Reinforcement Learning

CS 4641 B: Machine Learning (Summer 2020)

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07/06/2020

Outline

Opening

Introduction to Reinforcement Learning

Markov Decision Process

Planning Methods

Bandit Problems

Prediction Problem

Control Problem

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Opening

- Syllabus has been updated
  - Only 3 homework assignments (no enough time for more).
  - Only 1 attendance sheet—the remaining points for class participations will be coming from Piazza contributions. Hint: ask questions related to the lectures. Participate!
  - New website (I have write access to it and will be posting lecture material): https://mimoralea.github.io/cs4641B-summer2020/.
  - Office hours to be announced.
  - Remaining lectures are focused on Reinforcement Learning and Deep Reinforcement Learning. I hope you enjoy the material.

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> I visualize a time when we will be to robots what dogs are to humans, and I'm rooting for the machines.
>
> — Claude Shannon
> Father of the Information Age
> and contributor to the field of Artificial Intelligence

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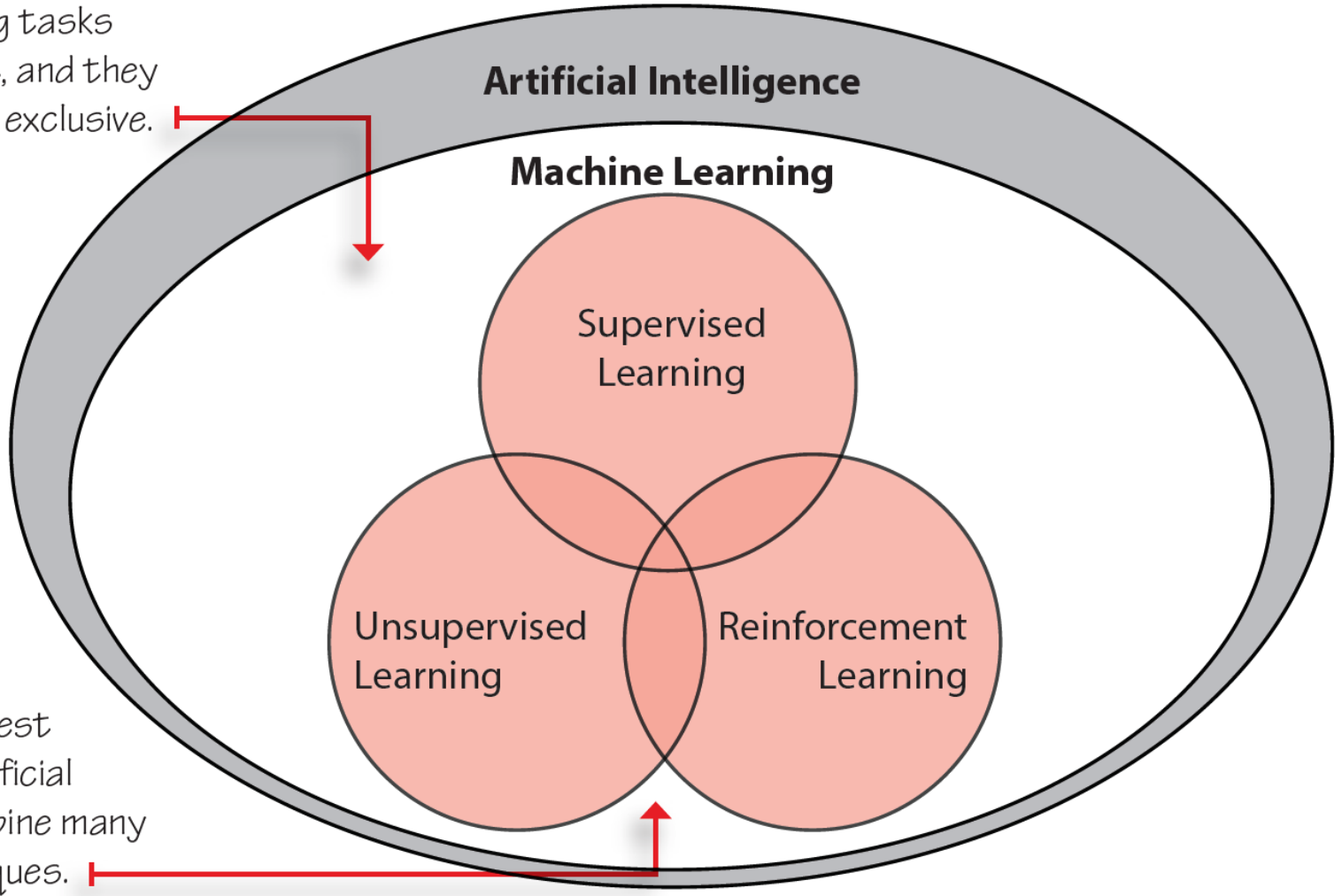
The big picture, you are here

(1) These types of Machine Learning tasks are all important, and they are not mutually exclusive.



(2) In fact, the best examples of Artificial Intelligence combine many different techniques.

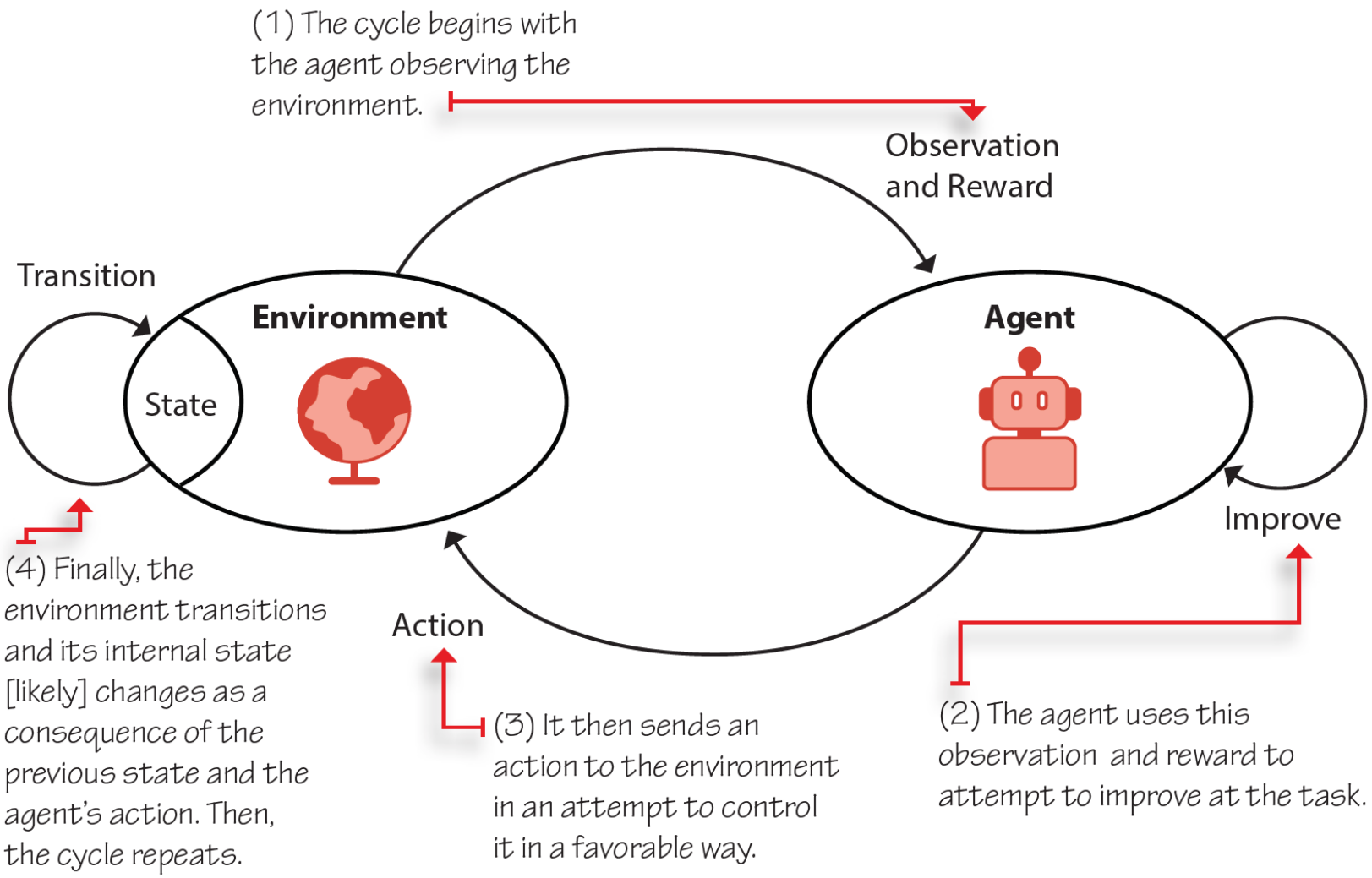
Comparison of ML areas

Supervised learning (SL) is the task of learning from labeled data. In SL, a human decides which data to collect and how to label it. The goal in SL is to generalize. A classic example of SL is a handwritten-digit recognition application; a human gathers images with handwritten digits, labels those images, and trains a model to recognize and classify digits in images correctly. The trained model is expected to generalize and correctly classify handwritten digits in new images.

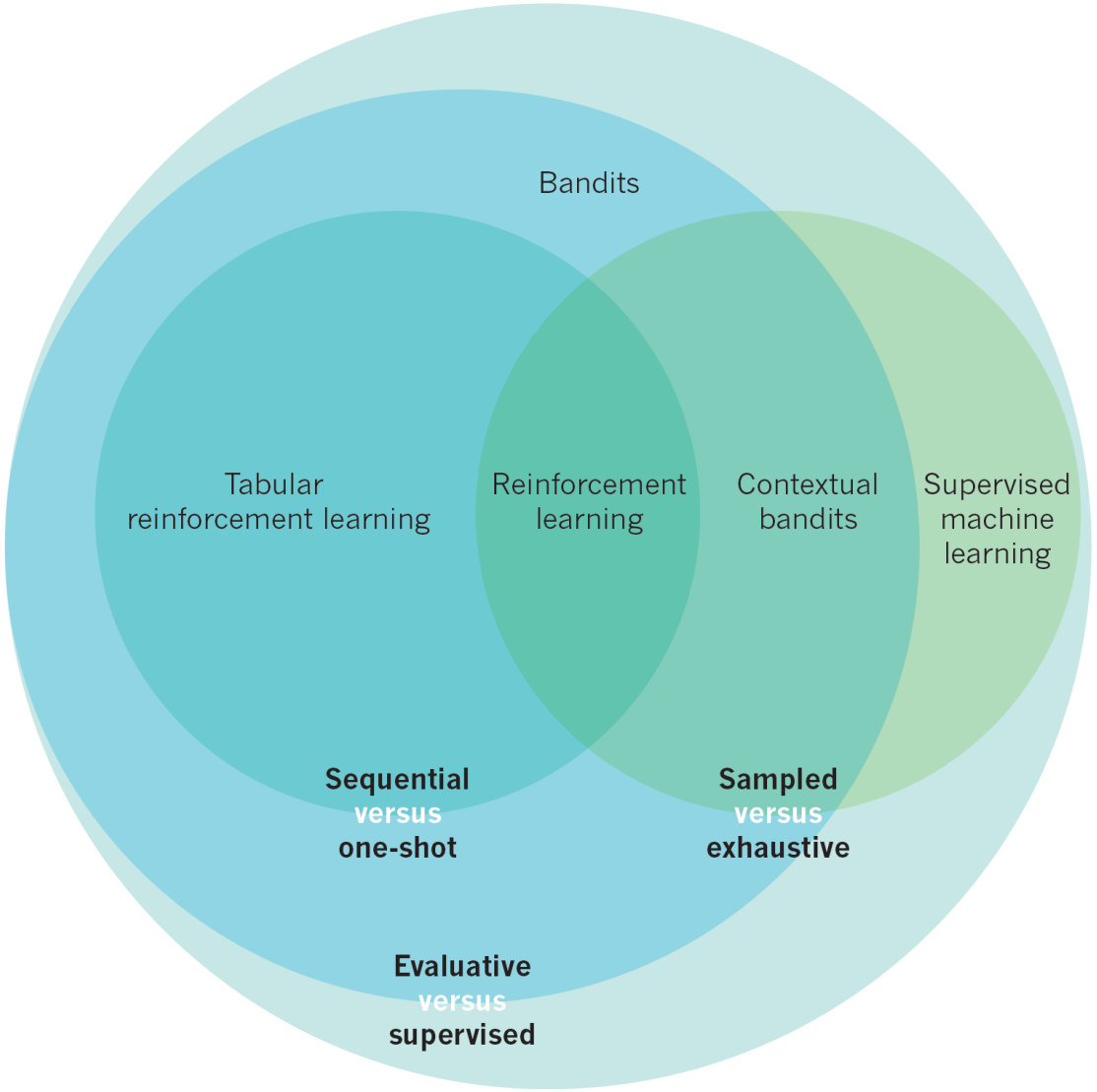
Unsupervised learning (UL) is the task of learning from unlabeled data. Even though data no longer needs labeling, the methods used by the computer to gather data still need to be designed by a human. The goal in UL is to compress. A classic example of UL is a customer segmentation application; a human collects customer data and trains a model to group customers into clusters. These clusters compress the information uncovering underlying relationships in customers.

Reinforcement learning (RL) is the task of learning through trial and error. In this type of task, no human labels data, and no human collects or explicitly designs the collection of data. The goal in RL is to act. A classic example of RL is a Pong-playing agent; the agent repeatedly interacts with a Pong emulator and learns by taking actions and observing its effects. The trained agent is expected to act in such a way that it successfully plays Pong.

The reinforcement learning cycle



Reinforcement Learning agents learn from feedback that is sequential, evaluative, and sampled

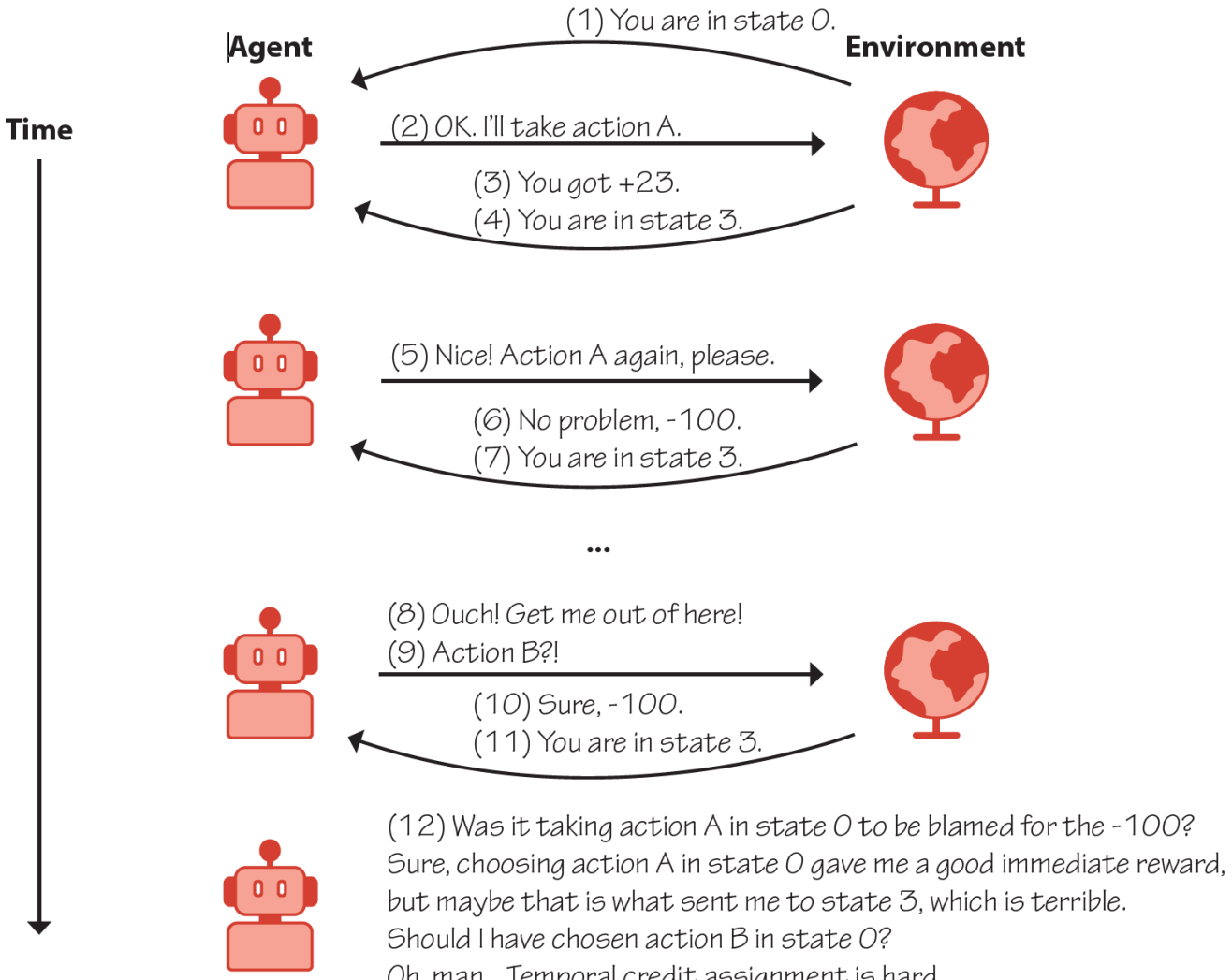


https://www.nature.com/articles/nature14540

Reinforcement Learning agents learn from sequential feedback

- The action taken by the agent may have delayed consequences.
- The reward may be sparse and only manifest after several time steps. Thus the agent must be able to learn from sequential feedback.
- Sequential feedback gives rise to a problem referred to as the temporal **credit assignment problem**.
- The temporal credit assignment problem is the challenge of determining which state and/or action is responsible for a reward.
- When there is a temporal component to a problem, and actions have delayed consequences, it becomes challenging to assign credit for rewards.

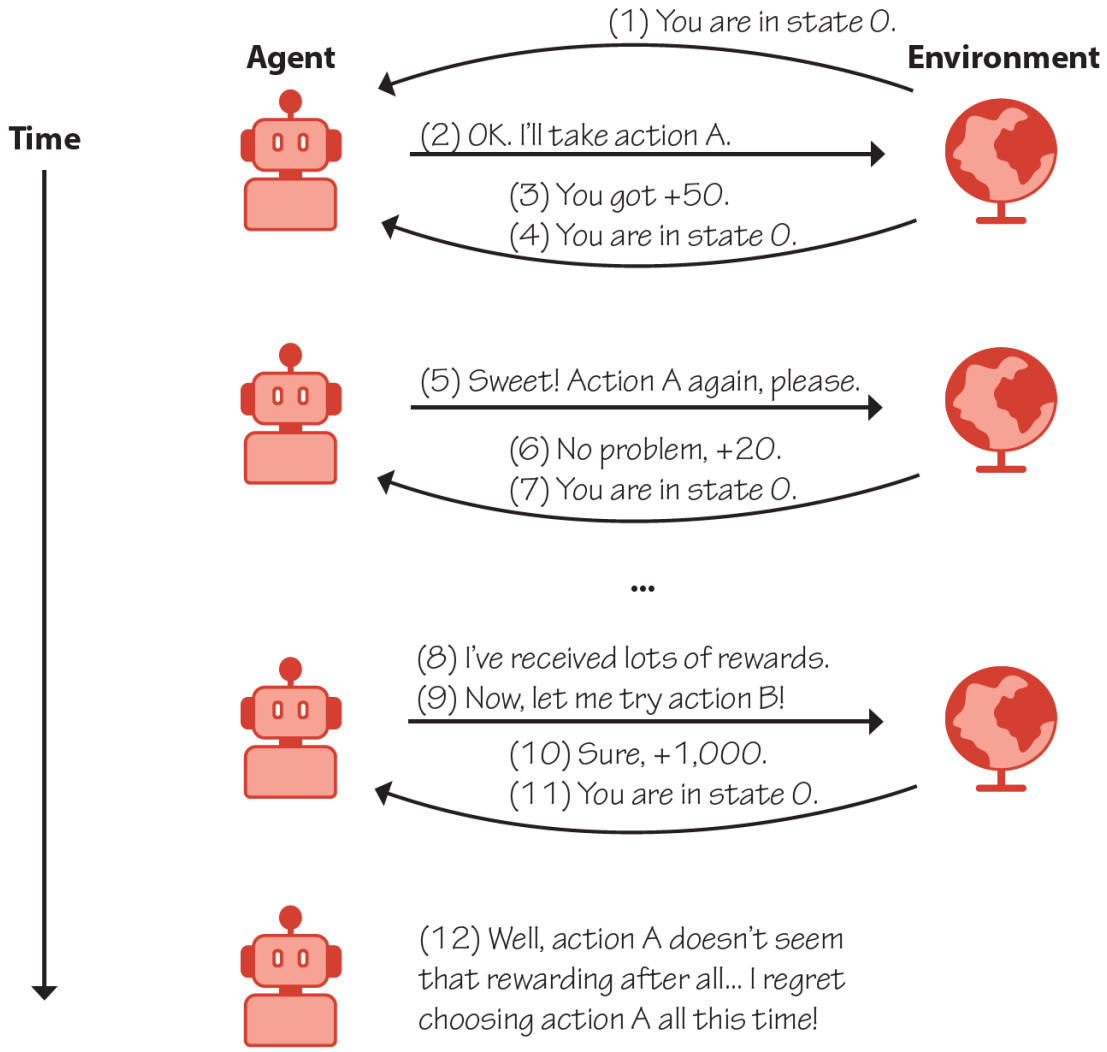
The difficulty of the temporal credit assignment problem



Reinforcement Learning agents learn from evaluative feedback

- The reward received by the agent may be weak, in the sense that it may provide no supervision.
- The reward may indicate goodness and not correctness, meaning it may contain no information about other potential rewards.
- Evaluative feedback gives rise to the need for exploration.
- The agent must be able to balance exploration, which is the gathering of information, with the exploitation of current information.
- This is also referred to as the **exploration vs. exploitation tradeoff**.

