좌극한, 우극한 (Definiton of One-Sided Limits)







• lim

















$$\bullet \lim_{x \to a} f(x) = L$$





$$\begin{array}{l}
\bullet \lim_{x \to a} f(x) = L \\
\forall \epsilon
\end{array}$$



$$\lim_{x \to a} f(x) = L$$

$$\forall \epsilon > 0$$

$$\lim_{x \to a} f(x) = L$$

$$\forall \epsilon > 0,$$



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\forall \epsilon > 0, \exists$$



$$\lim_{x \to a} f(x) = L
\forall \epsilon > 0, \exists \delta$$



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 $\forall \epsilon > 0, \exists \delta > 0$

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 $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t.}$

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➤ Start ➤ End

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- $\lim_{x \to a} f(x) = L \Leftrightarrow \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x$

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- $\lim_{x \to a^+} f(x) = L$ $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } a < x < a + \delta \Rightarrow |f(x) - L| < \epsilon$
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- $\lim_{x \to a} f(x) = L \Leftrightarrow \lim_{x \to a^{-}} f(x) = L$ and $\lim_{x \to a^{+}} f(x) = L$



Github:

https://min7014.github.io/math20240109001.html

Click or paste URL into the URL search bar, and you can see a picture moving.