

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

나눗셈의 극한은 극한의 나눗셈이다.

(The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).)

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\}$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right|$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right|$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t.}$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2}$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M|$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)|$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)|$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t.}$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$\delta$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$\delta = \min\{\delta_1, \delta_2\}$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$\delta = \min\{\delta_1, \delta_2\} \quad \therefore$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$\delta = \min\{\delta_1, \delta_2\} \quad \therefore \forall \epsilon > 0$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$\delta = \min\{\delta_1, \delta_2\} \quad \therefore \forall \epsilon > 0, \exists \delta > 0$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$\delta = \min\{\delta_1, \delta_2\} \quad \therefore \forall \epsilon > 0, \exists \delta > 0 \text{ s.t.}$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$\delta = \min\{\delta_1, \delta_2\} \quad \therefore \forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$\delta = \min\{\delta_1, \delta_2\} \quad \therefore \forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \Rightarrow$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$\delta = \min\{\delta_1, \delta_2\} \quad \therefore \forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \Rightarrow \left| \frac{1}{g(x)} - \frac{1}{M} \right| < \epsilon$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$\delta = \min\{\delta_1, \delta_2\} \quad \therefore \forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \Rightarrow \left| \frac{1}{g(x)} - \frac{1}{M} \right| < \epsilon$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\}$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$\delta = \min\{\delta_1, \delta_2\} \quad \therefore \forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \Rightarrow \left| \frac{1}{g(x)} - \frac{1}{M} \right| < \epsilon$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\}$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$\delta = \min\{\delta_1, \delta_2\} \quad \therefore \forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \Rightarrow \left| \frac{1}{g(x)} - \frac{1}{M} \right| < \epsilon$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \lim_{x \rightarrow a} \left\{ f(x) \cdot \frac{1}{g(x)} \right\}$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$\delta = \min\{\delta_1, \delta_2\} \quad \therefore \forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \Rightarrow \left| \frac{1}{g(x)} - \frac{1}{M} \right| < \epsilon$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \lim_{x \rightarrow a} \left\{ f(x) \cdot \frac{1}{g(x)} \right\} = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\}$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$\delta = \min\{\delta_1, \delta_2\} \quad \therefore \forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \Rightarrow \left| \frac{1}{g(x)} - \frac{1}{M} \right| < \epsilon$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \lim_{x \rightarrow a} \left\{ f(x) \cdot \frac{1}{g(x)} \right\} = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = L \cdot \frac{1}{M}$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$\delta = \min\{\delta_1, \delta_2\} \quad \therefore \forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \Rightarrow \left| \frac{1}{g(x)} - \frac{1}{M} \right| < \epsilon$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \lim_{x \rightarrow a} \left\{ f(x) \cdot \frac{1}{g(x)} \right\} = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = L \cdot \frac{1}{M} = \frac{L}{M}$$

The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M (\neq 0)$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{L}{M}$$

Proof.

$$\text{Show that } \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = \frac{1}{M}$$

$$\epsilon > 0$$

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M - g(x)}{Mg(x)} \right| = \frac{1}{|Mg(x)|} |g(x) - M|$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < \frac{|M|}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$|M| = |M - g(x) + g(x)| \leq |g(x) - M| + |g(x)| < \frac{|M|}{2} + |g(x)|, \quad \frac{|M|}{2} < |g(x)|, \quad \frac{1}{|g(x)|} < \frac{2}{|M|}$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{M^2}{2} \epsilon \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$\delta = \min\{\delta_1, \delta_2\} \quad \therefore \forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \Rightarrow \left| \frac{1}{g(x)} - \frac{1}{M} \right| < \epsilon$$

$$\lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \lim_{x \rightarrow a} \left\{ \frac{f(x)}{g(x)} \right\} = \lim_{x \rightarrow a} \left\{ f(x) \cdot \frac{1}{g(x)} \right\} = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} \left\{ \frac{1}{g(x)} \right\} = L \cdot \frac{1}{M} = \frac{L}{M}$$



The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

END