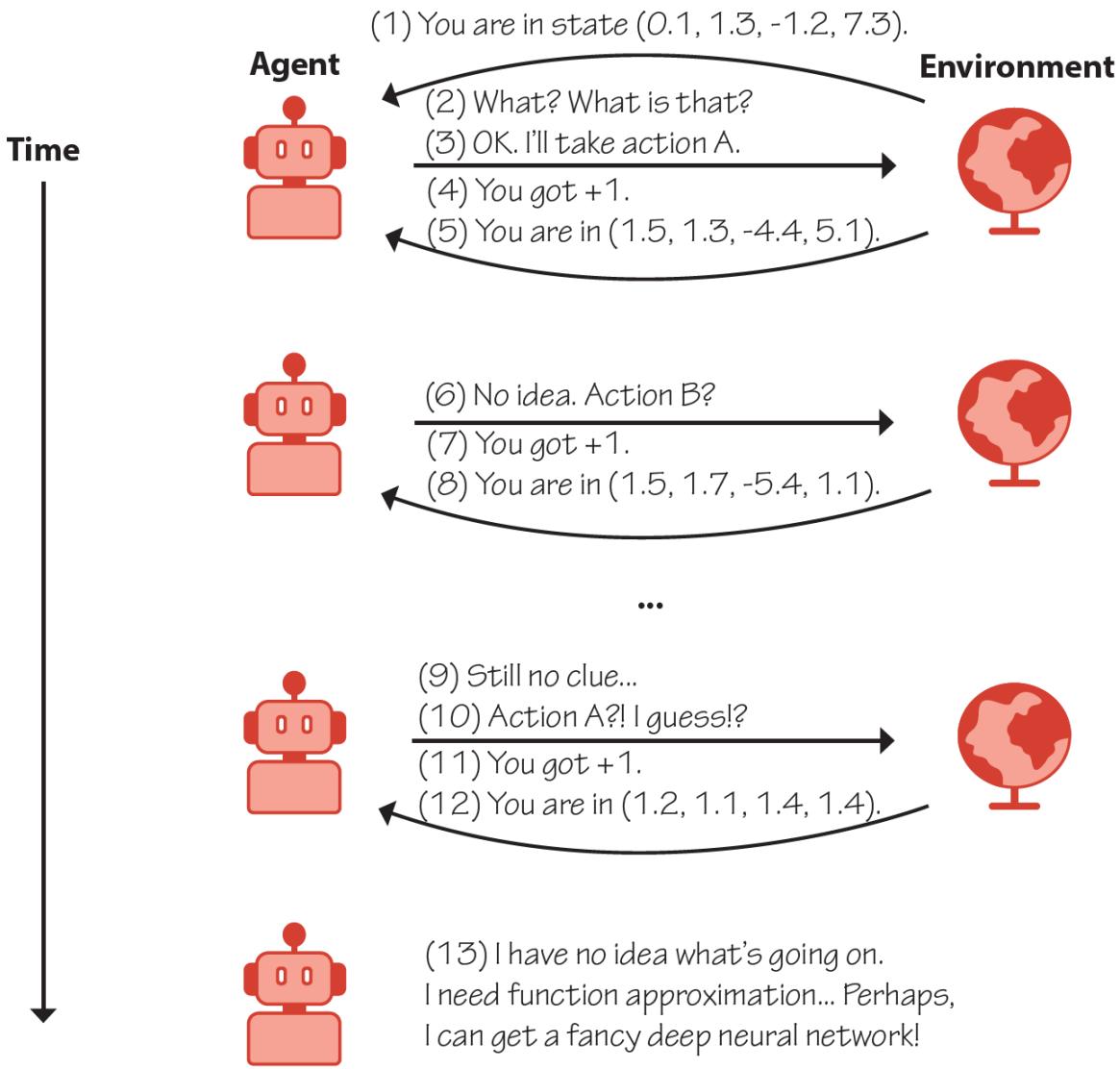
The difficulty of the exploration vs. exploitation tradeoff



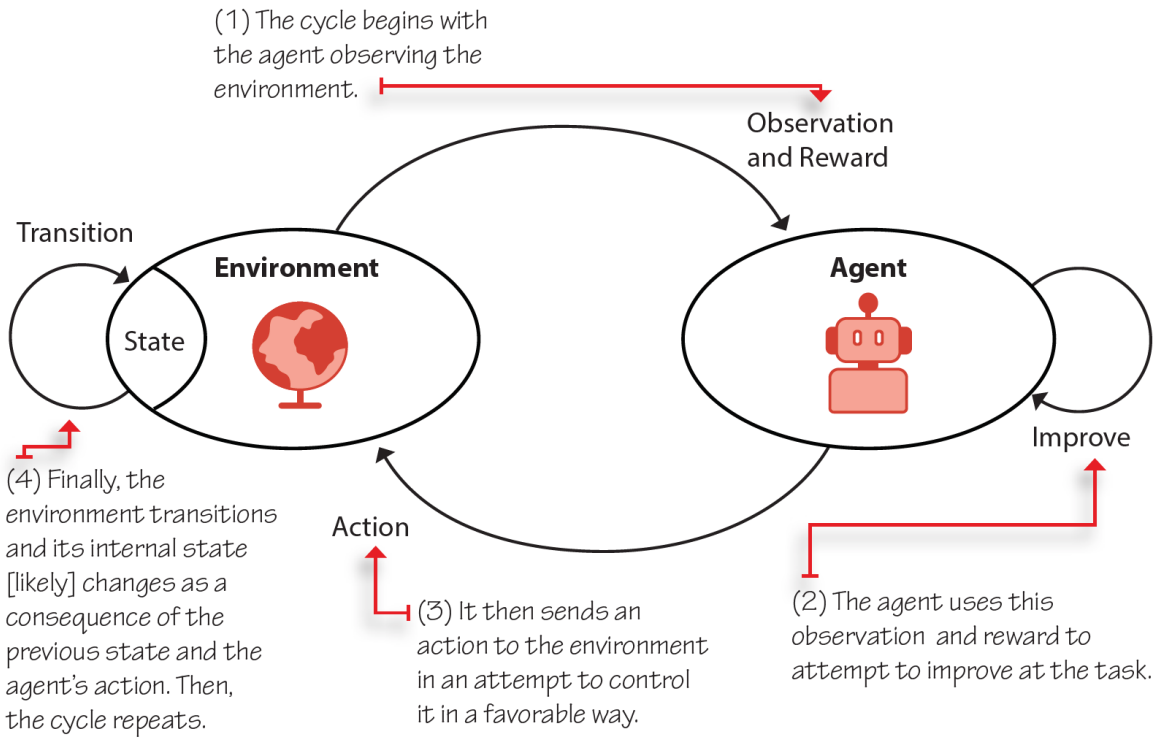
Reinforcement Learning agents learn from sampled feedback

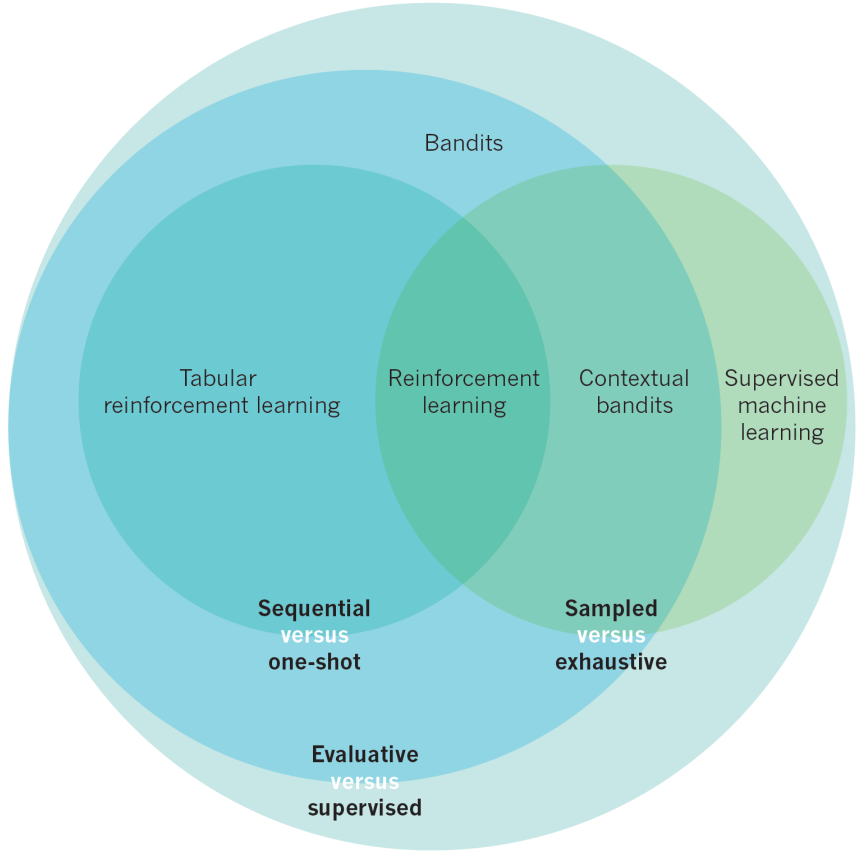
- The reward received by the agent is merely a sample, and the agent does not have access to the transition or reward function.
- Also, the state and action spaces are commonly large, even infinite, so trying to learn from sparse and weak feedback becomes a harder challenge when using samples.
- The agent must be able to learn from sampled feedback, it must be able to **generalize**.

The difficulty of learning from sampled feedback



Recap: Introduction to Reinforcement Learning





Recommended reading.
Reinforcement Learning: An introduction (chapters 1 and 16)
http://incompleteideas.net/book/the-book-2nd.html

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> Mankind's history has been a struggle against a hostile environment. We finally have reached a point where we can begin to dominate our environment [...]. As soon as we understand this fact, our mathematical interests necessarily shift in many areas from descriptive analysis to control theory.
>
> — Richard Bellman
> American applied mathematician
> an IEEE medal of honor recipient

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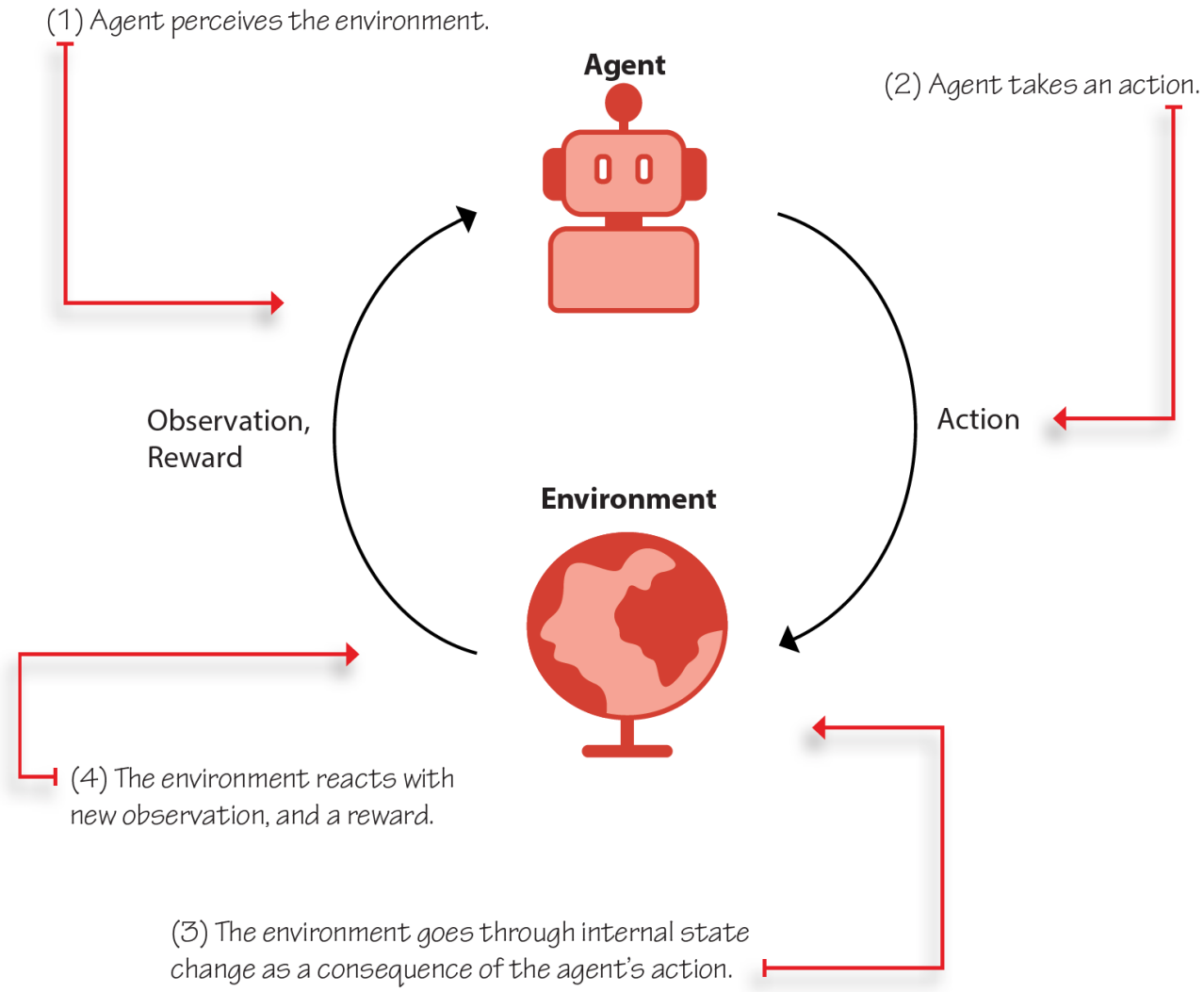
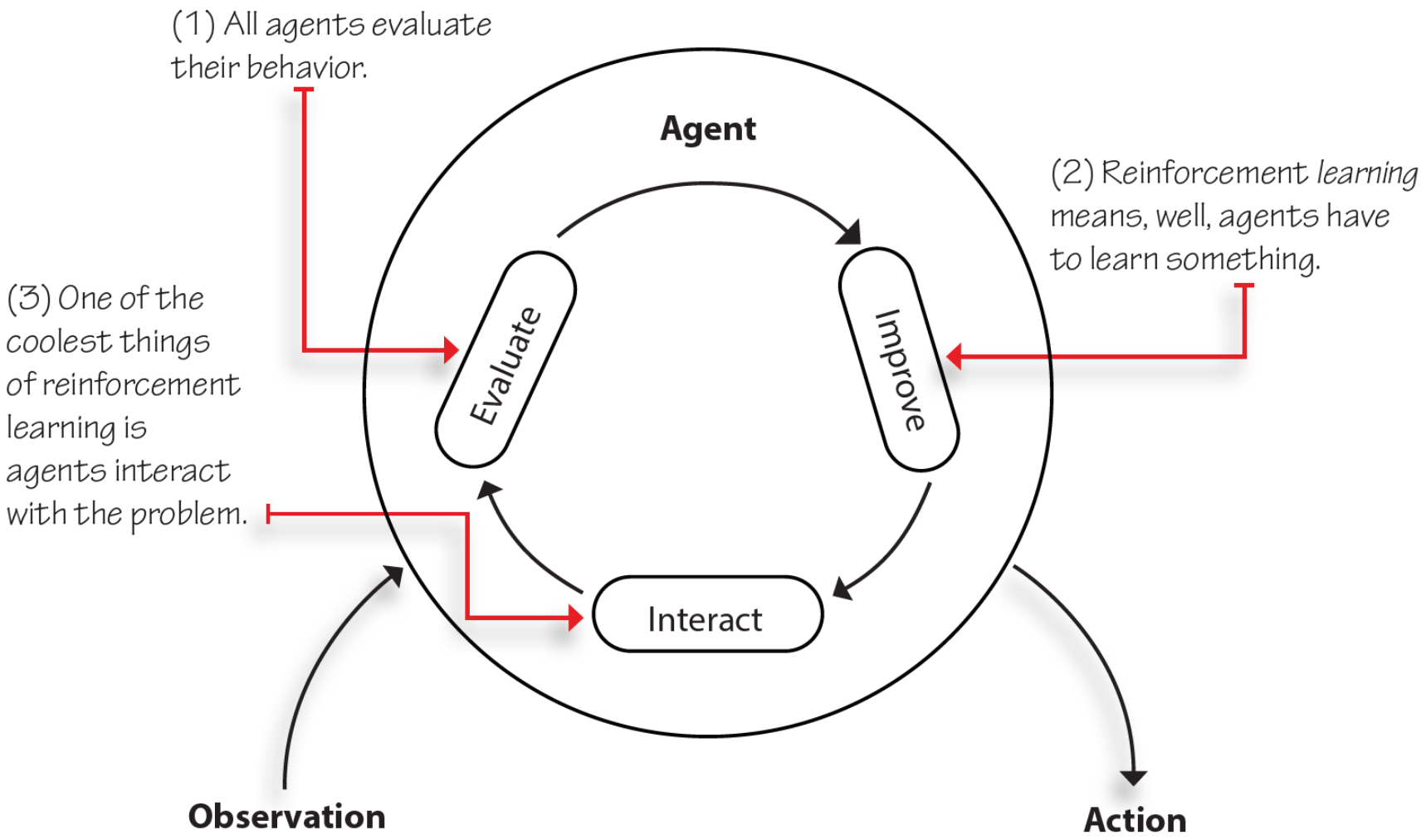
The reinforcement learning interaction cycle, again



Examples of problems, agents, and environments

• **Problem**: you are training your dog to sit. **Agent**: the part of your brain that makes decisions. **Environment**: your dog, the treats, your dog's paws, the loud neighbor, etc. **Actions**: Talk to your dog. Wait for dog's reaction. Move your hand. Show treat. Give treat. Pet. **Observations**: Your dog is paying attention to you. Your dog is getting tired. Your dog is going away. Your dog sat on command.
• **Problem**: your dog wants the treats you have. **Agent**: the part of your dog's brain that makes decisions. **Environment**: you, the treats, your dog's paws, the loud neighbor, etc. **Actions**: Stare at owner. Bark. Jump at owner. Try to steal the treat. Run. Sit. **Observations**: Owner keeps talking loud at me. Owner is showing the treat. Owner is hiding the treat. Owner gave me the treat.
• **Problem**: a trading agent investing in the stock market. **Agent**: the executing DRL code in memory and in the CPU. **Environment**: your Internet connection, the machine the code is running on, the stock prices, the geopolitical uncertainty, other investors, day-traders, etc. **Actions**: Sell n stocks of y company. Buy n stocks of y company. Hold. **Observations**: Market is going up. Market is going down. There are economic tensions between two powerful nations. There is danger of war in the continent. A global pandemic is wreaking havoc in the entire world.
• **Problem**: you are driving your car. **Agent**: the part of your brain that makes decisions. **Environment**: the make and model of your car, other cars, other drivers, the weather, the roads, the tires, etc. **Actions**: Steer by x, Accelerate by y. Break by z. Turn the headlights on. Defog windows. Play music. **Observations**: You are approaching your destination. There is a traffic jam on Main Street. The car next to you is driving recklessly. It's starting to rain. There is a police officer driving in front of you.

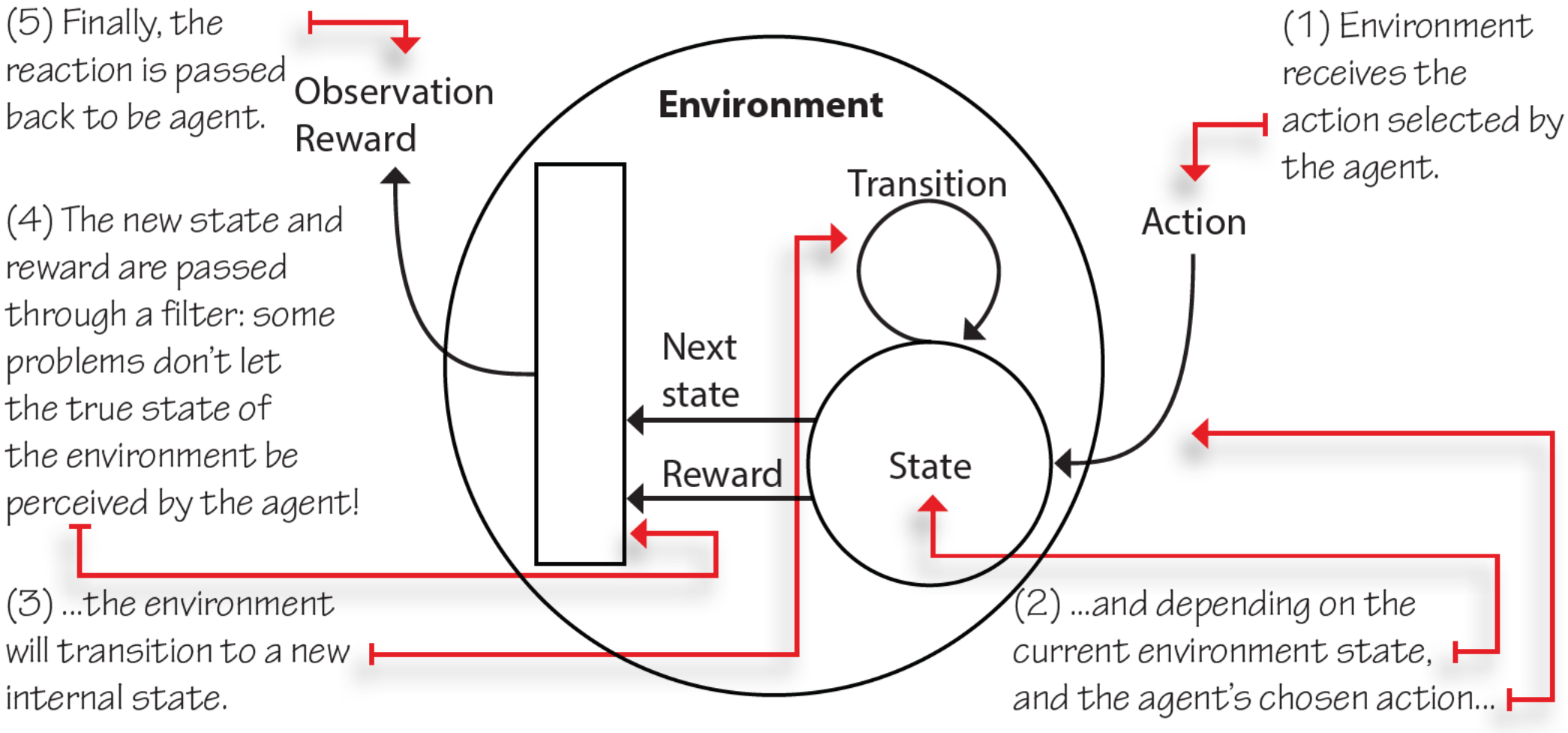
The agent: the decision-maker



Notes about the agent

- For now, the only important thing for you to know about agents is that there are agents and that they are the decision-makers in the RL picture.
- They have internal components and processes of their own, and that is what makes each of them unique and good at solving specific problems.
- More importantly, they are general, for the most part. They can solve a variety of problems if the problems provide the same interface (hint: Markov Decision Process).
- If we were to zoom into agents, we would notice that most agents have a three-step process:
  - All agents have an interaction component, a way to gather data for learning.
  - All agents evaluate their current behavior.
  - All agents improve something in their inner components that allows them to improve their overall performance (or at least attempt to improve).

The environment: Everything else



Notes about the environment

- Most real-world decision-making problems can be expressed as RL environments. A common way to represent decision-making processes in RL is by modeling the problem using a mathematical framework known as Markov Decision Processes (MDPs.)
- In RL, we assume all environments have an MDP working under the hood. Whether an ATARI game, the stock market, a self-driving car, your significant other, you name it, every problem has an MDP running under the hood (at least in the RL world, whether right or wrong.)

Markov Decision Process

Show Me the Math

MDPs vs. POMDPs

$$MDP(\mathcal{S}, \mathcal{A}, \mathcal{T}, \mathcal{R}, \mathcal{S}_\theta, \gamma, \mathcal{H})$$

(1) MDPs have state space *S*, action space *A*, transition function *T*, reward signal *R*. It also has a set of initial states distribution S_θ, the discount factor γ, and the horizon *H*.

$$POMDP(\mathcal{S}, \mathcal{A}, \mathcal{T}, \mathcal{R}, \mathcal{S}_\theta, \gamma, \mathcal{H}, \mathcal{O}, \mathcal{E})$$

(2) To define a POMDP you just add the observation space *O* and an emission probability *E* that defines the probability of showing an observation o_t given a state s_t. Very simple.

Useful definitions to navigate the RL lingo

- The environment is represented by a set of variables related to the problem. The combination of all the possible values this set of variables can take is referred to as the **state space**. A **state** is a specific set of values the variables of the state space take at any given time.
- Agents may or may not have access to the actual environment's state; however, one way or another, agents can observe something from the environment. The set of variables the agent perceives at any given time is called an **observation**.
- The combination of all possible values these variables can take is the **observation space**. Know that "state" and "observation" are terms used interchangeably in the RL community. This is because, very often, agents can see the internal state of the environment, but this is not always the case.
- At every state, the environment makes available a set of **actions** the agent can choose from. Often the set of actions is the same for all states, but this is not required. The set of all actions in all states is referred to as the **action space**.
- The agent attempts to influence the environment through these actions. The environment may change states as a response to the agent's action. The function that is responsible for this **transition** is called the **transition function**.
- After a transition, the environment emits a new observation. The environment may also provide a **reward signal** as a response. The function responsible for this mapping is called the **reward function**. The set of transition and reward function is referred to as the **model** of the environment.

