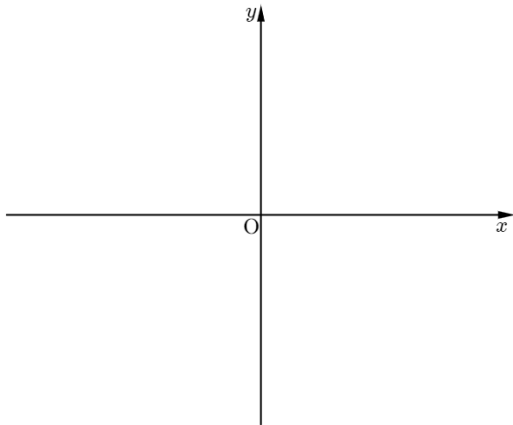


The hyperbolic equation when the two focal points are $(0, k)$, $(0, -k)$ and the difference in length is given by $2b$

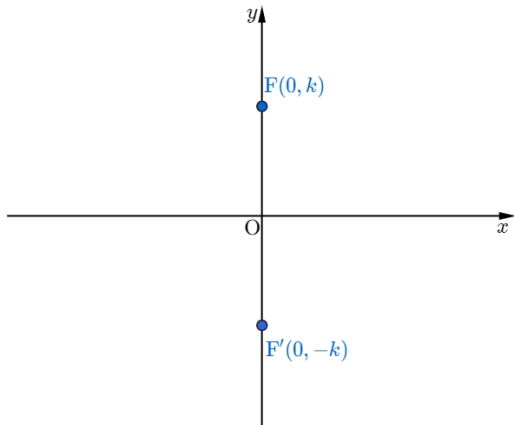
두 초점이 $(0, k)$, $(0, -k)$ 이고 길이의 차이가 $2b$ 로 주어 졌을 때 쌍곡선의 방정식

(The hyperbolic equation when the two focal points are $(0, k)$, $(0, -k)$ and the difference in length is given by $2b$)

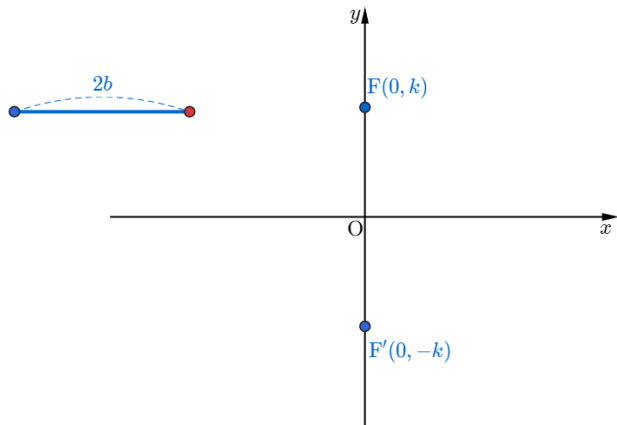
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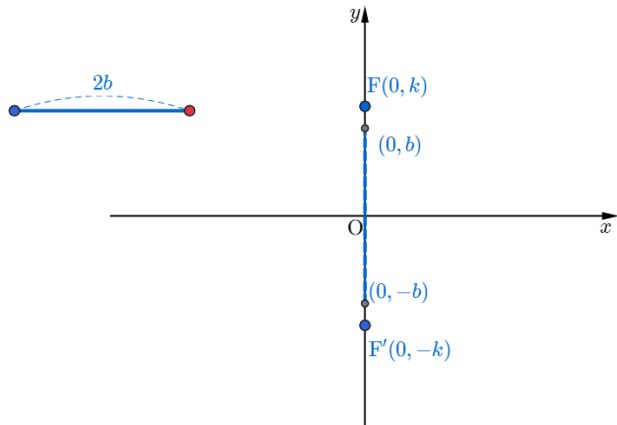
The hyperbolic equation when the two focal points are $(0, k)$, $(0, -k)$ and the difference in length is given by $2b$



