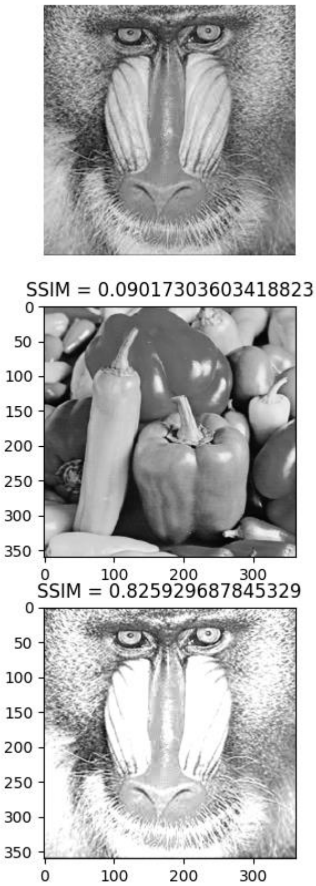
ADSP HW4 P10942A05 鄭閔軒

1. SSIM Result



(2)

$$X[m] = \sum_{n=0}^{4} \cos\left(\frac{\pi}{5} m \left(n+\frac{1}{2}\right)\right) x[n] \qquad \begin{array}{l} n=0,1,2,3,4 \\ m=0,1,2,3,4 \end{array}$$

5 point DCT with the least number of nontrivial multiplication

$$\begin{array}{c} \\ m=0 \\ m=1 \\ m=2 \\ m=3 \\ m=4 \end{array} \begin{array}{c} \begin{array}{ccccc} n=0 & n=1 & n=2 & n=3 & n=4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ \cos(\frac{\pi}{10})\ 18 & \cos\frac{3}{10}\pi\ 54 & \cos\frac{5}{10}\pi\ 90 & \cos\frac{7}{10}\pi\ 126 & \cos\frac{9}{10}\pi\ 162 \\ \cos(\frac{2}{10}\pi)\ 36 & \cos\frac{3}{5}\pi\ 108 & \cos\pi\ 180 & \cos\frac{7}{5}\pi\ 252 & \cos\frac{9}{5}\pi\ 324 \\ \cos(\frac{3}{10}\pi)\ 54 & \cos\frac{9}{10}\pi\ 162 & \cos\frac{3}{2}\pi\ 270 & \cos\frac{21}{10}\pi\ 18 & \cos\frac{27}{10}\pi\ 126 \\ \cos(\frac{4}{10}\pi)\ 72 & \cos\frac{6}{5}\pi\ 216 & \cos 2\pi\ 0 & \cos\frac{14}{5}\pi\ 144 & \cos\frac{18}{5}\pi\ 288 \end{bmatrix} \end{array}$$

$a = \cos 18°$

$b = \cos 54°$

$c = \cos 36°$

$d = \cos 72°$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 18° & 54° & 0 & -(54°) & -(18°) \\ +36° & 108° & -1 & 108° & 36° \\ 54° & -(18°) & 0 & 18° & -(54°) \\ 72° & 144° & 1 & 144° & 72° \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ a & b & 0 & -b & -a \\ c & -d & -1 & -d & c \\ b & -a & 0 & a & -b \\ d & -c & 1 & -c & d \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \cancel{x_2} \\ x_3 \\ x_4 \end{bmatrix}$$

$$\begin{bmatrix} a & b & -b & -a \\ c & -d & -d & c \\ b & -a & a & -b \\ d & -c & -c & d \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} a & b \\ b & -a \end{bmatrix} \begin{bmatrix} x_0 - x_4 \\ x_1 - x_3 \end{bmatrix} = \begin{bmatrix} b & b \\ b & b \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \begin{bmatrix} a-b & 0 \\ 0 & -a-b \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \quad \underline{3\text{muls}}$$

$$\begin{bmatrix} z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} c & d \\ d & c \end{bmatrix} \begin{bmatrix} x_0 + x_4 \\ -x_1 - x_3 \end{bmatrix} = 2\text{muls}$$

$$3 + 2 = \underline{5 \text{ multiplications}}$$

#

3.

x is complex number

$$e^{j\theta} = \cos\theta + j\sin\theta$$

If $c+jd = \exp(j\theta)$ $\quad c = \cos\theta \quad d = \sin\theta$

$$\begin{bmatrix} c & -d \\ d & c \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

when $\theta = \frac{\pi}{4}$ $\quad \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} + \begin{bmatrix} 0 & -\frac{2}{\sqrt{2}} \\ 0 & 0 \end{bmatrix}$

