

Bit Manipulation

⇒ Content :-

Basic no. system

Decimal \rightleftharpoons Binary

Add 2 decimal nos.

Negative no. in binary

MSB (most significant bit)

Bitwise Operators.

Properties of operators

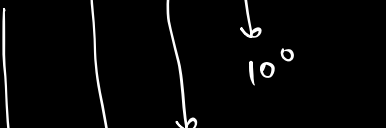
Probleme.

⇒ Number system :-

$$734 \Rightarrow 700 + 30 + 4$$

$$\Rightarrow 7 \times 100 + 3 \times 10 + 4 \times 1$$

$$\Rightarrow 7 \times 10^2 + 3 \times 10^1 + 4 \times 10^0$$

$2 \quad 4 \quad 5 \quad 6 \rightarrow 2 \times 10^3 + 4 \times 10^2 + 5 \times 10^1 + 6 \times 10^0$

 This is decimal no. system
 \Rightarrow base $\rightarrow 10$ [power of 10]
 \Rightarrow digits $\rightarrow 0-9$

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\Rightarrow base $\rightarrow 10$ [power of 10]

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Other no. systems:-

binary, octal, hexadecimal, ternary, decimal

1) octal \rightarrow power of 8 $\Rightarrow (0 \ 1 \ 3 \ 2)_8$

$$\begin{array}{cccc} 0 & 1 & 3 & 2 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 8^3 & 8^2 & 8^1 & 8^0 \end{array}$$

$$\Rightarrow 0 \times 8^3 + 1 \times 8^2 + 3 \times 8^1 + 2 \times 8^0$$

$$\Rightarrow 0 + 64 + 24 + 2$$

$$\Rightarrow \underline{\underline{90}}$$

2) $(125)_8 \rightarrow$

$$\begin{array}{ccc} 1 & 2 & 5 \\ \downarrow & \downarrow & \downarrow \\ 8^2 & 8^1 & 8^0 \end{array} \Rightarrow 1 \times 8^2 + 2 \times 8^1 + 5 \times 8^0$$
$$\Rightarrow 64 + 16 + 5$$
$$\Rightarrow \underline{\underline{85}}$$

: base $\rightarrow 8$

: digits $\rightarrow [0-7]$

10001 \rightarrow ✓

6784 \rightarrow ✗ ✗
8

a) ternary \rightarrow

base \Rightarrow 3.

digits \Rightarrow [0-2].

(0 2 1 0 1)₃

$$\begin{array}{ccccc} 0 & 2 & 1 & 0 & 1 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 3^4 & 3^3 & 3^2 & 3^1 & 3^0 \end{array} \Rightarrow 0 \times 3^4 + 2 \times 3^3 + 1 \times 3^2 + 0 \times 3^1 + 1 \times 3^0$$

$$\Rightarrow 0 + 54 + 9 + 0 + 1$$

$$\Rightarrow \underline{\underline{64}}$$

b) binary

base \rightarrow 2

digits \Rightarrow 0, 1

$$\begin{array}{ccccc} (1 & 0 & 1 & 1 & 0)_2 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array} \Rightarrow 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

$$\Rightarrow 16 + 4 + 2$$

$$\Rightarrow \underline{\underline{22}}$$

$$\begin{array}{ccccccc}
 1 & 0 & 1 & 1 & 1 & & \\
 \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\
 2^4 & & 2^2 & & 2^0 & & \\
 & & & & & & \\
 & & & & 2^1 & &
 \end{array}
 \Rightarrow 2^4 + 2^2 + 2^1 + 2^0$$

$$\Rightarrow 16 + 4 + 2 + 1$$

$$\Rightarrow \underline{\underline{23}}$$

∴ Decimal to binary :-

$$\begin{array}{rcl}
 2 \overline{) 28} & \rightarrow & 0 \\
 2 \overline{) 14} & \rightarrow & 0 \\
 2 \overline{) 7} & \rightarrow & 1 \\
 2 \overline{) 3} & \rightarrow & 1 \\
 2 \overline{) 1} & \rightarrow & 1 \\
 & & 0
 \end{array}$$

$$(11100)_2$$

$$\begin{array}{ccccc}
 1 & 1 & 1 & 0 & 0 \\
 \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
 2^4 & 2^3 & 2^2 & 2^1 & 2^0
 \end{array}$$

