곱의 극한은 극한의 곱이다. (The limit of a product is the product of the limits.)



► Start

$$\lim_{x \to a} f(x) = L$$

► Start

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

▶ Start

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \}$$

▶ Start

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\epsilon > 0$$

$$|f(x)g(x) - LM|$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\epsilon > 0$$

$$|f(x)g(x) - LM| = |f(x)g(x) - f(x)M + f(x)M - LM|$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{array}{ll} |f(x)g(x)-LM| & = & |f(x)g(x)-f(x)M+f(x)M-LM| \\ & \leq & |f(x)g(x)-f(x)M|+|f(x)M-LM| \end{array}$$



Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{array}{lcl} |f(x)g(x)-LM| & = & |f(x)g(x)-f(x)M+f(x)M-LM| \\ & \leq & |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)} \end{array}$$



Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{array}{lcl} |f(x)g(x)-LM| & = & |f(x)g(x)-f(x)M+f(x)M-LM| \\ & \leq & |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ (\cdot} \cdot \text{ The Triangle Inequality)} \\ & = & |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \end{array}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

 $\epsilon > 0$

$$\begin{array}{lcl} |f(x)g(x)-LM| & = & |f(x)g(x)-f(x)M+f(x)M-LM| \\ \leq & |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ (\cdot` The Triangle Inequality)} \\ = & |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \\ \exists \delta_1>0 \end{array}$$

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▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

 $\epsilon > 0$

$$\begin{array}{lcl} |f(x)g(x)-LM| & = & |f(x)g(x)-f(x)M+f(x)M-LM| \\ & \leq & |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ (`.' The Triangle Inequality)} \\ & = & |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \end{array}$$

 $\exists \delta_1 > 0 \text{ s.t.}$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{array}{lll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM|\\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ (\cdot` The Triangle Inequality)}\\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \end{array}$$

$$\exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1$$



Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{array}{lll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM|\\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ (\cdot` The Triangle Inequality)}\\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \end{array}$$

$$\exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow$$



Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{array}{lll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM|\\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ (\cdot'$ The Triangle Inequality)}\\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML|\\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \end{array}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{array}{lcl} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM|\\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \ (\because \text{ The Triangle Inequality})\\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \end{array}$$

$$\exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \ (\because \lim_{x\to a}f(x)=L)$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{array}{lll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM|\\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \ (\because \text{ The Triangle Inequality})\\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \end{array}$$

$$\exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \ (\because \lim_{x\to a}f(x)=L) \ (\because \text{ The Triangle Inequality})$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

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Proof.

$$\begin{array}{lll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM|\\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ (\cdot` The Triangle Inequality)}\\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML|\\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1\text{ (\cdot` $\lim_{x\to a}f(x)=L$) (\cdot` The Triangle Inequality)}\\ |f(x)| \end{array}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{array}{lll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM|\\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)}\\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ ($\cdot\cdot$ } \lim_{x\to a}f(x)=L) \text{ ($\cdot\cdot$ The Triangle Inequality)}\\ |f(x)|=|f(x)-L+L| \end{array}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{array}{lll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM|\\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)}\\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ ($\cdot\cdot$ } \lim_{x\to a}f(x)=L) \text{ ($\cdot\cdot$ } \text{The Triangle Inequality)}\\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L| \end{array}$$

▶ Start

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$$\begin{array}{lll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM|\\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)}\\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ ($\cdot\cdot$ $\lim_{x\to a}f(x)=L$) ($\cdot\cdot$ $ The Triangle Inequality)}\\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L| \end{array}$$

▶ Start

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$$\begin{array}{lll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM|\\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ (\cdot'$ The Triangle Inequality)}\\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \text{ (\cdot'$ The Triangle Inequality)}\\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ (\cdot'$ $\lim_{x\to a}f(x)=L$) (\cdot'$ The Triangle Inequality)}\\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L|\\ \exists \delta_2>0 \end{array}$$

▶ Start

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$$\begin{array}{lll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM|\\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)}\\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \text{ ($\cdot\cdot$ The Triangle Inequality)}\\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ ($\cdot\cdot$ $\lim_{x\to a}f(x)=L$) ($\cdot\cdot$ The Triangle Inequality)}\\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L|\\ \exists \delta_2>0 \text{ s.t.} \end{array}$$

