

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

나눗셈의 극한은 극한의 나눗셈이다.

(The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).)

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► Start

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