CSE-170 Computer Graphics

Lecture 24

Representations and Manifolds

Dr. Renato Farias
rfarias2@ucmerced.edu

Representation Schemes

Representations of Solids

- Stages for defining a representation:
  - Physical Universe

    Real solid object
  - Mathematical Universe

    Surface equations, geometric primitives, etc.
  - Representation Universe

    Boundary elements, surface control points, primitive parameters, etc.
  - Implementation Universe

    Data structures, number representation format, algorithms, etc.

Representations

- Typical properties to consider:
  - Unambiguous
  - Unique
  - Expressive Power
  - Validity
  - Compact
  - Efficient for algorithms
  - Closure under operations
  - Should make it impossible to be invalid
  - etc.

Taxonomy

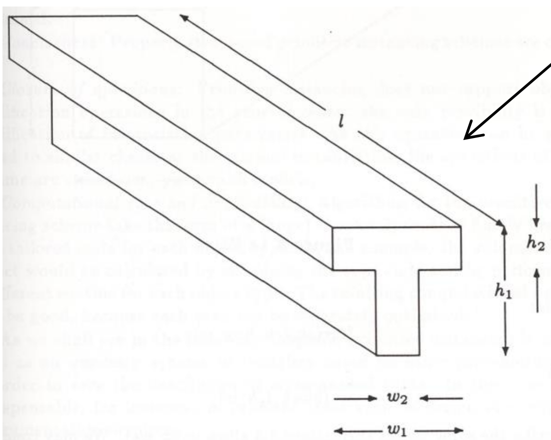
- Special purpose schemes
  – Primitive instancing
    - Including operations, for ex.: sweeps
- Generic
  – Decomposition Models
  – Constructive Models
  – Boundary Models
- Hybrid Schemes
  – More than one representation used
  – Scene graphs

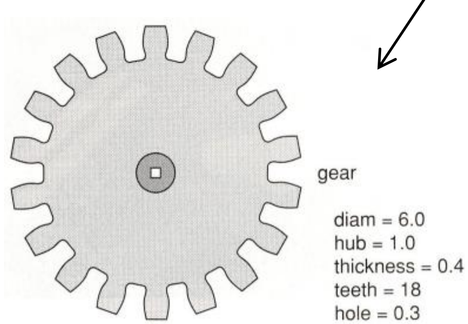
We have already seen most of these approaches (more about primitive instancing in next slide)

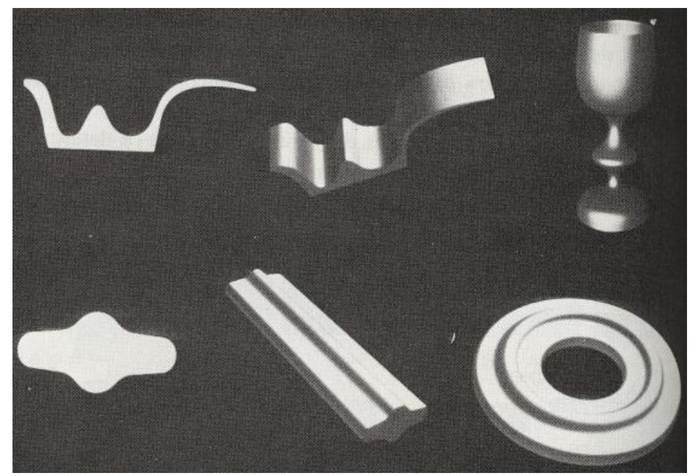
Taxonomy

– Primitive instancing

- Parameterized primitives and operations (like sweeps) are perfect for some specific applications
  - » Ex: defining t-bricks and gears, or generic sweeps







Definition of Solid Models

Mathematical Models of Solids

- What does "solidity" mean?