Include $(45 \times N)^\circ$ which $N = 1, 3, 5$ (odd number)

且 $\theta = \begin{cases} \cos^{-1}\left(\frac{1}{2^n}\right) & n \in \mathbb{Z} \\ \sin^{-1}\left(\frac{1}{2^n}\right) & n \in \mathbb{Z} \end{cases}$ 皆可藉由 bitwise operation

求解其中一對角之值

4、

220 , 231 , 245

(1) 220 points $2^2 \times 5 \times 11$

$$MUL_{220} = 55\, MUL_4 + 4\, MUL_{55}$$

$$= 55\, MUL_4 + 4\,(11\, MUL_5 + 5\, MUL_{11})$$

$$= 4 \times (\overset{110}{11 \times 10} + \overset{200}{5 \times 40})$$

$$= \underline{1240} \ \#$$

(2) 231 points

$11 \times 3 \times 7$

$$MUL_{231} = 77 MUL_3 + 3\,(11 MUL_7 + 7 MUL_{11})$$

$$= 77 \times 2 + 3\,(\overset{176+280}{11 \times 16 + 7 \times 40})$$

$$= 154 + 1368 = \underline{1522} \ \#$$

(3) 245 points

$$MUL_{245} = 49 MUL_5 + 5 MUL_{49}$$

$$= 49 \times 10 + 5\,(\underset{112}{7\overset{16}{MUL_7}} + \underset{112}{7 MUL_7} + \underset{108}{3 \times 6 \times 6})$$

$$= 490 + 1660 = \underline{2150} \ \#$$

5、

1. $O(n)$ computation complexity

2. fixed hardware architecture

(Only p_point DFT is required P is fixed

6.

$x_s[n] = x[n] * h[n]$ $h[1] = h[-1] = 0.24$ $h[2] = h[-2] = 0.06$

$h[3] = h[-3] = 0.03$ $h[0] = 0.34$ $h[n] = 0$ otherwise

$$x_s[n] = x[n] * h[n] = \sum_m x[n-m]\, h[m]$$

$$= 0.34^{①}\, x[0] + 0.03^{②} \left[8(h[1]+h[-1]) + 2(h[2]+h[-2]) + (h[3]+h[-3]) \right]$$

We only need 2 MUL. Bitwise operation in $n = \pm 1, \pm 2, \pm 3$ and combine them to reduce numbers of multiplication.

7. $len(x[n]) = 1100$ $\qquad$ $N+M-1$

(a) $length(y[n]) = 200$ $\qquad$ $3NM$

$P \geq 1299$ $\qquad$ $MUL_{1344} = 8252$

$2MUL_{1344} + 3 \times 1344 = 16504 + 4032$

$= \underline{20536}$ #

(b) $length(y[n]) = 20$

$S = ceil(N/L)$ $\qquad$ $P \geq L + M - 1$

$N = 1100$

$M = 20$

$L_0 = 105$

$P_0 = 124$

② $P = 84$ $\quad$ $34 \times 248 + 51 \times 84$

$L = 65$ $\quad$ $= 8432 + 4284$

$S = 17$ $\quad$ $= 12716$

① $P = 120$

$L = 101$

$S = \frac{N}{L} = 11$

$25 \times MUL_{120} + 35 \times 120$

$= 22 \times 380 + 33 \times 120$

$= 8360 + 3960 = 12320$

③ $P = 72$ $\quad$ $42 \times 164 + 63 \times 72$

$L = 53$ $\quad$ $= 6888 + 4536$

$S = 21$ $\quad$ $= \underline{11424}$ #

④ $P = 48$ $\quad$ $76 \times 92 + 114 \times 72$

$L = 29$

$S = 38$

(C)

P=16/24/28/36

length(y[n]) = 7 N=1100 M=7 L_0=25 P_0=31

① P=36 74×64+111×36
L=30 = 4736+3996
S = N/L = 37 = 8732

② P=28 100×64+150×28
L=22
S=50

③ P=24 124×28+186×24
L=18 =3472+4464
S=62 =7936 #

(d) length(y[n])=2

3×M×N = 3×2×1100 = 6600 #

Bonus 0.5

N=256 M=50 P=312 ≥ 305

We use 312-Point