

1. Python NTT

Handle calculate large N

```
for i in range(N):
    for j in range(N):
        cell_val = 1
        for _ in range(int(i*j)):
            cell_val = (cell_val * a % M)
        fwd NTT[i, j] = cell_val
```

Result

```
Forward NTTs
[[1. 1. 1. 1.]
 [1. 2. 4. 3.]
 [1. 4. 1. 4.]
 [1. 3. 4. 2.]]
Inverse NTTs
[[4. 4. 4. 4.]
 [4. 2. 1. 3.]
 [4. 1. 4. 1.]
 [4. 3. 1. 2.]]
```

2. (a)

$$\text{DFT: } F[m] = \sum_{n=0}^{N-1} f[n] e^{-j \frac{2\pi}{N} mn} \quad \text{when } f[n] \text{ is real}$$

$$F[m] = F^*[N-m]$$

$$f_3[n] = f_1[n] + j f_2[n]$$

$$\bar{F}_3[m] = \bar{F}_1[m] + j \bar{F}_2[m] \quad \text{又} \quad \bar{F}_1[m] = F_1^*[N-m] \quad \bar{F}_2[m] = F_2^*[N-m]$$

$$\begin{aligned} \bar{F}_3[m] + \bar{F}_3^*[N-m] &= \bar{F}_1[m] + j \bar{F}_2[m] + F_1^*[N-m] - j F_2^*[N-m] \\ &= 2 F_1[m] \end{aligned}$$

$$\bar{F}_3[m] - \bar{F}_3^*[N-m] = j 2 F_2[m]$$

$$F_1[m] = \frac{\bar{F}_3[m] + \bar{F}_3^*[N-m]}{2}$$

$$F_2[m] = \frac{\bar{F}_3[m] - \bar{F}_3^*[N-m]}{2j} \quad \text{只需一個DFT}$$

(b) if $x_1[n]$ $x_2[n]$ is real and even
 $x_3[n]$ $x_4[n]$ is real and odd

When N is odd If $z_1[n] = -z_1[N-n]$
 and $z_2[n] = (-1)^n z_2[n]$
 then $z_2[n] = z_2[N-n]$

$$y[n] = \underbrace{x_1[n] + x_3[n]}_{y_1[n]} + j \underbrace{(x_2[n] + x_4[n])}_{y_2[n]} = y_1[n] + j y_2[n]$$

$$y_1[m] = \frac{y[m] - y^*[N-m]}{2}$$

$$y_1[m] + y_1[N-m] = x_1[m] + x_3[N-m] + \cancel{x_2[m]} + \cancel{x_4[N-m]} = 2x_1[m]$$

$$y_1[m] = x_1[m] + x_3[m]$$

$$x_1[m] = x_1[N-m]$$

$$x_3[m] = -x_3[N-m]$$

$$x_1[m] = \frac{y_1[m] + y_1[N-m]}{2}$$

$$x_2[m] = \frac{y_2[m] + y_2[N-m]}{2}$$

and use (a) two real input approach

we can use one DFT handling two odd and real
 and two even and real signals

3. (a) 23 row of 32 point Haar

17 18 19 20 21 22 23

$$\begin{bmatrix} 00 & 00 & 00 & 00 & 00 & 00 & 00 & 00 & | & -1 & 00 & 00 & 00 & 00 & 00 & 00 & 00 & 00 & 00 & 00 \end{bmatrix}$$

(b) Edge detection problem

4. Walsh Transform

proper (b) step like signal expansion

No multiplication required, and use 1, -1 as matrix component and step like shape.

(c) Modulation

can use the Modulation property

improper (a) LTI system can always expressed a convolution formae, but Walsh Transform may not be suitable handling convolution.

(d) localized feature extraction

Although the walsh transform can be used in feature extraction problem, but localized features are difficult

5.

