imaging_ai

Kasra Eskandarizanjani Mina Naseh Paria Ighanian

```python
import sys
sys.executable
```

```
'/opt/hostedtoolcache/Python/3.10.15/x64/bin/python3'
```

---

title: Task 1

Task 1

The Fourier transformation $f(x, y) \rightarrowtail F(u, v)$ of a greyscale image $f(x, y)$ results in a band-limited signal in the spatial frequency range with maximum frequencies f_{umax} and $_{vmax}$. For representation in the computer, the (partial) image is sampled in x direction with 20 sampling points per mm and in y direction with 10 sampling points per mm.

1. What is the theoretical maximum value of f_{umax} and f_{vmax} if error-free image reconstruction from the digital image should be possible (not using any compressive-sensing techniques)? (6pts)

According to the Nyquist sampling theorem, the maximum representable frequency (Nyquist frequency) in each direction is half the sampling frequency. The sampling frequency can be derived from the given sampling points per mm.

- Sampling frequency in x is f_{sx} and the Nyquist frequency in x is f_{umax}:

$$f_{sx} = 20\,\text{points/mm} = 20 \times 10^3\,\text{points/m}$$
$$\Longrightarrow f_{umax} = \frac{f_{sx}}{2} = 10.0\,\text{kHz}$$

- Sampling frequency in y is f_{sy} and the Nyquist frequency in y is f_{vmax}:

$$f_{sy} = 10\,\text{points/mm} = 10 \times 10^3\,\text{points/m}$$
$$\Longrightarrow f_{vmax} = \frac{f_{sy}}{2} = 5.0\,\text{kHz}$$

This ensures error-free reconstruction, as the digital image will contain all frequency components of the original image within the Nyquist limit. Frequencies above these limits would result in aliasing, violating error-free reconstruction conditions.

What is the minimum memory requirement for the color image $f_F(x, y)$ when stored in a conventional computer system, if 1024 values are to be distinguished per color channel. Describe the image format to be used.

To start lets find the number of ixels

Let the image dimensions in mm be L_x (width) and L_y (height).
- Pixels in x-direction: $N_x = 10.0 \cdot L_x$ - Pixels in y-direction: $N_y = 5.0 \cdot L_y$ - Total number of pixels:

$$N_{\text{pixels}} = N_x \cdot N_y = 50.0 \cdot L_x \cdot L_y$$

Each pixel in a color image has values for three color channels: Red, Green, and Blue (RGB). Each channel can store 1024 distinct values, which means $log_2^{1024} = 10.0$ bits per channel.

Total bits per pixel: $b = 10.0 \times 3 = 30.0$ bits/pixel.

The memory requirement is the product of the number of pixels and bits per pixel:

$$\text{Used Memory} = N_{\text{pixels}} \cdot b = (50.0 \cdot L_x \cdot L_y) \cdot b\,\text{bits} = 6.25 \cdot L_x \cdot L_y \cdot 30.0\,\text{bytes}$$

TODO: Im not sure about the answer, I think it should not depend on L_x and L_y.

How many colors could be represented with the quantization chosen in sub-task 3? (2pts)

Where is sub-task 3? :(

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title: “task 2”

Task 2

For the subjective enhancement of a greyscale image $G = g(x, y)$, a transformation T_G is performed as a so-called gamma correction in the form $T_G : g \to f$ with $f(x, y) = cg^{\gamma}(x, y)$ where $g, f \in [0, 255]$.

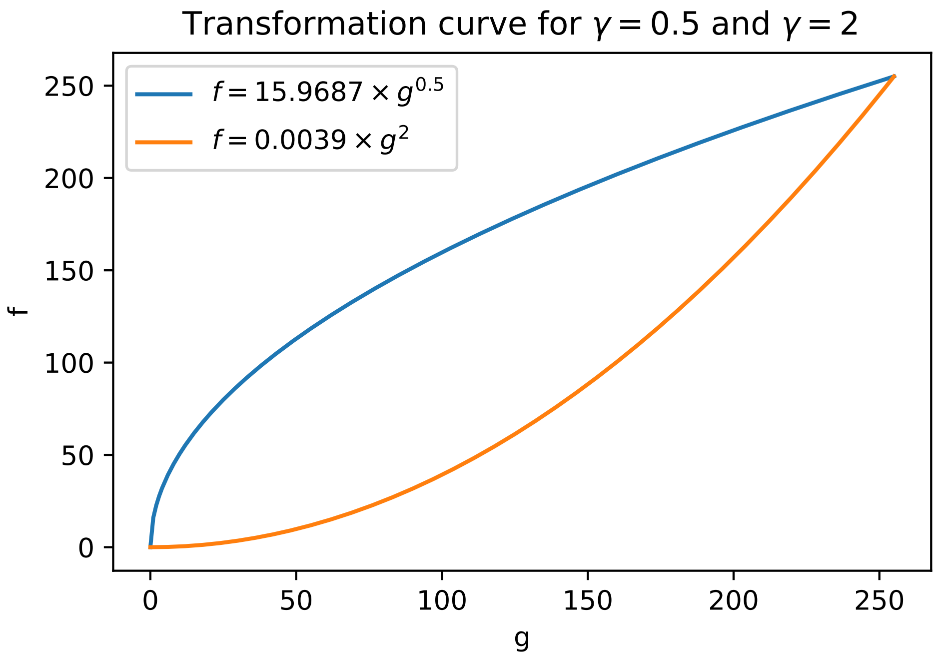
Sketch the transformation curve T_G for $\gamma_1 = 0.5$ and $\gamma_2 = 2$

