$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

$$\left[f(x) \le g(x)(0 < |x - a| < \delta_0) , \lim_{x \to a} f(x) = L , \lim_{x \to a} g(x) = M \right] \Rightarrow L \le M$$

$$\left[f(x) \le g(x)(0 < |x - a| < \delta_0) , \lim_{x \to a} f(x) = L , \lim_{x \to a} g(x) = M \right] \Rightarrow L \le M$$

► Start

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

$$f(x) \leq g(x) \ (0 < |x-a| < \delta_0)$$

$$\left[f(x) \le g(x)(0 < |x - a| < \delta_0) , \lim_{x \to a} f(x) = L , \lim_{x \to a} g(x) = M \right] \Rightarrow L \le M$$

$$f(x) \le g(x) \ (0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L$$

$$\left[f(x) \le g(x)(0 < |x - a| < \delta_0) , \lim_{x \to a} f(x) = L , \lim_{x \to a} g(x) = M \right] \Rightarrow L \le M$$

$$f(x) \le g(x) (0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

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$$f(x) \le g(x) \ (0 < |x-a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
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Theorem

$$f(x) \le g(x) \ (0 < |x-a| < \delta_0), \\ \lim_{x \to a} f(x) = L, \\ \lim_{x \to a} g(x) = M$$

$$L \le M$$

Proof.

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
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$$f(x) \le g(x) \ (0 < |x-a| < \delta_0), \\ \lim_{x \to a} f(x) = L, \\ \lim_{x \to a} g(x) = M$$

$$L \le M$$

Proof.

$$\exists \delta_1 > 0$$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

Theorem

$$f(x) \le g(x) \ (0 < |x-a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$L \le M$$

Proof.

 $\epsilon > 0$

 $\exists \delta_1 > 0 \text{ s.t.}$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
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Theorem

$$f(x) \le g(x) \ (0 < |x-a| < \delta_0), \\ \lim_{x \to a} f(x) = L, \\ \lim_{x \to a} g(x) = M$$

$$L \le M$$

Proof.

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1$$

$$\left[f(x) \le g(x)(0 < |x - a| < \delta_0) , \lim_{x \to a} f(x) = L , \lim_{x \to a} g(x) = M \right] \Rightarrow L \le M$$

Theorem

$$f(x) \le g(x) \ (0 < |x-a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$L \le M$$

Proof.

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow$$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

Theorem

$$f(x) \leq g(x) \; (0 < |x-a| < \delta_0), \\ \lim_{x \to a} f(x) = L, \\ \lim_{x \to a} g(x) = M$$

$$L \leq M$$

Proof.

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |f(x) - L| < \frac{\epsilon}{2}$$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

Theorem

$$f(x) \leq g(x) \; (0 < |x-a| < \delta_0), \\ \lim_{x \to a} f(x) = L, \\ \lim_{x \to a} g(x) = M$$

$$L \leq M$$

Proof.

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |f(x) - L| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} f(x) = M)$$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

Theorem

$$f(x) \le g(x) (0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

 $L \le M$

Proof.

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} f(x) = M) \ , \ L - \frac{\epsilon}{2} < f(x) < L + \frac{\epsilon}{2}$$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
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Theorem

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Proof.

$$\exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow |f(x)-L|<\frac{\epsilon}{2} \ (\because \lim_{x\to a}f(x)=M) \ , \ L-\frac{\epsilon}{2}< f(x)< L+\frac{\epsilon}{2} \ \exists \delta_2>0$$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
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Theorem

$$f(x) \le g(x) \ (0 < |x-a| < \delta_0), \\ \lim_{x \to a} f(x) = L, \\ \lim_{x \to a} g(x) = M$$

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$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
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Theorem

$$f(x) \le g(x) \ (0 < |x-a| < \delta_0), \\ \lim_{x \to a} f(x) = L, \\ \lim_{x \to a} g(x) = M$$

$$L \le M$$

Proof.

$$\begin{array}{l} \exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < \frac{\epsilon}{2} \text{ } (\because \lim_{x \to a} f(x) = M) \text{ , } L - \frac{\epsilon}{2} < f(x) < L + \frac{\epsilon}{2} \\ \exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \end{array}$$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

Theorem

$$f(x) \le g(x) \ (0 < |x-a| < \delta_0), \\ \lim_{x \to a} f(x) = L, \\ \lim_{x \to a} g(x) = M$$

$$L \le M$$

Proof.

$$\begin{array}{l} \exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < \frac{\epsilon}{2} \text{ } (\because \lim_{x \to a} f(x) = M) \text{ , } L - \frac{\epsilon}{2} < f(x) < L + \frac{\epsilon}{2} \\ \exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \Rightarrow \end{array}$$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

Theorem

$$f(x) \le g(x) \ (0 < |x-a| < \delta_0), \\ \lim_{x \to a} f(x) = L, \\ \lim_{x \to a} g(x) = M$$

$$L \le M$$

Proof.

