How have countries performed in the IMO over the past 30 years, and what do problem difficulties reveal about both high-performing and emerging countries?

Nguyen Huy Tung Bui Phuong Nam Dang Hoang Anh Khoa
Subhashree Panneer Bui Phuong Linh Pham Ngoc Minh

Contents

Introduction

In this project, we explore the International Mathematical Olympiad dataset[1], published in September 2024. The dataset contains results from the International Mathematical Olympiad (IMO) spanning from 1959 to 2024. It includes various aspects of the competition, such as participating countries, the composition of each team, and the timeline of the event. It also captures both individual and team results, including scores for each problem, individual rankings, and medal counts. The dataset consists of three main tables:

Table	Description	Key Variables
`country_results_df.csv`	Summarizes each country's yearly performance at the IMO, including team composition, problem scores, and medal counts	`year`, `country`, `awards_gold`
`individual_results_df.csv`	Contains detailed performance data for each IMO participant, including their country, problem scores, individual rank, and any award received	score for each problem
`timeline_df.csv`	Provides an overview of each IMO edition, including the year, host country, number of participating countries and contestants, and the duration of the events	

Our main objective is to analyze the performance of countries in the IMO over the past 30 years. We then explore how problem difficulty levels (Easy, Medium, Hard) influence the performance of both high-performing and emerging countries. Specifically, we aim to address the following questions:

1. Which countries have performed exceptionally well over the last 30 years, based on country rankings and gold medals?
2. Which countries show strong potential to achieve higher scores and win more medals in the future?
3. How do problem difficulty levels help differentiate the capabilities of the top-performing and high-potential countries?

Data Cleaning and Summary

Data Cleaning

Step 1: Import Packages and Load Data

We intend to use the following external packages:

1. `janitor`: For cleaning data. Install with: `install.packages("janitor")`
2. `patchwork`: For combining multiple `ggplot2` plots. Install with: `install.packages("patchwork")`

3. `rnaturalearth`: For accessing world map data. Install with: `install.packages("rnaturalearth")`
4. `rnaturalearthdata`: Provides map data used by `rnaturalearth`. Install with: `install.packages("rnaturalearthda`
5. `ggrepel`: For creating non-overlapping text labels in `ggplot2`. Install with: `install.packages("ggrepel")`
6. `sf`: For handling spatial (geographic) data. Install with: `install.packages("sf")`
7. `ggtext`: For rich text formatting in `ggplot2` plots, including Markdown and HTML support. Install with: `install.packages("ggtext")`
8. `countrycode`: For converting between different country name/code formats. Install with: `install.packages("countrycode")`
9. `kableExtra`: For creating attractive, well-formatted HTML and LaTeX tables. Install with: `install.packages("kableExtra")`

```r
# Load required packages
library(tidyverse)
library(tidytuesdayR)
library(janitor)
library(patchwork)
library(rnaturalearth)
library(rnaturalearthdata)
library(ggrepel)
library(sf)
library(ggtext)
library(countrycode)
library(kableExtra)

# Load the TidyTuesday dataset
tuesdata <- tidytuesdayR::tt_load("2024-09-24")

# Extract individual data frames
country_results_df <- tuesdata$country_results_df
individual_results_df <- tuesdata$individual_results_df
timeline_df <- tuesdata$timeline_df
```

Step 2: Remove Variables Not Needed for Analysis

```r
country_results_df <- country_results_df %>%
  select(-c(team_size_male, team_size_female, leader, deputy_leader))

individual_results_df <- individual_results_df %>%
  select(-c(contestant, total, individual_rank, award))
```

Step 3: Handle Missing Values

In this project, we focus only on the first two datasets: `country_results_df` and `individual_results_df`. Therefore, we will not address the missing values in the third dataset, `timeline_df`.

```r
# Function to summarize percentage of missing values
summarize_missing <- function(df) {
  df %>%
```

```r
    summarize(across(everything(), ~mean(is.na(.)) * 100)) %>%
    pivot_longer(everything(), names_to = "Column", values_to = "Missing (%)") %>%
    arrange(desc(`Missing (%)`)) %>%
    knitr::kable(digits = 2)
}

# Display percentage of missing values
summarize_missing(country_results_df)
```

Step 3a: Visualize the Percentage of Missing Values

Column	Missing (%)
p7	100.00
awards_honorable_mentions	13.62
p1	2.91
p2	2.91
p3	2.91
p4	2.91
p5	2.91
p6	2.91
awards_gold	0.05
awards_silver	0.05
awards_bronze	0.05
year	0.00
country	0.00
team_size_all	0.00

```r
summarize_missing(individual_results_df)
```

Column	Missing (%)
p7	99.93
p1	5.04
p2	5.04
p3	5.04
p4	5.04
p5	5.04
p6	5.04
year	0.00
country	0.00

Step 3b: Inspect Missing Values in the Two Datasets

Dataset 1: `country_results_df`

```r
# Summarize number of rows with missing problem scores (p1 to p6)
missing_summary_country <- tibble(
  Metric = c(
    "All problem scores (p1 to p6) missing",
    "p1 missing",
    "p2 missing",
    "p3 missing",
```

```r
    "p4 missing",
    "p5 missing",
    "p6 missing"
  ),
  Rows_Missing = c(
    country_results_df %>% filter(if_all(p1:p6, is.na)) %>% nrow(),
    country_results_df %>% filter(is.na(p1)) %>% nrow(),
    country_results_df %>% filter(is.na(p2)) %>% nrow(),
    country_results_df %>% filter(is.na(p3)) %>% nrow(),
    country_results_df %>% filter(is.na(p4)) %>% nrow(),
    country_results_df %>% filter(is.na(p5)) %>% nrow(),
    country_results_df %>% filter(is.na(p6)) %>% nrow()
  )
)

# Display missing score summary table
missing_summary_country %>% knitr::kable()
```

Metric	Rows_Missing
All problem scores (p1 to p6) missing	110
p1 missing	110
p2 missing	110
p3 missing	110
p4 missing	110
p5 missing	110
p6 missing	110

```r
# Filter rows with any missing medal counts (gold, silver, bronze)
missing_medals <- country_results_df %>%
  filter(if_any(c(awards_gold, awards_silver, awards_bronze), is.na))

# Filter rows with missing honorable mention awards
missing_honorable_mentions <- country_results_df %>%
  filter(is.na(awards_honorable_mentions))
missing_honorable_mentions
```

```
## # A tibble: 515 x 14
##     year country          team_size_all    p1    p2    p3    p4    p5    p6 p7
##    <dbl> <chr>                    <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <lgl>
##  1  2010 Democratic Peo~              6    NA    NA    NA    NA    NA    NA NA
##  2  1991 Democratic Peo~              6    NA    NA    NA    NA    NA    NA NA
##  3  1987 Romania                      6    42    42    40    42    42    42 NA
##  4  1987 Germany                      6    42    42    42    42    42    38 NA
##  5  1987 Union of Sovie~              6    42    42    28    42    42    39 NA
##  6  1987 German Democra~              6    42    42    42    38    42    25 NA
##  7  1987 United States ~              6    42    42    35    36    36    29 NA
##  8  1987 Hungary                      6    42    42    35    40    38    21 NA
##  9  1987 Bulgaria                     6    38    42    28    36    41    25 NA
## 10  1987 People's Repub~              6    31    42    27    38    40    22 NA
## # i 505 more rows
## # i 4 more variables: awards_gold <dbl>, awards_silver <dbl>,
## #   awards_bronze <dbl>, awards_honorable_mentions <dbl>
```

Dataset 2: `individual_results_df`

```
# Summarize number of rows with missing problem scores
missing_summary_individual <- tibble(
  Metric = c(
    "All problem scores (p1 to p6) missing",
    "p1 missing",
    "p2 missing",
    "p3 missing",
    "p4 missing",
    "p5 missing",
    "p6 missing"
  ),
  Rows_Missing = c(
    individual_results_df %>% filter(if_all(p1:p6, is.na)) %>% nrow(),
    individual_results_df %>% filter(is.na(p1)) %>% nrow(),
    individual_results_df %>% filter(is.na(p2)) %>% nrow(),
    individual_results_df %>% filter(is.na(p3)) %>% nrow(),
    individual_results_df %>% filter(is.na(p4)) %>% nrow(),
    individual_results_df %>% filter(is.na(p5)) %>% nrow(),
    individual_results_df %>% filter(is.na(p6)) %>% nrow()
  )
)

missing_summary_individual %>% knitr::kable()
```

Metric	Rows_Missing
All problem scores (p1 to p6) missing	1093
p1 missing	1093
p2 missing	1093
p3 missing	1093
p4 missing	1093
p5 missing	1093
p6 missing	1093

Step 3c: Remove Unnecessary Data and Drop Missing Values The `country_results_df` dataset contains 100% missing values in the `p7` column, which is expected since the IMO consists of only six problems. As this column provides no useful information, we will remove it from all datasets.

Additionally, there are 110 rows in `country_results_df` where all six problem scores (`p1` to `p6`) are missing. Since these scores are essential for our analysis and visualizations, we will remove these rows to maintain data integrity. We also observe that when any of the medal counts (`awards_gold`, `awards_silver`, or `awards_bronze`) is missing, the corresponding row also has all six problem scores missing. Therefore, after removing rows with missing `p1` to `p6` scores, no further action is needed for the medal columns, as those rows are removed along with them.

Similarly, in the `individual_results_df` dataset, the `p7` column is almost entirely missing and will be dropped. There are 1093 rows where all six problem scores are missing; these will also be removed for the same reason mentioned above.

