TFW HW3、 P10942A05 鄭閔軒

1、(a) Depends on signal frequency, Gaussian window size would be changed.
So S tranform have better performance than STFT in differenc frequency signal.

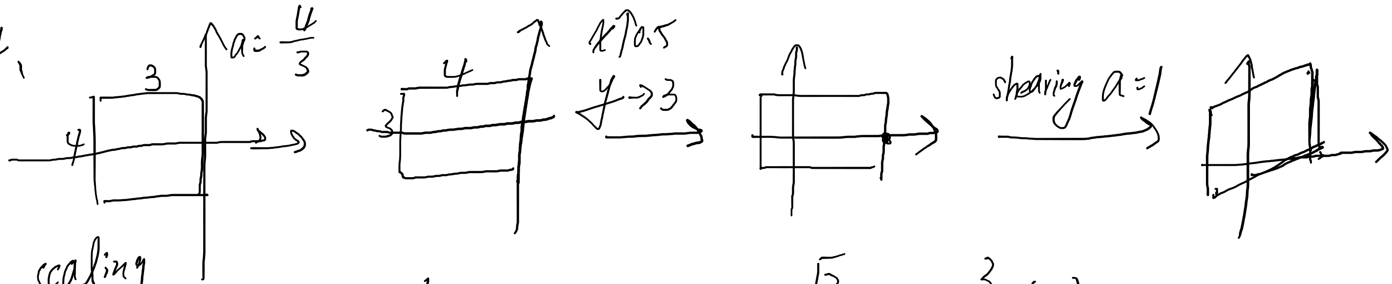
(b) Use windows w_1, w_2 one is narrow another is wider to prevent uncertainty principle。Have better time and frequency domain resolution than original spectrogram.

(c) t_0 central time f_0 frequency σ controls the scaling factor
Compare with Fourier series 3-parameters atom can use time and scaling factor to reconstruct or described a signal well. Also reduce use terms.

2、 (c) is most suitable for S tranform window function
due to (a) in high frequency, the time window would be too narrow to get properly resoluting in frequency domain
(b) function can't change window size properly depends on frequency.
(c) is most suitable when high frequency signal window (c) will have better frequency resolution than (a).

3.
(1) Due to use LCT can twist rectangular filter to other parallelograms you want when $\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = 1$

(2) To specific case generalized modulation (Shearing) can transform rectangular filter to sinsoid or other customized shape to filter specific signal. 

4. 

i scaling

$$x_{scaling}(t) = \frac{1}{\sqrt{\frac{4}{3}}}\, x\left(\frac{t}{\frac{4}{3}}\right) = \frac{\sqrt{3}}{2}\, x\left(\frac{3}{4}t\right)$$

ii shifting

$$t - \frac{1}{2}$$

$$e^{j6\pi t}\, x_{scaling}\left(t - \frac{1}{2}\right) = e^{j6\pi t}\, \frac{\sqrt{3}}{2}\, x\left(\frac{3}{4}\left(t - \frac{1}{2}\right)\right)$$

iii Shearing

$$e^{j\pi t^2} e^{j6\pi t}\, \frac{\sqrt{3}}{2}\, x\left(\frac{3}{4}\left(t - \frac{1}{2}\right)\right)$$

$$= \frac{\sqrt{3}}{2}\, e^{j\pi t(6+t)}\, x\left(\frac{3}{4}t - \frac{3}{8}\right)$$

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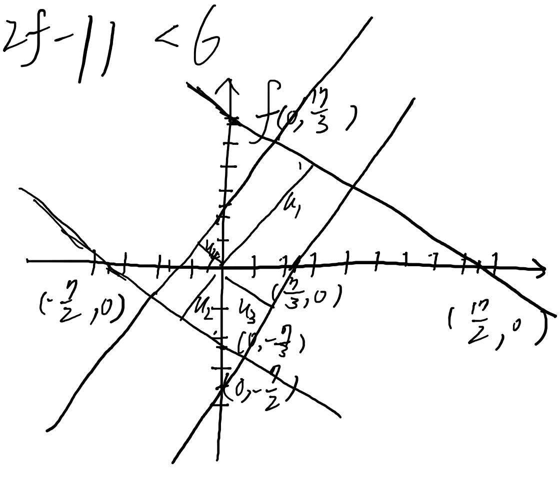
5、 If set $(3t-2f-1)=0$ then $|2t+3f-5|<12$

$$-7 < 2t+3f < 17$$

If set $2t+3f-5=0$ $|3t-2f-1|<6$

$$-5 < 3t-2f < 7$$



$$\phi_1 = \arctan\left(\frac{\frac{17}{2}}{\frac{17}{3}}\right) = \arctan\left(\frac{3}{2}\right)$$

