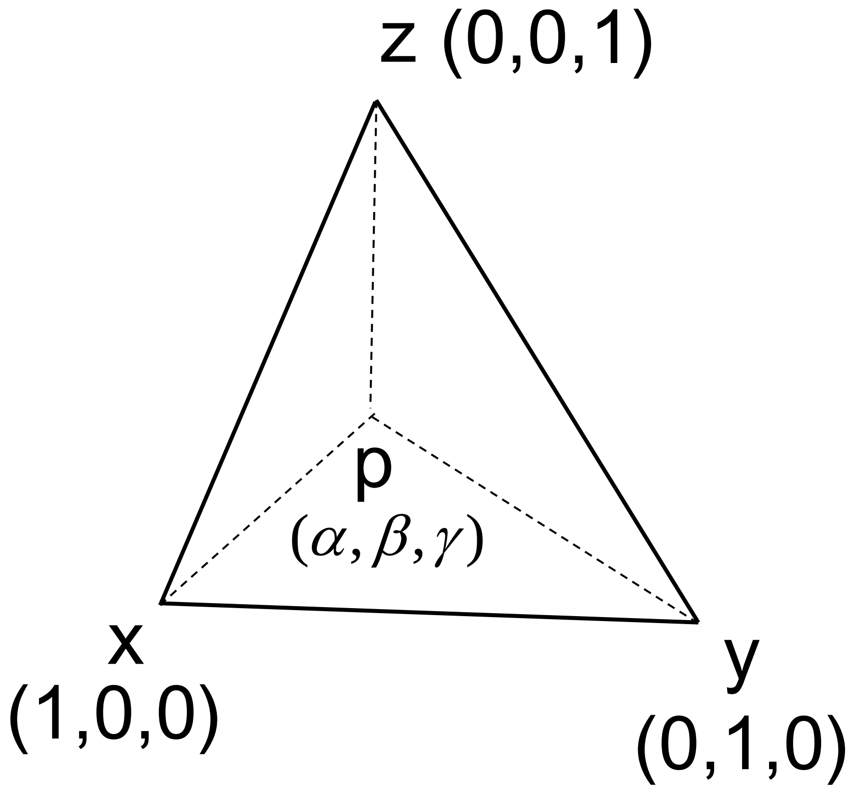
Barycentric Coordinates

- Triangle interpretation:
  - Barycentric coordinates describe **p** with respect to the triangle
    - The triangle becomes its "frame of reference"



Barycentric Coordinates

- Question:
  - How to compute the barycentric coordinates for a given point and triangle?

