음에 아닌 수에 대한 산술, 기하, 조화평균에 대한 부등식

(Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers)

Start End

Start End $x_1 > 0, \ldots, x_n > 0$

Start End $x_1 > 0, \ldots, x_n > 0$

$$\frac{x_1 + \cdots + x_n}{n}$$

Start End $x_1 > 0, \ldots, x_n > 0$

$$\frac{x_1 + \cdots + x_n}{n} \geq$$

Start End $x_1 > 0, \ldots, x_n > 0$

$$\frac{x_1 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 \times \cdots \times x_n}$$

Start End $x_1 > 0, \ldots, x_n > 0$

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Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start End $x_1 > 0, \ldots, x_n > 0$

$$\frac{x_1 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 \times \cdots \times x_n} \geq \frac{n}{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}$$

Start End $x_1 > 0, \ldots, x_n > 0$

$$\frac{x_1 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 \times \cdots \times x_n} \geq \frac{n}{\dfrac{1}{x_1} + \cdots + \dfrac{1}{x_n}}$$

Proof of first inequality

Proof of second inequality

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Home Start End

- Proof of first inequality

Home Start End

- Proof of first inequality
  If $n = 2$.

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- Proof of first inequality

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  $$\left(\frac{x_1 + x_2}{2}\right)^2 - \left(\sqrt{x_1 x_2}\right)^2$$

Home Start End

- Proof of first inequality

If $n = 2$.

$$\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2 = \frac{x_1^2 + 2x_1x_2 + x_2^2}{4} - x_1x_2$$

Home Start End

- Proof of first inequality

If $n = 2$.

$$\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2 = \frac{x_1^2 + 2x_1x_2 + x_2^2}{4} - x_1x_2$$
$$= \frac{x_1^2 - 2x_1x_2 + x_2^2}{4}$$

Home Start End

- Proof of first inequality

If $n = 2$.

$$\begin{aligned}\left(\frac{x_1 + x_2}{2}\right)^2 - \left(\sqrt{x_1 x_2}\right)^2 &= \frac{x_1^2 + 2x_1x_2 + x_2^2}{4} - x_1x_2 \\ &= \frac{x_1^2 - 2x_1x_2 + x_2^2}{4} \\ &= \frac{(x_1 - x_2)^2}{4}\end{aligned}$$

▸ Home ▸ Start ▸ End

- Proof of first inequality

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$$\begin{aligned}\left(\frac{x_1 + x_2}{2}\right)^2 - \left(\sqrt{x_1 x_2}\right)^2 &= \frac{x_1^2 + 2x_1 x_2 + x_2^2}{4} - x_1 x_2 \\ &= \frac{x_1^2 - 2x_1 x_2 + x_2^2}{4} \\ &= \frac{(x_1 - x_2)^2}{4} \geq \end{aligned}$$

Home Start End

- Proof of first inequality

If $n = 2$.

$$\begin{aligned}
\left(\frac{x_1 + x_2}{2}\right)^2 - \left(\sqrt{x_1 x_2}\right)^2 &= \frac{x_1^2 + 2x_1x_2 + x_2^2}{4} - x_1x_2 \\
&= \frac{x_1^2 - 2x_1x_2 + x_2^2}{4} \\
&= \frac{(x_1 - x_2)^2}{4} \geq 0
\end{aligned}$$

Home Start End

- Proof of first inequality

If $n = 2$.

$$\begin{aligned}\left(\frac{x_1+x_2}{2}\right)^2-\left(\sqrt{x_1x_2}\right)^2 &= \frac{x_1^2+2x_1x_2+x_2^2}{4}-x_1x_2\\ &= \frac{x_1^2-2x_1x_2+x_2^2}{4}\\ &= \frac{(x_1-x_2)^2}{4}\geq 0\end{aligned}$$

$$\therefore \frac{x_1+x_2}{2}$$

Home Start End

- Proof of first inequality

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$$\therefore \frac{x_1 + x_2}{2} \geq$$

Home Start End

- Proof of first inequality

If $n = 2$.