The first step is to find the values of c for both cases. Since $\max(f) = \max(g) = 255$, we have $c = 255/255^{\gamma}$.

```
from matplotlib import pyplot as plt
import numpy as np


def draw_transform_curve(gamma: float, ax: plt.Axes = None, label: bool = True):
    if not ax:
        fig, ax = plt.subplots()
    x = np.linspace(0, 255, 256)
    c = 255 / 255**gamma
    y = c * x**gamma
    message = f"$f = {c:0.4f} \\times g^{{{gamma}}}$"
    if label:
        ax.plot(x, y, label=message)
    else:
        ax.plot(x, y)
    ax.set_xlabel("g")
    if label:
        ax.set_ylabel("f")
    else:
        ax.set_ylabel(message)


fig, ax = plt.subplots()
for gamma in [0.5, 2]:
    draw_transform_curve(gamma, ax)
ax.set_title(f"Transformation curve for $\\gamma=0.5$ and $\\gamma=2$")
ax.legend()
plt.show()
```



How is the coeficient c typically determined? (2pts)

The coefficient c is typically determined such that the maximum value of the input image is mapped to the maximum value of the output image. This is done to ensure that the full dynamic range of the output image is used.

As mentioned above, $c = 255/255^{\gamma}$.

In which respect and for which type of input images G do the two gamma values γ_1, γ_2 lead to an image enhancement respectively? (2pts)

For $\gamma < 1$, the transformation curve is concave, which means that the lower intensity values are stretched more than the higher intensity values. This leads to a brighter image with more contrast. This is useful for images with low contrast.

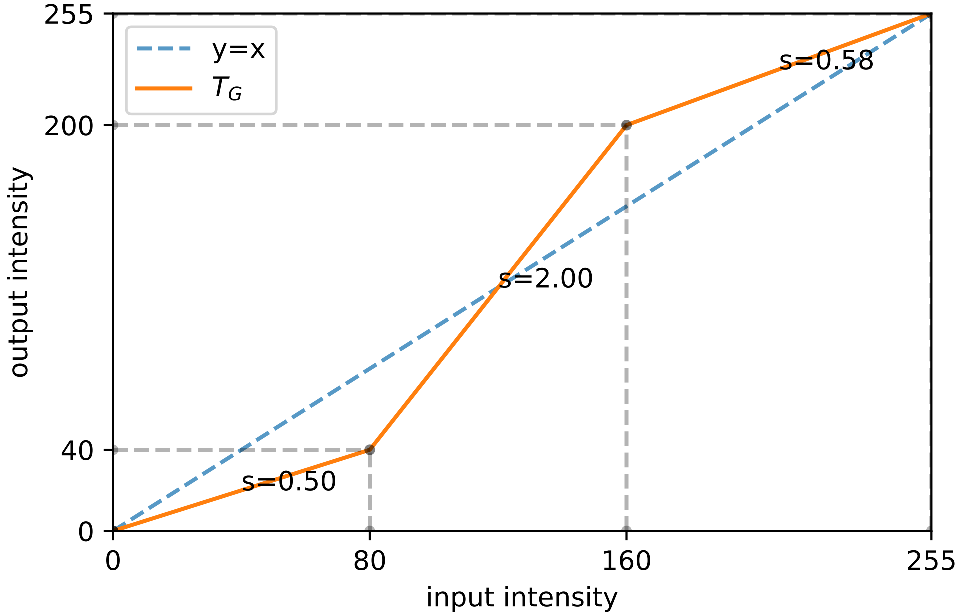
For $\gamma > 1$, the transformation curve is convex, which means that the higher intensity values are stretched more than the lower intensity values. This leads to a darker image with more contrast. This is useful for images with high contrast.

What should be the minimum slope of the transform function?

1. for a grey value spread (2pts)
2. for a grey value compression (2pts)

It's important to note that a slope of exactly 1 implies no change in contrast, as the transformation function becomes an identity mapping. Also, a slope of 0 implies that the output image will be a constant value, which is not useful for image enhancement.

2. For a grey value compression, the minimum slope of the transform function should be 0 (and smaller than 1). For instance, in this function:



As we can see, the gray values between `{python} spread_range[0]` are streched between `{python} spread_range[1]` which has a slope greater than 1. On the other hand, the gray values between `{python} compress_range[0]` are compressed between `{python} compress_range[1]` which has a slope smaller than 1.