Moreover, we note that missing values in the `awards_honorable_mentions` column of `country_results_df` indicate that the country received no Honorable Mentions. As such, we impute these missing values with 0.

```r
# Remove p7 and drop rows where all scores from p1 to p6 are missing
country_results_df <- country_results_df %>%
  select(-p7) %>%
  filter(!if_all(p1:p6, is.na))

individual_results_df <- individual_results_df %>%
  select(-p7) %>%
  filter(!if_all(p1:p6, is.na))

# Replace missing values in 'awards_honorable_mentions' with 0
country_results_df <- country_results_df %>%
  mutate(awards_honorable_mentions = replace_na(awards_honorable_mentions, 0))
```

Step 4: Filter Data for The Past 30 Years

Since the original dataset spans from 1959 to 2024, we choose to focus our analysis on the most recent 30 years (1995 to 2024) to better capture current trends and performances of countries.

```r
country_results_df <- country_results_df %>%
  filter(year >= 1995)

individual_results_df <- individual_results_df %>%
  filter(year >= 1995)
```

Step 5: Identify "Special" Countries

Upon examining the data, we identify 18 distinct country codes-'C01' to 'C06' (in 2022), 'C11' to 'C16' (in 2023), and 'C21' to 'C26' (in 2024). Further investigation reveals that these individuals are from Russia but were registered as private participants, following Russia's ban from the Olympiad in 2022 due to its invasion of Ukraine[2]. However, since these participants are officially listed as private participants, we do not classify them as Russian contestants in this project. We include only those entries explicitly labeled as 'Russian Federation' under the country field.

```r
individual_results_df %>%
  distinct(country)
```

```
## # A tibble: 151 x 1
##    country
##    <chr>
##  1 People's Republic of China
##  2 C21
##  3 United States of America
##  4 Japan
##  5 Hungary
##  6 India
##  7 Republic of Korea
##  8 United Kingdom
##  9 Belarus
## 10 Lithuania
## # i 141 more rows
```

Key Summary Statistics

Overview of Key Variables

Here is a summary table of key variables that we are investigating: `country`, `p1`, `p2`, `p3`, `p4`, `p5`, `p6`, `awards_gold`, `awards_silver`, `awards_bronze`, `awards_honorable_mentions`:

country The dataset includes 133 unique countries, but only 59 of them participated in all 30 years.

```
country_results_df %>%
  distinct(country) %>%
  count() %>%
  pull(n)
```

```
## [1] 133
```

```
country_results_df %>%
  count(country, name = "appearances") %>%
  mutate(percentage = appearances / 30 * 100) %>%
  arrange(desc(percentage)) %>%
  filter(percentage == 100) %>%
  summarize(count = n())
```

```
## # A tibble: 1 x 1
##   count
##   <int>
## 1    59
```

p1 The average score for Problem 1 over the 30-year period is 4.6 out of 7.0, indicating moderate performance. Palestine has an average score of 7.0 for this problem; however, this is based on a single contestant over the entire 30-year period, who scored a perfect 7.0.

```
individual_results_df %>%
  summarize(mean_p1_all = mean(p1))
```

```
## # A tibble: 1 x 1
##   mean_p1_all
##         <dbl>
## 1        4.65
```

```
individual_results_df %>%
  group_by(country) %>%
  summarize(mean_p1 = mean(p1)) %>%
  arrange(desc(mean_p1))
```

```
## # A tibble: 151 x 2
##    country mean_p1
##    <chr>     <dbl>
##  1 C01           7
##  2 C02           7
```

```
##   3 C03           7
##   4 C04           7
##   5 C05           7
##   6 C06           7
##   7 C11           7
##   8 C12           7
##   9 C13           7
##  10 C14           7
## # i 141 more rows
```

```
individual_results_df %>%
  filter(country == "Palestine")
```

```
## # A tibble: 1 x 8
##     year country      p1    p2    p3    p4    p5    p6
##    <dbl> <chr>     <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1  2022 Palestine     7     1     0     0     6     0
```

p2 The average score for Problem 2 over the 30-year period is 2.5 out of 7.0, indicating relatively low performance. Among all countries, China has the highest average score for Problem 2 at approximately 6.3, with no other country averaging above 6.0.

```
individual_results_df %>%
  summarize(mean_p2_all = mean(p2))
```

```
## # A tibble: 1 x 1
##   mean_p2_all
##         <dbl>
## 1        2.45
```

```
individual_results_df %>%
  group_by(country) %>%
  summarize(mean_p2 = mean(p2)) %>%
  arrange(desc(mean_p2))
```

```
## # A tibble: 151 x 2
##    country mean_p2
##    <chr>     <dbl>
##   1 C01          7
##   2 C02          7
##   3 C03          7
##   4 C04          7
##   5 C05          7
##   6 C06          7
##   7 C11          7
##   8 C12          7
##   9 C13          7
##  10 C14          7
## # i 141 more rows
```

p3 The average score for Problem 3 over the 30-year period is 0.8 out of 7.0, indicating very low performance. Among all countries, China again leads with the highest average score of approximately 4.5, while no other country averages above 4.0. This suggests that most countries struggle significantly with this problem.

```
individual_results_df %>%
  summarize(mean_p3_all = mean(p3))
```

```
## # A tibble: 1 x 1
##   mean_p3_all
##         <dbl>
## 1       0.839
```

```
individual_results_df %>%
  group_by(country) %>%
  summarize(mean_p3 = mean(p3)) %>%
  arrange(desc(mean_p3))
```

```
## # A tibble: 151 x 2
##    country mean_p3
##    <chr>     <dbl>
##  1 C01           7
##  2 C05           7
##  3 C06           7
##  4 C11           7
##  5 C12           7
##  6 C13           7
##  7 C14           7
##  8 C15           7
##  9 C16           7
## 10 C21           6
## # i 141 more rows
```

p4 The average score for Problem 4 over the 30-year period is 4.3 out of 7.0, indicating moderate performance. China has the highest average score for this problem at around 6.8, followed closely by Russia and the USA, each with an average of approximately 6.7.

```
individual_results_df %>%
  summarize(mean_p4_all = mean(p4))
```

```
## # A tibble: 1 x 1
##   mean_p4_all
##         <dbl>
## 1        4.25
```

```
individual_results_df %>%
  group_by(country) %>%
  summarize(mean_p4 = mean(p4)) %>%
  arrange(desc(mean_p4))
```

```
## # A tibble: 151 x 2
```

```
##    country mean_p4
##    <chr>     <dbl>
##  1 C01           7
##  2 C02           7
##  3 C03           7
##  4 C04           7
##  5 C05           7
##  6 C06           7
##  7 C11           7
##  8 C12           7
##  9 C13           7
## 10 C14           7
## # i 141 more rows
```

p5 The average score for Problem 5 over the 30-year period is 2.2 out of 7.0, indicating relatively low performance. China and Palestine are the only two countries with an average score of around 6.0; however, Palestine's score is based on a single contestant over the entire 30-year period and therefore does not reflect the country's overall performance.

```
individual_results_df %>%
  summarize(mean_p5_all = mean(p5))
```

```
## # A tibble: 1 x 1
##   mean_p5_all
##         <dbl>
## 1        2.21
```

```
individual_results_df %>%
  group_by(country) %>%
  summarize(mean_p5 = mean(p5)) %>%
  arrange(desc(mean_p5))
```

```
## # A tibble: 151 x 2
##    country mean_p5
##    <chr>     <dbl>
##  1 C01           7
##  2 C02           7
##  3 C04           7
##  4 C05           7
##  5 C06           7
##  6 C11           7
##  7 C12           7
##  8 C13           7
##  9 C14           7
## 10 C15           7
## # i 141 more rows
```

p6 The average score for Problem 6 over the 30-year period is 0.6 out of 7.0, indicating extremely low performance. China leads with an average score of 4.1, followed by the USA with 3.2. No other country achieves an average score above 3.0, highlighting the high level of difficulty associated with Problem 6.

```r
individual_results_df %>%
  summarize(mean_p6_all = mean(p6))
```

```
## # A tibble: 1 x 1
##   mean_p6_all
##         <dbl>
## 1       0.582
```

```r
individual_results_df %>%
  group_by(country) %>%
  summarize(mean_p6 = mean(p6)) %>%
  arrange(desc(mean_p6))
```

```
## # A tibble: 151 x 2
##    country                    mean_p6
##    <chr>                        <dbl>
##  1 C05                           7
##  2 C21                           7
##  3 People's Republic of China    4.13
##  4 C01                           4
##  5 C06                           4
##  6 United States of America      3.17
##  7 Russian Federation            2.65
##  8 Republic of Korea             2.51
##  9 C11                           2
## 10 C16                           2
## # i 141 more rows
```

awards_gold China has earned the most gold medals over the past 30 years, with a total of 150. The USA follows with 114 gold medals, and Russia ranks third with 97.

```r
country_results_df %>%
  group_by(country) %>%
  summarize(total_gold = sum(awards_gold)) %>%
  arrange(desc(total_gold))
```

```
## # A tibble: 133 x 2
##    country                    total_gold
##    <chr>                           <dbl>
##  1 People's Republic of China        150
##  2 United States of America          114
##  3 Russian Federation                 97
##  4 Republic of Korea                  94
##  5 Vietnam                            57
##  6 Romania                            50
##  7 Japan                              46
##  8 Islamic Republic of Iran           45
##  9 Taiwan                             45
## 10 Ukraine                            42
## # i 123 more rows
```