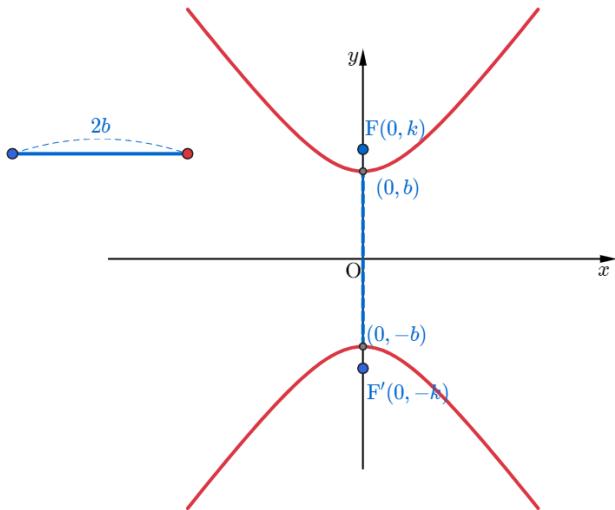
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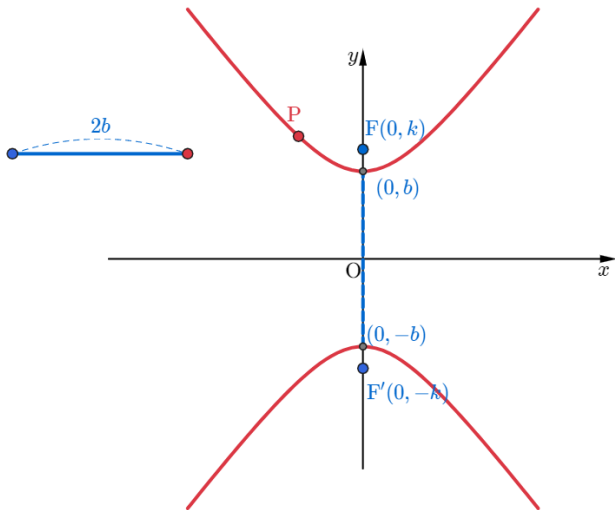
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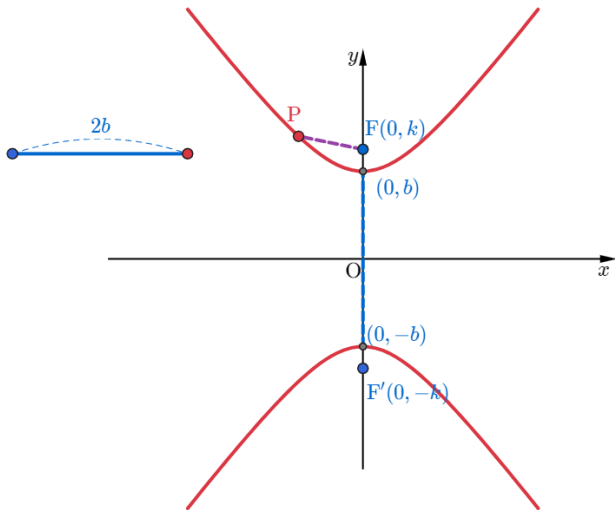
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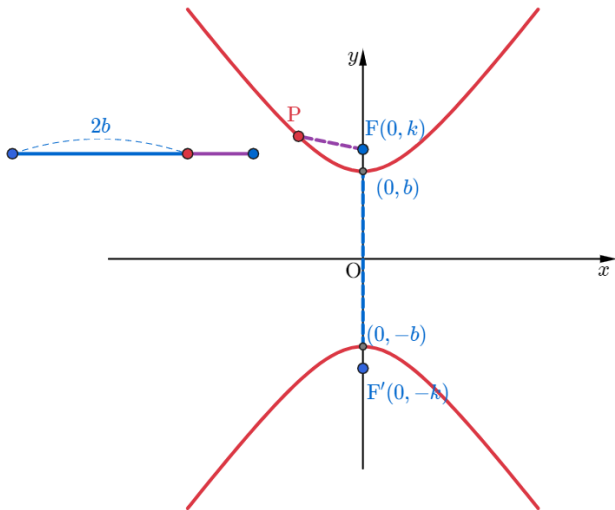
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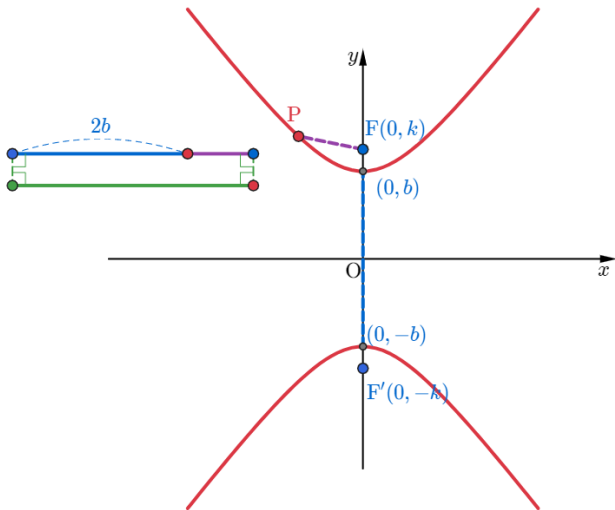
The hyperbolic equation when the two focal points are $(0, k)$, $(0, -k)$ and the difference in length is given by $2b$



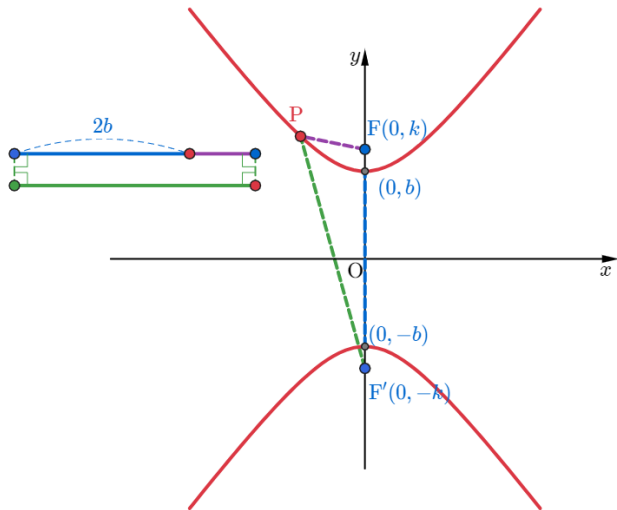
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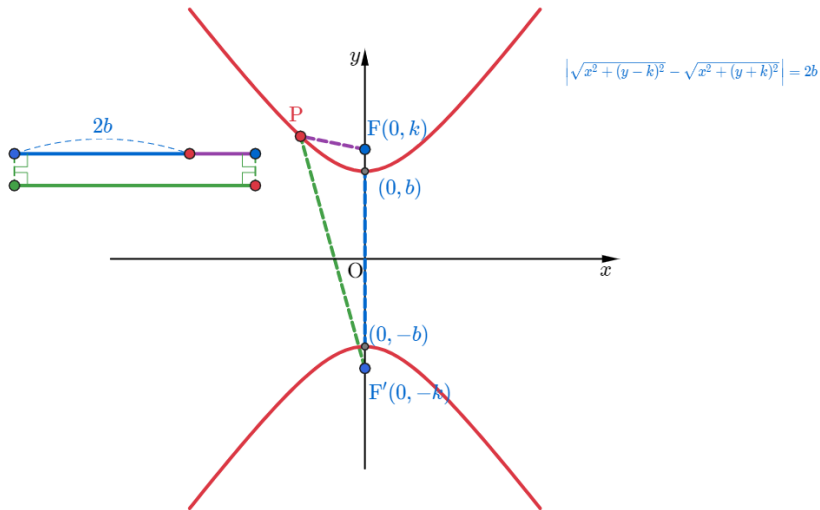
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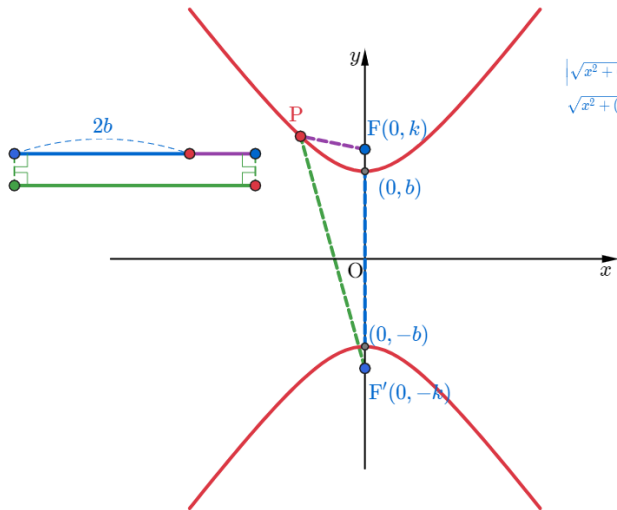
The hyperbolic equation when the two focal points are $(0, k)$, $(0, -k)$ and the difference in length is given by $2b$