Markov property

Show Me the Math

The Markov property

(1) The probability of the next state.

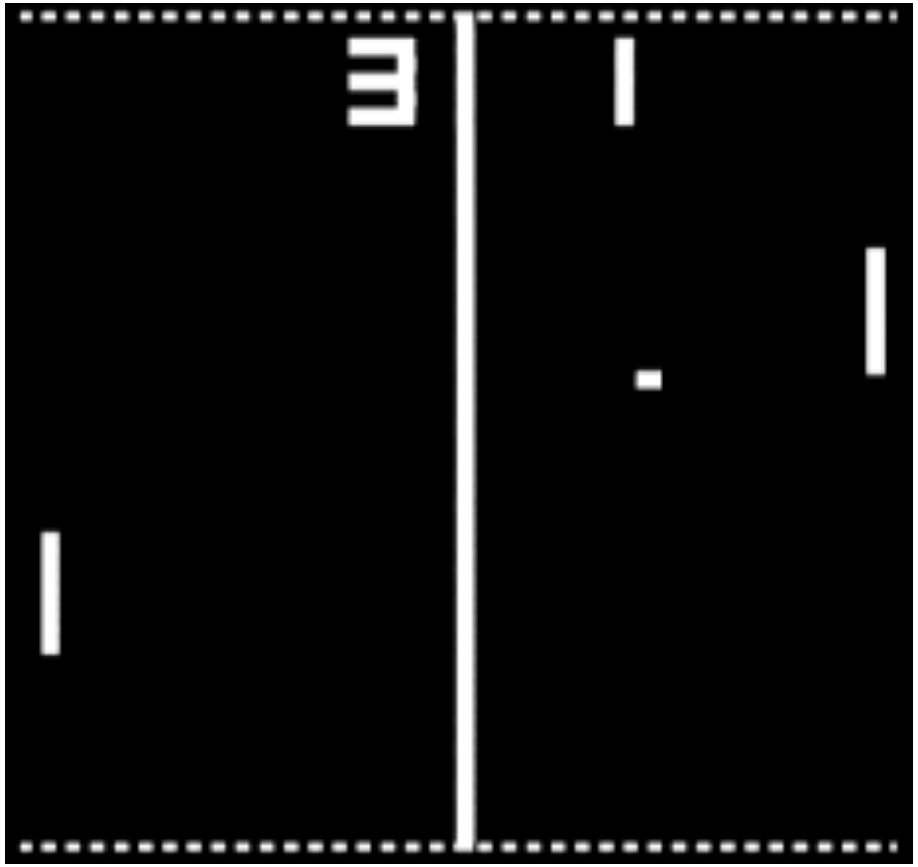
(3) Will be the same.

$$P(S_{t+1}|S_t, A_t) = P(S_{t+1}|S_t, A_t, S_{t-1}, A_{t-1}, \ldots)$$

(2) Given the current state and current action.

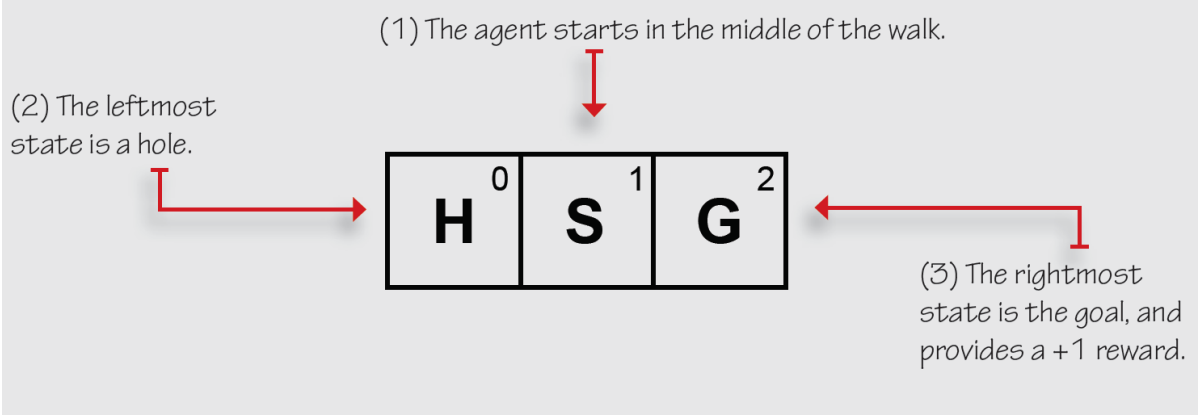
(4) As if you give it the entire history of interactions.

Why is the Markov property important?

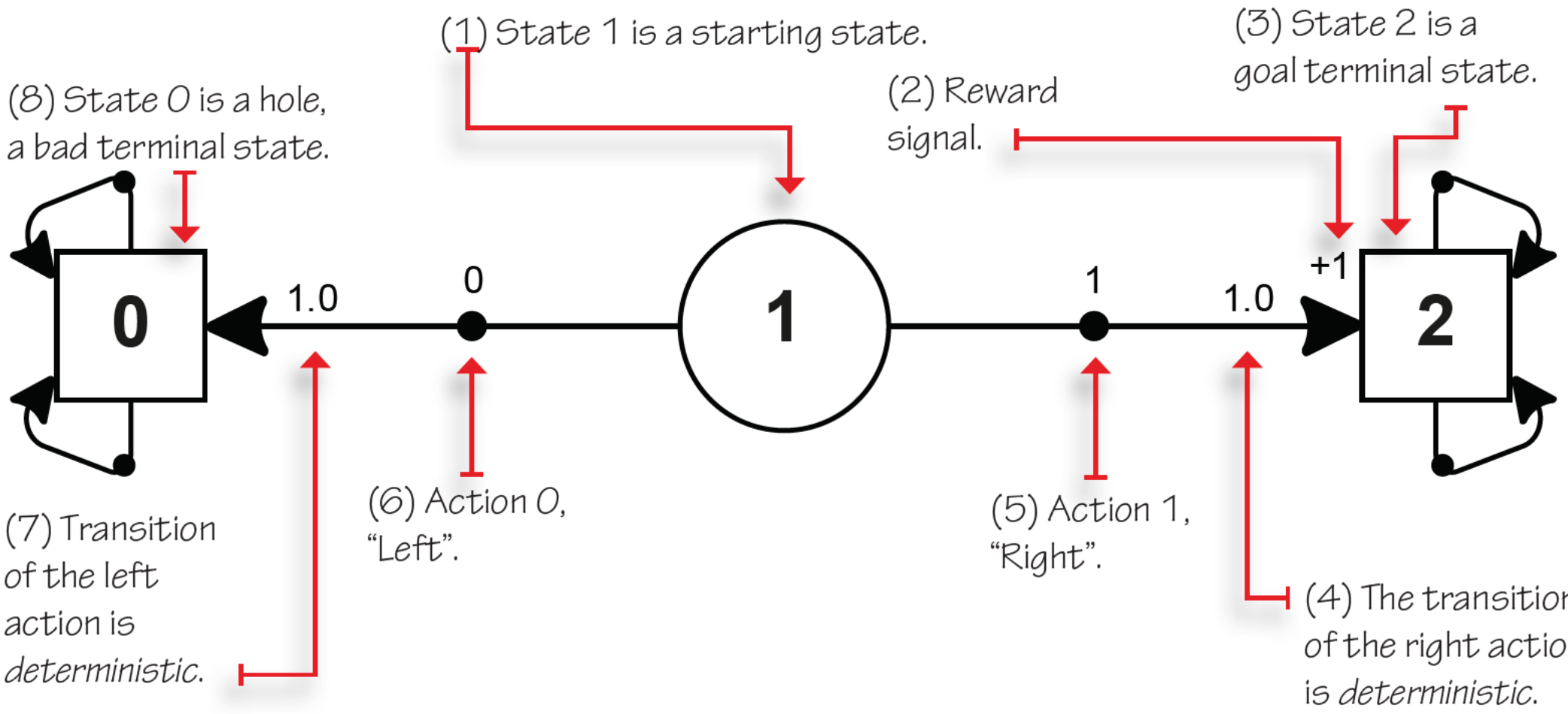


Example environment

- Imagine a simple environment. Let's call it the Bandit Walk.
- There are three states representing the cell ids.
- There are two actions, left, and right.
- The transition function is deterministic and as you expect (E.g. left moves the agent left, right moves it right). State 0 (H—hole) and state 2 (G—goal) are terminal states.
- The reward function is a +1 when landing the goal state "G", 0 otherwise.
- The agent starts in the middle cell, labeled S.



Respective MDP graphical representation



Respective MDP table:

State	Action	Next state	Transition probability	Reward signal
0 (Hole)	0 (Left)	0 (Hole)	1.0	0
0 (Hole)	1 (Right)	0 (Hole)	1.0	0
1 (Start)	0 (Left)	0 (Hole)	1.0	0
1 (Start)	1 (Right)	2 (Goal)	1.0	+1
2 (Goal)	0 (Left)	2 (Goal)	1.0	0
2 (Goal)	1 (Right)	2 (Goal)	1.0	0

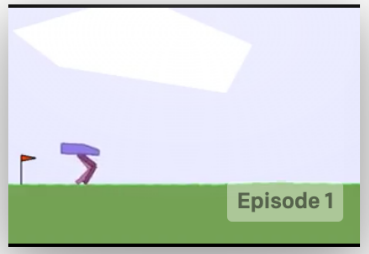
OpenAI Gym: A Python package that provides a variety of reinforcement learning environments

Description	Observation space	Sample observation	Action space	Sample action	Reward function
Hotter Colder: Guess a randomly selected number using hints.	Int range 0-3. 0 means no guess yet submitted, 1 means guess is lower than the target, 2 means guess is equal to the target and 3 means guess is higher than the target.	2	Float from -2000.0-2000.0. The float number the agent is guessing.	-909.37	The reward is the squared percentage of the way the agent has guessed toward the target.
Cart Pole: Balance a pole in a cart.	A 4-element vector with ranges: from [-4.8, -Inf, -4.2, -Inf] to [4.8, Inf, 4.2, Inf]. First element is the cart position, second is the cart velocity, third is pole angle in radians, fourth is the pole velocity at tip.	[-0.16, -1.61, 0.17, 2.44]	Int range 0-1. 0 means push cart left, 1 means push cart right.	0	The reward is 1 for every step taken, including the termination step.
Lunar Lander: Navigate a lander to its landing pad.	An 8-element vector with ranges: from [-Inf, -Inf, -Inf, -Inf, -Inf, -Inf, 0, 0] to [Inf, Inf, Inf, Inf, Inf, Inf, 1, 1]. First element is the x position, the second the y position, the third is the x velocity, the fourth is the y velocity, fifth is the vehicle's angle, sixth is the angular velocity, last two values are booleans indicating legs contact with the ground.	[0.36 , 0.23, -0.63, -0.10, -0.97, -1.73 , 1.0, 0.0]	Int range 0-3. No-op (do nothing), fire left engine, fire main engine, fire right engine.	2	Reward for landing is 200. There is reward for moving from the top to the landing pad, for crashing or coming to rest, for each leg touching the ground, and for firing the engines.
Pong: Bounce the ball past the opponent, and avoid letting the ball pass you.	A tensor of shape 210, 160, 3. Values ranging 0-255. Represents a game screen image.	[[[246, 217, 64], [55, 184, 230], [46, 231, 179], ..., [28, 104, 249], [25, 5, 22], [173, 186, 1]],...]]	Int range 0-5. Action 0 is No-op, 1 is Fire, 2 is up, 3 is right, 4 is left, 5 is down. Notice how some actions don't affect the game in any way. In reality the paddle can only move up, down or not move.	3	The reward is a 1 when the ball goes beyond the opponent, and a -1 when your agent's paddle misses the ball.
Humanoid: Make robot run as fast as possible and not fall.	A 44-element (or more, depending on the implementation) vector. Values ranging from -Inf to Inf. Represents the positions and velocities of the robot's joints.	[0.6, 0.08, 0.9, 0. , 0., 0., 0., 0., 0.045, 0., 0.47, ..., 0.32, 0., -0.22,..., 0.]	A 17-element vector. Values ranging from -Inf to Inf. Represents the forces to apply to the robot's joints.	[-0.9, -0.06, 0.6, 0.6, 0.6, -0.06, -0.4, -0.9, 0.5, -0.2, 0.7, -0.9, 0.4, -0.8, -0.1, 0.8, -0.03]	The reward is calculated based on forward motion with a small penalty to encourage a natural gait.

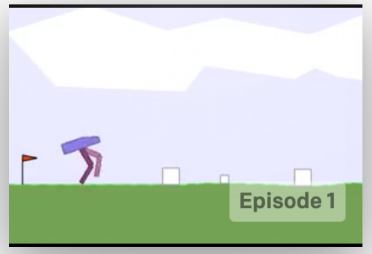
OpenAI Gym: A Python package that provides a variety of reinforcement learning environments

Box2D

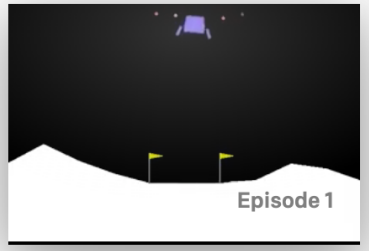
Continuous control tasks in the Box2D simulator.



BipedalWalker-v2
Train a bipedal robot to walk.



BipedalWalkerHardcore-v2
Train a bipedal robot to walk over rough terrain.



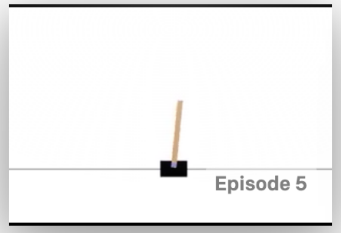
LunarLander-v2
Navigate a lander to its landing pad.

LunarLanderContinuous-v2
Navigate a lander to its landing pad.

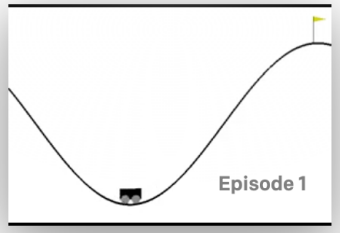
Classic control

Control theory problems from the classic RL literature.

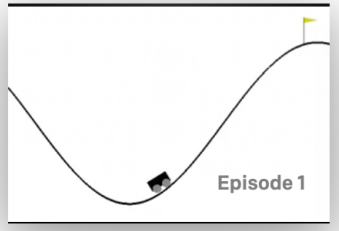
Acrobot-v1
Swing up a two-link robot.



CartPole-v1
Balance a pole on a cart.



MountainCar-v0
Drive up a big hill.



MountainCarContinuous-v0
Drive up a big hill with continuous control.

Pendulum-v0
Swing up a pendulum.

Recap: Markov Decision Process

SHOW ME THE MATH

MDPs vs. POMDPs

$$MDP(\mathcal{S}, \mathcal{A}, \mathcal{T}, \mathcal{R}, \mathcal{S}_\theta, \gamma, \mathcal{H})$$

(1) MDPs have state space S, action space A, transition function T, reward signal R. It also has a set of initial states distribution S_θ, the discount factor γ, and the horizon H.

$$POMDP(\mathcal{S}, \mathcal{A}, \mathcal{T}, \mathcal{R}, \mathcal{S}_\theta, \gamma, \mathcal{H}, \mathcal{O}, \mathcal{E})$$

(2) To define a POMDP you just add the observation space O and an emission probability E that defines the probability of showing an observation o_t given a state s_t. Very simple.

Recommended reading.
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> In preparing for battle I have always found that plans are useless, but planning is indispensable.
>
> — Dwight D. Eisenhower
> United States Army five-star general and
> 34th President of the United States

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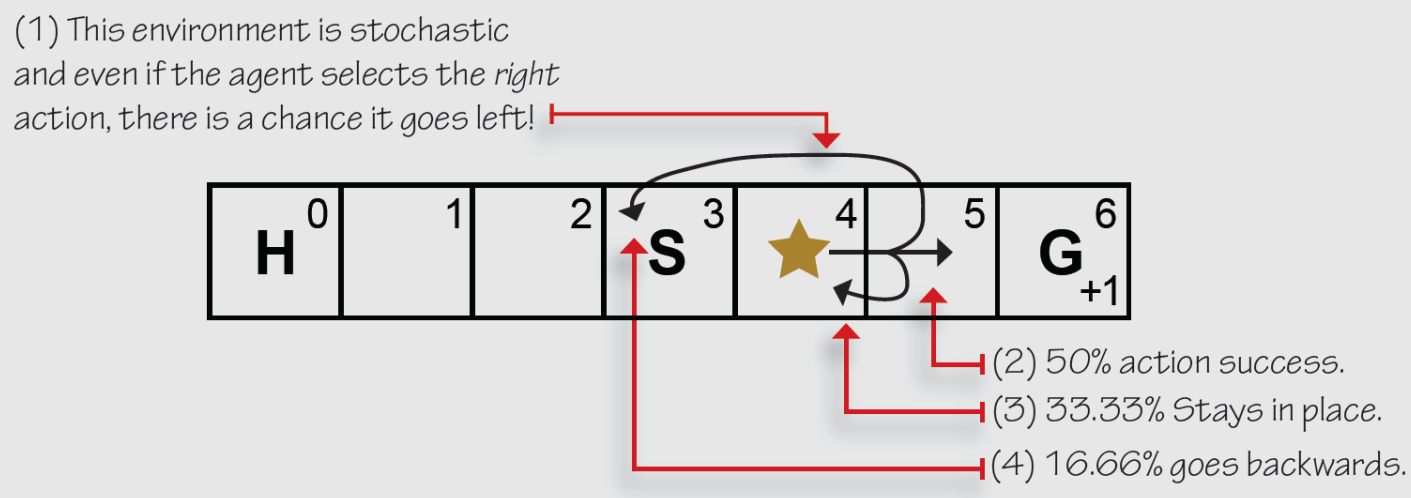
Let's obtain try to solve this decision-making problem

Concrete Example

The Slippery Walk Five (SWF) environment

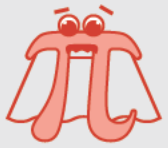
The Slippery Walk Five (SWF) is a one-row grid-world environment (a walk), that is stochastic, similar to the Frozen Lake, and it has only five non-terminal states (seven total if we count the two terminal).

The slippery walk five environment



The agent starts in *S*, *H* is a hole, *G* is the goal and provides a +1 reward.

The return G



Show Me the Math

The return *G*

(1) The return is the sum of rewards encounter from step *t*, until the final step *T*.

$$G_t = R_{t+1} + R_{t+2} + R_{t+3} + \ldots + R_T$$

(2) As I mentioned in the previous chapter, we can combine the return and time using the discount factor, gamma. This is then the discounted return, which prioritizes early rewards.

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots + \gamma^{T-1} R_T$$

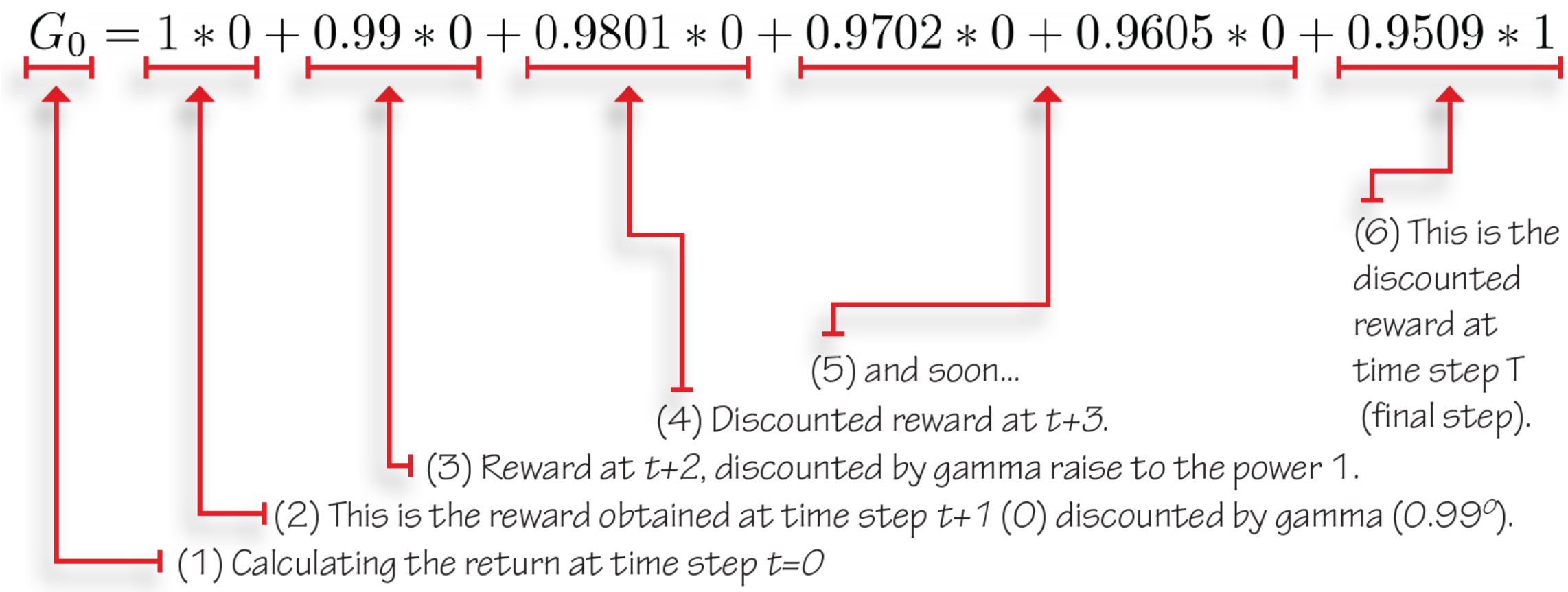
(3) We can simplify the equation and have a more general equation, such as this one.

$$G_t = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

$$G_t = R_{t+1} + \gamma G_{t+1}$$

(4) And stare at this recursive definition of *G* for a while.

Calculating the return G



The state-value function V

Show Me the Math

The state-value function *V*

(1) The value of a state *s*.

(3) Is the expectation over π.

(2) Under policy π.

$$v_\pi(s) = \mathbb{E}_\pi[G_t|S_t = s]$$

(5) Given you select state *s* at time step *t*.

(4) Of returns at time step *t*.

(6) Remember that returns are the sum of discounted rewards.

$$v_\pi(s) = \mathbb{E}_\pi[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + ...|S_t = s]$$

(7) And that we can defined them recursively like so.

$$v_\pi(s) = \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1}|S_t = s]$$

(8) This equation is called the Bellman equation and it tells us how to find the value of states.

$$v_\pi(s) = \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a)[r + \gamma v_\pi(s')], \forall s \in S$$

(9) We get the action (or actions,if the policy is stochastic) prescribed for state *s*. And do a weighted sum...

(10) We also weight the sum over the probability of next states and rewards.

(11) We add the reward and the discounted value of the landing state, then weight that by the probability of that transition occurring.

(12) Do this for all states in the state space.

The action-value function Q

Show Me the Math

The action-value function Q

(1) The value of action *a* in state *s* under policy π.

