TFW HW5 P10942A05 鄭閔軒

1、(a) admissibility criterion make continuous wavelet transform is reversible

(b) Scaling function simplify the inverse wavelet transform.

2、(a) $\frac{d^{10}}{dt^{10}} e^{-\pi t^2}$ $\int_{-\infty}^{\infty} t^k \psi(t)\, dt$ since $FT[t^k \psi(t)] = (\frac{j}{2\pi})^k \frac{d^k}{df^k} \psi(f)$

$= (-\frac{1}{2\pi j})^k (-2\pi j f)^{10} \frac{d^k}{df^k} f^{10} e^{-\pi f^2} \Big|_{f=0} = 0$ for $k<10$

Vanish moment is 10

(b) the vanish moment in sinc function is infinity

(c) the 12-point symlet vanish moment is $12/2 = 6$

(d) $H(f) = \frac{(1-e^{(-j2\pi f)})^4 \cos^2(\pi f)}{16} = \frac{1-4e^{-j2\pi f}+6e^{-j4\pi f}-4e^{-j6\pi f}+e^{-j8\pi f}}{16} \cos^2(\pi f)$

$\frac{1}{2} - \frac{3}{2} + \frac{3}{2} - \frac{1}{2}$

$\frac{d}{df} H \Big|_{f=0} = \left[0 - \frac{1}{4}(-2\pi j) + \frac{3}{8}(-4\pi j) - \frac{1}{4}(-6\pi j) + \frac{1}{16}(-8\pi j)\right] \cos^2(\pi f)' = 0$

$-1+6-9+4$

$\frac{d^2}{df^2} H \Big|_{f=0} = \left[-\frac{1}{4}(-2\pi j)^2 + \frac{3}{8}(-4\pi j)^2 - \frac{1}{4}(-6\pi j)^2 + \frac{1}{16}(-8\pi j)^2\right] = 0$

$\frac{d^3}{df^3} H \Big|_{f=0} = 2 - 24 + 54 - 32 = 0$ $\frac{d^4}{df^4} H \Big|_{f=0} = -4 + 96 - 324 + 256 \neq 0$

V.M. = 4

3、

1-D discrete wavelet transform time complexity

因輸入資料長度 $N >> L$，L代表濾波器之大小。

在此前提下可利用分段 convolution 去降低其時間

無利用分段 convolution 前時間複雜度為

$S(N_1+L-1)\log_2(N_1+L-1)$ 在 $N>>L$ 的前提下

$\approx SN_1\log_2(N_1+L-1)$ $\quad S=\frac{N}{N_1}$ 段

$= N\log_2(N_1+L-1)$

$\approx N\underbrace{\log_2 N_1}_{\text{constant}}$

So the complexity of the 1 WT is linear wit N

O(N)

4、

wavelet

(a) adaptive filter design

小波轉換可對於高低頻的信號，以高低通濾波做各別處理。並且可利用多次的 wavelet filter 的架構有效區分充滿雜訊的邊緣，來解決一般 low pass filter 造成邊緣平滑的問題。

b. image compression

如 JPEP 2000 就是以 wavelet 做 image compression

首先將影像轉至 YC_bC_r 然後利用 Discrete Wavelet transform 將影像分解出高低頻部份，存取其高頻部份資訊，因高頻部份為邊緣，通常占較少像素。所以只需存取其邊緣位置而非整張圖來達成影像壓縮的效果。

5、

(a) the advantage of symlet is the filter is more symmetic compare withe wavelet。 The shift after symlet transform is small.

(b) Coiflet's scaling function also has the vanish moment.
the scaling function is low-frequency like signal.

6、 $g[0]=\frac{4}{5}$ $g[1]=a$ $g[n]=0$

$$G(z)=\sum_{n=-\infty}^{\infty} g[n]z^{-n}=\frac{4}{5}+az^{-1}$$

(a) Quadratic Mirror

$$G^2(z)-G^2(-z)=2z^k \quad k \text{ is odd}$$

$$\left(\frac{4}{5}+az^{-1}\right)^2-\left(\frac{4}{5}-az^{-1}\right)^2=\frac{16}{5}az^{-1} \qquad a=\frac{5}{8}$$

b. orthonormal

$$G_1(z)G_1(z^{-1}) + G_1(-z)G_1(-z^{-1}) = 2$$

$$G_1(z) = G(z^{-1}) = \frac{4}{5} + az$$

$$\left(\frac{4}{5}+az\right)\left(\frac{4}{5}+az^{-1}\right) + \left(\frac{4}{5}-az\right)\left(\frac{4}{5}-az^{-1}\right)$$

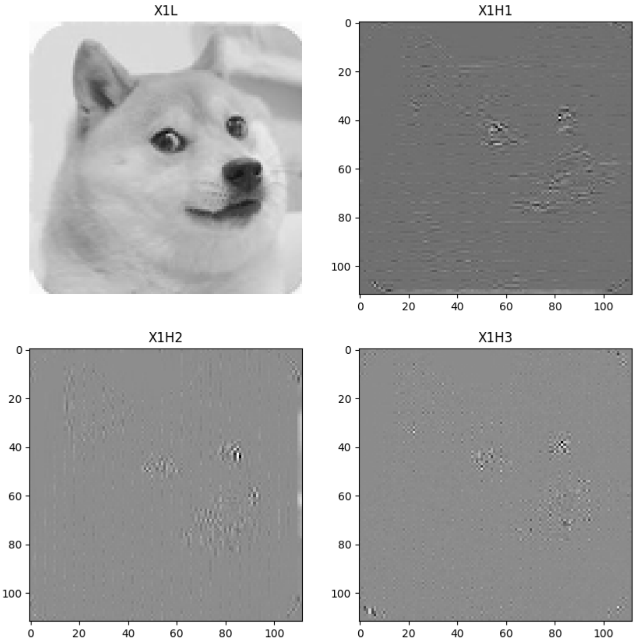
$$= \frac{16}{25} + \cancel{\frac{4}{5}az + \frac{4}{5}az^{-1}} + a^2 + \frac{16}{25} \cancel{- \frac{4}{5}az - \frac{4}{5}az^{-1}} + a^2$$

$$= \frac{32}{25} + 2a^2 = 2\left(a^2 + \frac{16}{25}\right) = 2$$

$$a^2 = \frac{9}{25} \qquad a = \pm\frac{3}{5}$$

7.

wavedbc10



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