---

title: "task 3" bibliography: references.bib # csl: nature.csl

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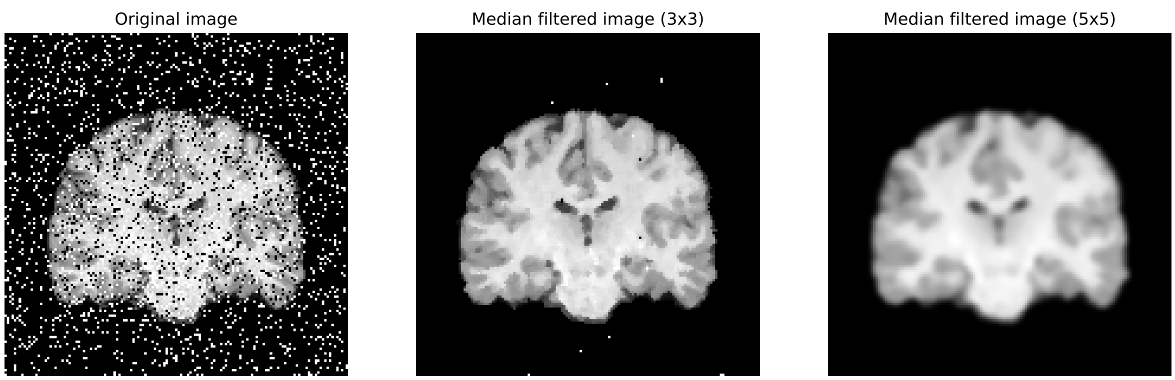
Task 3

In this task you will need to perform threshold-based image analysis:

Read the greyscale image brain.png, which is provided on the lecture homepage. Reduce the salt and pepper noise in the image using a median filter. (3pts)

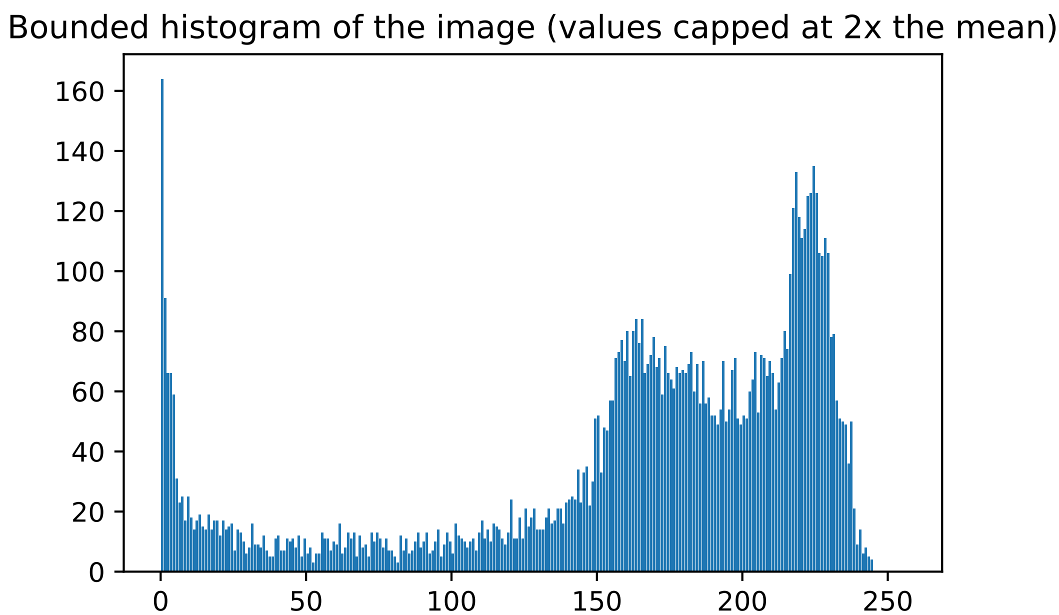
```python
import cv2
import numpy as np
import matplotlib.pyplot as plt

img_noise = cv2.imread("brain-noisy.png", cv2.IMREAD_GRAYSCALE)
assert img_noise is not None, "Image not found"
img = cv2.medianBlur(img_noise, 5)
img = cv2.GaussianBlur(img, (5, 5), 0)

fig, ax = plt.subplots(1, 3, figsize=(15, 5))
ax[0].imshow(img_noise, cmap="gray")
ax[0].set_title("Original image")
ax[0].axis("off")
ax[1].imshow(cv2.medianBlur(img_noise, 3), cmap="gray")
ax[1].set_title("Median filtered image (3x3)")
ax[1].axis("off")
ax[2].imshow(img, cmap="gray")
ax[2].set_title("Median filtered image (5x5)")
ax[2].axis("off")

plt.show()
```



As we can see the kernel size of 3×3 is not enough to remove the noise, while the kernel size of 5×5 is sufficient.