  "three-dimensional solidity" with point-set topology:

  $\Rightarrow$possible description of a solid model:

  - a subset of E^3 (3D Euclidian space)

  - and the subset has to be "limited"

Mathematical Models of Solids

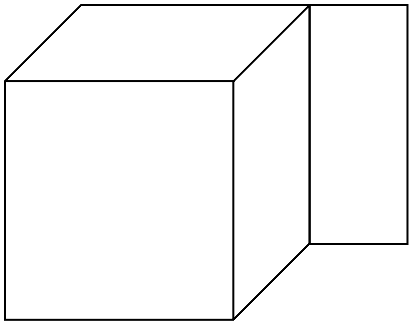
- Definition 1
  - A solid is a bounded, closed subset of E^3

- Is this enough?

Mathematical Models of Solids

- Definition 1
  - A solid is a bounded, closed subset of E^3

- Is this enough?



Mathematical Models of Solids

- Definition 1
  - A solid is a bounded, closed subset of E^3
- Is this enough?
  - What about isolated points, lines or faces?
  - Need notion of regularity

Mathematical Models of Solids

- Definition 2
  - A solid is a bounded regular set of E^3

- Regularization

  $r(A) = c(i(A))$ (c is closure, i is interior)

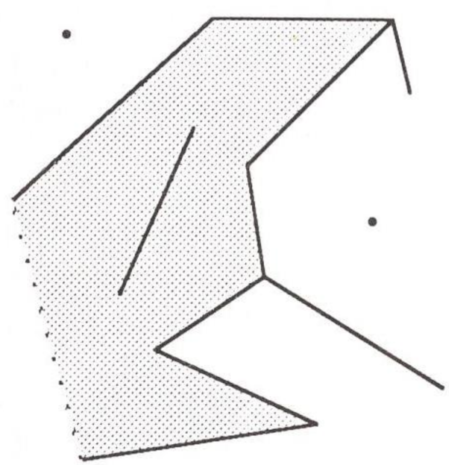
- Sets that satisfy $r(A) \equiv A$ are regular

- Is this enough?

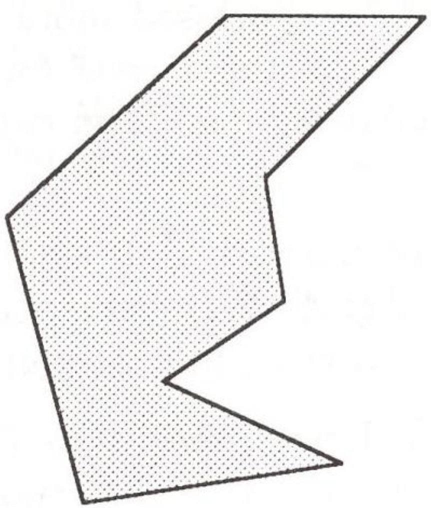
Mathematical Models of Solids

- Mostly yes, definition 2 is a well adopted point-set definition for a solid
  - A solid is a bounded regular set of E^3

A



$r(\mathrm{A}) = c(i(\mathrm{A}))$

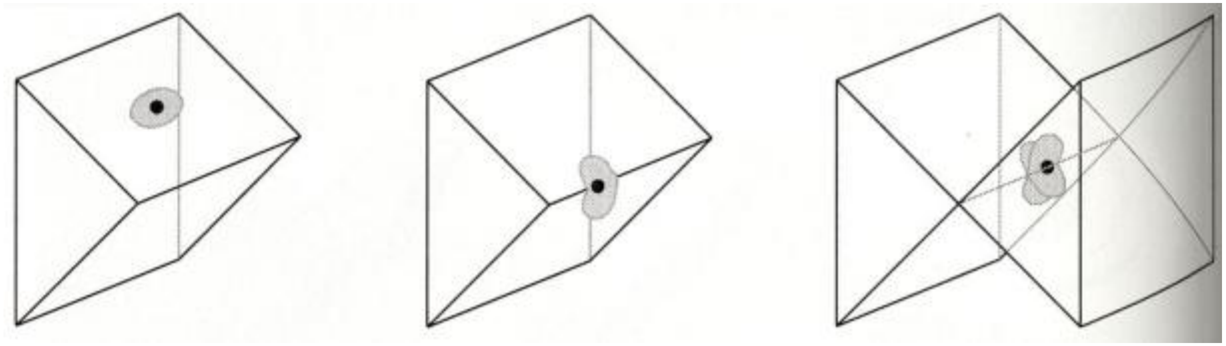


Manifolds

- A 2-manifold M is a topological space where every point has a neighborhood topologically equivalent to an open disk of E^2
  - topologically equivalent:
    - when you can "elastically deform" one space to the other, both spaces share several properties
  - 2-manifolds can be used to define the boundary of solid models!

