양수에 대한 산술, 기하, 조화평균에 대한 부등식
(Inequality of Arithmetic and Geometric and Harmonic Means for *n* positive real numbers)

▸ Start ▸ End

▸ Start ▸ End $\quad x_1 > 0, \ldots, x_n > 0$

▸ Start ▸ End $x_1 > 0, \ldots, x_n > 0$

$$\frac{x_1 + \cdots + x_n}{n}$$

▸ Start ▸ End $x_1 > 0, \ldots, x_n > 0$

$$\frac{x_1 + \cdots + x_n}{n} \geq$$

▸ Start ▸ End $x_1 > 0, \ldots, x_n > 0$

$$\frac{x_1 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 \times \cdots \times x_n}$$

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▸ Start   ▸ End $x_1 > 0, \ldots, x_n > 0$

$$\frac{x_1 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 \times \cdots \times x_n} \geq \frac{n}{\dfrac{1}{x_1} + \cdots + \dfrac{1}{x_n}}$$

▸ Start ▸ End $x_1 > 0, \ldots, x_n > 0$

$$\frac{x_1 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 \times \cdots \times x_n} \geq \frac{n}{\dfrac{1}{x_1} + \cdots + \dfrac{1}{x_n}}$$

▸ Proof of first inequality

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  If $n = 2$.

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  $$\left(\frac{x_1 + x_2}{2}\right)^2 - \left(\sqrt{x_1 x_2}\right)^2$$

Inequality of Arithmetic and Geometric and Harmonic Means for n positive real numbers

- Proof of first inequality
  If $n = 2$.
  $$\left(\frac{x_1 + x_2}{2}\right)^2 - \left(\sqrt{x_1 x_2}\right)^2 \quad = \quad \frac{x_1^2 + 2x_1 x_2 + x_2^2}{4} - x_1 x_2$$

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$$= \frac{x_1^2 - 2x_1 x_2 + x_2^2}{4}$$

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$$= \frac{x_1^2 - 2x_1 x_2 + x_2^2}{4}$$

$$= \frac{(x_1 - x_2)^2}{4}$$

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$$\therefore \frac{x_1 + x_2}{2}$$

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Inequality of Arithmetic and Geometric and Harmonic Means for n positive real numbers

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$$n = 2 \text{ is true.}$$

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Inequality of Arithmetic and Geometric and Harmonic Means for n positive real numbers

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  Assume $n = 2^{k-1}$ is true.

  $$\frac{x_1 + \cdots + x_{2^k}}{2^k} \quad = \quad \frac{\dfrac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + }{}$$

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  $$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2}$$

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$$\geq \sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}$$

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  $$\geq \quad \sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}$$

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  $$\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2}$$

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$$\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2}$$

$$\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} \, \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}$$

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$$\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2}$$

$$\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} \; \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}$$

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$$\therefore \frac{x_1 + \cdots + x_{2^k}}{2^k}$$

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$$\therefore \frac{x_1 + \cdots + x_{2^k}}{2^k} \geq \sqrt[2^k]{x_1 \cdots x_{2^k}}$$

$$n = 2^k \text{ is true.}$$

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  Let $m = 2^l$

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Inequality of Arithmetic and Geometric and Harmonic Means for n positive real numbers

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$$\alpha \quad = \quad \frac{x_1 + \cdots + x_n}{n}$$

Inequality of Arithmetic and Geometric and Harmonic Means for n positive real numbers

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$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{m}{n}(x_1 + \cdots + x_n)$$

Inequality of Arithmetic and Geometric and Harmonic Means for n positive real numbers

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  Let $m = 2^l$ such that $n < m$.

  $$\alpha \;=\; \frac{x_1 + \cdots + x_n}{n} = \frac{\dfrac{m}{n}(x_1 + \cdots + x_n)}{m}$$

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  $$\begin{aligned} \alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\ &= x_1 + \cdots + x_n + \frac{m-n}{n} \end{aligned}$$

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&= x_1 + \cdots + x_n + \frac{m-n}{n}(x_1 + \cdots + x_n)
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&= x_1 + \cdots + x_n +
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Inequality of Arithmetic and Geometric and Harmonic Means for n positive real numbers

- Proof of first inequality

  Let $m = 2^l$ such that $n < m$.

$$
\begin{aligned}
\alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\dfrac{m}{n}(x_1 + \cdots + x_n)}{m} \\
&= \frac{x_1 + \cdots + x_n + \dfrac{m-n}{n}(x_1 + \cdots + x_n)}{m} \\
&= x_1 + \cdots + x_n + (m - n)\alpha
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  $$\alpha^m \geq x_1 \cdots x_n \alpha^{m-n} \ ,$$

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\alpha^m \geq x_1 \cdots x_n \alpha^{m-n} \ , \quad \alpha^n \geq x_1 \cdots x_n \ ,
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\alpha^m \geq x_1 \cdots x_n \alpha^{m-n} \ , \ \ \alpha^n \geq x_1 \cdots x_n \ , \ \ \alpha \geq \sqrt[n]{x_1 \cdots x_n}
$$

$$
\therefore \frac{x_1 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 \cdots x_n} \quad (n \text{ is true.})
$$

- Proof of second inequality

Inequality of Arithmetic and Geometric and Harmonic Means for n positive real numbers

- Proof of second inequality

$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n}$$

- Proof of second inequality

$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq$$

Inequality of Arithmetic and Geometric and Harmonic Means for n positive real numbers

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Inequality of Arithmetic and Geometric and Harmonic Means for *n* positive real numbers

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$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1} \cdots \frac{1}{x_n}}$$

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$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1} \cdots \frac{1}{x_n}}$$

$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \frac{1}{\sqrt[n]{x_1 \cdots x_n}}$$

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$$\therefore \sqrt[n]{x_1 \times \cdots \times x_n} \geq$$

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Github:
https://min7014.github.io/math20191104001.html

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