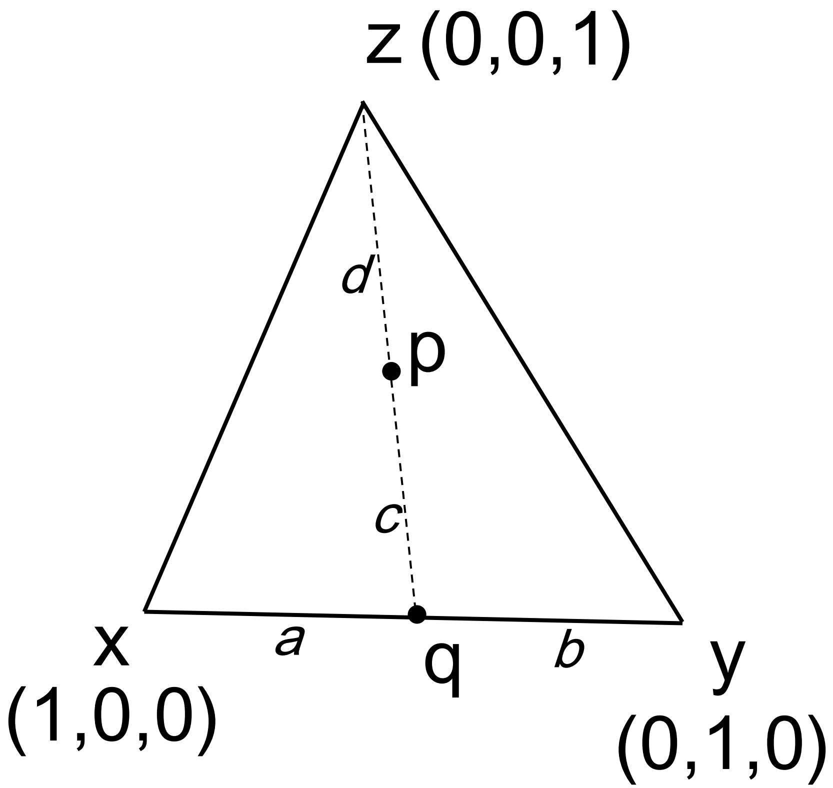


Barycentric Coordinates

- The coordinates can be achieved by two linear interpolations:

$$\mathbf{p} = \alpha\ \mathbf{x} + \beta\ \mathbf{y} + \gamma\ \mathbf{z}$$

$$\alpha + \beta + \gamma = 1$$



Barycentric Coordinates

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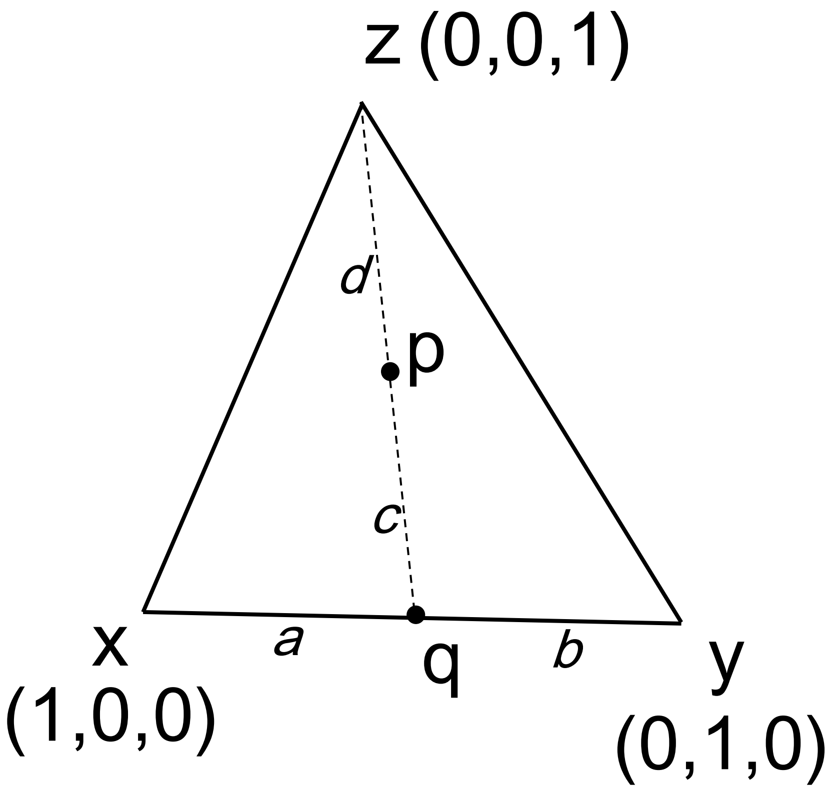