(2) Is the expectation of returns given we select action *a* in state *s* and follow policy π thereafter.

$$q_\pi(s,a) = \mathbb{E}_\pi[G_t | S_t = s, A_t = a]$$

(3) And just as before we can define this equation recursively like so.

$$q_\pi(s,a) = \mathbb{E}_\pi[R_t + \gamma G_{t+1} | S_t = s, A_t = a]$$

(4) The Bellman equation for action values is defined as follows.

$$q_\pi(s,a) = \sum_{s',r} p(s',r|s,a)[r + \gamma v_\pi(s')], \forall s \in S, \forall a \in A(s)$$

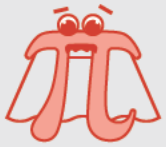
(5) Notice we don't weigh over actions because we are interested only in a *specific* action.

(6) We do weigh, however, by the probabilities of next states and rewards.

(7) What do we weigh? The sum of the reward and the discounted value of the next state.

(8) We do that for all state-action pairs.

The action-advantage function A



Show Me the Math

The action-advantage function *A*

(1) The *advantage* of action *a* in state *s* under policy π.

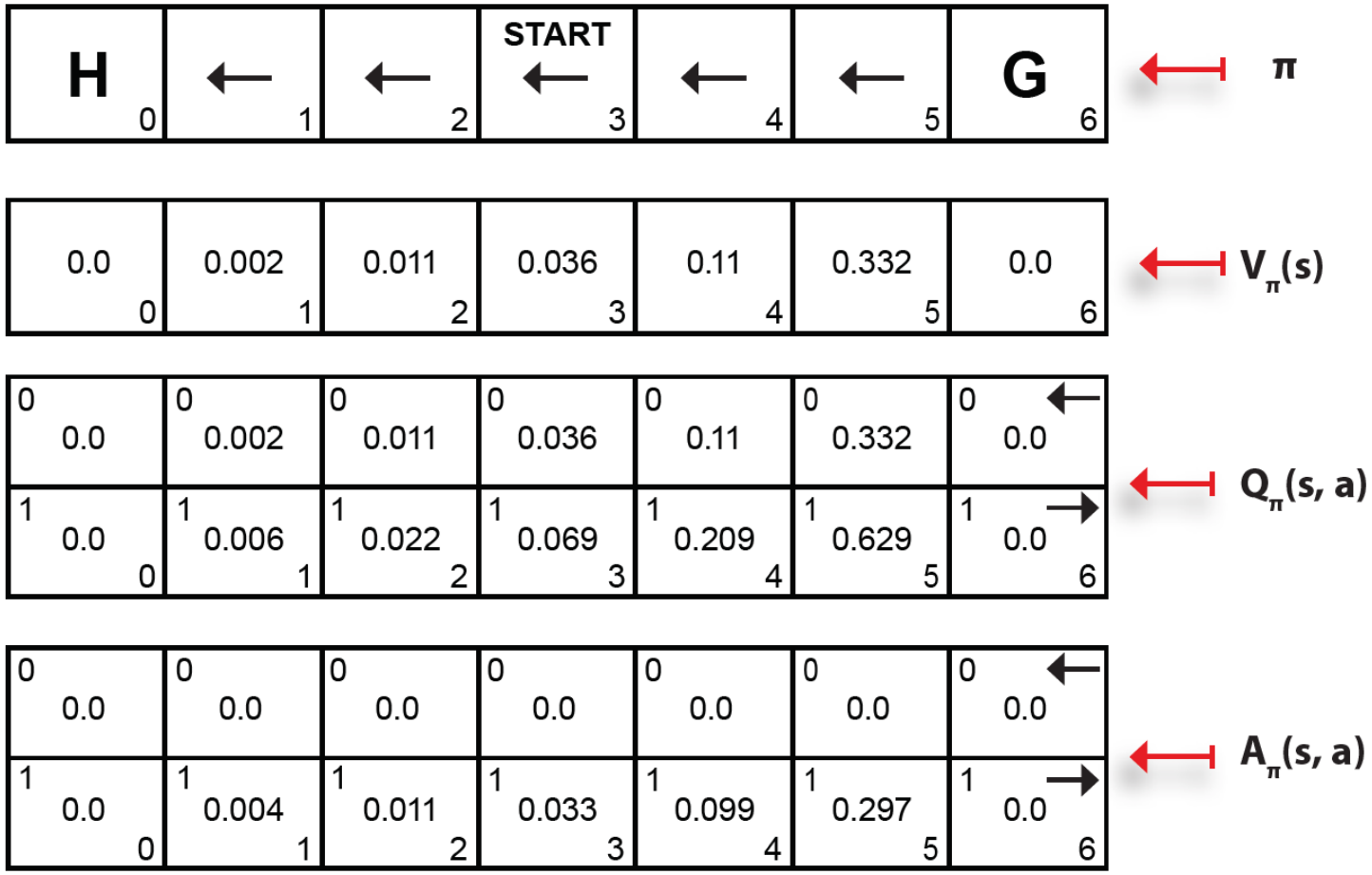
$$a_\pi(s,a) = q_\pi(s,a) - v_\pi(s)$$

(2) Is the *difference* between the value of that action, and the value of the state *s*, both under policy π.

Examples of V, Q, and A

(1) Notice how $Q_\pi(s,a)$ allows us to improve policy π, by showing the highest valued action under the policy.

(2) Also notice there is no advantage for taking the same action as policy π recommends.



The Bellman optimality equations

Show Me the Math

The Bellman optimality equations

(1) The optimal state-value function.

$$v_*(s) = \max_\pi v_\pi(s), \forall s \in S$$

(2) Is the state-value function with the highest value across all policies.

(3) Likewise, the optimal action-value function is the action-value function with the highest values.

$$q_*(s,a) = \max_\pi q_\pi(s,a), \forall s \in S, \forall a \in A(s)$$

(4) The optimal state-value function can be obtained this way.

$$v_*(s) = \max_a \sum_{s',r} p(s',r|s,a)[r + \gamma v_*(s')]$$

(5) We take the max action.

(6) Of the weighted sum of the reward and discounted optimal value of the next state.

(7) Similarly, the optimal action-value function can be obtained this way.

$$q_*(s,a) = \sum_{s',r} p(s',r|s,a)\left[r + \gamma \max_{a'} q_*(s',a')\right]$$

(8) Notice how the max is now on the inside.

Policy evaluation equation

Show Me the Math

The policy-evaluation equation

(1) The policy evaluation algorithm consist on the iterative approximation of the state-value function of the policy under evaluation. The algorithm converges as k approaches infinity.

(2) Initialize $v_0(s)$ for all s in S arbitrarily, and to 0 if s is terminal. Then, increase k and iteratively improve the estimates simply by following the equation below.

$$v_{k+1}(s) = \sum_{a} \pi(a|s) \sum_{s',r} p(s', r|s, a) \Big[r + \gamma v_k(s') \Big]$$

(3) Calculate the value of a state s as the weighted sum of the reward and the discounted estimated value of the next state s'.

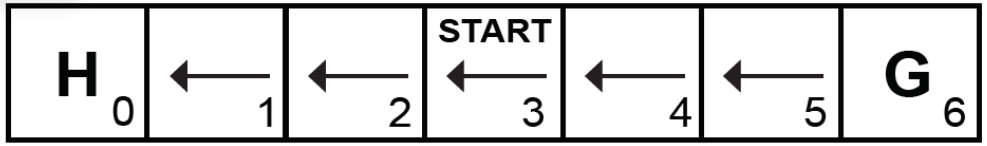
Initial calculations of policy evaluation

$$v_{k+1}(s) = \sum_{a} \pi(a|s) \sum_{s',r} p(s', r|s, a) \left[r + \gamma v_k(s') \right]$$

(1) We have a deterministic policy, so this part here is 1.

(2) Let's use gamma of 1.

(3) An "Always LEFT" policy. → π



State 5, Iteration 1 (initialized to 0 in iteration 0):

$$v_1^{\pi}(5) = p(s'=4 \mid s=5, a=\text{LEFT}) * [\, R(5, \text{LEFT}, 4) + v_0^{\pi}(4) \,] +$$
$$p(s'=5 \mid s=5, a=\text{LEFT}) * [\, R(5, \text{LEFT}, 5) + v_0^{\pi}(5) \,] +$$
$$p(s'=6 \mid s=5, a=\text{LEFT}) * [\, R(5, \text{LEFT}, 6) + v_0^{\pi}(6) \,]$$

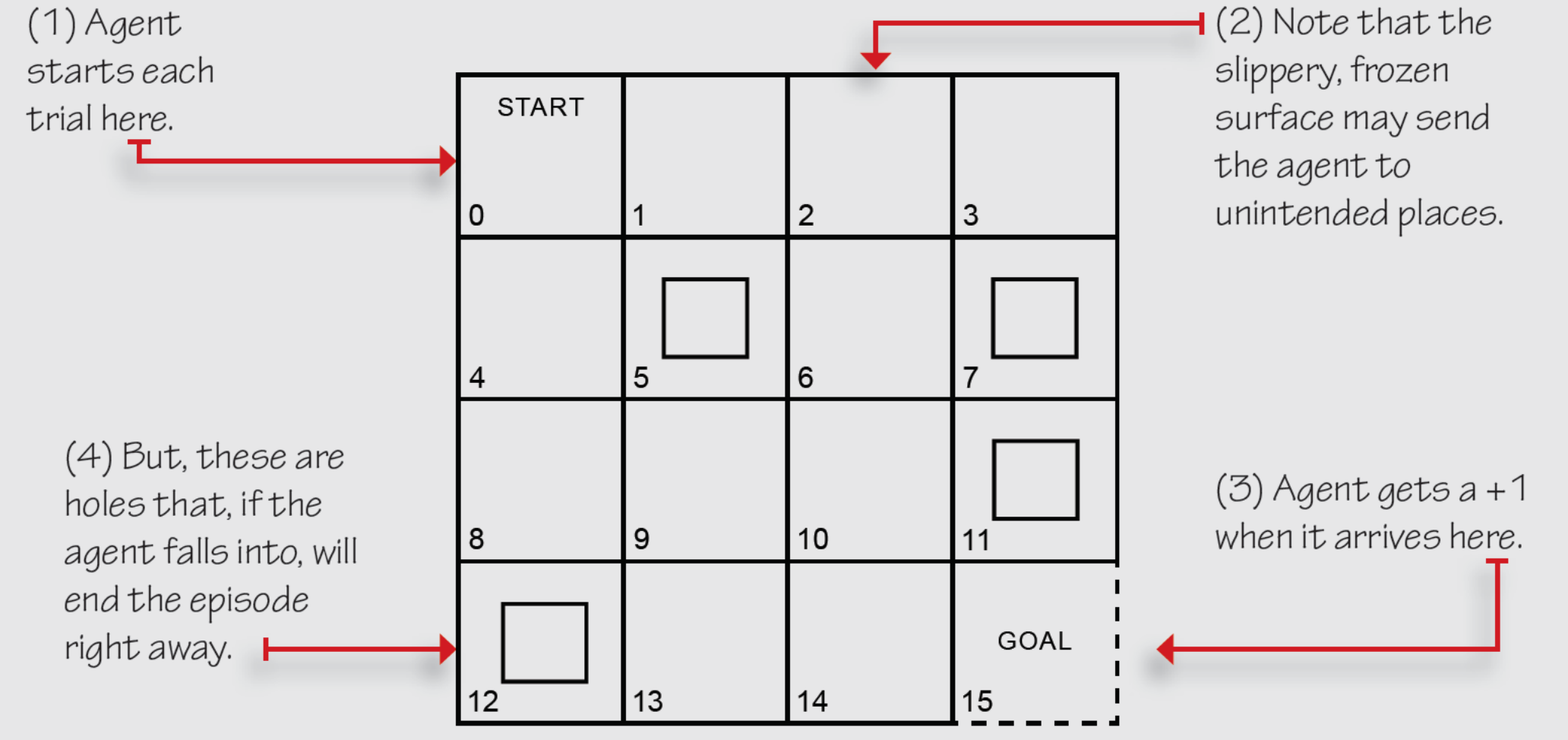
$$v_1^{\pi}(5) = 0.50 * (0+0) + 0.33 * (0+0) + 0.166 * (1+0) = 0.166$$

(4) Yep, this is the value of state 5 after 1 iteration of policy evaluation ($v_1^{\pi}(5)$).

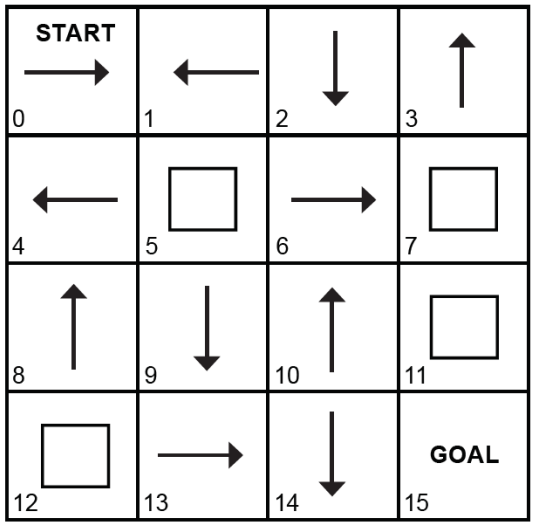
Policy evaluation detailed results

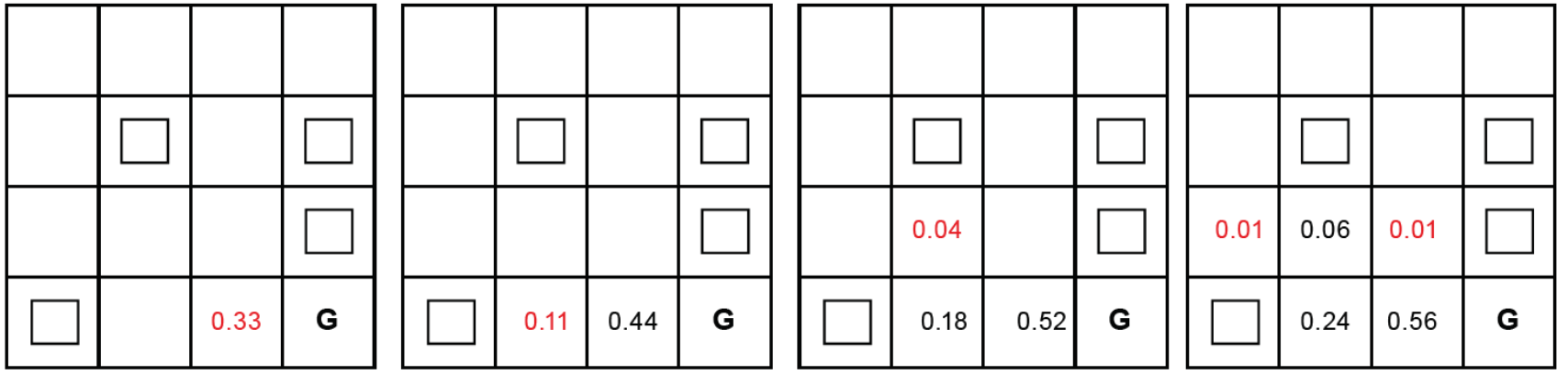
k	$V^{\pi}(0)$	$V^{\pi}(1)$	$V^{\pi}(2)$	$V^{\pi}(3)$	$V^{\pi}(4)$	$V^{\pi}(5)$	$V^{\pi}(6)$
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0.1667	0
2	0	0	0	0	0.0278	0.2222	0
3	0	0	0	0.0046	0.0463	0.2546	0
4	0	0	0.0008	0.0093	0.0602	0.2747	0
5	0	0.0001	0.0018	0.0135	0.0705	0.2883	0
6	0	0.0003	0.0029	0.0171	0.0783	0.2980	0
7	0	0.0006	0.0040	0.0202	0.0843	0.3052	0
8	0	0.0009	0.0050	0.0228	0.0891	0.3106	0
9	0	0.0011	0.0059	0.0249	0.0929	0.3147	0
10	0	0.0014	0.0067	0.0267	0.0959	0.318	0
...	...	...	...	...	...	...	...
104	0	0.0027	0.011	0.0357	0.1099	0.3324	0

Quick detour: The Frozen Lake environment

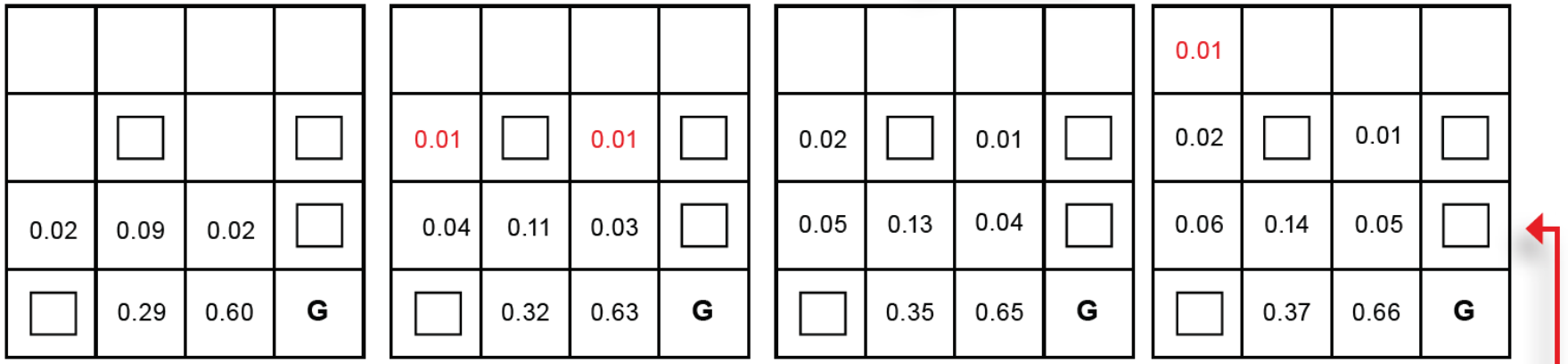


Policy evaluation in the FL environment



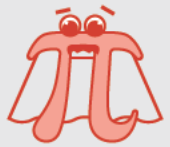


(1) Values start propagating with every iteration.



(2) The values continue to propagate and become more and more accurate.

Policy Improvement equation



Show Me the Math

The policy-improvement equation

(1) To improve a policy, we use a state-value function and an MDP to get a one-step lookahead and determine which of the actions lead to the highest value. This is policy improvement equation.

(2) We obtain a new policy π' by taking the highest-valued action.

(3) How, do we get the highest-valued action?

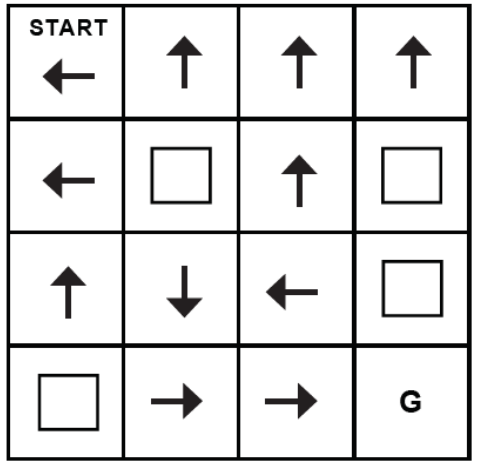
$$\pi'(s) = \underset{a}{\operatorname{argmax}} \sum_{s',r} p(s', r|s, a)\left[r + \gamma v_\pi(s')\right]$$

(4) By calculating, for each action, the weighted sum of all rewards and values of all possible next states.

(5) Notice that this is simply using the action with the highest-valued Q-function.

Policy Improvement example

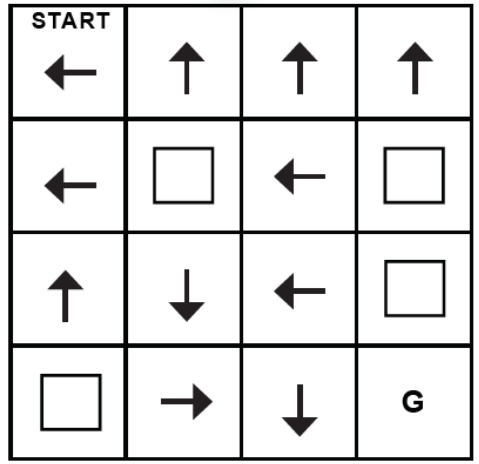
(1) This is the "Careful" policy.



(2) Action-value function of the "Careful" policy.

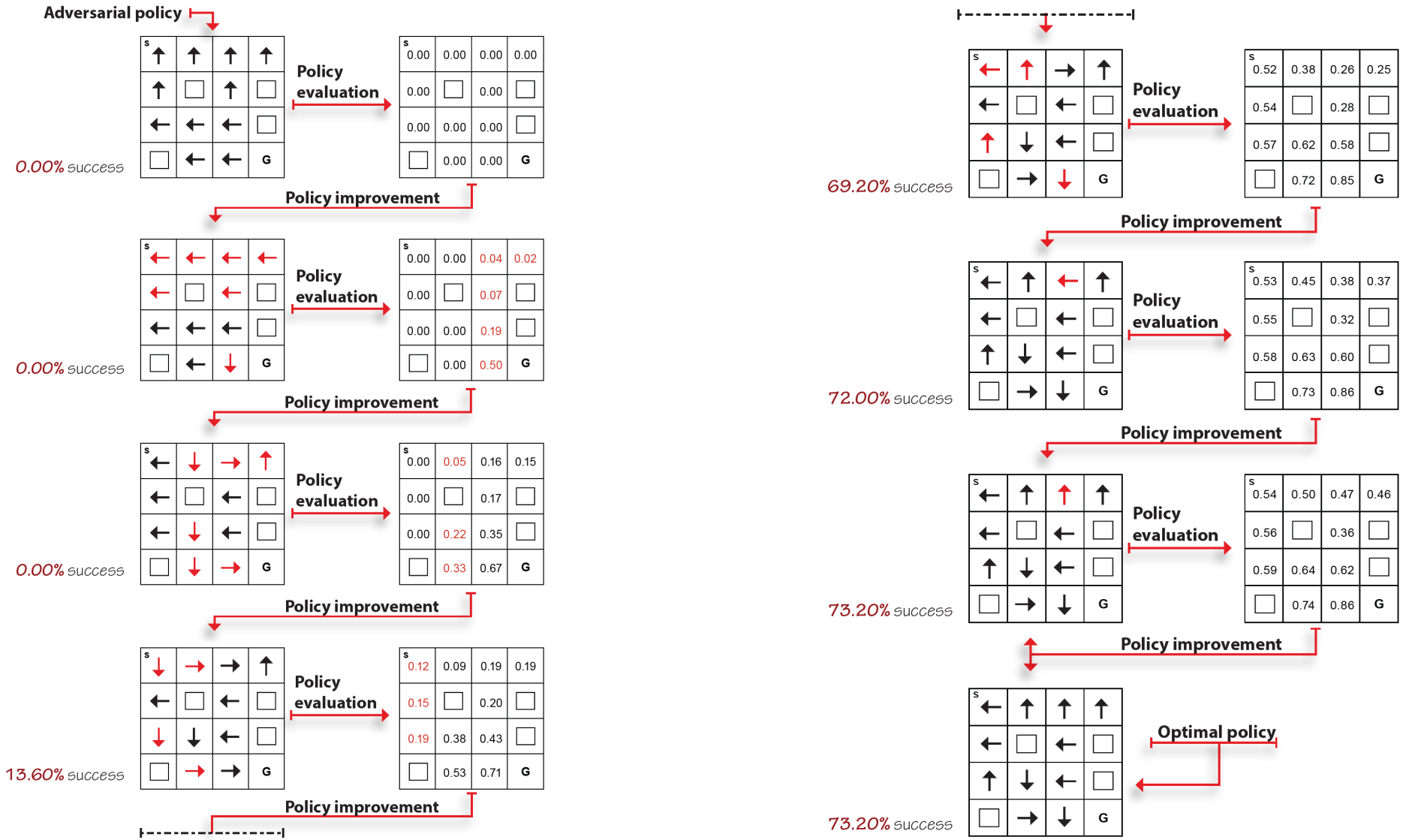
START 0.39 0.41 0.40 0.40	0.38 0.26 0.24 0.25	0.35 0.28 0.27 0.28	0.34 0.23 0.23 0.23
0.27 0.42 0.28 0.29		0.12 0.26 0.26 0.14	
0.45 0.29 0.30 0.31	0.29 0.34 0.34 0.48	0.2 0.43 0.27 0.39	
	0.39 0.35 0.59 0.43	0.67 0.57 0.71 0.76	GOAL

(3) The greedy policy over the "Careful" Q-function.

