GRÖBNER BASIS FOR POLYNOMIAL SYSTEMS IN ROBOTICS

Mina Khan[1], Frank Permenter[2] and Russ Tedrake[3]

[1]Mount Holyoke College [2, 3]Electrical Engineering and Computer Science, Massachusetts Institute of Technology

MSRP MIT Summer Research Program

CSAIL

Motivation: ***To efficiently find exact solutions to multivariate polynomial equations in robot control and motion planning.***

I. ROBOT CONTROL

- Robot control involves solving polynomial equations in inverse kinematics and dynamics problems for robots.
- Kinematics
  - ⬇
  - Positions
- Dynamics
  - ⬇
  - Forces

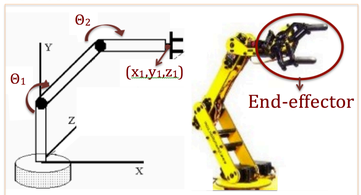


Fig 1: The serial N-link manipulator model (*left*) for a robotic arm (*right*)

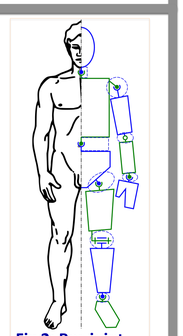


Fig 2: Bar-joint model for human

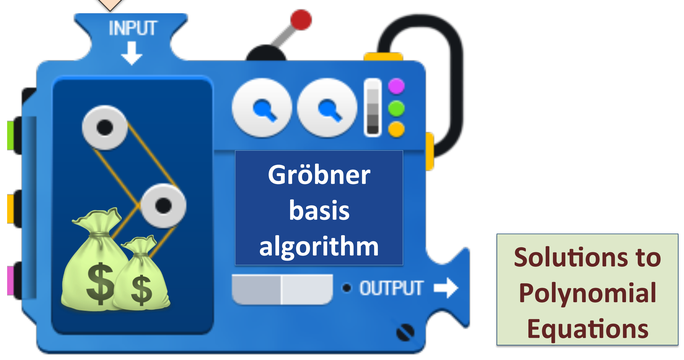
II. GRÖBNER BASIS

$$f_1(x_1, x_2, x_3, \ldots, x_n) = 0$$
$$f_2(x_1, x_2, x_3, \ldots, x_n) = 0$$
$$\vdots$$
$$f_m(x_1, x_2, x_3, \ldots, x_n) = 0$$

Fig 3: Gröbner basis algorithms

✔ Gröbner basis can be used to solve multivariate polynomial equations.

✖ Gröbner basis algorithms have double exponential worst-case complexity.



- Gröbner basis algorithms generalize Gaussian elimination for multivariate polynomials.
- Alternatives to Gröbner basis algorithms include numerical methods (such as homotopy-continuation) that do not reliably yield exact solutions.

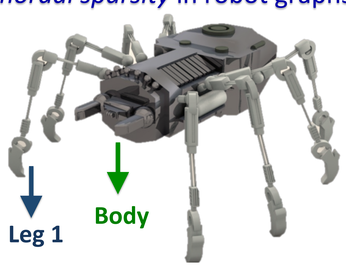
III. ROBOT STRUCTURES

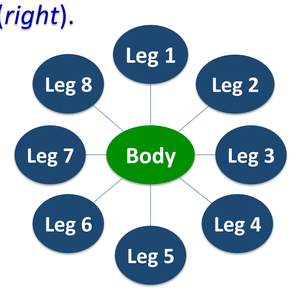
- Computations for sparse matrices may be simpler than computations for dense matrices.

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{vmatrix} = 24$$

Determinant of a sparse diagonal matrix

- Fig 4: We created a multi-leg spider robot (*left*). Each spider leg is connected to the body, but not to other legs, yielding *chordal sparsity* in robot graphs (*right*).





IV. COMPUTATIONAL METHODS

- We analyze Gröbner basis algorithms that leverage the sparsity of the polynomial matrices.
- Software packages that have slow Gröbner basis computations

Maple 18 MATLAB Macaulay2

- Sage is math software based on Python. Sage uses Gröbner basis algorithms in Singular and Magma, and most of these algorithms are in C/C++.
- **Sage allows fast Gröbner basis computations**

sage python MAGMA COMPUTER•ALGEBRA C C++ SINGULAR

- Gröbner basis algorithms include Buchberger's algorithm, Faugère's F4, F5 and FGLM.

V. FAST COMPUTATIONS

- We achieved a *50 milliseconds* computation time for Gröbner basis for a system of a spider robot with 32 legs. This system had *128 polynomials in 67 variables* and we did the computations in Sage using a 1.3 GHz processor.

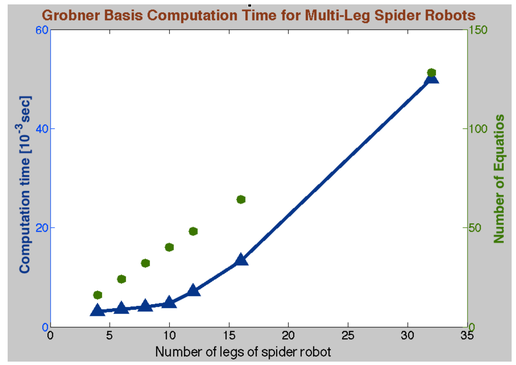


Fig 6: Computation time for Gröbner basis using Sage. The polynomial systems for the spider robot had 16 to 128 equations.

VI. FUTURE WORK

- Apply our advances in Gröbner basis computations to improve the performance of robots, especially Boston Dynamics "Atlas" robot. We are currently integrating Sage's algorithms in our existing MATLAB polynomial solvers.
- Investigate chordal graphs to further leverage polynomial structure in Gröbner basis algorithms.

Fig 6: Atlas humanoid robot

Acknowledgements

Diego Cifuentes, Robot Locomotion Group and MSRP Staff

Picture Courtesy

ROSARIO, J. M. et al. Proposal of methodology for the modeling and control of manipulators. J. Braz. Soc. Mech. Sci. 2002, vol.24, n.3 (Fig 1); http://www.societyofrobots.com/robot_arm_tutorial.shtml (Fig 1); "Modele cinematique corps humain" Wikimedia Commons (Fig 2); http://www.mathwarehouse.com/algebra/relation/evaluating-function.php (Fig 3); http://physicssum.deviantart.com/art/3D-Robot-Spider-280498884 (Fig 4); Logos: MATLAB, Maple, Macaulay2, Sage, Python, Magma, Singular, C/C++ (http://www.techhui.com/group/candcppdevelopers); http://www.fairfaxunderground.com/forum/file.php?40,file=113550,filename=Atlas-p2_nt.jpg (Fig 6)