정수 나누기 계산법
(The Integer Division Algorithm)

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z}$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t.$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R$$

$$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

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[Existence]

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] [Uniqueness]

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] [Uniqueness]

$ex) \;\; 7$

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$
[Existence] [Uniqueness]

$$ex) \ \ 7 \ \ =$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] [Uniqueness]

$$ex) \;\; 7 \;\; = \qquad 2$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$$

[Existence] [Uniqueness]

$$ex) \ 7 \ \ = \ \ \ 2 \ \ \ \times$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] [Uniqueness]

$$ex) \;\; 7 \;\; = \quad 2 \quad \times \quad 3$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] [Uniqueness]

$$ex) \;\; 7 \;\; = \quad 2 \quad \times \quad 3 \quad +$$

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z}\ \ s.t.\ \ A = BQ + R,\ \ 0 \leq R < |B|$
[Existence] [Uniqueness]

$ex)\ 7 \quad = \quad 2 \quad \times \quad 3 \quad + \quad 1$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists ! Q, \exists ! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] [Uniqueness]

$$ex) \;\; 7 \;\; = \quad 2 \quad \times \quad 3 \quad + \;\; 1 \;\; , \;\; 0 \;\; \leq \;\; 1 \;\; < \quad |2|$$

$$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] [Uniqueness]

$$ex) \;\; 7 \;\; = \;\; 2 \;\; \times \;\; 3 \;\; + \;\; 1 \;\; , \;\; 0 \;\; \leq \;\; 1 \;\; < \;\; |2|$$

$$7$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] [Uniqueness]

$$ex) \;\; 7 = 2 \times 3 + 1 \;, \; 0 \leq 1 < |2|$$

$$7 =$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] [Uniqueness]

$$ex) \;\; 7 = 2 \times 3 + 1 \;, \;\; 0 \leq 1 < |2|$$

$$7 = (-2)$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] [Uniqueness]

$$ex) \;\; 7 = 2 \times 3 + 1, \;\; 0 \leq 1 < |2|$$

$$7 = (-2) \times$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] [Uniqueness]

$$ex) \;\; 7 = 2 \times 3 + 1, \;\; 0 \leq 1 < |2|$$

$$7 = (-2) \times (-3)$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] [Uniqueness]

$$ex) \; 7 = 2 \times 3 + 1 \; , \; 0 \leq 1 < |2|$$

$$7 = (-2) \times (-3) +$$

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[Existence] [Uniqueness]

$$ex) \;\; 7 = 2 \times 3 + 1 \;,\; 0 \leq 1 < |2|$$

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[Existence] [Uniqueness]

$$ex) \; 7 = 2 \times 3 + 1 \;,\; 0 \leq 1 < |2|$$

$$7 = (-2) \times (-3) + 1 \;,\; 0 \leq 1 < |-2|$$

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[Existence] [Uniqueness]

$$ex) \;\; 7 = 2 \times 3 + 1 \;, \;\; 0 \leq 1 < |2|$$

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[Start]

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

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$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Let

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[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \ \ \cdots\cdots(1)$

[Start]
$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
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Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \quad \cdots\cdots (1)$

$\exists R \in S$

[Start]

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$\exists R \in S \ \ s.t.$

[Start]

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$\exists R \in S \ \ s.t. \ \ x \in S$

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$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
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$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x$

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$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

[Start]
$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
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Assume

[Start]
$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
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Assume $R \geq |B|$

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$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
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Assume $R \geq |B|$
$A - BQ \geq |B|$

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$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
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Assume $R \geq |B|$
$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$

[Start]
$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
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Assume $R \geq |B|$
$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$
$A - B\left(Q + \dfrac{|B|}{B}\right)$

[Start]

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

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$\exists Q \in \mathbb{Z} \ \ s.t. \ \ R = A - BQ \ \ \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \ \ , \ A - BQ - |B| \geq 0$

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$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
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Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

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$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$
$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$

$$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B|$$

[Start]
$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
[Existence]
Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

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$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$
$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$

$$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| <$$

[Start]
$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
[Existence]
Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$
$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$
$A - BQ \geq |B| \;\; , \; A - BQ - |B| \geq 0$
$$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ$$

[Start]

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$

$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore \textit{contradiction}$

[Start]

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \ \ \cdots\cdots (1)$

$\exists R \in S \ \ s.t. \ \ x \in S \Rightarrow R \leq x \ \ \ (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \ \ s.t. \ \ R = A - BQ \ \ \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \ \ , \ A - BQ - |B| \geq 0$