▶ Start

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$$\begin{array}{lll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM|\\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)}\\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \text{ ($\cdot\cdot$ The Triangle Inequality)}\\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ ($\cdot\cdot$ $\lim_{x\to a}f(x)=L$) ($\cdot\cdot$ The Triangle Inequality)}\\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L|\\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2 \end{array}$$

▶ Start

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Proof.

$$\begin{array}{lll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM| \\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ ($\cdot\cdot$ $\lim_{x\to a}f(x)=L$) ($\cdot\cdot$ The Triangle Inequality)} \\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L| \\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow \\ \end{array}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{array}{lll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM| \\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ ($\cdot\cdot$ $\lim_{x\to a}f(x)=L$) ($\cdot\cdot$ The Triangle Inequality)} \\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L| \\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow|g(x)-M|<\frac{\epsilon}{2(1+|L|)} \end{array}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{array}{lll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM| \\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ ($\cdot\cdot$ $\lim_{x\to a}f(x)=L$) ($\cdot\cdot$ The Triangle Inequality)} \\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L| \\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow|g(x)-M|<\frac{\epsilon}{2(1+|L|)} \text{ ($\cdot\cdot$ $\lim_{x\to a}g(x)=M$)} \end{array}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{array}{ll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM|\\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)}\\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \text{ ($\cdot\cdot$ The Triangle Inequality)}\\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ ($\cdot\cdot$ $\lim_{x\to a}f(x)=L$) ($\cdot\cdot$ The Triangle Inequality)}\\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L|\\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow|g(x)-M|<\frac{\epsilon}{2(1+|L|)} \text{ ($\cdot\cdot$ $\lim_{x\to a}g(x)=M$)}\\ \exists \delta_3>0 \end{array}$$

▶ Start

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$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

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$$\begin{array}{ll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM| \\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ ($\cdot\cdot$ $\lim_{x\to a}f(x)=L$) ($\cdot\cdot$ The Triangle Inequality)} \\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L|\\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow|g(x)-M|<\frac{\epsilon}{2(1+|L|)} \text{ ($\cdot\cdot$ $\lim_{x\to a}g(x)=M$)} \\ \exists \delta_3>0 \text{ s.t.} \end{array}$$

▶ Start

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$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

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Proof.

$$\begin{array}{lll} |f(x)g(x)-LM| & = & |f(x)g(x)-f(x)M+f(x)M-LM| \\ & \leq & |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ & = & |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ \exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < 1 \text{ ($\cdot\cdot$ $\lim_{x\to a} f(x)=L$) ($\cdot\cdot$ The Triangle Inequality)} \\ |f(x)| = |f(x)-L+L| \leq |f(x)-L|+|L| < 1+|L| \\ \exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \Rightarrow |g(x)-M| < \frac{\epsilon}{2(1+|L|)} \text{ ($\cdot\cdot$ $\lim_{x\to a} g(x)=M$)} \\ \exists \delta_3 > 0 \text{ s.t. } 0 < |x-a| < \delta_3 \end{array}$$

▶ Start

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$$\begin{array}{lll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM| \\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ ($\cdot\cdot$ $\lim_{x\to a}f(x)=L$) ($\cdot\cdot$ The Triangle Inequality)} \\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L|\\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow|g(x)-M|<\frac{\epsilon}{2(1+|L|)} \text{ ($\cdot\cdot$ $\lim_{x\to a}g(x)=M$)} \\ \exists \delta_3>0 \text{ s.t. } 0<|x-a|<\delta_3\Rightarrow \\ \end{array}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

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Proof.

$$\begin{array}{lll} |f(x)g(x)-LM| & = & |f(x)g(x)-f(x)M+f(x)M-LM| \\ & \leq & |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ (\cdot'$ The Triangle Inequality)} \\ & = & |f(x)|\cdot|g(x)-M|+|Mf(x)-LM| \text{ (\cdot'$ The Triangle Inequality)} \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ (\cdot'$ $\lim_{x\to a}f(x)=L$) (\cdot'$ The Triangle Inequality)} \\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L|\\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow|g(x)-M|<\frac{\epsilon}{2(1+|L|)} \text{ (\cdot'$ $\lim_{x\to a}g(x)=M$)} \\ \exists \delta_3>0 \text{ s.t. } 0<|x-a|<\delta_3\Rightarrow|Mf(x)-ML|<\frac{\epsilon}{2} \end{array}$$

→ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

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Proof.

$$\begin{array}{ll} |f(x)g(x)-LM| &=& |f(x)g(x)-f(x)M+f(x)M-LM| \\ &\leq& |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ (\cdot'$ The Triangle Inequality)} \\ &=& |f(x)|\cdot|g(x)-M|+|Mf(x)-LM| \text{ (\cdot'$ The Triangle Inequality)} \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ (\cdot'$ $\lim_{x\to a}f(x)=L)$ (\cdot'$ The Triangle Inequality)} \\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L|\\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow|g(x)-M|<\frac{\epsilon}{2(1+|L|)} \text{ (\cdot'$ $\lim_{x\to a}g(x)=M)$} \\ \exists \delta_3>0 \text{ s.t. } 0<|x-a|<\delta_3\Rightarrow|Mf(x)-ML|<\frac{\epsilon}{2} \text{ (\cdot'$ $\lim_{x\to a}Mf(x)=ML)$} \end{array}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{split} |f(x)g(x)-LM| &= |f(x)g(x)-f(x)M+f(x)M-LM| \\ &\leq |f(x)g(x)-f(x)M|+|f(x)M-LM| \ (\because \text{ The Triangle Inequality}) \\ &= |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \ (\because \text{ The Triangle Inequality}) \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \ (\because \lim_{x\to a}f(x)=L) \ (\because \text{ The Triangle Inequality}) \\ |f(x)|&=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L| \\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow|g(x)-M|<\frac{1}{2(1+|L|)} \ (\because \lim_{x\to a}g(x)=M) \\ \exists \delta_3>0 \text{ s.t. } 0<|x-a|<\delta_3\Rightarrow|Mf(x)-ML|<\frac{\epsilon}{2} \ (\because \lim_{x\to a}Mf(x)=ML) \\ \delta \end{split}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{split} |f(x)g(x)-LM| &= |f(x)g(x)-f(x)M+f(x)M-LM| \\ &\leq |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ (\cdot The Triangle Inequality)} \\ &= |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \text{ (\cdot The Triangle Inequality)} \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ (\cdot $\limbda_{x\to a} f(x)=L$) (\cdot \cdot The Triangle Inequality)} \\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L| \\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow|g(x)-M|<\frac{\epsilon}{2(1+|L|)} \text{ (\cdot $\limbda_{x\to a} g(x)=M$)} \\ \exists \delta_3>0 \text{ s.t. } 0<|x-a|<\delta_3\Rightarrow|Mf(x)-ML|<\frac{\epsilon}{2} \text{ (\cdot \cdot $\lim_{x\to a} Mf(x)=ML$)} \\ \delta=\min(\delta_1,\delta_2,\delta_3) \end{split}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{split} |f(x)g(x)-LM| &= |f(x)g(x)-f(x)M+f(x)M-LM| \\ &\leq |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ (\cdot The Triangle Inequality)} \\ &= |f(x)| \cdot |g(x)-M|+|Mf(x)-ML| \text{ (\cdot The Triangle Inequality)} \\ &= |f(x)| \cdot |g(x)-M|+|Mf(x)-ML| \text{ (\cdot \cdot $im} f(x)=L) \text{ ($\cdot$ The Triangle Inequality)} \\ |f(x)| &= |f(x)-L+L| \leq |f(x)-L|+|L| < 1+|L| \\ &\exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \Rightarrow |g(x)-M| < \frac{\epsilon}{2(1+|L|)} \text{ (\cdot \cdot $\lim_{x\to a} g(x)=M)} \\ &\exists \delta_3 > 0 \text{ s.t. } 0 < |x-a| < \delta_3 \Rightarrow |Mf(x)-ML| < \frac{\epsilon}{2} \text{ (\cdot \cdot $\lim_{x\to a} Mf(x)=ML)} \\ &\delta = \min(\delta_1,\delta_2,\delta_3) \\ &0 < |x-a| < \delta \end{split}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{split} |f(x)g(x)-LM| &= |f(x)g(x)-f(x)M+f(x)M-LM| \\ &\leq |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ &= |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ ($\cdot\cdot$ $\lim_{x\to a}f(x)=L$) ($\cdot\cdot$ The Triangle Inequality)} \\ |f(x)|&=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L|\\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow|g(x)-M|<\frac{\epsilon}{2(1+|L|)} \text{ ($\cdot\cdot$ $\lim_{x\to a}g(x)=M$)} \\ \exists \delta_3>0 \text{ s.t. } 0<|x-a|<\delta_3\Rightarrow|Mf(x)-ML|<\frac{\epsilon}{2} \text{ ($\cdot\cdot$ $\lim_{x\to a}Mf(x)=ML$)} \\ \delta=\min(\delta_1,\delta_2,\delta_3) \\ 0<|x-a|<\delta\Rightarrow \end{split}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{array}{ll} |f(x)g(x)-LM|&=&|f(x)g(x)-f(x)M+f(x)M-LM|\\ &\leq&|f(x)g(x)-f(x)M|+|f(x)M-LM| \ (\because \text{ The Triangle Inequality})\\ &=&|f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \ (\because \text{ The Triangle Inequality})\\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \ (\because \lim_{x\to a}f(x)=L) \ (\because \text{ The Triangle Inequality})\\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L|\\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow|g(x)-M|<\frac{\epsilon}{2(1+|L|)} \ (\because \lim_{x\to a}g(x)=M)\\ \exists \delta_3>0 \text{ s.t. } 0<|x-a|<\delta_3\Rightarrow|Mf(x)-ML|<\frac{\epsilon}{2} \ (\because \lim_{x\to a}Mf(x)=ML)\\ \delta=\min(\delta_1,\delta_2,\delta_3)\\ 0<|x-a|<\delta\Rightarrow|f(x)g(x)-LM|<\epsilon \end{array}$$

