

BM20A2901 DISCRETE OPTIMIZATION

Exercise 3 (B2)

1. Formulate a linear IP model for the following problem: n tasks are to be performed with m identical parallel processors. Each task is to be performed with one processor only but each processor may perform many tasks one after another. Task i takes t_i time units, $i=1, \dots, n$. How to assign the tasks to the processors to minimize so called *makespan*, the completion time of the last task?

Hint: You can use e.g. the following decision variables:

$$x_{ij} = \begin{cases} 1 & \text{if task } i \text{ is performed with processor } j \\ 0 & \text{if not} \end{cases}$$

2. Consider the scheduling problem instance with $n = 20$ tasks and $m = 3$ processors and processing times $t = (6, 69, 5, 8, 53, 10, 82, 14, 73, 15, 66, 52, 98, 65, 81, 46, 44, 83, 9, 82)$. Solve this instance with the following heuristic. It takes the tasks in decreasing order of time, and assigns each in turn to the least used processor. T_i = time used by processor i .

LPT algorithm (Longest Processing Time algorithm):

1. Sort t to descending order.
2. Initialize $T_1 = T_2 = \dots = T_m = 0$.
3. For $i = 1$ to n
 - choose j for which $T_j = \min \{T_1, \dots, T_m\}$
 - set $S(i) = j$ and $T_j = T_j + t_i$

In the resulting vector $S(i)$ = processor that is used for task i .

3. Solve the previous problem, using IP formulation of problem 1 and Matlab *intlinprog* or Excel Solver. Compare this correct solution to the solution given by the previous heuristic.