$\mathbf{q} = a\mathbf{x} + (1\text{-}a)\mathbf{y}$

$\mathbf{q} = a\mathbf{x} + b\mathbf{y}$, $a + b = 1$

Barycentric Coordinates

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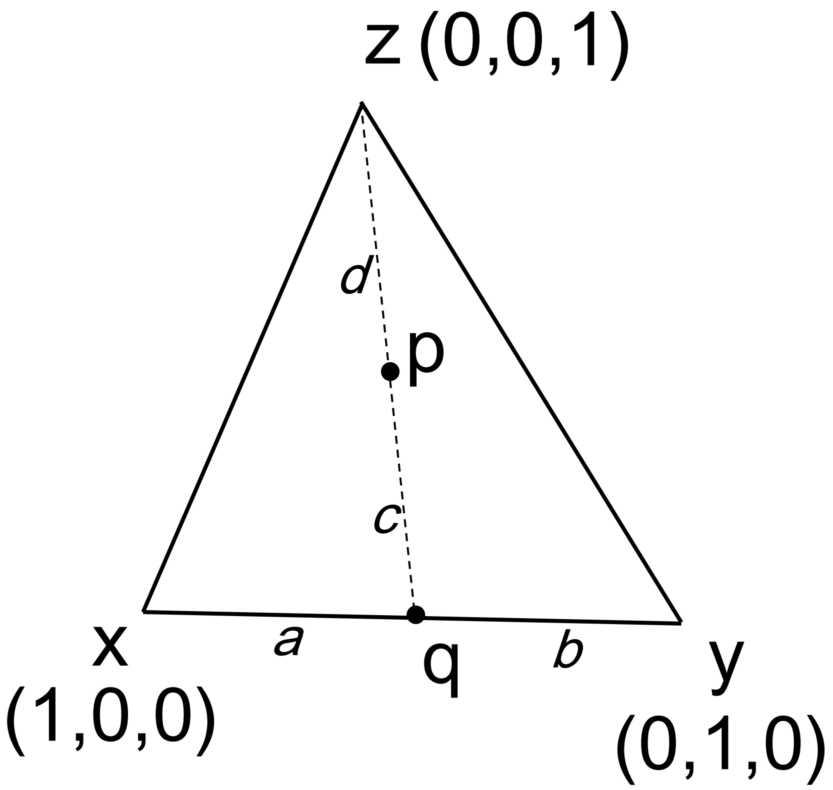

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$\mathbf{p} = c\mathbf{q} + d\mathbf{z}$, $c + d = 1$

Barycentric Coordinates

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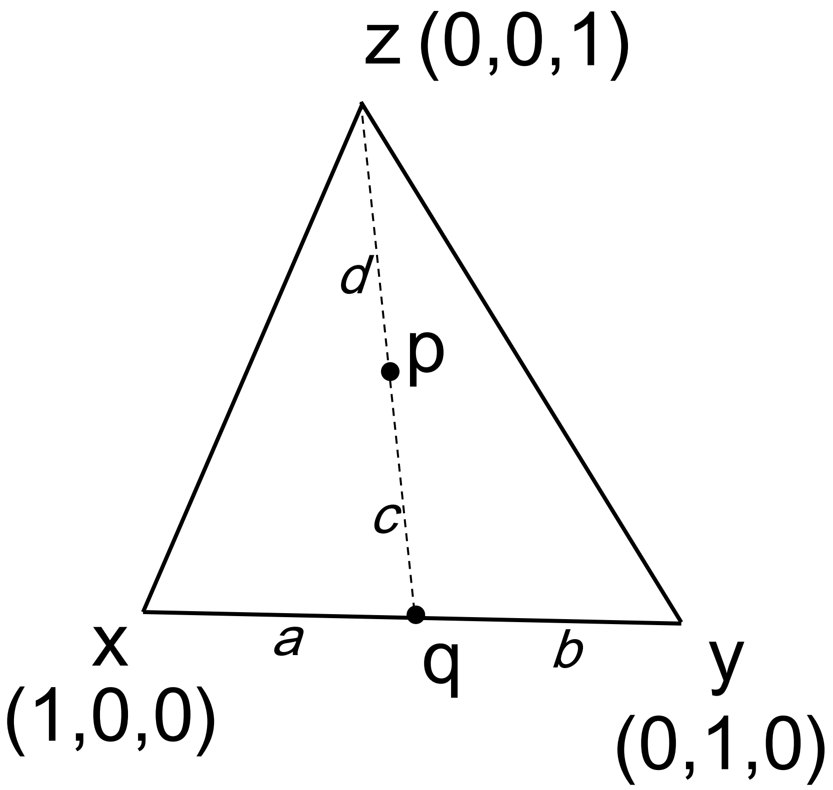