Otsu thresholding is a histogram-based method for image segmentation. Use it to find an intensity threshold to segment brain pixels from background. Use Otsu thresholding again to find the threshold only over the brain pixels to segment brain's grey matter from the white matter. Using the two thresholds create three binary masks brain-bg.png, brain-gm.png, brain-wm.png, which should be white in regions of background, grey matter, and white matter, respectively, and black elsewhere. (4pts)

```python
values, bin_edge = np.histogram(img, bins=256, range=(0, 256))
bin_centers = (bin_edge[:-1] + bin_edge[1:]) / 2
# values = values[1:]
# bin_centers = bin_centers[1:]
m = values.mean() * 2
values[values > m] = m

plt.bar(bin_centers, values, lw=2)
plt.title("Bounded histogram of the image (values capped at 2x the mean)")
plt.show()
```



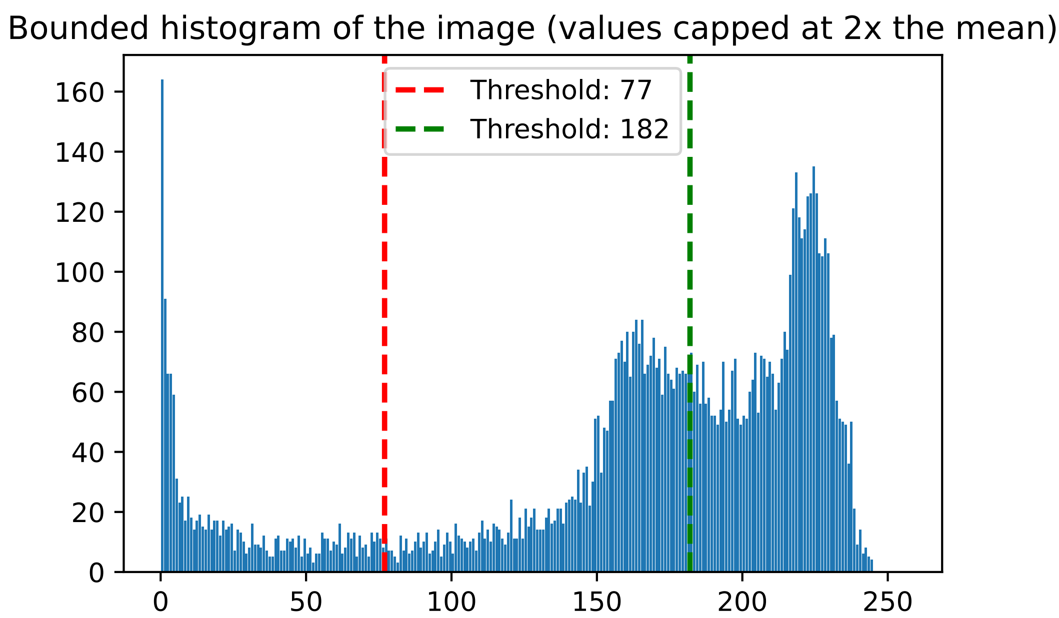
The correct way to use Otsu thresholding with several values is to use (Arora et al. 2008), which is not implemented in OpenCV. However, we can use the implementation in the `skimage` library (which implemented based on (Liao et al. 2001))

```python
from skimage.filters import threshold_multiotsu


def otsu_threshold(
    img: np.ndarray, classes: int
) -> tuple[list[np.ndarray], np.ndarray]:
    threshold = threshold_multiotsu(img, classes=classes).tolist()
    threshold = [0] + threshold + [255]
```

```python
    assert (
        len(threshold) == classes + 1
    ), "The number of thresholds should be equal to the number of classes - 1"
    masks = [(img >= t1) & (img < t2) for t1, t2 in zip(threshold, threshold[1:])]
    # masks.append(img >= threshold[-1])
    assert all(mask.dtype == np.bool for mask in masks), "Masks should be boolean"
    assert (
        len(masks) == classes
    ), "The number of masks should be equal to the number of classes"
    return masks, threshold[1:-1]


(brain_bg, brain_gm, brain_wm), threshold = otsu_threshold(img, 3)
```

```python
colors = ["r", "g", "y"]
(brain_bg, brain_gm, brain_wm), threshold = otsu_threshold(img, 3)

print(f"Threshold for the whole image: {threshold}")

values, bin_edge = np.histogram(img, bins=256, range=(0, 256))
bin_centers = (bin_edge[:-1] + bin_edge[1:]) / 2
m = values.mean() * 2
values[values > m] = m

plt.bar(bin_centers, values, lw=2)
for th, color in zip(threshold, colors):
    plt.axvline(th, color=color, lw=2, ls="--", label=f"Threshold: {th}")
plt.legend()
plt.title("Bounded histogram of the image (values capped at 2x the mean)")
plt.show()
```

