A Welfare Analysis of Long-Term Care Insurance in Japan *

Minamo Mikoshiba†

February 15, 2023

Abstract

This study examines how long-term care risks affect individuals' life-cycle behavior and analyzes the role of the LTCI system in Japan, the world's fastest aging society. This study builds a structural overlapping generations model with two-sided altruism introducing two-stage care arrangements between an older parent generation and an adult child generation, quantifies the economic and welfare effects of the insurance system, and evaluates Japan's LTCI with benefits-in-kind, compared to alternative long-term care policies. First, this study shows that universal coverage of the LTCI system protects families well against long-term care risks in old age. Due to the significant burden of care and the absence of a universal LTCI system, families turn to informal care or means-tested welfare programs. The welfare effects are strictly negative even if the government provides a lump-sum subsidy to each family. Second, I demonstrate that LTCI with a benefits-in-kind policy is more expensive than LTCI with cash benefits, even though LTCI with a benefits-in-kind policy positively impacts caregivers' labor supply. However, the welfare effects of LTCI with cash benefits depend on the generosity of the cash benefits.

Keywords: Social security, Long-term care, Long-term care insurance, Overlapping generations model, Japan.
JEL Classification: D15, E6, E14, I10, I13

*I would like to thank Nirei Makoto, Sagiri Kitao, Noguchi Haruko, So Kubota, and Hideki Hashimoto for their fruitful discussions. I thank the participants of the seminars at Waseda University, Hitotsubasi University, Kobe University, and Nagoya University for their helpful comments. This study was financially supported by the Japan Society for the Promotion of Science (20J20006). The Ministry of Health, Labour and Welfare (MHLW) approved the use of the Comprehensive Survey of Living Conditions under Tohatsu-1130-2 as of November 30, 2020. The MHLW also approved the use of the Statistics of Long-term Care Benefit Expenditures under Tohatsu-0507-3 as of May 7, 2018.

†University of Tokyo, Email: minamo.mikoshiba@gmail.com.

1 Introduction

Developed countries have been experiencing population aging. Among them, Japan faces unprecedented demographic aging, with the highest proportion of aging population in the world. This ongoing demographic aging has increased the risk of disability. For example, older adults have difficulty performing activities of daily living (ADLs) and instrumental activities of daily living (IADLs) (Christensen et al. 2009; Chatterji et al. 2015). In Japan, at least about 18.4% of people 65 and older and at least approximately 60% of people over 85 years of age are certified as needed long-term care or support, according to the Report Survey on Situation of Long-term Care Insurance Service of the Ministry of Health, Labour and Welfare (MHLW) in 2019. With the increasing proportion of older adults in the late stage, the Japanese government anticipates a further increase in the burden on families and expenditures in social security programs. This growing number of older adults has put immense pressure on the government to find ways to deal with their caregiving needs.

In this context, this study establishes the following two objectives. The first objective is to analyze how the risk of long-term care affects individual behavior over the life-cycle. Once older adults need long-term care, older adults and their families face significant care burdens through time and/or long-term care expenditures. In other words, older adults with disabilities and their families face a trade-off between unpaid family care from family members (informal care, hereafter, IC) and expenditures for long-term care services provided by the market (formal care), such as at-home care (formal home care, hereinafter, FHC)[1] and institutional care.[2] The former imposes a significant time burden on families, which disrupts the labor supply of family caregivers when caregivers are of working-age. The latter imposes a significant financial burden on caregivers. Then, this study examines care arrangements with three types of care options: IC, FHC, and public institutional care. Since IC and FHC are available at home, I consider the substitutability between IC and FHC when considering care arrangements. Additionally, an accurate assessment of the risk of long-term care becomes critical for analysis. Then, I use one-year interval transition probabilities of disability and mortality estimated by Mikoshiba et al. (2023) from nationwide long-term care claims data in Japan. To my best knowledge, this study is the first to apply estimated transition probabilities using long-term administrative claims

[1] As documented in Fu et al. (2017), FHC services include housekeeping, bathing, visiting nurses, rehabilitation, day services, short-stay services, medical care management counseling, welfare device leasing/purchasing, and home renovation. In contrast, institutional care, as well as chronic care hospitals, is included in institutional services.

[2] This study does not consider private institutional care services as long-term care options since public institutional care accounts for most of total facility capacity, and information on occupancy rates in private facilities is not available in sufficient detail. Website: `https://www.mhlw.go.jp/file/05-Shingikai-12601000-Seisakutoukatsukan-Sanjikanshitsu_Shakaihoshoutantou/0000171814.pdf` (in Japanese) (Accessed July 10, 2022).

data in a rich structural model. Since Japan's long-term care insurance (LTCI) system determines eligibility for LTCI purely based on the level of care demanded by unique objective and scientific measures of disability, this study shows significant advantages over studies using survey data with self-reported disability status.

The second objective is to quantify the welfare cost of the LTCI system, relative to alternative policies, in order to evaluate the role of the LTCI system. This study focuses on the characteristics of Japan's LTCI system, which is a universal coverage insurance system and a benefits-in-kind system. As mentioned above, eligibility for LTCI is determined solely based on the demand for long-term care, regardless of socioeconomic attributes, such as family structure, income and savings, without means tests. Japan's LTCI system covers all citizens and provides them with an identical set of benefits-in-kind. Japan's LTCI system provides only services and no cash family-care allowances, although other countries with social insurance for long-term care policies, such as Germany and South Korea, provide both services and cash allowances.

To analyze how long-term care risks affect an individual's life-cycle behavior, I first present distinct trajectory patterns of disability and mortality risks using estimated transition probabilities and show care arrangements using the Comprehensive Survey of Living Conditions (CSLC) by the MHLW. The estimated transition probabilities show some quantitatively important patterns of the risks of disability and mortality. The risk of long-term care is highly persistent almost regardless of age or sex. The ratio of eligibility is higher for females than males in general while the risk of mortality is higher for males than females for all ages and disability status. From the CSLC, I find that approximately 80% of the recipients use long-term care services at home, and approximately 70% use a mix of IC and FHC at home. Although cross-country comparisons on long-term care should be considered carefully as discussed in Ikegami (2019), more older adults use both IC and FHC services at home in Japan, relative to care arrangements in the United States (U.S.) and European countries as shown in Barczyk and Kredler (2019). This is consistent with Japan's LTCI system, which emphasizes FHC rather than IC or public institutional care. However, as observed in the U.S. and European countries, the family is an important caregiving source in Japan: IC accounts for most primary caregivers, especially children and spouses, and the burdens of caregivers are concentrated on them. It should be noted that the availability of IC services is highly dependent on the existence of those who can provide it; however, IC represents the most common form of primary caregivers. This study focuses on the care arrangements between widowed females and their female adult children as their primary caregivers. It is crucial to analyze how long-term care risk and LTCI system affect the labor supply of working-age children under population aging with the rapid decline in the labor force and a rising fiscal burden.

To capture these empirical facts, I develop a structural overlapping generations model with two-sided altruism. This model is populated by heterogeneous families differing in

various dimensions. The sources of uncertainty in this model are disability and longevity risk in old age, permanent skill shocks, and idiosyncratic wage shocks during working-age. There is no insurance market for risk, and families face a no-borrowing constraint. The adult child and older parent generation from families make decisions jointly to maximize the same objective function in the sense of two-sided altruism, following Fuster et al. (2007), İmrohoroğlu and Zhao (2018), and Gao (2020). The family chooses optimal allocations for life-cycle consumption, caregiver labor supply, savings, and care arrangements. Based on empirical findings, my model focuses on the disability status of the female in the older parent generation and the care arrangements between the female older parent and the female adult child as her primary caregiver. To develop a richer model of care arrangements, I incorporate three types of care options: IC from her adult child, FHC, and public institutional care. Furthermore, this model endogenizes care arrangements by introducing two-stage family decisions.

For the family, the opportunity cost of caregivers and the family savings are crucial in making care arrangements. The caregiver's opportunity cost contributes significantly to the labor market since IC involves a substantial time burden. The amount of savings is also important for care arrangements because savings are a source of insurance against long-term care risks in old age. Once disability shocks hit the female older parent, the family uses the savings to cover substantial expenditures on formal care services. When families have sufficient savings, they face a trade-off between a reduction in the current labor income of working-age female adult children because of the use of IC and a smaller bequest from a savings cutoff to purchase formal care services. In contrast, when families lack the savings required to purchase formal care services, they turn to either IC or a means-tested welfare program.

I calibrate the model parameters to the Japanese economy in 2015. Relative to the pattern of care arrangements at home in the data, the model replicates the overall pattern of care arrangements well. This study evaluates the universal insurance system with a benefits-in-kind policy by quantifying the welfare cost and the burden of LTCI relative to alternative long-term care policies.

I find that universal LTCI protects families well against disability risks in old age. When the government eliminates LTCI system, the cost of formal care services increases, and families cope with the burden of care by providing higher IC. Disability risks can induce precautionary savings. However, in poorer families, the massive burden of care would deplete their savings, and they would then need to turn to means-tested welfare programs. In this case, the reductions in government expenditure from eliminating LTCI might be offset by higher expenditures on means-tested welfare programs. Therefore, even when a lump-sum tax is adjusted to balance the government budget, the welfare effects remain strictly negative because the compensation does not cover significant long-term care burdens.

Furthermore, I consider the roles of the LTCI system with a benefits-in-kind policy by simulating the alternative scenario wherein the LTCI system provides only cash transfers. When the government provides sufficient cash transfers to purchase the average FHC services in the baseline model, the ratio of IC in total hours increases, and correspondingly caregivers' labor force participation and families' average savings fall. Although the cost of formal care exceeds the cost of providing IC, cash transfers compensate for the reduction in the labor income of caregivers, who are middle-aged married females. Therefore, the compensation increases the welfare effects for all combinations of skill types. Moreover, when a tax is adjusted to balance the government budget, the government imposes a lump-sum tax on each family because of the reduction in the tax revenues from the labor income of middle-aged married females and capital income. However, the impact of a lump-sum tax is modest because the labor income of middle-aged married females is significantly lower than that of married males. Therefore, LTCI with cash transfers—instead of a benefits-in-kind policy—still results in positive welfare effects, but the welfare effects are smaller than those from no tax adjustment.

This study builds on multiple lines of literature. This study is related to a huge body of literature investigating the roles of government insurance policies for older adults.[3] In particular, this study contributes to the growing literature investigating the roles of long-term care policies. Previous studies focus mainly on means-tested transfer using U.S. data. For example, Barczyk and Kredler (2017) focus on means-tested Medicaid and evaluate non-means-tested IC and formal care subsidies as alternative scenarios. They show that the combination of IC and formal care subsidies precipitates a large welfare gain, a reduction in fiscal spending on Medicaid, and a decline in labor income tax revenues. This is because low-income earners respond to IC subsidies, and a decline in their labor income tax revenues modestly impacts total tax revenues. In other streams of literature, some studies analyze the interaction between Medicaid and the long-term care market. Brown and Finkelstein (2008), Mommaerts (2015), Ko (2022), and Braun et al. (2019) discuss the demand for Medicaid and the private LTCI market in the U.S. Koreshkova and Lee (2020) study the interactions of Medicaid and the institutional care market.

These studies on means-tested transfer are helpful for understanding the role of means-tested transfer as a long-term care policy in macroeconomics. However, most developed countries do not have means-tested transfers. Instead, they have a universal LTCI system with a benefits-in-kind policy or/and cash benefits for the long-term care policy, as shown

[3]The extensive macroeconomic literature has analyzed the role of public pension systems. Auerbach and Kotlikoff (1987) are an earlier work shows that the public pension system reduces incentives for people to save and work, and these distortions generate welfare loss. Fuster et al. (2007) reveal consistent results under a dynastic framework in which households have family insurance. Moreover, recent studies have focused on insurance policies' roles in health and medical expenditure risks. In earlier studies, Attanasio et al. (2010) consider the role of Medicare, Braun et al. (2017) study the optimal size of means-tested Medicaid, and Pashchenko (2013) evaluate the implications of the Affordable Care Act.

in Barczyk and Kredler (2019). This study contributes to the literature by evaluating long-term care policies, such as universal LTCI systems with a benefits-in-kind policy, by considering means-tested welfare program and cash transfers as alternative policies. Furthermore, this study contributes to the literature by incorporating three types of care options, that is, IC from her adult child, FHC, and public institutional care, and by endogenizing care arrangements by introducing two-stage family decisions. Previous studies using U.S. data usually focus only on IC and institutional care, and dismiss FHC. However, it is important to consider FHC when analyzing the role of long-term care policies because institutional care is more expensive than FHC for the government.

This study also contributes to the literature on the effect of long-term care risk on households' life-cycle behavior. The macroeconomics literature on the risk in old age highlights the existence of precautionary savings against long-term care risks in old age. Kopecky and Koreshkova (2014), Ameriks et al. (2020), and Bueren (2022) show that the impending risk of future long-term care spending induces older parents to hold onto assets for self-insurance. De Nardi et al. (2010) find that spending on health care later in life is important in explaining the slow decline in spending during retirement. Lockwood (2018) shows that precautionary savings against long-term care risk precipitate large bequests when individuals do not need long-term care. İmrohoroğlu and Zhao (2018) introduce a dynastic framework and show that households cope with long-term care risk by increasing savings when the family insurance channel weakens.[4]

This study also relates to the empirical applied microeconomic literature regarding care arrangements. Various studies have revealed that IC negatively impacts caregivers' labor force participation. Van Houtven et al. (2013) and Skira (2015) find that caregivers in the U.S. are less likely to work than non-caregivers. Furthermore, Sugawara and Nakamura (2014) reveal the negative impact using Japanese data.[5] Fu et al. (2017) reveal that Japan's LTCI system with benefits-in-kind exerts significant and positive spillover effects on caregivers' labor force participation. On the contrary, Geyer and Korfhage (2015) demonstrate that the LTCI system with cash transfers negatively impacts the labor force participation using German data. Moreover, several studies in the U.S. and European countries report that IC is substituted for formal care. Charles and Sevak (2005) provide strong evidence of substitution between IC and institutional care. Bonsang (2009) reveals that IC and FHC are partial substitutes.[6] Although the FHC policy would mitigate the burden on family caregivers, I must carefully consider the substitutability or

[4]Several studies have analyzed housing and LTCI. Davidoff (2010) studies the substitution relation between home equity and LTCI. Barczyk et al. (2022) examine the interactions between housing and the family for the saving and intergenerational transfer behavior of the elderly.

[5]See also Shimizutani et al. (2008), Hanaoka and Norton (2008), Yamada and Shimizutani (2015), and Ando et al. (2021) for more studies in Japan.

[6]See also Van Houtven and Norton (2004), Bolin et al. (2008), and Mommaerts (2018) for more studies.

complementarity between IC and FHC to evaluate the policy.

There are recent studies that build a life-cycle model including medical and long-term care expenditures to analyze fiscal sustainability in Japan under demographic aging. These studies usually use publicly available data and incorporate a deterministic process of their expenditures.[7] In contrast, a few papers incorporate stochastic health risks in life-cycle models using administrative claims data in Japan due to the difficulty of access. Fukai, Ichimura, Kitao, and Mikoshiba (2021) use nationwide health insurance claims data and show the welfare effects of universal health insurance reform depend on household income levels and the generosity of the means-tested welfare program. Hsu and Yamada (2019) and Hagiwara (2022) analyze health insurance reform using claims data covering working-age individuals enrolled in employer-based health insurance. The analysis in this study parallels that of Fukai et al. (2021). It should be noted, however, that long-term care differs from medical care in terms of required treatment and duration of treatment. The long-term care risk is highly persistent and almost irreversible regardless of age or sex, as shown in Mikoshiba et al. (2023), but the degree of expertise in that treatment is less required than in medical treatment. Then, individuals with disabilities receive not only formal care but also informal care at home by family members, while medical treatment is performed in hospitals and medical facilities by qualified individuals. This study incorporates both informal and formal care into the model, focusing on and endogenizing care arrangements between the female older parent and the female adult child based on current disability status, while Fukai et al. (2021) consider exogenous medical expenditures based on current health status. In addition, this study endogenizes the labor supply of family caregivers.

The remainder of this study is organized as follows: Section 2 describes Japan's LTCI system and my data sources, and documents empirical findings on long-term care risks over the life-cycle and care arrangements; Section 3 presents my quantitative life-cycle model; Section 4 describes the model's parametrization; Section 5 presents numerical results; Section 6 presents the concluding remarks; and section **??** presents the appendix.

2 Institutional Settings and Empirical Facts

2.1 Public LTCI in Japan

Japan implemented LTCI in 2000 and became one of the first countries to develop mandatory public insurance schemes for long-term care both in the facility and at home.[8] The

[7] See, for example, Braun and Joines (2015), Kitao (2015), and İmrohoroğlu et al. (2016).

[8] For details on Japan's LTCI system, including its history, see Campbell and Ikegami (2000) and Tamiya et al. (2011). For further details of the program, see the description on the website of the MHLW. Website: `https://www.mhlw.go.jp/english/topics/elderly/care/index.html` (As of July 2002) (Accessed January 15, 2023).

official purpose of Japan's LTCI system was to help older adults with frailness "to maintain dignity and an independent daily life routine according to each person's own abilities."[9] Japan's LTCI system also aims to (i) relieve family caregiver's burden; (ii) emphasize at-home care rather than institutional care; (iii) allow the free choice of providers to increase consumer choice and competition; and (iv) separate long-term care from coverage of health care insurance and unified financing to integrate health and social services.[10]

The insured persons under the LTCI do not necessarily coincide with the recipients. Insured persons consist of persons 65 years and older (Category-I insured persons) and people 40 to 64 years of age who are insured by the health care insurance system (Category-II insured persons).[11] Older adults 65 years and older are eligible for LTCI services after receiving certification of needed long-term care or support, regardless of the cause.[12] In contrast, older adults aged 40 and 64 can use LTCI only if they are certified as needed long-term care or support due to specific age-related diseases, such as presenile dementia and cerebrovascular disease.[13] In March 2020, the number of older adults 65 years and older who are certified as needed long-term care or support was approximately 6.69 million, according to approximately 98% of those certified as long-term care or support (about 6.82 million).[14] This study focuses on those 65 years and older.

It is important that eligibility for LTCI is determined solely based on the level of care demanded, regardless of socioeconomic attributes, such as family status, income, and savings, without means tests.[15] LTCI quantifies the level of care demanded by calculating

[9]According to the Long-term Care Insurance Act (*Kaigo hoken hō*) from the Ministry of Justice. See the description on the website of the Ministry of Justice, `https://www.japaneselawtranslation.go.jp/ja/laws/view/3807` (Accessed October 12, 2022).

[10]According to Campbell and Ikegami (2000) and Tamiya et al. (2011).

[11]For further details of the program, see the description on the website of the MHLW. Website: `https://www.mhlw.go.jp/english/topics/elderly/care/2.html` (As of July 2002) (Accessed January 15, 2023).

[12]For details, see the description on the website of the MHLW. Website: `https://www.mhlw.go.jp/content/12300000/000614772.pdf` (Accessed January 16, 2023) and `https://www.mhlw.go.jp/english/topics/elderly/care/2.html` (As of July 2002) (Accessed January 15, 2023).

[13]For details on specific age-related diseases, see the description on the website of the MHLW. Website: `https://www.mhlw.go.jp/topics/kaigo/nintei/gaiyo3.html` (in Japanese) (Accessed January 16, 2023).

