Function Composition

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Function composition

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Function composition is

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Function Composition

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Function Composition

Start

Function Composition

Start

X

Function Composition

▸ Start

$$X \xrightarrow{f}$$

Start

$$X \xrightarrow{f} Y$$

Function Composition

Start

$$X \xrightarrow{f} \quad Y \xrightarrow{g}$$

Function Composition

▸ Start

$$X \xrightarrow{f} Y \xrightarrow{g} Z$$

Function Composition

Start

$$X \xrightarrow{f} \quad Y \xrightarrow{g} \quad Z$$

x

Function Composition

Start

$$X \xrightarrow{f} Y \xrightarrow{g} Z$$

$$x \xrightarrow{f}$$

Function Composition

Start

$$X \xrightarrow{f} Y \xrightarrow{g} Z$$

$$x \xrightarrow{f} y$$

Function Composition

Start

$$X \xrightarrow{f} Y \xrightarrow{g} Z$$

$$x \xrightarrow{f} y \xrightarrow{g}$$

Function Composition

▸ Start

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z
\end{array}
$$

Function Composition

Start

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & & & &
\end{array}
$$

Function Composition

▸ Start

$$X \xrightarrow{f} Y \xrightarrow{g} Z$$

$$x \xrightarrow{f} y \xrightarrow{g} z$$

$$x \xrightarrow{f}$$

Function Composition

Start

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & &
\end{array}
$$

Function Composition

Start

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} &
\end{array}
$$

Function Composition

▸ Start

$$\begin{array}{ccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array}$$

Function Composition

Start

$$
\begin{array}{ccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & &
\end{array}
$$

Function Composition

Start

$$
\begin{array}{ccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \qquad & \mathrm{X} \xrightarrow{g\circ f} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & &
\end{array}
$$

Function Composition

Start

$$X \xrightarrow{f} Y \xrightarrow{g} Z \qquad X \xrightarrow{g \circ f} Z$$

$$x \xrightarrow{f} y \xrightarrow{g} z$$

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x))$$

Function Composition

▸ Start

$$\begin{array}{ccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array}$$

$$\begin{array}{ccc} X & \xrightarrow{g \circ f} & Z \\ x & & \end{array}$$

Function Composition

Start

$$X \xrightarrow{f} Y \xrightarrow{g} Z \qquad X \xrightarrow{g \circ f} Z$$

$$x \xrightarrow{f} y \xrightarrow{g} z \qquad x \xrightarrow{g \circ f}$$

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x))$$

Function Composition

▸ Start

$$\begin{array}{ccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array} \qquad \begin{array}{ccc} X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{g\circ f} & z \end{array}$$

Function Composition

▸ Start

$$\begin{array}{ccccccccc} \mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & & \end{array}$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} &
\end{array}
$$

Function Composition

Start

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x))
\end{array}
\qquad
\begin{array}{ccc}
X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

Function Composition

▸ Start

$$X \xrightarrow{f} Y \xrightarrow{g} Z \qquad X \xrightarrow{g \circ f} Z$$

$$x \xrightarrow{f} y \xrightarrow{g} z \qquad x \xrightarrow{g \circ f} z$$

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x)) \qquad x \xrightarrow{g \circ f} (g \circ f)(x)$$

$$\therefore$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & \qquad & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & \qquad & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore \ \forall x$$

Function Composition

Start

$$\begin{array}{ccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array} \qquad \begin{array}{ccc} X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,$$

Function Composition

▸ Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & \quad & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & \quad & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ ($$

Function Composition

▸ Start

$$\begin{array}{ccccc} \mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array} \qquad \begin{array}{ccc} \mathrm{X} & \xrightarrow{g \circ f} & \mathrm{Z} \\ x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in \mathrm{X},\ (g$$

Function Composition

Start

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x))
\end{array}
\qquad
\begin{array}{ccc}
X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \qquad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
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\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ \ (g \circ f$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & \quad & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & \quad & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x)$$

Function Composition

▸ Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) =$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g \circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g \circ f)(x) = g$$