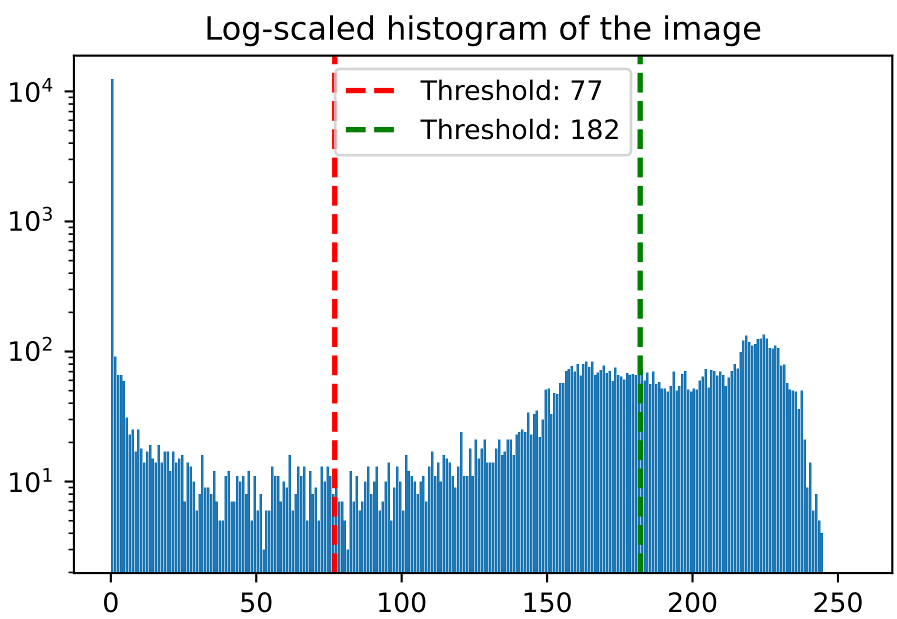
```
Threshold for the whole image: [77, 182]
```




```python
fig, ax = plt.subplots(1, 4, figsize=(15, 5))
ax[0].imshow(brain_bg, cmap="gray")
ax[0].set_title("Background")
ax[0].axis("off")

ax[1].imshow(brain_gm, cmap="gray")
ax[1].set_title("Grey matter")
ax[1].axis("off")

ax[2].imshow(brain_wm, cmap="gray")
ax[2].set_title("White matter")
ax[2].axis("off")

ax[3].imshow(brain_bg * 1 + brain_gm * 2 + brain_wm * 3, cmap="gray")
ax[3].set_title("All")
ax[3].axis("off")

plt.show()
```

Plot a log-scaled histogram of the image, which should show how frequently different intensity values occur in the image. How could you roughly estimate the two thresholds you found in the previous task just by looking at the histogram? (3pts)

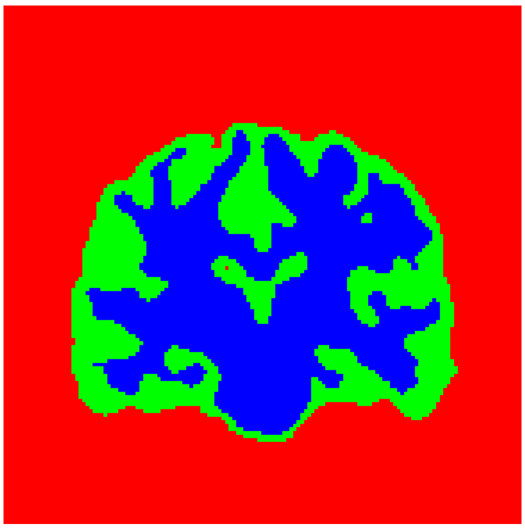
```
values, bin_edge = np.histogram(img, bins=256, range=(0, 256))
bin_centers = (bin_edge[:-1] + bin_edge[1:]) / 2
plt.bar(bin_centers, values, lw=2)
plt.yscale("log")

for th, color in zip(threshold, colors):
    plt.axvline(th, color=color, lw=2, ls="--", label=f"Threshold: {th}")

plt.legend()
plt.title("Log-scaled histogram of the image")
plt.show()
```



As we can see, the histogram has two peaks, which correspond to the grey matter and white matter. The two thresholds can be estimated by finding the two peaks in the histogram. (The purpose of otsu thresholding is to find the optimal threshold for the two peaks)

Combine the three masks into a single colour image so that background, grey matter, and white matter are mapped to red, green and blue, respectively. (3pts)

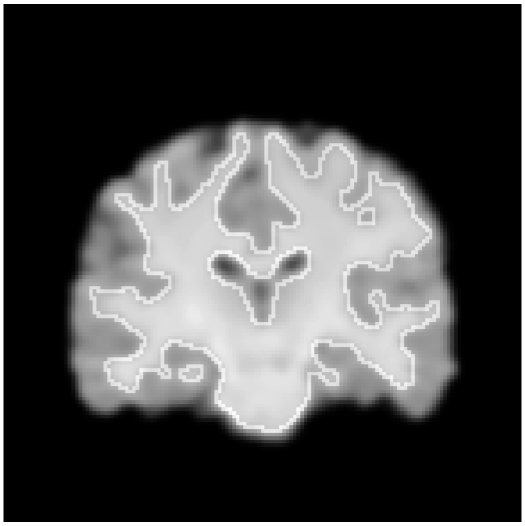
```
combined_brain = np.stack([brain_bg, brain_gm, brain_wm], axis=-1).astype(np.uint8) * 255

plt.imshow(combined_brain)
plt.axis("off")
plt.show()
```



