Приближение кривых. Среднеквадратичное приближение функции

Дз №6

Варинт 5 Доскоч Роман 3 курс 13 группа

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
from numpy import prod
import random
```

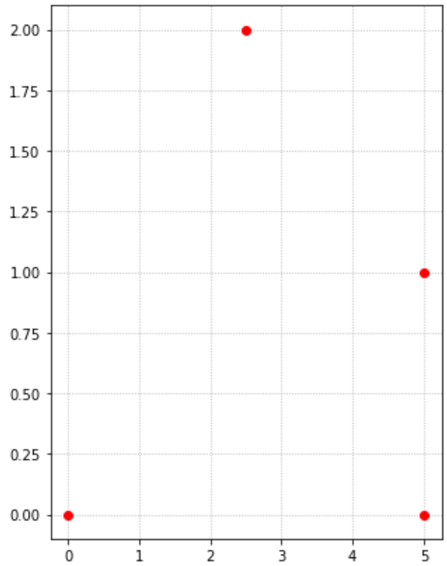
рисование графика для сплайна

In [2]:
```
partitions=30
def spline_show_graf(Sx_i, Sy_i):
    plt.figure(figsize=(5,7))
    for i in range(1,n):
        t_i = np.linspace(t[i-1], t[i], partitions)
        plt.plot(Sx_i(i, t_i), Sy_i(i, t_i))
    plt.grid(ls=':')
    plt.plot(x,y,'ro')
```

рисование графика для интерполирующего алгебраического многочлена

In [3]:
```
partitions=30
def alg_show_graf(g_x, g_y):
    plt.figure(figsize=(5,7))
    for i in range(1,n):
        t_i = np.linspace(t[i-1], t[i], partitions)
        _x = [g_x(t_i[j]) for j in range(partitions)]
        _y = [g_y(t_i[j]) for j in range(partitions)]
        plt.plot(_x, _y)
    plt.grid(ls=':')
    plt.plot(x,y,'ro')
```

In [4]:
```
variant = 5
x = np.array([0, variant, variant, variant/2])
y = np.array([0, 0, 1, 2])
plt.figure(figsize=(5,7))
plt.plot(x,y,'ro')
plt.grid(ls=':')
```



In [5]:
```
#Инициализация начальных значений
n=len(x)
a_y = y.copy();a_x = x.copy()
t = np.arange(0,n)
h = np.zeros(n)
h[1:] = t[1:] - t[:-1]
print(f'h = {h[1:]}')
print(f'a_x = {a_x}')
print(f'a_y = {a_y}')
print(f't = {t}')
```

```
h = [1. 1. 1.]
a_x = [0.  5.  5.  2.5]
a_y = [0 0 1 2]
t = [0 1 2 3]
```

Кубический сплайн

$$S_i(t) = \begin{bmatrix} \alpha_i^1 \\ \alpha_i^2 \end{bmatrix} + (t - t_i) \begin{bmatrix} \beta_i^1 \\ \beta_i^2 \end{bmatrix} + \frac{1}{2}(t - t_i)^2 \begin{bmatrix} \gamma_i^1 \\ \gamma_i^2 \end{bmatrix} + \frac{1}{6}(t - t_i)^3 \begin{bmatrix} \delta_i^1 \\ \delta_i^2 \end{bmatrix}$$

In [6]:
```
def Sx_i_3(i, t_i):
    return a_x[i] + b_x[i]*(t_i - t[i]) + 1/2*g_x[i]*(t_i - t[i])**2  + 1/6*d_x[i]*(t_i - t[i])**3

def Sy_i_3(i, t_i):
    return a_y[i] + b_y[i]*(t_i - t[i]) + 1/2*g_y[i]*(t_i - t[i])**2  + 1/6*d_y[i]*(t_i - t[i])**3
```

Метод встречной прогонки для назождения γ

In [7]:
```
def method_vstr_prog(m, n, a, b, c, f):
        alpha, beta, mu, nu = ([0] * n for _ in range(4))

        alpha[0] = b[0] / c[0]
        beta[0] = f[0] / c[0]
        for i in range(m-1):
            denom = c[i+1] - a[i] * alpha[i]
            alpha[i+1] = b[i+1] / denom
            beta[i+1] = (f[i+1] - a[i] * beta[i]) / denom

        mu[n-1] = a[n-2] / c[n-1]
        nu[n-1] = f[n-1] / c[n-1]
        for i in range(n-2, m-1, -1):
            denom = c[i] - b[i] * mu[i+1]
            mu[i] = a[i-1] / denom
            nu[i] = (f[i] - b[i] * nu[i+1]) / denom