$$\begin{array}{l} \exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} f(x) = M) \ , \ L - \frac{\epsilon}{2} < f(x) < L + \frac{\epsilon}{2} \\ \exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \Rightarrow |g(x)-M| < \frac{\epsilon}{2} \end{array}$$

$$\left[f(x) \le g(x)(0 < |x - a| < \delta_0) , \lim_{x \to a} f(x) = L , \lim_{x \to a} g(x) = M \right] \Rightarrow L \le M$$

Theorem

$$f(x) \le g(x) \ (0 < |x-a| < \delta_0), \\ \lim_{x \to a} f(x) = L, \\ \lim_{x \to a} g(x) = M$$

$$L \le M$$

Proof.

$$\begin{array}{l} \exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} f(x) = M) \ , \ L - \frac{\epsilon}{2} < f(x) < L + \frac{\epsilon}{2} \\ \exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \Rightarrow |g(x)-M| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} g(x) = M) \end{array}$$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

→ Start

Theorem

$$f(x) \le g(x) \ (0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

 $L \le M$

Proof.

$$\begin{array}{l} \exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < \frac{\epsilon}{2} \; (\because \lim_{x \to a} f(x) = M) \; , \; L - \frac{\epsilon}{2} < f(x) < L + \frac{\epsilon}{2} \\ \exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \Rightarrow |g(x)-M| < \frac{\epsilon}{2} \; (\because \lim_{x \to a} g(x) = M) \; , \; M - \frac{\epsilon}{2} < g(x) < M + \frac{\epsilon}{2} \end{array}$$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

Theorem

$$f(x) \le g(x) \ (0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

 $L \le M$

Proof.

 $\epsilon > 0$

$$\begin{split} &\exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} f(x) = M) \ , \ L - \frac{\epsilon}{2} < f(x) < L + \frac{\epsilon}{2} \\ &\exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \Rightarrow |g(x)-M| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} g(x) = M) \ , \ M - \frac{\epsilon}{2} < g(x) < M + \frac{\epsilon}{2} \end{cases}$$

δ

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

Theorem

$$f(x) \le g(x) \ (0 < |x-a| < \delta_0), \\ \lim_{x \to a} f(x) = L, \\ \lim_{x \to a} g(x) = M$$

$$L \le M$$

Proof.

$$\begin{split} &\exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} f(x) = M) \ , \ L - \frac{\epsilon}{2} < f(x) < L + \frac{\epsilon}{2} \\ &\exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \Rightarrow |g(x)-M| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} g(x) = M) \ , \ M - \frac{\epsilon}{2} < g(x) < M + \frac{\epsilon}{2} \\ &\delta = \min\{\delta_0, \delta_1, \delta_2\} \end{split}$$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

Theorem

$$f(x) \le g(x) \ (0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

 $L \le M$

Proof.

$$\begin{split} &\exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < \frac{\epsilon}{2} \text{ } (\because \lim_{x \to a} f(x) = M) \text{ } , \text{ } L - \frac{\epsilon}{2} < f(x) < L + \frac{\epsilon}{2} \\ &\exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \Rightarrow |g(x)-M| < \frac{\epsilon}{2} \text{ } (\because \lim_{x \to a} g(x) = M) \text{ } , \text{ } M - \frac{\epsilon}{2} < g(x) < M + \frac{\epsilon}{2} \\ &\delta = \min\{\delta_0, \delta_1, \delta_2\} \end{split}$$

$$f(x) \le g(x)$$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

Theorem

$$f(x) \le g(x) \ (0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

 $L \le M$

Proof.

$$\begin{split} &\exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} f(x) = M) \ , \ L - \frac{\epsilon}{2} < f(x) < L + \frac{\epsilon}{2} \\ &\exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \Rightarrow |g(x)-M| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} g(x) = M) \ , \ M - \frac{\epsilon}{2} < g(x) < M + \frac{\epsilon}{2} \\ &\delta = \min\{\delta_0, \delta_1, \delta_2\} \end{split}$$

$$f(x) \le g(x)$$
 , $L - \frac{\epsilon}{2} < f(x) \le g(x) < M + \frac{\epsilon}{2}$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

Theorem

$$f(x) \le g(x) \ (0 < |x-a| < \delta_0), \\ \lim_{x \to a} f(x) = L, \\ \lim_{x \to a} g(x) = M$$

$$L \le M$$

Proof.

$$\begin{array}{l} \exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} f(x) = M) \ , \ L - \frac{\epsilon}{2} < f(x) < L + \frac{\epsilon}{2} \\ \exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \Rightarrow |g(x)-M| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} g(x) = M) \ , \ M - \frac{\epsilon}{2} < g(x) < M + \frac{\epsilon}{2} \end{array}$$

$$\delta = \min\{\delta_0, \delta_1, \delta_2\}$$

$$f(x) \leq g(x)$$
 , $L - \frac{\epsilon}{2} < f(x) \leq g(x) < M + \frac{\epsilon}{2}$, $L - \frac{\epsilon}{2} < M + \frac{\epsilon}{2}$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

Theorem

$$f(x) \le g(x) \ (0 < |x-a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$L \le M$$

Proof.