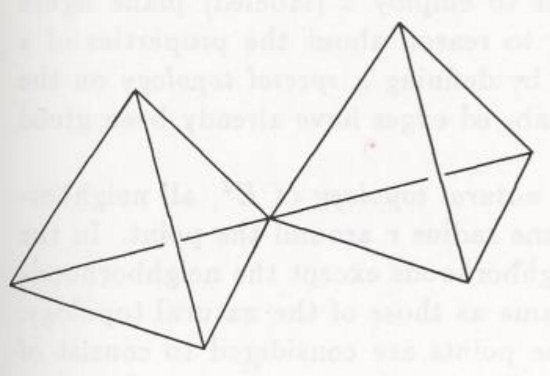
Manifolds

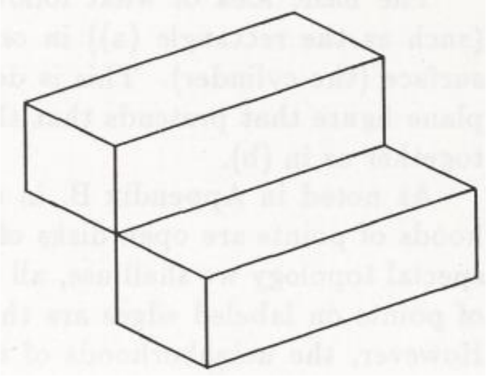
- A 2-manifold M is a topological space where every point has a neighborhood topologically equivalent to a 2D open disk
  - Ex: the third case below is not a 2-manifold:

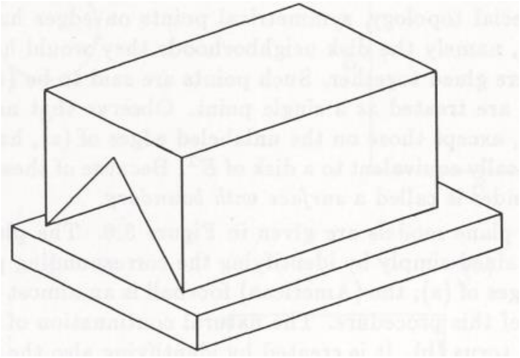


Manifolds

- There is a theoretical mismatch in our solidity notion between
  – r-sets and
  – 2-manifold of the boundary of an r-set





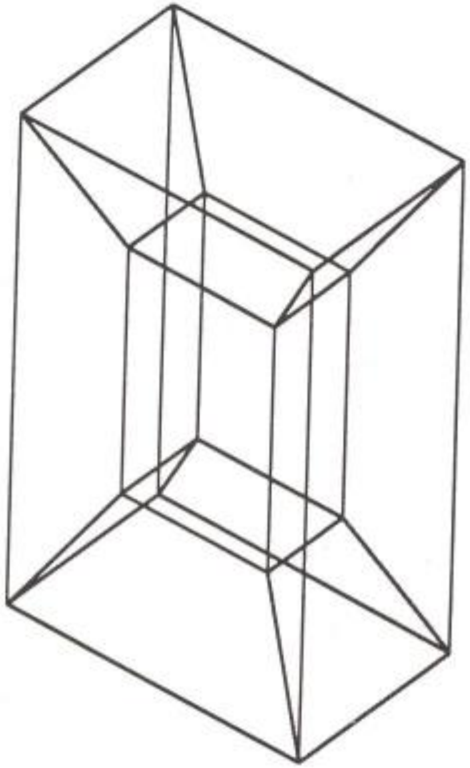


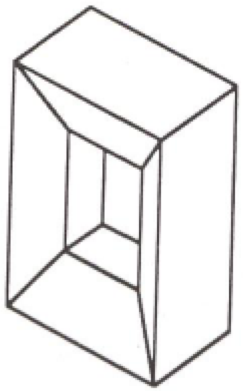
  – Examples above do not represent solid objects!
    - They are r-sets but boundaries are not 2-manifolds!
  – This mismatch remains and is handled for each modeling situation when/if needed