        y = [0] * n
        y[m] = (nu[m] - mu[m] * beta[m - 1]) / (1 - mu[m] * alpha[m-1])

        for i in range(m-1, -1, -1):
            y[i] = beta[i] - alpha[i] * y[i+1]

        for i in range(m, n-1):
            y[i+1] = nu[i+1] - mu[i+1] * y[i]

        return y
```

$$\text{Доп условие } S_0''(0) = S_n''(0) = 0 \Rightarrow \gamma_0 = \gamma_n = 0$$

$$h_i\gamma_{i-1} + 2(h_i + h_{i+1})\gamma_i + h_{i+1}\gamma_{i+1} = 6\Big(\frac{\alpha_{i+1} - \alpha_i}{h_{i+1}} - \frac{\alpha_i - \alpha_{i-1}}{h_i}\Big)$$

$$c_i\gamma_{i-1} + 2\gamma_i + e_i\gamma_{i+1} = b_i, i = \overline{1, \mathrm{n} - 1}$$

$$c_i = \frac{h_i}{x_{i+1} - x_{i-1}}$$

$$e_i = \frac{h_{i+1}}{x_{i+1} - x_{i-1}}$$

$$b_i = 6f[x_{i+1}, x_i, x_{i-1}]$$

In [8]:
```
b_x, g_x, d_x, b_y, g_y, d_y = [np.zeros(n) for _ in range(6)]

e_x = h[3:]/(t[3:]-t[:-3])
e_y = e_x.copy()

c_x = h[2:-1]/(t[3:]-t[:-3])
c_y = c_x.copy()

b_x_ = 6*((a_x[2:] - a_x[1:-1]) / h[2:] - (a_x[1:-1] - a_x[:-2]) / h[1:-1])/(t[2:]-t[:-2])
b_y_ = 6*((a_y[2:] - a_y[1:-1]) / h[2:] - (a_y[1:-1] - a_y[:-2]) / h[1:-1])/(t[2:]-t[:-2])

print(f'e_x=e_x={e_x}')
print(f'c_x=c_x={c_x}')
print(f'b_x_={b_x_}')
print(f'b_y_={b_y_}')
```

```
e_x=e_x=[0.33333333]
c_x=c_x=[0.33333333]
b_x_=[-15.   -7.5]
b_y_=[3. 0.]
```

Расчитываю через метод прогонки значения γ

In [9]:
```
print(f'{g_x=}')
g_x[1:-1] = method_vstr_prog(0, n-2, c_x, e_x, [2]*(n-2), b_x_)
g_y[1:-1] = method_vstr_prog(0, n-2, c_y, e_y, [2]*(n-2), b_y_)
print(f'g_x={g_x}')
print(f'g_y={g_y}')
```

```
g_x=array([0., 0., 0., 0.])
g_x=[ 0.         -7.07142857 -2.57142857  0.        ]
g_y=[ 0.          1.54285714 -0.25714286  0.        ]
```

$$\delta_i = \frac{\gamma_i - \gamma_{i-1}}{h_i}$$

In [10]:
```
d_x[1:] = (g_x[1:] - g_x[:-1]) / h[1:]
d_y[1:] = (g_y[1:] - g_y[:-1]) / h[1:]
print(f'd_x={d_x}')
print(f'd_y={d_y}')
```

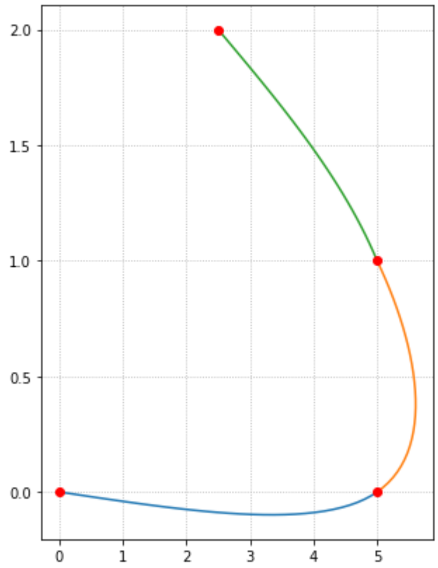
```
d_x=[ 0.         -7.07142857  4.5         2.57142857]
d_y=[ 0.          1.54285714 -1.8         0.25714286]
```

$$\beta_i = \frac{\alpha_i - \alpha_{i-1}}{h_i} - \frac{2\gamma_i + \gamma_{i-1}}{6} h_i, i = \overline{1, \mathrm{n}}$$

In [11]:
```
b_x[1:] = (a_x[1:] - a_x[:-1]) / h[1:] + (2*g_x[1:] + g_x[:-1]) / 6 * h[1:]
b_y[1:] = (a_y[1:] - a_y[:-1]) / h[1:] + (2*g_y[1:] + g_y[:-1]) / 6 * h[1:]
print(f'b_x={b_x}')
print(f'b_y={b_y}')
```