Start |

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

$$\epsilon > 0$$

$$\begin{array}{ll} |f(x)g(x)-LM|&=&|f(x)g(x)-f(x)M+f(x)M-LM|\\ &\leq&|f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)}\\ &=&|f(x)|\cdot|g(x)-M|+|Mf(x)-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)}\\ \exists \delta_1>0\text{ s.t. }0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1\text{ ($\cdot\cdot$ $\lim_{x\to a}f(x)=L$) ($\cdot\cdot$ The Triangle Inequality)}\\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L|\\ \exists \delta_2>0\text{ s.t. }0<|x-a|<\delta_2\Rightarrow|g(x)-M|<\frac{\epsilon}{2(1+|L|)}\text{ ($\cdot\cdot$ $\lim_{x\to a}g(x)=M$)}\\ \exists \delta_3>0\text{ s.t. }0<|x-a|<\delta_3\Rightarrow|Mf(x)-ML|<\frac{\epsilon}{2}\text{ ($\cdot\cdot$ $\lim_{x\to a}Mf(x)=ML$)}\\ \delta=\min(\delta_1,\delta_2,\delta_3)\\ 0<|x-a|<\delta\Rightarrow|f(x)g(x)-LM|<\epsilon\\ \end{array}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

$$\epsilon > 0$$

$$|f(x)g(x)-LM| = |f(x)g(x)-f(x)M+f(x)M-LM| \leq |f(x)g(x)-f(x)M| + |f(x)M-LM| \text{ (\cdot The Triangle Inequality)} \\ \leq |f(x)g(x)-f(x)M| + |f(x)M-LM| \text{ (\cdot The Triangle Inequality)} \\ = |f(x)| \cdot |g(x)-M| + |Mf(x)-ML| \text{ (\cdot The Triangle Inequality)} \\ |\exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow |f(x)-L|<1 \text{ (\cdot $\lim_{x\to a} f(x)=L$) (\cdot The Triangle Inequality)} \\ |f(x)| = |f(x)-L+L| \leq |f(x)-L|+|L|<1+|L| \\ |\exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow |g(x)-M|<\frac{\epsilon}{2(1+|L|)} \text{ (\cdot $\lim_{x\to a} g(x)=M$)} \\ |\exists \delta_3>0 \text{ s.t. } 0<|x-a|<\delta_3\Rightarrow |Mf(x)-ML|<\frac{\epsilon}{2} \text{ (\cdot \cdot $x\to a} Mf(x)=ML$)} \\ \delta=\min(\delta_1,\delta_2,\delta_3) \\ 0<|x-a|<\delta\Rightarrow |f(x)g(x)-LM|<\epsilon \\ |\cdot \forall \epsilon>0$$