(a) $16 \times \sqrt{16} = 64$ in Walsh

(b) Check whether use LUT

the maximum can use 256 addition

but can reduce to 64 addition use fast algorithm

even can be 0 use look up table

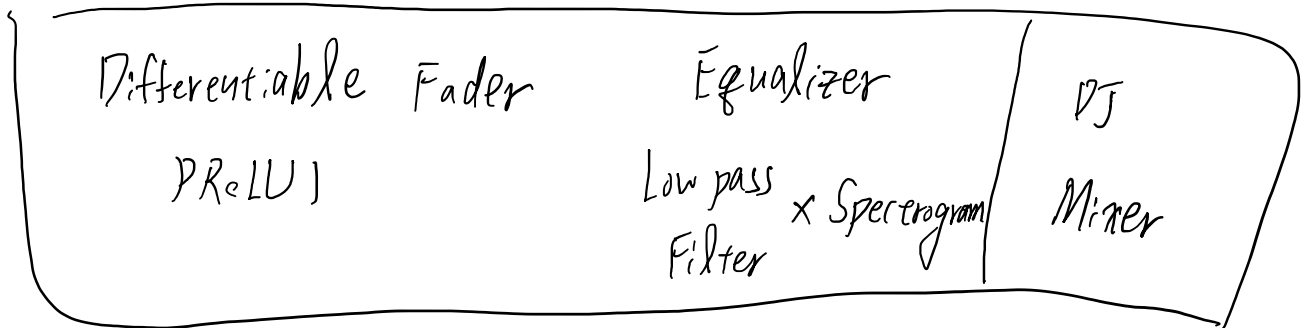
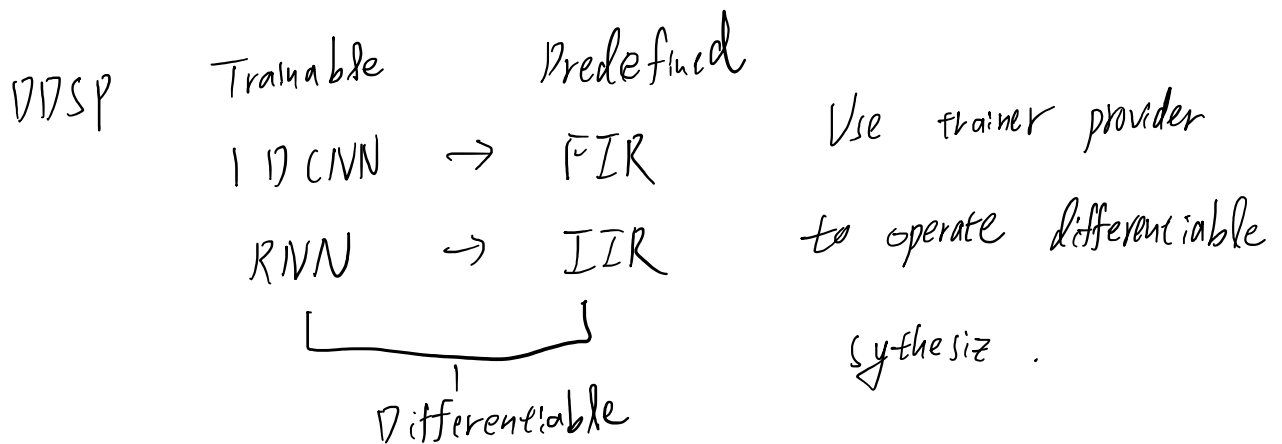
6. (1) orthogonal is useful characteristics

(2) OFDM can be performed by fast algorithm of FFT.

7.

(1) SSN characteristics is an critical issue in SI/PI simulation. In current IC packaging, we can use decoupling capacitors method to prevent SSN.

(2)



δ , $W_8 =$
 (a) $N=8$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \end{bmatrix}$$

$$N=16 \begin{bmatrix} W_8 & W_8 \\ W_8 & -W_8 \end{bmatrix}$$

0
 3
 4
 7
 11
 12
 1
 2
 5
 6
 9
 10
 14

row-0 $[1 1 1 1 1 1 1 1 1 1 1 1 1 1]$
 row-4 $[1 1 -1 -1 -1 -1 1 1 -1 -1 -1 -1 1 1]$
 row-9 $[1 -1 -1 1 1 -1 -1 -1 1 1 -1 -1 1 1]$
 200220020220 0220

$[200220020220 0220 \quad 1 1 -1 -1 -1 -1 1 1 \quad 1 1 -1 -1 -1 -1 1 1 \quad 22 0 0 0 0 22 22 0 0 0 0 22]$

(b)

NTT 因存在多個不同值的情況，所以在解調時會較難處理錯誤及延遲問題