Boundary-Based Models

Solid Modeling

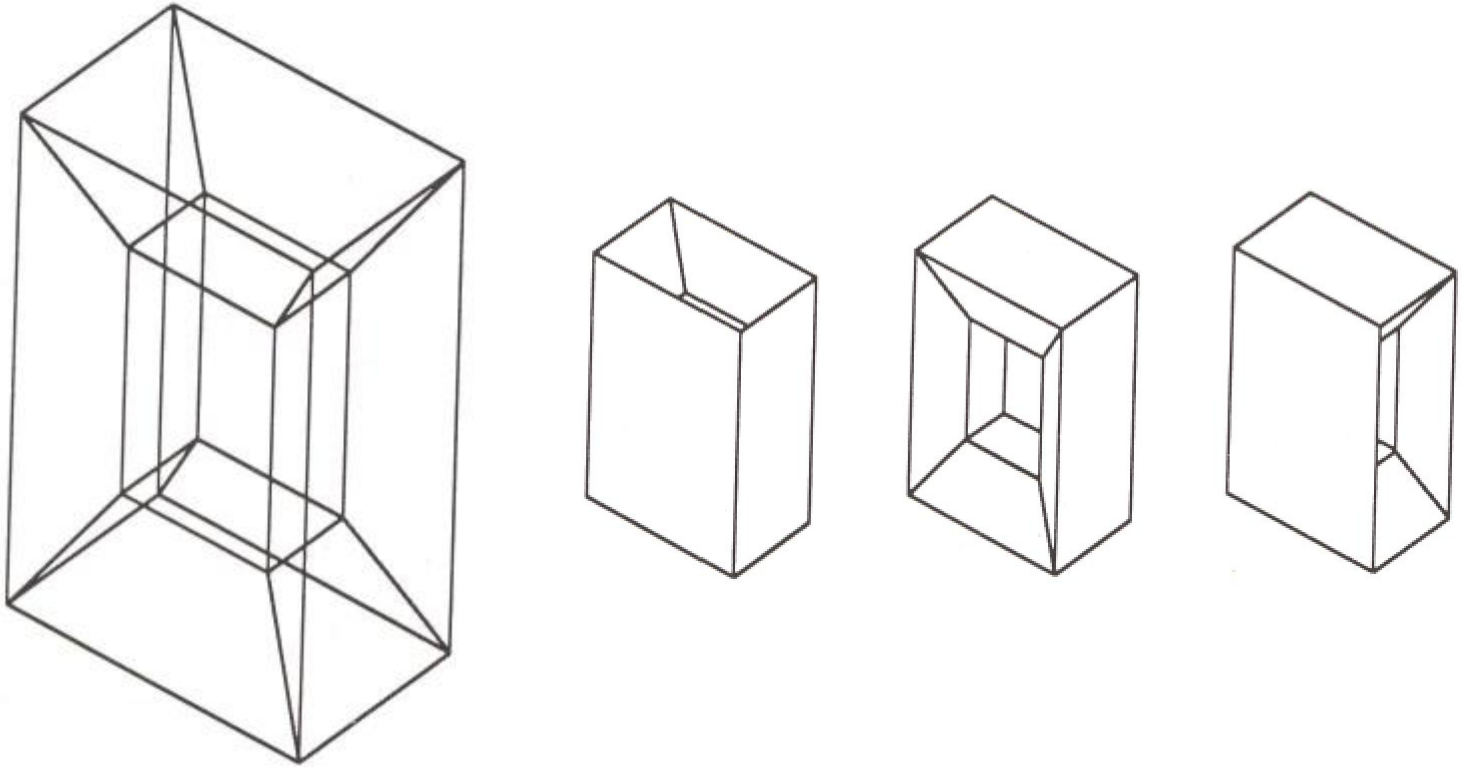
- Are simple line connections between points enough to precisely define a model?