Start |

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

$$\epsilon > 0$$

$$|f(x)g(x)-LM| = |f(x)g(x)-f(x)M+f(x)M-LM| \\ \leq |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ (\cdot'$ The Triangle Inequality)} \\ = |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \text{ (\cdot'$ The Triangle Inequality)} \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ (\cdot'$ $\lim_{x\to a}f(x)=L$) (\cdot'$ The Triangle Inequality)} \\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L|\\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow|g(x)-M|<\frac{\epsilon}{2(1+|L|)} \text{ (\cdot'$ $\lim_{x\to a}g(x)=M$)} \\ \exists \delta_3>0 \text{ s.t. } 0<|x-a|<\delta_3\Rightarrow|Mf(x)-ML|<\frac{\epsilon}{2} \text{ (\cdot'$ $\lim_{x\to a}Mf(x)=ML$)} \\ \delta=\min(\delta_1,\delta_2,\delta_3) \\ 0<|x-a|<\delta\Rightarrow|f(x)g(x)-LM|<\epsilon \\ \vdots \forall \epsilon>0,\exists \delta>0$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

$$\epsilon > 0$$

$$\begin{split} |f(x)g(x)-LM| &= |f(x)g(x)-f(x)M+f(x)M-LM| \\ &\leq |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ &= |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ ($\cdot\cdot$ $\lim_{x\to a}f(x)=L$) ($\cdot\cdot$ The Triangle Inequality)} \\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L|\\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow|g(x)-M|<\frac{\epsilon}{2(1+|L|)} \text{ ($\cdot\cdot$ $\lim_{x\to a}g(x)=M$)} \\ \exists \delta_3>0 \text{ s.t. } 0<|x-a|<\delta_3\Rightarrow|Mf(x)-ML|<\frac{\epsilon}{2} \text{ ($\cdot\cdot$ $\lim_{x\to a}Mf(x)=ML$)} \\ \delta=\min(\delta_1,\delta_2,\delta_3) \\ 0<|x-a|<\delta\Rightarrow|f(x)g(x)-LM|<\epsilon \\ \vdots \forall \epsilon>0 \text{ s.t. } \delta>0 \text{ s.t.} \end{split}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

$$\epsilon > 0$$

$$\begin{split} |f(x)g(x)-LM| &= |f(x)g(x)-f(x)M+f(x)M-LM| \\ &\leq |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ (\cdot The Triangle Inequality)} \\ &= |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \text{ (\cdot The Triangle Inequality)} \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ (\cdot $\lim_{x\to a}f(x)=L$) (\cdot The Triangle Inequality)} \\ |f(x)|&=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L|\\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow|g(x)-M|<\frac{\epsilon}{2(1+|L|)} \text{ (\cdot $\lim_{x\to a}g(x)=M$)} \\ \exists \delta_3>0 \text{ s.t. } 0<|x-a|<\delta_3\Rightarrow|Mf(x)-ML|<\frac{\epsilon}{2} \text{ (\cdot $\lim_{x\to a}Mf(x)=ML$)} \\ \delta=\min(\delta_1,\delta_2,\delta_3) \\ 0<|x-a|<\delta\Rightarrow|f(x)g(x)-LM|<\epsilon \\ \therefore \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a|<\delta \end{split}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$\begin{split} |f(x)g(x)-LM| &= |f(x)g(x)-f(x)M+f(x)M-LM| \\ &\leq |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ &= |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \text{ ($\cdot\cdot$ The Triangle Inequality)} \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ ($\cdot\cdot$ $\lim_{x\to a}f(x)=L$) ($\cdot\cdot$ The Triangle Inequality)} \\ |f(x)|&=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L|\\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow|g(x)-M|<\frac{\epsilon}{2(1+|L|)} \text{ ($\cdot\cdot$ $\lim_{x\to a}g(x)=M$)} \\ \exists \delta_3>0 \text{ s.t. } 0<|x-a|<\delta_3\Rightarrow|Mf(x)-ML|<\frac{\epsilon}{2} \text{ ($\cdot\cdot$ $\lim_{x\to a}Mf(x)=ML$)} \\ \delta=\min(\delta_1,\delta_2,\delta_3) \\ 0<|x-a|<\delta\Rightarrow|f(x)g(x)-LM|<\epsilon \\ \therefore \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a|<\delta\Rightarrow \end{split}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

