1. Both RGB and HSV are useful ways to represent colors. Consider the HSV representation such that the hue value is an angle in degrees and the other two parameters are in $[0,1]$. a) Convert each triplet from RGB to HSV format: (1,0,0), (0,1,0), (0,0,1), (0.5,0,0), (0,0.5,0),(0,0,0.5), (0.5,0.5,0.5).
b) Which line in the HSV diagram corresponds to the line (0,0,0),(1,1,1) in the RGB diagram?
c) From a perceptual point of view, which color format is most appropriate to perform color interpolation? Why?

Solution:

a) (0,1,1), (120,1,1), (240,1,1), (0,1,.5), (120,1,.5),(240,1,0.5), (0,0,0.5).

b) The vertical V axis.

c) HSV, because interpolation in this space will interpolate meaningful perceptual values independently from each other. It is also easy to, for example, interpolate the hue while leaving the saturation and value unchanged.

□

2. Consider the following Phong illumination equation: $I = I_a k_a + I_d k_d(\hat{\mathbf{l}} \cdot \hat{\mathbf{n}}) + I_s k_s(\hat{\mathbf{v}} \cdot \hat{\mathbf{r}})^f$.
a) Which parameters in this equation are properties of the material?
b) Which parameters are properties of the light source?
c) Which parameters depend on the position of the light source?
d) What is the effect that f controls?
e) Sketch in a diagram vectors $\hat{\mathbf{l}}$, $\hat{\mathbf{n}}$, $\hat{\mathbf{v}}$, and $\hat{\mathbf{r}}$ with respect to a point being illuminated, also depicting the considered light and camera positions.

Solution:

a) k_a, k_d, k_s.
b) I_a, I_d, I_s.
c) $\hat{\mathbf{l}}$ and $\hat{\mathbf{r}}$.
d) How large the specular highlight is. The larger f is the larger the highlights are.
e) The diagram appears several times in the illumination class notes.

□

3. What is the main difference between Gauraud shading and Phong shading? Describe the typical example when Phong shading produces better results.

Solution:

Gauraud shading interpolates colors computed at the vertices, while Phong computes the illumination equation at every pixel. Phong produces better specular reflections.

□

4. Let $f(\mathbf{p}, \hat{\mathbf{n}})$ be a function that computes the Phong illumination model for point $\mathbf{p}$ with normal vector $\hat{\mathbf{n}}$. Now consider a triangle with vertices $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, and normals $\hat{\mathbf{n}}_1$, $\hat{\mathbf{n}}_2$, $\hat{\mathbf{n}}_3$. Given an interior point with barycentric coordinates (α, β, γ) with respect to vertices $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, respectively, define the color of the point using a) the Phong shading model and b) the Gouraud shading model.

Solution:

a) $f(\alpha\mathbf{p}_1 + \beta\mathbf{p}_2 + \gamma\mathbf{p}_3, \alpha\hat{\mathbf{n}}_1 + \beta\hat{\mathbf{n}}_2 + \gamma\hat{\mathbf{n}}_3)$.
b) $\alpha f(\mathbf{p}_1, \hat{\mathbf{n}}_1) + \beta f(\mathbf{p}_2, \hat{\mathbf{n}}_2) + \gamma f(\mathbf{p}_3, \hat{\mathbf{n}}_3)$.

□

5. Consider a 2D triangle T with vertices $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and points $\mathbf{p}_i$ defined in barycentric coordinates with respect to T, $i \in \{1, 2, 3\}$. a) Draw in a sketch the estimated position for each point considering the following coordinates: $\mathbf{p}_1 = (0, 1, 0)$, $\mathbf{p}_2 = (0.5, 0, 0.5)$, $\mathbf{p}_3 = (-0.2, 0.6, 0.6)$. b) What is the relationship involving barycentric coordinates that has always to be verified?

