아폴로니우스의 원 (Circle of Apollonius)











$$\overline{\mathrm{AP}} : \overline{\mathrm{BP}} = 1 : r \ (0 < r < 1)$$

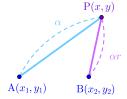
$$\begin{array}{ccc} \bullet & \bullet \\ \mathbf{A}(x_1, y_1) & & \mathbf{B}(x_2, y_2) \end{array}$$

$$\overline{\mathrm{AP}} : \overline{\mathrm{BP}} = 1 : r \ (0 < r < 1)$$

$$\begin{array}{ccc} \bullet & \bullet \\ \mathbf{A}(x_1, y_1) & & \mathbf{B}(x_2, y_2) \end{array}$$



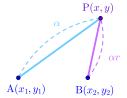
$$\overline{AP} : \overline{BP} = 1 : r \ (0 < r < 1)$$





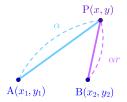
$$\overline{AP} : \overline{BP} = 1 : r \ (0 < r < 1)$$

$$\overline{\rm BP} = r\overline{\rm AP}$$



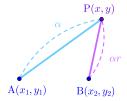


$$\overline{\text{AP}}: \overline{\text{BP}} = 1: r \ (0 < r < 1)$$
 
$$\sqrt{\left(x - x_2\right)^2 + \left(y - y_2\right)^2} = r\sqrt{\left(x - x_1\right)^2 + r^2(y - y_1)^2}$$





$$\overline{\text{AP}}: \overline{\text{BP}} = 1: r \ (0 < r < 1)$$
 
$$(x - x_2)^2 + (y - y_2)^2 = r^2 (x - x_1)^2 + r^2 (y - y_1)^2$$



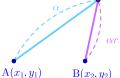


▶ End

$$\overline{AP} : \overline{BP} = 1 : r$$

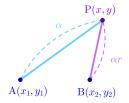
$$\left(x - \frac{x_2 - x_1 r^2}{1 - r^2}\right)^2 + \left(y - \frac{y_2 - y_1 r^2}{1 - r^2}\right)^2 = \frac{\left\{\left(x_1 - x_2\right)^2 + \left(y_1 - y_2\right)^2\right\}r^2}{\left(1 - r^2\right)^2}$$





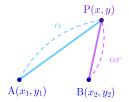
$$\overline{\mathrm{AP}} : \overline{\mathrm{BP}} = 1 : r \ (0 < r < 1)$$

$$(1-r^2)x^2 + (1-r^2)y^2 - 2(x_2 - x_1r^2)x - 2(y_2 - y_1r^2)y + (x_2^2 + y_2^2 - x_1^2r^2 - y_1^2r^2) = 0$$



$$\overline{\mathrm{AP}} : \overline{\mathrm{BP}} = 1 : r \ (0 < r < 1)$$

$$x^2 + y^2 - 2\frac{x_2 - x_1 r^2}{1 - r^2}x - 2\frac{y_2 - y_1 r^2}{1 - r^2}y + \frac{x_2^2 + y_2^2 - r^2 x_1^2 - r^2 y_1^2}{1 - r^2} = 0$$



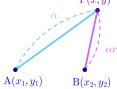
$$\begin{split} \overline{\text{AP}} : \overline{\text{BP}} &= 1 : r \quad (0 < r < 1) \\ \left( x - \frac{x_2 - x_1 r^2}{1 - r^2} \right)^2 + \left( y - \frac{y_2 - y_1 r^2}{1 - r^2} \right)^2 \\ &= \left( \frac{x_2 - x_1 r^2}{1 - r^2} \right)^2 + \left( \frac{y_2 - y_1 r^2}{1 - r^2} \right)^2 - \frac{x_2^2 + y_2^2 - r^2 x_1^2 - r^2 y_1^2}{1 - r^2} \\ P(x, y) \\ &\alpha \\ A(x_1, y_1) & B(x_2, y_2) \end{split}$$



