



## Data Structures

### Chapter 5 Tree

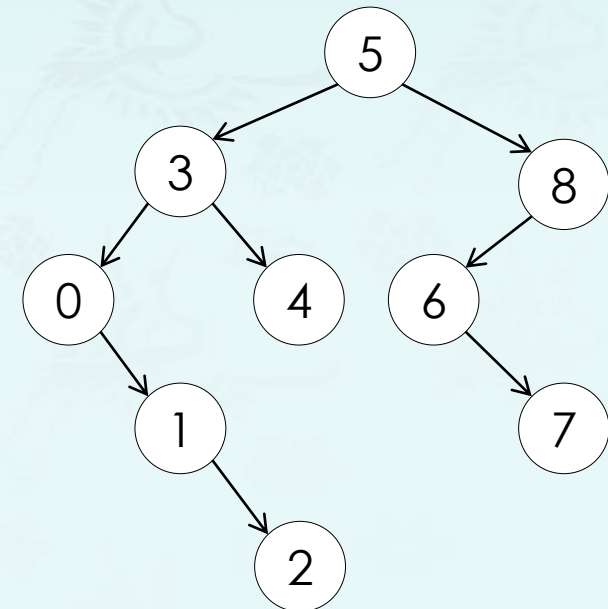
1. Introduction
2. Binary Tree
- 3. Binary Search Tree**
  - Introduction
  - Operations
  - **Demo & Coding**
4. Balancing Tree

## Minimum, Maximum:

- `Minimum()` and `maximum()` returns the node with min or max key.
  - Note that the entire tree does not need to be searched.
  - The minimum key is always located at the left most node, the maximum at the right most node.
  - Complexity of algorithm to find the maximum or minimum will be  $O(\log N)$  in almost balanced binary tree. If tree is skewed, then we have worst case complexity of  $O(N)$ .

```
tree minimum(tree node) { // returns left-most node key
    if (node->left == nullptr) return node;
    return minimum(node->left);
}
```

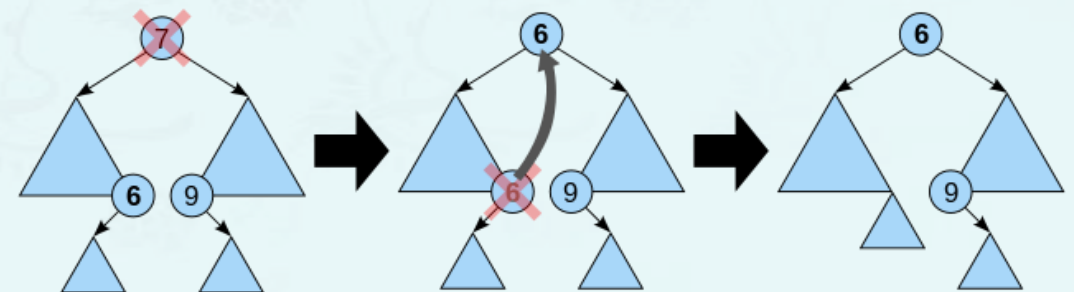
```
tree maximum(tree node) { // returns right-most node key
    if (node->right == nullptr) return node;
    return maximum(node->right);
}
```