$A - B\left(Q + \dfrac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore contradiction$

$\therefore R < |B| \ \ \cdots\cdots (3)$

[Start]
$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
[Existence]
Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\;\; (\because S \subset \mathbb{N} \cup \{0\})$
$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\;\; \cdots\cdots (2)$

Assume $R \geq |B|$
$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$
$$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore \textit{contradiction}$$
$\therefore R < |B| \;\;\; \cdots\cdots (3)$

By (1), (2), (3)
$\therefore$

[Start]

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$

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$\therefore R < |B| \;\; \cdots\cdots (3)$

By (1), (2), (3)

$\therefore \exists Q, \exists R \in \mathbb{Z}$

[Start]
$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
[Existence]
Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$
$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$
$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$
$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore contradiction$
$\therefore R < |B| \;\; \cdots\cdots (3)$

By (1), (2), (3)
$\therefore \exists Q, \exists R \in \mathbb{Z} \;\; s.t.$

[Start]

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$

$$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore \textit{contradiction}$$

$\therefore R < |B| \;\; \cdots\cdots (3)$

By (1), (2), (3)

$\therefore \exists Q, \exists R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R,$

[Start]
$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
[Existence]
Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$
$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$
$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$
$A - B\left(Q + \dfrac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore contradiction$
$\therefore R < |B| \;\; \cdots\cdots (3)$

By (1), (2), (3)
$\therefore \exists Q, \exists R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Start]

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniquness]

[Start]

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniquness]

$Let \; A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

[Start]
$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
[Uniqupness]
$Let \; A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$
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[Start]

$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

[Uniquness]

$Let \ A = BQ_1 + R_1, \ \ 0 \leq R_1 < |B|$

$Let \ A = BQ_2 + R_2, \ \ 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B|$

[Start]
$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
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$-|B| < R_2 - R_1 < |B| \;\; , \;\; |R_2 - R_1| < |B| \;\; \cdots\cdots (1)$

[Start]

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

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$BQ_1 + R_1 = BQ_2 + R_2$

[Start]
$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
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$BQ_1 + R_1 = BQ_2 + R_2 \;\; , \;\; B(Q_1 - Q_2) = R_2 - R_1 \;\; \cdots\cdots (2)$

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$by \ (1), (2)$

[Start]

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

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$BQ_1 + R_1 = BQ_2 + R_2 \;\; , \;\; B(Q_1 - Q_2) = R_2 - R_1 \;\; \cdots\cdots (2)$

$by \; (1), (2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

[Start]

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[Start]
$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
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$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$
$|B||Q_1 - Q_2| < |B| \;\; , \;\; |Q_1 - Q_2| < 1$

[Start]
$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
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$-|B| < R_2 - R_1 < |B| \;\; , \;\; |R_2 - R_1| < |B| \;\; \cdots\cdots (1)$
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$by \; (1), (2)$
$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$
$|B||Q_1 - Q_2| < |B| \;\; , \;\; |Q_1 - Q_2| < 1$
$|Q_1 - Q_2| = 0$

[Start]

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniquness]

$Let \; A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

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$BQ_1 + R_1 = BQ_2 + R_2 \;\; , \;\; B(Q_1 - Q_2) = R_2 - R_1 \;\; \cdots\cdots (2)$

$by \; (1), (2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

$|B||Q_1 - Q_2| < |B| \;\; , \;\; |Q_1 - Q_2| < 1$

$|Q_1 - Q_2| = 0 \;\; (\because Q_1, Q_2 \in \mathbb{Z})$

[Start]
$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
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$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$
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$|Q_1 - Q_2| = 0 \;\; (\because Q_1, Q_2 \in \mathbb{Z})$
$\therefore$

[Start]
$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
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$Let \; A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$
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$BQ_1 + R_1 = BQ_2 + R_2 \;\;,\;\; B(Q_1 - Q_2) = R_2 - R_1 \;\; \cdots\cdots(2)$

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$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$
$|B||Q_1 - Q_2| < |B| \;\;,\;\; |Q_1 - Q_2| < 1$
$|Q_1 - Q_2| = 0 \;\; (\because Q_1, Q_2 \in \mathbb{Z})$
$\therefore \;\; Q_1 = Q_2$

[Start]
$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$
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$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$
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$|Q_1 - Q_2| = 0 \;\; (\because Q_1, Q_2 \in \mathbb{Z})$
$\therefore \;\; Q_1 = Q_2 \;\; , \;\; R_1 = R_2$

Github:
https://min7014.github.io/math20201204001.html

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