[14]The data are taken from the Report Survey on Situation of Long-term Care Insurance Service conducted by the MHLW in 2020. Website: `https://www.mhlw.go.jp/topics/kaigo/osirase/jigyo/20/index.html` (in Japanese) (Accessed January 15, 2023).

[15]Recipients of the welfare transfer program who are 65 years and older can use long-term care services covered by LTCI if they are certified as needed long-term care or support. Category-I insured persons include welfare recipients who are aged 65 or over. LTCI premiums for welfare recipients aged 65 or over are financed by livelihood assistance. Out-of-pocket long-term care service expenditures for welfare recipients aged 65 or over are covered by long-term care assistance. On the contrary, welfare transfer program recipients between the ages of 40 and 64 are not included in Category-II insured persons. Welfare recipients aged 40 to 64 years do not pay LTCI premiums. However, welfare recipients aged between 40

standard hours of total care demanded. Standard hours of care are estimated based on 74-item questionnaires on ADLs, IADLs, behavioral and psychological symptoms of dementia, and the use of medical care. Then each older person is classified into one of eight levels of care needs by a computer algorithm and an expert committee based on standard hours of care.[16] The eight levels of care needs consist of ineligible or independent, support-required level (SL)1, SL2, care-required level (CL)1, CL2, CL3, CL4, and CL5. If classified as ineligible or independent, the older person is not eligible for the LTCI. SL1 is the mildest and CL5 is the severest level of care needs. SL refers to recipients who live independently but need help with IADLs. Care recipients in SL1-2 are eligible for preventive long-term care services to keep recipients from reaching a more severe care level. CL refers to recipients who require greater help with ADLs and IADLs than SL recipients. CL1 and CL2 include those who might be able to live alone if provided partial assistance with basic activities, such as walking in the room, eating, and using the toilet, but who are observed with lower levels of thinking and comprehension and some problematic behavior. On the contrary, CL3-5 are assigned to more severe old persons who are unable to live without full support for daily life, and some have difficulties in thinking, understanding and even communication, have frequent problematic behaviors, and are bedridden.

Japan's LTCI system provides only services and no cash family-care allowances. Although other countries with social insurance for long-term care policies, such as Germany and South Korea, provide both services and cash allowances, Japan provides only services due to concerns that family caregivers would continue to be exploited if cash allowances are given.[17] Japan's LTCI system establishes the maximum amount of services that can be purchased as benefits for each level of care. Within the ceiling amount, the LTCI covers 90% of expenditures and sets the copayment rate at 10%.[18] Eligible individuals can

---

and 64 are eligible for long-term care services only if they are certified as needed long-term care or support due to specific age-related diseases. Total long-term care service expenditures for welfare recipients aged 40 to 64 years are covered by long-term care assistance.

For details on welfare recipients and the LTCI, see the description on the website of the MHLW. Website: `https://www.mhlw.go.jp/shingi/2004/04/s0426-6c2.html` (in Japanese) (Accessed January 16, 2023). Also, see the description on the Welfare and Medical Service Agency website. Website: `https://www.wam.go.jp/content/wamnet/pcpub/kaigo/handbook/qa/` (in Japanese) (Accessed January 16, 2023).

[16]Reassessments are conducted every year in principle. For those who are certified for the first time, reassessments are performed six months following the first certification. Individuals can request a reassessment if they experience a decline in health or have questions about the assessment's results. See Tsutsui and Muramatsu (2005) for further details of the certification process.

[17]According to Campbell and Ikegami (2000) and Tamiya et al. (2011).

[18]The ratio of recipients exceeding the ceiling number of benefits is extremely low—1.3% of all the recipients in 2015—according to the MHLW. `https://www.mhlw.go.jp/file/06-Seisakujouhou-12300000-Roukenkyoku/201602kaigohokenntoha_2.pdf` (in Japanese, page 22) (Accessed November 4, 2022).

choose the type of long-term care services and facilities from the long-term care market, such as FHC and institutional care. The LTCI is then designed to emphasize at-home care over institutional care due to a growing need for long-term care and concerns about the fiscal burden. The primary public institutional care services, that is, welfare care facilities for the elderly (special nursing homes, *tokubetsu-yogo-rojin-home* in Japanese), are only available to older adults with CL3-5—except when the level of care improves after entering the institution or the family circumstances are serious. The LTCI does not cover living expenses and meal costs in institutional care.

2.2 Data Description

I use two main data sources to capture the profiles of the risks of disability, mortality in old age, and the care arrangements patterns of older adults with disabilities: Statistics of Long-term Care Benefit Expenditures (SLBE) and CSLC, both conducted by the MHLW.[19]

First, the SLBE is the nationwide long-term care claims data. The SLBE is administrated on a fiscal year basis, including monthly-based claims for 144 months from April 2006 to March 2018. The SLBE covers all residents in Japan as long as they are eligible for the LTCI.[20] This study uses transition risks of disability and mortality in an interval of one year by age, sex, and current level of care needs for those aged 65-94 estimated in Mikoshiba et al. (2023) using the SLBE. Mikoshiba et al. (2023) constructs the one-year interval panel data (30,347,066) for the cohort from 1912 to 1951 (813,532 individuals) and estimates the average transition probabilities from 2007 to 2018 for each age, sex, and current level of care needs. To my best knowledge, this study is the first to apply estimated transition probabilities using long-term administrative claims data in a rich structural model.

Second, the CSLC is a nationally representative repeated cross-sectional micro survey of the non-institutionalized population. The CSLC covers families and family members nationwide. The 2016 household questionnaires cover approximately 710,000 individuals, randomly sampled in 5,410 districts from the 2010 National Census, with a high response rate (78.5%).[21] The 2016 long-term care questionnaires complementarily cover approxi-

[19]Further detailed information can be found here: `https://www.mhlw.go.jp/english/database/db-hss/soltcbe.html` (for the SLBE) (Accessed November 4, 2022). `https://www.mhlw.go.jp/english/database/db-hss/cslc.html` (for the CSLC) (Accessed November 4, 2022).

[20]Note that the SLBE includes recipients of the welfare transfer program who are eligible for long-term care assistance free. Of the LTCI recipients in 2020 (about 5.67 million), the recipients of the welfare transfer program account for approximately 6.3% (about 0.32 million), according to the National Survey on Public Assistance Recipients by the MHLW in 2020. Website: `https://www.mhlw.go.jp/toukei/list/74-16.html` (in Japanese) (Accessed September 30, 2022).

[21]Information on the response rate can be found here: `https://www.mhlw.go.jp/stf/shingi2/0000192658.html` (in Japanese) (Accessed February 8, 2023).

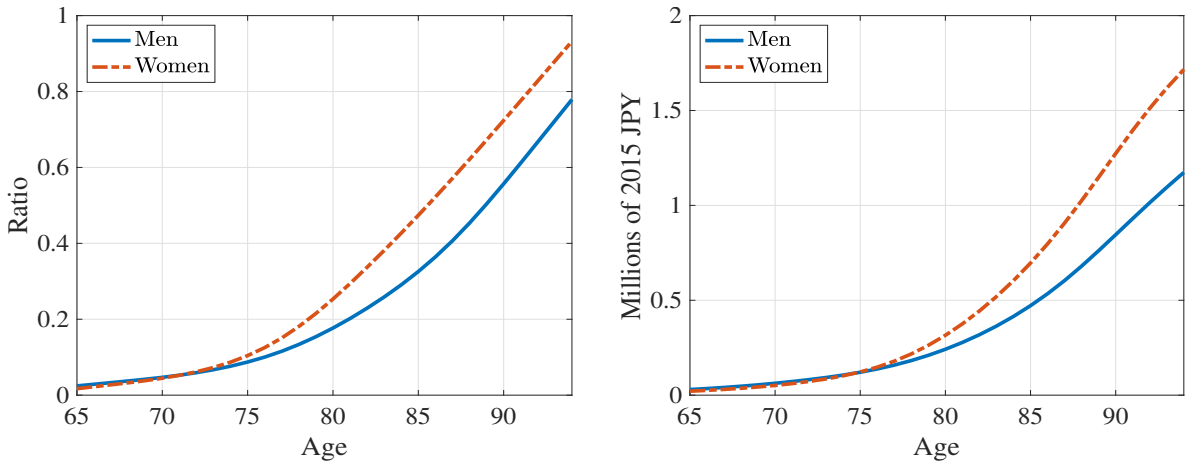
mately 8,000 LTCI certified individuals from 2,446 of the 5,410 districts mentioned above.

This study uses the CSLC to capture the care arrangements patterns of older adults with disabilities because the SLBE does not contain sociodemographic and socioeconomic information.[22] The CSLC contains basic information on living conditions and consists of five questionnaires: household questionnaires, long-term care questionnaires, health questionnaires, income questionnaires, and savings questionnaires. Although household and income questionnaires are conducted annually, long-term care, health, and income questionnaires are conducted only once every three years in the large-scale survey year. In this study, I use data from a large-scale survey conducted in 2016 and two questionnaires: household and long-term care questionnaires. I construct my sample by matching the household questionnaires with the long-term care questionnaires. Household questionnaires contain information about families and family members, including family structure, age, sex, marital status, number of children, living status with children, certification as needed long-term care or support, and primary caregivers. Long-term care questionnaires contain information on those certified as needed long-term care or support, including the level of care, the type of long-term care services, and the expenses of long-term care services. I provide the details of how to construct my sample and show patterns of care arrangements for older adults with disabilities in section 2.4.

2.3 Empirical Facts: Long-Term Care Risk in Old Age

This section describes the profiles of the risks of disability and mortality in old age. Figure 1 shows the average of older adults eligible for long-term care services covered by the LTCI and the average annual gross long-term care expenditures per capita by age and sex in 2015. As shown in Figure 1a, the average ratio of eligible LTCI recipients increases nearly monotonically with age. On average, the eligibility ratio is relatively low and remains less than 10% until the mid-70s for box sexes. However, the average ratio grows sharply from their mid-70s and reaches 77.89% and 93.22% at 94 years old for males and females. Until the mid-70s, the eligibility ratio is slightly higher for males than females but subsequently reversed after the mid-70s, with the ratio for females greatly exceeding that for males. From Figure 1b, it is apparent that the average annual gross long-term care expenditures per capita increase with the average eligibility ratio, particularly after the mid-70s.

[22]In this study, we calibrate the care arrangements in the steady state economy to that in the 2015 Japanese economy. Using the CSLC, we can capture care arrangements at home by objective LTC-status under the LTCI because the CSLC contains information about LTCI recipients, including not only LTC-status but also sociodemographic and socioeconomic information, such as family structure and primary caregivers, with large sample size (8,000 LTCI certified individuals) and a high response rate (78.5%).



(a) Ratio of Eligibility for LTCI (b) Annual Expenditures on LTCI per capita

Figure 1: Ratio of Eligibility and Annual Expenditures on LTCI per capita by Age and Sex

Note: Figure 1a shows the average of older adults eligible for long-term care services covered by the long-term care insurance system by age and sex. Figure 1b shows the average annual gross long-term care expenditures per capita by age and sex. The data of Figure 1a are from the Report Survey on Situation of Long-term Care Insurance Service by the Ministry of Health, Labour and Welfare (MHLW) in 2015 and the Population Statistics of Japan 2017 by the National Institute of Population and Social Security Research (NIPSSR). The data for Figure 1b are obtained from the Statistics of Long-term Care Benefit Expenditures by the MHLW in 2015 and the Population Statistics of Japan by the NIPSSR in 2017.

Although providing suitable information on expected risks of disability and expenditures in old age, these average profiles do not provide information on the heterogeneity of individual risks. Then, I use transition probabilities by age, sex, and current level of care needs estimated by Mikoshiba et al. (2023). To visualize the dispersion of risk, Mikoshiba et al. (2023) classified long-term care status (LTC-status) into four categories based on the eight levels of care needs; no-disability if independent or ineligible for LTCI; light if the levels range from the mildest support-required level (SL1) to the care-required level 2 (CL2); heavy if the levels range from the care-required level 3 (CL3) to the most severe care-required level 5 (CL5); and death if deceased.

These estimated transition probabilities show significant heterogeneity in the risks of disability and mortality in old age across different age, sex, and LTC-status. However, they also indicate distinct trajectory patterns in the transitions. Now, I describe these patterns that are important in this study. As an example, Table 1 shows the transition matrices of LTC-status for those aged 80 years. This table represents the distribution of their LTC-status in the next year when they are 81 years old, conditional on their LTC-status at 80 years.

Table 1: LTC-status Transition from Age t to $t+1$: Samples of Individuals Aged 80 Years

	LTC-status in $t+1$				
Males	No-Disability	Light	Heavy	Death	Total
No-Disability	0.9161	0.0372	0.0067	0.0400	1.000
Light	0.0001	0.7754	0.1378	0.0867	1.000
Heavy	0.0000	0.0664	0.7196	0.2139	1.000
	LTC-status in $t+1$				
Females	No-disability	Light	Heavy	Death	Total
No-Disability	0.9231	0.0567	0.0053	0.0148	1.000
Light	0.0001	0.8585	0.1049	0.0366	1.000
Heavy	0.0000	0.0675	0.8105	0.1220	1.000

Note: Table 1 shows the transition matrices of long-term care status (LTC-status) for those aged 80 years. The LTC-status corresponds to no-disability if independent or ineligible for long-term care insurance; light if the levels range from the mildest support-required level (SL1) to the care-required level 2 (CL2); and heavy if the levels range from the care-required level 3 (CL3) to the most severe care-required level 5 (CL5). The data are obtained from Mikoshiba et al. (2023).

First, the risk of long-term care is highly persistent and almost irreversible regardless of age or sex. Individuals of any current LTC-status will most likely remain in the same LTC-status in the next year for all ages and sexes. For example, at 80 years of age, the probabilities of staying in the same LTC-status range between 71.96% to 91.61% and 81.05% to 92.31% for males and females, respectively. Additionally, once an individual becomes disabled and eligible for LTCI, the person requires continuous care until death. The probability of transitioning to a no-disability status is nearly zero for all ages and sexes.

Second, I compare the risks of disability and mortality by age and current LTC-status for males and females. Females are more likely than males to remain in the same LTC-status in the next year for all ages. On the contrary, males have higher mortality rates than females for all ages and LTC-status. For example, at age 80, the transition probabilities from no-disability, light, and heavy to death status are 4.00%, 8.67%, and 21.39% for males and 1.48%, 3.65%, and 12.20% for females. As shown in Figure 1a, the eligibility ratio is higher for females than males. It may seem contradictory that females have a higher survival probability than males, while mortality risks tend to be higher for those eligible for the LTCI. This can be explained by the fact that mortality rates are higher for males than for females across ages and LTC-status.

2.4 Empirical Facts: Care Arrangements

In this section, I describe the care arrangements patterns of older adults with disabilities. The main source of data is the CSLC in 2016. I construct my sample by matching

household questionnaires with long-term care questionnaires and limiting those eligible for long-term care services covered by LTCI aged between 65 and 94 years with information on the level of care and primary caregivers (hereafter, "care sample"). The "care sample" has 5,181 observations.

Table 2: Characteristics of "care sample" from the CSLC

	Mean
Age and Sex	
Age	83.36
Female	0.65
Long-term care status	
Light	0.73
Heavy	0.27
Marital Status	
Widowed	0.51
Married	0.44
Divorce	0.03
Single	0.02
Children	
Having at least one child	0.91
% Living together or in the same municipality	0.87

Note: Table 2 presents the characteristics of the "care sample." The data are from the Comprehensive Survey of Living Conditions (CSLC) by the Ministry of Health, Labour and Welfare (MHLW). I use two questionnaires for 2016: household and long-term care questionnaires. The sample includes those eligible for long-term care services covered by long-term care insurance aged between 65 and 94, limiting to those who provide information on the level of care and primary caregivers. The sample has 5,181 observations. Long-term care status (LTC-status) is defined as light if the levels of care range from the mildest support-required level (SL1) to care-required level 2 (CL2); and heavy if the levels range from the care-required level 3 (CL3) to the most severe care-required level 5 (CL5). The numbers in the table are derived from the author's calculation and may not correspond to the numbers published by the MHLW.

Table 2 presents descriptive statistics on the characteristics of the "care sample." The average age of the "care sample" is 83.06 years, and female account for 65.01% of the "care sample." The sample's proportions of individuals with light and heavy LTC-status are 72.79% and 27.71%, respectively. The proportion of individuals with light LTC-status in the "care sample" is greater than in the long-term care claims data: the proportion of older adults 65 years and above who are eligible for LTCI services with light and heavy LTC-status accounts for approximately 65.30% and 34.70%, respectively.[23] This might be explained by the fact that the CLSC is limited to the non-institutionalized population only, and the primary public institutional care services (i.e., welfare care facilities for the

[23]The data are taken from the Report Survey on Situation of Long-term Care Insurance Service conducted by the MHLW in 2016. Website: `https://www.mhlw.go.jp/topics/kaigo/osirase/jigyo/16/index.html` (in Japanese) (Accessed January 15, 2023).

elderly) are only available to older adults with heavy LTC-status. As shown in Table 2, widowed and married individuals comprise approximately 94.66% of the "care sample." When restricting the "care sample" to those who provide information on children, 4,562 individuals (90.39%) have at least one child (out of 5,017). Furthermore, of the 4,372 individuals who have at least one child and provide information on living status with children, 3,787 individuals (86.62%) have at least one child who lives together or in the same municipality.

Table 3 represents the distribution of care arrangements by LTC-status in the "care sample." First, I distinguish long-term care services in the facility and at home. Since the CSLC does not include information on the institutionalized population, I use the SLBE, which has information on the number of recipients in the facility and at home, and calculate the proportion of recipients at home and in the facility to total recipients.[24] As shown in Table 3, most recipients (approximately 81.82%) use long-term care services at home. Although only about 18.18% of recipients use institutional care, the proportion of institutional care increases from 4.91% for light LTC-status to 38.97% for heavy LTC-status. These increases in institutional care are attributed to the institutional regulation of Japan's LTCI system, in which older adults with light LTC-status are ineligible to use the main public institutional care services, that is, welfare care facilities for the elderly (special nursing homes).

[24]The number of recipients in the facility consists of welfare care facilities for the elderly (special nursing homes), healthcare facilities for the elderly (*rojin-hoken-shisetsu*), nursing care medical facilities(*kaigo-ryoyogata-shisetsu*), etc. Website: `https://www.e-stat.go.jp/stat-search/files?tclass=000001094538&cycle=8` (in Japanese) (Accessed May 16, 2021).

Table 3: Distribution of Care Arrangements by Long-Term Care Status

	At-home care (a)				Institutional care (b)
		Care arrangements at home (% at home)			
	Total	Only IC	Mix IC-FHC	Only FHC	Total
All	81.82%	16.72%	56.22%	8.88%	18.18%
		(20.44%)	(68.71%)	(10.85%)	
By long-term care status					
Light	95.09%	21.60%	63.26%	10.23%	4.91%
		(22.72%)	(66.53%)	(10.75%)	
Heavy	61.03%	8.84%	45.41%	6.78%	38.97%
		(14.48%)	(74.41%)	(11.11%)	

Note: Table 3 shows the distribution of care arrangements by long-term care status (LTC-status) in 2016. The data for (a) are from the Comprehensive Survey of Living Conditions by the Ministry of Health, Labour and Welfare (MHLW). I use two questionnaires for 2016: household and long-term care questionnaires. The sample includes those eligible for long-term care services covered by long-term care insurance aged between 65 and 94 with information on the level of care and primary caregivers. I limit the sample to those who provide information on other caregivers and the use of formal home care services. The sample has 5,145 observations, consisting of 3,752 observations with light LTC-status and 1,393 with heavy LTC-status. LTC-status is defined as light if the levels of care range from the mildest support-required level (SL1) to care-required level 2 (CL2); and heavy if the levels range from the care-required level 3 (CL3) to the most severe care-required level 5 (CL5). The numbers in the table are derived from the author's calculation and may not correspond to the numbers published by the MHLW. The data for (b) are from the Statistics of Long-term Care Benefit Expenditures (SLBE) by the MHLW.

