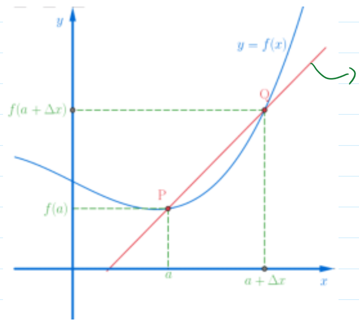
미분계수

2019년 7월 22일 월요일 오전 8:42

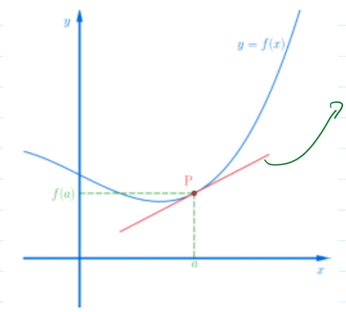
$$\left[\frac{\Delta y}{\Delta x}\right]_{P.Q,\ y=f(x)} = \boxed{\left[\frac{\Delta y}{\Delta x}\right]_{x=a,\ \Delta x,\ y=f(x)}}$$



$\to y = mx + n$

$\to$ 평균변화율

$$\left[\frac{\Delta y}{\Delta x}\right]_{P.Q,\ y=f(x)} = \left[\frac{\Delta y}{\Delta x}\right]_{x=a,\ \Delta x,\ y=f(x)}$$



$$\left[\frac{dy}{dx}\right]_{x=a,\ y=f(x)} = \lim_{\Delta x \to 0}\left[\frac{\Delta y}{\Delta x}\right]_{x=a,\ \Delta x,\ y=f(x)}$$

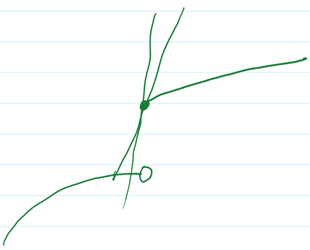
$$\parallel$$

$$\left[\frac{dy}{dx}\right]_{x=a} = y'_{x=a} = f'(a)$$

$$\left[\frac{\Delta y}{\Delta x}\right]_{x=a,\ y=f(x)} \qquad \lim_{\Delta x \to 0}\left[\frac{\Delta y}{\Delta x}\right]_{x=a,\ y=f(x)} = \left[\frac{dy}{dx}\right]_{x=a}$$

$$= f'(a)$$

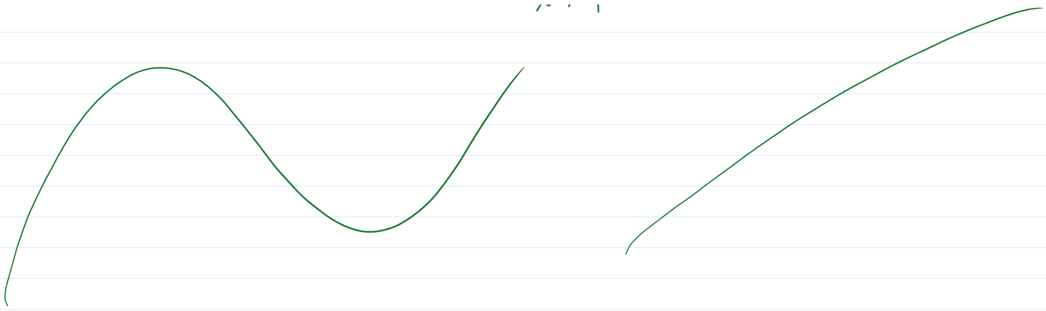
$$= y'_{x=a}$$

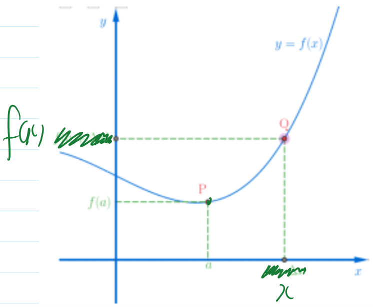


$y = \sqrt{x}$

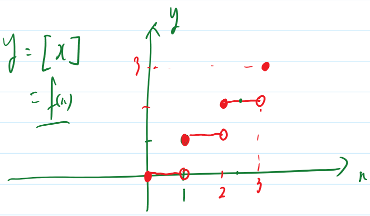
$$\frac{\sqrt{x} - 0}{x - 0} = \frac{1}{\sqrt{x}}$$

$$\lim_{x \to 0+} \frac{1}{\sqrt{x}} = \infty$$





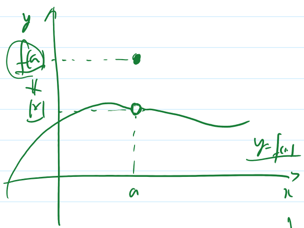
$\boxed{\lim_{x\to a} f(x) = f(a)} \Leftarrow$ 함수 f가 a에서 연속이다