$$\mathbf{q} = a\mathbf{x} + (1-a)\mathbf{y}$$

$$\mathbf{q} = a\mathbf{x} + b\mathbf{y},\ a + b = 1$$

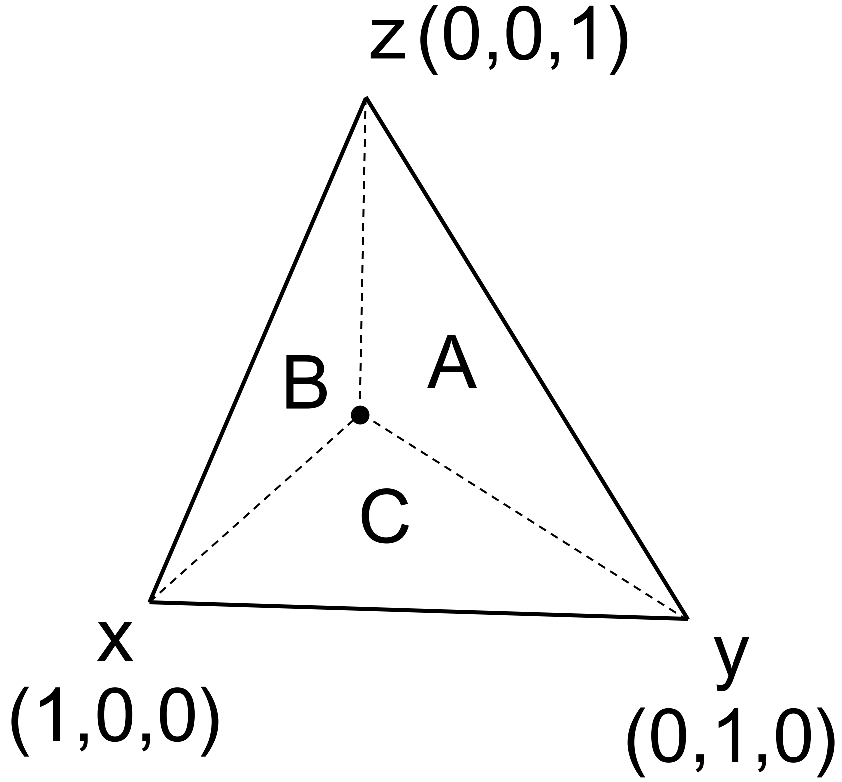
$$\mathbf{p} = c\mathbf{q} + d\mathbf{z},\ c + d = 1$$

$$\mathbf{p} = c(a\mathbf{x} + b\mathbf{y}) + d\mathbf{z}$$

$$\mathbf{p} = \underbrace{ca}_{\alpha}\mathbf{x} + \underbrace{cb}_{\beta}\mathbf{y} + \underbrace{d}_{\gamma}\mathbf{z}$$

Barycentric Coordinates

- Coordinates at vertices
  - Note that barycentric coordinates at the vertices will always have one ‘1’ and two ‘0’s



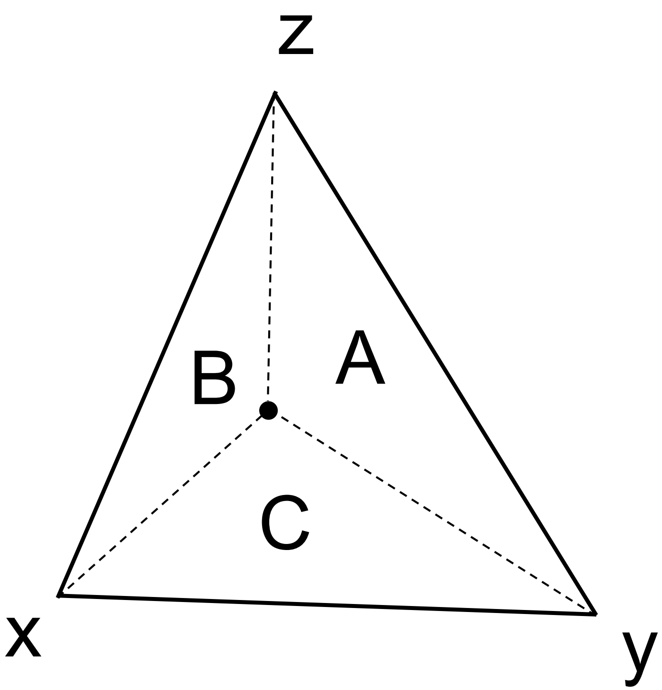
Barycentric Coordinates

- Sub-areas relations are also represented by barycentric coordinates:

$$\alpha = \frac{A}{A+B+C}$$

$$\beta = \frac{B}{A+B+C}$$

$$\gamma = \frac{C}{A+B+C}$$



Barycentric Coordinates

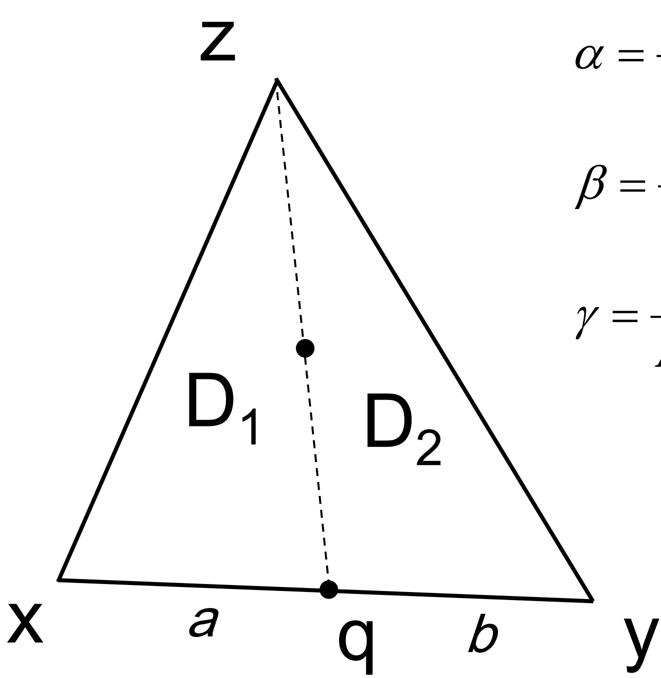
- Sub-areas relations:

D=D1+D2, D1=aD, D2=bD, (a+b=1)

C=E1+E2, E1=cD1, B=dD1, (c+d=1)

E2=cD2, A=dD2

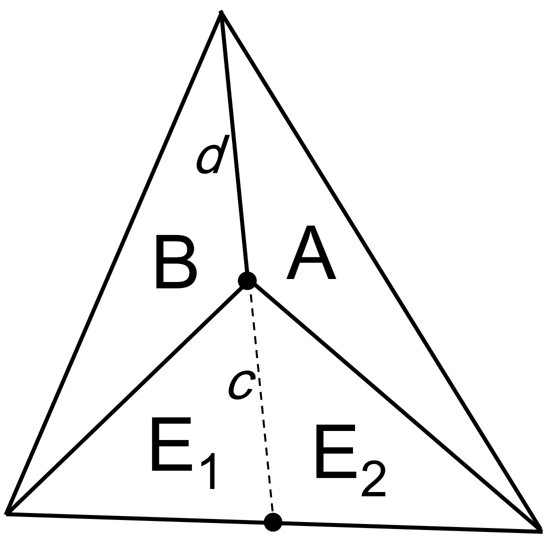
A=dbD, B=daD, C=c(a+b)D=cD



$$\alpha = \frac{A}{A+B+C}$$

$$\beta = \frac{B}{A+B+C}$$

$$\gamma = \frac{C}{A+B+C}$$



(C=E1+E2)

Barycentric Coordinates

- Coordinates can be also computed using the Cramer's rule
  - Ex. Cramer's rule for solving a 3x3 system:

$$ax + by + cz = j,$$
$$dx + ey + fz = k \text{ and}$$
$$gx + hy + iz = l,$$

which in matrix format is

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} j \\ k \\ l \end{bmatrix}$$

x, y and z can be found like so:

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}, \text{ and } z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}$$

Barycentric Coordinates

- Using the Cramer's rule
  - Solve the linear system:

$$p = \alpha x + \beta y + \gamma z$$
$$\alpha + \beta + \gamma = 1$$

  - Which can be written as:

$$\begin{pmatrix} x_x & y_x & z_x \\ x_y & y_y & z_y \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} p_x \\ p_y \\ 1 \end{pmatrix}$$

Barycentric Coordinates

- So:

$$\alpha = \frac{\begin{vmatrix} \mathbf{p} & \mathbf{y} & \mathbf{z} \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} \mathbf{x} & \mathbf{y} & \mathbf{z} \\ 1 & 1 & 1 \end{vmatrix}},$$

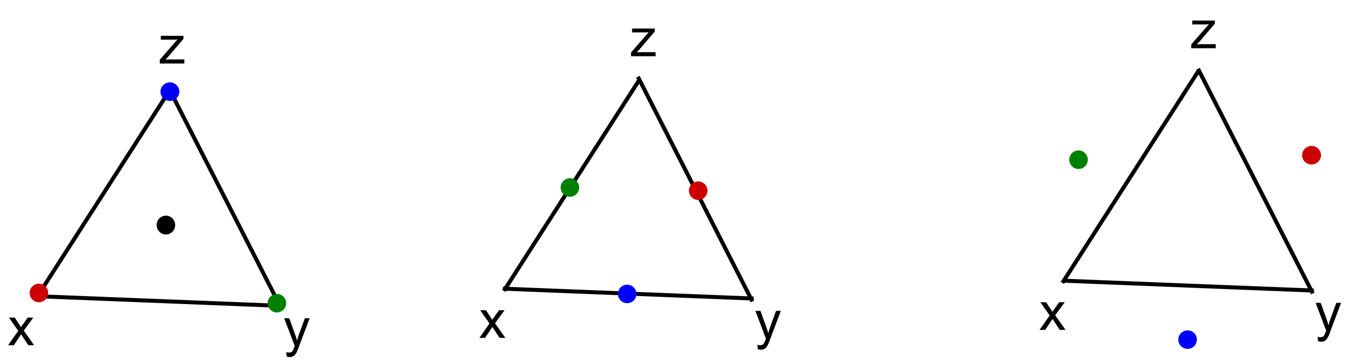
$$\beta = \frac{\begin{vmatrix} \mathbf{x} & \mathbf{p} & \mathbf{z} \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} \mathbf{x} & \mathbf{y} & \mathbf{z} \\ 1 & 1 & 1 \end{vmatrix}},$$

$$\gamma = \alpha - \beta - 1.$$

Barycentric Coordinates

- Interpretation
  - For each identified point below, determine how the barycentric coordinates should look like:

    remember that: $\mathbf{p} = \alpha\ \mathbf{x} + \beta\ \mathbf{y} + \gamma\ \mathbf{z}, \quad \alpha + \beta + \gamma = 1$



Barycentric Coordinates

- Interpretation
  - Now determine how the barycentric coordinates should look like when the identified points are in each of the delimited regions: