Численные методы решения систем нелинейных уравнений

Дз №8

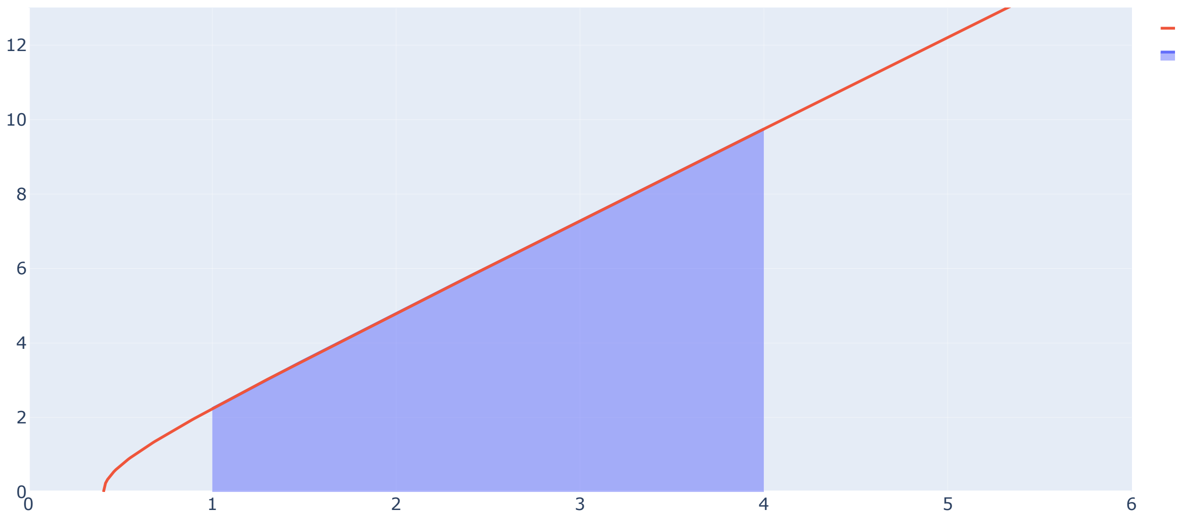
Варинт 5 Доскоч Роман 3 курс 13 группа

$$\int_1^4 \sqrt{6x^2 - 1}dx$$

In [1]:
```
import plotly as pl
import numpy as np
import plotly.graph_objs as go
import plotly.express as px
from numpy import sqrt,log
```

In [2]:
```
f = lambda x: sqrt(6*x**2-1)
```

In [3]:
```
x=np.linspace(1, 4, 100)
fig = go.Figure()
fig.add_trace(go.Scatter(x=x, y=f(x), fill='tonexty'))
fig.update_layout(xaxis_range = [0,6], yaxis_range = [0,13])
x=np.linspace(1/sqrt(6), 6, 500)
fig.add_trace(go.Scatter(x=x, y=f(x)))
```



предварительные расчеты

$$\int \sec x dx = \int \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}dx =$$
$$= \left[\begin{array}{c} t = \sec x + \tan x \\ dt = (\sec x \tan x + \sec^2 x)dx \end{array} \right] =$$
$$= \int \frac{dt}{t} = ln|\sec x + \tan x| + C$$

$$\int \sec^3 x dx = \int \sec^2 x \sec x dx =$$
$$= \begin{bmatrix} u = \sec x & du = \sec x \tan x \\ dv = \sec^2 x dx & v = \tan x \end{bmatrix} =$$
$$\sec x \tan x - \int \tan^2 x \sec x dx = \sec x \tan x - \int (\sec^2 x - 1) \sec x dx =$$
$$\sec x \tan x - \int \sec^3 x dx + \int \sec x dx;$$
$$2 \int \sec^3 x dx = \sec x \tan x + \int \sec x dx$$
$$\int \sec^3 x dx = \frac{1}{2}(\sec x \tan x + ln|\sec x + \tan x|) + C$$

