TFW HW2 P10942A05 鄭閔軒

1. We choose larger σ due to the scaled Gabor transform is more sensitive in time domain, and person don't need too high resolution in frequency domain

2. (a) If $x(t) = \exp(-\pi t^2)$

$$\frac{-2\pi t}{2\pi} = -t$$

$$W_x(t,f) = \int_{-\infty}^{\infty} x(t+\tfrac{\tau}{2})\, x^*(t-\tfrac{\tau}{2})\, e^{-j2\pi f\tau}\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-\pi(t+\frac{\tau}{2})^2} e^{-\pi(t-\frac{\tau}{2})^2} e^{-j2\pi f\tau}\, d\tau$$

$$= e^{-2\pi f^2} \int_{-\infty}^{\infty} e^{-2\pi\frac{\tau^2}{4}} e^{-j2\pi f\tau}\, d\tau$$

$$= e^{-2\pi f^2} \int_{-\infty}^{\infty} e$$

(b) If $x(t) = \delta(2t-1)$

$$W_x(t,f) = \int_{-\infty}^{\infty} x(t+\tfrac{\tau}{2})\, x^*(t-\tfrac{\tau}{2})\, e^{-j2\pi f\tau}\, d\tau$$

$$= \int_{-\infty}^{\infty} \delta(2t+\tau-1)\, \delta(2t-\tau-1)\, e^{-j2\pi f\tau}\, d\tau \qquad (\tau \to -2t+1)$$

$$= \delta(4t-2)\, e^{-j2\pi f(-2t+1)} = \frac{1}{4}\delta(t-\tfrac{1}{2})\, e^{j4\pi ft} e^{-j2\pi f}$$

$$t_0 = \frac{1}{2}$$

$$= \frac{1}{4}\delta(t-\tfrac{1}{2})$$

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3.

	(a) complexity	(b) constraints	
Direct implement	$O(TFQ)$	all method need $\Delta t < \frac{1}{2(\Omega_x+\Omega_w)}$	no other constraints
DFT-based	$O(TN\log N)$		$\frac{1}{\Delta t \Delta f} = N$ must be integer, $N \geq 2Q+1 = \frac{2B}{\Delta t}+1$
Recursive	$O(TF)$		rectangular window, accumulated error
Chirp Z transform	$O(TN\log N)$		no other constraints. $2Q+1 = \frac{2B}{\Delta t}+1$

(C) Direct implement

DFT-based

chirp Z transform

4.

(a) windowed WDF cross term appear between t_1, t_2 when viewing case $x(t) = \delta(t-t_1)+\delta(t-t_2)$

so If we implement windows fuction, we can reduce select cross term area, then avoid cross term.

(b)

In Cohen's class distribution, the center of auto term will be $(0, 0)$, when $|t_2 - t_1|$, $|f_2 - f_1|$ are large, we can use low pass mask to avoid cross term

(C) First we use Gabor transform will not have cross term problem, second we multiply WDF the cross term area will multiply zero to avoid cross term.

5.

In Fourier transform

If $x(t) \leftrightarrow F(f)$ $\quad x^*(-t) \leftrightarrow F^*(f)$

when $F(f) = F^*(f)$ means $F(f)$ is real

In Cohen's class distribution

$$A_x(\tau,\eta)\Phi(\tau,\eta) = A_x^*(-\tau,-\eta)\Phi^*(-\tau,-\eta)$$

when fulfill $\Phi(\tau,\eta) = \Phi^*(-\tau,-\eta)$ will alway be real.

上課問答

WDF 在除了二個以上物件及 order 超過二次時
會導致 cross term problem