$\lim_{x\to 2+}[x] = 2$

$\lim_{x\to 2-}[x] = 1$

$\nexists \lim_{x\to 2}[x]$



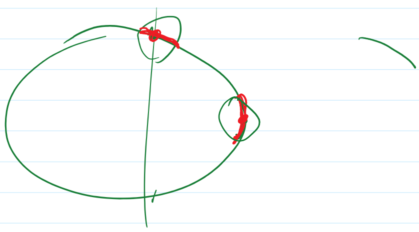
$\lim_{x\to a} f(x) = M \neq f(a)$

$f(x) = \begin{cases} x, & x\neq 0 \\ 1, & x=0 \end{cases}$ $\quad \lim_{x\to 0} f(x) = 0 \neq f(0) = 1$

연속 $\left\{ \begin{array}{l} \text{극한값 존재} \\ \text{함숫값 존재} \\ \text{극한값} = \text{함숫값} \end{array} \right.$

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a} = f'(a)$$

$\forall \varepsilon>0,\ \exists \delta>0$ s.t. $0<|x-a|<\delta \Rightarrow \left|\frac{f(x)-f(a)}{x-a} - f'(a)\right| < \varepsilon$



$f(x)-f(a)$

$$\lim_{x\to a}\{f(x)-f(a)\} = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}\cdot(x-a)$$

$$\lim_{x\to a}\{f(x)-f(a)\} \cancel{=} \lim_{x\to a}f(x) - \lim_{x\to a}f(a)$$

$$= \lim_{x\to a}f(x) - f(a)$$

$$= f'(a)\cdot 0 = 0$$

$$\therefore \lim_{x\to a}f(x) = f(a)$$

$\lim_{x\to a}f(x) = L$ $\quad \forall\varepsilon>0, \exists\delta>0$ s.t. $0<|x-a|<\delta \Rightarrow |f(x)-L|<\varepsilon$

$\lim_{x\to a}\{f(x)-f(a)\}=0$ $\quad \forall\varepsilon>0, \exists\delta>0$ s.t. $0<|x-a|<\delta \Rightarrow |\{f(x)-f(a)\}-0|<\varepsilon$

$$\parallel$$

$$|f(x)-f(a)|<\varepsilon$$

$\lim_{x\to a}f(x) = f(a)$

$\forall\varepsilon>0, \exists\delta>0$ s.t. $0<|x-a|<\delta \Rightarrow |f(x)-f(a)|<\varepsilon$

$\exists\lim_{x\to a}f(x)$, $\exists\lim_{x\to a}g(x)$

$\Rightarrow \lim_{x\to a}f(x)\cdot g(x) = \{\lim_{x\to a}f(x)\}\cdot\{\lim_{x\to a}g(x)\}$

$$\lim_{x\to a}\{f(x)-g(x)\} = \lim_{x\to a}f(x) - \lim_{x\to a}g(x)$$

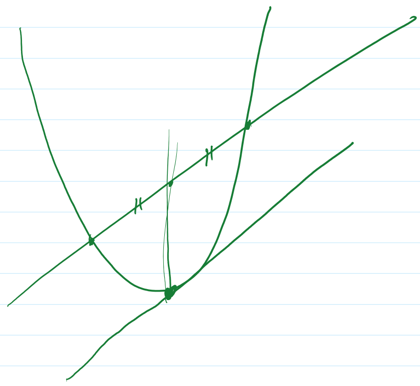
$$\lim_{x\to 0}1 = \lim_{x\to 0}x\frac{1}{x} \cancel{=} \lim_{x\to 0}x\cdot\lim_{x\to 0}\frac{1}{x}$$

$$\lim_{x\to 0}0 = \lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{x}\right) \cancel{=} \lim_{x\to 0}\frac{1}{x} - \lim_{x\to 0}\frac{1}{x}$$

$$\boxed{\lim_{x\to\infty}\frac{1}{x}=0}$$

$\forall\varepsilon>0, \exists M$ s.t. $x>M \Rightarrow \left|\frac{1}{x}-0\right|<\varepsilon$