awards_silver Iran has earned the most silver medals over the past 30 years, with 96, followed by Taiwan with 94 and Japan with 86.

```
country_results_df %>%
  group_by(country) %>%
  summarize(total_silver = sum(awards_silver)) %>%
  arrange(desc(total_silver))
```

```
## # A tibble: 133 x 2
##    country                   total_silver
##    <chr>                            <dbl>
##  1 Islamic Republic of Iran            96
##  2 Taiwan                              94
##  3 Japan                               86
##  4 Romania                             84
##  5 Bulgaria                            80
##  6 Hungary                             77
##  7 Republic of Korea                   75
##  8 Vietnam                             75
##  9 Germany                             72
## 10 Türkiye                             72
## # i 123 more rows
```

awards_bronze Slovakia, Croatia, and France have earned the most bronze medals, with 86, 84, and 82 medals respectively.

```
country_results_df %>%
  group_by(country) %>%
  summarize(total_bronze = sum(awards_bronze)) %>%
  arrange(desc(total_bronze))
```

```
## # A tibble: 133 x 2
##    country        total_bronze
##    <chr>                 <dbl>
##  1 Slovakia                 86
##  2 Croatia                  84
##  3 France                   82
##  4 Belarus                  79
##  5 Georgia                  76
##  6 Israel                   76
##  7 Czech Republic           75
##  8 Mexico                   75
##  9 Hong Kong                74
## 10 Kazakhstan               74
## # i 123 more rows
```

awards_honorable_mentions An Honorable Mention is awarded to participants who do not win a medal but score 7 points on at least one problem[2]. South Africa, Slovenia, and Latvia have earned the most honorable mentions, with 77, 72, and 71 times respectively.

```r
country_results_df %>%
  group_by(country) %>%
  summarize(total_honorable_mentions = sum(awards_honorable_mentions)) %>%
  arrange(desc(total_honorable_mentions))
```

```
## # A tibble: 133 x 2
##    country      total_honorable_mentions
##    <chr>                           <dbl>
##  1 South Africa                       77
##  2 Slovenia                           72
##  3 Latvia                             71
##  4 Estonia                            68
##  5 Azerbaijan                         67
##  6 Morocco                            67
##  7 Sri Lanka                          67
##  8 Finland                            65
##  9 Lithuania                          65
## 10 Macau                              64
## # i 123 more rows
```

Visualization 1: Which countries have performed exceptionally well over the last 30 years, based on country rankings and gold medals?

The first visualization explores country-level performance at the International Mathematical Olympiad (IMO) from 1995 to 2024, focusing on consistent and excellent performance. After counting the number of times each country appears in the annual top 5 rankings based on total team score (**Consistency**), we select the top 10 countries and highlight their average number of gold medals (**Well-performance**). By combining these two metrics on a global map, this visualization identifies nations that have demonstrated both sustained participation at the top level and outstanding performance over time.

Metric Definition

Before interpreting the results, we define two key metrics: **consistency** and **well-performance**. Consistency refers to two aspects: a country's regular participation in the IMO and its recurring presence within a specific range of rankings (e.g., top 5, top 10, etc.). This metric highlights how stable a country's performance is over time, regardless of whether it wins top awards each year. Well-performance is defined as the average number of gold medals a country earns annually between 1995 and 2024. Since gold medals are typically awarded to the top 1/12 of all participants[2], this measure reflects the strength of a country's highest-performing individuals. Together, these two dimensions offer a broader picture of national performance at the IMO. A country with both consistent and high-level performance would frequently appear in the top 5 team rankings and earn a high average number of gold medals.

It's worth noting that consistency and high performance do not always go hand in hand. A country can exhibit consistency by regularly participating and consistently placing in the *middle* tier, even if it rarely secures gold medals. Conversely, a country might achieve outstanding results in a few specific years but lack regular participation or sustained strong performance over time.

These two metrics collectively help us address the broader research question by examining both country rankings (linked to **consistency**) and the distribution of gold medals (linked to **well-performance**).

Data Preparation

To calculate the consistency and performance metrics, we follow these steps:

1. Annual Total Scores: We calculate the total score each country earns each year by summing the scores of all six IMO problems.

2. Filter for Frequent Participants: We filter the dataset to include only countries that participate in the IMO at least 10 times over the past 30 years. This threshold reflects our definition of **consistency** as frequent and sustained involvement. With this cutoff, we still capture the performance of over 100 countries, providing a broad and representative overview.

3. Top 5 Appearances: We count how many times each country appears in the top 5 rankings and use this to identify the top 10 most **consistent** and **best-performing** countries. If two or more countries have the same number of top 5 appearances, we use the average number of gold medals as a tiebreaker. This approach prioritizes countries with stronger overall performance. For example, although both Iran and Romania appear five times in the top 5, Romania ranks higher due to a higher average gold medal count. A similar situation occurs between North Korea and Hungary, allowing North Korea to secure a spot in the top 10.

4. Gold Medal Average: We compute the average number of gold medals earned by the top 10 most **consistent** and **best-performing** countries.

```r
# Calculate total score per row
country_total_score_all_years <- country_results_df %>%
  mutate(total_score = p1 + p2 + p3 + p4 + p5 + p6)

# Filter countries with at least 10 appearances
country_results_df_filter <- country_results_df %>%
  count(country) %>%
  filter(n >= 10) %>%
  arrange(desc(n))

# Calculate number of years each country was in the top 5 total score
country_number_of_years_top_5 <- country_total_score_all_years %>%
  group_by(year) %>%
  slice_max(total_score, n = 5) %>%
  ungroup() %>%
  count(country, name = "number_of_years_top_5_score") %>%
  arrange(desc(number_of_years_top_5_score))

# Join with filtered country list and fill missing values with 0
country_number_of_years_top_5 <- country_results_df_filter %>%
  select(country) %>%
  left_join(country_number_of_years_top_5, by = "country") %>%
  mutate(number_of_years_top_5_score = coalesce(number_of_years_top_5_score, 0))

# Compute average number of gold medals per appearance
country_average_gold_medals <- country_results_df %>%
  group_by(country) %>%
  summarize(average_gold_medals = mean(awards_gold)) %>%
  arrange(desc(average_gold_medals))
```

```r
# Join and rank countries by top 5 appearances and average gold medals
full_data_set <- country_average_gold_medals %>%
  left_join(country_number_of_years_top_5, by = "country") %>%
  arrange(desc(number_of_years_top_5_score), desc(average_gold_medals)) %>%
  slice_head(n = 10)

# Extract top 10 countries
top_10_country_list <- full_data_set %>%
  pull(country)

# Filter average gold medals data to only include top 10 countries
country_average_gold_medals <- country_average_gold_medals %>%
  filter(country %in% top_10_country_list)
```

```r
top_10_performing <- full_data_set %>%
  rename(
    "Country" = 1,
    "Average Gold Medals per Year" = 2,
    "Number of Years in Top 5 Rankings" = 3
  )

# Render formatted table
kable(
  top_10_performing,
  align = c("l", "c", "c"),
  caption = "Top 10 Best-Performing Countries"
) %>%
  kable_styling(bootstrap_options = c("striped", "hover"))
```

Table 6: Top 10 Best-Performing Countries

Country	Average Gold Medals per Year	Number of Years in Top 5 Rankings
People's Republic of China	5.172414	28
United States of America	3.800000	27
Russian Federation	3.592593	22
Republic of Korea	3.133333	19
Vietnam	1.900000	10
Bulgaria	1.333333	7
Thailand	1.133333	6
Romania	1.666667	5
Islamic Republic of Iran	1.500000	5
Democratic People's Republic of Korea	2.200000	4

Additionally, some data cleaning and pre-labeling are necessary due to inconsistencies in country naming between the maps library and the IMO dataset, as well as a few special cases. One such case is Greenland, which is treated as separate from Denmark in mapping libraries, despite being part of the Kingdom of Denmark in the context of the IMO.

For pre-labeling, we categorize the average number of gold medals into three groups based on performance: countries earning 1 to 2 gold medals, those earning 3 to 4, and those averaging more than 5 gold medals annually.

