$$\int \sec x \; dx$$

$\sec x$

$$\sec x \quad =$$

$$\sec x = \frac{1}{\cos x}$$

$$\sec x = \frac{1}{\cos x} = \frac{\cos x}{\cos^2 x}$$

$$\sec x \quad = \quad \frac{1}{\cos x} = \frac{\cos x}{\cos^2 x} = \frac{\cos x}{1 - \sin^2 x}$$

$$\sec x \quad = \quad \frac{1}{\cos x} = \frac{\cos x}{\cos^2 x} = \frac{\cos x}{1 - \sin^2 x} = \frac{\cos x}{(1 - \sin x)(1 + \sin x)}$$

$$\begin{aligned}
\sec x &= \frac{1}{\cos x} = \frac{\cos x}{\cos^2 x} = \frac{\cos x}{1-\sin^2 x} = \frac{\cos x}{(1-\sin x)(1+\sin x)} \\
&= \frac{1}{2}\left(\frac{\cos x}{1-\sin x} + \frac{\cos x}{1+\sin x}\right)
\end{aligned}$$

$$\begin{aligned}\sec x &= \frac{1}{\cos x} = \frac{\cos x}{\cos^2 x} = \frac{\cos x}{1-\sin^2 x} = \frac{\cos x}{(1-\sin x)(1+\sin x)} \\ &= \frac{1}{2}\left(\frac{\cos x}{1-\sin x} + \frac{\cos x}{1+\sin x}\right)\end{aligned}$$

$$\int \sec x \, dx$$

$$\begin{aligned}\sec x &= \frac{1}{\cos x} = \frac{\cos x}{\cos^2 x} = \frac{\cos x}{1-\sin^2 x} = \frac{\cos x}{(1-\sin x)(1+\sin x)} \\ &= \frac{1}{2}\left(\frac{\cos x}{1-\sin x} + \frac{\cos x}{1+\sin x}\right)\end{aligned}$$

$$\int \sec x \, dx = \int \frac{1}{2}\left(\frac{\cos x}{1-\sin x} + \frac{\cos x}{1+\sin x}\right) dx$$

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$$\begin{aligned}\int \sec x \, dx &= \int \frac{1}{2}\left(\frac{\cos x}{1-\sin x} + \frac{\cos x}{1+\sin x}\right) dx \\ &= \frac{1}{2}\{-\ln(1-\sin x) + \ln(1+\sin x)\} + c\end{aligned}$$

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&= \frac{1}{2}\{-\ln(1-\sin x) + \ln(1+\sin x)\} + c \\
&= \frac{1}{2}\ln\left(\frac{1+\sin x}{1-\sin x}\right) + c
\end{aligned}$$

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&= \frac{1}{2}\{-\ln(1-\sin x) + \ln(1+\sin x)\} + c \\
&= \frac{1}{2}\ln\left(\frac{1+\sin x}{1-\sin x}\right) + c = \frac{1}{2}\ln\left\{\frac{(1+\sin x)^2}{1-\sin^2 x}\right\} + c
\end{aligned}$$

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&= \frac{1}{2}\ln\left\{\frac{(1+\sin x)^2}{\cos^2 x}\right\} + c = \frac{1}{2}\ln\left(\frac{1+\sin x}{\cos x}\right)^2 + c
\end{aligned}$$

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&= \frac{1}{2}\ln\left\{\frac{(1+\sin x)^2}{\cos^2 x}\right\} + c = \frac{1}{2}\ln\left(\frac{1+\sin x}{\cos x}\right)^2 + c \\
&= \frac{1}{2}\ln(\sec x + \tan x)^2 + c
\end{aligned}$$

$$
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$$

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\int \sec x \, dx &= \int \frac{1}{2}\left(\frac{\cos x}{1-\sin x} + \frac{\cos x}{1+\sin x}\right) dx \\
&= \frac{1}{2}\left\{-\ln(1-\sin x) + \ln(1+\sin x)\right\} + c \\
&= \frac{1}{2}\ln\left(\frac{1+\sin x}{1-\sin x}\right) + c = \frac{1}{2}\ln\left\{\frac{(1+\sin x)^2}{1-\sin^2 x}\right\} + c \\
&= \frac{1}{2}\ln\left\{\frac{(1+\sin x)^2}{\cos^2 x}\right\} + c = \frac{1}{2}\ln\left(\frac{1+\sin x}{\cos x}\right)^2 + c \\
&= \frac{1}{2}\ln(\sec x + \tan x)^2 + c = \ln|\sec x + \tan x| + c
\end{aligned}
$$

$$\therefore \int \sec x \, dx = \ln|\sec x + \tan x| + c$$

END