3-1-(3)



$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{f(a+\underbrace{x-a}) - f(a)}{\underbrace{x-a}}$$

$\Delta x = x-a$

$$= \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$= \lim_{\Delta x\to 0} \frac{f(a+\Delta x)-f(a)}{\Delta x}$$

$\lim_{x\to a} g(x) = L$ $\quad \forall \varepsilon>0,\ \exists \delta>0$ s.t. $0<|x-a|<\delta \Rightarrow |g(x)-L|<\varepsilon$

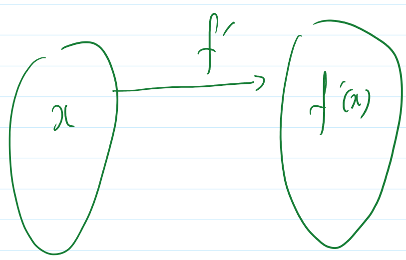
$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \qquad a\in[\alpha,\beta]$$

$a \Leftrightarrow x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \qquad x\in[\alpha,\beta]$$

$\boxed{A \xrightarrow{f} B}$ $\quad \forall x\in A,\ \exists y\in B$ s.t. $f(x)=y$

f를 함수라고 한다



$\{A \xrightarrow{f} B$

$$\begin{cases} A \xrightarrow{f} B \\ \quad f' \\ A' \longrightarrow B' \end{cases} \Rightarrow A' \subset A$$

* $f(x)=|x|$, $x\in\mathbb{R}$ $\underline{f'(x)?}$

답: $f'(x)=\begin{cases}-1 & , x<0 \\ 1 & , x>0\end{cases}$

* $f(x)=[x]$, $x\in\mathbb{R}$ $f'(x)?$

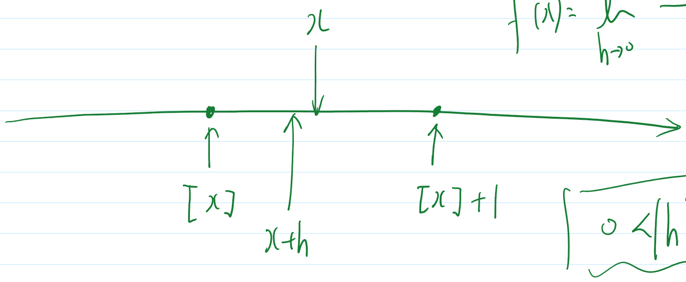
$\underline{x=n+a}$ $0\le a<1$, $n\in\mathbb{Z}$

답: $f'(x)=0$, $\mathbb{R}-\mathbb{Z}$

$$\lim_{h\to0}\frac{[x+h]-[x]}{h}=\lim_{h\to0}\frac{[n+a+h]-[n+a]}{h}=\lim_{h\to0}\frac{[n+a+h]-n}{h}$$

$$=\lim_{h\to0}\frac{\cancel{n}+[a+h]\cancel{-n}}{h}=\lim_{h\to0}\frac{[a+h]}{h}=\begin{cases}\lim_{h\to0}\dfrac{[h]}{h} & a=0 \\ \lim_{h\to0}\dfrac{0}{h}=0 & a\neq0 \quad a>0 \quad a+h>0\end{cases}$$

* if $x\notin\mathbb{Z}$



$$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to0}\frac{[x+h]-[x]}{h}$$

$$=\lim_{h\to0}\frac{[x]-[x]}{h}$$

$$0<|h|<\min(x-[x],\ [x]+1-x)$$

$y=\sqrt{x}$ 정의역 $x\ge0$

$y'=\dfrac{1}{2\sqrt{x}}$ 정의역 $x>0$

[함수의 정의역은 원래함수의 정의역의 부분집합이다.

$f(x) = (x^5+3)\cdot(x^3-2)$

$f'(x) = (x^5+3)'(x^3-2) + (x^5+3)\cdot(x^3-2)'$

$= 5x^4\cdot(x^3-2) + (x^5+3)\cdot 3x^2$

* $f\circ g(x) = f(g(x))$

$$\frac{f\circ g(x+h) - f\circ g(x)}{h} = \frac{f(g(x+h)) - f(g(x))}{\underbrace{g(x+h) - g(x)}} \times \frac{g(x+h) - g(x)}{h}$$

$\hookrightarrow$ $g(x+h) - g(x) = 0$ 인 경우도 있다.