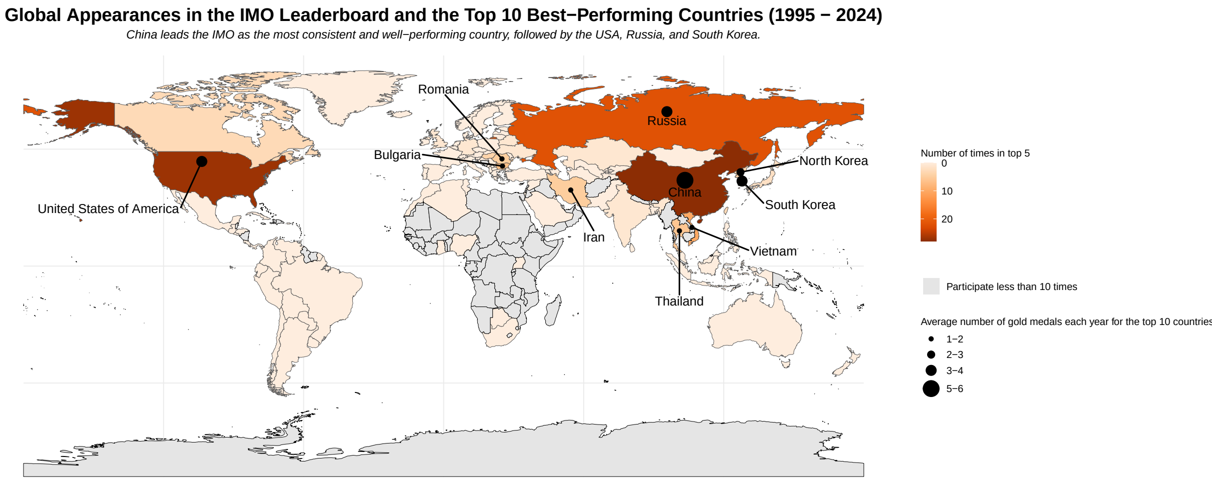
```r
# Standardize country names for consistency
standardize_country_names <- function(df) {
  df %>%
    mutate(country = case_when(
      country == "People's Republic of China" ~ "China",
      country == "Russian Federation" ~ "Russia",
      country == "Islamic Republic of Iran" ~ "Iran",
      country == "Republic of Korea" ~ "South Korea",
      country == "Czech Republic" ~ "Czechia",
      country == "Türkiye" ~ "Turkey",
      country == "Macau" ~ "Macao",
      country == "Republic of Moldova" ~ "Moldova",
      country == "Democratic People's Republic of Korea" ~ "North Korea",
      TRUE ~ country
    ))
}

# Apply standardization
country_number_of_years_top_5_rename <- standardize_country_names(country_number_of_years_top_5)
country_average_gold_medals_rename <- standardize_country_names(country_average_gold_medals)

# Load world map and fix country names
world <- ne_countries(scale = "medium", returnclass = "sf")
world$name[world$name == "Greenland"] <- "Denmark"

# Join data to world map
country_number_of_years_top_5_map <- world %>%
  left_join(country_number_of_years_top_5_rename, by = c("name" = "country"))

country_average_gold_medals_map <- world %>%
  left_join(country_average_gold_medals_rename, by = c("name" = "country"))

# Create centroids and categorize average gold medals
centroids <- st_centroid(country_average_gold_medals_map)

centroids$average_gold_medals_cat <- cut(
  centroids$average_gold_medals,
  breaks = c(0, 1, 2, 3, 4, 5, Inf),
  labels = c("0-1", "1-2", "2-3", "3-4", "4-5", "5-6"),
  include.lowest = TRUE
)

# Keep only valid data
centroids_clean <- centroids %>%
  filter(!is.na(average_gold_medals))

centroids_clean_coords <- cbind(centroids_clean, st_coordinates(centroids_clean))

# Create label nudges for overlapping countries
nudge_table <- tibble::tibble(
  name = c(
    "China", "United States of America", "Russia", "South Korea", "North Korea",
    "Vietnam", "Iran", "Thailand", "Bulgaria", "Romania"
```

```
  ),
  nudge_x = c(0, -40, 0, 25, 40, 35, 10, 0, -45, -25),
  nudge_y = c(-5, -20, -1, -10, 5, -10, -20, -30, 5, 30)
)

centroids_label <- centroids_clean_coords %>%
  left_join(nudge_table, by = "name")

# Split map data by presence of values for highlighting
country_number_of_years_top_5_map_not_na <- country_number_of_years_top_5_map %>%
  filter(!is.na(number_of_years_top_5_score))

country_number_of_years_top_5_map_na <- country_number_of_years_top_5_map %>%
  filter(is.na(number_of_years_top_5_score))
```

Plot

```
plot_1 <- ggplot() +
  # Map: countries with available top 5 data
  geom_sf(
    data = country_number_of_years_top_5_map_not_na,
    aes(fill = number_of_years_top_5_score)
  ) +
  scale_fill_distiller(
    palette = "Oranges",
    trans = "reverse",
    na.value = "grey90"
  ) +

  # Map: countries with no top 5 data (less than 10 participations)
  geom_sf(
    data = country_number_of_years_top_5_map_na,
    fill = "grey90",
    color = "black",
    size = 0.2,
    show.legend = FALSE,
    inherit.aes = FALSE
  ) +

  # Dummy point to show legend for countries with <10 participations
  geom_point(
    data = data.frame(dummy = "Participate less than 10 times", x = 0, y = 0),
    aes(x = x, y = y, color = dummy),
    size = 0,
    show.legend = TRUE
  ) +
  scale_color_manual(
    name = "",
    values = c("Participate less than 10 times" = "grey90")
  ) +
  guides(
```

```r
    color = guide_legend(override.aes = list(shape = 15, size = 8))
  ) +

  # Bubble sizes for gold medals
  geom_sf(
    data = centroids_clean,
    aes(size = average_gold_medals_cat),
    color = "black"
  ) +
  scale_size_manual(
    values = c("0-1" = 1, "1-2" = 2, "2-3" = 3.5, "3-4" = 5, "4-5" = 6.5, "5-6" = 8)
  ) +

  # Country labels with nudges
  geom_text_repel(
    data = centroids_label,
    aes(x = X, y = Y, label = name),
    size = 5,
    color = "black",
    segment.size = 0.8,
    nudge_x = centroids_label$nudge_x,
    nudge_y = centroids_label$nudge_y
  ) +

  # Theming
  theme_minimal() +
  theme(
    plot.title = element_text(size = 20, hjust = 0.5, face = "bold"),
    plot.subtitle = element_text(size = 13, hjust = 0.5, face = "italic", margin = margin(b = 10)),
    legend.text = element_text(size = 11),
    axis.title = element_blank(),
    axis.text = element_blank(),
    axis.ticks = element_blank()
  ) +

  # Titles and labels
  labs(
    title = "Global Appearances in the IMO Leaderboard and the Top 10 Best-Performing Countries (1995 -
    subtitle = "China leads the IMO as the most consistent and well-performing country, followed by the
    fill = "Number of times in top 5",
    size = "Average number of gold medals each year for the top 10 countries"
  ) +

  # Ordering of legends
  guides(
    fill  = guide_colorbar(order = 1),
    color = guide_legend(order = 2, override.aes = list(shape = 15, size = 8)),
    size  = guide_legend(order = 3)
  )

plot_1
```



Discussion

Overall, more than 100 countries have participated in the IMO at least 10 times over the past 30 years. Most European countries, as well as the majority of nations in the Americas and Asia, have taken part in the contest at various points. Africa, however, is the only continent with relatively low participation in the IMO. This disparity reflects broader challenges such as limited investment in education, underdeveloped mathematics programs, and the persistent impact of political instability, economic constraints, and social issues across many African nations[3].

A standout group of 'Big 4' countries - **China**, **the USA**, **Russia**, and **South Korea** - leads the IMO in both performance and consistency. China dominates with an average of 5.2 gold medals per year, nearly a full team's worth. The USA averages 3.8 gold medals annually, followed by Russia with 3.6 and South Korea with over 3. All four countries maintain strong averages and frequent top-five finishes - especially Russia, despite its ban in 2022[4]. Other high-performing countries such as Vietnam, Bulgaria, Thailand, Iran, Romania, and North Korea also show consistent results, averaging 1.3 to 2.2 golds annually, with several achieving multiple top-five placements.

China's IMO dominance can be attributed to a combination of strong motivation, a large population, and intensive early training. Olympiad success offers major incentives-top university placements, scholarships, and national recognition-making it a compelling alternative to the highly competitive Gaokao exam. A vast talent pool, paired with a culture that deeply values education, fosters early and focused mathematical development. This is further reinforced by specialized instructors, rigorous selection processes, and dedicated school time for Olympiad preparation[5].

In conclusion, countries like China, the USA, Russia, and South Korea exemplify both consistency and excellence in mathematical education. Their success reflects not only deep talent pools, but also robust national infrastructures that support mathematics training and competition. More broadly, these patterns highlight how a few countries continue to lead the way in the global pursuit of mathematical excellence at the International Mathematical Olympiad.

Visualization 2: Which countries show strong potential to achieve higher scores and win more medals in the future?

The second visualization examines country-level performance at the International Mathematical Olympiad (IMO) from 1995 to 2024, focusing on each country's potential to achieve higher average scores per team member and secure more medals. It highlights how improvements in individual scores can translate into greater medal counts.

A scatter plot is used to visualize countries based on their rate of increase in average individual scores per year versus the rate of increase in medals per year. Potential countries are colored, while non-potential countries are left uncolored. This visualization aims to identify countries that have consistently increased both their average individual scores and medal tallies over time.

Metric Definition

We define **potential** countries as nations that exhibit statistically significant positive trends in either (or both) the rate of increase in total medals per year and the average score per team member per year, but have not yet established themselves as top-performing in terms of total medals achieved. These countries demonstrate positive momentum, with trends suggesting the possibility of converting improved scores into more medals in the future. To identify such countries, we compute linear regression slopes over a 30-year period (1995 - 2024), considering only those with at least 10 years of IMO participation to ensure the stability and reliability of trend estimation.

Data Preparation

a. Derived Metrics

To quantify trends in performance:

`avg_score_per_member`: Calculated by summing a country's total IMO scores across problems `p1-p6` in a given year and dividing by the team size. This accounts for differences in team sizes, enabling fairer comparisons across countries.

`total_medals`: The total number of gold, silver, and bronze medals won by a country in a given year.

b. Linear Trend Analysis

We apply two separate linear regressions to each country's data across the years: one models the trend in total medals (`total_medals` ~ `year`), and the other models the trend in average score per team member (`avg_score_per_member` ~ `year`). The slope coefficients from these regressions represent the rate of change in performance over time-that is, how quickly a country's performance is improving or declining in terms of medal count and average individual score.

c. Average Medal Count

We also compute each country's average total medals across all years of participation. This provides context for interpreting improvement trends and helps identify countries that are improving but have not yet won many medals-signaling strong potential for future success.

d. Country Abbreviations

To improve visual clarity, we use the `countrycode` R package to convert full country names into three-letter ISO codes. This significantly reduces the data-to-ink ratio by minimizing text clutter, allowing the chart to better highlight underlying data patterns.