$$\begin{aligned}\left(\frac{x_1+x_2}{2}\right)^2-\left(\sqrt{x_1x_2}\right)^2 &= \frac{x_1^2+2x_1x_2+x_2^2}{4}-x_1x_2\\ &= \frac{x_1^2-2x_1x_2+x_2^2}{4}\\ &= \frac{(x_1-x_2)^2}{4}\geq 0\end{aligned}$$

$$\therefore \frac{x_1+x_2}{2}\geq\sqrt{x_1x_2}$$

Home Start End

- Proof of first inequality

If $n = 2$.

$$\begin{aligned}\left(\frac{x_1 + x_2}{2}\right)^2 - \left(\sqrt{x_1 x_2}\right)^2 &= \frac{x_1^2 + 2x_1x_2 + x_2^2}{4} - x_1x_2 \\ &= \frac{x_1^2 - 2x_1x_2 + x_2^2}{4} \\ &= \frac{(x_1 - x_2)^2}{4} \geq 0\end{aligned}$$

$$\therefore \frac{x_1 + x_2}{2} \geq \sqrt{x_1 x_2}$$

$$n = 2$$

Home Start End

- Proof of first inequality

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$n = 2$ is true.

- Proof of first inequality

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  Assume

Home Start End

- Proof of first inequality
  Assume $n = 2^{k-1}$

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Home Start End

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Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}}$$

Home Start End

- Proof of first inequality
  Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} +$$

Home Start End

- Proof of first inequality

  Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{}$$

Home Start End

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Home Start End

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  Assume $n = 2^{k-1}$ is true.

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$$\geq$$

Home Start End

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$$\geq \sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}$$

Home Start End

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$$\geq \sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}$$

Home Start End

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Home Start End

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$$\begin{aligned}
\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{\dfrac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \dfrac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2} \\
&\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\
&\geq
\end{aligned}$$

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\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{\dfrac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \dfrac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2} \\
&\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\
&\geq \sqrt{\phantom{x_1 \cdots x_{2^k}}}
\end{aligned}$$

- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2}$$

$$\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2}$$

$$\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}\qquad\qquad}$$

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$$\begin{aligned}
\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2} \\
&\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\
&\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}\ \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}
\end{aligned}$$

Home Start End

- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$
\begin{aligned}
\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2} \\
&\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\
&\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}\ \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}} \\
&=
\end{aligned}
$$

Home Start End

- Proof of first inequality

  Assume $n = 2^{k-1}$ is true.

$$
\begin{aligned}
\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2} \\
&\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\
&\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}\ \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}} \\
&= \sqrt[2^k]{x_1 \cdots x_{2^k}}
\end{aligned}
$$

- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{\dfrac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \dfrac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2}$$

$$\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2}$$

$$\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}\ \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}$$

$$= \sqrt[2^k]{x_1 \cdots x_{2^k}}$$

$$\therefore \frac{x_1 + \cdots + x_{2^k}}{2^k}$$

- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\begin{aligned}
\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2} \\
&\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\
&\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}\ \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}} \\
&= \sqrt[2^k]{x_1 \cdots x_{2^k}}
\end{aligned}$$

$$\therefore \frac{x_1 + \cdots + x_{2^k}}{2^k} \geq$$

Home Start End

- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\begin{aligned}
\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2} \\
&\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\
&\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}\ \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}} \\
&= \sqrt[2^k]{x_1 \cdots x_{2^k}}
\end{aligned}$$

$$\therefore \frac{x_1 + \cdots + x_{2^k}}{2^k} \geq \sqrt[2^k]{x_1 \cdots x_{2^k}}$$

- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$
\begin{aligned}
\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2} \\
&\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\
&\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}\ \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}} \\
&= \sqrt[2^k]{x_1 \cdots x_{2^k}}
\end{aligned}
$$

$$
\therefore \frac{x_1 + \cdots + x_{2^k}}{2^k} \geq \sqrt[2^k]{x_1 \cdots x_{2^k}}
$$

$$
n = 2^k
$$

- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\begin{aligned}
\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{\dfrac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \dfrac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2} \\
&\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\
&\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}\ \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}} \\
&= \sqrt[2^k]{x_1 \cdots x_{2^k}}
\end{aligned}$$

$$\therefore \frac{x_1 + \cdots + x_{2^k}}{2^k} \geq \sqrt[2^k]{x_1 \cdots x_{2^k}}$$

$$n = 2^k \text{ is true.}$$

Home Start End

Home Start End

- Proof of first inequality

Home Start End

- Proof of first inequality
  Let $m = 2^l$

- Proof of first inequality
  Let $m = 2^l$ such that

- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

Home Start End

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  α

- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

$$\alpha =$$

Home Start End

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  Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n}$$