* $h(x) = \{g(x)\}^n$

$f(x) = x^n$ $\qquad f'(x) = nx^{n-1}$

$$\{f\circ g(x)\}' = f'(g(x))\,g'(x)$$

$$= n\{g(x)\}^{n-1} g'(x)$$

연습문제 3-2

$$\lim_{n\to\infty}\sqrt{\frac{1}{n}} \overset{h=\frac{1}{n}}{=} \lim_{h\to 0+}\sqrt{h}, \quad \boxed{\lim_{h\to 0-}\sqrt{h}\ (X)} \quad \lim_{h\to 0}\sqrt{h}\ (X)$$

$h>0$

연습문제 3-3

$$\lim_{x\to 0} x|x| = \begin{cases} \lim_{x\to 0+} x^2 = 0 \\ \lim_{x\to 0-} -x^2 = 0 \end{cases}$$

연속이므로

3-4 f: 기함수 $\qquad f'$: 우함수

f: 우함수 $\qquad f'$: 기함수

$f'(0) = 0$ ← x가 0에서 미분가능 하면

3-5 * $\lim_{x\to a}|f(x)| = 0 \Leftrightarrow \lim_{x\to a} f(x) = 0$?

$$\begin{cases} \lim_{x\to a} |f(x)| = 0 & \forall\varepsilon>0,\ \exists\delta>0 \text{ s.t. } 0<|x-a|<\delta \Rightarrow \big||f(x)|-0\big| < \varepsilon \\ & \big||f(x)|\big| = |f(x)| = |f(x)-0| \\ \lim_{x\to a} f(x) = 0 & \forall\varepsilon>0,\ \exists\delta>0 \text{ s.t. } 0<|x-a|<\delta \Rightarrow |f(x)-0| < \varepsilon \end{cases}$$

$*$ $f(x) \le g(x) \le h(x)$, $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L \Rightarrow \lim_{x\to a} g(x) = L$

$\lim_{x\to a} f(x) \le \lim_{x\to a} g(x) \le \lim_{x\to a} h(x)$ (포괄 정리)

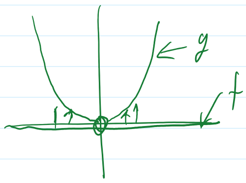
3-9 $f(x) = (x-a)^m (x-b)^n$ $\quad f'\left(\dfrac{mb+na}{m+n}\right) = 0$

m, n 자연수

a, b 사이의 내분점 $\quad$ 1509 장민준

3-10
$$\left\{\begin{array}{l} f(x) < g(x)\ (0<|x-a|<\delta) \\ \exists \lim_{x\to a} f(x),\ \exists \lim_{x\to a} g(x) \end{array}\right. \Rightarrow \lim_{x\to a} f(x) \le \lim_{x\to a} g(x)$$

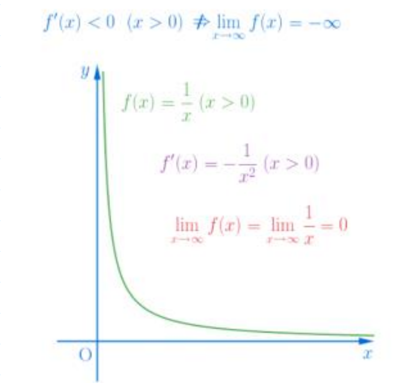
ex) $f(x)=0$, $g(x)=x^2$, $a=0$



3-11 $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ $\quad (a_n \ne 0,\ a_i \in \mathbb{R})$

$\lim_{x\to\infty} f(x) = +\infty$ or $-\infty$ 임을 보여라.

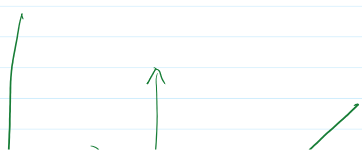
$*$ $f'(x) < 0$ $(x>0)$ $\Rightarrow \lim_{x\to\infty} f(x) = -\infty$? $\times$



https://min7014.github.io/2019/20190802906.pdf

3-13 $f(x) = 4x^2 - 12x + 5$

$f \circ g(x) = f(x)$ $\quad g(x)$?



