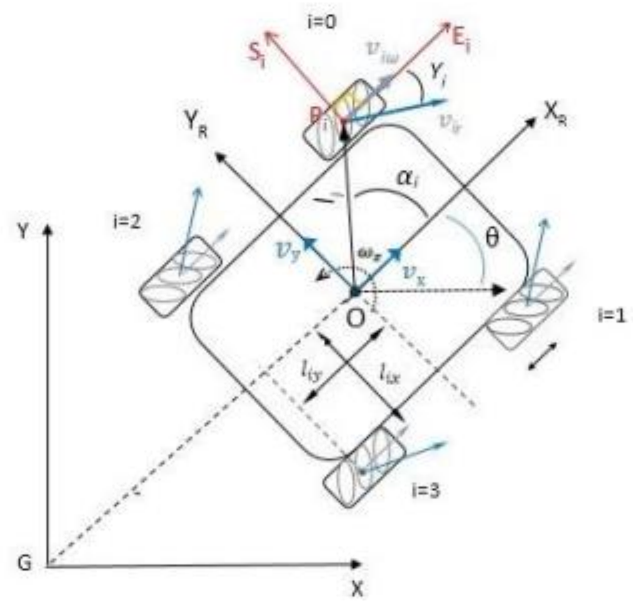
2. KINEMATIC

Figure 2 shows the configuration of a robot with four omnidirectional wheels.



•x, y, θ, robot's position (x, y) and its orientation angle θ (The angle between X and XR);

•X G Y, inertial frame; x,y are the coordinates of the reference point O in the inertial basis;

•$XROYR$, robot's base frame; Cartesian coordinate system associated with the movement of the body center; •$SiPiEi$, coordinate system of ith wheel in the wheel's center point Pi ;

•O, Pi , the inertial basis of the Robot in Robot's frame and Pi = {XPi , YPi } the center of the rotation axis of the wheel i

•OPi , is a vector that indicates the distance between Robot's center and the center of the wheel ith; •lix , liy , lix , half of the distance between front wheels and liy half of the distance between front wheel and the rear wheels.

•li , distance between wheels and the base (center of the robot O);

•ri , denotes the radius of the wheel i (Distance of the wheel's center to the roller center)

•rr , denotes the radius of the rollers on the wheels.

•αi , the angle between OPi and XR ;

•βi , the angle between S i and XR ; •γi , the angle between vir and Ei ;

•ωi [rad/s], wheels angular velocity;

•$vi\omega$ [m/s], i = 0,1,2,3 ∈ R , is the velocity vector corresponding to wheel revolutions

•vir , the velocity of the passive roller in the wheel i;

•.[wsi wEi ωi] T , Generalized velocity of point Pi in the frame $SiPiEi$;

•[vSi vEi ωi] T , Generalized velocity of point Pi in the frame $XROYR$; •vx , vy [m/s] - Robot linear velocity; •ωz [rad/s] - Robot angular velocity;

forward kinematic:

$$\begin{bmatrix} v_X \\ v_Y \\ \omega_z \end{bmatrix} = T^{+} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix}$$

$$T^{+} = \frac{r}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ -\frac{1}{(l_x+l_y)} & \frac{1}{(l_x+l_y)} & -\frac{1}{(l_x+l_y)} & \frac{1}{(l_x+l_y)} \end{bmatrix};$$

the system inverse kinematic:

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \frac{-1}{r} \begin{bmatrix} \frac{\cos(\beta_1-\gamma_1)}{\sin\gamma_1} & \frac{\sin(\beta_1-\gamma_1)}{\sin\gamma_1} & \frac{l_1 \sin(\beta_1-\gamma_1-\alpha_1)}{\sin\gamma_1} \\ \frac{\cos(\beta_2-\gamma_2)}{\sin\gamma_2} & \frac{\sin(\beta_2-\gamma_2)}{\sin\gamma_2} & \frac{l_2 \sin(\beta_2-\gamma_2-\alpha_2)}{\sin\gamma_2} \\ \frac{\cos(\beta_3-\gamma_3)}{\sin\gamma_3} & \frac{\sin(\beta_3-\gamma_3)}{\sin\gamma_3} & \frac{l_3 \sin(\beta_3-\gamma_3-\alpha_3)}{\sin\gamma_3} \\ \frac{\cos(\beta_4-\gamma_4)}{\sin\gamma_4} & \frac{\sin(\beta_4-\gamma_4)}{\sin\gamma_4} & \frac{l_4 \sin(\beta_4-\gamma_4-\alpha_4)}{\sin\gamma_4} \end{bmatrix} \begin{bmatrix} v_X \\ v_Y \\ \omega_z \end{bmatrix}.$$

Therefore:

$$\begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix} = \frac{r}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ -\frac{1}{(l_x+l_y)} & \frac{1}{(l_x+l_y)} & -\frac{1}{(l_x+l_y)} & \frac{1}{(l_x+l_y)} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix}$$

Longitudinal Velocity:

v_x(t)=(ω1+ ω2+ ω3+ ω4) . $\frac{r}{4}$

Transversal Velocity:

v_y(t)=(-ω1+ ω2+ ω3- ω4) . $\frac{r}{4}$

Angular velocity:

ω_z(t)=(-ω1+ ω2-ω3+ω4) . $\frac{r}{4(l_x+l_y)}$

CONCLUSION

A mobile platform with four omnidirectional wheels was introduced in this paper. The results were systematically obtained by using kinematic equations that were similar to those achieved from the experimental results. The results show that the platform performs full omnidirectional motions. This shows that by using Mecanum wheels in the platform the robot can achieve any direction between $\mathbf{0}$ ° to $\mathbf{360}$ °without changing its orientation.