Second, I classify long-term care services at home into three types of care provision—only IC, mixed use of IC and FHC, and only FHC. I limit the care samples to those who provide information on primary caregivers, other caregivers, and the use of FHC services. I classify care arrangements based on information on relationships with caregivers and the use of FHC services: I consider individuals as receiving IC services if they receive long-term care from either their own child or child-in-law or spouse or other family members or others; receiving FHC services if they use FHC services. As can be seen in Table 3, the mixed use of IC and FHC is the most common care arrangement at home, accounting for approximately 68.71% of the sample. Although cross-country comparisons on long-term care should be considered carefully for very heterogeneous definitions of what long-term care covers as discussed in Ikegami (2019), more older adults use both IC services and FHC services at home in Japan, relative to care arrangements in the U.S. and European countries shown in Barczyk and Kredler (2019). This is consistent with Japan's LTCI system, which emphasizes at-home care rather than institutional care.

Under Japan's LTCI system, most recipients use at-home care. To account for the burdens of caregivers at home, Table 4 presents the total annual care hours, primary caregivers, and their intensity of care by LTC-status. It can be seen from the data in Table 4 that the total annual hours vary significantly depending on the LTC-status. The entire care hours in heavy LTC-status are approximately twice as long as those in the

light LTC-status. Those eligible for the LTCI receive about 2.78 hours of total care per day, while individuals in the heavy LTC-status receive about 5.53 hours of care per day.

Table 4: Caregiver Burden at Home by Long-Term Care Status

	Annual care hours	Distribution of primary caregivers				Ratio of care hours by primary caregivers
		IC (Children)	IC (Spouse)	IC (Others)	FHC	
All	1291.96	50.68%	30.96%	4.07%	14.29%	68.59%
By Long-term care status						
Light	1013.09	52.49%	29.24%	4.27%	14.00%	70.00%
Heavy	2017.58	45.94%	35.44%	3.55%	15.07%	65.15%

Note: Table 4 shows caregiver burden at home by long-term care status (LTC-status) in 2016. The data are from the Comprehensive Survey of Living Conditions by the Ministry of Health, Labour and Welfare (MHLW). I use two questionnaires for 2016: household and long-term care questionnaires. The sample includes those eligible for long-term care services covered by long-term care insurance aged between 65 and 94 with information on the level of care and primary caregivers. The samples of annual care hours, distribution of primary caregivers, and care intensity of primary caregivers have 4,066, 5,145, and 4,262 observations, respectively. The samples consist of 2,862, 3752, and 3,051 observations with light LTC-status, and 1,204, 1,393, and 1,211 observations with heavy LTC-status, respectively. LTC-status is defined as light if the levels of care range from the mildest support-required level (SL1) to care-required level 2 (CL2); and heavy if the levels range from the care-required level 3 (CL3) to the most severe care-required level 5 (CL5). The numbers in the table are derived from the author's calculation and may not correspond to the numbers published by the MHLW.

I split the primary caregivers at home into four groups: children if they receive IC from their own child or child-in-law; spouses if they receive IC from their spouse; others if they receive IC from other family members or others; and FHC if they receive FHC. Table 4 shows that IC comprises most primary caregivers, especially children and spouses, who account for about 81.64% of the primary caregivers. In contrast, individuals who receive FHC as primary caregivers account for only 14.29% of the sample. Furthermore, primary caregivers provide approximately 68.59% of total care hours, suggesting that caregiver burdens are concentrated on primary caregivers.

Although IC represents the most preferred long-term care option, its availability depends heavily on family structure. More accurately speaking, availability depends on the existence of those who can provide IC. For example, IC by children is not available to childless older adults with disabilities. Even if older adults with disabilities have at least one child, they cannot receive IC from their child if their child lives far away. Table 2 shows that approximately 95% of the "care sample" is widowed or married. Moreover, about 91% have at least one child, and for about 87% of their child who lives together or in the same municipality as their elderly parents with disabilities. Then, I limit the "care sample" to those widowed or married, with at least one child living together or in the same municipality (hereafter, "family sample") and describe the care arrangements by sex and marital status. The "family sample" has 3,692 observations, accounting for 71.26% of the "care sample." More than half of the "family sample" consists of widowed females,

accounting for about 50.76% of the sample. Then followed by married males, married females, and widowed males, comprising 24.86%, 16.74%, and 7.91% of the "family sample," respectively. Figure 2 shows the distribution of the "family sample" by five-year age-group, sex, and marital status.

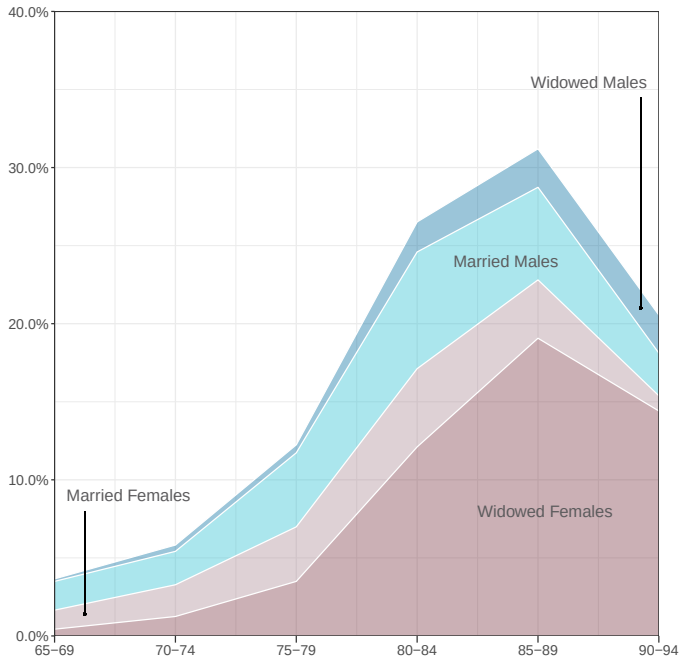


Figure 2: Distribution of "family sample" by Five-Year Age-Group, Marital Status, and Sex

Note: Figure 2 shows the distribution of the "family sample" by five-year age-group, marital status, and sex. The data are from the Comprehensive Survey of Living Conditions by the Ministry of Health, Labour and Welfare (MHLW). I use two questionnaires for 2016: household and long-term care questionnaires. The sample includes those eligible for long-term care services covered by long-term care insurance aged between 65 and 94. I limit the sample to those who provide information on the level of care and primary caregivers; those who are widowed or married; and those with at least one child living together or in the same municipality. The sample has 3,692 observations. The figure is derived from the author's calculation may not correspond to the numbers published by the MHLW.

As shown in Figure 2, until their late-70s, married males account for the largest share of the age group, but after the late-70s the proportion of widowed females increases sharply and exceeds that of married males. This is consistent with the empirical findings in section 2.3 that the eligibility ratio is slightly higher for males until the mid-1970s but reversed, and males have higher mortality rates than females for all ages and LTC-status. Moreover, the number of observations in the "family sample" increases with age until the early 90s. This is also consistent with the empirical findings in section 2.3 that the risk of disability increases with age. The fewer observations in the 90-94 age group could be explained by the increase in mortality rates with age.

Table 5 presents the care arrangements for widowed women and married men, who make up the majority (75.62%) of the "family sample." To calculate the care arrangements

by LTC-status, I limit the "family sample" to those who provide information on primary caregivers, other caregivers, and the use of FHC services. I will show more details of the distribution and characteristics of primary caregivers for the four groups by sex and marital status, including married females and widowed males, in section Appendix A.

Table 5: Care Arrangements for Widowed Females and Married Males, Having at least One Child Living Together or in the Same Municipality by Long-Term Care Status

	Distribution of primary caregivers				Ratio of care hours by primary caregivers	Care Arrangements		
	IC (Children)	IC (Spouse)	IC (Others)	FHC		Only IC	Mix IC-FHC	Only FHC
Widowed Females								
All	88.40%	0.00%	1.88%	9.71%	65.11%	18.06%	75.42%	6.52%
By long-term care status								
Light	99.21%	0.00%	2.20%	7.60%	65.83%	20.87%	75.49%	4.64%
Heavy	83.30%	0.00%	1.00%	15.70%	63.27%	10.10%	78.05%	11.85%
Married Males								
All	26.14%	68.89%	0.73%	5.05%	66.19%	26.25%	71.30%	2.45%
By long-term care status								
Light	27.34%	67.81%	0.78%	4.07%	68.12%	28.84%	68.50%	2.65%
Heavy	23.97%	68.58%	0.64%	6.81%	62.55%	21.57%	76.35%	2.08%

Note: Table 5 shows care arrangements for widowed females and married males, who have at least one child living together or in the same municipality by long-term care status (LTC-status). The data are from the Comprehensive Survey of Living Conditions by the Ministry of Health, Labour and Welfare (MHLW). I use two questionnaires for 2016: household and long-term care questionnaires. The sample includes those eligible for long-term care services covered by long-term care insurance aged between 65 and 94. I limit the sample to those who provide information on the level of care, primary caregivers, other caregivers, and the use of formal home care services; those who are widowed females or married males; and those who have at least one child living together or in the same municipality. The sample of primary caregivers and distribution of primary caregivers have 1,865 and 915 observations for widowed females and married males, respectively. The sample of care intensity has 1,552 and 783 observations for widowed females and married males, respectively. LTC-status is defined as light if the levels of care range from the mildest support-required level (SL1) to care-required level 2 (CL2); and heavy if the levels range from the care-required level 3 (CL3) to the most severe care-required level 5 (CL5). The numbers in the table are derived from the author's calculation and may not correspond to the numbers published by the MHLW.

It can be seen from the data in Table 5 that about 90% of widowed females are cared for by their children as primary caregivers. Considering the characteristics of their children who are primary caregivers, I find that they are of working-age (average age is 58.67 years) and predominantly female (67.63%). Furthermore, Table 5 shows that the primary caregivers for married males are their wives, who represent approximately 68.89% of married males. I find that they are already retired from the labor market: the average age is 76.67 years; and they have lower labor force participation rates (11.86%, versus 17.46% among all 70s in the 2016 CSLC). These trends for widowed females and married males are also observed for widowed males and married females, respectively. Although about 70% people use IC and FHC services at home, regardless of marital status, sex, or LTC-status, primary caregivers provide high care intensity to recipients,

which is consistent with the existing literature that IC and FHC are substitutes rather than complements (e.g., Bonsang (2009)).

From these empirical findings, the model in this study focuses primarily on widowed females and their working-age female children. It is important to analyze how long-term care risk and LTCI affect the labor supply of working-age children under population aging with the rapid decline in the labor force and a rising fiscal burden. This study also captures the essence of married males whose caregivers are their wives who have already retired from the labor market.

3 Model

In this section, I build an overlapping generations model with two-sided altruism. The family comprises two generations, and each generation exhibits altruism toward each other. I quantify the effects of long-term care risks on individuals' life-cycle consumption, female adult child labor supply, and savings through care arrangements. I further evaluate the roles of the LTCI. The sources of uncertainty in this model are disability and longevity risk in old age, permanent skill shocks, and idiosyncratic wage shocks during working-age. The LTC-status affects the total hour requirement for care, disability and mortality risk in the next periods, and the composition of the older parent generation. There is no insurance market for risk: there are uninsurable idiosyncratic risks and individuals face a no-borrowing constraint. This is a partial equilibrium model: individuals take as given the paths of factor prices and various social security policies. The model time is discrete, and the model frequency is annual.

3.1 Demographics

There is a dynastic framework with two stages: an adult child stage and an older parent stage. An individual lives as an adult child during the first J periods. At age $J+1$, an individual becomes an older parent in the next-generation household of the dynasty. At this time, an individual leaves their labor force. From age $J+1$, an individual faces disability and mortality risk in each period. The maximum possible age is $2J$.

Then a family is made up of two generations: an adult parent generation (indexed by k) of age $j^k \in \{1, \ldots, J\}$ and an older parent generation (indexed by p) of age $j^p = j^k + J$. An individual's life overlaps with his or her older parent generation households during the first J periods and with his or her adult child generation households in the last J periods. In each family, an older parent generation consists of one household, while an adult child generation consists of a measure of $(1+\nu)$ households. The annual population growth rate is v_g and $v = (1+v_g)^J - 1$. In particular, a new generation in a family line is born only in every J period, while a new generation in the economy is born in every period.

At the initial period of each family ($j^k = 1$), both households consist of one married couple: each family comprises four individuals from two generations of households, that is, household members $i \in \{kf, km, pf, pm\}$, each representing a female adult child, a male adult child, a female older parent, and a male older parent.[25] Although an adult child generation household has two members during J periods,[26] an older parent generation household has $n^p \in [1, 2]$ members. A detailed explanation of the household members in the older parent generation is presented in section 3.3. To simplify the model, this study assumes that each household member within the same generation has the same age and skill because the average age difference between couples from 1975 to 2015 is 2.4 years[27] and there is a high degree of sorting in Japanese married couples.[28] This study does not model the marriage decision and abstracts from divorce and remarriage.

The formalization of the household follows Fuster et al. (2007), İmrohoroğlu and Zhao (2018), and Gao (2020), in the sense of two-sided altruism.[29] In this setting, living household members make a joint decision to maximize the same objective functions.

3.2 Skill

Individuals differ by skill $z = \{L, H\}$, each referring to the state of low- and high-skilled individuals. I define individuals as high-skilled if they have a college degree or higher and low-skilled if they do not. At birth, each individual stochastically inherits a skill z from his or her parents. Individuals' skill state z is fixed throughout the life-cycle and affects their age-specific deterministic labor productivity, $\epsilon(j^k, z)$. Notably, an individual's permanent lifetime labor efficiency is deterministic within their entire life, while the individual's permanent labor productivity is stochastic between parents and children. z follows a first-order Markov chain of two states with transition probabilities $\Omega(z' \mid z)$. Since household members within the same generation have the same skill, there are four types of the family

[25]The average completed number of children per married couple is stable at around 2.2 from 1970 to the early 2000s, and it drops to 1.94 in 2015, according to the Annual Population and Social Security Surveys (The National Fertility Survey) from the National Institute of Population and Social Security Research. Website: `https://www.ipss.go.jp/ps-doukou/e/doukou15/Nfs15R_points_eng.pdf` (Table II-2) (Accessed June 27, 2022).

[26]This study does not consider the risk of longevity in middle adulthood since the mortality rates for those 35–65 years are quite low, at 0.26%, according to the Japanese Mortality Database of the National Institute of Population of Social Security Research. Website: `http://www.ipss.go.jp/p-toukei/JMD/index-en.asp` (Downloaded on June 27, 2022).

[27]The data come from the Vital Statistics conducted by the MHLW. `https://www.mhlw.go.jp/toukei/saikin/hw/jinkou/tokusyu/konin16/dl/01.pdf` (in Japanese) (Accessed November 4, 2022).

[28]For further detail, see Fukuda et al. (2021), which uses Census data between 1980 and 2010.

[29]Previous empirical facts on bequest motives and children's help with long-term care are consistent with dynastic motives (two-sided altruism). For example, Hamaaki et al. (2019) find that older parents give a larger share to individuals of the family line and/or those who provide long-term care to their parents. I discuss the modeling choice of two-sided altruism in section Appendix B.

with skill combinations of the older parent generation and the adult child generation.

3.3 Long-Term Care and Mortality Risk

At the beginning of each period, individuals in the older parent generation face disability and mortality risks. In this study, disability and mortality risks are summarized in h, denoting an individual's LTC-status. This study classifies LTC-status h into four categories based on the eight levels of care needs; no-disability ($h = 1$) if independent or ineligible for LTCI; light ($h = 2$) if the levels range from the mildest support required level to the care level 2; heavy ($h = 3$) if the levels range from the care level 3 to the most severe care level 5; and death ($h = 4$) if deceased.

This model considers only the LTC-status of females in the older parent generation to focus primarily on care arrangements between a widowed female and her working-age female adult child—the most common family structure and care arrangements for older adults with disabilities—while keeping the dimensions of the state space manageable. However, my model also captures quantitatively important aspects of the risks of disability and mortality for males in the older parent generation by assuming that the LTC-status of females affects the composition of the older parent generation as in Barczyk and Kredler (2017). The older parent generation household has $n^p(j^p, h) \in [1, 2]$ members. First, when the female in the older parent generation is independent or ineligible for the LTCI ($h = 1$), I assume that there are $n^p(j^p, h) \in [1, 2]$ members in the older parent generation, consisting of one female and a male of measure $n^p(j^p, h) - 1 \in [0, 1]$. The measure of males decreases deterministically with age j^p. This assumption comes from the empirical fact that males have higher mortality rates than females for all ages. In addition, I assume that the male is at a deterministic risk of long-term care when the female is independent or ineligible for the LTCI. Then, the male receives IC from his wife and pays the average out-of-pocket long-term care expenditures $H_{pm}(j^p)$ to receive FHC services. This assumption reflects the empirical fact that males tend to need long-term care first compared to females and that his primary caregiver is his wife, who is already retired from the labor market. Second, I assume that once the female needs care ($h = 2, 3$), the male dies, and the female becomes widowed. Then, the household of the older parent generation has only one female, $n^p(j^p, h) = 1$. Finally, when death shocks hit the female older parent ($h = 4$), both the female and the male (if still alive) die. This assumption also comes from the empirical fact that males have higher mortality rates than females for all ages. The older parent generation household has no members, and the family consists of only one generation, that is, the adult child generation.

The LTC-status of the female older parent in the next periods h' depends on her current LTC-status h and age j^p. LTC-status h follows a first-order Markov chain with transition probabilities $\Psi(h' \mid h, j^p)$ of being LTC-status h' from age j^p to $j^p + 1$. Death

is the absorbing state for all age j^p, $\Psi(h' = 4 \mid h = 4, j^p) = 1$. For simplicity, this study assumes that once an individual becomes disabled and eligible for LTCI, the probability of transitioning to a no-disability status is zero for all ages because Mikoshiba et al. (2023) shows extremely low probabilities (i.e., close to zero).

$$\Psi(h' = 1 \mid h = 2, j^p) = \Psi(h' = 1 \mid h = 3, j^p) = 0 \quad \text{for all} \quad j^p$$

I also assume that the types of long-term care services do not affect the mortality risk, as shown in Applebaum et al. (1988).

3.4 Care Arrangements

When the female in the older parent generation is eligible for LTCI ($h = 2$ or $h = 3$), the family makes care arrangements from three types of long-term care options: IC from her female adult child; FHC; and public institutional care, that is, welfare care facilities for the elderly (special nursing homes).[30] This study models care arrangements as two-stage decision making.