Function Composition

▸ Start

$$X \xrightarrow{f} Y \xrightarrow{g} Z \qquad X \xrightarrow{g \circ f} Z$$

$$x \xrightarrow{f} y \xrightarrow{g} z \qquad x \xrightarrow{g \circ f} z$$

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x)) \qquad x \xrightarrow{g \circ f} (g \circ f)(x)$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g($$

Function Composition

▸ Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f$$

Start

$$\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \qquad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

Function Composition

▸ Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

Function Composition

▸ Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \qquad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

(

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

((

Function Composition

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$((h$

Function Composition

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \qquad & \mathrm{X} & \xrightarrow{g \circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g \circ f)(x) = g(f(x))$$

$$(\,(h \circ$$

Function Composition

Start

$$\begin{array}{ccccccccc} \mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$(\ (h \circ g$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$(\ (h \circ g)$$

Function Composition

Start

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$(\,(h \circ g)\ \circ$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$(\, (h \circ g) \ \circ f$$

Function Composition

Start

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$(\ (h \circ g)\ \circ f)$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g \circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g \circ f)(x) = g(f(x))$$

$$\left((h \circ g) \circ f \right)(x)$$

Function Composition

▸ Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \qquad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$\left(\left(h\circ g\right)\circ f\right)(x) =$$

Function Composition

Start

$$X \xrightarrow{f} Y \xrightarrow{g} Z \qquad X \xrightarrow{g\circ f} Z$$

$$x \xrightarrow{f} y \xrightarrow{g} z \qquad x \xrightarrow{g\circ f} z$$

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x)) \qquad x \xrightarrow{g\circ f} (g\circ f)(x)$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = ($$

Function Composition

Start

$$\begin{array}{ccccccccc} \mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g \circ f} & \mathrm{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in \mathrm{X},\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h$$

Function Composition

▸ Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g \circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g \circ f)(x) = g(f(x))$$

$$\left((h \circ g) \circ f \right) (x) = (h \circ g$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)$$

Function Composition

▸ Start

$$\begin{array}{ccccccccc} \mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$(\,(h\circ g)\ \circ f)\,(x) =\ (h\circ g)\ ($$

Function Composition

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g \circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)\ (f$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \qquad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)\,(f\,(x)$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x))$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)\ (f\,(x)\,) = h$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g \circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore \ \forall x \in \mathrm{X}, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h($$

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g($$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$\left((h \circ g) \circ f \right) (x) = (h \circ g) \ (f\ (x)\ ) \ = h\,(g\,(f\,(x)$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

Function Composition

▸ Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g \circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore \ \forall x \in \mathrm{X}, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$=$$

Function Composition

Start

$$\begin{array}{ccccccccc} \mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g \circ f} & \mathrm{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in \mathrm{X},\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h$$

Function Composition

Start

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h($$

Function Composition

Start

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h(($$

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)\,(f(x)) = h(g(f(x)))$$

$$= h\,((g$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ$$

Function Composition

▸ Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)\ (f(x)) = h(g(f(x)))$$

$$= h((g \circ f$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x))$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x))$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & \qquad & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & \qquad & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) =$$

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = ($$

Function Composition

Start

$$X \xrightarrow{f} Y \xrightarrow{g} Z \qquad X \xrightarrow{g \circ f} Z$$

$$x \xrightarrow{f} y \xrightarrow{g} z \qquad x \xrightarrow{g \circ f} z$$

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x)) \qquad x \xrightarrow{g \circ f} (g \circ f)(x)$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ$$

Function Composition

▸ Start

$$\begin{array}{ccccccccc} \mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g \circ f} & \mathrm{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in \mathrm{X},\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ ($$

Function Composition

▸ Start

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g$$

Function Composition

Start

$$\begin{array}{ccccccccc} \mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f$$

Function Composition

▸ Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f)$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & \qquad & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & \qquad & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))$$

Function Composition

Start

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))($$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

Function Composition

Start

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ(g\circ f))(x)$$

$\therefore$

Function Composition

▸ Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x$$

Function Composition

Start

$$\begin{array}{ccccccccc} \mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x \in \mathrm{X}, ($$