```
b_x=[ 0.          2.64285714 -2.03571429 -2.92857143]
b_y=[0.         0.51428571 1.17142857 0.95714286]
```

In [12]:
```
spline_show_graf(Sx_i_3, Sy_i_3)
```



Ответ:

$$S(t) = \begin{cases} \begin{bmatrix} 5 \\ 0 \end{bmatrix} + (t-1)\begin{bmatrix} 2.64 \\ 0.52 \end{bmatrix} + \frac{1}{2}(t-1)^2\begin{bmatrix} -7.07 \\ 1.54 \end{bmatrix} + \frac{1}{6}(t-1)^3\begin{bmatrix} -7.07 \\ 1.54 \end{bmatrix} t \in [1,2] \\ \begin{bmatrix} 5 \\ 1 \end{bmatrix} + (t-2)\begin{bmatrix} -2.04 \\ 1.17 \end{bmatrix} + \frac{1}{2}(t-2)^2\begin{bmatrix} -2.57 \\ -0.25 \end{bmatrix} + \frac{1}{6}(t-2)^3\begin{bmatrix} 4.5 \\ -1.8 \end{bmatrix} t \in [2,3] \\ \begin{bmatrix} 2.5 \\ 2 \end{bmatrix} + (t-3)\begin{bmatrix} -2.93 \\ 0.96 \end{bmatrix} + \frac{1}{6}(t-3)^3\begin{bmatrix} 2.57 \\ 0.25 \end{bmatrix} t \in [3,4] \end{cases}$$

Алгебраическая интерполяция

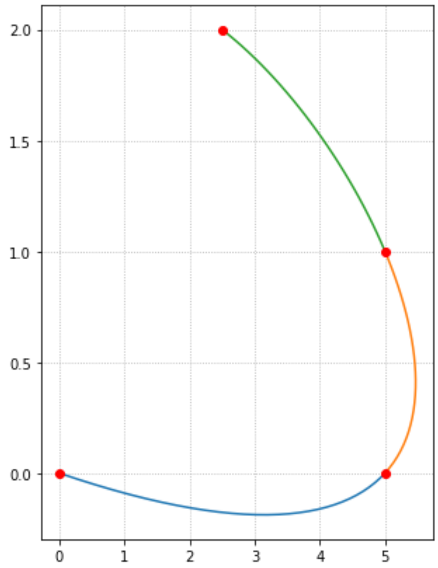
$$g(t) = \sum_{i=0}^{n}\left(\prod_{i\neq j}\frac{t-t_j}{t_i-t_j}\right)\begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

In [13]:
```
def g_x(t_):
    return sum([L(t_,i)*x_i for i, x_i in enumerate(x)])

def g_y(t_):
    return sum([L(t_,i)*y_i for i, y_i in enumerate(y)])

def L(t_,i):
    return prod([(t_-t_j)/(t[i] - t_j) for j, t_j in enumerate(t) if i != j])
```

In [14]:
```
alg_show_graf(g_x, g_y)
```



$$g(t) = \frac{(t-1)(t-2)(t-3)}{(0-1)(0-2)(0-3)}\begin{bmatrix}0\\0\end{bmatrix} + \frac{(t-0)(t-2)(t-3)}{(1-0)(1-2)(1-3)}\begin{bmatrix}5\\0\end{bmatrix} + \frac{(t-0)(t-1)(t-3)}{(2-0)(2-1)(2-3)}\begin{bmatrix}5\\1\end{bmatrix} + \frac{(t-0)(t-1)(t-2)}{(3-0)(3-1)(3-2)}\begin{bmatrix}2.5\\2\end{bmatrix} =$$
$$\frac{t(t-2)(t-3)}{2}\begin{bmatrix}5\\0\end{bmatrix} - \frac{t(t-1)(t-3)}{2}\begin{bmatrix}5\\1\end{bmatrix} + \frac{t(t-1)(t-2)}{6}\begin{bmatrix}2.5\\2\end{bmatrix}$$

Среднеквадратичное приближение параболой

вариант л)

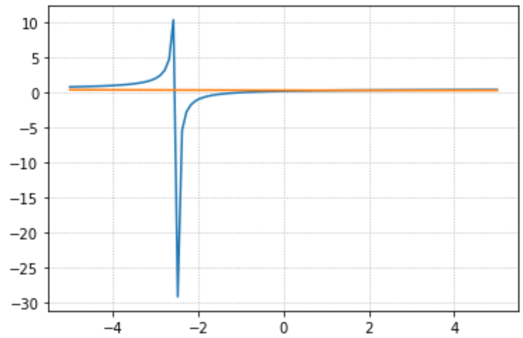
$$f(x) = \frac{x+1}{2x+5}, x \in [1;2]$$

In [15]:
```
def f(x):
    return (x+1)/(2*x+5)
def phi(x):
    return .001*x**2-.0089*x+0.32