3.4.1 First Stage: At-home Care versus Institutional Care

The family has to choose long-term care services at home and in the facility. The family choice is denoted by ι, which can be either at-home care services ($\iota = 0$) or public institutional care services ($\iota = 1$). If the family chooses public institutional care services, the family incurs the cost ξ when entering the public nursing home. In particular, as mentioned before, individuals with light LTC-status ($h = 2$) are not eligible to use public institutional care under Japan's LTCI system. The residential choice of the family occurs only for people with a heavy LTC-status ($h = 3$) in this model.

3.4.2 Second Stage: Care Arrangements at Home

If the family chooses at-home care services ($\iota = 0$) in the first stage, the family has to determine both the time of use of IC ϕ and FHC q at the same time to meet a minimum requirement of total care hours χ_h.

$$A\left(\theta_h(q/p_{\text{LTC}})^\rho + (1-\theta_h)(T(\phi)\times 365)^\rho\right)^{\frac{1}{\rho}} \geq \chi_h \tag{1}$$

[30]This study does not consider private institutional care services as long-term care options. Private institutional care services include fee-based homes for the elderly, residences with health and welfare services for the elderly, group homes, etc.According to the MHLW, public facilities account for the majority in terms of capacity, with special nursing homes in particular accounting for the highest proportions, although the capacity of private facilities has been increasing. In addition, information on occupancy rates in private facilities is insufficiently available. Then, this study focuses only on public institutional care. Website: `https://www.mhlw.go.jp/file/05-Shingikai-12601000-Seisakutoukatsukan-Sanjikanshitsu_Shakaihoshoutantou/0000171814.pdf` (in Japanese) (Accessed July 10, 2022).

where A is the return to care input hours, p_{LTC} is the price of FHC per hour,[31] ρ and θ_h represent substitutability between IC and FHC, and χ_h is the minimum requirement of total long-term care hours depending on the level of care, $\chi_{h=2} \leq \chi_{h=3}$. Using the formalization above, this model can capture the heterogeneity of the burden of care and the substitutability between IC and FHC by the level of care.[32] The time of use of IC ϕ is a discrete choice, and the corresponding IC hours are as follows.

$$T(\phi) = \begin{cases} 8 \text{ hrs per day} & \text{if } \phi = 1 \\ 4 \text{ hrs per day} & \text{if } \phi = 1/2 \\ 1 \text{ hrs per day} & \text{if } \phi = 1/8 \\ 0 \text{ hrs per day} & \text{if } \phi = 0 \end{cases}$$

The female older parent with disabilities exhibits a preference for IC, ω. The more hours IC is used, the more utility the female older parent gains, as described in section 3.6.

On the contrary, once the family chooses public institutional care services ($\iota = 1$) in the first stage, the female older parent with disabilities spends her entire life in the public facility. The family is also required to pay both the long-term care services cost $\bar{q}$ and the facility fee use $\bar{c}$ until she dies.

3.5 Endowments

Individuals in the adult child generation work in the labor market. A female adult child allocates her disposable time to labor supply, leisure, and IC hours if her female older parent needs care. At the beginning of each period, a female adult child faces an idiosyncratic wage shock, $\mu(j)$. The earnings of a female adult child are defined as follows.

$$y_{kf}(j,z) = \epsilon(j,z)\mu(j)\frac{1}{\overline{WH}_{kf}(j)}\left(\overline{DH}_{kf}(j) - \mathbb{1}_{h\in\{2,3\}}T(\phi) - l\right)$$

where $\{\epsilon(j,z)\}_{j=1}^{J}$ are deterministic age-specific efficiency profiles, $\overline{WH}_{kf}(j)$ is the average working hours, $\overline{DH}_{kf}(j)$ is disposable time, and l is leisure. The idiosyncratic wage shock $\mu(j)$ follows the autoregressive AR(1) process.

$$\log(\mu(j)) = \Theta \log(\mu(j-1)) + \zeta(j),\ \zeta(j) \sim N(0, \sigma_\zeta^2)$$

where $\zeta(j)$ is distributed normally with a mean of zero, the variance is σ_ζ^2, and $\Theta < 1$ captures the persistence of the shock. I discretize this process into a three-state Markov chain

[31] In Japan's LTCI system, the central government established the fees for each long-term care service and revised them every three years. For further details on the fee, see, for example, `https://www.mhlw.go.jp/topics/kaigo/housyu/housyu.html` (in Japanese) (Accessed July 5, 2022).

[32] I follow the formulation of Daruich (2018), which analyzes early childhood investments, and Gao (2020), which considers the model of child care.

using Tauchen (1986)'s method. Let $\Lambda(\mu, \mu')$ be the transition matrix of the idiosyncratic wage shocks. Furthermore, it is assumed that μ at the time of birth is determined by a random draw from an initial distribution $\overline{\Lambda}(\mu)$.

I consider a male adult child supplies labor inelasticity because the average participation in the labor force of 35-64-year-old married males is approximately 95.3%.[33] The earnings of a male adult child are denoted by y_{km}, which evolves deterministically throughout the life-cycle and depends on age and skill $y_{km}(j, z)$.

3.6 Preferences

The utility for the family is the sum of the adult child generation's utility u_k and the older parent generation's utility u_p in the sense of two-sided altruism.

An individual in the adult child generation derives utility from his or her generation's consumption c_k and leisure l_i for $i = \{kf,\ km\}$. The instantaneous utility of the adult child generation is given as follows.

$$u_k(c_k, l_{kf}) = \frac{(1+\nu)}{1-\sigma}\left(\left(\frac{c_k}{(1+\nu)\eta(2)}\right)^{1-\gamma}\bar{l}_{km}^{\gamma}\right)^{1-\sigma} + \frac{(1+\nu)}{1-\sigma}\left(\left(\frac{c_k}{(1+\nu)\eta(2)}\right)^{1-\gamma} l_{kf}^{\gamma}\right)^{1-\sigma}$$

where l_{kf} denotes the leisure of the female adult child. $\bar{l}_{km}$ is a fixed parameter that represents the exogenous leisure time of the male adult child. $\eta(n)$ is the equivalence scale and varies with the size of the family. The utility of the older parent generation is given as follows.

$$\begin{aligned} u_p(c_p) \ = \ & \frac{n^p(j^p, h) - 1}{1-\sigma}\left(\left(\frac{c_p}{\eta(n^p(j^p, h))}\right)^{1-\gamma}\bar{l}_{pm}^{\gamma}\right)^{1-\sigma} \\ & + \frac{1}{1-\sigma}\left(\left(\frac{c_p}{\eta(n^p(j^p, h))}\right)^{1-\gamma}\bar{l}_{pf}^{\gamma}\right)^{1-\sigma} + \mathbb{1}_{h\in\{2,3\}}\mathbb{1}_{\iota=0}(\omega\phi) \end{aligned}$$

where c_p is the older parent generation's consumption, and leisure l_i for $i = \{pf,\ pm\}$ is the exogenous leisure time of the individual in the older parent generation. ω represents the preference parameter for IC when the female older parent with disabilities chooses at-home care.

3.7 Government

The government operates the following social insurance programs: LTCI, health insurance, pay-as-you-go public pension, and means-tested welfare transfer program.

[33] According to the Employment Status Survey of the Ministry of Internal Affairs and Communications in 2017. Website: `https://www.stat.go.jp/data/shugyou/2017/index.html` (in Japanese) (Accessed July 5, 2022).

Public LTCI: The government provides mandatory public LTCI based on the level of care demanded, regardless of socioeconomic attributes, such as family status, income, and savings. All individuals 65 years and older receive long-term care services covered by the LTCI if they are certified as needed long-term care or support. Out-of-pocket long-term care expenditures paid by recipients are denoted as H_i^{op} for $i \in \{pf, pm\}$ and expressed as follows.

$$H_{pf}^{op} = \lambda^h q$$
$$H_{pm}^{op} = \lambda^h H_{pm}$$

where λ^h is the copayment of the LTCI. The government covers the remaining long-term care expenditures.

Public Health Insurance: The government also offers a mandatory public health insurance program. This study assumes that medical expenditures are required when an individual is an older parent.[34][35] The average annual gross medical expenditures are given exogenously, $M_i(j^p)$ for $i \in \{pf, pm\}$, depending on age and sex. Out-of-pocket medical expenditures are defined similarly to $M_i^{op} = \lambda_{j^p}^m M_i(j^p)$, where $\lambda_{j^p}^m$ is the copayment of public health insurance depending on the age. The government covers the remaining medical expenditures.

Public Pension: The government operates a pay-as-you-go public pension system. Individual receive public pension benefits once they become older parents. Let $\text{pen}_i(j^p, z)$ denote the public pension benefits for individuals $i \in \{pf, pm\}$ with age j^p and permanent skill z. I assume that the benefits of a male older parent are determined as follows.

$$\text{pen}_{pm}(j^p, z) = \kappa \cdot \frac{\bar{y}_m(z)}{J-1} \tag{2}$$

where

$$\bar{y}_m = \begin{cases} y_{km}(j,z) & \text{if } j = 1 \\ y_{km}(j,z) + \bar{y}_{km}(j-1,z) & \text{if } 1 < j \leq J \\ \bar{y}_{km}(j-1,z) & \text{if } J < j \end{cases}$$

[34]This study does not consider medical expenditures in the adult child stage. The average annual medical expenses are relatively low, remaining close to 200,000 yen until 50 years, then subsequently increasing. However, it stays under 500,000 yen and 400,000 yen for males and females, respectively, until 65 years, according to the National Medical Expenses of the MHLW in 2015. Website: `https://www.e-stat.go.jp/stat-search/files?stat_infid=000031622557` (in Japanese) (Accessed November 8, 2021).

[35]This study assumes no correlation between long-term care and medical expenditures. Suzuki et al. (2012) report no correlation between long-term care and medical expenditures after controlling for inpatients and nursing home residents. This study uses the complete set of insurance claims data provided by public insurers of Fukui Prefecture in Japan. This study is written in Japanese.

where κ is the public pension replacement rate. I further assume that the benefits of a female older parent depend on the average earnings of the skill group instead of the past individual earnings.[36]

$$\text{pen}_{pf}(j^p, z) = \kappa \cdot \frac{1}{J-1} \sum_{j=1}^{J} \mathbf{E}\left[\epsilon(j,z)\mu(j)\frac{1}{\overline{WH}_{kf}}\left(\overline{DH}_{kf} - \mathbb{1}_{h\in\{2,3\}}T(\phi_j) - l_j\right)\right] \tag{3}$$

Means-tested Welfare Transfer Program: Individuals with low income and savings can be eligible for the means-tested welfare program (i.e., *seikatsu-hogo*). This covers their minimum living expenses, and the means-tested welfare program covers their long-term and medical care expenditures. A means-tested transfer tr is provided to guarantee a minimum consumption level $\underline{c}$ for each generation. This level differs by marital status. The transfer amount for a family is given as follows.

$$tr = \left\{0, (1+\tau^c)(\underline{c}_k + \underline{c}_p) - \left(Ra + \sum_{i\in\{kf,km\}}(1-\tau^l)(1+\nu)y_i + \sum_{i\in\{pf,pm\}}(\text{pen}_i - M_i^{op} - H_i^{op})\right)\right\}$$

As in De Nardi et al. (2010), this study imposes that if transfers are positive, the family consumes all of its resources, $a' = 0$.

Taxes: The government imposes proportional taxes on consumption at rate τ^c, labor income at τ^l, capital income at τ^a, and lump-sum tax τ^{ls} on each individual. The net-of-tax gross return on capital is denoted as $R = 1 + (1-\tau^a)r$, where r is an interest rate. The government budget constraint is given as follows.

$$\tau^l Y_l + \tau^a Y_a + \tau^c(C_k + C_p) + \tau^{ls}N = SS + HI + LTC + TR + G \tag{4}$$

where Y_l, Y_a, C_k, and C_p denote aggregate labor income, capital income, and consumption for the adult child and older parent generation, respectively. N denotes the total number of individuals. SS, HI, LTC, and TR each denote the total government expenditures for a public pension, public health insurance, public LTCI, and the means-tested welfare program, respectively. G denotes the government's consumption expenditures.

In the baseline model, I assume that τ^{ls} is zero and let G absorb the imbalance and satisfy equation (4) to isolate the effects of governmental long-term care expenditure and focus on changes from different risks individuals face over the life-cycle. In the numerical experiments in section 5, I consider various policy scenarios and adjust τ^{ls} to account for a change in the net government revenues to balance the government budget in equation (4).

[36] Although pension benefits depend on past individual earnings in the actual economy, there is a substantial additional burden for computation by introducing a new state variable, such as average lifetime earnings. To keep the state space dimensions manageable, this study follows the formalization of Attanasio et al. (2010).

3.8 Problems of Families

State: I summarize the state as $\mathbf{x} = \{j^k, a, z, z', h, \iota_{-1}, \mu\}$. Families are heterogeneous in terms of the age of the adult child generation j^k; asset a; skill of the older parent generation z; skill of the adult child generation z'; current LTC-status of the female older parent h; use of institutional care for the female older parent in the previous period ι_{-1}; and idiosyncratic wage shock for the female adult child μ. I define the problem of families with six value functions.

Case 1. Value function of no parents $(h = 4)$ The state vector of a family without parents is given as $(j^k, a, z, z', h = 4, \mu)$. Given the states, a family optimally chooses the consumption of the adult child generation c_k, the leisure of a female in the adult child generation l_{kf}, and savings a' to maximize utility over the life-cycle. The value function is expressed as follows.

$$V_{j^k}^K(a, z, z', h = 4, \mu) = \max_{c_k, l_{kf}, a'} \left\{ u_k(c_k, l_{kf}) + \beta \, \mathbf{E} \, \tilde{V}_{j^k+1}(\mathbf{x}') \right\}$$

subject to

$$(1 + \tau^c)c_k + a' = Ra + (1 - \tau^l)(1 + \nu)(y_{kf} + y_{km}) + tr$$

where

$$\begin{aligned} & y_{kf} = \epsilon \, \mu \left(\left(\overline{DH}_{kf} - l_{kf} \right) / \overline{WH}_{kf} \right) \\ & a' \geq 0, \;\; c_k \geq 0 \\ & 0 \leq \;\; l_{kf} \leq \overline{DH}_{kf} \end{aligned}$$

$$\mathbf{E} \, \tilde{V}_{j^k+1}(\mathbf{x}') = \begin{cases} \sum_{\mu'} \Lambda(\mu', \mu) V_{j^k+1}^K(a', z, z', h' = 4, \mu') & \text{if } j^k < J \\ (1 + \nu) \sum_{z''} \Omega_{z''|z'} \sum_{\mu'} \overline{\Lambda}(\mu') V_1^{ND} \left(\frac{a'}{(1+\nu)}, z', z'', h' = 1, \iota_{-1} = 0, \mu' \right) & \text{if } j^k = J \end{cases}$$

Case 2. Value function of heavy LTC-status in the facility $(h = 3$ **and** $\iota_{-1} = 1)$ The state vector of a family is given as $(j^k, a, z, z', h = 3, \iota_{-1} = 1, \mu)$. Given the states, a family optimally chooses the consumption of the adult child generation c_k, leisure of a female in the adult child generation l_{kf}, and savings a' to maximize utility over the life-cycle. Note that once the family chooses public institutional care services, the female older parent with disabilities spends her entire life in the public facility. Since the female older parent used institutional care in previous periods, the family pays both the cost of long-term care services $\bar{q}$ and the fee for the use of the facility $\bar{c}$, including the residence fee, the food fee, and the expenses of daily living.

$$V_{j^k}^{HI}(a, z, z', h = 3, \iota_{-1} = 0, \mu) = \max_{c_k, l_{kf}, a'} \left\{ u_k(c_k, l_{kf}) + u_p(\bar{c}) + \beta \, \mathbf{E} \, \tilde{V}_{j^k+1}(\mathbf{x}') \right\}$$

subject to

$$(1+\tau^c)(c_k+\bar{c})+a'+H_{pf}^{op}=Ra+(1-\tau^l)(1+\nu)(y_{kf}+y_{km})+\text{pen}_{pf}-M_{pf}^{op}+tr$$

where

$$\begin{aligned}
&y_{kf}=\epsilon\ \mu\left(\left(\overline{DH}_{kf}-l_{kf}\right)/\overline{WH}_{kf}\right)\\
&H_{pf}^{op}=\lambda^h\bar{q}\\
&a'\geq 0,\ c_k\geq 0\\
&0\leq\ l_{kf}\leq\overline{DH}_{kf}
\end{aligned}$$

$$\mathbf{E}\,\tilde{V}_{j^k+1}(\mathbf{x}')=\begin{cases}
\sum_{\mu'}\Lambda(\mu',\mu)[\Psi(h'=2\mid h=3,j^p)V_{j^k+1}^{LI}(a',z,z',h'=2,\iota_{-1}=1,\mu') & \text{if } j^k<J\\
\qquad+\Psi(h'=3\mid h=3,j^p)V_{j^k+1}^{HI}(a',z,z',h'=3,\iota_{-1}=1,\mu')\\
\qquad+\Psi(h'=4\mid h=3,j^p)V_{j^k+1}^{K}(a',z,z',h'=4,\mu')] & \\
(1+\nu)\sum_{z''}\Omega_{z''|z'}\sum_{\mu'}\overline{\Lambda}(\mu')V_1^{ND}\left(\frac{a'}{(1+\nu)},z',z'',h'=1,\iota_{-1}=0,\mu'\right) & \text{if } j^k=J
\end{cases}$$

Case 3. Value function of heavy LTC-status at home ($h=3$ **and** $\iota_{-1}=0$) The state vector of a family is expressed as $(j^k,a,z,z',h=3,\iota_{-1}=0,\mu)$. Given the states, a family optimally chooses the long-term care services between institutional care and at-home care in the next period ι, the consumption of the adult child generation c_k, the consumption of the older parent generation c_p, the leisure of a female in the adult child generation l_{kf}, and savings a' to maximize utility over the life-cycle. The value function is expressed as follows.

$$\begin{aligned}
&V_{j^k}^{HC}(a,z,z',h=3,\iota_{-1}=0,\mu)\\
&=\max_{\iota\in\{0,1\}}\left\{(1-\iota)\left(H_{j^k}(a,z,z',h=3,\iota_{-1}=0,\mu)\right)+\iota\left(V_{j^k}^{HI}(a,z,z',h=3,\iota_{-1}=0,\mu)+\xi\right)\right\}
\end{aligned}$$

If the family chooses institutional care in the next periods ($\iota=1$), see case 2. If the family chooses at-home care ($\iota=0$), the family determines both the time of use of IC ϕ and FHC q at the same time to satisfy equation (1). Note that q is determined by minimizing out-of-pocket long-term care expenditures when ϕ is given as follows.

$$q^*(\phi)=\begin{cases}
0 & \text{if } (\chi_{h=3}/A)^\rho-(1-\theta_{h=3})(T(\phi)\times 365)^\rho\leq 0\\
\left(\frac{(\chi_{h=3}/A)^\rho-(1-\theta_{h=3})(T(\phi)\times 365)^\rho}{\theta_{h=3}}\right)^{\frac{1}{\rho}}p_{\text{LTC}} & \text{if } (\chi_{h=3}/A)^\rho-(1-\theta_{h=3})(T(\phi)\times 365)^\rho>0
\end{cases}$$

Thereafter, the value function can be rewritten as follows.