$f \circ g(x) = f(x) \quad g(x)\,?$

$4\{g(x)\}^2 - 12\{g(x)\} + 5 = 4x^2 - 12x + 5$

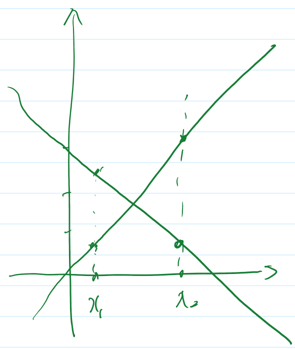
$4\{g(x)\}^2 - 12\{g(x)\} - 4x^2 + 12x = 0$

$\{g(x)\}^2 - 3\{g(x)\} - x(x-3) = 0$

$\{g(x) - x\}\{g(x) + (x-3)\} = 0$

$\therefore g(x) = x, \ 3-x$ ✗

$g(x) = x$ 또는 $g(x) = 3-x$



3-14 $g(x) = \begin{cases} f(x) & (x < a) \\ m - f(x) & (a \le x < b) \\ n + f(x) & (b \le x) \end{cases}$

$\lim_{x \to a-} \frac{g(x) - g(a)}{x - a} = \lim_{x \to a-} \frac{f(x) - \{m - f(a)\}}{x - a} = \lim_{x \to a-} \frac{f(x) - f(a)}{x - a} = f'(a)$

$\lim_{x \to a+} \frac{g(x) - g(a)}{x - a} = \lim_{x \to a+} \frac{\{m - f(x)\} - \{m - f(a)\}}{x - a} = \lim_{x \to a+} \left\{ - \frac{f(x) - f(a)}{x - a} \right\} = -f'(a)$

$\lim_{x \to a+} g(x) = \lim_{x \to a+} \{m - f(x)\} = m - f(a)$

$\lim_{x \to a-} g(x) = \lim_{x \to a-} f(x) = f(a)$

3-15. f : 미분가능 $\not\Rightarrow$ f' : 연속

$y = x \sin \frac{1}{x}$

3-16 $f(x) = (x-a)^n Q(x) + R(x) \Rightarrow f'(x) = (x-a)^{n-1} Q_1(x) + R'(x)$

3-18 $\frac{d}{dx}(f(x))$

필수 4-1 (2) 기하적 의미
(1)

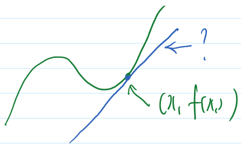
* 기울기 m, 한점 (x_1, y_1) 이면 $y - y_1 = m(x - x_1)$

* 함수 $y=f(x)$, $(x_1, f(x_1))$

접선: 기울기 $f'(x_1)$

$y - f(x_1) = f'(x_1)(x - x_1)$

$* f'(x_1)x - y + f(x_1) - f'(x_1)x_1 = 0$

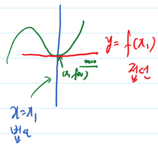


법선: i) $f'(x_1) \neq 0$ ii) $f'(x_1) = 0$

$m \times m' = -1$

기울기 $-\frac{1}{f'(x_1)}$

$y - f(x_1) = -\frac{1}{f'(x_1)}(x - x_1)$



$x = x_1$

$* x + f'(x_1)y - f'(x_1)f(x_1) - x_1 = 0$

$\begin{cases} ax+by+c=0 \\ a'x+b'y+c'=0 \end{cases}$ 수직 $\iff aa'+bb'=0$

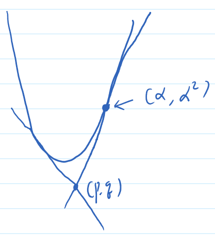
필수예제 4-4

* 한 점을 지나고 포물선과 접하는 직선의 개수에 대하여 논하여라.

$y=x^2$ (∵ 모든 포물선은 닮음이다)

(α, α^2) 의 접선의 방정식

$y-\alpha^2 = 2\alpha(x-\alpha)$



(p, q)를 지난다고 한다면

$q-\alpha^2 = 2\alpha(p-\alpha) = 2p\alpha - 2\alpha^2$

$\alpha^2 - 2p\alpha + q = 0$

0개: $D<0$ $p^2<q$ (p,q) $(x^2<y)$

1개: $D=0$ $p^2-q=0$ (p,p^2) $(x^2=y)$

2개: $D>0$ $p^2>q$ (p,q) $(x^2>y)$

* 포물선의 두 접선이 수직일때 교점을 모두 구하여라.

