2.1

$$\begin{aligned}
p(X=0) &= 0.18 \\
p(X=1) &= 0.27 \\
p(X=2) &= 0.34 \\
p(X=3) &= 0.14 \\
p(X=4) &= 0.07 \\
p(X>4) &= 0
\end{aligned} \tag{1}$$

2.5

(a) X: number of the packets in first slot
Y: number of the packets in second slot

$$Y = X - min\{X, c\}$$

for k=0,1,...,b-1

$$p_X(k) = e^{-\lambda\frac{\lambda^k}{k!}}$$

$$p_X(b) = \sum_{k=b}^{\infty} e^{-\lambda\frac{\lambda^k}{k!}} = 1 - \sum_{k=0}^{b-1} e^{-\lambda\frac{\lambda^k}{k!}}$$

$$p_Y(k) = p_X(k+c) = e^{-\lambda}\frac{\lambda^{k+c}}{(k+c)!}, \quad k = 1, \dots, b-c-1$$

$$p_Y(b-c) = p_X(b) = 1 - \sum_{k=0}^{b-1} e^{-\lambda}\frac{\lambda^k}{k!}$$

(b) It is equal to the probability of more than b packets being generated by the source

$$\sum_{k=b+1}^{\infty} e^{-\lambda}\frac{\lambda^k}{k!} = 1 - \sum_{k=0}^{b} e^{-\lambda}\frac{\lambda^k}{k!}$$

2.9

for k=1,...,n:

$$\frac{p_X(k)}{p_X(k-1)} = \frac{\begin{pmatrix} n \\ k \end{pmatrix} p^k(1-p)^{n-k}}{\begin{pmatrix} n \\ k-1 \end{pmatrix} p^{k-1}(1-p)^{n-k+1}} = \frac{(n+1)p - kp}{k - kp}$$

for $k <= k^*$

$$k - kp <= (n+1)p - kp$$

so the ratio is more than one and $p_X(k)$ is monotically nondecreasing
for $k > k^*$

$$k - kp > (n+1)p - kp$$

so the ratio is less than one and $p_X(k)$ is monotically decreasing

2.13

N: number of natural
G: number of girls
$G = N + 2$

$$p_N(k) = \begin{cases} \begin{pmatrix} 5 \\ k \end{pmatrix} \cdot (0.5)^5 & 0 \le k \le 5 \\ 0 & \text{else} \end{cases}$$

$$p_G(g) = \sum_{n:n+2=g} p_N(n) = p_N(g-2)$$

$$p_G(g) = \begin{cases} \begin{pmatrix} 5 \\ g-2 \end{pmatrix} \cdot (0.5)^5, & 2 \le g \le 7 \\ 0 & \text{else} \end{cases}$$

2.17

$$Y = 32 + \frac{9X}{5}$$

$$E[Y] = 32 + \frac{9E[X]}{5} = 32 + 18 = 50$$

$$var(Y) = (\frac{9}{5})^2 var(X)$$

$$var(X) = 100$$

so

$$var(Y) = 18$$

$$range = [50 - 18, 50 + 18] = [32, 68]$$

2.21

$$E[X] = \sum_{k=1}^{\infty} 2^k \cdot 2^{-k} = \sum_{k=1}^{\infty} 1 = \infty$$

the game has gain in any case
so it doesn't matter how much we pay

2.25

(a)

$$p_{I,J}(i,j) = \begin{cases} \frac{1}{\sum_{k=1}^{n} m_k}, & \text{if } j \le m_i \\ 0, & \text{otherwise.} \end{cases}$$

$$p_I(i) = \sum_{j=1}^{m} p_{I,J}(i,j) = \frac{m_i}{\sum_{k=1}^{n} m_k} \quad i = 1, \ldots, n$$

$$p_J(j) = \sum_{i=1}^{n} p_{I,J}(i,j) = \frac{l_j}{\sum_{k=1}^{n} m_k} \quad j = 1, \ldots, m$$

(b)

$$\sum_{j=1}^{m_i} \left(p_{ij} a + (1 - p_{ij})\, b\right)$$

2.41

(a)
$p = 0.02$
$n = 250$
$E[X] = np = 250 \cdot 0.02 = 5$

$$P(X = 5) = \binom{250}{5} (0.02)^5 (0.98)^{245} = 0.1773$$

(b) $\lambda = np = 5$

$$e^{-\lambda} \frac{\lambda^5}{5!} = 0.1755$$

(c) M: money paid in a year

$$E[M] = \sum_{i=1}^{5} 50 E\left[M_i\right]$$

$$P\left(M_i = m\right) = \begin{cases} 0.98 & \mathrm{m} = 0 \\ 0.01 & \mathrm{m} = 10 \\ 0.006 & \mathrm{m} = 20 \\ 0.004 & \mathrm{m} = 50 \end{cases}$$

$$E\left[m_i\right] = 0.01 \times 10 + 0.006 \times 20 + 0.004 \times 50 = 0.42$$

$$var\left(M_i\right) = E\left[M_i^2\right] - \left(E\left[M_i\right]\right)^2 = 13.22$$

$$E[M] = 250 E\left[M_i\right] = 105,$$

$$var(M) = 250 var\left(M_i\right) = 3305.$$

(d)

$$var = \frac{p(1-p)}{250}$$

p should be between [0,1] and satisfy:

$$(p - 0.02)^2 \leq \frac{25p(1-p)}{250}$$