$$H_{j^k}(a,z,z',h=3,\iota_{-1}=0,\mu)=\max_{\phi\in\{0,1/8,1/2,1\}}\left\{\max_{c_k,c_p,l_{kf},a'}\{u_k(c_k,l_{kf})+u_p(c_p)+\beta\,\mathbf{E}\,\tilde{V}_{j^k+1}(\mathbf{x}')\}\right\}$$

subject to

$$(1+\tau^c)(c_k+c_p)+a'+H^{op}_{pf}=Ra+(1-\tau^l)(1+\nu)(y_{kf}+y_{km})+\text{pen}_{pf}-M^{op}_{pf}+tr$$

where

$$\begin{aligned}
&y_{kf}=\epsilon\ \mu\left(\left(\overline{DH}_{kf}-T(\phi)-l_{kf}\right)/\overline{WH}_{kf}\right)\\
&H^{op}_{pf}=\lambda^h q^*(\phi)\\
&a'\geq 0,\ c_k,c_p\geq 0\\
&0\leq\ l_{kf}\leq\overline{DH}_{kf}-T(\phi)
\end{aligned}$$

$$\mathbf{E}\,\tilde{V}_{j^k+1}=\begin{cases}
\sum_{\mu'}\Lambda(\mu',\mu)[\Psi(h'=2\mid h=3,j^p)V^{LC}_{j^k+1}(a',z,z',h'=2,\iota_{-1}=0,\mu') & \text{if } j^k<J\\
\qquad+\Psi(h'=3\mid h=3,j^p)V^{HC}_{j^k+1}(a',z,z',h'=3,\iota_{-1}=0,\mu')\\
\qquad+\Psi(h'=4\mid h=3,j^p)V^{K}_{j^k+1}(a',z,z',h'=4,\mu')]\\
\\
(1+\nu)\sum_{z''}\Omega_{z''|z'}\sum_{\mu'}\overline{\Lambda}(\mu')V^{ND}_1\left(\frac{a'}{(1+\nu)},z',z'',h'=1,\iota_{-1}=0,\mu'\right) & \text{if } j^k=J
\end{cases}$$

Case 4. Value function of light LTC-status in the facility ($h=2$ **and** $\iota_{-1}=1$) The state vector of a family is given as $(j^k,a,z,z',h=2,\iota_{-1}=1,\mu)$. Given the states, a family optimally chooses the consumption of the adult child generation c_k, the leisure of a female in the adult child generation l_{kf}, and savings a' to maximize utility over the life-cycle. The value function is expressed as follows.

$$V^{LI}_{j^k}(a,z,z',h=2,\iota_{-1}=0,\mu)=\max_{c_k,l_{kf},a'}\left\{u_k(c_k,l_{kf})+u_p(\bar{c})+\beta\,\mathbf{E}\,\tilde{V}_{j^k+1}(\mathbf{x}')\right\}$$

subject to

$$(1+\tau^c)(c_k+\bar{c})+a'+H^{op}_{pf}=Ra+(1-\tau^l)(1+\nu)(y_{kf}+y_{km})+\text{pen}_{pf}-M^{op}_{pf}+tr$$

where

$$\begin{aligned}
&y_{kf}=\epsilon\ \mu\left(\left(\overline{DH}_{kf}-l_{kf}\right)/\overline{WH}_{kf}\right)\\
&H^{op}_{pf}=\lambda^h\bar{q}\\
&a'\geq 0,\ c_k\geq 0\\
&0\leq\ l_{kf}\leq\overline{DH}_{kf}
\end{aligned}$$

$$\mathbf{E}\,\tilde{V}_{j^k+1}(\mathbf{x}')=\begin{cases}
\sum_{\mu'}\Lambda(\mu',\mu)[\Psi(h'=2\mid h=2,j^p)V^{LI}_{j^k+1}(a',z,z',h'=2,\iota_{-1}=1,\mu') & \text{if } j^k<J\\
\qquad+\Psi(h'=3\mid h=2,j^p)V^{HI}_{j^k+1}(a',z,z',h'=3,\iota_{-1}=1,\mu')\\
\qquad+\Psi(h'=4\mid h=2,j^p)V^{K}_{j^k+1}(a',z,z',h'=4,\mu')]\\
\\
(1+\nu)\sum_{z''}\Omega_{z''|z'}\sum_{\mu'}\overline{\Lambda}(\mu')V^{ND}_1\left(\frac{a'}{(1+\nu)},z',z'',h'=1,\iota_{-1}=0,\mu'\right) & \text{if } j^k=J
\end{cases}$$

Case 5. Value function of light LTC-status at home ($h = 2$ **and** $\iota_{-1} = 0$) The state vector of a family is given as $(j^k, a, z, z', h = 2, \iota_{-1} = 0, \mu)$. Given the states, a family optimally chooses both times of use of IC ϕ and FHC hours q at the same time to satisfy equation (1), consumption of the adult child generation c_k, consumption of the older parent generation c_p, leisure of a female in the adult child generation l_{kf}, and savings a' to maximize utility over the life-cycle. The value function is expressed in the following way.

$$V^{LC}_{j^k}(a, z, z', h = 2, \iota_{-1} = 0, \mu) = \max_{\phi \in \{0,1/8,1/2,1\}} \left\{ \max_{c_k, c_p, l_{kf}, a'} \{u_k(c_k, l_{kf}) + u_p(c_p) + \beta \, \mathbf{E} \, \tilde{V}_{j^k+1}(\mathbf{x}')\} \right\}$$

subject to

$$(1 + \tau^c)(c_k + c_p) + a' + H^{op}_{pf} = Ra + (1 - \tau^l)(1 + \nu)(y_{kf} + y_{km}) + \text{pen}_{pf} - M^{op}_{pf} + tr$$

where

$$\begin{aligned}
&y_{kf} = \epsilon \, \mu \left(\left(\overline{DH}_{kf} - T(\phi) - l_{kf} \right) / \overline{WH}_{kf} \right) \\
&H^{op}_{pf} = \lambda^h q^*(\phi) \\
&a' \geq 0, \;\; c_k, c_p \geq 0 \\
&0 \leq \;\; l_{kf} \leq \overline{DH}_{kf} - T(\phi)
\end{aligned}$$

$$\mathbf{E} \, \tilde{V}_{j^k+1} = \begin{cases} \sum_{\mu'} \Lambda(\mu', \mu) [\Psi(h' = 2 \mid h = 2, j^p) V^{LC}_{j^k+1}(a', z, z', h' = 2, \iota_{-1} = 0, \mu') & \text{if } j^k < J \\ \qquad + \Psi(h' = 3 \mid h = 2, j^p) V^{HC}_{j^k+1}(a', z, z', h' = 3, \iota_{-1} = 0, \mu') \\ \qquad + \Psi(h' = 4 \mid h = 2, j^p) V^{K}_{j^k+1}(a', z, z', h' = 4, \mu')] \\ \\ (1 + \nu) \sum_{z''} \Omega_{z''|z'} \sum_{\mu'} \overline{\Lambda}(\mu') V^{ND}_1 \left(\frac{a'}{(1+\nu)}, z', z'', h' = 1, \iota_{-1} = 0, \mu' \right) & \text{if } j^k = J \end{cases}$$

Case 6. Value function of no disability ($h = 1$) The state vector of a family is given as $(j^k, a, z, z', h = 1, \iota_{-1} = 0, \mu)$. Given the states, a family optimally chooses the consumption of the adult child generation c_k, the consumption of the older parent generations c_p, the leisure of a female in the adult child generation l_{kf}, and savings a' to maximize utility over the life-cycle. The value function is expressed in the following way.

$$V^{ND}_{j^k}(a, z, z', h = 1, \iota_{-1} = 0, \mu) = \max_{c_k, c_p, l_{kf}, a'} \left\{ u_k(c_k, l_{kf}) + u_p(c_p) + \beta \, \mathbf{E} \, \tilde{V}_{j^k+1}(\mathbf{x}') \right\}$$

subject to

$$\begin{aligned}
&(1 + \tau^c)(c_k + c_p) + a' \\
=&Ra + (1 - \tau^l)(1 + \nu)(y_{kf} + y_{km}) + \sum_{i \in \{pf, pm\}} \text{pen}_i - \sum_{i \in \{pf, pm\}} M^{op}_i - H^{op}_{pm} + tr
\end{aligned}$$

where

$$
\begin{aligned}
& y_{kf} = \epsilon\ \mu\left(\left(\overline{DH}_{kf} - l_{kf}\right)/\overline{WH}_{kf}\right) \\
& a' \geq 0,\ \ c_k, c_p \geq 0 \\
& 0 \leq\ \ l_{kf} \leq \overline{DH}_{kf}
\end{aligned}
$$

$$
\mathbf{E}\,\tilde{V}_{j^k+1}(\mathbf{x}') = \begin{cases}
\sum_{\mu'} \Lambda(\mu', \mu)[\Psi(h'=1 \mid h=1, j^p) V^{ND}_{j^k+1}(a', z, z', h'=1, \iota_{-1}=0, \mu') & \text{if } j^k < J \\
\qquad +\Psi(h'=2 \mid h=1, j^p) V^{LC}_{j^k+1}(a', z, z', h'=2, \iota_{-1}=0, \mu') & \\
\qquad +\Psi(h'=3 \mid h=1, j^p) V^{HC}_{j^k+1}(a', z, z', h'=3, \iota_{-1}=0, \mu') & \\
\qquad +\Psi(h'=4 \mid h=1, j^p) V^{K}_{j^k+1}(a', z, z', h'=4, \mu')] & \\
 & \\
(1+\nu) \sum_{z''} \Omega_{z''|z'} \sum_{\mu'} \overline{\Lambda}(\mu') V^{ND}_1\left(\frac{a'}{(1+\nu)}, z', z'', h'=1, \iota_{-1}=0, \mu'\right) & \text{if } j^k = J
\end{cases}
$$

3.9 Equilibrium

Stationary Recursive Competitive Equilibrium: Given the interest rate r, and a set of government policies $\{\lambda^h, \lambda^m, \tau^c, \tau^a, \tau^l\}$, a stationary recursive competitive is a set of value functions $\{V^{ND}_{j^k}(\mathbf{x}), V^{LC}_{j^k}(\mathbf{x}), V^{LI}_{j^k}(\mathbf{x}), V^{HC}_{j^k}(\mathbf{x}), V^{HI}_{j^k}(\mathbf{x}), V^{K}_{j^k}(\mathbf{x})\}^J_{j^k=1}$, family decision rules $\{c_{k,j^k}(\mathbf{x}), c_{p,j^k}(\mathbf{x}), l_{kf,j^k}(\mathbf{x}), a_{j^k+1}(\mathbf{x}), \iota_{j^k}(\mathbf{x}), \phi_{j^k}(\mathbf{x}), q_{j^k}(\mathbf{x})\}^J_{j^k=1}$, time-invariant measures of families $X_{j^k}(\mathbf{x})$ with age-j^k families with the state vector $\mathbf{x} = \{a, z, z', h, \iota_{-1}, \mu\}$, and lump-sum transfer τ^{ls}, such that the following conditions are satisfied.[37]

1. Given factor prices and the government policies, the family decision rules solve the family decision problem in section 3.8.

2. The government budget is balanced in equation (4).

3. Individuals and aggregate behavior are consistent as follows.

$$
\begin{aligned}
Y_l &= \sum_{j^k=1}^{J} \sum_{\mathbf{x}} \left[y_{kf}(\mathbf{x}) + y_{km}(\mathbf{x})\right] X_{j^k}(\mathbf{x}) \\
&= \sum_{j^k=1}^{J} \sum_{\mathbf{x}} \left[\frac{\epsilon(j^k, z')\mu(j^k)}{\overline{WH}_{kf}(j^k)} \left(\overline{DH}_{kf}(j^k) - \mathbb{1}_{h\in\{2,3\}} T(\phi_{j^k}) - l_{kf,j^k}(\mathbf{x})\right) + y_{km}(j^k, z')\right] X_{j^k}(\mathbf{x}) \\
Y_a &= \sum_{j^k=1}^{J} \sum_{\mathbf{x}} Ra_{j^k}(\mathbf{x}) X_{j^k}(\mathbf{x}) \\
C_k &= \sum_{j^k=1}^{J} \sum_{\mathbf{x}} c_{k,j^k}(\mathbf{x}) X_{j^k}(\mathbf{x})
\end{aligned}
$$

[37] See section Appendix C for further details of the numerical procedures.

$$C_p = \sum_{j^k=1}^{J} \sum_{\mathbf{x}} c_{p,j^k}(\mathbf{x}) X_{j^k}(\mathbf{x})$$

$$N = \sum_{j^k=1}^{J} \sum_{\mathbf{x}} \left[2 + n^p(j^p, h)\right] X_{j^k}(\mathbf{x})$$

$$SS = \sum_{j^k=1}^{J} \sum_{\mathbf{x}} \left[\text{pen}_{pf}(j^p, z') + (n^p(j^p, h) - 1) pen_{pm}(j^p, z')\right] X_{j^k}(\mathbf{x})$$

$$HI = \sum_{j^k=1}^{J} \sum_{\mathbf{x}} (1 - \lambda^m(j^p)) \left[M_{pf}(j^p) + (n^p(j^p, h) - 1) M_{pm}(j^p)\right] X_{j^k}(\mathbf{x})$$

$$LTC = \sum_{j^k=1}^{J} \sum_{\mathbf{x}} (1 - \lambda^h) \left[q_{j^k}(\mathbf{x}) + (n^p(j^p, h) - 1) H_{pm}(j^p)\right] X_{j^k}(\mathbf{x})$$

$$TR = \sum_{j^k=1}^{J} \sum_{\mathbf{x}} tr(\mathbf{x}) X_{j^k}(\mathbf{x})$$

4. The public pension benefit system is balanced in equations (2) and (3).

5. The set of age-dependent measures of families satisfies the following conditions.

   – For $j^k < J$,

$$\begin{aligned} & X_{j^k+1}(a', z, z', h', \iota, \mu') \\ =& \frac{1}{(1+\nu)^{1/J}} \sum_{\{a,h,\iota_{-1},\mu:a',\iota\}} \Psi(h' \mid h, j^p) \Lambda(\mu', \mu) X_{j^k}(a, z, z', h, \iota_{-1}, \mu) \end{aligned} \tag{5}$$

   where a' and ι are the optimal choices in the later periods.

   – For $j^k = J$,

$$\begin{aligned} & X_1(a', z', z'', h' = 1, \iota = 0, \mu') \\ =& (1+\nu) \sum_{\{a,z,h,\iota_{-1},\mu:a'\}} \Omega_{z''|z'} \overline{\Lambda}(\mu') X_J(a, z, z', h, \iota_{-1}, \mu) \end{aligned} \tag{6}$$

   where a' is the optimal choice in the next periods.

3.10 Model Discussion

Before describing the model's calibration, I discuss several elements of the model which are critical to determining the care arrangements. I focus on the implications for the two main mechanisms in making care arrangements—caregivers' opportunity cost and the family savings.

The cost of caregiver opportunities in the labor market plays a significant role, as shown in Van Houtven et al. (2013) and Skira (2015). The opportunity cost of IC

depends on wage rates and the value of leisure.[38] If the opportunity cost of the female adult child is relatively low, the IC cost would be less than the FHC cost for the family. Since my model is a partial equilibrium model, permanent labor productivity $\epsilon(j^k, z)$ and idiosyncratic wage shocks μ among working-age married females determine wage rates. Furthermore, in the two-sided altruism model, the preference of the female older parent also affects the cost of providing IC by the female adult child. If the preference for IC is relatively high, the demand for IC would exceed the demand for formal care services that the LTCI covers. Although the preference of the female older parent for IC ω affects the choice between IC and FHC at home, the cost of the female older parent for public institutional care ξ affects the determination of care between public institutional care and at-home care.

The family savings is important for care arrangements because savings provide a source of insurance against long-term care risks in old age. Once a female older parent faces disability shocks, the family uses the savings to cover the substantial expenses of formal care services. Furthermore, the altruism of the female older parent also affects the decision to use savings as insurance against the risk of disability in old age. The desire to leave a bequest increases the family savings, as shown in Lockwood (2018). In the two-sided altruism model, the older parent generation can increase the future resources of their descendants by leaving a bequest, and the adult child generation also can prevent the cutting-off of their bequest of the older parent generation by providing IC, as discussed in Groneck (2017).

Then the family makes care arrangements, depending on the caregiver's opportunity cost and the family savings. When the family has sufficient savings, the family faces a trade-off between a reduction in current labor income because of using IC and a smaller bequest from a savings cut-off to purchase formal care services. On the contrary, when family savings are not sufficient to purchase formal care services, they turn to an IC or a welfare transfer program.

4 Calibration

This section describes the calibration of the model parameters. I calibrate the steady state economy to the Japanese economy in 2015. The parameters in this model are of two groups. The model parameters in the first group are external parameters directly estimated from the data and literature. Table 6 summarizes their values. The model parameters in the second group are internal parameters calibrated by matching the model-generated targets' value to their data counterparts. Table 7 summarizes the description and values of the parameters. My model is a partial equilibrium model, and the interest

[38] If the female adult child values leisure more, the cost of providing IC increases because the female adult child allocates her disposal time to labor supply, leisure, and IC.

rate r is exogenous and set to 2% based on Aoki et al. (2016).

4.1 Demographics

I let individuals enter the economy at age $j = 1$, which corresponds to 35 years. I set the age difference between an adult child generation and an older parent generation at 30 years because the average age difference between mother and children from 1975–2015 is 30.052 years, according to the Vital Statistics of the MHLW in 2019.[39] Then, individuals retire from the labor market at 65 years and live to the maximum possible age of 94. I set the annual population growth rate at zero. The equivalence scale η adjusts the consumption of each generation according to the size of the household, which assigns $\eta(n) = 1 + 0.7(n-1)$ to the size of the family n, based on Bick and Choi (2013).

4.2 Long-Term Care Risk and Medical Expenditure Risk

I use transition probabilities by age, sex, and current level of care needs estimated in Mikoshiba et al. (2023), described in section 2.3. I assume that the number of household members in the older parent generation $n^p(j^p, h)$ depends on both the age and LTC-status discussed in section 3.3: the deterministic measure of the male older parent $n^p(j^p, h = 1) - 1$ is calibrated based on their survival probabilities, estimated by Mikoshiba et al. (2023).

As shown in Figure 1b, I calculate the average annual gross long-term care expenditures per capita for male older parents H_{pm} from the Statistics of Long-term Care Benefit Expenditures (SLBE) of the MHLW in 2015 and the Population Statistics of Japan of the National Institute of Population and Social Security Research (NIPSSR) in 2017.[40] Further, I calculate the average annual gross medical expenditures for older parents $M_i(j^p)$ for $i \in \{pf, pm\}$ from the National Medical Expenses (NME) of the MHLW in 2015.

4.3 Skill

The transition probabilities of skill inheritance Ω are calibrated to match both the ratio of high-skilled individuals in the working-age population and the correlation between the income of children and parents, as in İmrohoroğlu and Zhao (2018). The proportion of high-skilled individuals is 31%, as reported in the Employment Status Survey (ESS) of the

[39]Data are available here: `https://www.e-stat.go.jp/stat-search/database?statdisp_id=0003411609` (Accessed July 2, 2022).

[40]Note that the Population Statistics of Japan 2017 provides the annual population by age and sex in 2015. Website: `https://www.ipss.go.jp/syoushika/tohkei/Popular/Popular2017RE.asp?chap=0` (Accessed July 30, 2020).