$$2^4 + 2^3 + 2^2$$

$$\Rightarrow 16 + 8 + 4$$

$$\Rightarrow \underline{\underline{28}}$$

$$\begin{array}{rcl}
 2 \overline{) 25} & \rightarrow & 1 \\
 2 \overline{) 12} & \rightarrow & 0 \\
 2 \overline{) 6} & \rightarrow & 0 \\
 2 \overline{) 3} & \rightarrow & 1 \\
 2 \overline{) 1} & \rightarrow & 1 \\
 & & 0
 \end{array}$$

$$(11001)_2$$

$$\begin{array}{ccccc}
 1 & 1 & 0 & 0 & 1 \\
 \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
 2^4 & 2^3 & 2^2 & 2^1 & 2^0
 \end{array}$$

$$2^4 + 2^3 + 2^0$$

$$= 16 + 8 + 1 = \underline{\underline{25}}$$

: addition of two nos:- [decimal]

$$\begin{array}{r}
 3 \quad 4 \quad 5 \quad 9 \\
 + 2 \quad 8 \quad 4 \quad 7 \\
 \hline
 6 \quad 3 \quad 0 \quad 6
 \end{array}$$

$$\begin{array}{r}
 0 \quad 1 \quad 1 \quad 1 \quad 8 \\
 \leq 2 \quad 4 \quad 5 \quad 8 \\
 + 3 \quad 7 \quad 6 \quad 4 \\
 \hline
 6 \quad 2 \quad 2 \quad 2
 \end{array}$$

$$8 + 4 = \underline{\underline{12}}$$

$$\text{value} \Rightarrow 2 \Rightarrow 12 \% 10$$

$$\text{Carry} \Rightarrow 1 \Rightarrow \underline{\underline{12 / 10}}$$

$$\text{sum} = 6 + 5 + 1 = 12$$

$$\text{value} \Rightarrow 2 \Rightarrow \text{sum} \% 10$$

$$\text{Carry} \Rightarrow 1 \Rightarrow \text{sum} / 10$$

$$\text{sum} = 7 + 4 + 1 = 12$$

$$\text{value} \Rightarrow \text{sum} \% 10 \Rightarrow 2$$

$$\text{Carry} \Rightarrow \text{sum} / 10 \Rightarrow 1$$

$$\text{decimal} \Rightarrow \text{sum} \downarrow$$

$$\text{value} \Rightarrow \text{sum} \% 10$$

$$\text{Carry} \Rightarrow \text{sum} / 10$$

∴ add binary nos:-

$$\text{value} = \text{sum} \% 2$$

Conv \Rightarrow sum/2.

$$\begin{array}{cccccc}
 0 & 0 & 1 & 1 & 0 & \\
 & 1 & 0 & 1 & 1 & 0 \\
 & & & & & \\
 & 0 & 0 & 1 & 1 & 1 \\
 \hline
 0 & 1 & 1 & 1 & 0 & 1
 \end{array}$$

$$\text{Sum} = 1$$

$$1\frac{1}{2} \times 2 = 1$$

$$1/2 = 0$$

$$\text{Sum} = 0^4 1^4 1 = 2$$

24.2 20

$$2/2 = 1$$

Sum 2 14 141

23

$$301.2 = 1$$

$$3/2 \approx 1$$

Sum = 04170

21

$$1/2 = 1$$

11220

Sum 2 14090

21

$$17,2 = 1$$

$$1/2 = 0$$

\Rightarrow

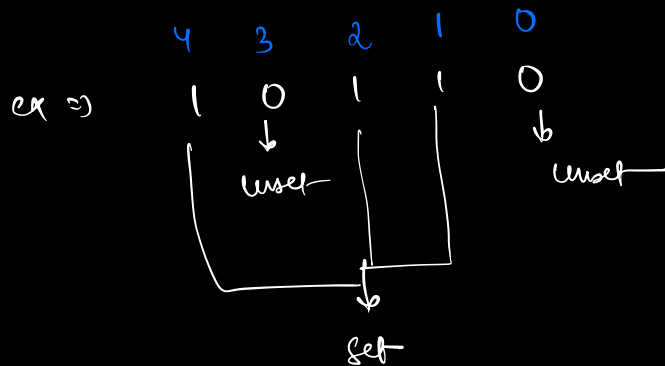
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{2}$	$\frac{2}{2}$	
0	0	0	1	1	
	1	0	0	1	1
	0	1	0	0	1
<hr/>					
	$1\%2$	$1\%2$	$1\%2$	$2\%2$	$2\%2$
0	1	1	1	0	0
<hr/>					

$$(11100)_2$$

: Naming convention:

bit \rightarrow 0 : unset

\rightarrow 1 : Set



bit position \Rightarrow right to left
[0 to N]