$$\epsilon > 0$$

$$|f(x)g(x)-LM| = |f(x)g(x)-f(x)M+f(x)M-LM| \\ \leq |f(x)g(x)-f(x)M|+|f(x)M-LM| \text{ (\cdot'$ The Triangle Inequality)} \\ = |f(x)|\cdot|g(x)-M|+|Mf(x)-ML| \text{ (\cdot'$ The Triangle Inequality)} \\ \exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow|f(x)-L|<1 \text{ (\cdot'$ $\lim_{x\to a}f(x)=L$) (\cdot'$ The Triangle Inequality)} \\ |f(x)|=|f(x)-L+L|\leq|f(x)-L|+|L|<1+|L|\\ \exists \delta_2>0 \text{ s.t. } 0<|x-a|<\delta_2\Rightarrow|g(x)-M|<\frac{\epsilon}{2(1+|L|)} \text{ (\cdot'$ $\lim_{x\to a}g(x)=M$)} \\ \exists \delta_3>0 \text{ s.t. } 0<|x-a|<\delta_3\Rightarrow|Mf(x)-ML|<\frac{\epsilon}{2} \text{ (\cdot'$ $\lim_{x\to a}Mf(x)=ML$)} \\ \delta=\min(\delta_1,\delta_2,\delta_3) \\ 0<|x-a|<\delta\Rightarrow|f(x)g(x)-LM|<\epsilon \\ \therefore \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a|<\delta\Rightarrow|f(x)g(x)-LM|<\epsilon \\ \end{cases}$$

▶ Start

Theorem

$$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a} \{ f(x)g(x) \} = LM$$

Proof.

$$|f(x)g(x)-LM| = |f(x)g(x)-f(x)M+f(x)M-LM| \leq |f(x)g(x)-f(x)M| + |f(x)M-LM| \text{ (\cdot The Triangle Inequality)} \\ = |f(x)| \cdot |g(x)-M| + |f(x)M-LM| \text{ (\cdot The Triangle Inequality)} \\ = |f(x)| \cdot |g(x)-M| + |Mf(x)-ML| \text{ (\cdot The Triangle Inequality)} \\ |\exists \delta_1>0 \text{ s.t. } 0<|x-a| < \delta_1\Rightarrow |f(x)-L| < 1 \text{ (\cdot $\lim_{x\to a} f(x)=L$) (\cdot The Triangle Inequality)} \\ |f(x)| = |f(x)-L+L| \leq |f(x)-L| + |L| < 1 + |L| \\ |\exists \delta_2>0 \text{ s.t. } 0<|x-a| < \delta_2\Rightarrow |g(x)-M| < \frac{\epsilon}{2(1+|L|)} \text{ (\cdot $\lim_{x\to a} g(x)=M$)} \\ |\exists \delta_3>0 \text{ s.t. } 0<|x-a| < \delta_3\Rightarrow |Mf(x)-ML| < \frac{\epsilon}{2} \text{ (\cdot $\lim_{x\to a} Mf(x)=ML$)} \\ \delta=\min(\delta_1,\delta_2,\delta_3) \\ 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| < \epsilon \\ |\cdot \forall \epsilon>0, \exists \delta>0 \text{ s.t. } 0<|x-a| < \delta\Rightarrow |f(x)g(x)-LM| <$$