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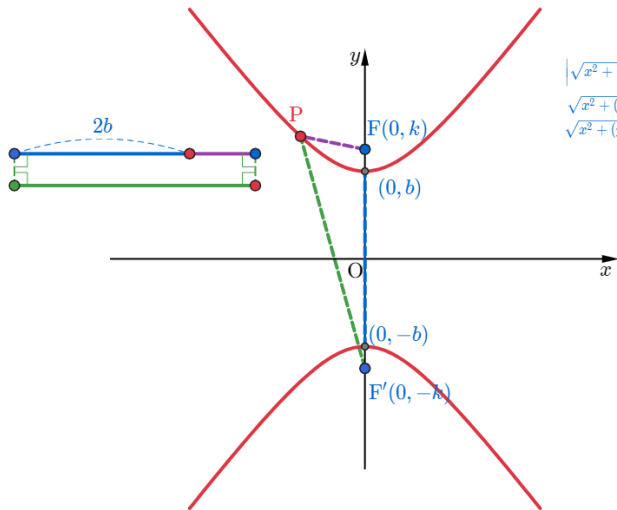


The hyperbolic equation when the two focal points are $(0, k)$, $(0, -k)$ and the difference in length is given by $2b$



$$\left| \sqrt{x^2 + (y - k)^2} - \sqrt{x^2 + (y + k)^2} \right| = 2b$$
$$\sqrt{x^2 + (y - k)^2} - \sqrt{x^2 + (y + k)^2} = \pm 2b$$

The hyperbolic equation when the two focal points are $(0, k)$, $(0, -k)$ and the difference in length is given by $2b$

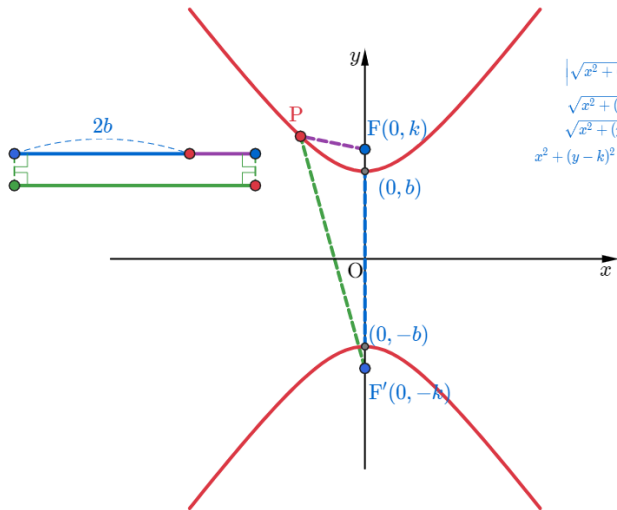


$$\left| \sqrt{x^2 + (y - k)^2} - \sqrt{x^2 + (y + k)^2} \right| = 2b$$

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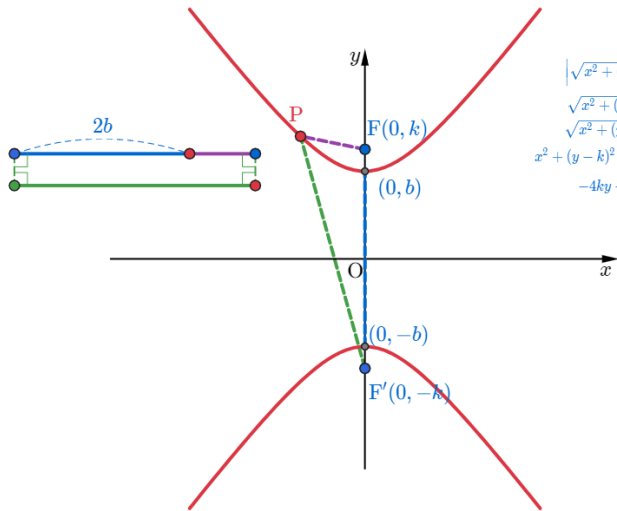
$$\sqrt{x^2 + (y - k)^2} = \sqrt{x^2 + (y + k)^2} \pm 2b$$