: -ve nos. in binary:-

8 bit binary representation [only store 8 bits]

	7	6	5	4	3	2	1	0
10 \Rightarrow	0	0	0	0	1	0	1	0

-10 \Rightarrow	1	0	0	0	1	0	1	0
-------------------	---	---	---	---	---	---	---	---

1st bit from left is used as
sign bit.

$$\begin{array}{rcl}
 10 & \rightarrow & \underline{0} \ \underline{0} \ \underline{0} \ \underline{0} \ \overset{0}{\underline{1}} \ \overset{0}{\underline{0}} \ \underline{1} \ \underline{0} \\
 -4 & \rightarrow & \underline{1} \ \underline{0} \ \underline{0} \ \underline{0} \ \underline{0} \ \underline{1} \ \underline{0} \ \underline{0}
 \end{array}$$

$$\begin{array}{cccccccc}
 \textcircled{1} & 0 & 0 & 0 & 1 & 1 & 1 & 0
 \end{array}$$

indicating 8 4 4 2
-ve = 14
-14

$$0 \Rightarrow 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$$

$$\underline{-0} \leftarrow \boxed{1} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$$

↓
 Can't be used directly as
 a sign bit

$$\therefore a \Rightarrow -a \text{ [2's complement of } a \text{]}$$

↓
 1's complement + 1

↓
 flipping
 all bits
 $0 \rightleftharpoons 1$

$$\text{sum} = 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0$$

$$\Rightarrow 1 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$$

sum of GP \Rightarrow $a = 1$
 $r = 2$
 $N = 7$

$$\Rightarrow \text{sum} = \frac{1 \times (2^7 - 1)}{2 - 1}$$

$$\Rightarrow \text{sum} = \underline{\underline{2^7 - 1}}$$

left most bit > all other bits combined.



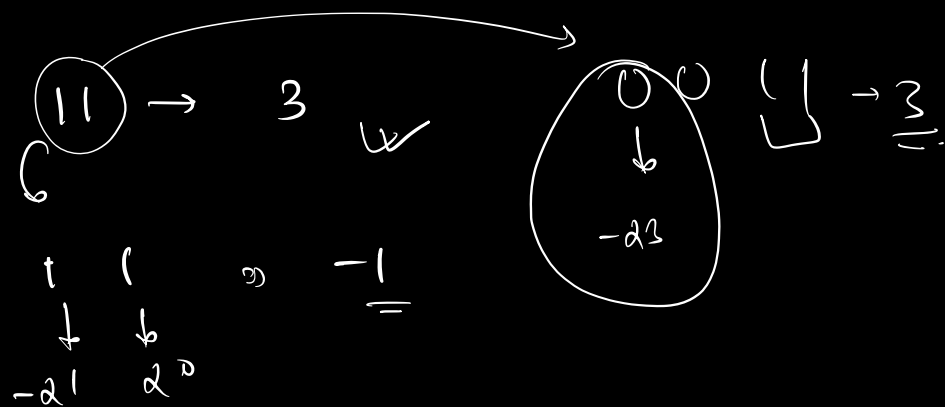
MOST SIGNIFICANT
BIT

MSB

NOTE \Rightarrow MSB is always (-ve)
 ↓
 base value

8 bits \Rightarrow base value $\Rightarrow 2^7$

↓
-2⁷



8 bit

$10 \Rightarrow$	0	0	0	0	1	0	1	0
$-4 \Rightarrow$	1	1	1	1	1	1	0	0
Worst	<hr/>							
	0	0	0	0	0	1	1	0

$$a \Rightarrow -a [2^s \text{ comp} \rightarrow 1^s \text{ comp} + 1]$$

$4 \Rightarrow$	0	0	0	0	0	1	0	0
$15 \Rightarrow$	1	1	1	1	1	0	1	1
$41 \Rightarrow$	0	0	0	0	0	0	0	1
$2^3 \Rightarrow -4 \Rightarrow$	1	1	1	1	1	1	0	0
(4)	<hr/>							

0	0	0	0	0	1	1	0
-2^5	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$$2^2 + 2^1$$

$$= \underline{\underline{6}}$$

$$10 + (-4) = \underline{\underline{6}}$$

\Rightarrow 8 bit :- $\begin{matrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ -2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{matrix}$