Function Composition

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \qquad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ(g\circ f))(x)$$

$$\therefore\ \forall x \in \mathrm{X}, ((\ $$

Function Composition

Start

$$\begin{array}{ccccccccc} \mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ(g\circ f))(x)$$

$$\therefore\ \forall x \in \mathrm{X}, ((h$$

Function Composition

Start

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \ \forall x \in X, ((h \circ$$

Function Composition

Start

$$\begin{array}{ccccccccc} \mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \qquad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ(g\circ f))(x)$$

$$\therefore\ \forall x \in \mathrm{X}, ((h\circ g$$

Start

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)$$

Function Composition

Start

$$\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \ \forall x \in X, ((h \circ g) \circ$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)\circ f$$

Function Composition

Start

$$X \xrightarrow{f} Y \xrightarrow{g} Z \qquad X \xrightarrow{g\circ f} Z$$

$$x \xrightarrow{f} y \xrightarrow{g} z \qquad x \xrightarrow{g\circ f} z$$

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x)) \qquad x \xrightarrow{g\circ f} (g\circ f)(x)$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)\circ f)$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ(g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)\circ f)(x)$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) =$$

Function Composition

Start

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x))
\end{array}
\qquad
\begin{array}{ccc}
X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = ($$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ(g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)\circ f)(x) = (h$$

Function Composition

Start

$$\begin{array}{ccccccccc} \mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g \circ f} & \mathrm{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore \ \forall x \in \mathrm{X}, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \ \forall x \in \mathrm{X}, ((h \circ g) \circ f)(x) = (h \circ$$

Function Composition

Start

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ ($$

Function Composition

▸ Start

$$\begin{array}{ccccccccc} \mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x \in \mathrm{X}, ((h\circ g)\circ f)(x) = (h\circ (g$$

Function Composition

▸ Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x \in \mathrm{X}, ((h\circ g)\circ f)(x) = (h\circ (g\circ$$

Function Composition

▸ Start

$$\begin{array}{ccccccccc} \mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ(g\circ f))(x)$$

$$\therefore\ \forall x \in \mathrm{X}, ((h\circ g)\circ f)(x) = (h\circ (g\circ f$$

Function Composition

▸ Start

$$X \xrightarrow{f} Y \xrightarrow{g} Z \qquad X \xrightarrow{g \circ f} Z$$

$$x \xrightarrow{f} y \xrightarrow{g} z \qquad x \xrightarrow{g \circ f} z$$

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x)) \qquad x \xrightarrow{g \circ f} (g \circ f)(x)$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f)$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ(g\circ f))(x)$$

$$\therefore\ \forall x \in \mathrm{X},\ ((h\circ g)\circ f)(x) = (h\circ(g\circ f))(x)$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g \circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & \quad & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & \quad & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore \ \forall x \in \mathrm{X}, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \ \forall x \in \mathrm{X}, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

(

Function Composition

Start

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x))
\end{array}
\qquad
\begin{array}{ccc}
X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)\circ f)(x) = (h\circ (g\circ f))(x)$$

$$(h$$

Function Composition

▸ Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x \in \mathrm{X}, ((h\circ g)\circ f)(x) = (h\circ (g\circ f))(x)$$

$$(h\circ$$

Function Composition

▸ Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g$$

Function Composition

▸ Start

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g)$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ$$

Function Composition

▸ Start

$$\begin{array}{ccccccccc} \mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g \circ f} & \mathrm{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in \mathrm{X},\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in \mathrm{X}, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ(g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)\circ f)(x) = (h\circ(g\circ f))(x)$$

$$(h\circ g)\circ f =$$

Function Composition

▸ Start

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g \circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore \ \forall x \in \mathrm{X}, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \ \forall x \in \mathrm{X}, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f = h$$

Function Composition

Start

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f = h \circ$$

Function Composition

▸ Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore \; \forall x \in X, \; (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \; \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f = h \circ ($$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)\circ f)(x) = (h\circ (g\circ f))(x)$$

$$(h\circ g)\circ f = h\circ (g$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f = h \circ (g \circ$$

Function Composition

Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f = h \circ (g \circ f$$

Function Composition

▸ Start

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f = h \circ (g \circ f)$$

Function Composition

Home

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f = h \circ (g \circ f)$$