* 필수예제 4-5의 기하적 의미?

기하적으로 계산의 비가 일정한 연속함수 있을까?

* 필수예제 4-6 의 기하적 의미!

뉴턴 근사

기하적으로 계산의 비가 일정한 연속함수 있을까?

$[a,b] = \{x \mid a \le x \le b\}$

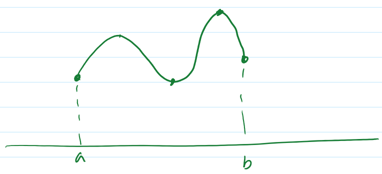
$(a,b) = \{x \mid a < x < b\}$

f가 $[a,b]$에서 연속 $\begin{cases} c \in (a,b) \quad \lim_{x\to c} f(x) = f(c) \\ \lim_{x\to a+} f(x) = f(a) \\ \lim_{x\to b-} f(x) = f(b) \end{cases}$

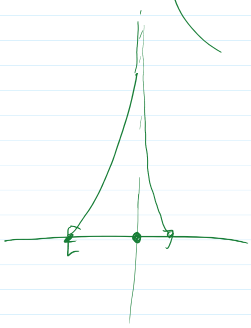
* 최대 최소 정리

f는 $[a,b]$에서 연속 $\Rightarrow$ 함수 f는 $[a,b]$에서 최댓값, 최솟값이 존재

$\begin{cases} \exists\, c_1 \in [a,b] \text{ s.t. } f(c_1) \ge f(x),\ {}^\forall x \in [a,b] \\ \text{and} \\ \exists\, c_2 \in [a,b] \text{ s.t. } f(c_2) \le f(x),\ {}^\forall x \in [a,b] \end{cases}$

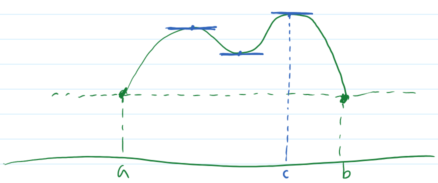


$y = \begin{cases} \frac{1}{x} & x \in [-1, 1] \\ 0 & x = 0 \end{cases}$



* 롤의 정리

$\begin{cases} f\text{가 } [a,b]\text{에서 연속} \\ f\text{가 } (a,b)\text{에서 미분가능} \\ f(a) = f(b) \end{cases} \Rightarrow \exists\, c \in (a,b) \text{ s.t. } f'(c) = 0$



proof) f가 $[a,b]$에 연속이므로

최대 최소 정리에 의해 최댓값 최솟값이 존재 된다.

proof) f가 [a,b]에 연속이므로
최대 최소 정리에 의해 최댓값 최솟값이 존재한다.

Case 1, 최댓값과 최솟값이 f(a) . $f(x) = f(a)$, $x \in [a,b]$

$f'(x) = 0$, $x \in (a,b)$

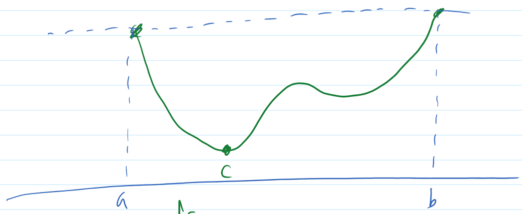
Case 2, Case 1이 아닌 경우

최소가 (a, b)에 있는 경우

$\exists c \in (a,b)$ s.t. $f(c) \le f(x)$, $x \in [a,b]$

$$\lim_{x\to c-} \frac{f(x)-f(c)^{\leftarrow \ge 0}}{\underbrace{x-c}_{<0}} \le 0 \qquad \lim_{x\to c+} \frac{f(x)-f(c)^{\leftarrow \ge 0}}{\underbrace{x-c}_{>0}} \ge 0$$

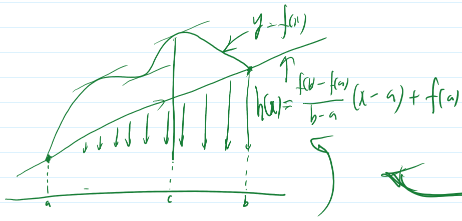
$f'(c)$

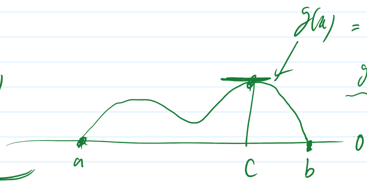


$\therefore f'(c) = 0$

* 평균값 정리

$\left\{ \begin{array}{l} f\text{가 } [a,b]\text{에서 연속} \\ f\text{가 } (a,b)\text{에서 미분가능} \end{array} \right. \Rightarrow \exists c \in (a,b)$ s.t. $f'(c) = \dfrac{f(b)-f(a)}{b-a}$





증명) $g(x) = f(x) - \left\{ \underbrace{(x-a)\cdot\frac{f(b)-f(a)}{b-a} + f(a)}_{h(x)} \right\}$

$g(a) = 0 \qquad g(b) = 0$

$\left\{ \begin{array}{l} g\text{가 } [a,b]\text{에서 연속} \\ g\text{가 } (a,b)\text{에서 미분가능} \\ g(a) = g(b) \end{array} \right. \Rightarrow$ 롤의 정리에 의해서 $\exists c \in (a,b)$ s.t. $g'(c) = 0$

$g'(x) = f'(x) - \dfrac{f(b)-f(a)}{b-a}$

$g'(c) = 0$

$f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$\therefore\ \exists c \in (a,b)$ s.t. $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

* $\begin{cases} f\text{가 } [a,b]\text{에서 연속} \\ f'\text{이 } (a,b)\text{에서 } f'=0 \end{cases} \Rightarrow f\text{가 } [a,b]\text{에서 } f(x)=f(a)$

proof) $\forall x \in (a,b)$

$\begin{cases} f\text{가 } [a,x]\text{에서 연속} \\ f\text{가 } (a,x)\text{에서 미분가능 } (\because f'(t)=0,\ t\in(a,b)) \end{cases}$

평균값 정리에 의해서

$\exists c \in (a,x)$ s.t. $f'(c) = \dfrac{f(x)-f(a)}{x-a}$

$\begin{cases} x-a \neq 0 \quad (\because c \in (a,x)) \\ f'(c)=0 \end{cases} \Rightarrow f(x)-f(a)=0$

$f(x)=f(a)\ ,\ x\in(a,b)$ ··· ①

$f(b) = \lim\limits_{x\to b-} f(x)$ ($\because f$가 $[a,b]$에서 연속)

$= \lim\limits_{x\to b-} f(a) = f(a)$ ·· ②

①, ②에 의해서

$f(x)=f(a)\ ,\ x\in[a,b]$

* 연습문제 4-3 초월이 길치할때

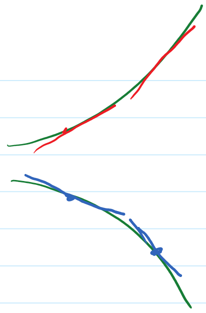
* 연습문제 4-4 이유리

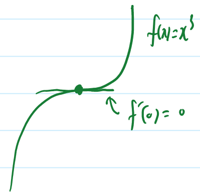
극대, 극소와 미분

2019년 9월 3일 화요일 오후 3:55

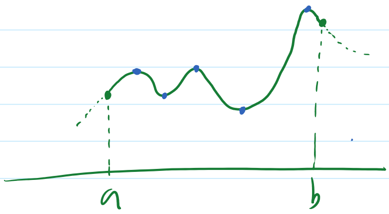
* 증가 $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

* 감소 $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$





극대

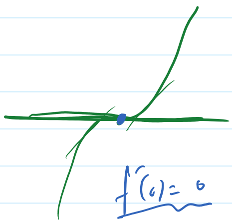


$f'(c_1)=0$

극소

$f(x)=x^3$

$f'(x)=3x^2$

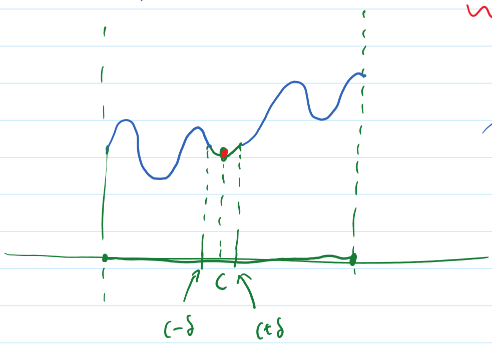


$f'(0)=0$

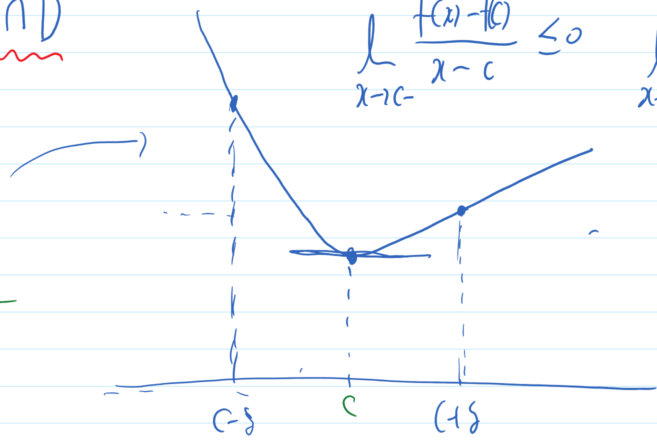
f가 정의역 D에서 정의된 함수일 때,

* $c \in D$ 에서 극솟값을 갖는다.

$\exists \delta>0$ s.t. $f(c) \le f(x)$, $x \in (c-\delta, c+\delta) \cap D$

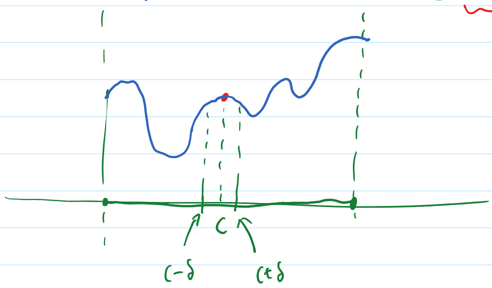


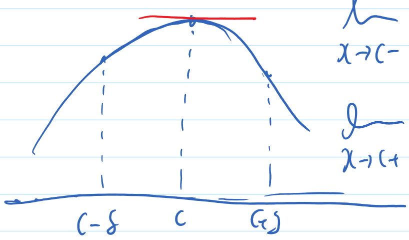
$\lim_{x \to c-} \frac{f(x)-f(c)}{x-c} \le 0$ $\lim_{x \to c+} \frac{f(x)-f(c)}{x-c} \ge 0$



* $c \in D$ 에서 극댓값을 갖는다.

$\exists \delta>0$ s.t. $f(c) \ge f(x)$, $x \in (c-\delta, c+\delta) \cap D$





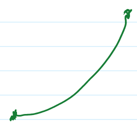
$\lim_{x \to c-} \frac{f(x)-f(c)}{x-c} \ge 0$

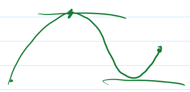
$\lim_{x \to c+} \frac{f(x)-f(c)}{x-c} \le 0$

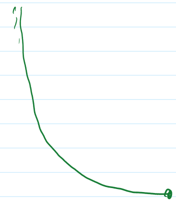
$*$ $\begin{cases} f \text{ 가 } (a,b) \text{ 에서 미분가능} \\ c \in (a,b) \\ f \text{ 는 } c \text{ 에서 극대 또는 극소} \end{cases} \Rightarrow f'(c) = 0$

최대 최소와 미분

2019년 9월 3일 화요일 오후 4:01







$$f(x) = \begin{cases} \frac{1}{x}, & (x \in (0, 1]) \\ 1, & x = 0 \end{cases}$$

* f가 [a,b]에서 연속 ⇒ 최댓값 $= \max_{x \in [a,b]} \{f(x)\}$ 최솟값 $= \min_{x \in [a,b]} \{f(x)\}$

* f가 [a,b]에서 연속 ⇒ $R = \{x \mid x$에서 극값을 갖는다., $x \in (a,b)\}$

최댓값 $= \max_{x \in R \cup \{a,b\}} \{f(x)\}$ 최솟값 $= \min_{x \in R \cup \{a,b\}} \{f(x)\}$

* f가 [a,b]에서 연속, f가 (a,b)에서 미분가능 ⇒ $R = \{x \mid f'(x) = 0, x \in (a,b)\}$

최댓값 $= \max_{x \in R \cup \{a,b\}} \{f(x)\}$

최솟값 $= \min_{x \in R \cup \{a,b\}} \{f(x)\}$