

1 задание

Доскоч Роман 3 курс ПИ ТП

4/1

1)  $f(x) = \lg x - \frac{1}{x+1} + e^{2x}$ ;  $x=0.9$

$x_0 = 0$  ВФорме Ньютона:

$x_1 = 0.25$

$x_2 = 0.5$

$x_3 = 0.75$

$x_4 = 1$

$x$	$y$	$f[x_0, x_1]$	$f[x_1, x_2]$	$f[x_2, x_3]$	$f[x_3, x_4]$
0	0				
0.25	1.104	4.416	3.119	3.843	2.672
0.5	2.597	5.975	6.000	6.519	
0.75	4.848	8.975	10.836		
1	8.444	14.418			

$\phi(x) = f(x_0) + (x-x_0)f[x_0, x_1] + (x-x_0)(x-x_1)f[x_0, x_1, x_2] + \dots + (x-x_0)(x-x_1)\dots(x-x_{n-1})f[x_0, x_1, \dots, x_n]$

$\phi(x) = 0 + x \cdot 4.416 + x \cdot (x-0.25) \cdot 3.119 + x \cdot (x-0.25)(x-0.5) \cdot 3.843 + x \cdot (x-0.25)(x-0.5)(x-0.75) \cdot 2.672$ , при  $x=0.9 \Rightarrow \phi(x) = 6.794792$

$f(x) = 6.783489$   $r_4(y) = 0.00738 < 1$

$|r_n| \leq \frac{\|f^{(n+1)}\|}{(n+1)!} \|w_{n+1}\|$

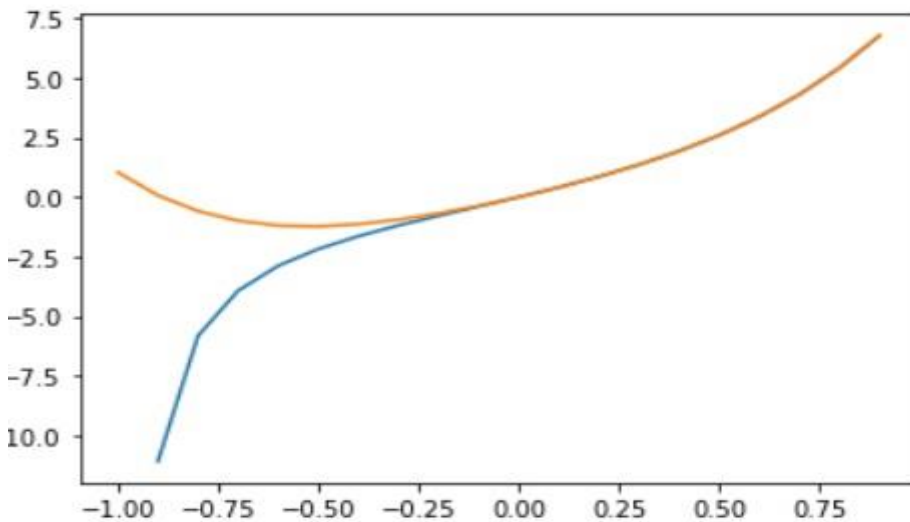
$f^{(n+1)} = \frac{d^5}{dx^5} \left( \lg x - \frac{1}{x+1} + e^{2x} \right) = 1513.87$

$(n+1)! = 720$

$\|w_{n+1}\| = |(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)| = 0.00351$

$|r_n| \leq \frac{1513.87}{720} \cdot 0.00351 = 0.00738$

График функции и аппроксимирующей ее функции



Код на питоне для построения графика.

```
import matplotlib.pyplot as plt
import math as m
import numpy as np

x = np.arange(-1.0, 1.0, 0.1)
f = [ m.tan(i) - 1/(i+1) + m.exp(2*i) for i in x]
phi = [ i*4.416 + i*(i-0.25)*3.118 + i*(i-0.25)*(i-0.5)*3.843 + i*(i-0.25)*(i-0.5)*(i-0.75)*2.672 for i in x]

plt.plot(x, f, x, phi)
plt.show()
```

Расчеты барицентрической формы проводились по алгоритму

$$P_n(x) = \frac{\sum_{i=0}^n y_i \frac{v_i}{x - x_i}}{\sum_{i=0}^n \frac{v_i}{x - x_i}},$$

```
import numpy as np

x = np.array([0, 0.25, 0.5, 0.75, 1.0])
y = np.array([0, 1.104, 2.598, 4.8418, 8.4464])
X = 0.9
j = np.array([10.67, -42.67, 64, -42.67, 10.57])

P_x = np.sum(y*j/(X-x))/np.sum(j/(X-x))
P_x
```

6.785910340426813

Барицентрическая форма

$$P_n(x) = \frac{\sum_{i=0}^n y_i \frac{v_i}{x - x_i}}{\sum_{i=0}^n \frac{v_i}{x - x_i}}, \quad v_i = \frac{1}{\omega'_{n+1}(x_i)}$$

$y_0 = 0$	$x_0 = 0$	$v_0 = 1/(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)(x_0 - x_4) = 10.67$
$y_1 = 1.104$	$x_1 = 0.25$	$v_1 = -11 = -42.67$
$y_2 = 2.598$	$x_2 = 0.5$	$v_2 = -11 = 64$
$y_3 = 4.8418$	$x_3 = 0.75$	$v_3 = -11 = -42.67$
$y_4 = 8.4464$	$x_4 = 1.0$	$v_4 = -11 = 10.67$

$$P_n(x) = \frac{0 \cdot 10.67}{0.9 - 0} + \frac{1.104 \cdot (-42.67)}{0.9 - 0.25} + \frac{2.598 \cdot 64}{0.9 - 0.5} + \frac{4.842 \cdot (-42.67)}{0.9 - 0.75} + \frac{8.446 \cdot 10.67}{0.9 - 1.0}$$

$$x=0.9 \quad \frac{10.67}{0.9 - 0} + \frac{-42.67}{0.9 - 0.25} + \frac{64}{0.9 - 0.5} + \frac{-42.67}{0.9 - 0.75}$$

$$\frac{4.842 \cdot (-42.67)}{0.9 - 0.75} + \frac{8.446 \cdot 10.67}{0.9 - 1.0} = 6.7859103$$

$$\frac{10.67}{0.9 - 1.0}$$

2 Задание При каком  $x$ ,  $f(x) = 3$

$K_2$

$f(x)$	$x$	$x[y_0, y_1]$	$x[y_0, y_2]$	$x[y_0, y_3]$
4	-2	$\left. \begin{array}{l} > -1/2 \\ > 2 \\ > -1 \\ > -3/2 \end{array} \right\}$	$\left. \begin{array}{l} > 2 \\ > -3/2 \end{array} \right\}$	$\frac{-3,5}{+5} = 0,7$
2	-1			
0	1			
-1	1.5			

$$\begin{aligned}
 x(3) &= x_0 + (3-y_0)x[y_0, y_1] + (3-y_0)(3-y_1)x[y_1, y_2] + \\
 &+ (3-y_0)(3-y_1)(3-y_2)x[y_2, y_3] = -2 + (-1) \cdot (-1/2) + (-1) \cdot 1 \cdot 2 \\
 &+ (-1) \cdot 1 \cdot 3 \cdot 0.7 = -2 + 0.5 - 2 - 0.21 = -3.71
 \end{aligned}$$

Ответ:  $x = -3.71$