$$\frac{17}{2}\times\frac{17}{3} = u_1\sqrt{(\tfrac{17}{2})^2+(\tfrac{17}{3})^2} \qquad u_1 = \frac{\sqrt{(\frac{17}{2})^2+(\frac{17}{3})^2}}{\frac{17}{2}\times\frac{17}{3}}$$

$$u_2 = -\frac{\sqrt{(\frac{7}{2})^2+(\frac{7}{3})^2}}{\frac{7}{2}\times\frac{7}{3}}$$

$$\text{FrFT1}: O_F^{-\phi_1}\left(O_F^{\phi_1}(x(t))\ H(u)\right) \qquad H(u) = \begin{cases} 1 & \text{for } u_1 > u > u_2 \\ 0 & \text{otherwise} \end{cases}$$

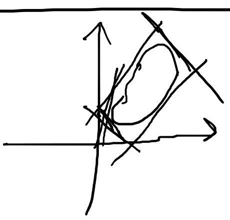
$$\phi_2 = \arctan\left(\frac{\frac{7}{3}}{-\frac{7}{2}}\right) = \arctan\left(-\frac{2}{3}\right)$$

$$u_3 = \frac{\sqrt{(\frac{7}{3})^2+(\frac{7}{2})^2}}{\frac{7}{3}\times\frac{7}{2}} \qquad u_4 = -\frac{\sqrt{(\frac{5}{3})^2+(\frac{5}{2})^2}}{\frac{5}{3}\times\frac{5}{2}}$$

$$\text{FrFT2}: O_F^{-\phi_2}\left(O_F^{\phi_2}(x(t))\ H(u)\right) \quad \text{for } H(u) = \begin{cases} 1 & \text{for } u_3 > u > u_4 \\ 0 & \text{other wise} \end{cases}$$

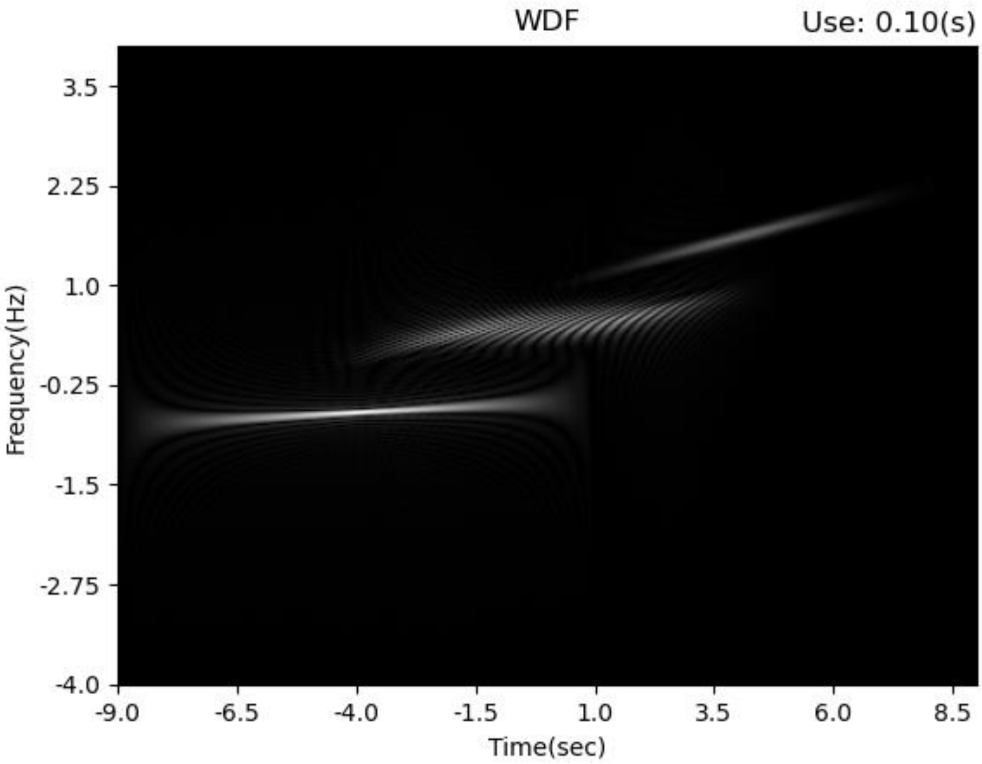
Use FrFT1 and FrFT2 to filter noise signal.

Bonus Question：依老師上課所述



需利用3次 FrFT Filter 來製造5條 cutoff line (2組平行 + 1條獨立)

程式題：


WDF
Use: 0.10(s)
3.5
2.25
1.0
-0.25
-1.5
-2.75
-4.0
Frequency(Hz)
-9.0
-6.5
-4.0
-1.5
1.0
3.5
6.0
8.5
Time(sec)