   Solution:

   a) Your sketch should show $\mathbf{p}_1$ coincident to $\mathbf{b}$, $\mathbf{p}_2$ as the midpoint of segment $\{\mathbf{a}, \mathbf{c}\}$, and $\mathbf{p}_3$ will be along the line passing by $\mathbf{a}$ and the midpoint of segment $\{\mathbf{b}, \mathbf{c}\}$, close to this segment but outside of the triangle.
   b) The sum of the three barycentric coordinates has always to be 1. □

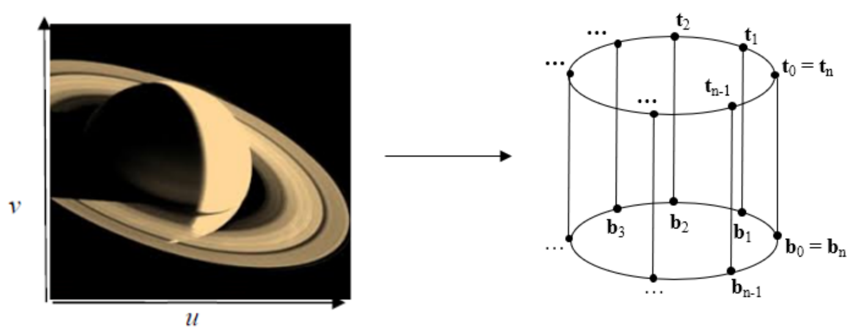
6. Let the colors at the vertices of a triangle be $\mathbf{a} = (0, 0, 1)$, $\mathbf{b} = (1, 0, 0)$, and $\mathbf{c} = (0, 1, 0)$. If a pixel has been Gouraud-shaded to have color (0.3, 0.1, 0.6), what are its barycentric coordinates?

   Solution:

   From the given information we can write $(0.3, 0.1, 0.6) = \alpha(0, 0, 1) + \beta(1, 0, 0) + \gamma(0, 1, 0)$, which is a system of three equations and three variables α, β, γ. From the first equation: $\alpha 0 + \beta 1 + \gamma 0 = 0.3$ $=> \beta = 0.3$; from the second equation: $\alpha 0 + \beta 0 + \gamma 1 = 0.1 => \gamma = 0.1$; from the third equation: $\alpha 1 + \beta 0 + \gamma 0 = 0.6 => \alpha = 0.6$; therefore the barycentric coordinates are (0.6,0.3,0.1).

   □

7. In the diagram below the texture is being mapped to a cylinder approximated with equally-spaced 3D vertices $\mathbf{t}_i$ and $\mathbf{b}_i$, $0 \leq i \leq n$. The curved face of the cylinder is approximated with n rectangular faces and each rectangular face R_i has CCW (counter-clockwise) vertices: $R_i = (\mathbf{b}_i, \mathbf{b}_{i+1}, \mathbf{t}_{i+1}, \mathbf{t}_i)$, $0 \leq i < n$. Give the equations determining the $(\mathbf{u}, \mathbf{v})$ texture coordinates for the four vertices of a given face R_i such that the image is completely mapped around the cylinder.

   Solution:

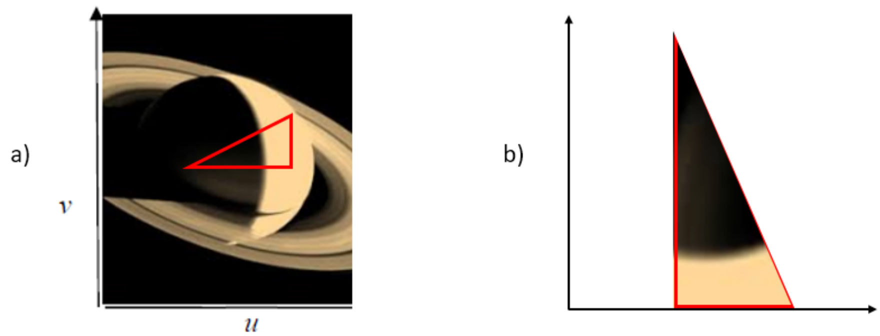
   $\mathbf{b}_i = (i/n, 0),\ \mathbf{b}_{i+1} = ((i+1)/n, 0),\ \mathbf{t}_i = (i/n, 1),\ \mathbf{t}_{i+1} = ((i+1)/n, 1).$

