سوال ۴ -

$L(\mu, \sigma \mid x_1, \dots, x_n)$ به دلیل استقلال

$$= L(\mu, \sigma \mid x_1)\, L(\mu, \sigma \mid x_2) \cdots L(\mu, \sigma \mid x_n)$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{x_1-\mu}{2\sigma^2}}$$

$$\xrightarrow[\text{لگاریتم بگیریم}]{Ln} -\frac{n}{2}\ln(2\pi) - n\ln(\sigma) - \sum \frac{(x_i-\mu)^2}{2\sigma^2}$$

$$\Rightarrow \begin{cases} \dfrac{\partial}{\partial \mu}\ln(L) = \dfrac{1}{\sigma^2}\left[(x_1+\cdots+x_n) - n\mu\right] \\ \\ \dfrac{\partial}{\partial \sigma}\ln(L) = \dfrac{-n}{\sigma} + \dfrac{1}{\sigma^3}\left[(x_1-\mu)^2 + \cdots (x_n-\mu)^2\right] \end{cases}$$

$$\Rightarrow \hat{\mu} = \frac{(x_1+\cdots+x_n)}{n} = \bar{X} \quad , \quad \hat{\sigma} = \sqrt{\frac{(x_1-\mu)^2+\cdots+(x_n-\mu)^2}{n}}$$

DATE / / SUBJECT:

$$x_1, \dots, x_n \sim N(\mu, \sigma^2)$$

$$\begin{cases} H_0 : \mu > \mu_0 \ (\circ) \\ H_1 : \mu \leqslant \mu_0 \ (\circ) \end{cases}$$

$$L = \left(\frac{1}{\frac{2\pi}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2} \right)^{\frac{n}{2}} e^{-\frac{n}{2}} \rightarrow \Lambda(x) = \left[\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{\sum_{i=1}^{n} (x_i - \mu_0)^2} \right]^{\frac{n}{2}}$$

$$= \frac{1}{1 + \frac{n(\bar{x} - \mu_0)^2}{(x_i - \bar{x})^2}} \leqslant c^{\frac{2}{n}} \rightarrow \frac{(\bar{x} - \mu_0)^2}{\frac{S^2}{n}} \geqslant (n-1)(c^{-\frac{2}{n}} - 1)$$

واریانس نمونه $\boxed{S^2}$ است

$$\xrightarrow{t\text{-Stat}} \frac{\bar{x} - \mu_0}{\frac{S}{\sqrt{n}}}$$

$$\rightarrow t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \rightarrow p \text{ value} = P(t_{n-1} \leqslant t_0)$$

رد : $[-\infty, -t_\alpha] = (-\infty, t_{1-\alpha}]$

پذیرش : $(-t_\alpha, +\infty) = (t_{1-\alpha}, +\infty)$