Solid Modeling

- Edges are not enough:



Boundary-Based (B-Rep) Models

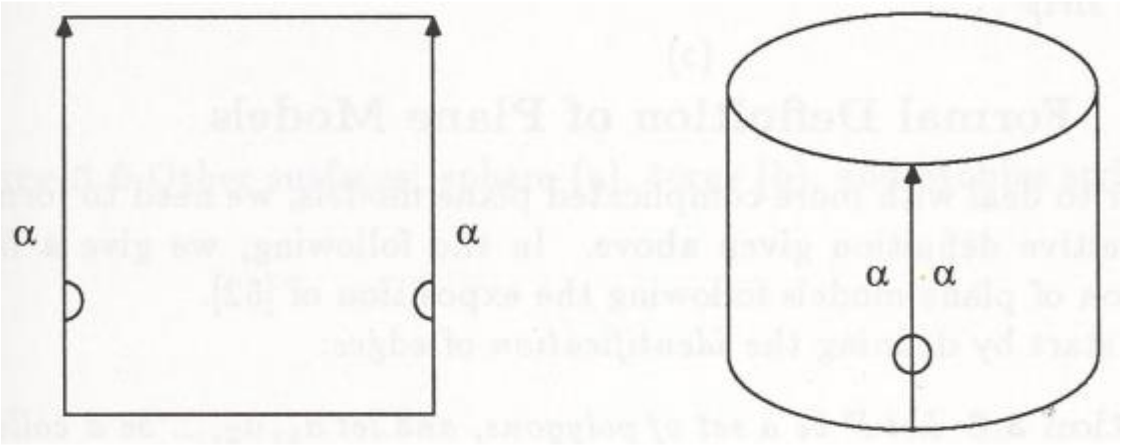
- In B-Rep we model the boundary of a solid object
  - Consistent collection of "faces" which are connected together
  - How to ensure the final boundary "skin" satisfies our notion of "solidity"?

Plane Models

– Since our 2-manifold boundaries are locally planar, it is possible to convert them to planar representations

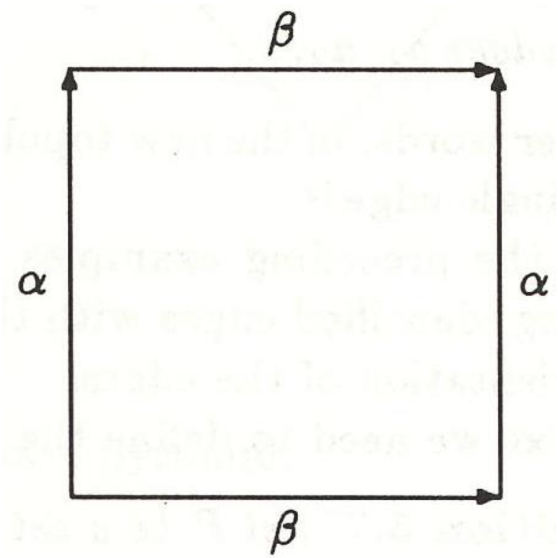
Plane Models

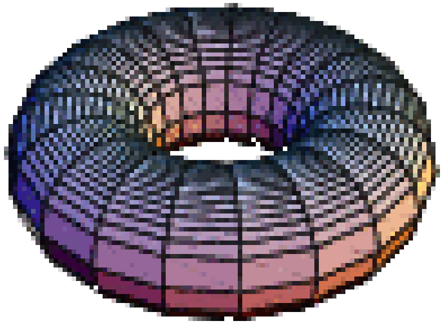
- A planar way of specifying topological properties of a 2-manifold
  - By the identification of edges
  - Cylinder



