# 실수에서의 코시 슈바르츠 부등식
(Cauchy Schwarz inequality in $\mathbb{R}$)

$a_i, b_i \in \mathbb{R}$

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$$\left(\sum_{i=1}^{n} a_i^2\right)$$

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$$\left(\sum_{i=1}^{n} a_i^2\right) \cdot \left(\sum_{i=1}^{n} b_i^2\right) \geq \left(\sum_{i=1}^{n} a_i b_i\right)^2$$

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proof.

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$$\left(\sum_{i=1}^{n} a_i^2\right) \cdot \left(\sum_{i=1}^{n} b_i^2\right) \geq \left(\sum_{i=1}^{n} a_i b_i\right)^2$$

proof.

If $(a_1, \ldots, a_n) = (0, \ldots, 0)$,then it is trivial.

Assume $(a_1, \ldots, a_n) \neq (0, \ldots, 0)$

$t \in \mathbb{R}$, $\sum_{i=1}^{n} (a_i t - b_i)^2 \geq 0 \ (\because a_i t - b_i \in \mathbb{R}) \cdots\cdots (1)$

$$\left(\sum_{i=1}^{n} a_i^2\right) t^2 - 2\left(\sum_{i=1}^{n} a_i b_i\right) t + \left(\sum_{i=1}^{n} b_i^2\right) \geq 0$$

$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 - \left(\sum_{i=1}^{n} a_i^2\right) \cdot \left(\sum_{i=1}^{n} b_i^2\right) \leq 0 \ \left(\because \sum_{i=1}^{n} a_i^2 > 0, D/4 \leq 0\right)$$

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The two sides are equal if and only if $(a_1, \cdots, a_n)$ and $(b_1, \cdots, b_n)$ are linearly dependent. ($\because$ (1) )

$a_i, b_i \in \mathbb{R}$

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