   □

8. Considering the same texture as in the previous question, consider now that a planar triangle to be rasterized has vertex coordinates: $\mathbf{a} = (1, 3), \mathbf{b} = (1, 0), \mathbf{c} = (2, 0)$, and texture coordinates: $\mathbf{r} = (0.25, 0.5), \mathbf{s} = (0.6, 0.5), \mathbf{t} = (0.6, 0.6)$.
   a) Draw in the texture the region that will be mapped to the triangle $(\mathbf{a}, \mathbf{b}, \mathbf{c})$.
   b) Now sketch in a Cartesian XY plane how the pattern of the texture will look like inside triangle $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$.

c) Consider point $\mathbf{e} = (1/5, 1/5, 3/5)$ described in barycentric coordinates with respect to the vertices of the triangle $(\mathbf{a}, \mathbf{b}, \mathbf{c})$. What are the texture coordinates that should be used for point $\mathbf{e}$?

Solution:



c) The texture coordinates will be: 1/5 r + 1/5 s + 3/5 t =
$1/5\begin{pmatrix} .25 \\ .5 \end{pmatrix} + 1/5\begin{pmatrix} .6 \\ .5 \end{pmatrix} + 3/5\begin{pmatrix} .6 \\ .6 \end{pmatrix} = 1/5\begin{pmatrix} 1/4 \\ 1/2 \end{pmatrix} + 1/5\begin{pmatrix} 3/5 \\ 1/2 \end{pmatrix} + 3/5\begin{pmatrix} 3/5 \\ 3/5 \end{pmatrix} = \begin{pmatrix} 1/20 \\ 1/10 \end{pmatrix} + \begin{pmatrix} 3/25 \\ 1/10 \end{pmatrix} + \begin{pmatrix} 9/25 \\ 9/25 \end{pmatrix} = \begin{pmatrix} 53/100 \\ 14/25 \end{pmatrix} = \begin{pmatrix} 0.53 \\ 0.56 \end{pmatrix}$, which can be verified in the image to be inside the triangle in texture space.

□

9. Describe how z_{near} and z_{far} projection parameters are important for a) visible surface determination and b) clipping operations in the rendering pipeline of OpenGL.

Solution:

a) Value $z_{far} - z_{near}$ will give the range of Z values that has to be mapped to the visible portion of the scene. If this value is too high the resolution of the Z-buffer may not be enough to correctly solve visibility tests.
b) The z_{far} and z_{near} values also determine the near and far planes that will clip the scene to the viewing frustum.

□

10. Considering the Painter's algorithm, BSP trees, and the Z-Buffer method as seen in class, which of these methods are most appropriate for each of the cases below:
a) Limited memory, static scene with several overlaps, dynamic camera.
b) Unlimited memory, dynamic scene with many overlaps, dynamic camera.
c) Limited memory, dynamic scene with triangles that can be sorted without ambiguities, dynamic camera.

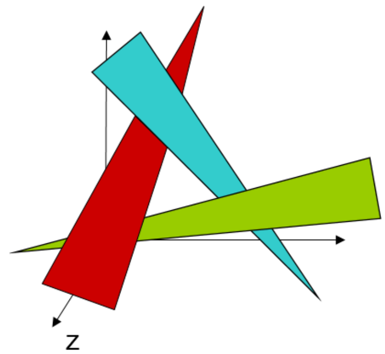
Solution:

a) BSP trees, b) Z-Buffer, c) Painter's algorithm.