Ministry of Internal Affairs and Communications (MIC) in 2017.[41] I use the estimated value of the correlation between the income of children and parents by Lefranc et al. (2014). The skill inheritance transition probabilities are given as follows.

$$\Omega = \begin{bmatrix} \Omega_{LL} & \Omega_{LH} \\ \Omega_{HL} & \Omega_{HH} \end{bmatrix} = \begin{bmatrix} 0.797 & 0.203 \\ 0.448 & 0.552 \end{bmatrix}$$

In the matrix, the generic element $\Omega_{zz'}$ with $z, z' \in \{L, H\}$ is the probability of the transition of inheriting skills from the older parent generation with skill z to the adult child generation with skill z'. In the steady state, the distribution of skill combinations between the older parent generation and the adult child generation becomes as follows.

$$\begin{bmatrix} 0.549 & 0.139 \\ 0.139 & 0.172 \end{bmatrix}$$

4.4 Endowments

The age-specific deterministic labor productivity $\epsilon(j^k, z)$ for working-age married females is calibrated from their earnings based on the ESS of the MIC. I use the ESS data in 2017 and adjust them to the 2015 level using the consumer price index (CPI). Figure 3 shows the life-cycle earnings profiles for working-age married females by age and skill to calibrate the labor productivity of working-age married females. From Figure 3, it is evident that high-skilled married females earn more than low-skilled married females over the working-age. As well documented in studies including Kitao and Mikoshiba (2020) and Mikoshiba et al. (2023), a large number of female workers leave the labor force at child-bearing ages and return to work after several years, yielding the so-called "M-shaped" patterns.[42] To capture the labor supply at extensive and intensive margins of married females, this study introduces both average working hours and disposable income for married females across ages. I normalize disposable time to 1.0 and calibrate the average working hours using the Time Use Survey of the MIC in 2016.[43] Based on Hsu and Yamada (2019), I take $\Theta = 0.98$ and variance $\sigma_\zeta = 0.09$ and discretize this process into a three-state Markov chain by Tauchen (1986). Subsequently, the resulting value of μ is $\{0.40, 1.00, 2.47\}$, and the initial distribution $\overline{\Lambda}(\mu)$ is $\{0.21, 0.58, 0.21\}$.

[41] Data are available here: `https://www.e-stat.go.jp/dbview?sid=0003222463` (Accessed March 5, 2020).

[42] As shown in Mikoshiba et al. (2023), low-skilled females tend to have children earlier than high-skilled females.

[43] Data are available here: `https://www.stat.go.jp/data/shakai/2016/pdf/gaiyou2.pdf` (in Japanese) (Accessed February 24, 2022).

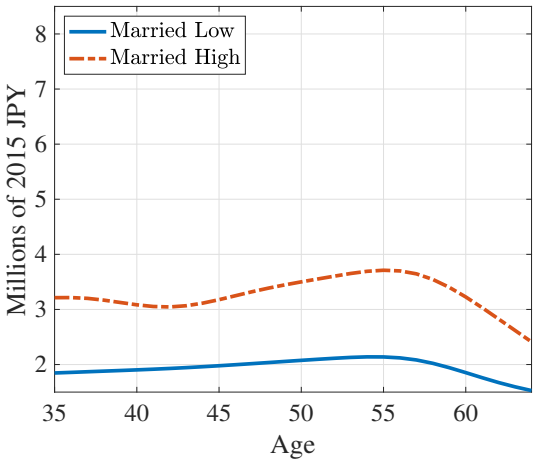


Figure 3: Earnings of Married Females by Age and Skill

Note: Figure 3 shows married females' earnings by age and skill. The married sample includes both widowed and divorced individuals. I define individuals as high-skilled if they have a college or higher degree and low-skilled otherwise. The data are obtained from the Employment Status Survey (ESS) of the Ministry of Internal Affairs and Communications (MIC) in 2017. I use the data from 2017 and adjust them to the 2015 level using the CPI.

The average earnings of married males $y_{km}(j^k, z)$ vary deterministically with age and skill. I compute them using the ESS data on married males' average earnings. I use the data for 2017 and adjust them to the 2015 level using the CPI. Figure 4 shows the life-cycle profiles of married males of average earnings by age and skill. As well documented in studies including Kitao and Mikoshiba (2020), there is a large difference in earnings between sexes and skill levels. Regardless of skill level, male earnings are much higher than female earnings. High-skilled married males earn the most among the four profiles.

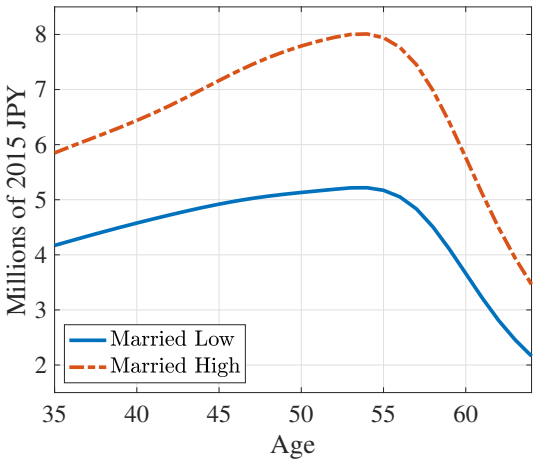


Figure 4: Average Earnings of Married Males by Age and Skill

Note: Figure 4 reveals the average earnings of married males by age and skill. Average earnings are calculated by multiplying earnings by the labor force participation of married males. The married sample includes both widowed and divorced individuals. I individuals as high-skilled if they have a college or higher degree and as low-skilled otherwise. The data are obtained from the Employment Status Survey (ESS) of the Ministry of Internal Affairs and Communications (MIC) in 2017. I use the data from 2017 and adjust them to the 2015 level using the CPI.

The leisure of the male adult child, the male older parent, and the female older parent are calculated using the 2016 Time Use Survey of the MIC. The values are 0.54, 0.54, and 0.50 for the male adult child, the male older parent, and the female older parent, respectively.

4.5 Care Arrangements

For the cost of institutional care services, I use the Survey of Institutions and Establishments for Long-term Care (SIEL) of the MHLW in 2016. The SIEL reports the average cost for different types of expenditures for institutional care services covered by the LTCI by different levels of long-term care. First, I calculate the weighted average cost of institutional care services in the welfare care facilities for the elderly (special nursing homes) and obtain 372.83 (10,000-yen, CPI adjusted it in 2015).[44] Secondly, I calculate the weighted average fee for the welfare care facilities for the elderly (special nursing homes). Since institutional care recipients in the facility must pay for their living costs, I calculate the weighted average living costs as the sum of the residence fee, food fee, and daily living expenses.[45]

For the parameters of at-home care, the average cost of FHC per hour p_{LTC} was first directly calibrated from the CSLC in 2016. I use the "care sample" and obtain 0.167 (10,000-yen, CPI adjusted in 2015) by using the information on the time and monthly expenditures of the FHC services. In the second step, I calibrate six parameters of at-home care: returns to care input hours A; substitutability of IC-FHC ρ; FHC productivity θ_h for $h \in \{2, 3\}$; the cost parameter for entering public institutional care relative to at-home care ξ; and the preference parameter for IC relative to FHC ω. I calibrate these parameters to ensure that the model achieves the target values from the data. The target value of A represents the average annual long-term care hours, and I obtain 1291.96 hours from the CSLC "care sample" in 2016, reported in Table 4. From the CSLC "family sample" in 2016, I calculate the target values of ρ and θ_h—that is, the correlation of IC–FHC hours and the ratio of FHC hours to total hours, respectively. For the IC-FHC hours, as in Daruich (2018), I group the eligible individuals by the quartile of FHC hours,[46] compute average annual hours for IC and FHC for each quartile and calculate

[44]Data are available here: `https://www.mhlw.go.jp/toukei/saikin/hw/kaigo/service16/dl/data28.xlsx` (in Japanese) (Downloaded on July 10, 2022).

[45]The average residence fee is set at 58.83 (10,000-yen), as the average standard amount of residence fee by different types of institutions. The average food fee is set at 50.37 (10,000-yen) by the standard amount of the food fee, and the living costs are set at 12 (10,000-yen). For details of the standard amount of living costs, see, for example, `https://www.kaigokensaku.mhlw.go.jp/commentary/fee.html` (in Japanese) (Accessed July 10, 2022). Data are available here: `https://www.e-stat.go.jp/stat-search/file-download?statInfId=000031627136&fileKind=1` (in Japanese) (Downloaded on July 10, 2022).

[46]I compute the average annual FHC hours by the total expenditures and the average cost of FHC per

the correlation between the two averages. The target values of ξ and ω are the share of institution users and the ratio of IC users, described in Tables 3 and 5. See Table 7 for the model generated and target values of the parameters.

4.6 Preference

The coefficient of relative risk aversion σ is set at 3.0, which is in the range of values used in the literature. For example, De Nardi et al. (2016) set the risk aversion at 2.83 by the estimation in the model. The subjective discount factor β is set at 0.9799 to ensure that the model achieves the average per adult equivalent wealth of 823.93 (10,000-yen, adjusted for by the CPI in 2015), based on Kitao and Yamada (2019), who use the National Survey of Family Income and Expenditure in 2014.

I set the intensity of leisure in the utility function γ at 0.5 to correspond to the average participation of working-age married males in the labor force, 70.71%, the ESS data in 2017 of the MIC. The calibrated value of γ is in the range of values used in the literature. For example, Fuster et al. (2007) set the leisure intensity at 0.63, and Gao (2020) estimates it at 0.42, as estimated in the model. See Table 7 for the model-generated and target values of the parameters.

4.7 Government

The government operates the public LTCI, public health insurance, pay-as-you-go public pension, and means-tested welfare transfer program. The copayment ratio of the LTCI λ^h is set to 10% for all ages. The LTCI covers 90% of the long-term care expenditures for both FHC and institutional care services. Public health insurance also covers part of the medical expenditure, and its copayment ratio λ^m_{jp} varies with age. I set λ^m_{jp} at 30%, 20%, and 10% for those aged 69 years and below, between 70 and 74, and above 75, respectively. The pension replacement rate κ is set at one-third, based on the OECD (2019)'s estimated average gross replacement rate of public pensions. The means-tested welfare program of my model provides means-tested transfers to eligible households. The consumption floor is set at 87 and 132 (10,000-yen) for widowed and married couples, respectively.[47]

I set the consumption tax rate at 8% based on the tax rate in 2015. Furthermore, I set the labor and capital tax rates at 30% and 35%, respectively, based on Gunji and Miyazaki (2011) and Kitao and Mikoshiba (2020)—consistent with literature estimates of

hour p_{LTC} using the CSLC data in 2016.

[47]The amount is set to be within the range of average public assistance (*seikatsu-hogo*) payments for the family size. The monthly amount is multiplied by 12. For more details on the program, see, for example, `https://www.mhlw.go.jp/file/05-Shingikai-12601000-Seisakutoukatsukan-Sanjikanshitsu_Shakaihoshoutantou/kijun23_05.pdf` (in Japanese) (Accessed June 6, 2022).

effective income tax rates.[48]

Table 6: External Parameters of the Model

Parameter	Description	Value/Source
Demographics		
J	Lifetime span	30 (initial age corresponds to 35)
ν_g	Population growth	0
η	Equivalence scale	Bick and Choi (2013)
Long-term care and Mortality risk, and Long-term care and Medical expenditures		
Ψ	LTC-status transition probabilities	Mikoshiba et al. (2023)
$n^p(j^p, h)$	# of household members in the older parent generation	Mikoshiba et al. (2023)
$H_{pm}(j^p)$	Average gross long-term care expenditure	SLBE (2015) by MHLW and Population Statistics 2017 by NIPSSR
$M_i(j^p)$	Average gross medical expenditure	NME (2015) by MHLW
Endowments		
$\epsilon(j^k, z)$	Average earnings of married female	ESS (2017) by MIC
$\overline{WH}_{yf}(j^k)$	Average working hour	Time use survey (2016) by MIC
$\overline{DH}_{yf}(j^k)$	Average disposal time	Time use survey (2016) by MIC
Θ	Shock of productivity	0.98, Hsu and Yamada (2019)
σ_ζ	Shock of productivity	0.09, Hsu and Yamada (2019)
$y_{km}(j^k, z)$	Average earnings of married male	ESS (2017) by MIC
Ω	Skill inheritance transition	Lefranc et al. (2014), ESS (2017) by MIC
$\bar{l}_{cm}, \bar{l}_{pm}, \bar{l}_{pf}$	Average leisure time	{0.54, 0.54, 0.50} Time use survey (2016) by MIC
Care Arrangement		
p_{LTC}	Average cost of FHC per hour	0.176 (10,000-yen), CSLC (2016) by MHLW
χ_h	Minimum requirement of care hours	$\{1013.09, 2017.58\}$, CSLC (20016) by MHLW
$\bar{c}$	Minimum consumption level in facility	121.20 (10,000-yen), SIEL (2016) by MHLW
$\bar{q}$	Average formal care cost in facility	327.83 (10,000-yen), SIEL (2016) by MHLW
Preference		
σ	Risk aversion parameter	3.0
Government		
λ^h	LTCI copayment rates	10%
$\lambda^m_{j^p}$	Public health insurance copayment rates	30, 20, 10% (varies by age)
τ^c	Consumption tax rates	8%
τ^a	Labor income tax rates	30%, Gunji and Miyazaki (2011)
τ^l	Capital income tax rate	35%, Kitao and Mikoshiba (2020)
κ	Public pension replacement rate	1/3, OECD (2019)
$\underline{c}$	Consumption floor	87 for widowed, 132 for married (10,000-yen)
Other Parameter		
r	Interest rate	2%, Aoki et al. (2016)

[48]For example, Hansen and İmrohoroğlu (2016) estimate the capital income tax rate in 2010 at 35.6%.

Table 7: Internal Parameters of the Model

Parameter	Values	Description	Target	Data	Model
Preference					
β	0.9799	Subjective discount factor	Average per adult equiv wealth	823.93	814.37
γ	0.5000	Intensity of leisure	Average FLFP rate	0.7071	0.7127
Care Arrangements					
A	2.5625	Returns to care input hours	Average annual long-term care hours	1291.9	1287.1
ρ	0.4100	IC-FHC substitutability	IC-FHC hours correlation	-0.317	-0.494
$\theta_{h=2}$	0.4100	FHC productivity (Light)	IC hours ratio in total (Light)	0.5992	0.5537
$\theta_{h=3}$	0.5300	FHC productivity (Heavy)	IC hours ratio in total (Heavy)	0.4790	0.4758
ω	1.5000	Preference for IC	Ratio of IC user	0.9348	0.9451
ξ	2.4922	Cost for entering public facility	Ratio of recipients at home	0.6103	0.6078

5 Numerical Analysis

In this section, I show the numerical results of my quantitative model. First, I review and discuss the results of the baseline model. I then analyze the roles of the LTCI and evaluate how the universal LTCI with benefits-in-kind policy affects individual behavior over the life-cycle and welfare by simulating policy experiments.

5.1 Baseline Model

In this section, I show the care arrangements at home in the baseline model. Table 8 presents the care arrangements at home according to LTC-status and illustrates the distribution of the care arrangements at home from three types of long-term care options—only IC from her female working-age adult child, mixed use of IC and FHC, and only FHC. Table 8 shows that my model replicates the overall pattern of care arrangements well relative to the distribution pattern in the data: the mixed use of IC and FHC is the most common care arrangements at home for each LTC-status; the use of FHC services increases with the level of care; and the total annual hours vary significantly depending on the LTC-status.

Table 8: Distribution of Care Arrangements in the Baseline Model: Model and Data

		Model	Data
Aggregate	Distribution		
	Only IC	16.53%	18.06%
	Mix IC-FHC	77.99%	75.42%
	Only FHC	5.49%	6.52%
	Total Care hours	1287.08 h	1291.96 h
Light	Distribution		
	Only IC	22.06%	20.87%
	Mix IC-FHC	73.62%	75.49%
	Only FHC	4.33%	4.64%
	Total Care hours	1066.30 h	1013.09 h
Heavy	Distribution		
	Only IC	0.00%	10.01%
	Mix IC-FHC	91.05%	78.05%
	Only FHC	8.95%	11.85%
	Total Care hours	1946.77 h	2017.58 h

Table 9 presents care arrangements at home by skill combinations between the older parent generation and the adult child generation (z_p, z_k) at the baseline. As shown in the first row in Table 9, the ratio of IC hours to total care hours is higher for the family with the high-skilled older parent generation and with the low-skilled adult child generation. This is because the skill affects two main mechanisms of care arrangements: the opportunity cost of working-age married females and the amount of family savings.

First, the level of permanent lifetime labor efficiency $\epsilon(j^k, z)$ affects the opportunity costs of married females during working-age. The opportunity cost of providing IC services becomes lower for the family with the low-skilled adult child generation. Then the use of IC services is only higher for the family with the low-skilled adult child generation, and the use of IC services only is high for the family with the high-skilled adult child generation.

Second, the saving of the family with the high-skilled older parent generation is much higher than that of the family with the low-skilled older parent generation because the labor earnings of high-skilled individuals are much higher than those of low-skilled individuals for both sexes, as shown in Figures 3 and 4. Then the cost of choosing IC services becomes lower for the family with the high-skilled older parent generation. This is because the higher amount of family savings leads to a higher level of consumption that the family would enjoy, and the family increases their overall utility by leaving a larger bequest for the offspring. In other words, the family with sufficient savings chooses to decline the current labor earnings of the female adult child through IC rather than decline the savings through IC services.

It can be seen from the first row in Table 9 that the mechanism of the amount of family savings dominates that of the opportunity cost of working-age married females. This is because of the exogenous sufficient labor earnings from the married male in the adult child generation. As the male-female difference in earnings is well documented in Kitao and Mikoshiba (2020), married males have extremely high labor force participation and earn more than unmarried males, married females, and single females.

Table 9: Distribution of Care Arrangements in the Baseline Model by Skill Type

		(High, Low)	(High, High)	(Low, Low)	(Low, High)	Average
Aggregate	Ratio of IC hours to total hours	68.17%	52.28%	52.22%	44.81%	53.42%
	Distribution					
	Only IC	33.39%	21.61%	9.86%	14.96%	16.53%
	Mix IC-FHC	66.59%	66.34%	88.70%	70.22%	77.99%
	Only FHC	0.02%	12.05%	1.44%	14.81%	5.49%
Light	Distribution					
	Only IC	46.88%	32.42%	12.02%	22.37%	22.06%
	Mix IC-FHC	53.10%	58.57%	86.22%	65.19%	73.62%
	Only FHC	0.02%	9.01%	1.75%	12.44%	4.33%
Heavy	Distribution					
	Only IC	0.00%	0.00%	0.00%	0.00%	0.00%
	Mix IC-FHC	100.00%	81.87%	100.00%	80.40%	91.05%
	Only FHC	0.00%	18.13%	0.00%	19.60%	8.95%

Finally, note that this model does not capture externally low-income families. Although the proportion of welfare transfer program recipients accounts for approximately 2% of the population in the real economy, the proportion of means-tested welfare transfer program recipients at baseline accounts for only 0.31% of the population. This is because this model focuses on the family of two married couples. However, this assumption seems reasonable to capture the characteristics of the family born between the 1950s and 1980s.