Home Start End

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Home Start End

- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{m}{n}$$

Home Start End

- Proof of first inequality

  Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{m}{n}(x_1 + \cdots + x_n)$$

- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m}$$

Home Start End

- Proof of first inequality

  Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m}$$

$$=$$

- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

$$\begin{aligned} \alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\ &= \frac{x_1 + \cdots + x_n}{} \end{aligned}$$

Home Start End

- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

$$\begin{aligned} \alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\ &= \quad x_1 + \cdots + x_n + \end{aligned}$$

- Proof of first inequality

  Let $m = 2^l$ such that $n < m$.

$$\begin{aligned}\alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\ &= \frac{x_1 + \cdots + x_n + \frac{m-n}{n}}{}\end{aligned}$$

Home Start End

- Proof of first inequality

  Let $m = 2^l$ such that $n < m$.

$$\begin{aligned} \alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\ &= \frac{x_1 + \cdots + x_n + \frac{m-n}{n}(x_1 + \cdots + x_n)}{} \end{aligned}$$

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Home Start End

- Proof of first inequality

  Let $m = 2^l$ such that $n < m$.

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Home Start End

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\alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\
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&= x_1 + \cdots + x_n +
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Home Start End

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Home Start End

- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

$$\begin{aligned}
\alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\
&= \frac{x_1 + \cdots + x_n + \frac{m-n}{n}(x_1 + \cdots + x_n)}{m} \\
&= \frac{x_1 + \cdots + x_n + (m-n)\alpha}{m}
\end{aligned}$$

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  Let $m = 2^l$ such that $n < m$.

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&= \frac{x_1 + \cdots + x_n + \frac{m-n}{n}(x_1 + \cdots + x_n)}{m} \\
&= \frac{x_1 + \cdots + x_n + (m-n)\alpha}{m} \\
&= x_1 + \cdots + x_n
\end{aligned}
$$

Home Start End

- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

$$
\begin{aligned}
\alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\
&= \frac{x_1 + \cdots + x_n + \frac{m-n}{n}(x_1 + \cdots + x_n)}{m} \\
&= \frac{x_1 + \cdots + x_n + (m-n)\alpha}{m} \\
&= \quad x_1 + \cdots + x_n +
\end{aligned}
$$

Home Start End

- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

$$\begin{aligned}
\alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\
&= \frac{x_1 + \cdots + x_n + \frac{m-n}{n}(x_1 + \cdots + x_n)}{m} \\
&= \frac{x_1 + \cdots + x_n + (m-n)\alpha}{m} \\
&= \frac{x_1 + \cdots + x_n + \overbrace{\alpha + \cdots + \alpha}^{m-n}}{m}
\end{aligned}$$

Home Start End

- Proof of first inequality

  Let $m = 2^l$ such that $n < m$.

$$\begin{aligned}
\alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\
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&= \frac{x_1 + \cdots + x_n + (m-n)\alpha}{m} \\
&= x_1 + \cdots + x_n + \overbrace{\alpha + \cdots + \alpha}^{m-n}
\end{aligned}$$

- Proof of first inequality
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- Proof of first inequality

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&= \frac{x_1 + \cdots + x_n + (m-n)\alpha}{m} \\
&= \frac{x_1 + \cdots + x_n + \overbrace{\alpha + \cdots + \alpha}^{m-n}}{m} \geq
\end{aligned}$$

Home Start End

- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

$$
\begin{aligned}
\alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\
&= \frac{x_1 + \cdots + x_n + \frac{m-n}{n}(x_1 + \cdots + x_n)}{m} \\
&= \frac{x_1 + \cdots + x_n + (m-n)\alpha}{m} \\
&= \frac{x_1 + \cdots + x_n + \overbrace{\alpha + \cdots + \alpha}^{m-n}}{m} \geq \sqrt[m]{x_1 \cdots x_n \alpha^{m-n}}
\end{aligned}
$$