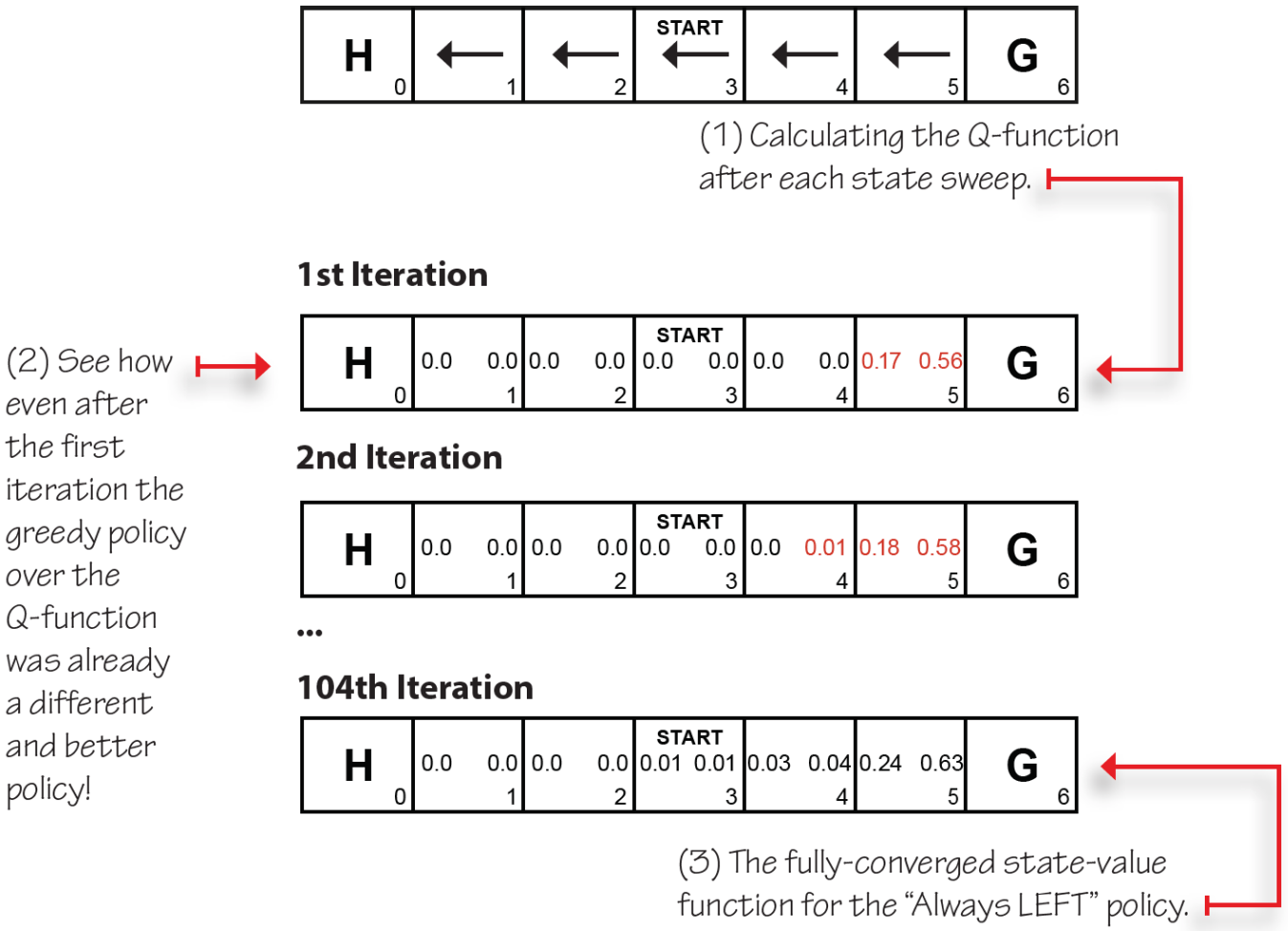


(4) I'm calling this new policy "Careful+"

Policy Iteration



Value Iteration Motivation



Value Iteration

Show Me the Math

The value-iteration equation

(1) We can merge a truncated policy evaluation step and a policy improvement into the same equation.

(2) We calculate the value of each action.

(3) Using the sum of the weighted sum...

(4) Of the reward and the discounted estimated value of the next state.

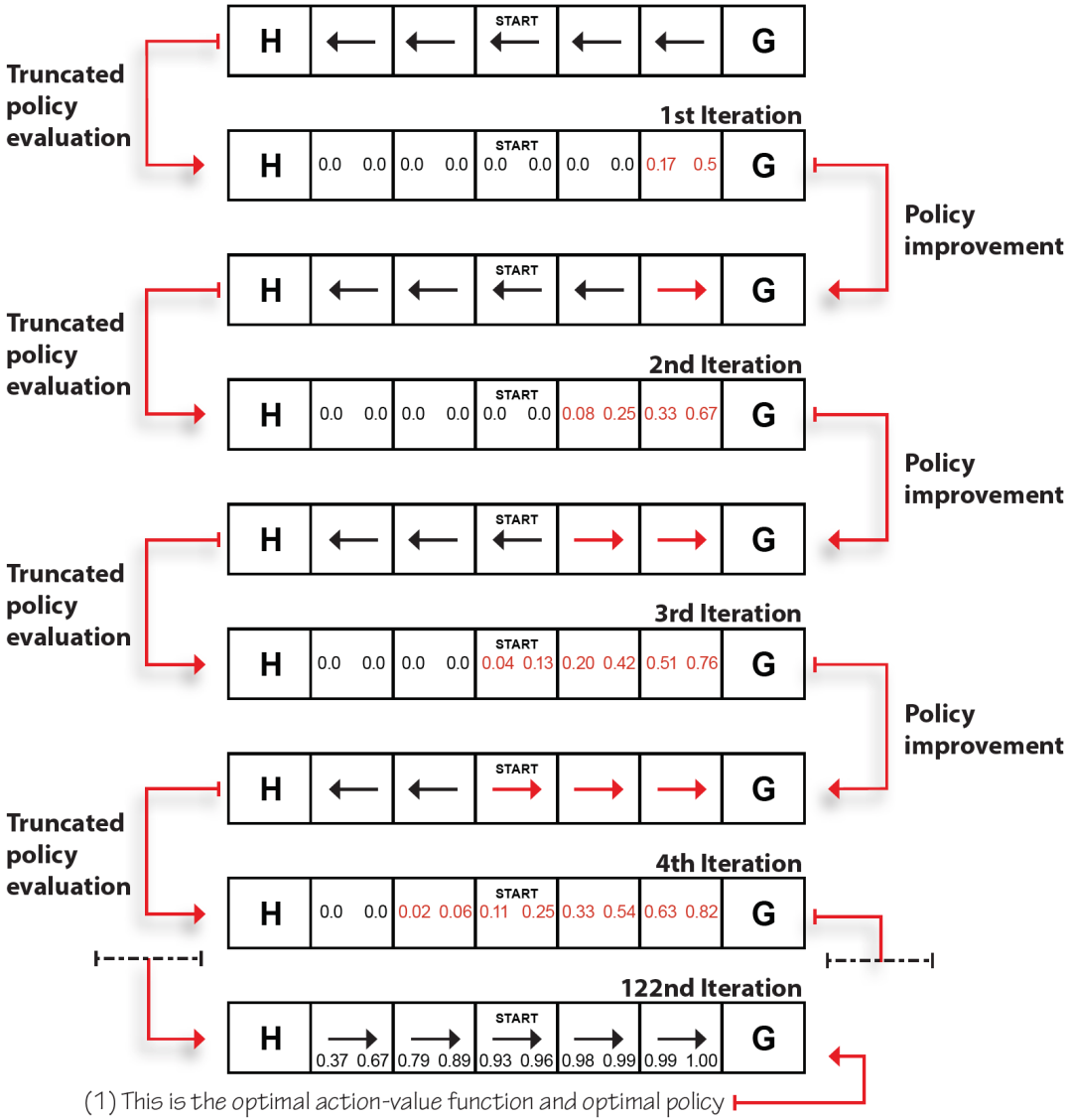
$$v_{k+1}(s) = \max_{a} \sum_{s',r} p(s', r|s, a)\left[r + \gamma v_k(s')\right]$$

(7) Then, we take the max over the values of actions.

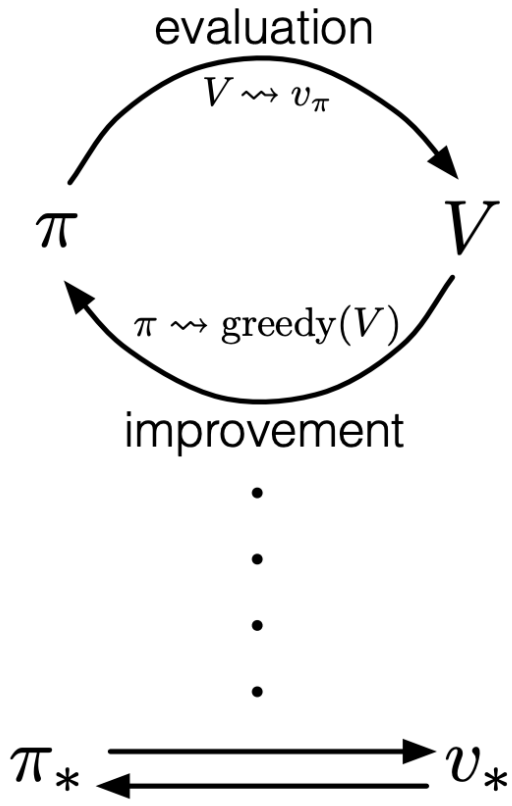
(6) And add for all transitions in the action.

(5) Multiply by the probability of each possible transition.

Value Iteration example



Recap: Planning methods



Recommended reading.
Reinforcement Learning: An introduction (chapter 4)
http://incompleteideas.net/book/the-book-2nd.html

Outline

Opening

Introduction to Reinforcement Learning

Markov Decision Process

Planning Methods

Bandit Problems

Prediction Problem

Control Problem

> Uncertainty and expectation are the joys of life.
> Security is an insipid thing.
>
> — William Congreve
> English playwright and poet of the Restoration period
> and political figure in the British Whig Party

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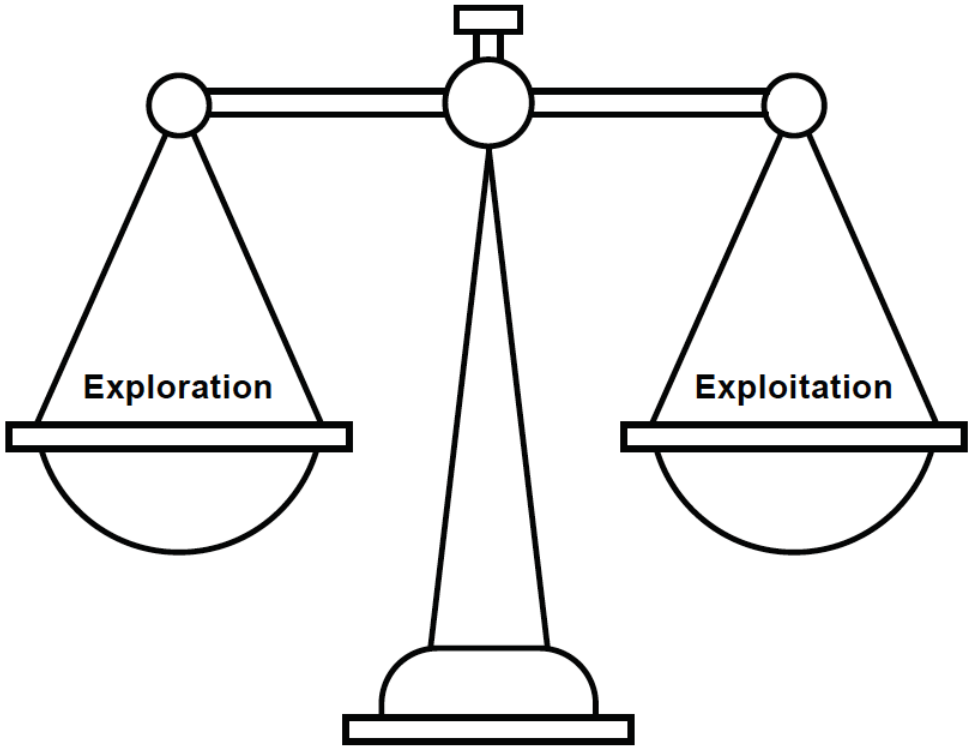
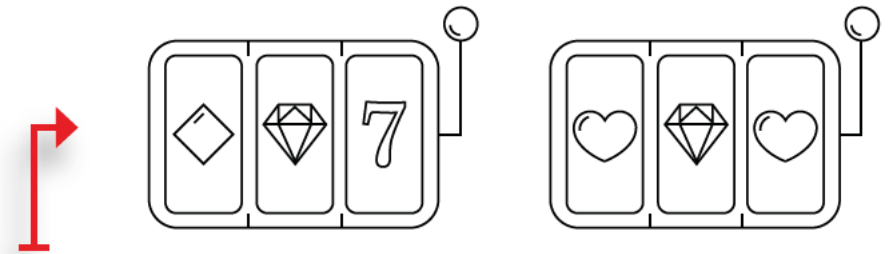
Exploration vs. Exploitation

- Planning methods assume we have a “map” of the environment. But what if we don’t?
- We need to explore to gain information about the environment.
- But exploring cause us to missed opportunities we could otherwise exploit.
- There is a tradeoff between exploration and exploration.



Exploration vs. Exploitation

- Multi-armed bandits (MAB) are a special case of a RL problem in which the size of the state space and horizon equal one.
- MAB have multiple actions, a single state, and a greedy horizon; you can also think of it as a “many-options single-choice” environment.
- The name comes from slot machines (bandits) with multiple arms to choose from (more realistically: multiple slot machines to choose from).



(1) A 2-armed bandit is a decision-making problem with two choices. You need to try them both sufficient to correctly asses each option. So, how do you best hand the exploration-exploitation tradeoff?

Bandit Problems

Show Me the Math

Multi-armed bandit

(1) MABs are MDPs with a single non-terminal state, and a single time step per episode.

$$MAB = \mathcal{MDP}(\mathcal{S} = \{s\}, \mathcal{A}, \mathcal{T}, \mathcal{R}, \mathcal{S}_\theta = \{s\}, \gamma = 1, \mathcal{H} = 1)$$

(2) The Q-function of action *a* is the expected reward given *a* was sampled.

$$q(a) = \mathbb{E}[R_t | A_t = a]$$

(3) The best we can do in a MAB is represented by the optimal V-function, or selecting the action that maximizes the Q-function.

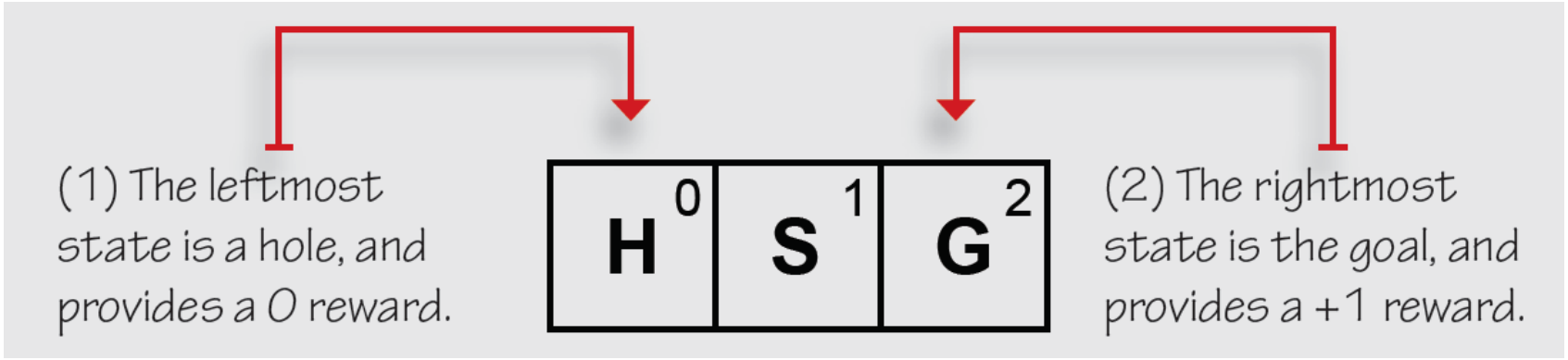
$$v_* = q(a_*) = \max_{a \in A} q(a)$$

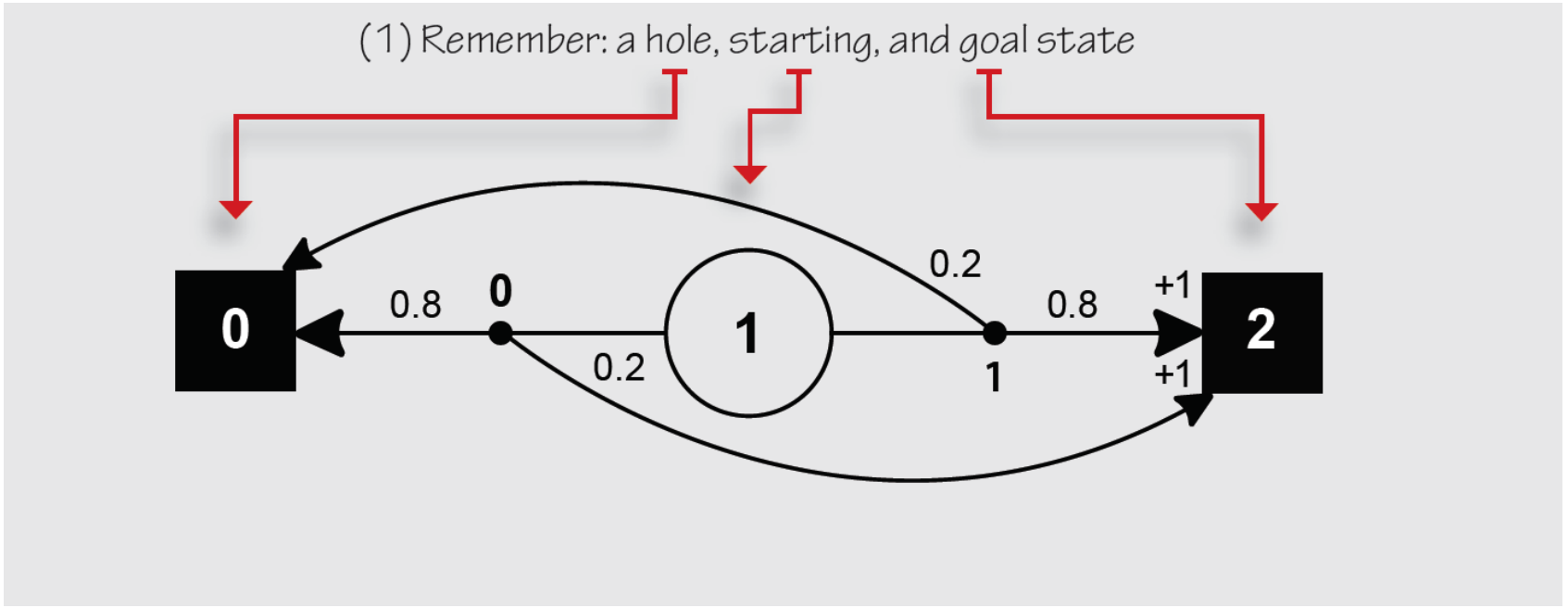
$$a_* = \operatorname*{argmax}_{a \in A} q(a)$$

(4) The optimal action, is the action that maximizes the optimal Q-function, and optimal V-function (only 1 state).