5.2 Policy Experiments

Japan's LTCI system is universal and covers all citizens aged 65 years and older eligible for LTCI. Japan's LTCI system provides only services and no cash allowance. Eligible individuals can choose their long-term care services from the market with a copayment ratio of 10%.

To understand the LTCI's roles, I simulate the model under different LTCI systems from the baseline and evaluate how the policy change affects the families' behavior and heterogeneous families' welfare. First, I evaluate universal LTCI's roles under the extreme scenario wherein no LTCI is provided. Second, I evaluate the roles of LTCI with a benefits-in-kind policy considering an alternative scenario in which the LTCI provides only cash benefits. The welfare measure is calculated as the percentage change in consumption

required in all possible states, ensuring that people are indifferent between the baseline and simulated scenarios.

5.2.1 An Economy without LTCI

To evaluate the universal LTCI's roles, I first consider an extreme scenario wherein no LTCI is provided. First, I simulate the scenario without tax adjustment to focus on changes in different family risks over the life-cycle and isolate governmental long-term care expenditure's effects. I set G in the first simulation to absorb the imbalance and satisfy the equation (4). In the second simulation, to balance the government budget in equation (4), I adjust a lump-sum tax rate τ^{ls} to account for a change in net government revenue.[49]

Table 10: An Economy without LTCI

	No tax change	Tax adjusted
Change in average IC ratio		
- Average	+ 60.717%	+ 61.255%
- (High, Low)	+ 29.605%	+ 29.746%
- (High, High)	+ 62.330%	+ 62.633%
- (Low, Low)	+ 65.070%	+ 65.912%
- (Low, High)	+ 85.921%	+ 86.070%
Change in average FLFP	- 8.929%	- 9.858%
Change in average savings	+ 10.133%	+ 9.540%
Welfare Program Recipients	1.000%	0.904%
	(+213.971%)	(+183.870%)
Lump-sum tax (JPY)	-	-80,674.423
Welfare effects		
- Average	- 2.522%	- 1.236%
- (High, Low)	- 2.331%	- 1.133%
- (High, High)	- 2.091%	- 1.095%
- (Low, Low)	- 2.690%	- 1.288%
- (Low, High)	- 2.382%	- 1.249%

Note: The table presents changes in variables relative to those in a baseline model.

Table 10 presents the extreme scenario of an economy without LTCI and the changes in the aggregate variables of the behavior of families and the welfare effects of heterogeneous families. In the scenario without tax adjustment, the average ratio of IC hours to total hours would be almost 60% higher due to the higher cost of FHC without LTCI. The increase in the IC ratio is particularly large for families that incur a high cost to provide

[49]Policy experiments in this study parallel that of Fukai, Ichimura, Kitao, and Mikoshiba (2021) in that it first performs simulations without tax adjustment, then simulations with tax adjustment, and compares the two.

IC in the baseline. The additional increase in the IC ratios would reduce the labor force participation of working-age married females by almost 10% on average. The average savings would increase by almost 10% because the family would likely accumulate more precautionary savings due to the higher cost of FHC. The number of people receiving welfare transfer would increase dramatically, from 0.31% of the population in the baseline to 1.00%. Given this, the welfare effects would be strictly negative in the scenario without tax adjustment. Welfare loss is substantial for families with the low-skilled adult child generation and the low-skilled older parent generation.

Adjusting the lump-sum tax to balance the budget, each family would receive nearly 80,000 yen annually. Compensation from the lump-sum transfer would increase the IC ratio in total hours because families would be incentivized to prevent cutting down on an additional bequest. Given this, females' labor force participation would be lower, and the average saving would be higher, relative to the scenario without tax adjustment. The increase in the number of means-tested welfare program recipients is higher relative to the baseline but smaller than the scenario without tax adjustment. Despite the compensation, the welfare effect would still be negative. This is because the number of lump-sum transfers would not be sufficient to compensate for the massive burden of care.

Experiments with no universal LTCI reveal that universal LTCI protects families well against long-term care risks in old age. When the government eliminates LTCI, the cost of FHC services would exceed that at the baseline, and families would cope with the burden of care by providing more IC, which would precipitate a decline in working-age married females' average labor force participation. In the absence of LTCI, the risk of long-term care may induce more significant precautionary savings. However, in poorer families, the massive burden of care would deplete savings, forcing these families to turn to the means-tested welfare program. Given this, the reductions in government expenditure from eliminating LTCI may be offset by higher expenditures for the means-tested welfare program. Consequently, the welfare effects would be strictly negative, even if a lump-sum tax is adjusted to balance the government budget, because the compensation through a lump-sum tax would be insufficient to cover significant long-term care burdens.

5.2.2 Roles of Benefits-in-kind

To understand the roles of LTCI with a benefits-in-kind policy, I simulate an alternative scenario in which the LTCI provides only cash benefits, and the copayment ratio is 100%. I set the number of annual cash benefits to allow the use of the average FHC services in the baseline model, which corresponds to 839,047.70 yen and 1,799,312.86 yen for light and heavy LTC-status, respectively. Table 11 presents the changes in the aggregate variables of the families' behavior and the welfare effects of heterogeneous families.

Table 11: An Economy with Cash Benefits to Allow Average FHC in the Baseline

	No tax change	Tax adjusted
Change in average IC ratio		
- Average	+ 63.740%	+ 63.743%
- (High, Low)	+ 28.834%	+ 28.844%
- (High, High)	+ 64.087%	+ 64.080%
- (Low, Low)	+ 70.350%	+ 70.357%
- (Low, High)	+ 85.935%	+ 85.923%
Change in average FLFP	- 8.920%	- 8.913%
Change in average savings	- 9.824%	- 9.796%
Welfare Program Recipients	0.000%	0.000%
	(-100.000%)	(-100.000%)
Lump-sum tax (JPY)	-	+ 1,763.566
Welfare effects		
- Average	+ 1.632%	+ 1.602%
- (High, Low)	+ 1.411%	+ 1.383%
- (High, High)	+ 1.030%	+ 1.006%
- (Low, Low)	+ 1.884%	+ 1.851%
- (Low, High)	+ 1.316%	+ 1.290%

Note: The table presents changes in variables relative to those in a baseline model.

When there is no tax adjustment, the IC ratios in total hours would increase by approximately 63% due to the higher price of FHC. Correspondingly, caregivers' labor force participation and families' average savings would fall by approximately 9.0% and 9.7%, respectively. The savings incentive would be smaller than that in the baseline because cash transfers compensate for the reduction in labor income of working-age married females, resulting from an increased IC. With the cash transfer compensation, the ratio of means-tested welfare program recipients would be lower than that in the baseline model, and the welfare effects would be positive for all combinations of skill types.

The second row in Table 11 reveals that LTCI with cash benefits would require about 1,800 yen from each family. The cash benefits would increase IC and, simultaneously, decrease tax revenues from the labor income of working-age married females and capital income, thereby precipitating the imposition of a lump-sum tax. Hence, the increase in welfare effects from cash transfers would be slightly mitigated by the lump-sum tax.

The overall IC ratio would be slightly higher with the lump-sum tax than with no tax adjustment. However, this increase in the ratio of IC can be attributed to a family with the low-skilled adult child generation and the low-skilled older parent generation. For a family with the low-skilled adult child generation and the low-skilled older parent generation, the lump-sum tax would reduce the savings required to purchase FHC services, which, in turn, would compel families to turn to IC or a means-tested welfare program. Regarding families, except for a family with the low-skilled adult child generation and the

low-skilled older parent generation, the lump-sum tax would slightly reduce IC because the lump-sum tax would weaken the mechanism of family savings.

Although only services are provided in Japan's LTCI system, universal LTCI systems, such as in Germany and South Korea, allow older adults in need of care to choose between benefits-in-kind and cash benefits.[50] For example, in Germany, its government aims to reduce government long-term care expenditures by encouraging IC at home. Although those eligible for LTCI can choose to receive only benefits-in-kind, only cash benefits, and a combination of the two, the number of cash benefits is less generous than the value of the corresponding benefits-in-kind services: the number of cash benefits is 40% to 50% lower than the value of benefits-in-kind, depending on the long-term care need group.[51] To evaluate the effects of the generosity of cash benefits, Table 12 presents the simulation results when I assume different degrees of this generosity. In the two experiments, I adjust the number of cash benefits to allow the use of 50% or 40% of FHC services in the baseline model.

Table 12: Alternative Generosity of Cash Benefits

	50% of average FHC in the baseline		40% of average FHC in the baseline	
	No tax change	Tax adjusted	No tax change	Tax adjusted
Change in average IC ratio				
- Average	+ 63.398%	+ 63.535%	+ 62.984%	+ 63.286%
- (High, Low)	+ 29.834%	+ 29.835%	+ 29.886%	+ 29.907%
- (High, High)	+ 63.025%	+ 63.214%	+ 62.868%	+ 63.088%
- (Low, Low)	+ 69.774%	+ 69.960%	+ 69.040%	+ 69.511%
- (Low, High)	+ 85.893%	+ 85.924%	+ 85.976%	+ 86.007%
Change in average FLFP	- 9.178%	- 9.678%	- 9.143%	- 9.632%
Change in average savings	- 0.933%	- 1.434%	+ 1.175%	+ 0.628%
Welfare Program Recipients	0.2867%	0.2619%	0.4084%	0.3612%
	(- 9.953%)	(- 17.729%)	(+ 28.280%)	(+ 13.451%)
Lump-sum tax (JPY)	-	- 42,136.738	-	- 50,186.014
Welfare effects				
- Average	- 0.479%	+ 0.191%	- 0.893%	- 0.094%
- (High, Low)	- 0.530%	+ 0.095%	- 0.898%	- 0.155%
- (High, High)	- 0.580%	- 0.064%	- 0.886%	- 0.272%
- (Low, Low)	- 0.435%	+ 0.299%	- 0.893%	- 0.019%
- (Low, High)	- 0.528%	+ 0.058%	- 0.896%	- 0.197%

Note: The table presents changes in variables relative to those in a baseline model.

Table 12 reveals that welfare gains depend on the generosity of cash benefits. Irrespective of the generosity of the cash benefits, the IC ratio in total hours would increase due to the higher price of the FHC than the baseline model. Compared to Table 11, I find that as generosity decreases, average savings and the number of means-tested welfare program

[50]For details on comparing universal LTCI in Germany and South Korea, see Rhee et al. (2015).

[51]For details on the LTCI in Germany, see Campbell et al. (2010) and Mori (2020).

recipients increase, and welfare effects worsen. Cash benefits are especially beneficial to the low-skilled adult child generation, but less beneficial for the high-skilled adult child generation.

These experiments reveal that universal LTCI with a benefits-in-kind policy is more expensive than universal long-term care with cash benefits, although LTCI with a benefits-in-kind policy does not significantly discourage the labor supply of working-age married females who are caregivers. Experiments suggest that less generous cash benefits can reduce government long-term care expenditures by replacing high-cost formal care services with care provided by family members. However, welfare gains depend on cash benefits' generosity: welfare effects would worsen as generosity decreases.

It should be noted that in the German LTCI, for the use of cash benefits, informal caregivers are officially qualified by MDK (Medizinischer Dienst der Krankenversicherung) to ensure the quality of care. Then, the results of this experiment may indicate an upper bound on the welfare effects of cash benefits because not all applicants may be able to receive cash benefits.

6 Conclusion

This study examines how the risk of long-term care affects individual behavior over the life-cycle and analyzes the role of LTCI in Japan, which has the oldest population in the world. This study quantifies the welfare effects of LTCI on heterogeneous households relative to alternative policies and focuses on the role of universal LTCI with a benefits-in-kind policy. I develop a structural overlapping generations model with two-sided altruism. Based on empirical evidence on disability and mortality risks and care arrangements, my model focuses on the LTC-status of the female in the older parent generation and the care arrangements between the female older parent and the female adult child as her primary caregiver. To develop a richer model of care arrangements, I incorporate three types of care options: IC from her adult child, FHC, and public institutional care. Additionally, this model endogenizes care arrangements by introducing two-stage family decisions.

In this study, I focus on the two features of Japan's LTCI system: mandatory universal insurance and a benefits-in-kind policy. This study removes the universal LTCI and examines the role of universal LTCI and its interaction with the means-tested welfare program. I find that universal LTCI protects households well against long-term care risks in old age. Due to the substantial burden of care and the absence of a universal LTCI, families turn to the IC or the means-tested welfare program. However, even when a lump-sum tax is adjusted to balance the government budget, the compensation is insufficient to cover the substantial long-term care burden. Thus, the welfare effects are strictly negative. Furthermore, the effects are not uniform between individuals. A family with a low-skilled adult child generation and a low-skilled older parent generation would have

the highest preference for IC services and the most significant loss in welfare.

Furthermore, I consider the role of universal LTCI with a benefits-in-kind policy by simulating the scenario in which universal LTCI provides only cash benefits. Universal LTCI with a benefits-in-kind policy is more expensive than universal LTCI with cash benefits, although universal LTCI with a benefits-in-kind policy does not significantly discourage caregiver labor supply. Regardless of tax adjustment, the welfare effects are positive if universal LTCI provides a level of cash benefits that allows the use of average FHC services in the baseline model. This is because the IC rate would increase due to the higher cost of formal care, but the cash benefits compensate for the reduction in labor income of working-age married females, which is significantly lower than that of working-age married males. However, the welfare gains depend on the generosity of the cash benefits, that is, the welfare effects would worsen as the generosity decreases.

However, the introduction of cash benefits requires careful discussion and further analysis. Several points should be considered when interpreting this study's results. First, this study focuses on the care arrangements between widowed females and their working-age female adult children as the primary caregivers. In assuming that working-age female adult children are the primary caregivers for widowed females, I probably underestimate the role of benefits-in-kind. According to the CSLC, female caregivers predominate in the sample, but male caregivers account for 32.37% of caregivers for widowed females in the family sample, which is not a negligible number. Since the positive welfare effects of the only cash benefits scenario are mainly due to the low productivity of working-age married females, the welfare effects of only cash benefits may be overestimated. Additionally, it is important to consider male as well as female caregivers, especially with the increasing number of male caregivers, such as unmarried sons and male spouses, as shown in Tokunaga et al. (2015). Second, experiments from the scenario of only cash benefits may indicate an upper bound on the welfare effects of cash benefits because the use of cash benefits in Germany and South Korea requires some qualification and/or condition (Mori 2020; Rhee et al. 2015).

Finally, I discuss the key concerns of—and omissions from—my model, which can be investigated in future research. The first concern is the impact of demographic changes. Family structures and informal caregivers are changing with declining birth rates, unmarried, late marriages, a declining trend in which daughters-in-law are primary caregiver, and an increasing trend of unmarried children and male spouses being the primary caregivers.[52] However, I leave the evaluation of the impact of demographic changes to future research because this study considers a steady state, and such a model cannot consider the

[52]The data on family structures are according to the Annual Report on the Declining Birthrates by the Cabinet Office, Government of Japan in 2022. The data are available here: `https://www8.cao.go.jp/shoushi/shoushika/whitepaper/measures/english/w-2022/pdf/gaiyoh.pdf` (in Japanese) (Accessed January 24, 2023). For informal caregivers trends, see, for example, Tokunaga et al. (2015).

impact of demographic changes. Changes in family structure are crucial to the economic and welfare evaluations of alternative policy reforms and the fiscal sustainability of the insurance system. Specifically, the care options available to individuals depend highly on family structure, which should be investigated in the future.

The second concern is institutional care services in the long-term care market. This study focuses only on care arrangements with three care options: IC, FHC, and public institutional care. This study does not consider private institutional care because public institutional care accounts for most of the total facility capacity, and information on occupancy rates in private facilities is not available in sufficient detail. However, it is important to consider private institutional care services when understanding the impact of LTCI on wealthy families. This is because private institutional care provides a higher quality of care and housing than public institutional care, and high-income households mainly use these services. Furthermore, not all applicants can enter public institutional care services (i.e., special nursing homes) because the demand for public institutional care exceeds the supply. Although I introduce the cost parameter ξ that captures the cost of entering the public facility in this model, this parameter does not capture the heterogeneity of excess demand in municipalities. If I take institutional care services seriously, I need to seriously consider both private institutional care services and the waiting times between application and enrollment in public institutional care services.

References

Ameriks, J., J. Briggs, A. Caplin, M. D. Shapiro, and C. Tonetti (2020). Long-term-care utility and late-in-life saving. *Journal of Political Economy 128*(6), 2375–2451.

Ando, M., M. Furuichi, and Y. Kaneko (2021). Does universal long-term care insurance boost female labor force participation? Macro-level evidence. *IZA Journal of Labor Policy 11*(1).

Aoki, K., M. Alexander, and N. Kalin (2016). Household portfolios in a secular stagnation world: Evidence from Japan. Bank of Japan Working Paper Series 16-E-4, Bank of Japan.

Applebaum, R. A., J. B. Christianson, M. Harrigan, and J. Schore (1988). The evaluation of the national long-term care demonstration. 9. The effect of channeling on mortality, functioning, and well-being. *Health Services Research 23*(1), 143–159.

Attanasio, O., S. Kitao, and G. L. Violante (2010). Financing medicare: A general equilibrium analysis. In J. B. Shoven (Ed.), *Demography and the Economy*, pp. 333–366. University of Chicago Press.

Auerbach, A. J. and L. J. Kotlikoff (1987). *Dynamic fiscal policy*. Cambridge University Press.

Barczyk, D., S. Fahle, and M. Kredler (2022). Save, spend or give? A model of housing, family insurance, and savings in old age. *Review of Economic Studies*. In Press. `https://doi.org/10.1093/restud/rdac081` (Available online December 17, 2022).

Barczyk, D. and M. Kredler (2017). Evaluating long-term-care policy options, taking the family seriously. *Review of Economic Studies 85*(2), 766–809.

Barczyk, D. and M. Kredler (2019). Long-term care across Europe and the United States: The role of informal and formal care. *Fiscal Studies 40*(3), 329–373.

Bernheim, B. D., A. Shleifer, and L. H. Summers (1985). The strategic bequest motive. *Journal of Political Economy 93*(6), 1045–1076.

Bick, A. and S. Choi (2013). Revisiting the effect of household size on consumption over the life-cycle. *Journal of Economic Dynamics and Control 37*(12), 2998–3011.

Boar, C. (2021). Dynastic precautionary savings. *Review of Economic Studies 88*(6), 2735–2765.

Bolin, K., B. Lindgren, and P. Lundborg (2008). Informal and formal care among single-living elderly in Europe. *Health Economics 17*(3), 393–409.

Bonsang, E. (2009). Does informal care from children to their elderly parents substitute for formal care in Europe? *Journal of Health Economics 28*(1), 143–154.

Braun, R. A. and D. H. Joines (2015). The implications of a graying Japan for government policy. *Journal of Economic Dynamics and Control 57*, 1–23.

Braun, R. A., K. A. Kopecky, and T. Koreshkova (2017). Old, sick, alone, and poor: A welfare analysis of old-age social insurance programmes. *Review of Economic Studies 84*(2), 580–612.

Braun, R. A., K. A. Kopecky, and T. Koreshkova (2019). Old, frail, and uninsured: Accounting for features of the U.S. long-term care insurance market. *Econometrica 87*(3), 981–1019.

Brown, J. R. and A. Finkelstein (2008). The interaction of public and private insurance: Medicaid and the long-term care insurance market. *American Economic Review 98*(3), 1083–1102.