The hyperbolic equation when the two focal points are $(0, k)$, $(0, -k)$ and the difference in length is given by $2b$



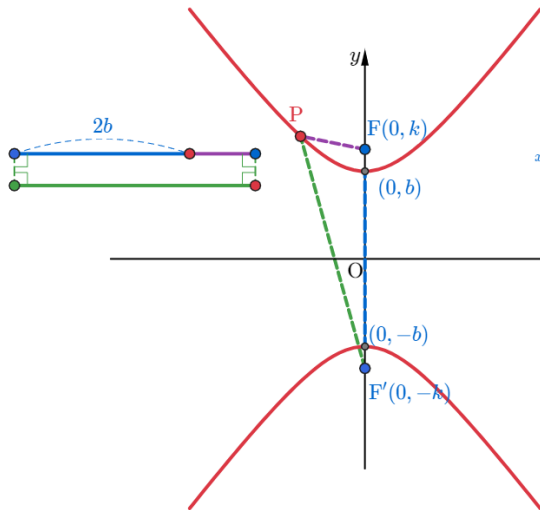
$$\begin{aligned} \left| \sqrt{x^2 + (y - k)^2} - \sqrt{x^2 + (y + k)^2} \right| &= 2b \\ \sqrt{x^2 + (y - k)^2} - \sqrt{x^2 + (y + k)^2} &= \pm 2b \\ \sqrt{x^2 + (y - k)^2} &= \sqrt{x^2 + (y + k)^2} \pm 2b \\ x^2 + (y - k)^2 &= x^2 + (y + k)^2 \pm 4b\sqrt{x^2 + (y + k)^2} + 4b^2 \end{aligned}$$

The hyperbolic equation when the two focal points are $(0, k)$, $(0, -k)$ and the difference in length is given by $2b$



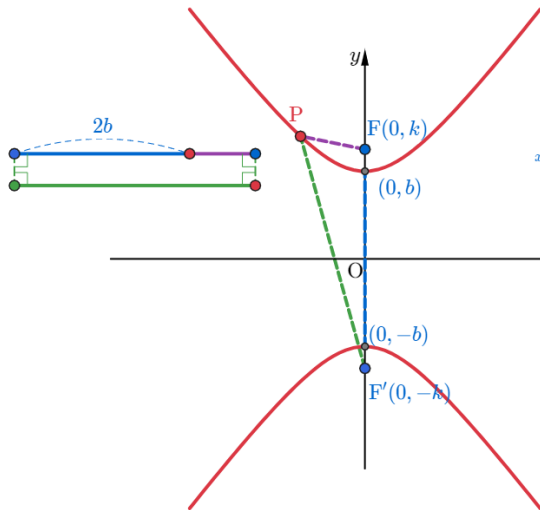
$$\begin{aligned} \left| \sqrt{x^2 + (y - k)^2} - \sqrt{x^2 + (y + k)^2} \right| &= 2b \\ \sqrt{x^2 + (y - k)^2} - \sqrt{x^2 + (y + k)^2} &= \pm 2b \\ \sqrt{x^2 + (y - k)^2} &= \sqrt{x^2 + (y + k)^2} \pm 2b \\ x^2 + (y - k)^2 &= x^2 + (y + k)^2 \pm 4b\sqrt{x^2 + (y + k)^2} + 4b^2 \\ -4ky - 4b^2 &= \pm 4b\sqrt{x^2 + (y + k)^2} \end{aligned}$$