$$\begin{array}{l} \exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} f(x) = M) \ , \ L - \frac{\epsilon}{2} < f(x) < L + \frac{\epsilon}{2} \\ \exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \Rightarrow |g(x)-M| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} g(x) = M) \ , \ M - \frac{\epsilon}{2} < g(x) < M + \frac{\epsilon}{2} \end{array}$$

$$\delta = \min\{\delta_0, \delta_1, \delta_2\}$$

$$f(x) \leq g(x) \ , L - \tfrac{\epsilon}{2} < f(x) \leq g(x) < M + \tfrac{\epsilon}{2} \ , \ L - \tfrac{\epsilon}{2} < M + \tfrac{\epsilon}{2} \ , \ 0 < M - L + \epsilon$$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

Theorem

$$f(x) \le g(x) \ (0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

 $L \le M$

Proof.

$$\begin{array}{l} \exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} f(x) = M) \ , \ L - \frac{\epsilon}{2} < f(x) < L + \frac{\epsilon}{2} \\ \exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \Rightarrow |g(x)-M| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} g(x) = M) \ , \ M - \frac{\epsilon}{2} < g(x) < M + \frac{\epsilon}{2} \end{array}$$

$$\delta = \min\{\delta_0, \delta_1, \delta_2\}$$

$$f(x) \leq g(x)$$
 , $L - \frac{\epsilon}{2} < f(x) \leq g(x) < M + \frac{\epsilon}{2}$, $L - \frac{\epsilon}{2} < M + \frac{\epsilon}{2}$, $0 < M - L + \epsilon$

$$\forall \epsilon > 0$$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

Theorem

$$f(x) \le g(x) \ (0 < |x-a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$L \le M$$

Proof.

$$\begin{array}{l} \exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < \frac{\epsilon}{2} \; (\because \lim_{x \to a} f(x) = M) \; , \; L - \frac{\epsilon}{2} < f(x) < L + \frac{\epsilon}{2} \\ \exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \Rightarrow |g(x)-M| < \frac{\epsilon}{2} \; (\because \lim_{x \to a} g(x) = M) \; , \; M - \frac{\epsilon}{2} < g(x) < M + \frac{\epsilon}{2} \end{array}$$

$$\delta = \min\{\delta_0, \delta_1, \delta_2\}$$

$$f(x) \leq g(x) \ , L - \tfrac{\epsilon}{2} < f(x) \leq g(x) < M + \tfrac{\epsilon}{2} \ , \ L - \tfrac{\epsilon}{2} < M + \tfrac{\epsilon}{2} \ , \ 0 < M - L + \epsilon$$

$$\forall \epsilon > 0, 0 < \mathit{M} - \mathit{L} + \epsilon$$

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

Theorem

$$f(x) \le g(x) \ (0 < |x-a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

$$L \le M$$

Proof.

 $\epsilon > 0$

$$\begin{array}{l} \exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} f(x) = M) \ , \ L - \frac{\epsilon}{2} < f(x) < L + \frac{\epsilon}{2} \\ \exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \Rightarrow |g(x)-M| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} g(x) = M) \ , \ M - \frac{\epsilon}{2} < g(x) < M + \frac{\epsilon}{2} \end{array}$$

$$\delta = \min\{\delta_0, \delta_1, \delta_2\}$$

$$f(x) \leq g(x) \ , L - \tfrac{\epsilon}{2} < f(x) \leq g(x) < M + \tfrac{\epsilon}{2} \ , \ L - \tfrac{\epsilon}{2} < M + \tfrac{\epsilon}{2} \ , \ 0 < M - L + \epsilon$$

$$\forall \epsilon > 0, 0 < M - L + \epsilon$$

.

$$f(x) \le g(x)(0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$
 $\Rightarrow L \le M$

Theorem

$$f(x) \le g(x) \ (0 < |x - a| < \delta_0), \lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$$

 $L \le M$

Proof.

$$\begin{array}{l} \exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} f(x) = M) \ , \ L - \frac{\epsilon}{2} < f(x) < L + \frac{\epsilon}{2} \\ \exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \Rightarrow |g(x)-M| < \frac{\epsilon}{2} \ (\because \lim_{x \to a} g(x) = M) \ , \ M - \frac{\epsilon}{2} < g(x) < M + \frac{\epsilon}{2} \end{array}$$

$$\delta = \min\{\delta_0, \delta_1, \delta_2\}$$

$$f(x) \leq g(x) \ , L - \tfrac{\epsilon}{2} < f(x) \leq g(x) < M + \tfrac{\epsilon}{2} \ , \ L - \tfrac{\epsilon}{2} < M + \tfrac{\epsilon}{2} \ , \ 0 < M - L + \epsilon$$

$$\forall \epsilon > 0, 0 < \mathit{M} - \mathit{L} + \epsilon$$

$$\therefore L \leq M$$



$$\left[f(x) \le g(x)(0 < |x - a| < \delta_0) , \lim_{x \to a} f(x) = L , \lim_{x \to a} g(x) = M \right] \Rightarrow L \le M$$

→ Home