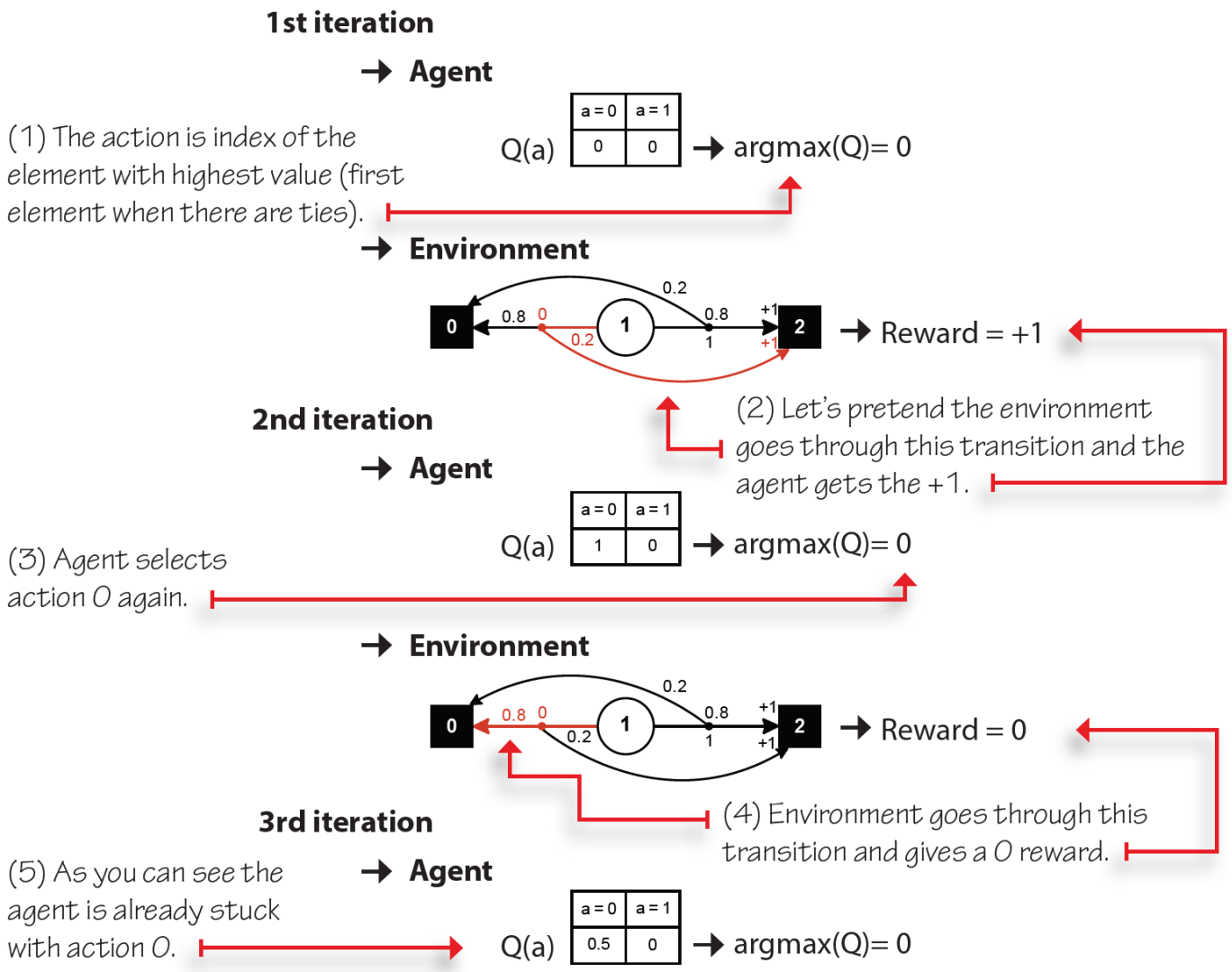
$$q(a_*) = v_*$$

Slippery Bandit Walk environment

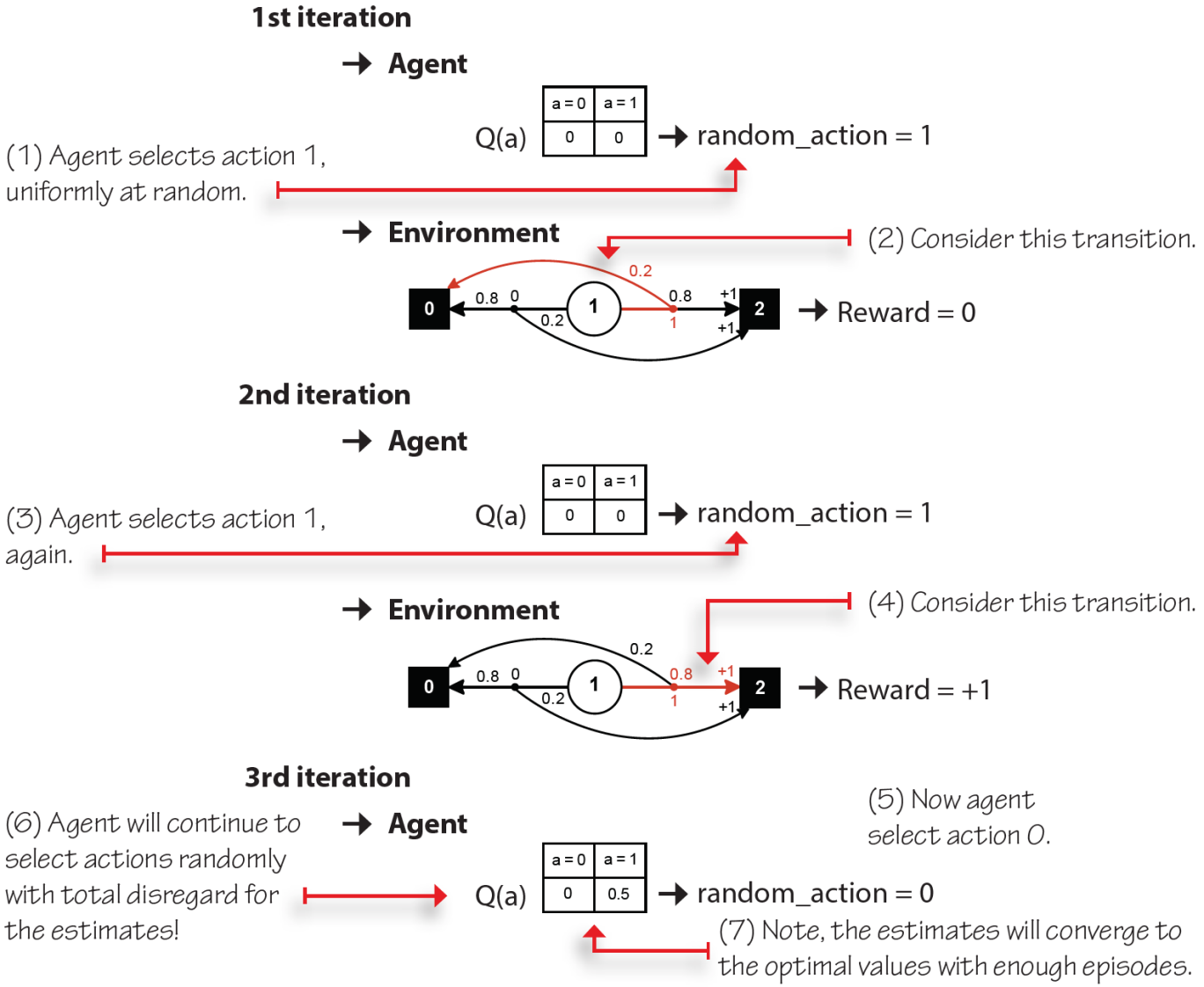




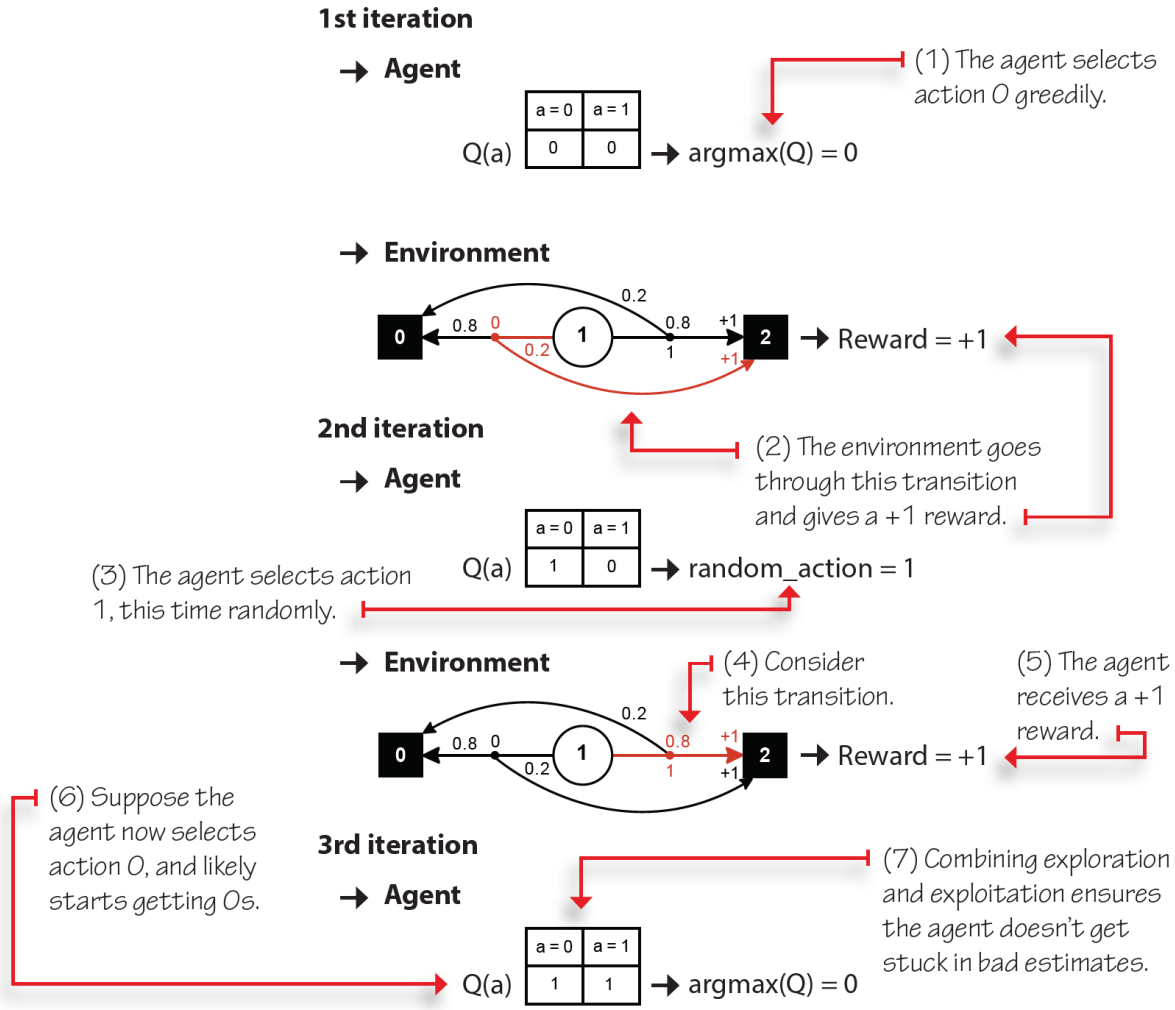
Greedy strategy: Always exploit



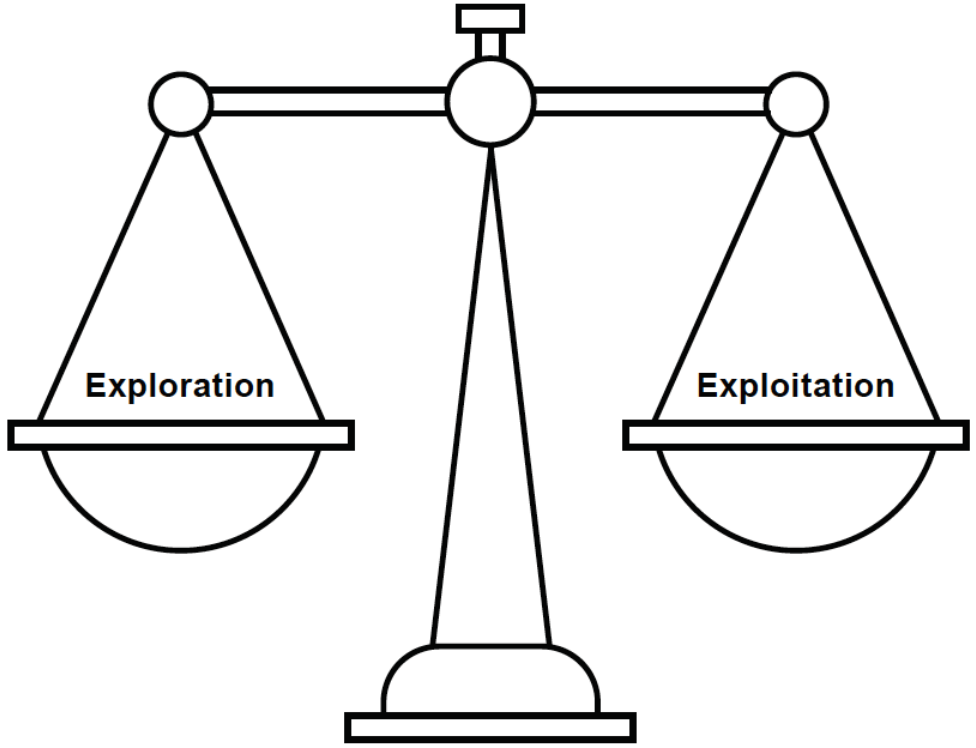
Random strategy: Always explore



Epsilon-Greedy strategy: Always always greedy, sometimes random



Recap: Bandit Problems



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> I conceive that the great part of the miseries of mankind are brought upon them by false estimates they have made of the value of things.
>
> — Benjamin Franklin
> Founding Father of the United States
> an author, politician, inventor, and a civic activist.

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Prediction Problem

- Estimate the value of policies; evaluate policies under feedback that is simultaneously sequential and evaluative.
- This is not policy optimization but is equally important for finding optimal policies.
- Having perfect estimates makes policy improvement trivial.

Terminology recap

ŘŁ With An RL Accent

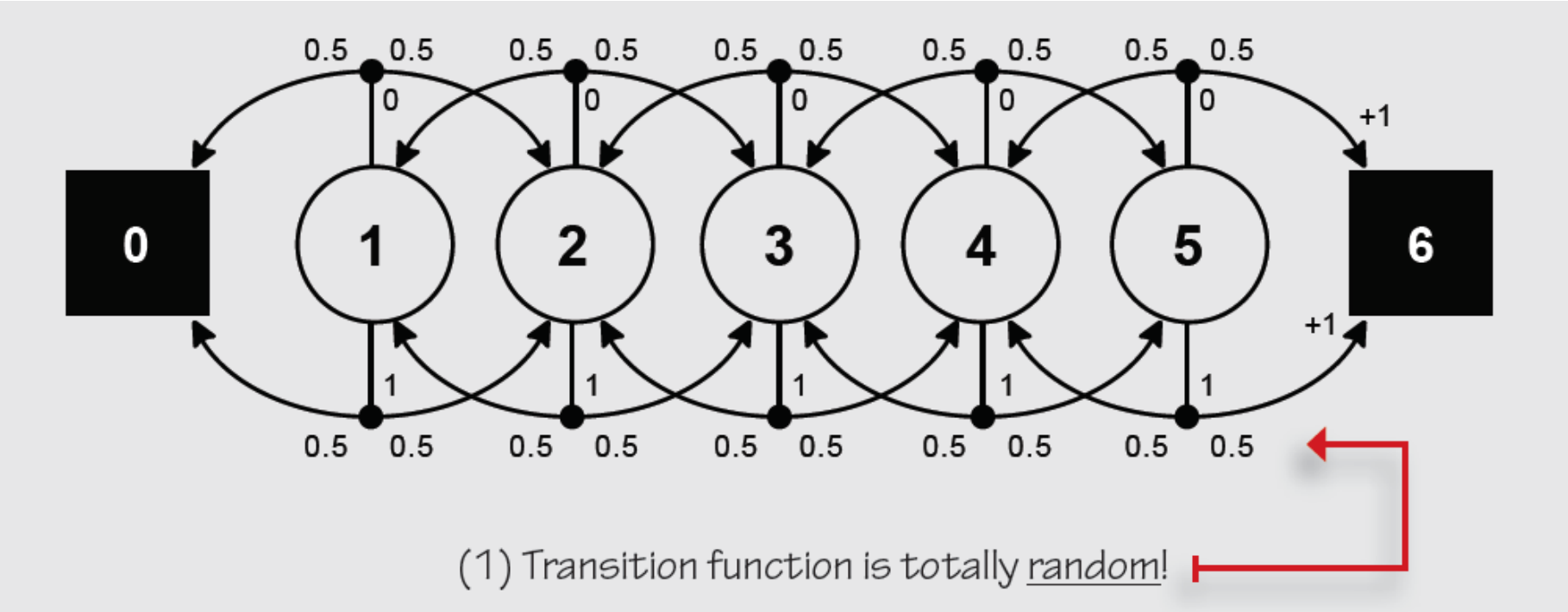
Reward vs. Return vs. Value function

Reward: Refers to the *one-step reward signal* the agent gets: the agent observes a state, selects an action, and it receives a reward signal. The reward signal is the core of RL, but it is *not* what the agent is trying to maximize! Again, the agent is not trying to maximize the reward! Realize that while your agent maximizes the one-step reward, in the long-term, is getting less than it could.

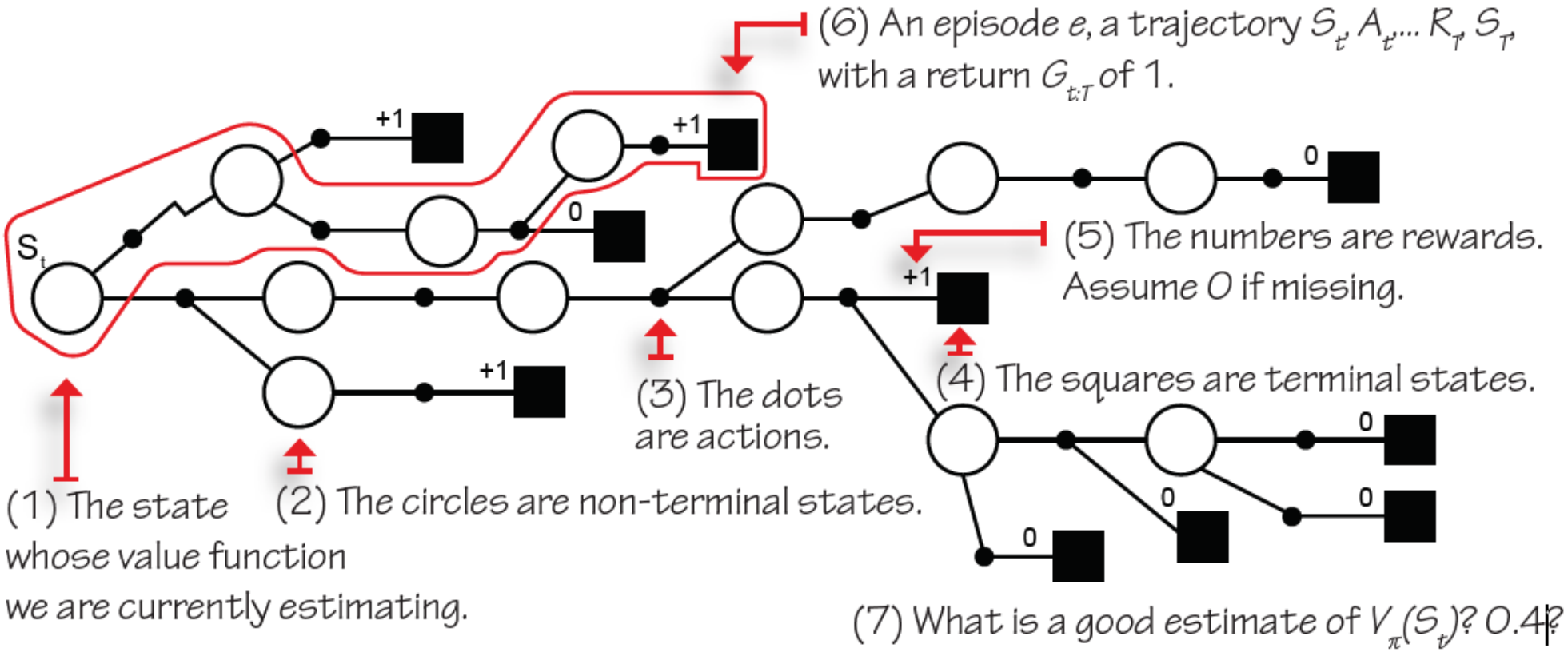
Return: Refers to the *total discounted rewards*. Returns are calculated from any state and usually go until the end of the episode. That is when a terminal state is reached the calculation stops. Returns are often referred to as *total reward*, *cumulative reward*, sum of rewards, and are commonly *discounted*: *total discounted reward*, *cumulative discounted reward, sum of discounted reward*. But, it is basically the same: a return tells you how much reward your agent *obtained* in an episode. As you can see, returns are better indicators of performance because they contain a long-term sequence, a single-episode history of rewards. But the return is *not* what an agent tries to maximize, either! An agent that attempts to obtain the highest possible return may find a policy that takes it through a noisy path; sometimes, this path will provide a high return, perhaps most of the time a low one.

Value function: Refers to the *expectation of returns*. That means, sure, we want high returns, but high in *expectation (on average)*. So, if the agent is in a very noisy environment, or if the agent is using a stochastic policy, it's all just fine. The agent is trying to maximize the *expected total discounted reward*, after all: value functions.

The Random Walk environment



Monte-Carlo prediction



Monte-Carlo prediction equations

(1) **WARNING:** I'm heavily abusing notation to make sure you get the whole picture. In specific, you need to notice when each thing is calculated. For instance, when you see a subscript *t:T*, that just means it is derived from time step *t* until the final time step, *T*. When you see *T*, that means it is computed at the end of the episode at the final time step *T*.

$$v_\pi(s) = \mathbb{E}_\pi[G_{t:T} \mid S_t = s]$$

(2) As a reminder, the action-value function is the expectation of returns. This is a *definition* good to remember.

(3) And the returns are the total discounted reward.

$$G_{t:T} = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{T-1} R_T$$

(4) So, in MC, the first thing we do is sample the policy for a trajectory.

$$S_t, A_t, R_{t+1}, S_{t+1}, \ldots, R_T, S_T \sim \pi_{t:T}$$

(5) Given that trajectory, we can calculate the return for all states encountered.

$$T_T(S_t) = T_T(S_t) + G_{t:T}$$

(6) Then, add up the per-state returns.

$$N_T(S_t) = N_T(S_t) + 1$$

(7) And, increment a count (more on this later.)

$$V_T(S_t) = \frac{T_T(S_t)}{N_T(S_t)}$$

(8) We can simply estimate the expectation using the empirical mean. So, the estimated state-value function for a state is just the mean return for that state.

$$N(s) \to \infty \quad V(s) \to v_\pi(s)$$

(9) As the counts approach infinity, the estimate will approach the true value

(10) But, notice that means can be calculated incrementally. So, there is no need to keep track of the sum of returns for all states. This equation is equivalent, just more efficient.

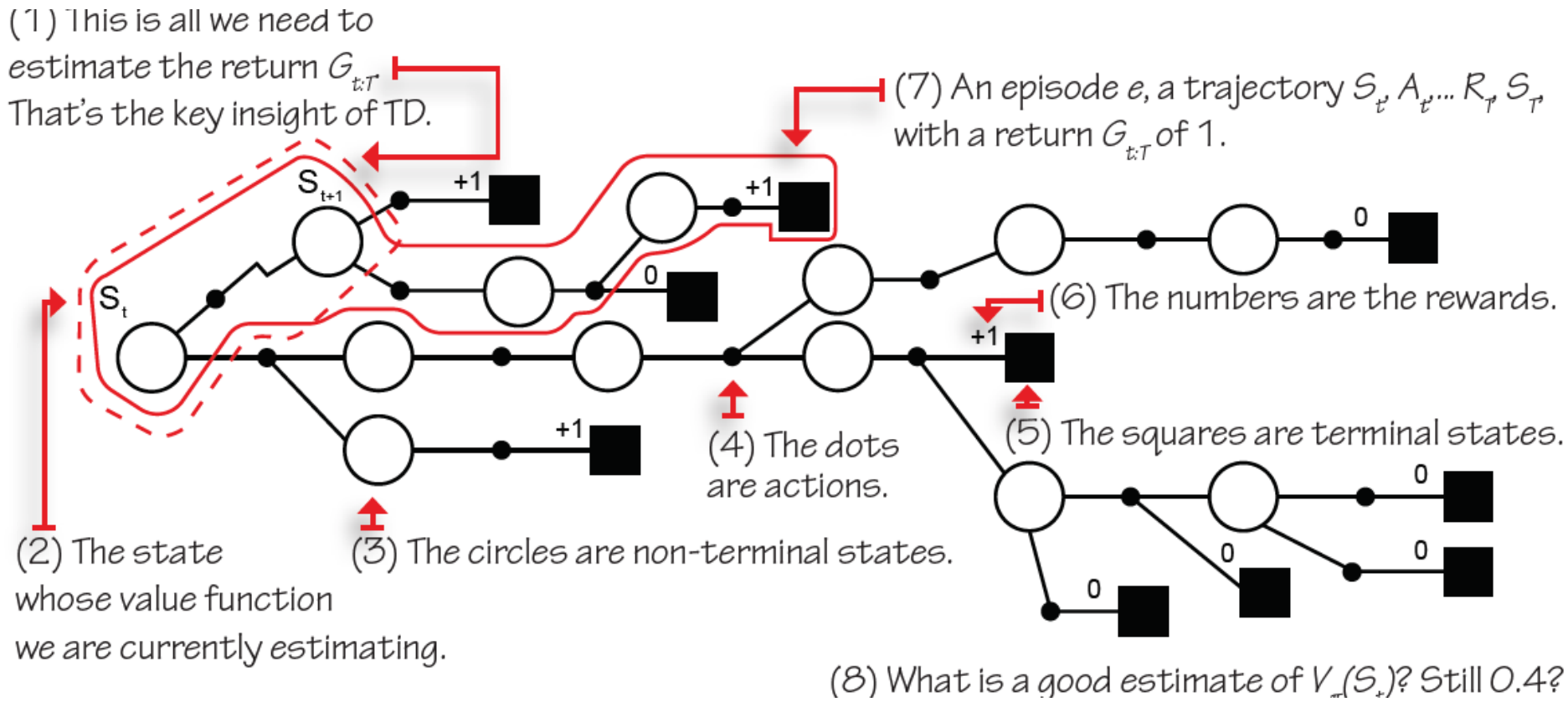
$$V_T(S_t) = V_{T-1}(S_t) + \frac{1}{N_t(S_t)}\left[G_{t:T} - V_{T-1}(S_t)\right]$$

(11) On this one, we just replace the mean for a learning value that can be time dependent, or constant.

$$V_T(S_t) = V_{T-1}(S_t) + \alpha_t\left[\overbrace{\underbrace{G_{t:T}}_{\text{MC target}} - V_{T-1}(S_t)}^{\text{MC error}}\right]$$

(12) Notice that V is calculated only at the end of an episode, time step T, because G depends on it.