Bueren, J. (2022). Long-term care needs and savings in retirement. *Review of Economic Dynamics*. In Press. `https://doi.org/10.1016/j.red.2022.08.004` (Available online August 30, 2022).

Campbell, J. C. and N. Ikegami (2000). Long-term care insurance comes to Japan. *Health Affairs 19*(3), 26–39.

Campbell, J. C., N. Ikegami, and M. J. Gibson (2010). Lessons from public long-term care insurance in Germany and Japan. *Health Affairs 20*(1), 87–95.

Charles, K. K. and P. Sevak (2005). Can family caregiving substitute for nursing home care? *Journal of Health Economics 24*(6), 1174–1190.

Chatterji, S., J. Byles, D. Cutler, T. Seeman, and E. Verdes (2015). Health, functioning, and disability in older adults–present status and future implications. *Lancet 385*(9967), 563–575.

Christensen, K., G. Doblhammer, R. Rau, and J. W. Vaupel (2009). Ageing populations: The challenges ahead. *Lancet 374*(9696), 1196–1208.

Cox, D. (1987). Motives for private income transfers. *Journal of Political Economy 95*(3), 508–546.

Daruich, D. (2018). The macroeconomic consequences of early childhood development policies. Working Paper 2018-029B, Federal Reserve Bank of St. Louis.

Davidoff, T. (2010). Home equity commitment and long-term care insurance demand. *Journal of Public Economics 94*(1–2), 44–49.

De Nardi, M., E. French, and J. B. Jones (2010). Why do the elderly save? The role of medical expenses. *Journal of Political Economy 118*(1), 39–75.

De Nardi, M., E. French, and J. B. Jones (2016). Medicaid insurance in old age. *American Economic Review 106*(11), 3480–3520.

Fu, R., H. Noguchi, A. Kawamura, H. Takahashi, and N. Tamiya (2017). Spillover effect of Japanese long-term care insurance as an employment promotion policy for family caregivers. *Journal of Health Economics 56*, 103–112.

Fukai, T., H. Ichimura, S. Kitao, and M. Mikoshiba (2021). Medical expenditures over the life cycle: Persistent risks and insurance. Discussion Paper Series 21-E-073, Research Institute of Economy, Trade and Industry (RIETI).

Fukuda, S., S. Yoda, and R. Mogi (2021). Educational assortative mating in Japan: Evidence from the 1980–2010 census. *The Journal of Population Studies (Zinkogaku Kenkyu) 57*, 1–20.

Fuster, L., A. İmrohoroğlu, and S. İmrohoroğlu (2007). Elimination of social security in a dynastic framework. *Review of Economic Studies 74*(1), 113–145.

Gao, H. (2020). Social security and female labor supply in China. Working Paper. `https://hangao.weebly.com/uploads/3/7/4/7/37479479/draft_ss_hangao.pdf` (Accessed August 27, 2021).

Geyer, J. and T. Korfhage (2015). Long-term care insurance and carers' labor supply - A structural model. *Health Economics 24*(9), 1178–1191.

Groneck, M. (2017). Bequests and informal long-term care: Evidence from HRS exit interviews. *Journal of Human Resources 52*(2), 531–572.

Gunji, H. and K. Miyazaki (2011). Estimates of average marginal tax rates on factor incomes in Japan. *Journal of the Japanese and International Economies 25*(2), 81–106.

Hagiwara, R. (2022). Welfare Effects of Health Insurance Reform: The Role of Elastic Medical Demand. Discussion Paper 2022-E-5, Institute for Monetary and Economic Studies, Bank of Japan.

Hamaaki, J., M. Hori, and K. Murata (2019). The intra-family division of bequests and bequest motives: Empirical evidence from a survey on Japanese households. *Journal of Population Economics 32*, 309–346.

Hanaoka, C. and E. C. Norton (2008). Informal and formal care for elderly persons: How adult children's characteristics affect the use of formal care in Japan. *Social Science and Medicine 67*(6), 1002–1008.

Hansen, G. D. and S. İmrohoroğlu (2016). Fiscal reform and government debt in Japan: A neoclassical perspective. *Review of Economic Dynamics 21*, 201–224.

Horioka, C. Y. (2002). Are the Japanese selfish, altruistic or dynastic? *Japanese Economic Review 53*(1), 26–54.

Hsu, M. and T. Yamada (2019). Population aging, health care, and fiscal policy reform: The challenges for Japan. *Scandinavian Journal of Economics 121*(2), 547–577.

Ikegami, N. (2019). Financing long-term care: Lessons from Japan. *International Journal of Health Policy and Management 8*(8), 462–466.

İmrohoroğlu, A. and K. Zhao (2018). The chinese saving rate: Long-term care risks, family insurance, and demographics. *Journal of Monetary Economics 96*, 33–52.

İmrohoroğlu, S., S. Kitao, and T. Yamada (2016). Achieving fiscal balance in Japan. *International Economic Review 57*(1), 117–154.

Kitao, S. (2015). Fiscal cost of demographic transition in Japan. *Journal of Economic Dynamics and Control 54*, 37–58.

Kitao, S. and M. Mikoshiba (2020). Females, the elderly, and also males: Demographic aging and macroeconomy in Japan. *Journal of the Japanese and International Economies 56*, 101064.

Kitao, S. and T. Yamada (2019). Dimensions of inequality in Japan: Distributions of earnings, income and wealth between 1984 and 2014. Discussion Paper Series 19-E-034, Research Institute of Economy, Trade and Industry (RIETI).

Ko, A. (2022). An equilibrium analysis of the long-term care insurance market. *Review of Economic Studies 89*(4), 1993–2025.

Kopecky, K. A. and T. Koreshkova (2014). The impact of medical and nursing home expenses on savings. *American Economic Journal: Macroeconomics 6*(3), 29–72.

Koreshkova, T. and M. Lee (2020). Nursing homes in equilibrium: Implications for long-term care policies. Working Paper wp414, Michigan Retirement Research Center, University of Michigan.

Laferrère, A. and F. Wolff (2006). Microeconomic models of family transfers. In S. Kolm and J. M. Ythier (Eds.), *Handbook of the Economics of Giving, Altruism and Reciprocity*, Volume 2, pp. 889–969. Elsevier.

Laitner, J. (1997). Intergenerational and interhousehold economic links. In M. R. Rosenzweig and S. Oded (Eds.), *Handbook of Population and Family Economics*, Volume 1, pp. 189–238. Elsevier.

Lefranc, A., F. Ojima, and T. Yoshida (2014). Intergenerational earnings mobility in Japan among sons and daughters: Levels and trends. *Journal of Population Economics 27*, 91–134.

Lockwood, L. M. (2018). Incidental bequests and the choice to self-insure late-life risks. *American Economic Review 108*(9), 2513–2550.

Mikoshiba, M., H. Noguchi, and A. Kawamura (2023). From No-Disability to Death for Old Population in Japan: Transition Profile of Care-demanding Status Based on Long-Term Care Claims Data. Working Paper.

Mommaerts, C. (2015). Long-term care insurance and the family. Working Paper. https://drive.google.com/file/d/0By9dlLrSuqrXREdldHYzUWN0MTQ/view?usp=sharing&resourcekey=0-lv89itVY-62i9QNpBs5s9g (Accessed May 11, 2021).

Mommaerts, C. (2018). Are coresidence and nursing homes substitutes? Evidence from medicaid spend-down provisions. *Journal of Health Economics 59*, 125–138.

Mori, C. (2020). Family care and care allowances in Germany. *The Japanese Journal of Labour Studies (Nihon Rodo Kenkyu Zasshi) 62*(6), 27–37. (in Japanese).

Nakamura, S. and S. Maruyama (2012). Intergenerational transfers from children to parents –A critical review–. *Economic Review (Keizai Kenkyu) 63*(4), 318–332. (in Japanese).

OECD (2019). *Pensions at a glance 2019*. Organization for Economic.

Pashchenko, S. (2013). Accounting for non-annuitization. *Journal of Public Economics 98*, 53–67.

Rhee, J. C., D. Nicolae, and G. F. Anderson (2015). Considering long-term care insurance for middle-income countries: Comparing South Korea with Japan and Germany. *Health Policy 119*(10), 1319–1329.

Shimizutani, S., W. Suzuki, and H. Noguchi (2008). The socialization of at-home elderly care and female labor market participation: Micro-level evidence from Japan. *Japan and the World Economy 20*(1), 82–96.

Skira, M. M. (2015). Dynamic wage and employment effects of elder parent care. *International Economic Review 56*(1), 63–93.

Sugawara, S. and J. Nakamura (2014). Can formal elderly care stimulate female labor supply? The Japanese experience. *Journal of the Japanese and International Economies 34*, 98–115.

Suzuki, W., Y. Iwamoto, and M. Yuda (2012). The distribution patterns of medical care and long-term care expenditures: Estimations based on administrative data in Fukui prefecture. *Japanese Journal of Health Economics and Policy (Iryo Keizai Kenkyu) 24*(2), 86–107. (in Japanese).

Tamiya, N., H. Noguchi, A. Nishi, M. R. Reich, N. Ikegami, H. Hashimoto, K. Shibuya, I. Kawachi, and J. C. Campbell (2011). Population ageing and wellbeing: Lessons from Japan's long-term care insurance policy. *Lancet 378*(9797), 1183–1192.

Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters 20*(2), 177–181.

Tokunaga, M., H. Hashimoto, and N. Tamiya (2015). A gap in formal long-term care use related to characteristics of caregivers and households, under the public universal system in Japan: 2001-2010. *Health Policy 119*(6), 840–849.

Tsutsui, T. and N. Muramatsu (2005). Care-needs certification in the long-term care insurance system of Japan. *Journal of the American Geriatrics Society 53*(3), 522–527.

Van Houtven, C. H., N. B. Coe, and M. M. Skira (2013). The effect of informal care on work and wages. *Journal of Health Economics 32*(1), 240–252.

Van Houtven, C. H. and E. C. Norton (2004). Informal care and health care use of older adults. *Journal of Health Economics 23*(6), 1159–1180.

Yamada, H. and S. Shimizutani (2015). Labor market outcomes of informal care provision in Japan. *The Journal of the Economics of Ageing 6*, 79–88.

Yamada, K. (2006). Intra-family transfers in Japan: Intergenerational co-residence, distance, and contact. *Applied Economics 38*(16), 1839–1861.

Appendix A The CSLC Data

This section describes the distribution and characteristics of primary caregivers by sex and marital status in the "family sample." I divide primary caregivers at home into five groups: their own children if they receive IC from their own child; children-in-law if they receive IC from their own child-in-law; spouses if they receive IC from their spouse; others if they receive IC from other family members; and FHC if they receive FHC.

As can be seen from Table 13, more than 80% widowed are cared for by their children, particularly their own children. The average age of their children ranges from the mid to late 50s for both their own children and children-in-law. Both widowed females and widowed males are often cared for by the females' own children, although the proportion of females is just over half. In contrast, their children-in-law are predominantly females, accounting for more than 95%. According to Tokunaga et al. (2015), the trend of traditional primary caregivers being daughters-in-law continues but is declining. However, the trend is toward more male caregivers, such as unmarried sons and male spouses. As shown in Table 13, their own children tend to be regular workers, while their children-in-law tend to be contingent workers.

Table 13 shows that the primary caregivers for married are their spouses. When comparing married males and females, married females are more likely to be cared for by their own children and children-in-law, respectively. The characteristics of the primary caregiver, their spouses, show that the labor participation rate is low (about 10%), indicating that care is provided primarily by those who have retired from the labor market.

Table 13: Characteristics of Main Caregivers by Marital Status and Sex

	Distribution of Primary Caregivers				
	IC (Own Children)	IC (Children-in-law)	IC (Spouse)	IC (Others)	FHC
Widowed Females (N=1,865, 50.71%)					
Distribution of primary caregivers	63.11%	25.29%	0.00%	1.88%	9.71%
Average age	58.77	58.46	-	33.01	-
Proportion of female	55.14%	98.73%	-	48.10%	-
Labor Force Participation	59.19%	58.44%	-	59.52%	-
% Regular Employment	43.94%	22.14%	-	87.64%	-
% Contingent Employment	32.52%	43.28%	-	6.13%	-
% Self Employment	16.40%	5.14%	-	6.24%	-
% Other Employment	7.15%	29.44%	-	0.00%	-
Married Males (N=915, 24.88%)					
Distribution of primary caregivers	21.20%	4.94%	68.09%	0.73%	5.05%
Average age	53.77	55.88	76.97%	31.28	-
Proportion of female	57.59%	98.39%	100.00%	56.31%	-
Labor Force Participation	72.01%	65.55%	11.86%	36.77%	-
% Regular Employment	39.56%	31.83%	19.12%	100.00%	-
% Contingent Employment	33.30%	49.85%	19.57%	0.00%	-
% Self Employment	12.48%	2.31%	32.34%	0.00%	-
% Other Employment	14.66%	16.02%	28.87%	0.00%	-
Married Females (N=606, 16.48%)					
Distribution of primary caregivers	29.52%	10.42%	52.70%	0.46%	6.90%
Average age	54.68	55.64	79.31	76.00	-
Proportion of female	60.40%	94.30%	0.00%	100.00%	-
Labor Force Participation	66.04%	59.71%	15.83%	0.00%	-
% Regular Employment	31.22%	36.75%	23.30%	-	-
% Contingent Employment	47.47%	37.47%	13.42%	-	-
% Self Employment	16.20%	6.61%	56.25%	-	-
% Other Employment	5.11%	19.17%	7.04%	-	-
Widowed Males (N=292, 7.94%)					
Distribution of primary caregivers	59.78%	21.61%	0.00%	2.08%	16.53%
Average age	55.36	54.00	-	26.00	-
Proportion of female	52.48%	96.99%	-	84.28%	-
Labor Force Participation	71.18%	50.55%	-	82.48%	-
% Regular Employment	50.19%	38.77%	-	100.00%	-
% Contingent Employment	24.67%	40.18%	-	0.00%	-
% Self Employment	22.83%	1.70%	-	0.00%	-
% Other Employment	2.31%	19.35%	-	0.00%	-

Note: Table 13 shows characteristics of primary caregivers for those who are widowed or married, having at least one child living together or in the same municipality. The data are from the Comprehensive Survey of Living Conditions by the Ministry of Health, Labour and Welfare (MHLW). I use two questionnaires for 2016: household questionnaires and long-term care questionnaires. The sample includes those eligible for long-term care services covered by long-term care insurance aged between 65 and 94. I limit the sample to those who provide information on the level of care, primary caregivers, other caregivers, and the use of formal home care services; those who are widowed females or married males; and those with at least one child living together or in the same municipality. The figures are derived from the author's calculation and may not correspond to the numbers published by the MHLW.

Appendix B Two-sided altruism and Strategic bequest motives

Previous empirical studies show a positive correlation between parental transfers, such as bequests and inter-vivo transfers, and IC provided by children. For example, in Japan, children who provide IC to their parents are more likely to receive a larger share of bequests than other children (Horioka 2002), and children who expect to receive bequests from their parents tend to live with them and have more contact with them (Yamada 2006). Various theoretical models have been discussed to explain these empirical facts. This section discusses two main theoretical models: (i) two-sided altruism and (ii) strategic bequest motives.

In two-sided altruism, parents and children exhibit altruism toward each other (Laitner 1997; Laferrère and Wolff 2006). The parental transfer of resources to children becomes an altruistic behavior to compensate for the loss of utility from the burden of IC by children. Additionally, altruistic children provide IC voluntarily. Models with two-sided altruism have dynastic structures: children inherit the family line and resources from their parents. In contrast, for strategic bequest reasons, parents provide transfers to receive IC from children (Bernheim et al. 1985; Cox 1987). When children do not voluntarily care for their parents as much as they would like, parents can transfer their resources to their child contingent on IC by children.

There are several studies on bequest motives and informal caregiving by children. For example, Hamaaki et al. (2019) show that bequest motives and children's help in long-term care are consistent with dynastic motives by comparing the patterns of bequest distribution when the first parent dies (primary inheritance) and the second parent dies later (secondary inheritance). In particular, empirical evidence from secondary inheritance suggests the existence of strong traditional family values. Note that Japanese parents divide their bequests unequally among their children and do not leave inter-vivo transfers or a written will. However, as discussed in Groneck (2017) and Nakamura and Maruyama (2012), the motives for the substantial impact of caregiving on received bequests can be difficult to identify because both theoretical models can interpret the positive correlation between parental transfers and informal care by children. Furthermore, unfortunately, as discussed in Nakamura and Maruyama (2012), many previous studies in Japan have concluded support for strategic bequest motives based solely on the positive correlation between parental transfers and children who provide IC and/or live with their parents.

Finally, I discuss the theoretical limitations of two-sided altruism and strategic bequest motives. The two-sided altruism model is tractable compared to strategic motives. However, as discussed in Mommaerts (2015) and Boar (2021), two-sided altruism makes it difficult to analyze the timing of parental inter-vivo transfers and the different wealth accumulation paths of parents and children. In the contrast, the estimation of strategic

bequest motives requires detailed data on the existence of inter-vivo transfers and the savings paths of children and parents. Both approaches have been used in previous structural model of long-term care. Since parents in the U.S. tend to divide their bequests equally and leave inter-vivo transfers and written wills for unequal transfers, Mommaerts (2015) considers a cooperative framework with limited commitments and Ko (2022) considers non-cooperative decision-making between parents and children. In addition, Barczyk and Kredler (2017) and Barczyk et al. (2022) consider both two-sided altruism and the bargaining process between parents and children by developing a dynamic non-cooperative framework. In contrast, İmrohoroğlu and Zhao (2018) use two-sided altruism by focusing on within-family saving behavior using Chinese data.

Two-sided altruism is chosen in this model because there is empirical evidence that bequest motives and IC by children are consistent with two-sided altruism and there are still strong traditional family norms in Japan. Additionally, inter-vivo transfers and/or written wills, which are important elements in estimating strategic bequests, are not widely observed in Japan. However, more research on the motives for the substantial impact of caregiving on received bequests is desirable for future research because identifying the motives is important to analyze the saving behavior of elderly parents.

Appendix C Computation Algorithm of Steady State

In this section, I present the algorithms used to compute the steady state following the five steps described in the following.

Step 1: Guess $\text{pen}_{pf}(j^p, z)$ and τ^{ls}.

Step 2: Given the interest rate r, and a set of government policies $\{\lambda^h, \lambda^m, \tau^c, \tau^a, \tau^l\}$, calculate the problem of the family.

(a) Guess the value function of No disability of age $j^k = 1$, $V_1(a, z, z', h = 1, \iota_{-1} = 0, \mu)$.

(b) Solve the family problem by backward induction.

(c) Update the guess of $V_1(a, z, z', h = 1, \iota_{-1} = 0, \mu)$ and iterate until convergence.

Step 3: Compute the set of age-dependent measures of family $\{X\}_{j^k=1}^{J}$ from the policy function in Step 2.

(a) Guess the age-dependent measures of age $j^k = 1$, $X_1(a, z, z', h, \iota_{-1}, \mu)$.

(b) Calculate the age-dependent measures to satisfy equations (5) and (6).

(c) Update the guess of $X_1(a, z, z', h, \iota_{-1}, \mu)$ and iterate until convergence.

Step 4: Use the policy function and set of age-dependent measures of family and calculate aggregate variables.

Step 5: Use equation (3) and government budget conditions to update the guesses $\text{pen}_{pf}(j^p, z)$ and τ^{ls}, if needed.