x=np.linspace(-5, 5, 100)

plt.grid(ls=':')
plt.plot(x,f(x))
plt.plot(x,phi(x))
```

Out[15]: [<matplotlib.lines.Line2D at 0x2a6bc6064f0>]



In [16]:
```
a=1;b=2;n=5
p=np.arange(a,b,(b-a)/n)
for i in range(n):
    print(f'{i} - {sum(p**i)}')
```

```
0 - 5.0
1 - 7.0
2 - 10.2
3 - 15.399999999999995
4 - 23.966399999999993
```

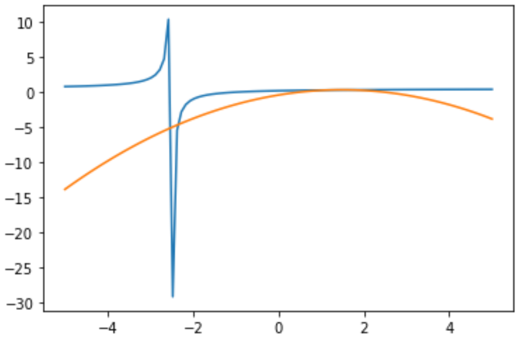
In [17]:
```
for i in range(n):
    print(f'{i} - {sum(f(p)*p**i)}')
```

```
0 - 1.5333584567844352
1 - 2.1666038580389118
2 - 3.1834903549027196
3 - 4.841274112743198
4 - 7.580014718141998
```

In [18]:
```
def phi(x):
    return -.3375*x**2+1.005*x-.4125