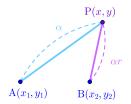
$$\begin{split} \overline{\text{AP}} : \overline{\text{BP}} &= 1 : r \quad (0 < r < 1) \\ \left( x - \frac{x_2 - x_1 r^2}{1 - r^2} \right)^2 + \left( y - \frac{y_2 - y_1 r^2}{1 - r^2} \right)^2 \\ &= \frac{(x_2 - x_1 r^2)^2 + (y_2 - y_1 r^2)^2 - (x_2^2 + y_2^2 - r^2 x_1^2 - r^2 y_1^2)(1 - r^2)}{(1 - r^2)^2} \\ &\qquad \qquad P(x, y) \\ &\qquad \qquad \qquad \\ A(x_1, y_1) \qquad \qquad B(x_2, y_2) \end{split}$$

➤ Start ➤ End

$$\begin{split} & \overline{\text{AP}} : \overline{\text{BP}} = 1 : r \quad (0 < r < 1) \\ & \left( x - \frac{x_2 - x_1 r^2}{1 - r^2} \right)^2 + \left( y - \frac{y_2 - y_1 r^2}{1 - r^2} \right)^2 \\ & = \frac{x_2^2 - 2x_1 x_2 r^2 + x_1^2 r^4 + y_2^2 - 2y_1 y_2 r^2 + y_1^2 r^4 - x_2^2 - y_2^2 + x_1^2 r^2 + y_1^2 r^2 + x_2^2 r^2 + y_2^2 r^2 - x_1^2 r^4 - y_1^2 r^4}{(1 - r^2)^2} \end{split}$$

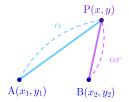


$$\begin{split} \overline{\text{AP}} : \overline{\text{BP}} &= 1 : r \ (0 < r < 1) \\ \left( x - \frac{x_2 - x_1 r^2}{1 - r^2} \right)^2 + \left( y - \frac{y_2 - y_1 r^2}{1 - r^2} \right)^2 &= \frac{-2 x_1 x_2 r^2 - 2 y_1 y_2 r^2 + x_1^2 r^2 + y_1^2 r^2 + x_2^2 r^2 + y_2^2 r^2}{(1 - r^2)^2} \end{split}$$

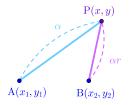


➤ Start ➤ End

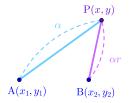
$$\begin{split} \overline{\text{AP}} : \overline{\text{BP}} &= 1 : r \ \left( 0 < r < 1 \right) \\ & \left( x - \frac{x_2 - x_1 r^2}{1 - r^2} \right)^2 + \left( y - \frac{y_2 - y_1 r^2}{1 - r^2} \right)^2 = \frac{x_1^2 r^2 - 2x_1 x_2 r^2 + x_2^2 r^2 + y_1^2 r^2 - 2y_1 y_2 r^2 + y_2^2 r^2}{\left( 1 - r^2 \right)^2} \end{split}$$



$$\begin{split} \overline{\text{AP}} : \overline{\text{BP}} &= 1 : r \quad (0 < r < 1) \\ & \left( x - \frac{x_2 - x_1 r^2}{1 - r^2} \right)^2 + \left( y - \frac{y_2 - y_1 r^2}{1 - r^2} \right)^2 = \frac{(x_1^2 - 2x_1 x_2 + x_2^2) r^2 + (y_1^2 - 2y_1 y_2 + y_2^2) r^2}{(1 - r^2)^2} \end{split}$$



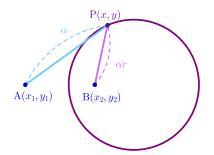
$$\begin{split} \overline{\text{AP}} : \overline{\text{BP}} &= 1 : r \quad (0 < r < 1) \\ \left( x - \frac{x_2 - x_1 r^2}{1 - r^2} \right)^2 + \left( y - \frac{y_2 - y_1 r^2}{1 - r^2} \right)^2 = \frac{(x_1 - x_2)^2 r^2 + (y_1 - y_2)^2 r^2}{(1 - r^2)^2} \end{split}$$



➤ Start ➤ End

$$\overline{\text{AP}} : \overline{\text{BP}} = 1 : r \quad (0 < r < 1)$$

$$\left(x - \frac{x_2 - x_1 r^2}{1 - r^2}\right)^2 + \left(y - \frac{y_2 - y_1 r^2}{1 - r^2}\right)^2 = \frac{(x_1 - x_2)^2 r^2 + (y_1 - y_2)^2 r^2}{(1 - r^2)^2}$$



#### Github:

https://min7014.github.io/math20210915001.html

Click or paste URL into the URL search bar, and you can see a picture moving.