Task 1

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The Fourier transformation $f(x, y) \rightarrowtail F(u, v)$ of a greyscale image $f(x, y)$ results in a band-limited signal in the spatial frequency range with maximum frequencies f_{umax} and $_{vmax}$. For representation in the computer, the (partial) image is sampled in x direction with 20 sampling points per mm and in y direction with 10 sampling points per mm.

1. What is the theoretical maximum value of f_{umax} and f_{vmax} if error-free image reconstruction from the digital image should be possible (not using any compressive-sensing techniques)? (6pts)

According to the Nyquist sampling theorem, the maximum representable frequency (Nyquist frequency) in each direction is half the sampling frequency. The sampling frequency can be derived from the given sampling points per mm.

- Sampling frequency in x is f_{sx} and the Nyquist frequency in x is f_{umax}:

$$f_{sx} = 20 \text{ points/mm} = 20 \times 10^3 \text{ points/m}$$
$$\implies f_{umax} = \frac{f_{sx}}{2} = 10.0 \, \text{cycles/mm}$$

- Sampling frequency in y is f_{sy} and the Nyquist frequency in y is f_{vmax}:

$$f_{sy} = 10 \, \text{points/mm} = 10 \times 10^3 \, \text{points/m}$$
$$\implies f_{vmax} = \frac{f_{sy}}{2} = 5.0 \, \text{cycles/mm}$$

This ensures error-free reconstruction, as the digital image will contain all frequency components of the original image within the Nyquist limit. Frequencies above these limits would result in aliasing, violating error-free reconstruction conditions.

What is the minimum memory requirement for the color image $f_F(x, y)$ when stored in a conventional computer system, if 1024 values are to be distinguished per color channel. Describe the image format to be used.

To start lets find the number of ixels

Let the image dimensions in mm be L_x (width) and L_y (height).
- Pixels in x-direction: $N_x = 10.0 \cdot L_x$ - Pixels in y-direction: $N_y = 5.0 \cdot L_y$ - Total number of pixels:

$$N_{\text{pixels}} = N_x \cdot N_y = 50.0 \cdot L_x \cdot L_y$$

Each pixel in a color image has values for three color channels: Red, Green, and Blue (RGB). Each channel can store 1024 distinct values, which means $log_2^{1024} = 10.0$ bits per channel.

Total bits per pixel: $b = 10.0 \times 3 = 30.0$ bits/pixel.

The memory requirement is the product of the number of pixels and bits per pixel:

$$\text{Used Memory} = N_{\text{pixels}} \cdot b = (50.0 \cdot L_x \cdot L_y) \cdot b \, \text{bits} = 6.25 \cdot L_x \cdot L_y \cdot 30.0 \, \text{bytes} = 187.5 \cdot L_x \cdot L_y \, \text{bytes}$$

How many colors could be represented with the quantization chosen in sub-task 3? (2pts)

Each channel (Red, Green, and Blue) can represent 1024 intensity levels. With 10 bits per channel and 3 channels, the total number of colors is: [Total colors = 1024^3 = 1,073,741,824]