Use erosion (or any other morphological) filter to produce a border between the grey and white matter. Overlay that border on the denoised input image. (3pts)

```
kernel = np.ones((3, 3), np.uint8)
brain_wm_eroded = cv2.erode(brain_wm.astype(np.uint8), kernel, iterations=1)
brain_wm_dilated = cv2.dilate(brain_wm_eroded, kernel, iterations=1)
border = (brain_wm_dilated - brain_wm_eroded) * 255
alpha = 0.85
bordered_img = cv2.addWeighted(img, alpha, border, 1 - alpha, 0)


# plt.imshow(img, cmap="gray")
# plt.imshow(border, cmap="gray", alpha=0.5)
plt.imshow(bordered_img, cmap="gray")
```

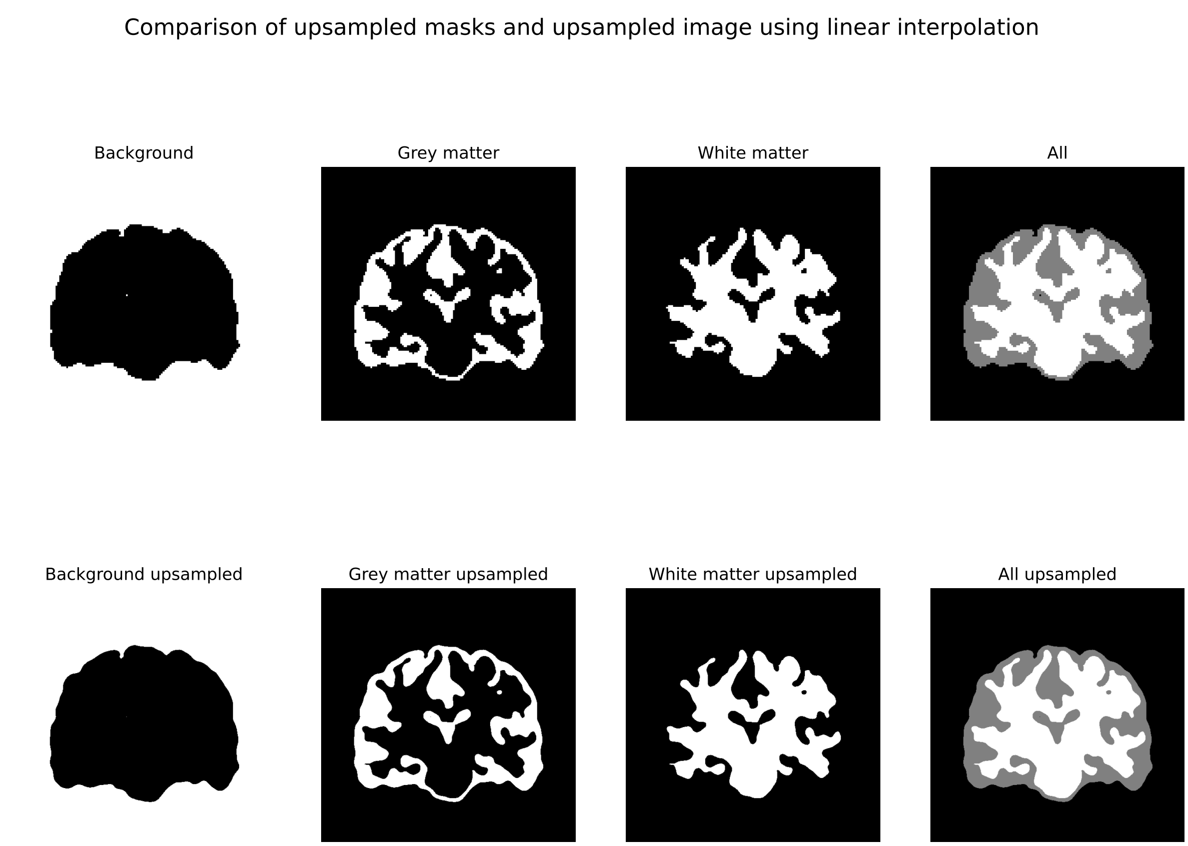
```
plt.axis("off")
plt.show()
```



Use bilinear interpolation to up-sample the image by a factor of four along each axis. Apply the same thresholds as in 2) to obtain a segmentation into background, grey matter, and white matter. Up-sample the masks from 2) in the same way and compare the up-sampled masks to the masks from the up-sampled image. Can you see a diference? Why? Repeat the same procedure using nearest neighbour interpolation. Can you see a diference now? (4pts)

```
def upsample(img: np.ndarray, factor: int, interpolation: int) -> np.ndarray:
    return cv2.resize(
        img, (img.shape[1] * factor, img.shape[0] * factor), interpolation=interpolation
    )


masks, threshold = otsu_threshold(img, 3)
img_upsampled = upsample(img, 4, cv2.INTER_LINEAR)
masks_upsampled, threshold_upsampled = otsu_threshold(img_upsampled, 3)

fig, ax = plt.subplots(2, 4, figsize=(15, 10))
fig.suptitle(
    "Comparison of upsampled masks and upsampled image using linear interpolation",
    fontsize=16,
)

titles = ["Background", "Grey matter", "White matter", "All"]
```

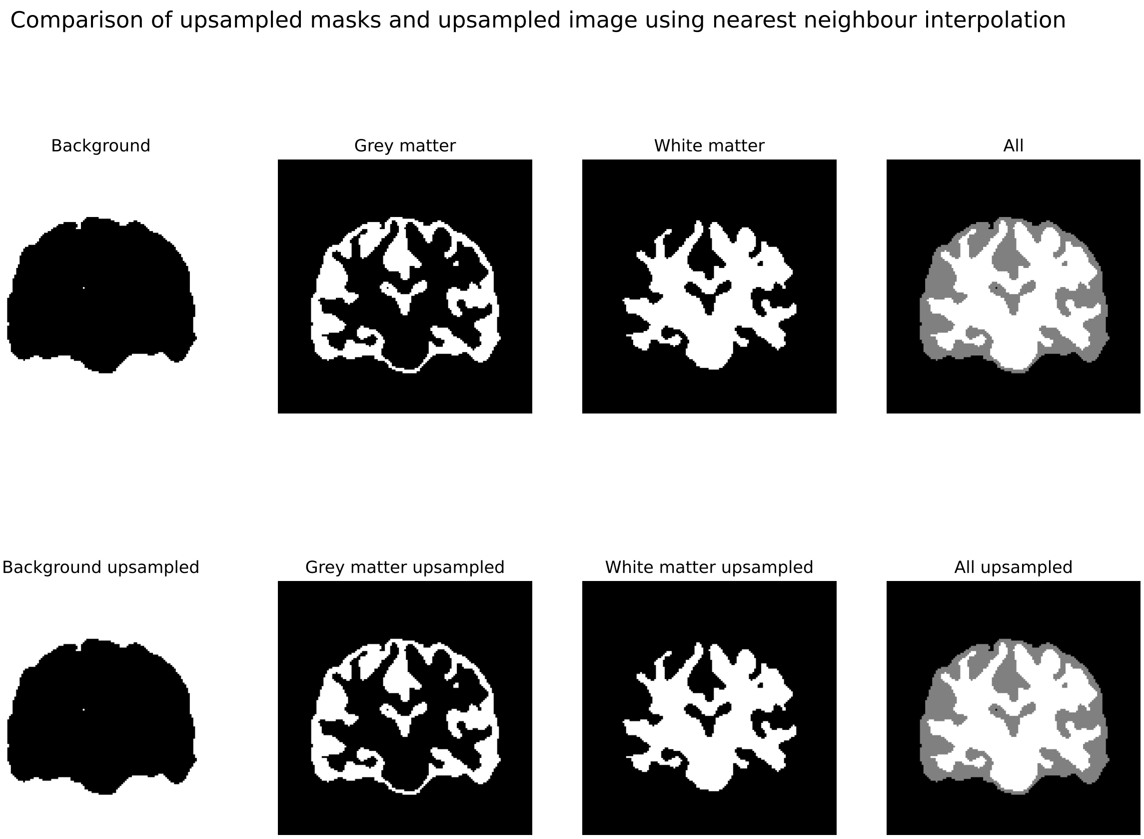
```python
masks.append(masks[0] * 1 + masks[1] * 2 + masks[2] * 3)
masks_upsampled.append(
    masks_upsampled[0] * 1 + masks_upsampled[1] * 2 + masks_upsampled[2] * 3
)

for i, (mask, mask_upsampled, title) in enumerate(zip(masks, masks_upsampled, titles)):
    ax[0, i].imshow(mask, cmap="gray")
    ax[0, i].set_title(title)
    ax[0, i].axis("off")

    ax[1, i].imshow(mask_upsampled, cmap="gray")
    ax[1, i].set_title(f"{title} upsampled")
    ax[1, i].axis("off")

plt.show()
```



Clearly, we can see much smoother edges in the upsampled masks compared to the upsampled image. This is because the interpolation method used in the up-sampling process is linear, which smooths the edges.