plt.plot(x,f(x))
plt.plot(x,phi(x))
```

Out[18]: [<matplotlib.lines.Line2D at 0x2a6bc666790>]



Вычисление коэффициентов интерполирующей параболы

$F(x) = \frac{x+1}{2x+5}, \; x \in [1;2]$

$\varphi(x) = a_2 x^2 + a_1 x + a_0, \quad p(x) = 1$

$\varphi_0(x) = 1, \; \varphi_1(x) = x, \; \varphi_2(x) = x^2$

Построим матрицу Грама

$(\varphi_0, \varphi_0) = \int_1^2 1 \cdot 1 \, dx = 1$

$(\varphi_0, \varphi_1) = \int_1^2 1 \cdot x \, dx = \frac{x^2}{2}\Big|_1^2 = 2 - 1/2 = 1.5 = 3/2$

$(\varphi_0, \varphi_2) = \int_1^2 1 \cdot x^2 \, dx = \frac{x^3}{3}\Big|_1^2 = \frac{8}{3} - \frac{1}{3} = 7/3$

$(\varphi_1, \varphi_1) = \int_1^2 x \cdot x \, dx = 3/2$

$(\varphi_1, \varphi_2) = \int_1^2 x \cdot x^2 \, dx = \frac{x^4}{4}\Big|_1^2 = 2 - 1/4 = 7/4$

$(\varphi_2, \varphi_2) = \int_1^2 x^2 \cdot x^2 \, dx = \frac{x^5}{5}\Big|_1^2 = \frac{16}{5} - \frac{1}{5} = 3$

$(\varphi_0, F) = \int_1^2 \frac{x+1}{2x+5} dx = \frac{1}{2}\int_1^2 \frac{2x+2}{2x+5} dx = \frac{1}{4}\int_{1}^{2} \frac{t-3}{t} dt = \frac{1}{4}\left(t - 3\ln(t)\right)\Big|_7^9 = \frac{1}{4}(9 - 3\ln 9 - 7 + 3\ln 7$

$2x+5$

$= \frac{1}{4}\left(2 + 3\ln\frac{7}{9}\right) \approx 0.3115$

$(\varphi_1, F) = \int_1^2 \frac{x^2+x}{2x+5} dx = \int_1^2 \frac{\frac{1}{2}x \cdot (2x+2) + \frac{3x}{2} - \frac{3x}{2}}{2x+5} dx = \int_1^2 \frac{\frac{1}{2}x(2x+5) - \left(\frac{3 \cdot 2x}{2 \cdot 2} + \frac{5 \cdot 3}{4} - \frac{5 \cdot 3}{4}\right)}{2x+5} dx$

$= \int_1^2 \frac{\frac{1}{2}x(2x+5) - \frac{3}{4}(2x+5) + \frac{15}{4}}{2x+5} dx = \frac{1}{2}\int_1^2 x\,dx - \frac{3}{4}\int_1^2 dx + \frac{15}{4}\int_1^2 \frac{d(2x+5)}{2x+5} =$

$= \frac{1}{2}\frac{x^2}{2} - \frac{3}{4}x + \frac{15}{4}\ln(2x+5)\Big|_1^2 = 0.4712$

$(\varphi_2, F) = \int_1^2 \frac{x^2 \cdot (x+1)}{2x+5} = \ldots = 0.7386$

$$\begin{bmatrix} 1 & 3/2 & 7/3 \\ 3/2 & 3/2 & 7/4 \\ 7/3 & 7/4 & 3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0.3115 \\ 0.4712 \\ 0.7386 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 18 & 28 \\ 18 & 18 & 21 \\ 28 & 21 & 24 \end{bmatrix} \begin{matrix} a_0 \\ a_1 \\ a_2 \end{matrix} = \begin{bmatrix} 12 \cdot 0.3115 \\ 12 \cdot 0.4712 \\ 12 \cdot 0.7386 \end{bmatrix}$$

$a_0 = \cancel{0.465} \quad 0.32$

$a_1 = \cancel{-0.298} \quad -0.0039$

$a_2 = \cancel{0.125} \quad 0.001$

Вычисление коэффициентов интерполирующей параболы по 5 узлам

узлы: [1.0, 1.2, 1.4, 1.6, 1.8, 2]

$\varphi(x)$ \ x	1	1.2	1.4	1.6	1.8	Σ
$\varphi_0(x)$	1	1	1	1	1	5
$\varphi_1(x)$	1	1.2	1.4	1.6	1.8	7
$\varphi_2(x)$	1	1.44	1.96	2.56	~~2.94~~ 3.24	10.2

Обозначим

$$(\varphi_i, \varphi_j) \to (i, j) = \sum_{k=0}^{n} x_k^{i+j} \quad \Big| \quad (f, \varphi_i) = \sum_{k=0}^{n} f(x_k)\, x_k^{i}$$

$(0,0) = 5$

$(0,1) = (1,0) = 7$

$(0,2) = (2,0) = (1,1) = 10.2$

$(1,2) = (2,1) = 15.4$

$(2,2) = 23.96$

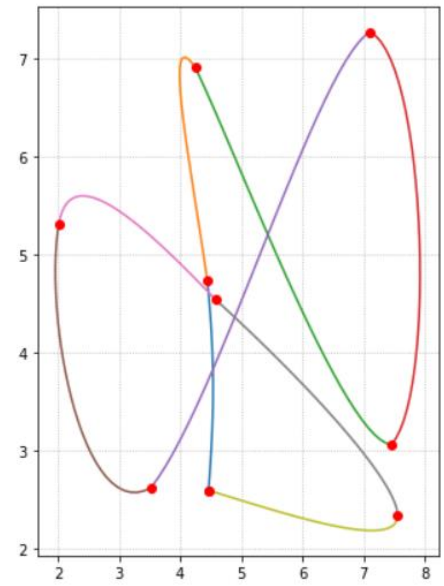
$(0,f) = 1.53$, $(1,f) = 2.166$, $(2,f) = 3.183$

$$\begin{bmatrix} 5 & 7 & 10.2 \\ 7 & 10.2 & 15.4 \\ 10.2 & 15.4 & 23.96 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1.53 \\ 2.166 \\ 3.183 \end{bmatrix}$$

$a_0 = -0.4125$

$a_1 = 1.005$

$a_2 = -0.3375$



бонус: кубический сплайн для 9 случайных точек с замыканием