Точное Значени интегралла

$$\int_1^4 \sqrt{6x^2 - 1} dx = \frac{1}{\sqrt{6}} \int_1^4 \sqrt{(\sqrt{6}x)^2 - 1} d\sqrt{6}x;$$
$$\left[t = \sqrt{6}x \right] = \frac{1}{\sqrt{6}} \int \sqrt{t^2 - 1} dt = \begin{bmatrix} t = \sec\theta \\ dt = \sec\theta \tan\theta d\theta \end{bmatrix} = \frac{1}{\sqrt{6}} \int \sqrt{\sec^2\theta - 1} \sec\theta \tan\theta d\theta =$$
$$= \frac{1}{\sqrt{6}} \int \tan^2\theta \sec\theta d\theta = \frac{1}{\sqrt{6}} \int (\sec^2\theta - 1) \sec\theta d\theta = \frac{1}{\sqrt{6}} \int \sec^3\theta - \sec\theta d\theta =$$
$$= \frac{1}{\sqrt{6}} \left(\frac{1}{2} \left(\sec\theta \tan\theta + ln|\sec\theta + \tan\theta| \right) - ln|\sec\theta + \tan\theta| \right) = \frac{1}{2\sqrt{6}} \left(\sec\theta \tan\theta - ln|\sec\theta + \tan\theta| \right) =$$
$$= \frac{1}{2\sqrt{6}} \left(t\sqrt{t^2 - 1} - ln\left|t + \sqrt{t^2 - 1}\right| \right) = \frac{1}{2\sqrt{6}} \left(\sqrt{6}x\sqrt{6x^2 - 1} - ln\left|\sqrt{6}x + \sqrt{6x^2 - 1}\right| \right) =$$
$$= \frac{1}{12} \left(6x\sqrt{6x^2 - 1} - \sqrt{6}ln\left|\sqrt{6}x + \sqrt{6x^2 - 1}\right| \right);$$
$$\int_1^4 \sqrt{6x^2 - 1} dx = \frac{1}{12} \left(6x\sqrt{6x^2 - 1} - \sqrt{6}ln\left(\sqrt{6}x + \sqrt{6x^2 - 1} \right) \right)\Big|_1^4 =$$
$$= \frac{1}{12} \left(24\sqrt{95} + \sqrt{6}ln\left(\frac{\sqrt{6} + \sqrt{5}}{4\sqrt{6} + \sqrt{95}} \right) - 6\sqrt{5} \right) \approx 18.084020121143315$$

In [4]:
```python
real_integral = (24*sqrt(95)+ sqrt(6)*log((sqrt(6)+sqrt(5))/(4*sqrt(6)+sqrt(95)))-6*sqrt(5))/12
real_integral
```

Out[4]: 18.084020121143315

Метод левых прямоугольников

In [5]:
```python
def left(a, b, iteration):
    res=0
    h=(b-a)/iteration
    for i in range(iteration):
        res+=f(a+i*h)
    return h*res
```

Метод правых прямоугольников

In [6]:
```python
def right(a, b, iteration):
    res=0
    h=(b-a)/iteration
    for i in range(1,iteration+1):
        res+=f(a+i*h)
    return h*res
```

Метод средних прямоугольников

In [7]:
```python
def midle(a, b, iteration):
    res=0
    h=(b-a)/iteration
    for i in range(iteration):
        res+=f(a + i*h - h/2)
    return h*res
```

Метод трапеций

In [8]:
```
def trapesium(a, b, iteration):
    res=(f(a)+f(b))/2
    h=(b-a)/iteration
    for i in range(1,iteration):
        res+=f(a+i*h)
    return h*res
```

In [9]:
```
b=4;a=1;iteration=5
Iright = right(a,b,iteration)
Ileft = left(a,b,iteration)
Imidle = midle(a,b,iteration)
Itrapesium = trapesium(a,b,iteration)
print(f'{real_integral} - {Iright}     = {real_integral - Iright}')
print(f'{real_integral} - {Ileft}      = {real_integral - Ileft}')
print(f'{real_integral} - {Imidle}     = {real_integral - Imidle}')
print(f'{real_integral} - {Itrapesium} = {real_integral - Itrapesium}')
```

```
18.084020121143315 - 20.330893891182267     = -2.2468737700389525
18.084020121143315 - 15.82445807079676      = 2.2595620503465543
18.084020121143315 - 13.518129650190494     = 4.565890470952821
18.084020121143315 - 18.077675980989515 = 0.006344140153800026
```

Сравнение точности от количества узлов

In [10]:
```
iterations=np.arange(1,22,2)
fig = go.Figure()
fig.add_trace(go.Scatter(x=iterations,y=[right(a,b,i) for i in iterations], name='right'))
fig.add_trace(go.Scatter(x=iterations,y=[left(a,b,i) for i in iterations], name='left'))
fig.add_trace(go.Scatter(x=iterations,y=[midle(a,b,i) for i in iterations], name='midl'))
fig.add_trace(go.Scatter(x=iterations,y=[trapesium(a,b,i) for i in iterations], name='trapesium'))
fig.add_hline(y=real_integral)
fig.update_layout(legend=dict(yanchor="top",y=0.99,xanchor="left",x=.8))
fig.show()
```