Description of Plot

We construct a scatter plot with the following axes:

X-axis: Rate of increase in total medals - represented by the slope of the linear model (`total_medals` ~ `year`).

Y-axis: Rate of increase in average score per team member - represented by the slope of the linear model (`avg_score_per_member` ~ `year`).

a. Quadrants

The scatter plot is divided into four quadrants to highlight distinct performance patterns:

- **Top-Right**: Countries improving in both metrics - these are our **key focus**, as they demonstrate strong and balanced growth.
- **Top-Left**: Countries with rising average individual scores but declining medal counts - suggesting improving team performance that has yet to translate into more medals.
- **Bottom-Right**: Countries gaining medals despite declining average individual scores - this may reflect performance imbalances, potentially driven by a few standout individuals rather than overall team strength.
- **Bottom-Left**: Countries declining in both metrics - indicating a lack of positive trajectory in recent years.

Our primary focus is the **Top-Right quadrant**, highlighting countries that are consistently improving in both scoring capability and medal acquisition - strong indicators of positive trajectory and future competitiveness.

b. Color Encoding

We color only the countries in the **Top-Right quadrant**. Each country is plotted as a point, colored according to its average total medal count. Notably, the color scale is intentionally reversed from the typical convention: darker shades represent countries with lower average medal counts, while lighter shades correspond to higher averages. This design choice supports the core objective of the visualization - to spotlight countries that are improving but have yet to achieve high medal counts. By assigning darker colors to these underperforming yet rising countries, we make it easier to identify promising candidates that may not have been historical top performers but are showing strong upward momentum.

c. Reference Line: $y = x$

We include a diagonal reference line (scaled appropriately) in the scatter plot to represent balanced improvement-that is, countries improving at similar rates in both average individual scores and total medals. Countries (in the **Top-Right quadrant**) positioned close to this line are likely converting score improvements into medals efficiently, while those farther away exhibit imbalances in their growth trajectories (e.g., improving in one metric but not the other).

To quantify this alignment, we calculate the absolute difference between the medal slope and score slope. When considered alongside each country's overall rate of improvement and average medal count, this provides a robust, multi-dimensional framework for ranking countries by future potential.

```r
# Calculate average individual scores and total medals per country per year
country_scores <- country_results_df %>%
  mutate(
    avg_score_per_member = rowSums(select(., p1:p6)) / team_size_all,
    total_medals = awards_gold + awards_silver + awards_bronze
  ) %>%
  select(year, country, total_medals, avg_score_per_member)

# Compute trends for countries with at least 10 years of data
trends_df <- country_scores %>%
  group_by(country) %>%
  filter(n() >= 10) %>%
  summarize(
    medals_slope = coef(lm(total_medals ~ year))[2],
    score_slope = coef(lm(avg_score_per_member ~ year))[2],
    avg_medals = mean(total_medals),
    .groups = "drop"
  ) %>%
  mutate(
    country_abbr = countrycode(country, "country.name", "iso3c"),
    is_top_right = medals_slope > 0 & score_slope > 0
  )

# Identify top potential countries
top_potential_countries <- trends_df %>%
  filter(is_top_right) %>%
  mutate(
    combined_rate = (medals_slope + score_slope) / 2,
    distance_to_yx = abs(medals_slope - score_slope)
  ) %>%
  arrange(
    desc(combined_rate),
    distance_to_yx
  ) %>%
  slice_head(n = 10) %>%
  select(
    Country = country,
    Country_Code = country_abbr,
    `Medal Trend` = medals_slope,
    `Score Trend` = score_slope,
    `Avg Medals` = avg_medals,
    `Improvement Balance` = distance_to_yx,
    `Combined Rate` = combined_rate
  ) %>%
  mutate(across(where(is.numeric), ~ round(., 4))) %>%
  arrange(`Avg Medals`) %>%
  pull(Country)

# Mark top 10 in trends data
trends_df <- trends_df %>%
```

```
  mutate(is_top10 = country %in% top_potential_countries)

# Step 5: Define quadrant labels for plot
quadrant_labels <- data.frame(
  x = c(0.10, 0.10, -0.05, -0.05),
  y = c(1.2, -0.8, -0.8, 1.2),
  label = c(
    "Improving \noverall",
    "More medals,\nlower scores",
    "Declining \nperformance",
    "Higher scores,\nfewer medals"
  ),
  hjust = 0.5
)

# Calculate axis ranges and slope-adjusted reference
x_range <- 0.25 - (-0.1)
y_range <- 1.5 - (-1.0)
slope_adjusted <- y_range / x_range
```

Sorting Rationale

We sort the data based on the **combined rate**, which considers both the rate of increase in average individual scores and total medals. This ensures that the top potential countries are those showing a high rate of increase in both metrics.

If two countries have the same `combined_rate`, we then consider the distance of each country's point to the line $y = x$ in our plot. This is because countries closer to this line are likely to have similar growth rates in total medals and average individual scores-indicating that their overall team performance is improving in a balanced way. In other words, an increase in individual scores is effectively translating into more medals, and vice versa.

Note: This secondary sorting criterion (distance to the line $y = x$) does not affect the current top 10 rankings, since no two countries share the same combined_rate. However, it may become relevant when analyzing the top 50 countries, where more ties in combined_rate may occur, especially among less prominent countries near the origin.

```
# Identify top 10 countries with strongest and most balanced positive trends
top_potential_countries <- trends_df %>%
  filter(is_top_right) %>%
  mutate(
    combined_rate = (medals_slope + score_slope) / 2,
    distance_to_yx = abs(medals_slope - score_slope)
  ) %>%
  arrange(
    desc(combined_rate),   # prioritize overall improvement
    distance_to_yx         # then favor balance (closer to y = x)
  ) %>%
  slice_head(n = 10) %>%
  select(
    Country = country,
    Country_Code = country_abbr,
    `Medal Trend` = medals_slope,
```

```r
    `Score Trend` = score_slope,
    `Avg Medals` = avg_medals,
    `Improvement Balance` = distance_to_yx,
    `Combined Rate` = combined_rate
  ) %>%
  mutate(across(where(is.numeric), round, 4))

# Render formatted table
kable(
  top_potential_countries,
  align = c("l", "c", "c", "c", "c", "c", "c", "c"),
  caption = "Top 10 Potential Countries"
) %>%
  kable_styling(bootstrap_options = c("striped", "hover"))
```

Table 7: Top 10 Potential Countries

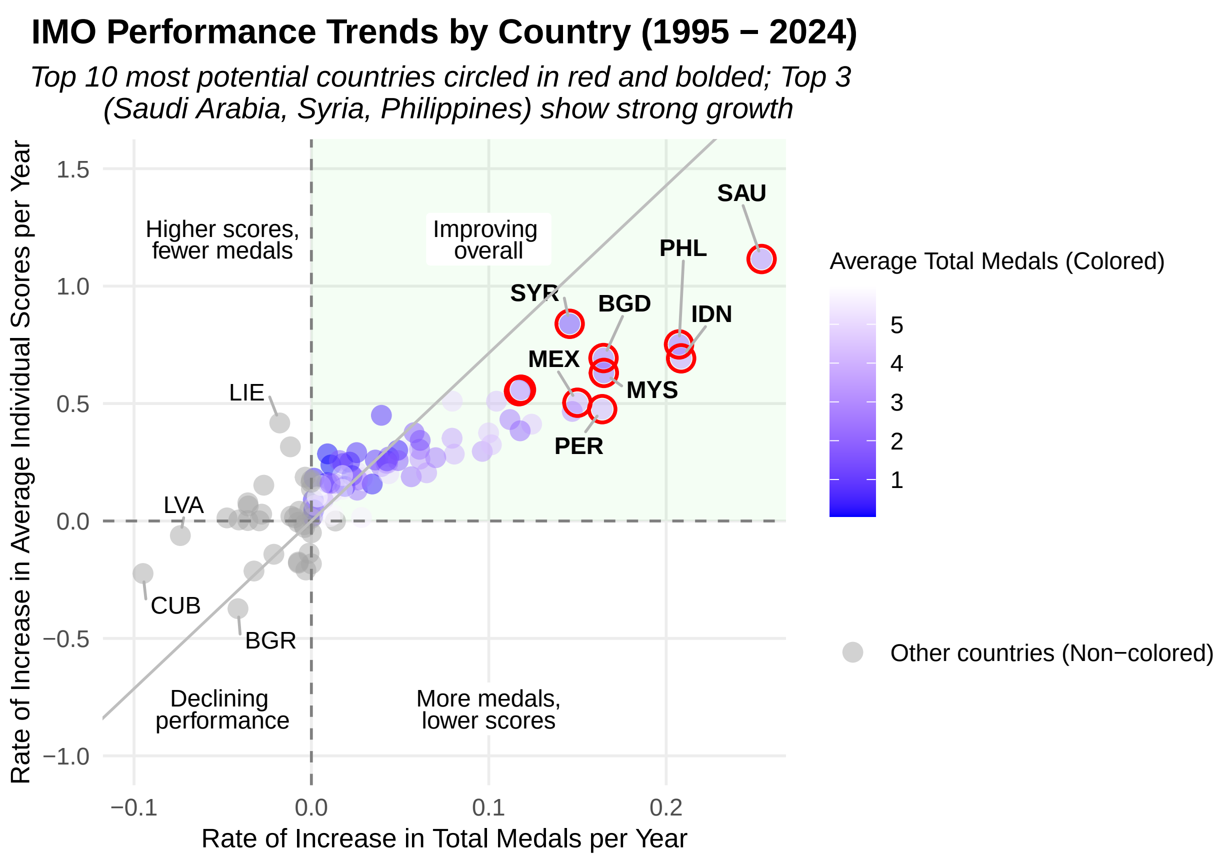
Country	Country_Code	Medal Trend	Score Trend	Avg Medals	Improvement Balance	Combined Rate
Saudi Arabia	SAU	0.2538	1.1154	2.8500	0.8616	0.6846
Syria	SYR	0.1456	0.8395	1.1875	0.6939	0.4925
Philippines	PHL	0.2072	0.7518	2.1034	0.5445	0.4795
Indonesia	IDN	0.2084	0.6925	3.3448	0.4841	0.4505
Bangladesh	BGD	0.1647	0.6938	2.2500	0.5291	0.4292
Malaysia	MYS	0.1648	0.6304	1.9667	0.4656	0.3976
Portugal	PRT	0.1181	0.5591	1.7000	0.4410	0.3386
Thailand	THA	0.1170	0.5525	4.8667	0.4355	0.3348
Mexico	MEX	0.1499	0.5034	3.8667	0.3535	0.3267
Peru	PER	0.1639	0.4760	3.9643	0.3120	0.3199