Now, let's repeat the same procedure using the nearest neighbour interpolation method.

```
masks, threshold = otsu_threshold(img, 3)
img_upsampled = upsample(img, 4, cv2.INTER_NEAREST)
masks_upsampled, threshold_upsampled = otsu_threshold(img_upsampled, 3)

fig, ax = plt.subplots(2, 4, figsize=(15, 10))
fig.suptitle(
    "Comparison of upsampled masks and upsampled image using nearest neighbour interpolation"
    fontsize=16,
)

titles = ["Background", "Grey matter", "White matter", "All"]
masks.append(masks[0] * 1 + masks[1] * 2 + masks[2] * 3)
masks_upsampled.append(
    masks_upsampled[0] * 1 + masks_upsampled[1] * 2 + masks_upsampled[2] * 3
)

for i, (mask, mask_upsampled, title) in enumerate(zip(masks, masks_upsampled, titles)):
    ax[0, i].imshow(mask, cmap="gray")
    ax[0, i].set_title(title)
    ax[0, i].axis("off")

    ax[1, i].imshow(mask_upsampled, cmap="gray")
    ax[1, i].set_title(f"{title} upsampled")
    ax[1, i].axis("off")

plt.show()
```



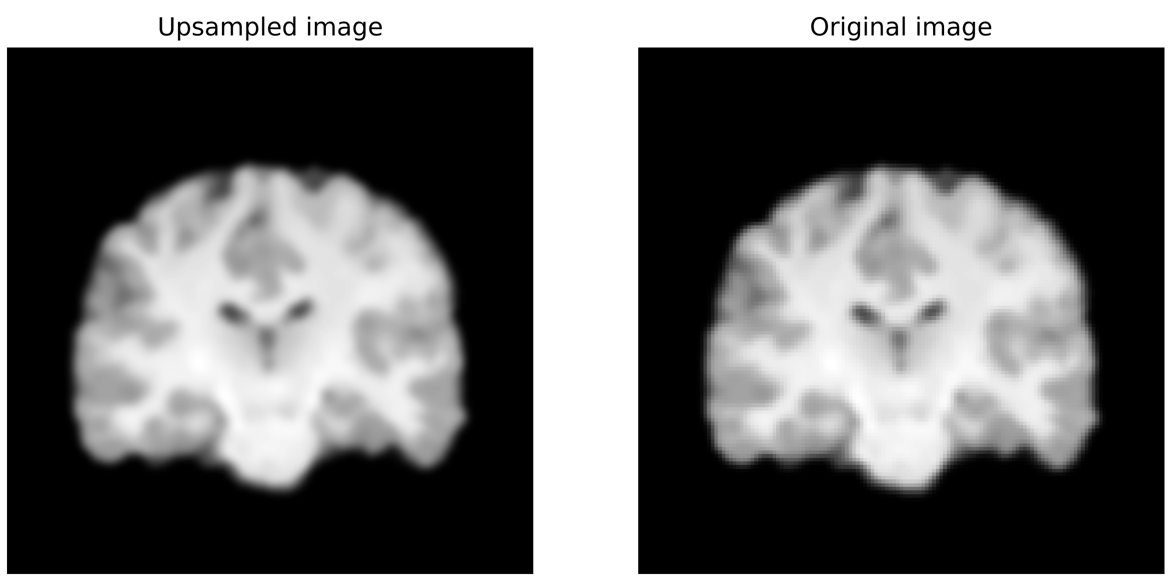
We can see the edges are much sharper in the upsampled masks compared to the upsampled image. This is because the nearest neighbour interpolation method does not smooth the edges.

TODO: Test same thing with pyrUp

```
def upsample_pyramid_(img: np.ndarray) -> np.ndarray:
    # return cv2.resize(
    #     img, (img.shape[1] * factor, img.shape[0] * factor), interpolation=interpolation
    # )
    return cv2.pyrUp(img, dstsize=(img.shape[1] * 2, img.shape[0] * 2))

def upsample_pyramid(img: np.ndarray, factor: int) -> np.ndarray:
    if factor <= 1:
        raise ValueError("Factor should be greater than 1")
    f = 1
    while f < factor:
        img = upsample_pyramid_(img)
        f *= 2
```

```
    return img


print(img.shape)
img_upsampled = upsample_pyramid(img, 4)
print(img_upsampled.shape)
fig, ax = plt.subplots(1, 2, figsize=(10, 5))
ax[0].imshow(img_upsampled, cmap="gray")
ax[0].set_title("Upsampled image")
ax[0].axis("off")

ax[1].imshow(img, cmap="gray")
ax[1].set_title("Original image")
ax[1].axis("off")

plt.show()
```

```
(145, 145)
(580, 580)
```



Arora, Siddharth, Jayadev Acharya, Amit Verma, and Prasanta K Panigrahi. 2008. "Multilevel Thresholding for Image Segmentation Through a Fast Statistical Recursive Algorithm." *Pattern Recognition Letters* 29 (2): 119–25.

Liao, Ping-Sung, Tse-Sheng Chen, Pau-Choo Chung, et al. 2001. "A Fast Algorithm for Multilevel Thresholding." *J. Inf. Sci. Eng.* 17 (5): 713–27.