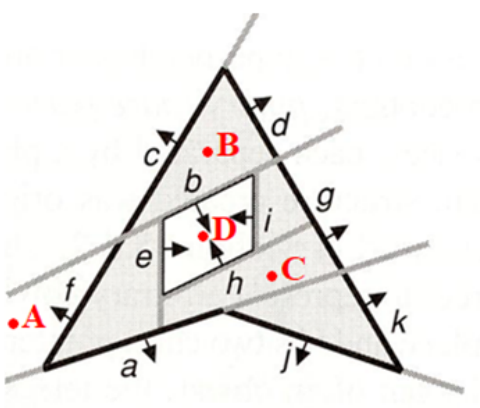
□

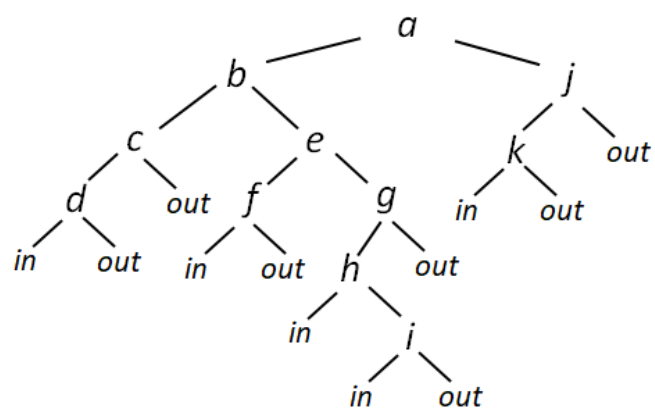
11. Draw an example scene composed of three triangles where, no matter the point of view, no triangle is completely in front of the others.

Solution: □



12. We have seen in lecture how BSP trees can be used for visible surface determination. (Please be sure to review the examples provided in the lecture slides.) Here we look into a different problem. The BSP tree representation below is used to compute if a point is inside the given polygon. For each segment of the polygon, its normal vector points to the outside region of the polygon with respect to the segment.
a) Write a pseudo code for a point classification algorithm using the shown BSP tree.
b) Give the nodes visited by your algorithm, in the correct order, for classifying points A, B, C, and D.
c) Is the tree shown the only possible BSP tree for the polygon? If not, show an alternative tree representation and check if points A, B, C and D can still be correctly classified.
d) Suppose that the corner delimited by segments j, k and the line passing by a is removed from the polygon. What changes will happen to the tree representation above? Draw the new tree.





Solution:

a) The function below is to be called with n as the root node and $\mathbf{p}$ the query point:

```
traverse ( n, p ):
if ( n is a leaf ) return leaf->label; // will return in or out
if ( p is in front of n->edge ) traverse ( n->right, p );
else traverse ( n->left, p );
```

b) A: a, b, e, f, out; B: a, b, c, d, in; C and D are left for you to do it.

c) No. You can build a different valid tree by selecting a different order of edges to be used as nodes in the tree.

d) The new tree will not have anymore the branch with root j.

$\square$

13. The grid below represents the Z-Buffer of a scene composed of two planes. Draw this scene.

9	8	7	6	5	4	3	2
9	8	7	6	5	4	3	2
9	8	7	7	7	7	3	2
9	8	7	7	7	7	3	2
9	8	7	7	7	7	3	2
9	8	7	7	7	7	3	2
9	8	7	6	5	4	3	2
9	8	7	6	5	4	3	2

Solution:

Your scene should have two planes. One of them will be a square parallel to the XY plane and with the Z coordinate as 7. This corresponds to the several 7 values we see at the center square of the Z-Buffer. The values outside indicate how the other plane should be: diagonally going from Z=9 to Z=2. □

14. Consider the midpoint algorithm being used to rasterize a 2D segment from (x_0, y_0) to (x_1, y_1), where $x_0 < x_1$ and $y_0 < y_1$.
a) Given the previous pixel (x_p, y_p) painted, which are the two next pixels being considered?
b) Why the two next pixels are labeled NE and E?
c) What is the implicit equation $f(x, y)$ of the line?
d) Based on f and the previous pixel (x_p, y_p), what is the midpoint criterion to determine the next pixel to paint?
e) Given the decision variable d_{cur} of the current pixel, what will be the decision variable of the next pixel?
f) List all multiplications and floating point operations required to implement the midpoint algorithm.

Solution:

a) $(x_p + 1, y_p + 0.5)$ and $(x_p + 1, y_p + 1.5)$
b) The labels denote the northeast and east neighbor pixels.
c) Write line equation $f(x, y) = ax + by + c$ and then derive a, b, and c from the given endpoints.
d) If $f(x_p + 1, y_p + 0.5) > 0$ the midpoint is below the line and the E pixel is painted, otherwise the NE pixel is painted.
e) If the E pixel is taken, $d_{next} = d_{cur} + (y_1 - y_0)$; if the NE pixel is taken, $d_{next} = d_{cur} + (y_1 - y_0) + (x_0 - x_1)$.
f) This one is easy to find out. □