Plot

```r
plot_2 <- ggplot() +
  # Quadrant backgrounds
  annotate("rect", xmin = 0, xmax = Inf, ymin = 0, ymax = Inf, fill = "palegreen", alpha = 0.1) +
  annotate("rect", xmin = 0, xmax = Inf, ymin = -Inf, ymax = 0, fill = "white", alpha = 0.1) +
  annotate("rect", xmin = -Inf, xmax = 0, ymin = -Inf, ymax = 0, fill = "white", alpha = 0.1) +
  annotate("rect", xmin = -Inf, xmax = 0, ymin = 0, ymax = Inf, fill = "white", alpha = 0.1) +

  # Quadrant labels
  geom_label(
    data = quadrant_labels,
    aes(x = x, y = y, label = label, hjust = hjust),
    size = 3, label.size = NA, lineheight = 0.8
  ) +

  # All countries (non-top-10)
  geom_point(
    data = trends_df,
    aes(
      x = medals_slope,
```

```
    y = score_slope,
    color = ifelse(is_top_right, avg_medals, NA),
    shape = ifelse(is_top_right, "Other countries (Non-colored)", " ")
  ),
  alpha = 0.5, size = 3
) +

# Highlight top 10 countries
geom_point(
  data = filter(trends_df, is_top10),
  aes(x = medals_slope, y = score_slope),
  shape = 1, size = 4, color = "red", stroke = 1, alpha = 1.0
) +

# Country labels
geom_text_repel(
  data = trends_df,
  aes(x = medals_slope, y = score_slope, label = country_abbr),
  size = 3,
  fontface = ifelse(trends_df$is_top10, "bold", "plain"),
  box.padding = 0.5,
  point.padding = 0.3,
  segment.color = "grey70",
  min.segment.length = 0.2,
  max.overlaps = 15
) +

# Reference lines
geom_vline(xintercept = 0, linetype = "dashed", color = "grey50") +
geom_hline(yintercept = 0, linetype = "dashed", color = "grey50") +
geom_abline(slope = slope_adjusted, intercept = 0, linetype = "solid", color = "grey") +

# Color and shape scales
scale_color_gradient(
  low = "blue", high = "white",
  name = "Average Total Medals (Colored)",
  na.value = "grey65"
) +
scale_shape_manual(
  name = "    ",
  values = c("Other countries (Non-colored)" = 19, " " = 19),
  guide = guide_legend(override.aes = list(
    color = c("white", "grey65"),
    alpha = c(0.5, 0.5)
  ))
) +

# Labels and titles
labs(
  title = "IMO Performance Trends by Country (1995 - 2024)",
  subtitle = "Top 10 most potential countries circled in red and bolded; Top 3 \n (Saudi Arabia, Syri
  x = "Rate of Increase in Total Medals per Year",
  y = "Rate of Increase in Average Individual Scores per Year"
```

```r
  ) +

  # Coordinate limits & theme
  coord_cartesian(xlim = c(-0.1, 0.25), ylim = c(-1.0, 1.5)) +
  theme_minimal(base_size = 11) +
  theme(
    panel.grid.minor = element_blank(),
    legend.position = "right",
    plot.title = element_text(face = "bold", hjust = 0.5),
    plot.subtitle = element_text(face = "italic", hjust = 0.5),
    axis.title = element_text(size = 10),
    legend.title = element_text(size = 9),
    legend.spacing.y = unit(0.0, "cm")
  )

plot_2
```



Discussion

These findings spotlight high-potential countries steadily improving in skill and results. Our Top 10 list highlights nations on a strong upward path, poised for future medal success-led notably by six Asian countries, reflecting the region's rising investment in math education and talent development.

Saudi Arabia stands out for its rapid rise, showing the highest increase in both total medals and average individual scores. Though its average of 2.85 medals isn't the highest, its exceptional score trend and perfect

6-medal haul in IMO 2024-despite only 21 appearances-highlight its impressive progress[6].

Syria and the Philippines follow with solid performances, though at lower rates than Saudi Arabia. Syria stands out with a respectable average medal count from just 16 IMO appearances-remarkable amid the challenges of an ongoing civil war. This reflects the resilience of its students, though the conflict casts uncertainty on its long-term competitive stability[7]. The Philippines exemplifies a nation dedicated to STEM education and academic growth, with a steadily rising IMO performance. Strong emphasis on math education and targeted initiatives have fueled this upward trajectory. While its overall rate is similar to Syria's, the Philippines' higher medal trend and consistent score gains suggest it may soon surpass Syria. Its stable political and educational landscape further strengthens its position as a rising competitor[8].

A key insight emerges from examining each country's position relative to the line $y = x$, which represents a one-to-one correspondence between score growth and medal gains. Countries below this line are increasing their medal counts faster than their individual scores. Interestingly, most of the potential countries (circled in red and bolded) fall below this reference line, suggesting they are particularly effective at converting higher individual scores into more medals. In contrast, countries above the $y = x$ line are improving their individual scores more rapidly than their medal counts. This trend may reflect intensifying global competition-where the bar for medal-winning performances is rising-implying that score improvements alone may no longer guarantee more medals. These countries often represent emerging talent that has yet to fully translate performance gains into podium finishes.

Lastly, examining the other quadrants reveals several non-potential countries. For instance, in the top-left quadrant, countries like Liechtenstein tend to achieve relatively high individual scores but earn fewer medals. This could be due to intense competition near the medal cutoffs-students may perform well but still fall just short of the threshold. Additionally, since medals are awarded based on relative rankings, small improvements in performance might not translate into medals if other countries are also progressing. In the bottom-left quadrant, nations such as Cuba and Bulgaria show declining performance, with negative trends in both individual scores and medal counts. Notably, very few countries fall into the bottom-right quadrant. This suggests that winning more medals while having lower average individual scores is extremely unlikely.

Visualization 3: How do problem difficulty levels help differentiate the capabilities of the top-performing and high-potential countries?

To better understand performance across problem difficulties, the third visualization examines how question difficulty levels (**Easy**, **Medium**, **Hard**) relate to the performance of the top high-performing and top potential countries identified earlier. It compares the distribution of average score percentiles across different difficulty levels for each of these countries.

Problem difficulty is inferred from question numbers, where Questions 1 & 4 are classified as **Easy**, Questions 2 & 5 as **Medium**, and Questions 3 & 6 as **Hard**[2][9]. The plot includes the top 3 high-performing countries (from Visualization 1) and the top 3 potential countries (from Visualization 2). For each country, average score percentiles are grouped by difficulty level.

A faceted density plot shows the distribution of average score percentiles for these six countries from 1995 to 2024, segmented by difficulty level. Dotted lines indicate the median average percentile for each difficulty level: blue for **Easy**, green for **Medium**, and red for **Hard**. This allows for a clear comparison of performance distributions across the three difficulty levels.

Acknowledgement: Actual difficulty levels may vary by year. This categorisation reflects general trends and does not represent official or definitive difficulty classifications.

Data Preparation

Calculate percentiles and categorize each problem as Easy, Medium, or Hard

Since each problem has its own difficulty level, comparing the raw score distributions across problems can be misleading. On average, contestants tend to score higher on easy problems, followed by medium and then hard ones. Therefore, raw scores alone cannot be reliably used to evaluate performance across different problems.

To address this, we use score percentiles in this visualization to assess each country's performance relative to others - independent of problem difficulty. Specifically, we calculate the score percentile for each contestant, for each problem, in each year. The percentile of a score is defined as:

$$\text{Percentile of a score in a problem in a year} = \text{Percentage of people with a score } \leq \text{ that score within the same problem in t}$$

Additionally, we assign a 0th percentile to all scores of zero. This ensures that the scale reflects the lack of progress on a problem - making the interpretation more intuitive: scoring nothing should not place a contestant high in the distribution. It also mitigates percentile inflation in harder problems, where zero is often the most common score. This approach highlights and rewards any progress made, however small, and aligns better with our goal of measuring relative performance.

We then compute the average score percentile for each country in each year, grouped by problem difficulty. These values are visualized to show the distribution of average score percentiles across the three difficulty levels - **Easy**, **Medium**, and **Hard** - over time.

