켤레 복소수 (The Complex Conjugate)







Complex conjugates a pair of complex numbers,



a pair of complex numbers, both having the same real part,



a pair of complex numbers, both having the same real part, but with imaginary parts of



a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude



a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.



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The conjugate of the complex number $a+bi$

 $\overline{z+w}$



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$$\overline{a+bi} = a-bi \ (a,\ b \in \mathbb{R})$$
The conjugate of the complex number $a+bi$

 $\overline{z+w} = \overline{z} + \overline{w} \longrightarrow Proof$

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.

ex) 1 + 2i, 1 - 2i are complex conjugates.

$$\overline{a+bi} = a-bi (a, b \in \mathbb{R})$$

- $\overline{z \pm w} = \overline{z} \pm \overline{w}$ Proof
- $oldsymbol{\overline{z}\cdot w}$

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- \bullet $\frac{z}{w}$

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- $\overline{z \pm w} = \overline{z} \pm \overline{w}$ Proof
- $\bullet \ \overline{z \cdot w} = \overline{z} \cdot \overline{w} \ \underline{\hspace{0.2cm}} \underline{\hspace{0.2cm}}$
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- $\bullet \ \overline{\left(\frac{z}{w}\right)} = \frac{\overline{z}}{\overline{w}} \bullet Proof$
- ullet $\overline{z} = z$

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$$\overline{z \pm w} = \overline{z} \pm \overline{w}$$

▶ Main ▶ Start ▶ End

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Let
$$z = a_1 + b_1 i$$
, $w = a_2 + b_2 i$ $(a_1, a_2, b_1, b_2 \in \mathbb{R})$

► Main ► Start ► End

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 $\overline{z \pm w} = \overline{(a_1 + b_1 i) \pm (a_2 + b_2 i)}$ (Double signs in same order)

► Main ► Start ► End

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▶ Main ▶ Start ▶ End

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 $= (a_1 \pm a_2) - (b_1 \pm b_2) i$

▶ Main ▶ Start ▶ End

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► Main ► Start ► End

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 $= (a_1 \pm a_2) - (b_1 \pm b_2) i$
 $= \underline{(a_1 - b_1 i) \pm (a_2 - b_2 i)}$
 $= \overline{a_1 + b_1 i \pm a_2 + b_2 i}$

▶ Main ▶ Start ▶ End

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► Main ► Start ► End

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► Main ► Start ► End

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$$\overline{z \cdot w} = \underbrace{(a_1 + b_1 i) \cdot (a_2 + b_2 i)}_{(a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1) i}$$

▶ Main ▶ Start ▶ End

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 $= \overline{(a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1) i}$
 $= (a_1 a_2 - b_1 b_2) - (a_1 b_2 + a_2 b_1) i$

► Main ► Start ► End

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► Main ► Start ► End

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$$\overline{z \cdot w} = \overline{(a_1 + b_1 i) \cdot (a_2 + b_2 i)}$$

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$$= \overline{(a_1 - b_1 i) \cdot (a_2 - b_2 i)}$$

$$= \overline{a_1 + b_1 i \cdot a_2 + b_2 i}$$

$$\bullet \ \overline{z \cdot w} = \overline{z} \cdot \overline{w}$$

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$$= \overline{(a_1 a_2 - b_1 b_2) - (a_1 b_2 + a_2 b_1) i}$$

$$= \overline{(a_1 - b_1 i) \cdot (a_2 - b_2 i)}$$

$$= \overline{a_1 + b_1 i} \cdot \overline{a_2 + b_2 i}$$

$$= \overline{z} \cdot \overline{w}$$

$$\bullet \overline{\left(\frac{z}{A}\right)} = \overline{\frac{\overline{z}}{A}}$$

Let
$$z = a_1 + b_1 i$$
, $w = a_2 + b_2 i (a_1, a_2, b_1, b_2 \in \mathbb{R})$

$$\begin{array}{c|c}
\bullet \text{ Main} & \bullet \text{ Start} & \bullet \text{ End} \\
\hline
\bullet & \overline{\left(\frac{z}{w}\right)} = \frac{\overline{z}}{\overline{w}}
\end{array}$$

Let
$$z = a_1 + b_1 i$$
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$$\left(\frac{z}{w}\right)$$

$$\begin{array}{c}
\bullet \text{ Main} & \bullet \text{ Start} & \bullet \text{ End} \\
\hline
\bullet & \overline{\left(\frac{z}{w}\right)} = \frac{\overline{z}}{\overline{w}}
\end{array}$$

Let
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$$\overline{\left(\frac{z}{w}\right)} = \overline{\frac{a_1 + b_1 i}{a_2 + b_2 i}}$$

$$\bullet \ \overline{\left(\frac{z}{w}\right)} = \frac{\overline{z}}{\overline{w}}$$

Let
$$z = a_1 + b_1 i$$
, $w = a_2 + b_2 i (a_1, a_2, b_1, b_2 \in \mathbb{R})$

$$\overline{\left(\frac{z}{w}\right)} = \overline{\frac{a_1 + b_1 i}{a_2 + b_2 i}} = \overline{\frac{a_1 a_2 - b_1 b_2}{a_2^2 + b_2^2} + \frac{-a_1 b_2 + a_2 b_1}{a_2^2 + b_2^2} i}$$

$$\bullet \ \overline{\left(\frac{z}{w}\right)} = \frac{\overline{z}}{\overline{w}}$$