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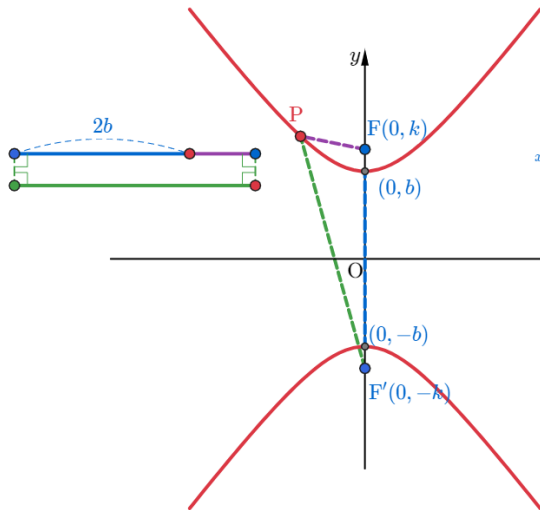
$$\begin{aligned} \left| \sqrt{x^2 + (y - k)^2} - \sqrt{x^2 + (y + k)^2} \right| &= 2b \\ \sqrt{x^2 + (y - k)^2} - \sqrt{x^2 + (y + k)^2} &= \pm 2b \\ \sqrt{x^2 + (y - k)^2} &= \sqrt{x^2 + (y + k)^2} \pm 2b \\ x^2 + (y - k)^2 &= x^2 + (y + k)^2 \pm 4b\sqrt{x^2 + (y + k)^2} + 4b^2 \\ -4ky - 4b^2 &= \pm 4b\sqrt{x^2 + (y + k)^2} \\ -ky - b^2 &= \pm b\sqrt{x^2 + (y + k)^2} \end{aligned}$$

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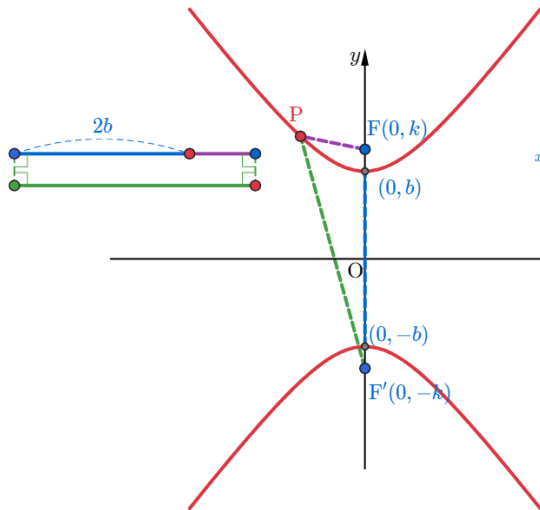
$$\begin{aligned} \left| \sqrt{x^2 + (y - k)^2} - \sqrt{x^2 + (y + k)^2} \right| &= 2b \\ \sqrt{x^2 + (y - k)^2} - \sqrt{x^2 + (y + k)^2} &= \pm 2b \\ \sqrt{x^2 + (y - k)^2} &= \sqrt{x^2 + (y + k)^2} \pm 2b \\ x^2 + (y - k)^2 &= x^2 + (y + k)^2 \pm 4b\sqrt{x^2 + (y + k)^2} + 4b^2 \\ -4ky - 4b^2 &= \pm 4b\sqrt{x^2 + (y + k)^2} \\ -ky - b^2 &= \pm b\sqrt{x^2 + (y + k)^2} \\ k^2y^2 + 2b^2ky + b^4 &= b^2x^2 + b^2y^2 + 2b^2ky + b^2k^2 \end{aligned}$$

The hyperbolic equation when the two focal points are $(0, k)$, $(0, -k)$ and the difference in length is given by $2b$



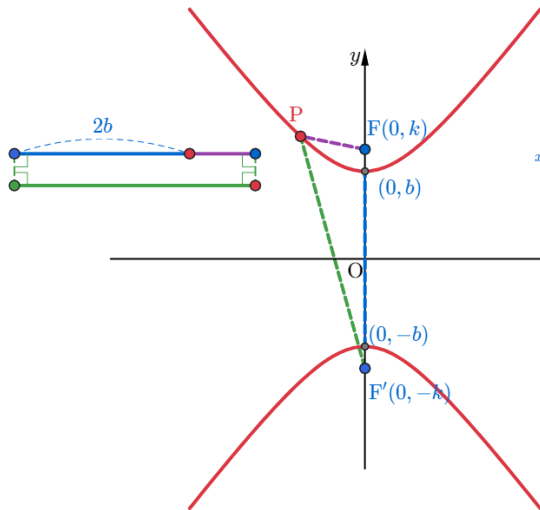
$$\begin{aligned}
 & \left| \sqrt{x^2 + (y - k)^2} - \sqrt{x^2 + (y + k)^2} \right| = 2b \\
 & \sqrt{x^2 + (y - k)^2} - \sqrt{x^2 + (y + k)^2} = \pm 2b \\
 & \sqrt{x^2 + (y - k)^2} = \sqrt{x^2 + (y + k)^2} \pm 2b \\
 & x^2 + (y - k)^2 = x^2 + (y + k)^2 \pm 4b\sqrt{x^2 + (y + k)^2} + 4b^2 \\
 & -4ky - 4b^2 = \pm 4b\sqrt{x^2 + (y + k)^2} \\
 & -ky - b^2 = \pm b\sqrt{x^2 + (y + k)^2} \\
 & k^2y^2 + 2b^2ky + b^4 = b^2x^2 + b^2y^2 + 2b^2ky + b^2k^2 \\
 & -b^2x^2 + (k^2 - b^2)y^2 = b^2(k^2 - b^2)
 \end{aligned}$$

The hyperbolic equation when the two focal points are $(0, k)$, $(0, -k)$ and the difference in length is given by $2b$



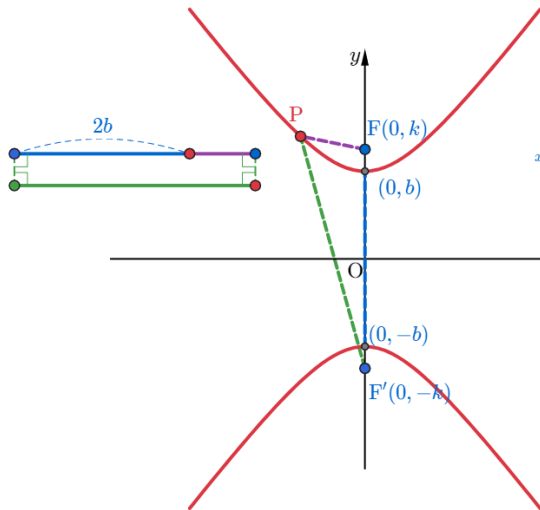
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 & k^2y^2 + 2b^2ky + b^4 = b^2x^2 + b^2y^2 + 2b^2ky + b^2k^2 \\
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The hyperbolic equation when the two focal points are $(0, k)$, $(0, -k)$ and the difference in length is given by $2b$



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$$k^2y^2 + 2b^2ky + b^4 = b^2x^2 + b^2y^2 + 2b^2ky + b^2k^2$$

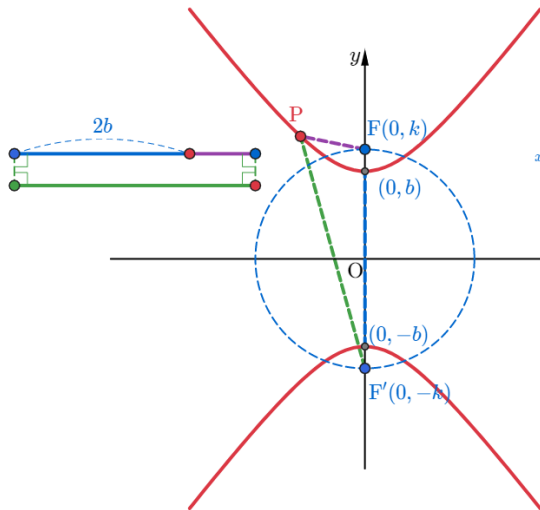
$$-b^2x^2 + (k^2 - b^2)y^2 = b^2(k^2 - b^2)$$