Plane Models

- Torus:

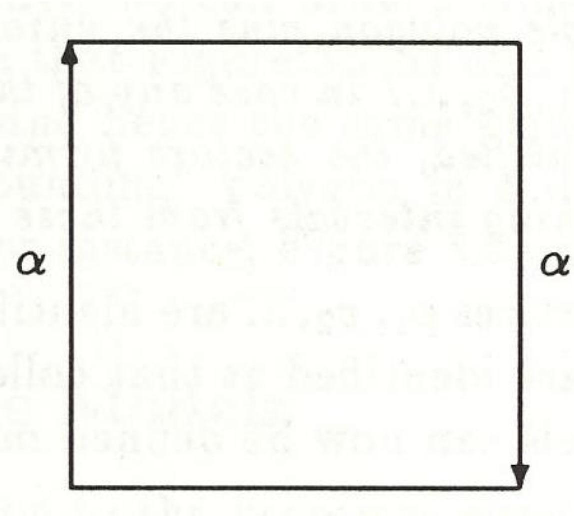


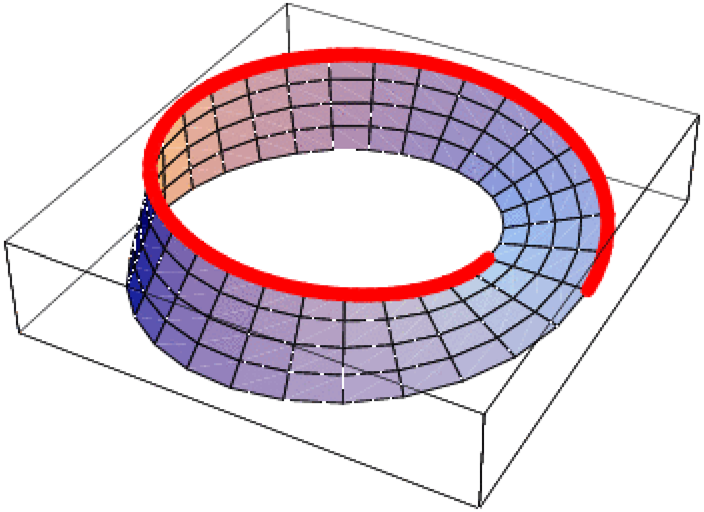


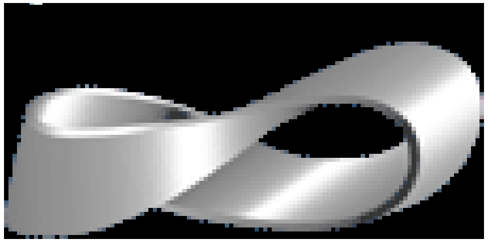
$$x(u,v) = (R + r\cos v)\cos u$$
$$y(u,v) = (R + r\cos v)\sin u$$
$$z(u,v) = r\sin v$$

Plane Models

- Möbius strip:



