GRÖBNER BASES FOR POLYNOMIAL SYSTEMS IN ROBOTICS

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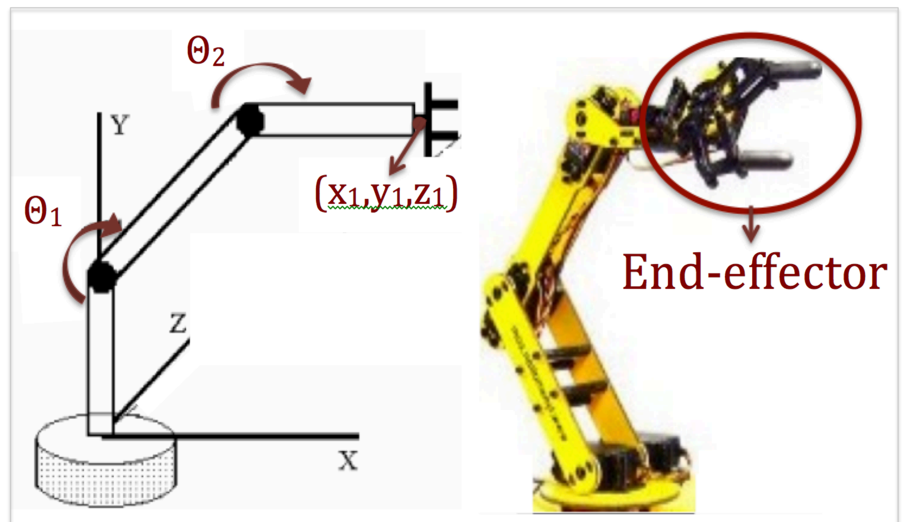
CSAIL

Motivation: *Efficiently* find *exact* solutions to multivariate polynomial equations in robot control and motion planning.

I. ROBOT CONTROL

- Robot control involves solving polynomial equations in inverse kinematics (positions) and dynamics (forces) problems for robots.



Serial N-link manipulator model (left) for a robot arm (right)

II. GRÖBNER BASES

✔ Gröbner bases can be used to find solutions to systems of multivariate polynomial equations.

✖ Gröbner basis algorithms have double exponential worst-case complexity, so we aim to find efficient Gröbner basis algorithms.

Polynomial System

$f_1(x1, x2, x3,....,xn) = 0$
$f_2(x1, x2, x3,....,xn) = 0$
$f_2(x1, x2, x3,....,xn) = 0$
:
$f_m(x1, x2, x3,....,xn) = 0$

Gröbner basis

$g_1(x1) = 0$
$g_2(x1, x2) = 0$
$g_3(x1, x2, x3) = 0$
:
$g_m(x1, x2, x3,....,xn) = 0$

INPUT → Gröbner basis algorithm → OUTPUT

When a system has a unique solution, Gröbner basis has at least one univariate equation and subsequent equations with an increasing number of variables[1]. Thus, we can solve Gröbner basis easily by back-substitution.

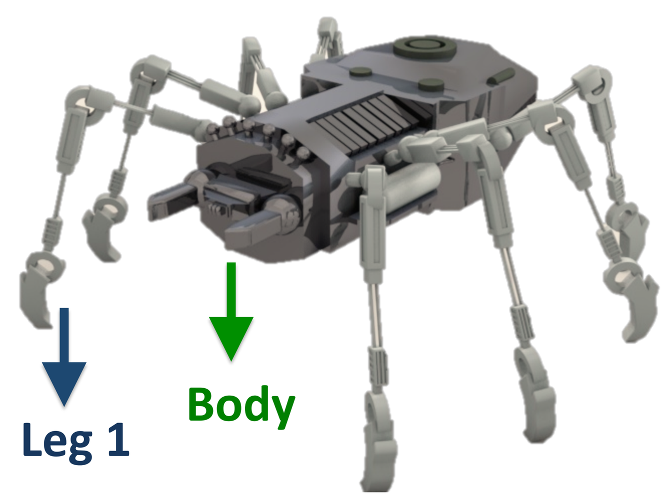
III. ROBOT STRUCTURES

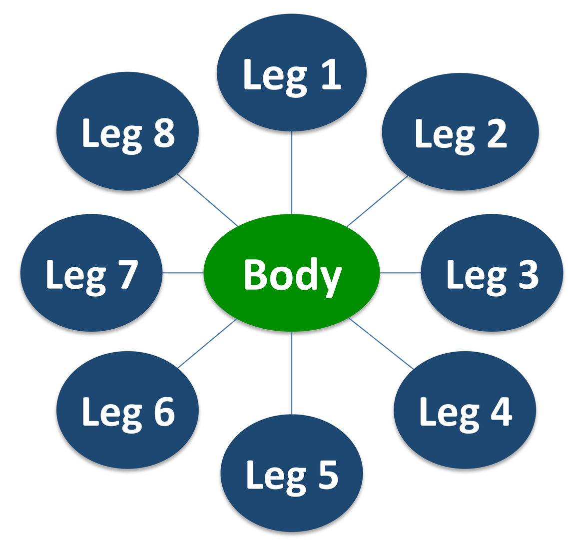
- Sparse matrix computations are simpler than dense matrix computations.

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{vmatrix} = 24$$

Determinant of a sparse diagonal matrix

We created a spider robot (left). Each spider leg is connected to the body, but not to other legs, yielding chordal sparsity in robot graphs (right).





