3.1

$$p_Y(1) = P(X \leq \frac{1}{3}) = \frac{1}{3}$$
$$p_Y(2) = 1 - p_Y(1) = \frac{2}{3}$$
$$E[Y] = \frac{1}{3} \times 1 + \frac{2}{3} \times 2 = \frac{5}{3}$$
$$E[Y] = \int_0^1 g(x) f_X(x) dx = \int_0^{\frac{1}{3}} dx + \int_{\frac{1}{3}}^1 2dx = \frac{5}{3}$$

3.5

$$F_X(x) = 1 - P(X > x) = 1 - \frac{A_x}{A} = 1 - \frac{b(h-x)^2/(2h)}{bh/2} = 1 - \left(\frac{h-x}{h}\right)^2,$$
$$f_X(x) = \frac{dF_X}{dx}(x) = \begin{cases} \frac{2(h-x)}{h^2}, & \text{if } 0 \leq x \leq h, \\ 0, & \text{otherwise.} \end{cases}$$

3.13

$$P(Y \leq 59) = P(X \leq 15) = P\left(Z \leq \frac{15 - E[X]}{\sigma_X}\right) = P(Z \leq 0.5) = \Phi(0.5)$$
$$\Phi(0.5) = 0.6915$$
$$P(Y \leq 59) = 0.6915$$

2.21

$$f_{X|Y}(x \mid y) = 1/y, \text{ for } 0 \leq x \leq y \tag{1}$$

$$f_{X,Y}(x,y) = f_Y(y) f_{X|Y}(x \mid y) = \begin{cases} \frac{1}{l} \times \frac{1}{y}, & 0 \leq x \leq y \leq l \\ 0, & \text{otherwise.} \end{cases}$$
$$f_X(x) = \int f_{X,Y}(x,y) dy = \int_x^l \frac{1}{ly} dy = \frac{1}{l} \ln(l/x), \quad 0 \leq x \leq l.$$
$$E[X] = \int_0^l x f_X(x) dx = \int_0^l \frac{x}{l} \ln(l/x) dx = \frac{l}{4}$$
$$E[Y] = l/2$$
$$E[X/Y] = 1/2$$
$$E[X] = E[Y] E\left[\frac{X}{Y}\right] = \frac{l}{2} \cdot \frac{1}{2} = \frac{l}{4}$$

2.25

$A:\ X^2 + Y^2 \geq c^2$.

$$\begin{aligned}
P(A) &= \frac{1}{2\pi\sigma^2}\int_0^{2\pi}\int_a^{\infty} re^{-r^2/2\sigma^2}drd\theta \\
&= \frac{1}{\sigma^2}\int_a^{\infty} re^{-r^2/2\sigma^2}dr \\
&= e^{-a^2/2\sigma^2}
\end{aligned}$$

$$f_{X,Y|A}(x,y) = \frac{f_{X,Y}(x,y)}{P(A)} = \frac{1}{2\pi\sigma^2}e^{-\frac{1}{2\sigma^2}\left(x^2+y^2-c^2\right)}$$