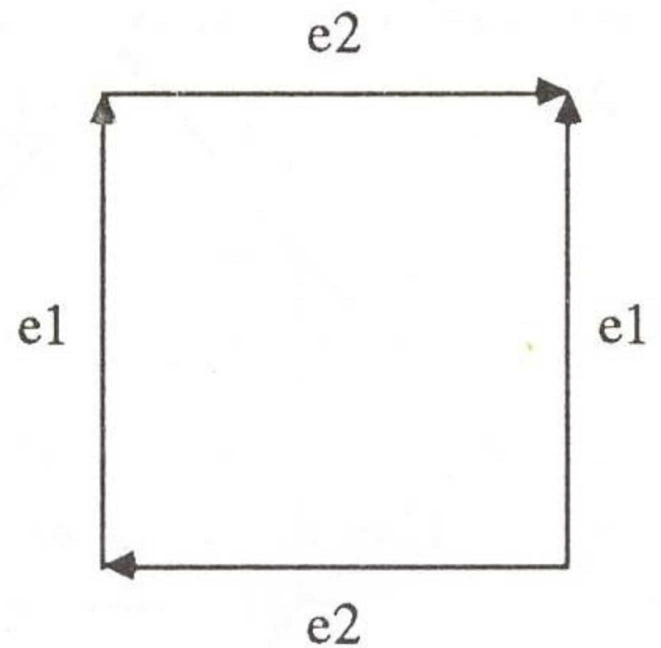


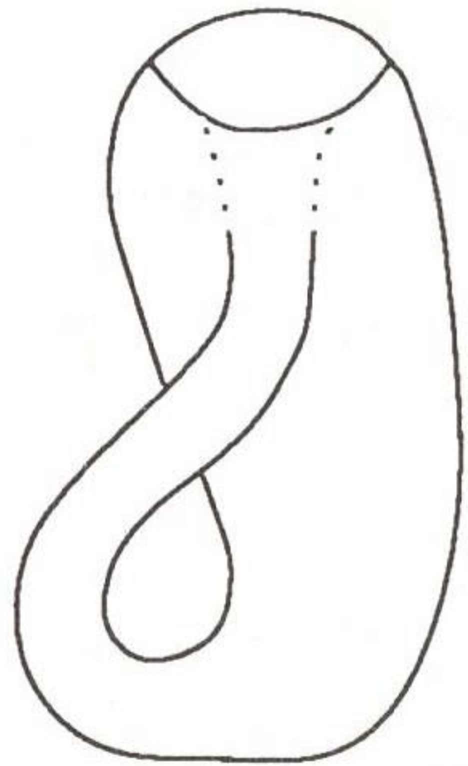


$$x(u,v) = \left(1 + \frac{v}{2}\cos\frac{u}{2}\right)\cos(u)$$
$$y(u,v) = \left(1 + \frac{v}{2}\cos\frac{u}{2}\right)\sin(u)$$
$$z(u,v) = \frac{v}{2}\sin\frac{u}{2}$$

Plane Models

- Klein bottle:





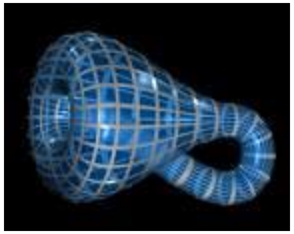
$$\begin{pmatrix} x[u,v] \\ y[u,v] \\ z[u,v] \end{pmatrix} = \begin{pmatrix} \mathrm{Cos}[u]\ \left(a + \mathrm{Cos}[\frac{u}{2}]\ \mathrm{Sin}[v] - \mathrm{Sin}[\frac{u}{2}]\ \mathrm{Sin}[2\,v]\right) \\ \mathrm{Sin}[u]\ \left(a + \mathrm{Cos}[\frac{u}{2}]\ \mathrm{Sin}[v] - \mathrm{Sin}[\frac{u}{2}]\ \mathrm{Sin}[2\,v]\right) \\ \mathrm{Sin}[\frac{u}{2}]\ \mathrm{Sin}[v] + \mathrm{Cos}[\frac{u}{2}]\ \mathrm{Sin}[2\,v] \end{pmatrix}$$

Topological Classes

- If one model can be transformed into another one by only "elastic deformations", then they are equivalent
  - i.e., in the same "topological class"
  - Ex. of classes: sphere, torus, klein bottles, double-torus, etc.

Klein Bottles: zero volume, one-sided surface

examples:



Designing B-Rep Data Structures

- Graph:
  - Connecting vertices, edges, and polygons
  - Polygons have an orientation
- 2-Manifold Restrictions:
  - Every edge is adjacent to 2 polygons/faces
  - Single consistent cycle around each vertex
    - Polygons sharing a vertex define a cycle where each consecutive pair of polygons are adjacent to one edge

Designing B-Rep Data Structures

- We will use these characteristics to design consistent graph-based data structures
  - Example: