

# 매개변수 함수의 2차 미분

(Second derivative of parametric functions)

## Second derivative of parametric functions

$$\frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}y$$

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2$$

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y$$

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left( \right.$$



$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}\right.$$

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}y\right)$$

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$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}y\right) = \frac{d}{dx} \left(\frac{dy}{dx}\right)$$

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}y\right) = \frac{d}{dx} \left(\frac{dy}{dx}\right)$$

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}y\right) = \frac{d}{dx} \left(\frac{dy}{dx}\right)$$

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$$= \frac{d}{dt} \left( \right)$$

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}y\right) = \frac{d}{dx} \left(\frac{dy}{dx}\right)$$

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$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}y\right) = \frac{d}{dx} \left(\frac{dy}{dx}\right)$$

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$$= \frac{\frac{d}{dt} \left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}y\right) = \frac{d}{dx} \left(\frac{dy}{dx}\right)$$

$$= \frac{\frac{d}{dt} \left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{d}{dt} \left( \right)$$

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}y\right) = \frac{d}{dx} \left(\frac{dy}{dx}\right) \\ &= \frac{\frac{d}{dt} \left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{d}{dt} \left( \frac{dy}{dx} \right)\end{aligned}$$

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}y\right) = \frac{d}{dx} \left(\frac{dy}{dx}\right) \\ &= \frac{\frac{d}{dt} \left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{d}{dt} \left( \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \right)\end{aligned}$$

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$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}y\right) = \frac{d}{dx} \left(\frac{dy}{dx}\right)$$

$$= \frac{\frac{d}{dt} \left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt} \left( \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \right)}{\frac{dx}{dt}} = \frac{d^2y}{dt^2}$$



$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}y\right) = \frac{d}{dx} \left(\frac{dy}{dx}\right)$$

$$= \frac{\frac{d}{dt} \left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt} \left( \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \right)}{\frac{dx}{dt}} = \frac{d^2y}{dt^2} \cdot$$

$$\begin{aligned}
 \frac{d^2y}{dx^2} &= \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}y\right) = \frac{d}{dx} \left(\frac{dy}{dx}\right) \\
 &= \frac{\frac{d}{dt} \left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt} \left( \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \right)}{\frac{dx}{dt}} = \frac{\frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dy}{dt} \cdot \frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^3}
 \end{aligned}$$

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 \frac{d^2y}{dx^2} &= \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}y\right) = \frac{d}{dx} \left(\frac{dy}{dx}\right) \\
 &= \frac{\frac{d}{dt} \left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt} \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)}{\frac{dx}{dt}} = \frac{\frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dy}{dt} \cdot \frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^3}
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$$= \frac{\frac{d}{dt} \left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt} \left( \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \right)}{\frac{dx}{dt}} = \frac{\frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dx}{dt} \cdot \frac{d^2y}{dt^2}}{\frac{dx}{dt}}$$

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 &= \frac{\frac{d}{dt} \left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt} \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)}{\frac{dx}{dt}} = \frac{\frac{\frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dx}{dt} \cdot \frac{d^2y}{dt^2}}{\left(\frac{dx}{dt}\right)^2}}{\frac{dx}{dt}}
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 &= \frac{\frac{d}{dt} \left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt} \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)}{\frac{dx}{dt}} = \frac{\frac{\frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dx}{dt} \cdot \frac{d^2y}{dt^2}}{\left(\frac{dx}{dt}\right)^2}}{\frac{dx}{dt}}
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 &= \frac{\frac{d}{dt} \left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt} \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)}{\frac{dx}{dt}} = \frac{\frac{\frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dx}{dt} \cdot \frac{d^2y}{dt^2}}{\left(\frac{dx}{dt}\right)^2}}{\frac{dx}{dt}} \\
 &= \frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dx}{dt} \cdot \frac{d^2y}{dt^2}
 \end{aligned}$$

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 \frac{d^2y}{dx^2} &= \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}y\right) = \frac{d}{dx} \left(\frac{dy}{dx}\right) \\
 &= \frac{\frac{d}{dt} \left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt} \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)}{\frac{dx}{dt}} = \frac{\frac{\frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dx}{dt} \cdot \frac{d^2y}{dt^2}}{\left(\frac{dx}{dt}\right)^2}}{\frac{dx}{dt}} \\
 &= \frac{\frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dx}{dt} \cdot \frac{d^2y}{dt^2}}{\left(\frac{dx}{dt}\right)^3}
 \end{aligned}$$

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 \frac{d^2y}{dx^2} &= \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}y\right) = \frac{d}{dx} \left(\frac{dy}{dx}\right) \\
 &= \frac{\frac{d}{dt} \left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt} \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)}{\frac{dx}{dt}} = \frac{\frac{\frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dx}{dt} \cdot \frac{d^2y}{dt^2}}{\left(\frac{dx}{dt}\right)^2}}{\frac{dx}{dt}} \\
 &= \frac{\frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dx}{dt} \cdot \frac{d^2y}{dt^2}}{\left(\frac{dx}{dt}\right)^3} \quad \therefore
 \end{aligned}$$

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 \frac{d^2y}{dx^2} &= \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}y\right) = \frac{d}{dx} \left(\frac{dy}{dx}\right) \\
 &= \frac{\frac{d}{dt} \left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt} \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)}{\frac{dx}{dt}} = \frac{\frac{\frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dx}{dt} \cdot \frac{d^2y}{dt^2}}{\left(\frac{dx}{dt}\right)^2}}{\frac{dx}{dt}} \\
 &= \frac{\frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dx}{dt} \cdot \frac{d^2y}{dt^2}}{\left(\frac{dx}{dt}\right)^3} \quad \therefore \frac{d^2y}{dx^2}
 \end{aligned}$$

$$\begin{aligned}
 \frac{d^2y}{dx^2} &= \frac{d^2}{dx^2}y = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx} \left(\frac{d}{dx}y\right) = \frac{d}{dx} \left(\frac{dy}{dx}\right) \\
 &= \frac{\frac{d}{dt} \left(\frac{dy}{dx}\right)}{\frac{dx}{\frac{dt}}}} = \frac{\frac{d}{dt} \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)}{\frac{dx}{\frac{dt}}}} = \frac{\frac{\frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dx}{dt} \cdot \frac{d^2y}{dt^2}}{\left(\frac{dx}{dt}\right)^2}}{\frac{dx}{\frac{dt}}} \\
 &= \frac{\frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dx}{dt} \cdot \frac{d^2y}{dt^2}}{\left(\frac{dx}{dt}\right)^3} \quad \therefore \frac{d^2y}{dx^2} = \frac{\frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dx}{dt} \cdot \frac{d^2y}{dt^2}}{\left(\frac{dx}{dt}\right)^3}
 \end{aligned}$$

YouTube: <https://youtu.be/HiM3Tvy8Mjo>

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