$\Rightarrow -2^7 + 2^4 + 2^1$

$\Rightarrow -128 + 16 + 2 \Rightarrow \underline{\underline{-110}}$

\Rightarrow 4 bit :- $\begin{matrix} 1 & 0 & 1 & 0 \\ -2^3 & 2^2 & 2^1 & 2^0 \end{matrix}$

$\Rightarrow -2^3 + 2^1$

$\Rightarrow -8 + 2 = \underline{\underline{-6}}$

\Rightarrow M bits \Rightarrow

$\begin{matrix} \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ & \downarrow & & & & & & \downarrow & \downarrow & \downarrow \\ & -(2^{M-1}) & & & & & & 2^2 & 2^1 & 2^0 \end{matrix}$

Ranges

bits	MIN	MAX
2 $[-2, 1]$	$\begin{matrix} \text{---} & \text{---} & \Rightarrow -2 \\ -2^1 & 2^0 \\ 1 & 0 \end{matrix}$	$\begin{matrix} \text{---} & \text{---} & \Rightarrow 1 \\ -2^1 & 2^0 \\ 0 & 1 \end{matrix}$
4 $[-8, 7]$	$\begin{matrix} \text{---} & \text{---} & \text{---} & \text{---} & \Rightarrow -8 \\ -2^3 & 2^2 & 2^1 & 2^0 \\ 1 & 0 & 0 & 0 \end{matrix}$	$\begin{matrix} \text{---} & \text{---} & \text{---} & \text{---} & \Rightarrow 7 \\ -2^3 & 2^2 & 2^1 & 2^0 \\ 0 & 1 & 1 & 1 \end{matrix}$

5	$[-16, 15]$	$\begin{array}{c c} \begin{array}{ccccccc} - & - & - & - & - & & \\ -2^4 & 2^3 & 2^2 & 2^1 & 2^0 & & \\ 1 & 0 & 0 & 0 & 0 & & \end{array} & \begin{array}{l} \Rightarrow -16 \\ 01111 \Rightarrow 15 \end{array} \end{array}$
6	$[-32, 31]$	$\begin{array}{c c} \begin{array}{c} 1000000 \Rightarrow -32 \end{array} & \begin{array}{l} 011111 \Rightarrow 31 \end{array} \end{array}$
8	$[-128, 127]$	$\begin{array}{c c} \begin{array}{c} 10000000 \\ \Rightarrow -2^7 \Rightarrow -128 \end{array} & \begin{array}{l} 0111111 \\ \underline{127} \end{array} \end{array}$

$$N \text{ bits} \Rightarrow \left[-2^{N-1}, 2^{N-1} - 1 \right]$$

Datatypes

$$\left[-2^{N-1}, 2^{N-1} - 1 \right]$$

byte \Rightarrow 1 byte \Rightarrow 8 bits \Rightarrow $-128, 127$

short \Rightarrow 2 byte \Rightarrow 16 bits \Rightarrow $-2^{15}, 2^{15}-1$

int \Rightarrow 4 byte \Rightarrow 32 bits \Rightarrow $-2^{31}, 2^{31}-1$

long \Rightarrow 8 byte \Rightarrow 64 bits \Rightarrow $-2^{63}, 2^{63}-1$

$$\Rightarrow \text{constant} \Rightarrow 1 \leq arr[i] \leq \underline{10^9}$$

$$2^{10} \Rightarrow 1024 \approx 1000 \Rightarrow 10^3$$

$$2^{10} \approx 10^3$$

$$(2^{10})^3 \leq (10^3)^3 \leftarrow \text{cubing both sides}$$

$$\Rightarrow 2^{30} \leq 10^9.$$

$$\Rightarrow 2 \times 2^{30} \leq 2 \times 10^9$$

$$\Rightarrow \underline{\underline{2^{31}}} \leq 2 \times 10^9 \text{] int range}$$

$$\text{constant } \Rightarrow 1 \leq arr[i] \leq 10^{18}.$$

$$(2^{10})^6 \leq (10^3)^6$$

$$\Rightarrow 2^{60} \leq 10^{18}$$

$$\Rightarrow 8 \times 2^{60} \leq 8 \times 10^{18}$$

$$\Rightarrow \underline{\underline{2^{63}}} \leq 8 \times 10^{18} \text{] long range}$$