This method allows for a fairer evaluation of each country's performance relative to its peers and enables meaningful comparisons across different difficulty levels to better understand trends.

```r
indi <- individual_results_df %>%
  pivot_longer(p1:p6, names_to = "question", values_to = "score") %>%
  clean_names() %>%
  group_by(year, question) %>%
  mutate(
    percentile = 100 * (rank(score, ties.method = "max") - 1) / (n() - 1)
  ) %>%
  ungroup() %>%
  mutate(
    level = case_when(
      question %in% c("p1", "p4") ~ "Easy",
      question %in% c("p2", "p5") ~ "Medium",
      question %in% c("p3", "p6") ~ "Hard"
    ),
    level = factor(level, levels = c("Easy", "Medium", "Hard"), ordered = TRUE),
    percentile = if_else(score == 0, 0, percentile)
  )
```

Compare the average score for each problem

```r
compared_level <- indi %>%
  group_by(question) %>%
  summarize(mean_score = mean(score))

compared_level
```

```
## # A tibble: 6 x 2
##   question mean_score
##   <chr>         <dbl>
## 1 p1            4.65
## 2 p2            2.45
## 3 p3            0.839
## 4 p4            4.25
## 5 p5            2.21
## 6 p6            0.582
```

As we can see, Problems 1 and 4 have the highest average scores of 4.6/7.0 and 4.3/7.0, respectively. Following that, Problems 2 and 5 show moderate average scores of 2.5/7.0 and 2.2/7.0. Lastly, Problems 3 and 6 have the lowest averages, at 0.8/7.0 and 0.6/7.0, respectively. These patterns suggest that Problems 1 & 4, 2 & 5, and 3 & 6 share similar difficulty levels. This aligns with the difficulty categories we defined earlier, supporting their reasonableness based on the observed trends.

Calculate the average score percentile for each country over time, grouped by difficulty level

```r
percentile_by_country <- indi %>%
  group_by(year, country, level) %>%
  summarize(mean_percentile = mean(percentile), .groups = "drop")
```

Filter data for top 3 high-performing countries

```r
top_3_high_performing <- full_data_set %>%
  slice_head(n = 3) %>%
  select(1)

top_3_high_performing
```

```
## # A tibble: 3 x 1
##   country
##   <chr>
## 1 People's Republic of China
## 2 United States of America
## 3 Russian Federation
```

The top three high-performing countries are **China**, **the USA**, and **Russia**.

```r
df_filtered_china <- percentile_by_country %>%
  filter(country == "People's Republic of China") %>%
  mutate(level = factor(level, levels = c("Hard", "Medium", "Easy"), ordered = TRUE))

df_filtered_usa <- percentile_by_country %>%
  filter(country == "United States of America") %>%
  mutate(level = factor(level, levels = c("Hard", "Medium", "Easy"), ordered = TRUE))

df_filtered_russia <- percentile_by_country %>%
  filter(country == "Russian Federation") %>%
  mutate(level = factor(level, levels = c("Hard", "Medium", "Easy"), ordered = TRUE))
```

Filter data for top 3 potential countries

```
top_3_potential <- top_potential_countries %>%
  slice_head(n = 3) %>%
  select(1)

top_3_potential
```

```
## # A tibble: 3 x 1
##   Country
##   <chr>
## 1 Saudi Arabia
## 2 Syria
## 3 Philippines
```

The top three potential countries are **Saudi Arabia**, **Syria**, and **Philippines**.

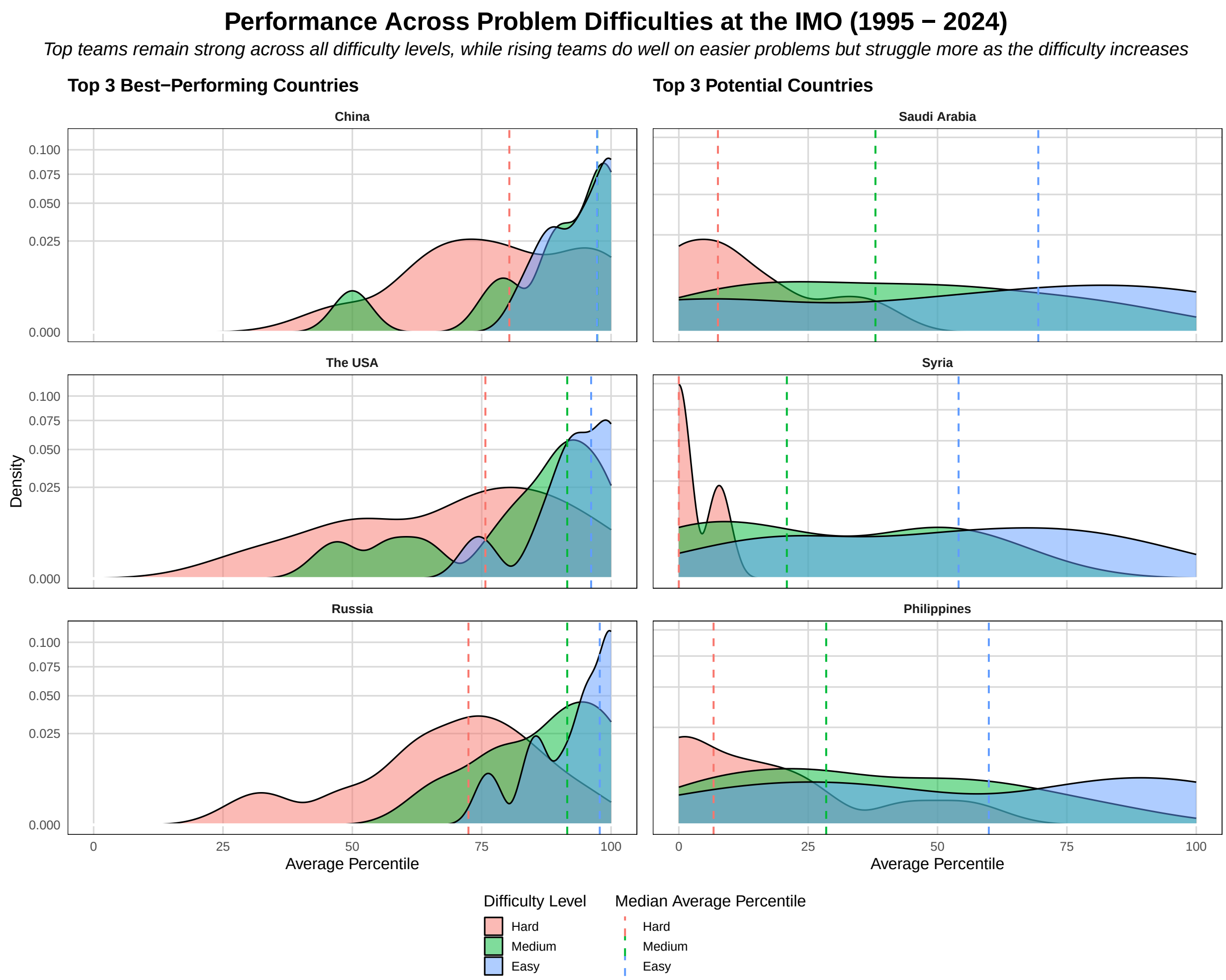
```
df_filtered_saudi_arabia <- percentile_by_country %>%
  filter(country == "Saudi Arabia") %>%
  mutate(level = factor(level, levels = c("Hard", "Medium", "Easy"), ordered = TRUE))

df_filtered_syria <- percentile_by_country %>%
  filter(country == "Syria") %>%
  mutate(level = factor(level, levels = c("Hard", "Medium", "Easy"), ordered = TRUE))

df_filtered_philippines <- percentile_by_country %>%
  filter(country == "Philippines") %>%
  mutate(level = factor(level, levels = c("Hard", "Medium", "Easy"), ordered = TRUE))
```

