압착정리 (The Squeeze Theorem)





#### Theorem

$$f(x) \le g(x) \le h(x)(0 < |x-a| < \delta_0)$$
 ,  $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$  
$$\lim_{x \to a} g(x) = L$$

### Proof.

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 $\epsilon > 0$ 

 $\exists \delta_1 > 0$ 

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▶ Start ▶ End

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▶ Start ▶ End

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$$\exists \delta_1>0 \text{ s.t. } 0<|x-a|<\delta_1\Rightarrow |f(x)-L|<\epsilon \ (\because \lim_{x\to a}f(x)=L) \ \ , \ L-\epsilon< f(x)< L+\epsilon \ \exists \delta_2>0 \text{ s.t.}$$

▶ Start ▶ End

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$$f(x) \le g(x) \le h(x)$$

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$$\therefore \forall \epsilon > 0, \exists \delta > 0$$

▶ Start ▶ End

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$$f(\lambda) \leq g(\lambda) \leq h(\lambda) + e + f(\lambda) \leq g(\lambda) \leq h(\lambda) + e + f(\lambda) + e + f(\lambda) \leq g(\lambda) + f(\lambda) + e + f(\lambda) \leq g(\lambda) + e + f(\lambda) \leq g(\lambda) + e + f(\lambda) \leq g(\lambda) + f(\lambda) + f(\lambda) \leq g(\lambda) + f(\lambda) \leq g(\lambda) + f(\lambda) + f(\lambda) \leq g(\lambda) + f(\lambda) + f(\lambda) + f(\lambda) \leq g(\lambda) + f(\lambda) + f(\lambda) + f(\lambda) + f(\lambda) + f(\lambda) \leq g(\lambda) + f(\lambda) +$$

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$$\therefore \forall \epsilon > 0$$
,  $\exists \delta > 0$  s.t. $0 < |x - a| < \delta \Rightarrow |g(x) - L| < \epsilon$ 

#### Github:

https://min7014.github.io/math20240108001.html

Click or paste URL into the URL search bar, and you can see a picture moving.