Let
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$$= \overline{\frac{a_1 a_2 - b_1 b_2}{a_2^2 + b_2^2}} - \overline{\frac{-a_1 b_2 + a_2 b_1}{a_2^2 + b_2^2}} i$$

$$\bullet \ \overline{\left(\frac{z}{w}\right)} = \frac{\overline{z}}{\overline{w}}$$

Let
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$$= \frac{a_1 a_2 - b_1 b_2}{a_2^2 + b_2^2} - \frac{-a_1 b_2 + a_2 b_1}{a_2^2 + b_2^2} i$$

$$= \frac{a_1 a_2 - b_1 b_2}{a_2^2 + b_2^2} + \frac{a_1 b_2 - a_2 b_1}{a_2^2 + b_2^2} i$$

Let
$$z = a_1 + b_1 i$$
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$$= \frac{a_1 a_2 - b_1 b_2}{a_2^2 + b_2^2} - \frac{-a_1 b_2 + a_2 b_1}{a_2^2 + b_2^2} i$$

$$= \frac{a_1 a_2 - b_1 b_2}{a_2^2 + b_2^2} + \frac{a_1 b_2 - a_2 b_1}{a_2^2 + b_2^2} i$$

$$= \frac{a_1 - b_1 i}{a_2 - b_2 i}$$

$$\bullet \ \overline{\left(\frac{z}{w}\right)} = \frac{\overline{z}}{\overline{w}}$$

Let
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$$= \frac{a_1 a_2 - b_1 b_2}{a_2^2 + b_2^2} - \frac{-a_1 b_2 + a_2 b_1}{a_2^2 + b_2^2} i$$

$$= \frac{a_1 a_2 - b_1 b_2}{a_2^2 + b_2^2} + \frac{a_1 b_2 - a_2 b_1}{a_2^2 + b_2^2} i$$

$$= \frac{a_1 - b_1 i}{a_2 - b_2 i} = \overline{\frac{a_1 + b_1 i}{a_2 + b_2 i}}$$

$$\bullet \ \overline{\left(\frac{z}{w}\right)} = \frac{\overline{z}}{\overline{w}}$$

Let
$$z = a_1 + b_1 i$$
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$$= \frac{a_1 a_2 - b_1 b_2}{a_2^2 + b_2^2} - \frac{-a_1 b_2 + a_2 b_1}{a_2^2 + b_2^2} i$$

$$= \frac{a_1 a_2 - b_1 b_2}{a_2^2 + b_2^2} + \frac{a_1 b_2 - a_2 b_1}{a_2^2 + b_2^2} i$$

$$= \frac{a_1 - b_1 i}{a_2 - b_2 i} = \overline{\frac{a_1 + b_1 i}{a_2 + b_2 i}} = \overline{\frac{z}{w}}$$



▶ Main → Start ▶ End

Let
$$z = a + bi(a, b \in \mathbb{R})$$

▶ Main → Start ▶ End

Let
$$z = a + bi(a, b \in \mathbb{R})$$

 $\overline{z} = z$

▶ Main ▶ Start ▶ End

Let
$$z = a + bi(a, b \in \mathbb{R})$$

 $\frac{\overline{z}}{a + bi} = z$
 $a + bi$

▶ Main → Start ▶ End

Let
$$z = a + bi(a, b \in \mathbb{R})$$

 $\overline{z} = z$
 $\overline{a + bi} = a + bi$
 $a - bi = a + bi$

▶ Main → Start → End

Let
$$z = a + bi(a, b \in \mathbb{R})$$

 $\frac{\overline{z}}{a + bi} = z$
 $a - bi = a + bi$
 $2bi = 0$

Let
$$z = a + bi(a, b \in \mathbb{R})$$

 $\frac{\overline{z}}{a + bi} = z$
 $a - bi = a + bi$
 $a - bi = a + bi$
 $2bi = 0$
 $b = 0$



▶ Main → Start ▶ End

Let
$$z = a + bi(a, b \in \mathbb{R})$$

▶ Main → Start ▶ End

Let
$$z = a + bi(a, b \in \mathbb{R})$$

 $\overline{z} = -z$

▶ Main → Start ▶ End

Let
$$z = a + bi(a, b \in \mathbb{R})$$

 $\overline{z} = -z$
 $\overline{a + bi} = -(a + bi)$

Let
$$z = a + bi(a, b \in \mathbb{R})$$

 $\overline{z} = -z$
 $\overline{a + bi} = -(a + bi)$
 $a - bi = -a - bi$

Let
$$z = a + bi(a, b \in \mathbb{R})$$

 $\frac{\overline{z}}{a + bi} = -(a + bi)$
 $a - bi = -a - bi$
 $2a = 0$

Let
$$z = a + bi(a, b \in \mathbb{R})$$

 $\overline{z} = -z$
 $\overline{a+bi} = -(a+bi)$
 $a-bi = -a-bi$
 $2a = 0$
 $a = 0$

Github:

https://min7014.github.io/math20210126001.html

Click or paste URL into the URL search bar, and you can see a picture moving.