Plot

```
# Define country groups
high_performance_levels <- c("China", "The USA", "Russia")
potential_levels <- c("Saudi Arabia", "Syria", "Philippines")

# Combine filtered data into one dataframe
df_all <- bind_rows(
  df_filtered_china        %>% mutate(country = "China"),
  df_filtered_usa          %>% mutate(country = "The USA"),
  df_filtered_russia       %>% mutate(country = "Russia"),
  df_filtered_saudi_arabia %>% mutate(country = "Saudi Arabia"),
  df_filtered_syria        %>% mutate(country = "Syria"),
  df_filtered_philippines  %>% mutate(country = "Philippines")
)

# Calculate median percentiles per country and level
medians_all <- df_all %>%
  group_by(country, level) %>%
  summarize(median_percentile = median(mean_percentile), .groups = "drop")
```

```r
# Scales and axis limits
x_limits <- c(0, 100)
y_limits <- c(0, 0.2)

fill_scale <- scale_fill_manual(values = c("Hard" = "#F8766D", "Medium" = "#00BA38", "Easy" = "#619CFF")
color_scale <- scale_color_manual(values = c("Hard" = "#F8766D", "Medium" = "#00BA38", "Easy" = "#619CFF

# Prepare data for high performance countries
high_performance_df <- df_all %>%
  filter(country %in% high_performance_levels) %>%
  mutate(country = factor(country, levels = high_performance_levels))

medians_high <- medians_all %>%
  filter(country %in% high_performance_levels) %>%
  mutate(country = factor(country, levels = high_performance_levels))

# Plot for high performance countries
g_left <- ggplot(high_performance_df, aes(x = mean_percentile, fill = level)) +
  geom_density(alpha = 0.5) +
  geom_hline(yintercept = 0, color = "white", linewidth = 1.2) +
  geom_vline(data = medians_high, aes(xintercept = median_percentile, color = level),
             linetype = "dashed", linewidth = 0.8) +
  facet_wrap(~country, ncol = 1) +
  fill_scale + color_scale +
  scale_x_continuous(limits = x_limits) +
  scale_y_sqrt() +
  labs(
    title = "Top 3 Best-Performing Countries",
    x = "Average Percentile", y = "Density",
    fill = "Difficulty Level", color = "Median Average Percentile"
  ) +
  theme_minimal(base_size = 14) +
  theme(
    strip.text      = element_text(face = "bold"),
    plot.title      = element_text(size = 16, face = "bold"),
    panel.border    = element_rect(color = "black", fill = NA, size = 0.4),
    panel.grid.major = element_line(color = "grey85"),
    panel.grid.minor = element_blank(),
    axis.line       = element_blank()
  )

# Prepare data for potential countries
potential_df <- df_all %>%
  filter(country %in% potential_levels) %>%
  mutate(country = factor(country, levels = potential_levels))

medians_potential <- medians_all %>%
  filter(country %in% potential_levels) %>%
  mutate(country = factor(country, levels = potential_levels))

# Plot for potential countries
g_right <- ggplot(potential_df, aes(x = mean_percentile, fill = level)) +
  geom_density(alpha = 0.5) +
```

```r
  geom_hline(yintercept = 0, color = "white", linewidth = 1.2) +
  geom_vline(data = medians_potential, aes(xintercept = median_percentile, color = level),
             linetype = "dashed", linewidth = 0.8) +
  facet_wrap(~country, ncol = 1) +
  fill_scale + color_scale +
  scale_x_continuous(limits = x_limits) +
  scale_y_sqrt() +
  labs(
    title = "Top 3 Potential Countries",
    x = "Average Percentile", y = NULL,
    fill = "Difficulty Level", color = "Median Average Percentile"
  ) +
  theme_minimal(base_size = 14) +
  theme(
    strip.text      = element_text(face = "bold"),
    plot.title      = element_text(size = 16, face = "bold"),
    axis.text.y     = element_blank(),
    axis.ticks.y    = element_blank(),
    panel.border    = element_rect(color = "black", fill = NA, size = 0.4),
    panel.grid.major = element_line(color = "grey85"),
    panel.grid.minor = element_blank(),
    axis.line       = element_blank()
  )

# Final combined plot
plot_3 <- g_left + g_right +
  plot_layout(ncol = 2, guides = "collect") +
  plot_annotation(
    title = "Performance Across Problem Difficulties at the IMO (1995 - 2024)",
    subtitle = "Top teams remain strong across all difficulty levels, while rising teams do well on eas
    theme = theme(
      plot.title      = element_text(size = 22, face = "bold", hjust = 0.5),
      plot.subtitle   = element_text(size = 16, face = "italic", margin = margin(b = 10), hjust = 0.5),
      legend.position = "bottom",
      legend.box      = "horizontal",
      panel.spacing.x = unit(1.5, "lines")
    )
  )

plot_3
```



Discussion

For the top three performing countries, the median average score percentiles across all difficulty levels are exceptionally high. This is expected, given their consistently outstanding performance at the IMO. Among the three countries, China appears to have the highest average score percentiles across all difficulty levels. This is possibly due to China's extensive training programs and investments in mathematics education for students from a young age[10]. Interestingly, the **Easy** average score percentiles show the highest medians among the three difficulty levels for all three countries, approaching the 100th percentile. This suggests that top-performing countries tend to excel at **Easy** problems, making minimal errors on simpler tasks. Minimizing mistakes on easier questions appears to be a key factor in high-level competition, likely because these problems are more predictable and rely on foundational skills that are thoroughly covered in training programs. Additionally, the slightly lower median average percentiles for **Medium** and **Hard** problems indicate that performance variability increases with problem difficulty, even among the best countries. Therefore, securing strong performance on **Easy** problems provides a reliable baseline score that top countries can consistently achieve, giving them a competitive edge even when their performance on more difficult problems fluctuates.

For the top three potential countries, they show skewed distributions toward the left (lower average percentiles) for all problem levels. Among these three, Saudi Arabia-identified as the most promising-shows the strongest performance across **Easy**, **Medium**, and **Hard** problems. This improvement can be attributed to the Mawhiba (King Abdulaziz Talent Care Program), which identifies and nurtures mathematically gifted

Saudi students through rigorous training programs, including preparation for the International Mathematical Olympiad (IMO). Students participate in specialized camps and workshops led by global experts to develop problem-solving skills. Since the mid-2010s, Mawhiba has sponsored Saudi Arabia's national IMO team, leading to improved international rankings, including bronze and silver medals in recent competitions[11]. Syria and the Philippines show similar distributions: the median average percentile for **Easy** problems is around the 60th percentile, **Medium** around the 25th percentile, and **Hard** around the 0-10th percentile. Notably, for all three of these potential countries, the **Hard** problem median average percentiles remain consistently low-around the 0-10th percentile-indicating persistent struggles with the most difficult problems. This may be due to gaps in exposure, training programs, and experience. In contrast to the top-performing countries, these potential nations also exhibit a wider performance gap across problem difficulties, suggesting less consistency. While top performers tend to maintain balanced scores regardless of difficulty, the potential countries show greater variability, highlighting key areas for growth.

To improve, potential countries should first focus on building strong mathematical foundations to consistently perform well on **Easy** problems. Once they have achieved mastery at this level, they can gradually invest more effort into tackling **Medium** and **Hard** problems, aiming for more balanced performance across the board. Encouragingly, many of these countries are already investing in mathematics education. For instance, Syria - the second most promising potential country identified - has launched intensive training initiatives such as the Syrian Mathematical Olympiad, which aims to provide mathematics training resources[12]. Notably, despite being in conflict for over a decade, Syria remains committed to the successful implementation of several flagship teacher development programmes. These include the School-Based Teacher Development (SBTD I and II) programmes for teachers, and the Leading for the Future (LftF) programme for school principals and deputy principals. These initiatives empower educators to explore new classroom approaches and reflect on their impact[13].

Summary

In summary, we have identified key trends in countries' performance at the IMO over the past 30 years (1995 - 2024). China, the USA, and Russia consistently lead the world, with the most frequent appearances in the top five rankings and the highest average number of gold medals. This dominance is likely due to their substantial investments in mathematics education and large talent pools. Additionally, Saudi Arabia, Syria, and the Philippines have emerged as countries with strong potential to earn more medals and improve their individual scores in the future. Beyond these three, many other nations also exhibit upward momentum, suggesting growing global investment in mathematical excellence. We also observed that the top-performing countries tend to excel across all difficulty levels (Easy, Medium, and Hard), particularly in Easy problems, reflecting not only strong and stable performance but also the ability to avoid simple mistakes. In contrast, emerging countries still display greater variability across difficulty levels, pointing to less consistency in performance.

Teamwork

- Member 1: Nguyen Huy Tung: Code (Visualization 1), Report (Visualization 1)
- Member 2: Bui Phuong Nam: Code (Visualization 1), Report (Visualization 1)
- Member 3: Dang Hoang Anh Khoa: Code (Visualization 2), Report (Visualization 2)
- Member 4: Subhashree Panneer: Code (Data Cleaning, Visualization 2), Report (Visualization 2)
- Member 5: Bui Phuong Linh: Code (Data Cleaning, Visualization 3), Report (Introduction, Visualization 3, Summary)
- Member 6: Pham Ngoc Minh: Code (Visualization 3), Report (Visualization 3)

References

1. R4DS TidyTuesday Project. (2024, September 24). *TidyTuesday: International Mathematical Olympiad Data*. https://github.com/rfordatascience/tidytuesday/blob/main/data/2024/2024-09-24/readme.md
2. Wikipedia. (n.d.). *International Mathematical Olympiad*. https://en.wikipedia.org/wiki/International_Mathematical_Olympiad
3. Kiseleva, J., et al. (2022). Transparency of AI in healthcare as a multilayered system of accountabilities. *PMC*. https://pmc.ncbi.nlm.nih.gov/articles/PMC8988107/
4. International Mathematical Olympiad. (Archived 2022). *Official Website*. https://web.archive.org/web/20220331101006/https://www.imo-official.org/
5. Postulante Universitario. (2016, July). *What Explains Chinese Domination at the Math Olympiad?* https://postulanteuniversitario.blogspot.com/2016/07/what-explains-chinese-domination-at.html
6. Arab News. (2024). *Saudi Arabia's Math Olympiad Achievements*. https://www.arabnews.com/node/2553956/saudi-arabia
7. Middle East Institute. (n.d.). *Syria's Education Crisis: A Sustainable Approach After 11 Years of Conflict*. https://www.mei.edu/publications/syrias-education-crisis-sustainable-approach-after-11-years-conflict
8. Good News Pilipinas. (n.d.). *How Did Filipino Students Outshine 45K Participants in Math Olympiad?* https://www.goodnewspilipinas.com/how-did-filipino-students-outshine-45k-participants-in-math-olympiad-with-an-unprecedented-430-awards/
9. Art of Problem Solving. (n.d.). *International Mathematical Olympiad*. https://artofproblemsolving.com/wiki/index.php/International_Mathematical_Olympiad
10. Wikipedia. (n.d.). *Chinese Mathematics*. https://en.wikipedia.org/wiki/Chinese_mathematics
11. Mawhiba Foundation. (n.d.). *King Abdulaziz and His Companions Foundation for Giftedness and Creativity*. https://www.mawhiba.org/en/
12. Syrian Mathematical Olympiad. (n.d.). *Official Website*. https://sites.google.com/view/syrian-mathematical-olympiad/home
13. ReliefWeb. (n.d.). *Teaching the World's Fastest Calculators: A Mathematical Triumph in Syria.* https://reliefweb.int/report/syrian-arab-republic/teaching-world-s-fastest-calculators-mathematical-triumph-syria-enar