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- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

$$\begin{aligned}
\alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\
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&= \frac{x_1 + \cdots + x_n + \overbrace{\alpha + \cdots + \alpha}^{m-n}}{m} \geq \sqrt[m]{x_1 \cdots x_n \alpha^{m-n}}
\end{aligned}$$

$$\alpha^m \geq x_1 \cdots x_n \alpha^{m-n} \quad ,$$

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- Proof of first inequality

  Let $m = 2^l$ such that $n < m$.

$$\begin{aligned}
\alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\
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&= \frac{x_1 + \cdots + x_n + \overbrace{\alpha + \cdots + \alpha}^{m-n}}{m} \geq \sqrt[m]{x_1 \cdots x_n \alpha^{m-n}}
\end{aligned}$$

$$\alpha^m \geq x_1 \cdots x_n \alpha^{m-n} \ , \ \ \alpha^n \geq x_1 \cdots x_n \ ,$$

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- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

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\alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\
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&= \frac{x_1 + \cdots + x_n + (m-n)\alpha}{m} \\
&= \frac{x_1 + \cdots + x_n + \overbrace{\alpha + \cdots + \alpha}^{m-n}}{m} \geq \sqrt[m]{x_1 \cdots x_n \alpha^{m-n}}
\end{aligned}$$

$$\alpha^m \geq x_1 \cdots x_n \alpha^{m-n} \ , \ \alpha^n \geq x_1 \cdots x_n \ , \ \alpha \geq \sqrt[n]{x_1 \cdots x_n}$$

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- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

$$\begin{aligned} \alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\ &= \frac{x_1 + \cdots + x_n + \frac{m-n}{n}(x_1 + \cdots + x_n)}{m} \\ &= \frac{x_1 + \cdots + x_n + (m-n)\alpha}{m} \\ &= \frac{x_1 + \cdots + x_n + \overbrace{\alpha + \cdots + \alpha}^{m-n}}{m} \geq \sqrt[m]{x_1 \cdots x_n \alpha^{m-n}} \end{aligned}$$

$$\alpha^m \geq x_1 \cdots x_n \alpha^{m-n} \ , \ \ \alpha^n \geq x_1 \cdots x_n \ , \ \ \alpha \geq \sqrt[n]{x_1 \cdots x_n}$$

$$\therefore \frac{x_1 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 \cdots x_n} \quad (n \text{ is true.})$$

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- Proof of second inequality

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- Proof of second inequality

$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n}$$

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- Proof of second inequality

$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq$$

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- Proof of second inequality

$$\frac{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1}\cdots\frac{1}{x_n}}$$

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- Proof of second inequality

$$\frac{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1}\cdots\frac{1}{x_n}}$$

$$\frac{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}{n}$$

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- Proof of second inequality

$$\frac{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1}\cdots\frac{1}{x_n}}$$

$$\frac{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}{n} \geq$$

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- Proof of second inequality

$$\frac{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1}\cdots\frac{1}{x_n}}$$

$$\frac{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}{n} \geq \frac{1}{\sqrt[n]{x_1\cdots x_n}}$$

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- Proof of second inequality

$$\frac{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1}\cdots\frac{1}{x_n}}$$

$$\frac{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}{n} \geq \frac{1}{\sqrt[n]{x_1\cdots x_n}}$$

$$\therefore \sqrt[n]{x_1 \times \cdots \times x_n} \geq$$

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- Proof of second inequality

$$\frac{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1}\cdots\frac{1}{x_n}}$$

$$\frac{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}{n} \geq \frac{1}{\sqrt[n]{x_1\cdots x_n}}$$

$$\therefore \sqrt[n]{x_1\times\cdots\times x_n} \geq \frac{n}{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}$$

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- Proof of second inequality

$$\frac{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1}\cdots\frac{1}{x_n}}$$

$$\frac{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}{n} \geq \frac{1}{\sqrt[n]{x_1\cdots x_n}}$$

$$\therefore \sqrt[n]{x_1\times\cdots\times x_n} \geq \frac{n}{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}$$

Github:
https://min7014.github.io/math20191104001.html

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