Temporal-Difference Learning



Temporal-Difference Learning equations

(1) We again start from the definition of the state-value function.
(2) And the definition of the return.

$$v_\pi(s) = \mathbb{E}_\pi[G_{t:T} \mid S_t = s]$$

$$G_{t:T} = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{T-1} R_T$$

(3) From the return, we can rewrite the equation by grouping up some terms. Check it out.

$$\begin{aligned} G_{t:T} &= R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots + \gamma^{T-1} R_T \\ &= R_{t+1} + \gamma(R_{t+2} + \gamma R_{t+3} + \ldots + \gamma^{T-2} R_T) \\ &= R_{t+1} + \gamma G_{t+1:T} \end{aligned}$$

(4) Now, the same return has a recursive style.

(5) We can use this new definition to also rewrite the state-value function definition equation.

$$\begin{aligned} v_\pi(s) &= \mathbb{E}_\pi[G_{t:T} \mid S_t = s] \\ &= \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1:T} \mid S_t = s] \\ &= \mathbb{E}_\pi[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s] \end{aligned}$$

(6) And because the expectation of the returns from the next state is simply the state-value function of the next state, we get.

(7) This means we could estimate the state-value function on every time step.
(8) We roll out a single interaction step.

$$S_t, A_t, R_{t+1}, S_{t+1} \sim \pi_{t:t+1}$$

(9) And can obtain an estimate $V(s)$ of the true state-value function $v_\pi(s)$ a different way than with MC.

(10) The key difference to realize is we are now estimating $v_\pi(s_t)$ with an estimate of $v_\pi(s_{t+1})$. We are using an estimated, not an actual return.

$$V_{t+1}(S_t) = V_t(S_t) + \alpha_t \Big[\overbrace{R_{t+1} + \gamma V_t(S_{t+1}) - V_t(S_t)}^{\text{TD error}} \Big]$$

$R_{t+1} + \gamma V_t(S_{t+1})$: TD target

(11) A big win is we can now make updates to the state-value function estimates $V(s)$ every time step.

MC vs. TD learning

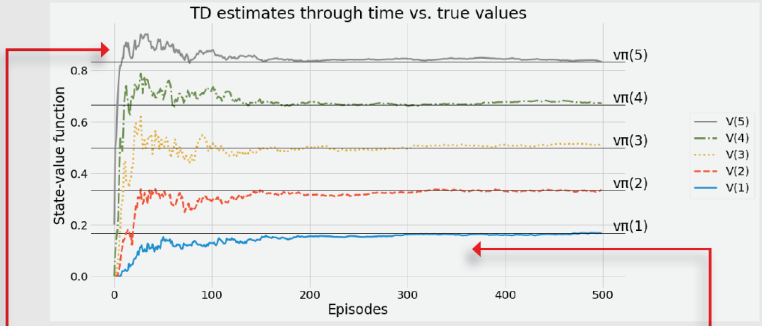
Tally It Up

MC and TD both nearly converge to the true state-value function

(1) Here I'll be showing only First-Visit Monte-Carlo prediction (FVMC) and Temporal-Difference Learning (TD). If you head to the Notebook for this chapter, you'll also see the results for Every-Visit Monte-Carlo prediction, and some additional plots that may be of interest to you!

(2) Take a close look at these plots. These are the running state-value function estimates $V(s)$ of an all-left policy in the Random Walk environment. As you can see in these plots, both algorithms show near-convergence to the true values.

(3) Now, see the difference trends of these algorithms. FVMC running estimates are very noisy, they jump back and forth around the true values.



(4) TD running estimates don't jump as much, but they are off center for most of the episodes. For instance $V(5)$ is usually higher than $v_\pi(5)$, while $V(1)$ is usually lower than $v_\pi(1)$. But if you compare those values with FVMC estimates, you notice a different trend.

Tally It Up

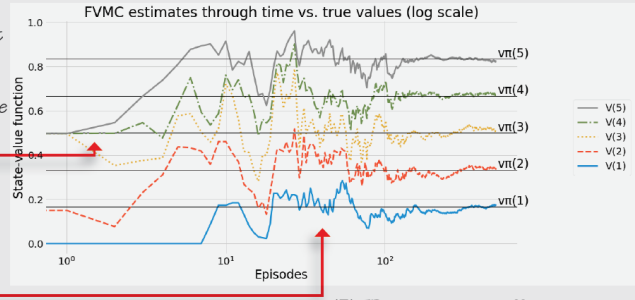
MC estimates are noisy, TD estimates off target

$$V_T(S_t) = V_{T-1}(S_t) + \alpha_t \left[\overbrace{G_{t:T} - V_{T-1}(S_t)}^{\text{MC error}} \right]$$

$G_{t:T}$: MC target

(1) If we get a close-up (log-scale plot) these trends, you will see what's happening. MC estimates jump around the true values. This is because of the high variance of the MC targets.

(2) A couple of pros though; first you can see all estimates get close to their true values very early on. Also, the estimates jump around the true values.

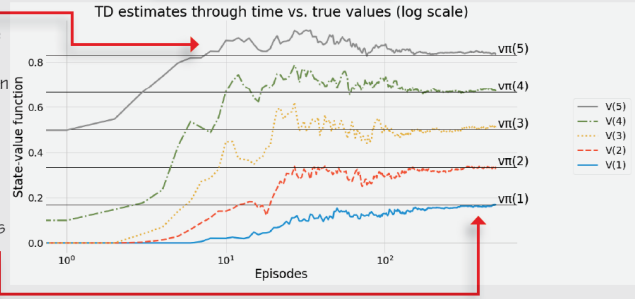


$$V_{t+1}(S_t) = V_t(S_t) + \alpha_t \left[\overbrace{G_{t:t+1} - V_t(S_t)}^{\text{TD error}} \right]$$

$G_{t:t+1}$: TD target

(3) TD estimates are off target most of the time, but they are less jumpy. This is because TD targets are low variance, though biased. They use an estimated return for target.

(4) The bias shows, too. In the end, TD targets give up accuracy in order to become more precise. Also, they take a bit long before estimates ramp up, at least in this environment.



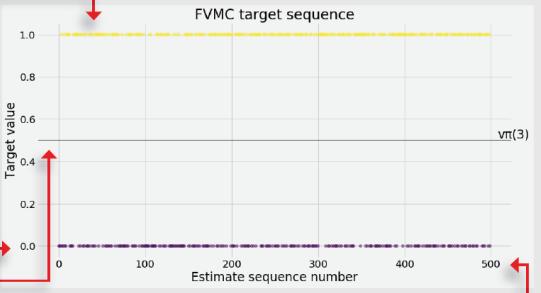
Tally It Up

MC targets high variance is evident, TD targets bias, too

$$G_{t:T} = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{T-1} R_T$$

(1) Here we can see the bias/variance tradeoff between MC and TD targets. Remember, the MC target is the return, which accumulates a lot of random noise. That means high variance *targets*.

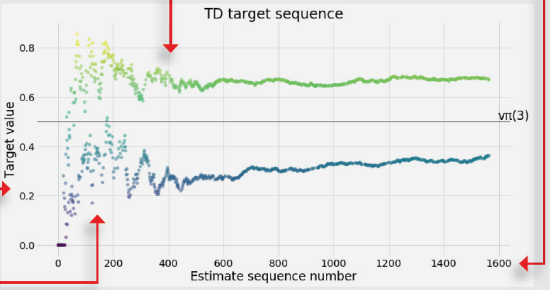
(2) These plots are showing the targets for the initial state in the RW environment. MC targets, the returns, are either 0 or 1 because the episode terminates either on the left, with a 0 return or on the right, with a 1 return, while the optimal value is 0.5!



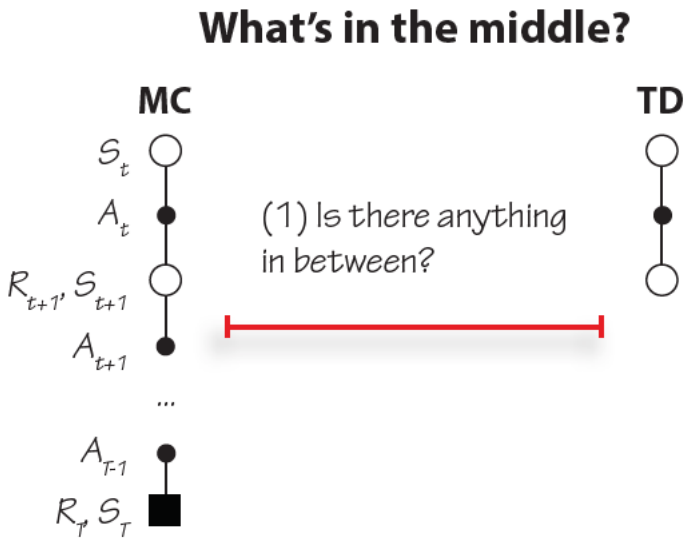
$$G_{t:t+1} = R_{t+1} + \gamma V_t(S_{t+1})$$

(3) TD targets are calculated using an estimated return. We use the value function to predict how much value we will get from the next state onwards. This helps us truncate the calculations and get more estimates per episode (as you can see on the x axis, we have ~1600 estimates in 500 episodes), but because we use $V_t(S_{t+1})$, which is an estimate and therefore likely wrong, TD targets are biased.

(4) Here you can see the range of the TD targets is much lower, MC alternates *exactly* between 1 and 0, TD jumps between *approximately* 0.7 and ~0.3, depending on which "next state" is sampled. But as the $V_t(S_{t+1})$ is an estimate, $G_{t:t+1}$ is biased, off target, inaccurate.



Is there anything in between?



n-step TD equations

Show Me the Math

N-step temporal-difference equations

$$S_t, A_t, R_{t+1}, S_{t+1}, \ldots, R_{t+n}, S_{t+n} \sim \pi_{t:t+n}$$

(1) Notice how in n-step TD we must wait n steps before we can update *V(s)*.
(2) Now, *n* doesn't have to be ∞ like in MC, or 1 like in TD. Here you get to pick. In reality *n* will be *n* or less if your agent reaches a terminal state. So, it could be less than *n*, but never more.

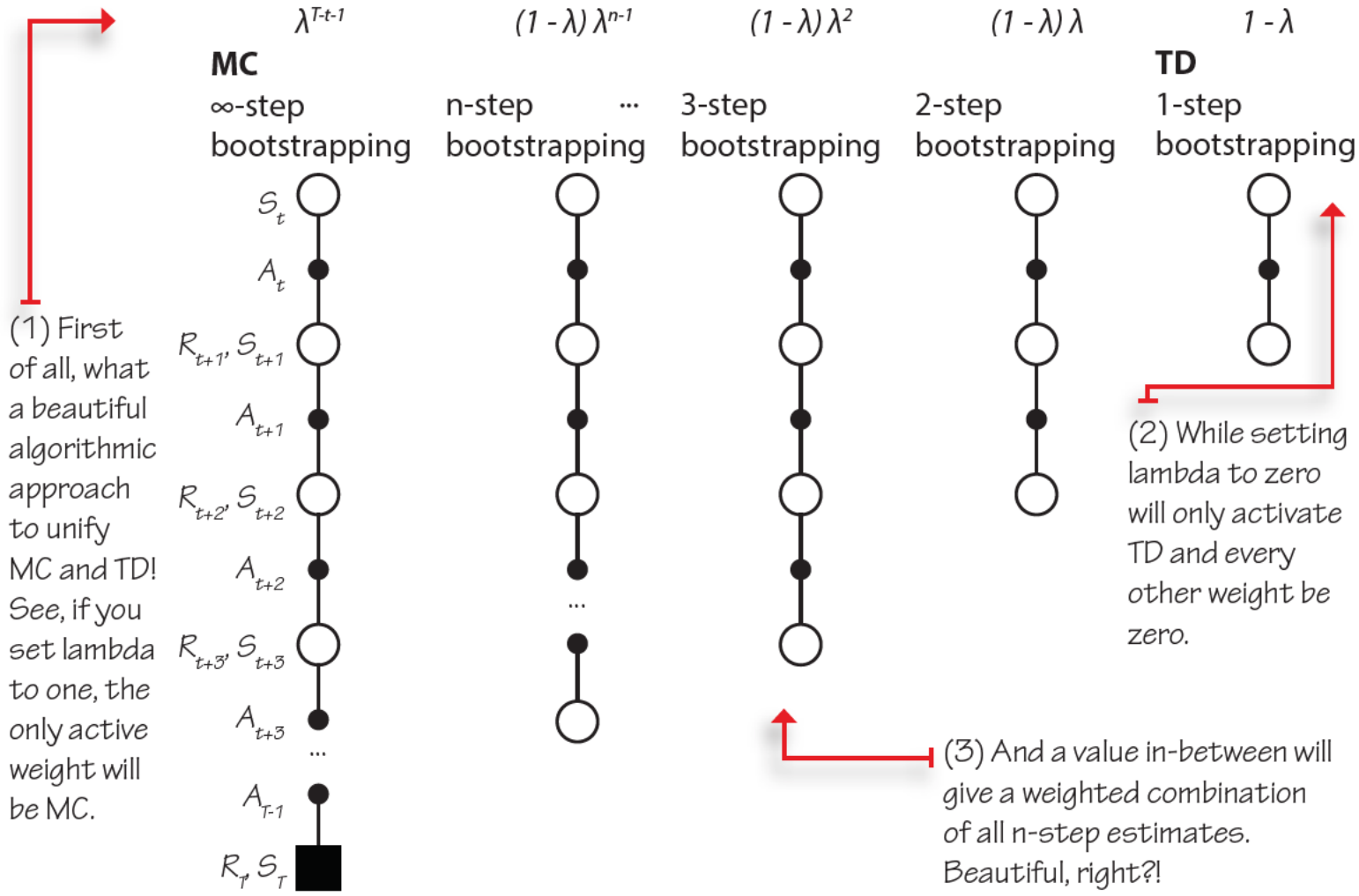
$$G_{t:t+n} = R_{t+1} + \ldots + \gamma^{n-1} R_{t+n} + \gamma^n V_{t+n-1}(S_{t+n})$$

(3) Here you see how the value function estimate gets updated approximately every *n* steps.

$$V_{t+n}(S_t) = V_{t+n-1}(S_t) + \alpha_t \Big[\overbrace{\underbrace{G_{t:t+n}}_{\text{n-step target}} - V_{t+n-1}(S_t)}^{\text{n-step error}} \Big]$$

(4) But after that, you can just plug-in that target as usual.

TD lambda


λ^{T-t-1}
MC
∞-step bootstrapping
$(1-\lambda)\lambda^{n-1}$
n-step bootstrapping
...
$(1-\lambda)\lambda^{2}$
3-step bootstrapping
$(1-\lambda)\lambda$
2-step bootstrapping
$1-\lambda$
TD
1-step bootstrapping
S_t
A_t
R_{t+1}, S_{t+1}
A_{t+1}
R_{t+2}, S_{t+2}
A_{t+2}
R_{t+3}, S_{t+3}
A_{t+3}
...
A_{T-1}
R_T, S_T
(1) First of all, what a beautiful algorithmic approach to unify MC and TD! See, if you set lambda to one, the only active weight will be MC.
(2) While setting lambda to zero will only activate TD and every other weight be zero.
(3) And a value in-between will give a weighted combination of all n-step estimates. Beautiful, right?!

TD lambda equations

Show Me the Math

Forward-view TD(λ)

(1) Sure, this is a loaded equation, but we will unpack it below. The bottom line is that we are using all n-step returns until the final step *T*, and weighting it with an exponentially decaying value.

$$G^{\lambda}_{t:T} = \underbrace{(1-\lambda)\sum_{n=1}^{T-t-1}\lambda^{n-1}G_{t:t+n}}_{\text{Sum of weighted returns from 1-step to T-1 steps}} + \underbrace{\lambda^{T-t-1}G_{t:T}}_{\text{Weighted final return (T)}}$$

(2) The thing is, because *T* is variable, we need to weight the actual return with a normalizing value so that all weights add up to 1.

(3) All this equation is saying is that we will calculate the one-step return and weight it with the following factor. $\mapsto 1-\lambda$

$$G_{t:t+1} = R_{t+1} + \gamma V_t(S_{t+1})$$

(4) And also the two-step return and weight it with this factor.

$$G_{t:t+2} = R_{t+1} + \gamma R_{t+2} + \gamma^2 V_{t+1}(S_{t+2}) \qquad (1-\lambda)\lambda$$

(5) Then the same for the three-step return, and this factor.

$$G_{t:t+3} = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \gamma^3 V_{t+2}(S_{t+3}) \qquad (1-\lambda)\lambda^2$$

(6) You do this for all n-steps...

$$G_{t:t+n} = R_{t+1} + \ldots + \gamma^{n-1}R_{t+n} + \gamma^n V_{t+n-1}(S_{t+n}) \qquad (1-\lambda)\lambda^{n-1}$$

(7) Until your agent reaches a terminal state. Then you weight by this normalizing factor.

$$G_{t:T} = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{T-1}R_T \qquad \lambda^{T-t-1}$$

(8) Notice the issue with this approach is that you must sample an entire trajectory before you can calculate these values.

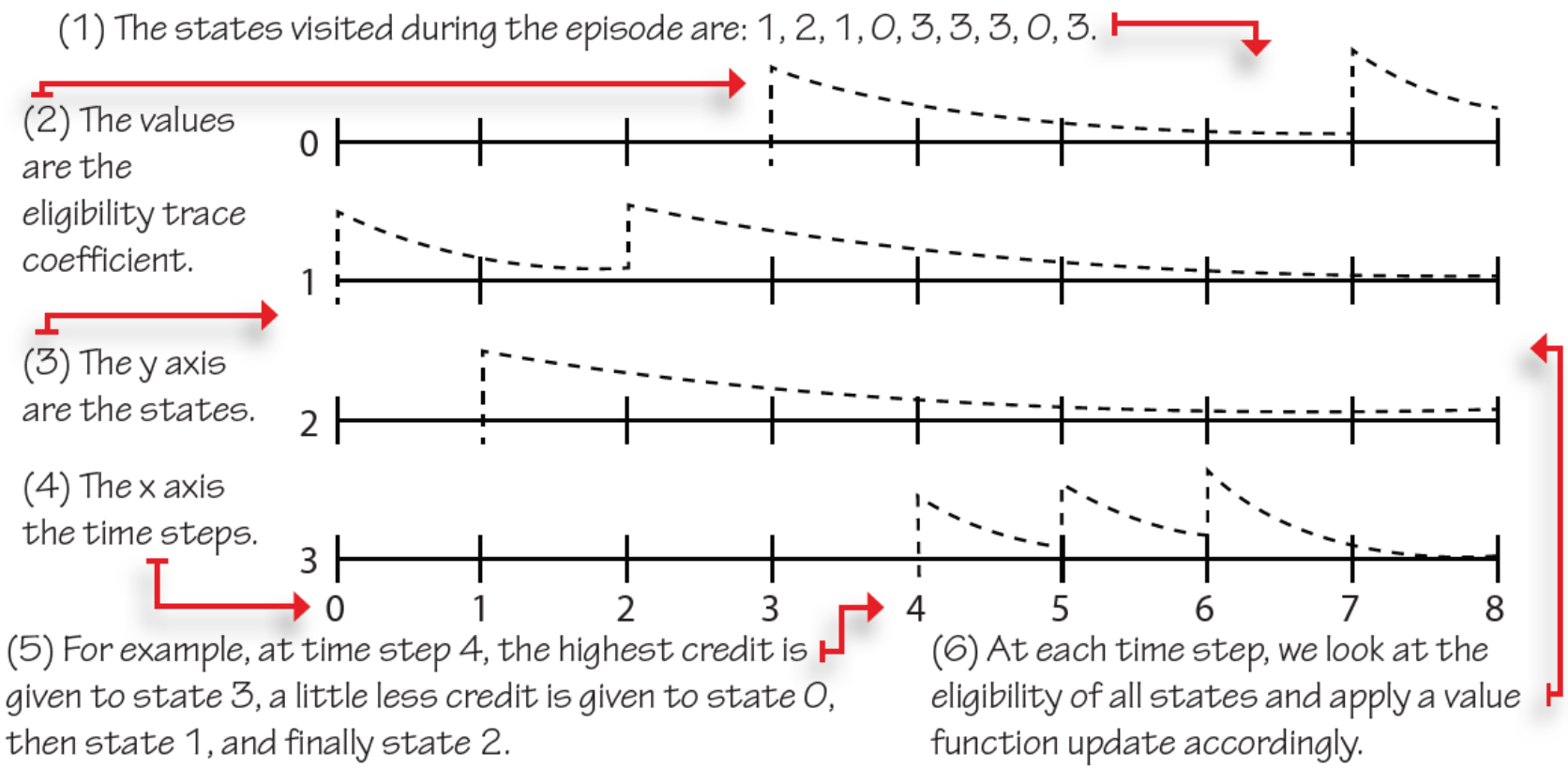
$$S_t, A_t, R_{t+1}, S_{t+1}, \ldots, R_T, S_T \sim \pi_{t:T}$$

(9) Here you have it, V will become available at time *T*.