IV. COMPUTATIONS

- Gröbner basis algorithms include Buchberger's algorithm, Faugère's F4, F5 and FGLM.

✖ Maple, MATLAB and Macaulay2 *cannot* compute Gröbner basis for a polynomial system representing even one robotic spider leg.

Maple 18 MATLAB Macaulay2

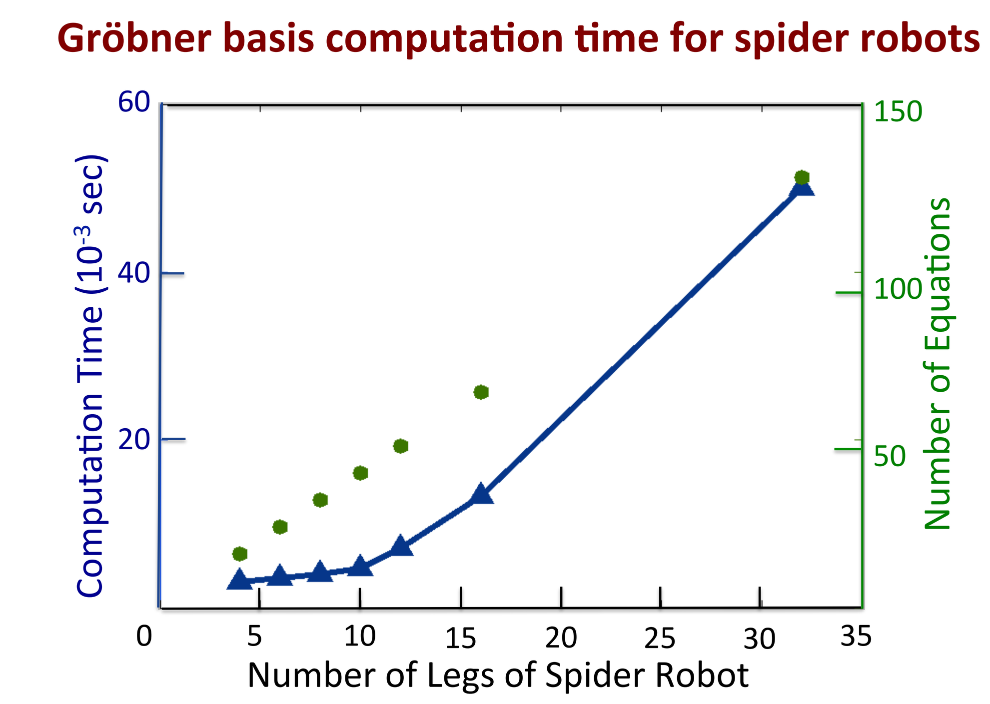
- Sage is Python-based math software that uses C++ libraries in Singular and Magma to compute Gröbner bases.

✔ Sage allows fast Gröbner basis computations.

sage python MAGMA COMPUTER • ALGEBRA C C++ SINGULAR

V. CONCLUSIONS

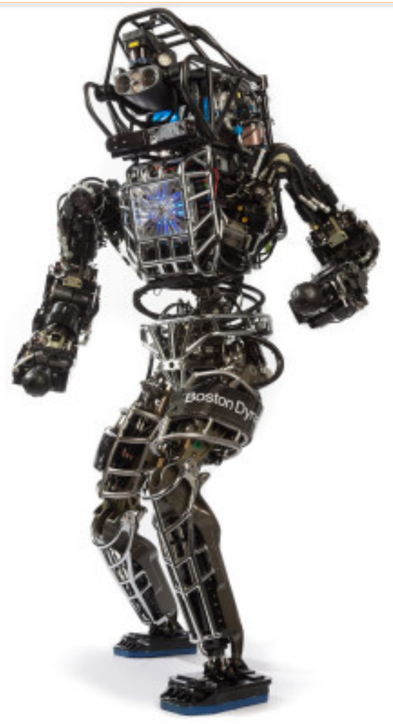
- Sage efficiently computes Gröbner bases using a 1.3 GHz Intel Core i5 processor. We achieved a 50 milliseconds computation time for Gröbner basis for a system of 128 polynomials in 67 variables for a 32-leg spider robot.



Computation time for Gröbner bases using Sage. Our polynomial systems for the spider robot had 16 to 128 equations.

VI. FUTURE WORK

- Integrate Sage's Gröbner basis algorithms in our existing polynomial solvers to improve the performance of robots, especially Boston Dynamics "Atlas".
- Investigate chordal graphs to further leverage polynomial sparsity in Gröbner basis algorithms. Our existing algorithm using chordal graphs is slower than Sage's algorithms.



Atlas robot

Acknowledgements

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Picture Courtesy: ROSARIO, J. M. et al. Proposal of methodology for the modeling and control of manipulators. J. Braz. Soc. Mech. Sci. 2002, vol.24, n.3; http://www.societyofrobots.com/robot_arm_tutorial.shtml; http://physicssum.deviantart.com/art/3D-Robot-Spider-280498884; Logos: MATLAB, Maple, Macaulay2, Sage, Python, Magma, Singular, C/C++ (http://www.techhui.com/group/candcppdevelopers); http://www.fairfaxunderground.com/forum/file.php?40,file=113550,filename=Atlas-p2_nt.jpg