삼각함수의 3배각 공식 (Triple Angle Formula for Trigonometric Functions)





 $\sin 3\alpha =$



$$\sin 3\alpha = 3\sin \alpha - 4\sin^3 \alpha$$



$$\sin 3\alpha = 3\sin \alpha - 4\sin^3 \alpha$$
• proof

$$\cos 3\alpha =$$



$$\sin 3\alpha = 3\sin \alpha - 4\sin^3 \alpha$$
Proof

$$\cos 3\alpha = 4\cos^3 \alpha - 3\cos \alpha$$



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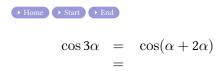
$$\therefore \sin 3\alpha = 3\sin \alpha - 4\sin^3 \alpha$$





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$$\therefore \cos 3\alpha = 4\cos^3 \alpha - 3\cos \alpha$$

Github:

https://min7014.github.io/math20230420001.html

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