$$b^2x^2 - (k^2 - b^2)y^2 = -b^2(k^2 - b^2)$$

$$\text{Let } a^2 = k^2 - b^2$$

$$\therefore \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

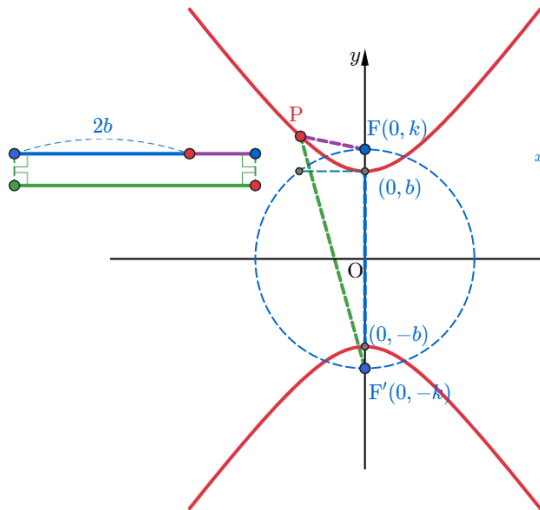
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$$\begin{aligned}
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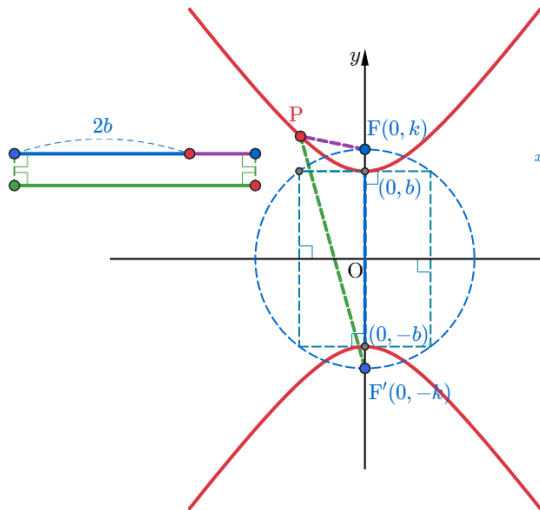
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The hyperbolic equation when the two focal points are $(0, k)$, $(0, -k)$ and the difference in length is given by $2b$

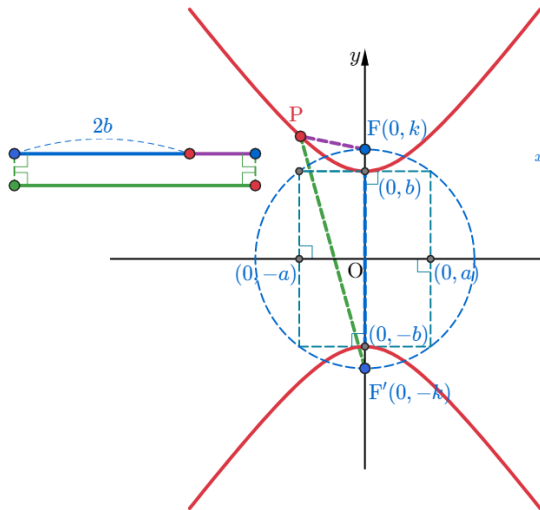


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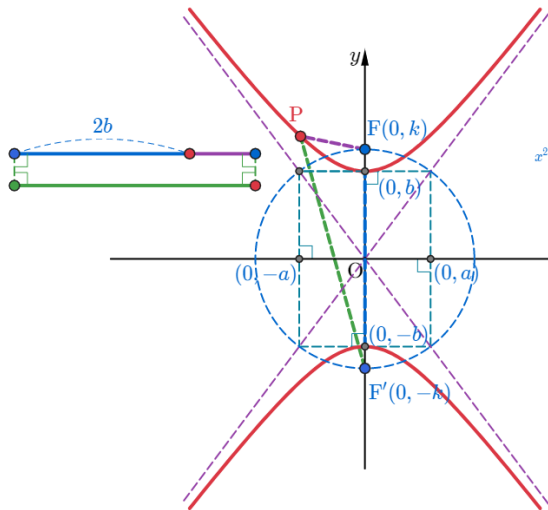
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The hyperbolic equation when the two focal points are $(0, k)$, $(0, -k)$ and the difference in length is given by $2b$



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The hyperbolic equation when the two focal points are $(0, k)$, $(0, -k)$ and the difference in length is given by $2b$

Github:

<https://min7014.github.io/math20200610001.html>

Click or paste URL into the URL search bar, and you can see a picture moving.