(10) Because of this.

$$V_T(S_t) = V_{T-1}(S_t) + \alpha_t \Big[\overbrace{\underbrace{G^{\lambda}_{t:T}}_{\lambda\text{-return}} - V_{T-1}(S_t)}^{\lambda-\text{error}} \Big]$$

Eligibility traces



TD(lambda) algorithm

Show Me the Math

Backward-view TD(λ) — TD(λ) with eligibility traces, "the" TD(λ)

(1) Every new episode we set the eligibility vector to 0. → $E_0 = 0$

(2) Then, we interact with the environment one cycle. → $S_t, A_t, R_{t+1}, S_{t+1} \sim \pi_{t:t+1}$

(3) When you encounter a state S_t, make it eligible for an update... Technically, you increment its eligibility by 1. → $E_t(S_t) = E_t(S_t) + 1$

(4) We then simply calculate the TD error just as we have been doing so far.

$$\delta^{TD}_{t:t+1}(S_t) = \underbrace{R_{t+1} + \gamma V_t(S_{t+1})}_{\text{TD target}} - V_t(S_t)$$

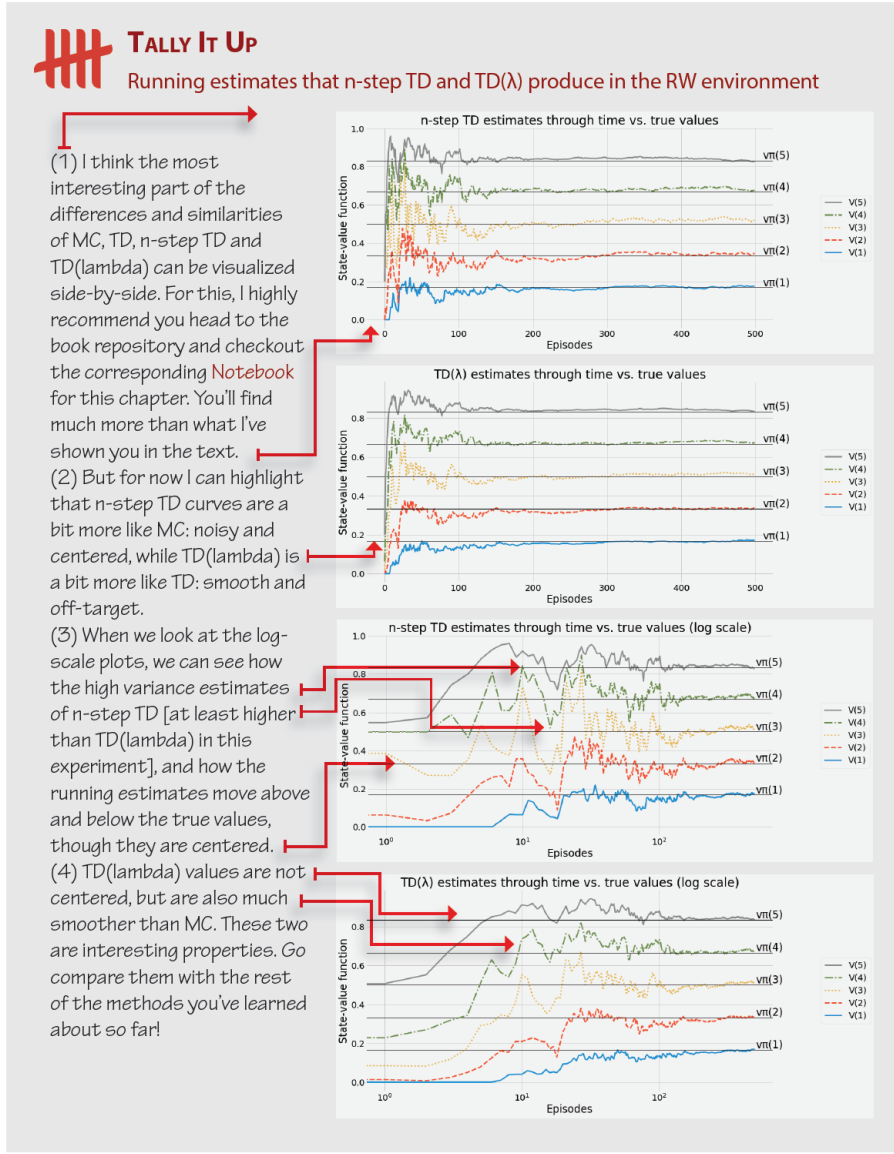
(5) However, unlike before, we update the estimated state-value function V, that is, the *entire* function at once, *every time step!* Notice I'm not using a $V_t(S_t)$, but a V_t instead. Because we are multiplying by the eligibility *vector*, all eligible states will get the corresponding credit.

$$V_{t+1} = V_t + \alpha_t \underbrace{\delta^{TD}_{t:t+1}(S_t)}_{\text{TD error}} E_t$$

(6) Finally, we decay the eligibility. → $E_{t+1} = E_t \gamma \lambda$

Recap: Prediction problem

- Allows us to accurately evaluate policies.
- Having accurate estimates makes policy improvement trivial.
- There are two core methods, Monte-Carlo prediction and Temporal-Difference Learning.
- One uses the actual returns and approximates the expectation by taking means. The other bootstraps on its own value estimates; uses partial or predicted returns.
- There are pros and cons in both!



Recommended reading.
Reinforcement Learning: An introduction (chapters 5, 6, 7, and 12)
http://incompleteideas.net/book/the-book-2nd.html

Outline

Opening

Introduction to Reinforcement Learning

Markov Decision Process

Planning Methods

Bandit Problems

Prediction Problem

Control Problem

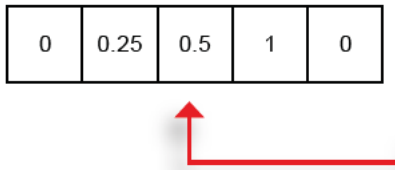
> When it is obvious that the goals cannot be reached, don't adjust the goals, adjust the action steps.
>
> — Confucius
> Chinese teacher, editor, politician, and philosopher of the Spring and Autumn period of Chinese history

Georgia Tech
CREATING THE NEXT

Control Problem

- Optimize policies.
- Find the best policies for any given environment.
- Needs accurate policy evaluation methods and exploration.
- This is the full reinforcement learning problem. Note: Not Deep Reinforcement Learning, which we'll discuss in next lecture.

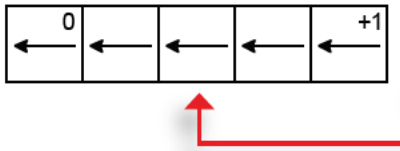
We need to estimate action-value functions



(1) Two actions, left and right, and the v -function as shown. Can you tell me the best policy?

(2) What if i told you left send you right with 70% chance?
(3) What do you think the best policy is now?
(4) See?! V-Function is not enough.

We need to explore

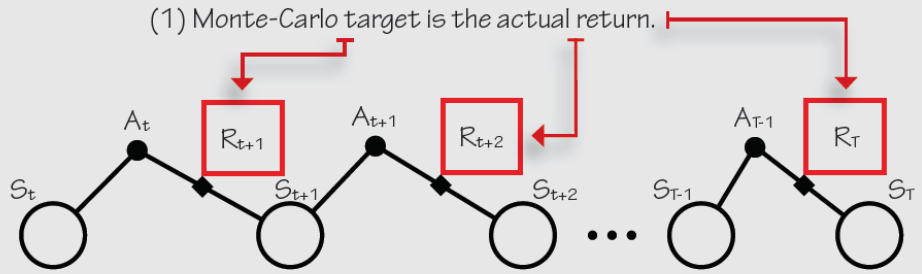


(1) Imagine you start with the following deterministic policy.

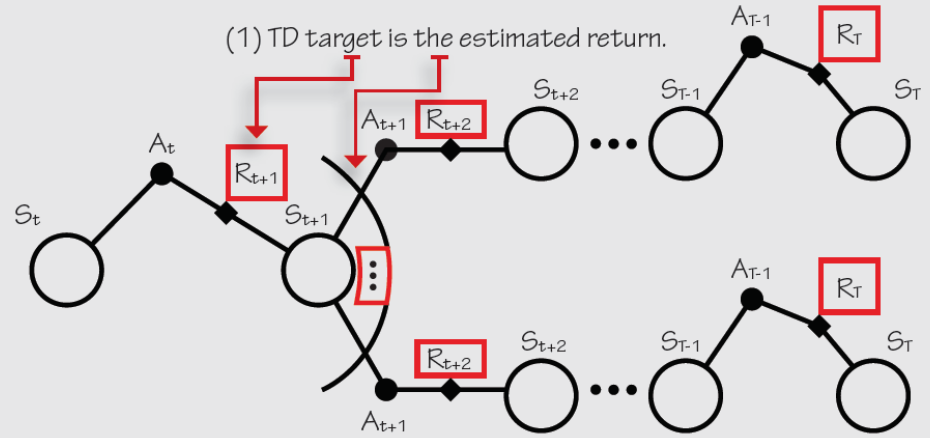
(2) How can you tell whether the right action is better than the left if all you estimate is the left action?
(3) See?! Your agent needs to explore.

What estimation method to use?

Other important concepts worth repeating are the different ways value functions can be estimated. In general, all methods that learn value functions progressively move estimates a fraction of the error towards the targets. The general equation that most learning methods follow is: *estimate = estimate + step * error*. The *error* is simply the difference between a *sampled target* and the *current estimate*: *(target - estimate)*. The two main and opposite ways for calculating these targets are Monte-Carlo and Temporal-Difference learning.

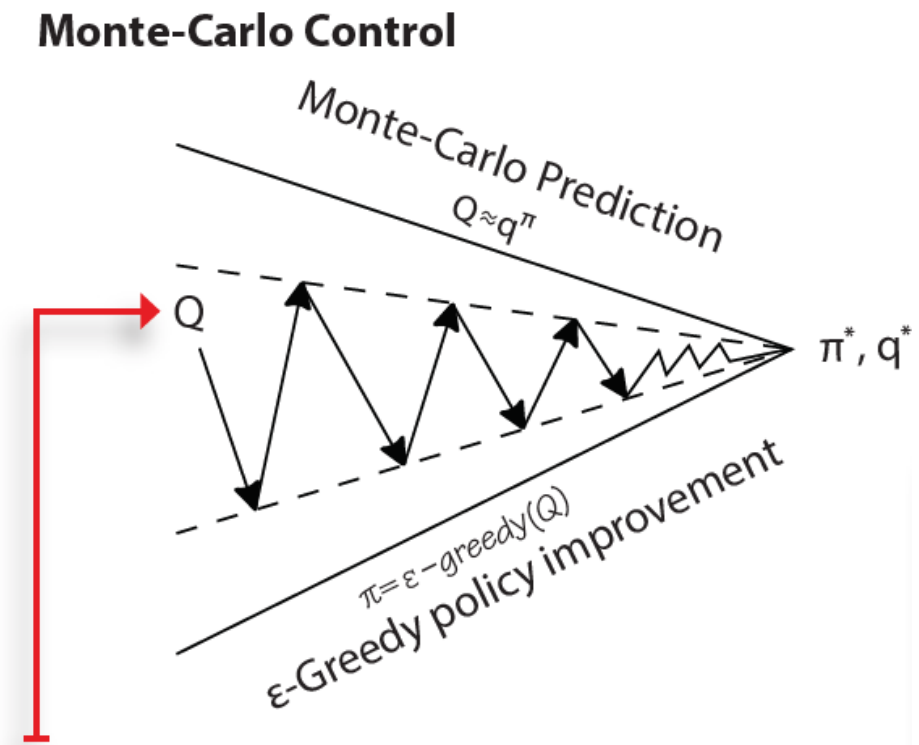


The Monte-Carlo target consists of the actual return. Really, nothing else. Monte-Carlo estimation consists of adjusting the estimates of the value functions using the empirical (observed) mean return in place of the expected (as if you could average infinite samples) return.



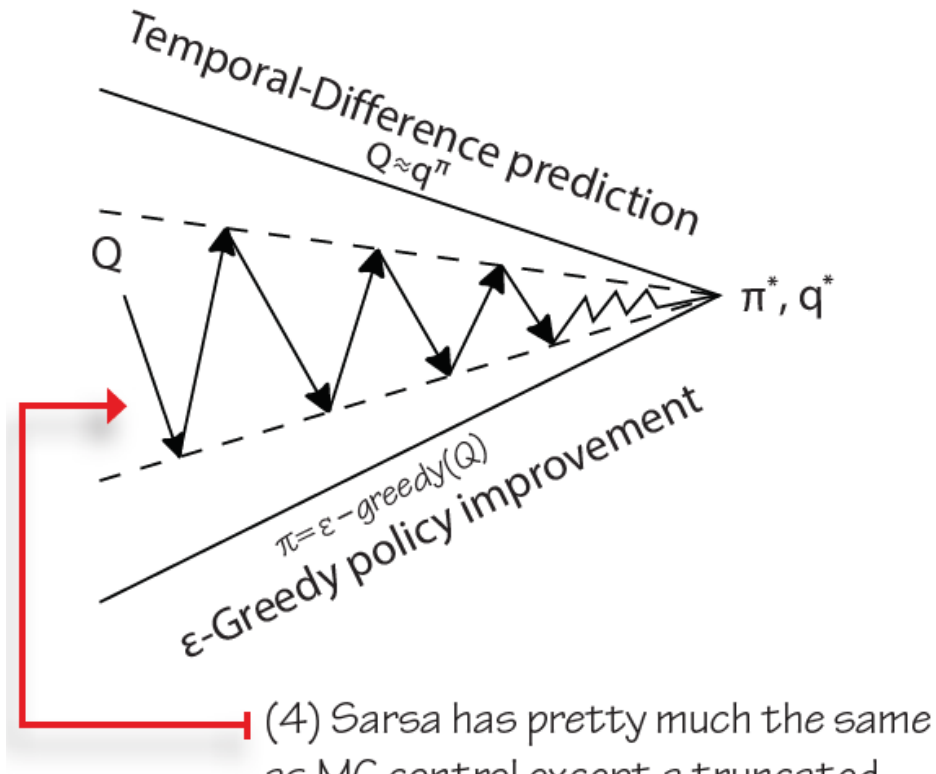
The Temporal-Difference target consists of an estimated return. Remember "bootstrapping"? It basically means using the estimated expected return from *later* states, for estimating the expected return from the *current* state. TD does that. Learning *a guess* from *a guess*. The TD target is formed by using a single reward and the estimated expected return from the next state using the running value function estimates.

Monte-Carlo Control



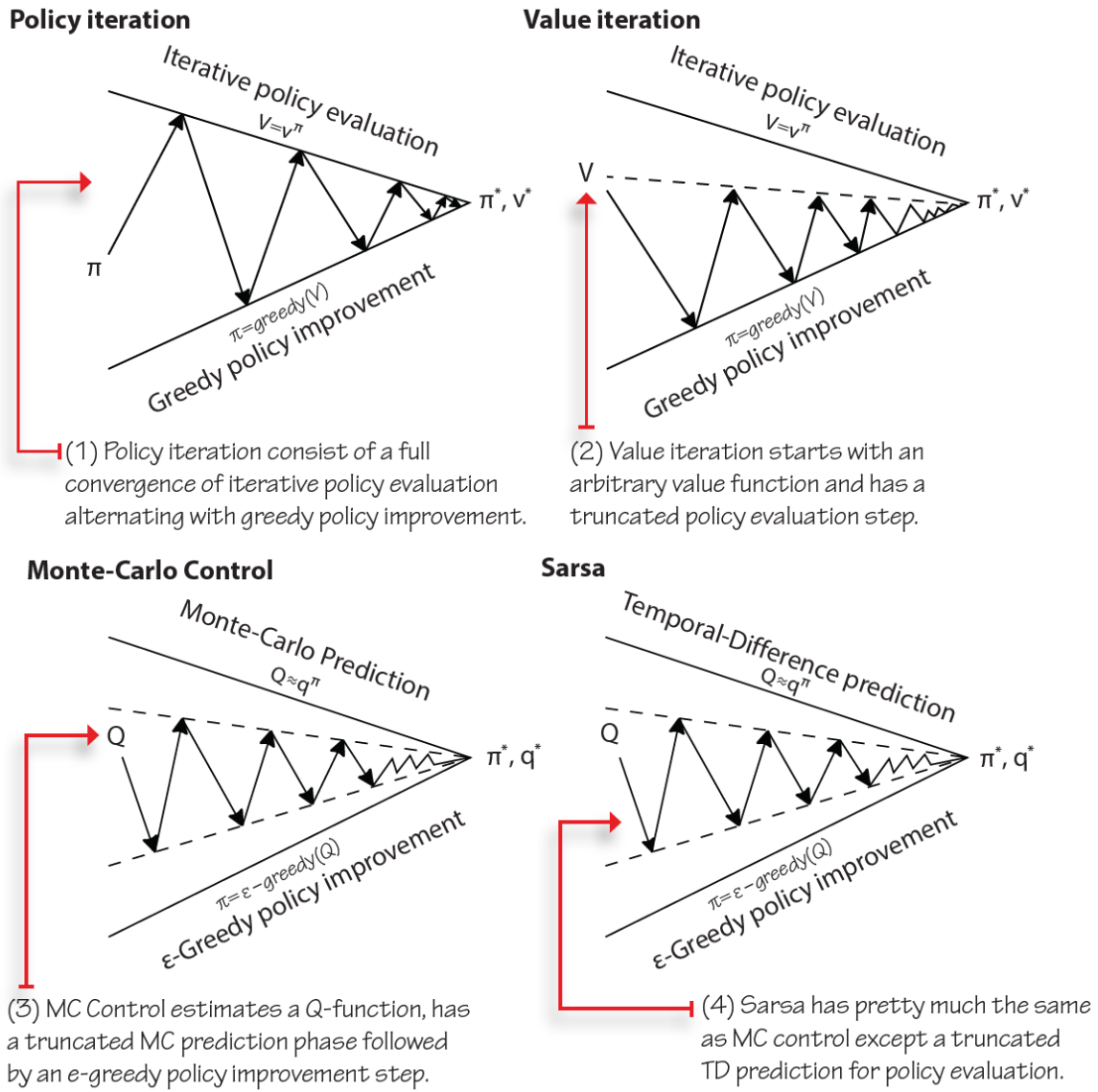
(3) MC Control estimates a Q-function, has a truncated MC prediction phase followed by an e-greedy policy improvement step.

TD Control: Sarsa



(4) Sarsa has pretty much the same as MC control except a truncated TD prediction for policy evaluation.

Planning methods, RL methods



Convergence: GLIE: Greedy in the Limit with Infinite Exploration and Stochastic Approximation theory.

Q-Learning: off-policy learning

Show Me the Math

Sarsa vs. Q-learning update equations

(1) The only difference between Sarsa and Q-learning is the action used in the target.

(2) This is Sarsa update equation.

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha_t \Big[\overbrace{R_{t+1} + \gamma Q(S_{t+1}, A_{t+1})}^{\text{Sarsa target}} - Q(S_t, A_t) \Big]$$

(3) It uses the action actually taken in the next state to calculate the target.

(4) This one is Q-learning's.

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha_t \Big[\overbrace{R_{t+1} + \gamma \max_a Q(S_{t+1}, a)}^{\text{Q-learning target}} - Q(S_t, A_t) \Big]$$

(5) Q-learning uses the action with the maximum estimated value in the next state, despite the action actually taken.

On-policy vs. Off-policy learning

ŘŁ **With An RL Accent**

On-policy vs. Off-policy learning

On-policy learning: Refers to methods that attempt to evaluate or improve the policy used to make decisions. It is straightforward; think about a single policy. This policy generates behavior. Your agent evaluates that behavior and select areas of improvement based on those estimates. Your agent learns to assess and improve the same policy it uses for generating the data.

Off-policy learning: Refers to methods that attempt to evaluate or improve a policy different from the one used to generate the data. This one is more complex. Think about two policies. One produces the data, the experiences, the behavior, but your agent uses that data to evaluate, improve, and overall learn about a different policy, a different behavior. Your agent learns to assess and improve a policy different than the one used for generating the data.

Convergence: GLIE: Greedy in the Limit with Infinite Exploration and Stochastic Approximation theory.

- GLIE:
  - All state-action pairs must be explored infinitely often.
  - The policy must converge on a greedy policy.
- What this means in practice is that an e-greedy exploration strategy, for instance, must slowly decay epsilon towards zero. If it goes down too quickly, the first condition may not be met, if it decays too slowly, well, it takes longer to converge.
- Notice that for off-policy RL algorithms, such as Q-learning, the only requirement of these two that holds is the first one. The second one is no longer a requirement because in off-policy learning, the policy learned about is different than the policy we are sampling actions from. Q-learning, for instance, only requires all state-action pairs to be updated sufficiently, and that is covered by the first condition above.

There is another set of requirements for general convergence based on Stochastic Approximation Theory that applies to all these methods. Because we are learning from samples, and samples have some variance, the estimates won't converge unless we also push the learning rate, alpha, towards zero:

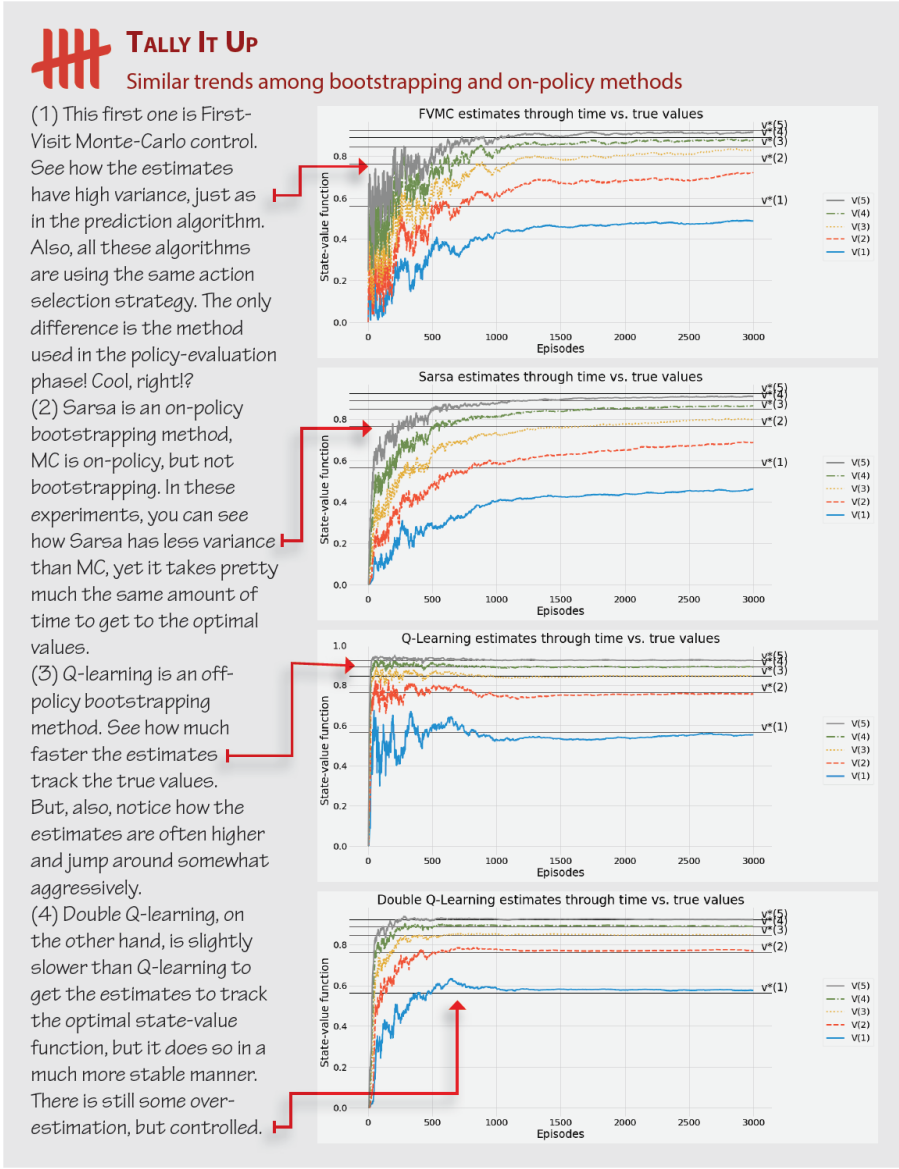
- The sum of learning rates must be infinite.
- The sum of squares of learning rates must be finite.

That means you must pick a learning rate that decays but never reaches zero. For instance, if you use 1/t or 1/e, the learning rate is initially large enough to ensure the algorithm doesn't follow only a single sample too tightly but becomes small enough to ensure it finds the signal behind the noise.

Recap: Control problem

- It's what we are looking, how to find optimal control policies.
- There is a profound synergy between policy evaluation and policy improvement.
- The control problem consists on estimation action-value functions, and correctly balancing exploration and exploitation.

Recommended reading.
Reinforcement Learning: An introduction (chapters 5, 6)
http://incompleteideas.net/book/the-book-2nd.html



Thank you!