## pred(), succ() – predecessor, successor:

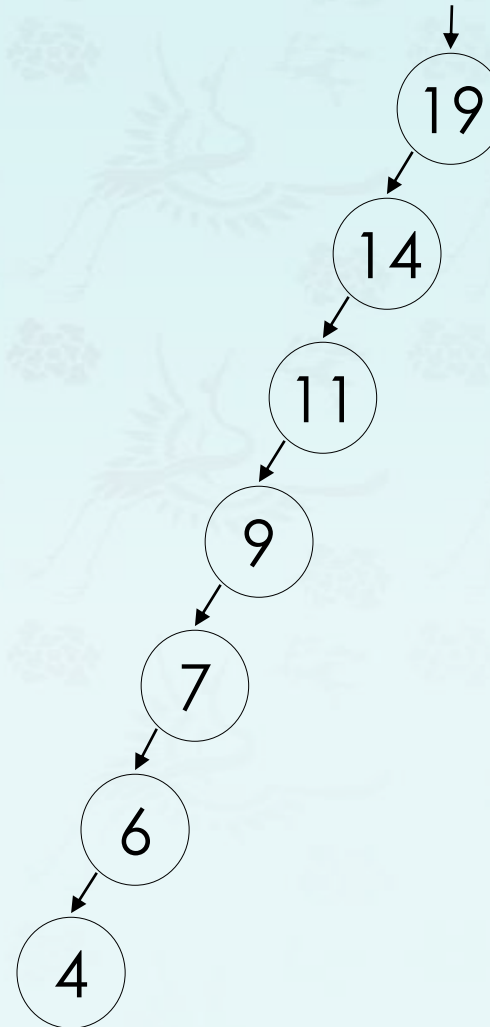
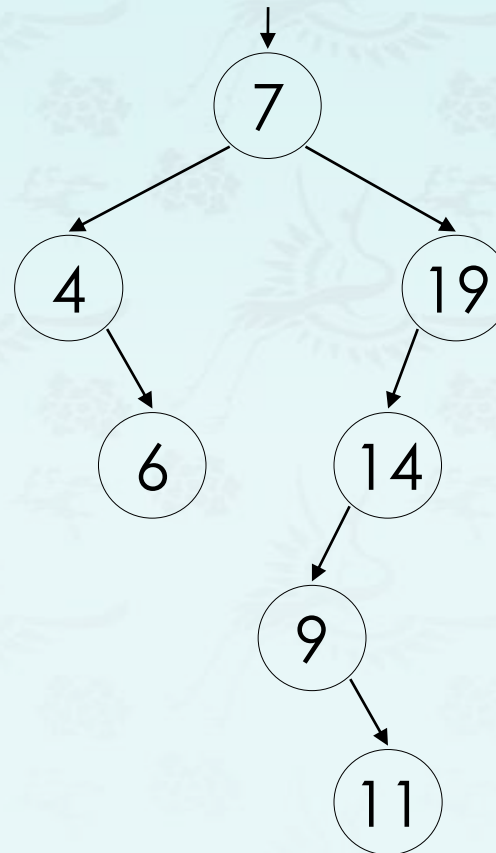
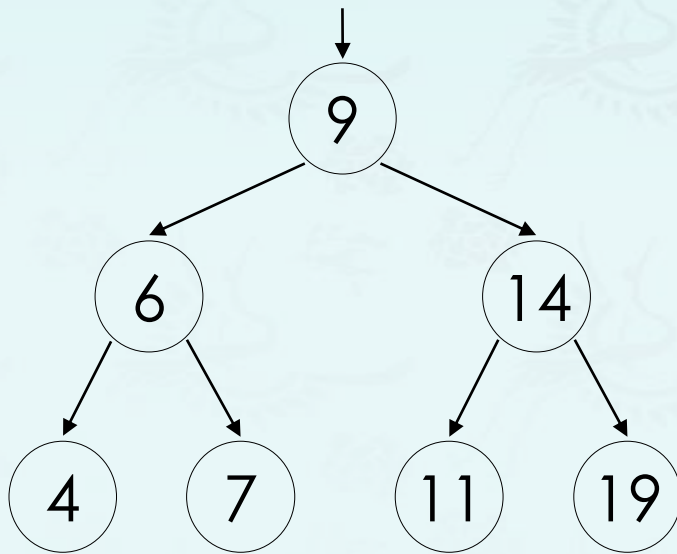
- Successor
  - If the given node has a right subtree then by the BST property the next larger key must be in the right subtree. Since all keys in a right subtree are larger than the key of the given node, the successor must be the smallest of all those keys in the right subtree.
- Predecessor
  - If the given node has a left subtree then by the BST property the next smaller key must be in the left subtree. Since all keys in a left subtree are smaller than the key of the given node, the predecessor must be the largest of all those keys in the left subtree.
- Complexity of algorithm
  - $O(\log N)$  in almost balanced binary tree. If tree is skewed, then we have worst case complexity of  $O(N)$ .

```
tree successor(tree node) {  
    if (node != nullptr && node->right != nullptr)  
        return minimum(node->right);  
    return nullptr;  
}
```



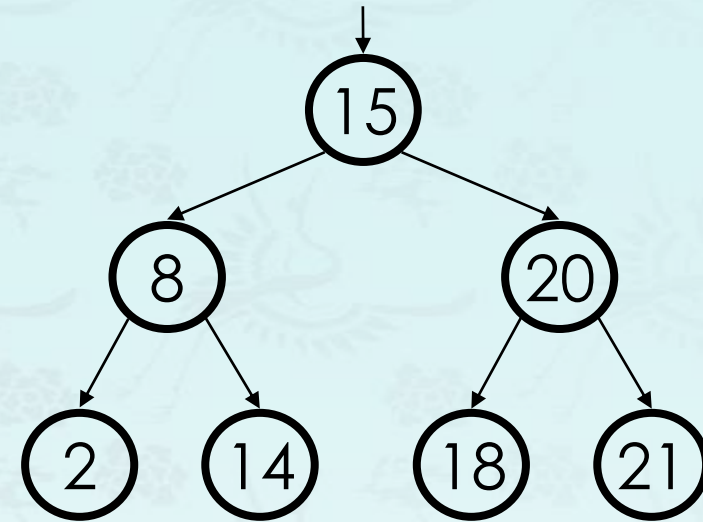
# Binary Search Trees: Observations

- What do you see in the following BSTs?
  - A **balanced** tree of  $N$  nodes has a height of  $\sim \log_2 N$ .
  - A very **unbalanced** tree can have a height close to  $N$ .



# Binary Search Trees: Observations

- For binary tree of height  $h$ :
  - max # of leaves:  $2^h$
  - max # of nodes:  $2^{h+1} - 1$
  - min # of leaves: 1
  - min # of nodes:  $h + 1$
- The shallower the BST the better.
  - Average case height is  $O(\log N)$
  - Worst case height is  $O(N)$
  - Simple cases such as adding  $(1, 2, 3, \dots, N)$ , or the opposite order, lead to the worst case scenario: height  $O(N)$ .



# Binary Search Trees: Observations

- Q: If you have a sorted sequence, and we want to design a data structure for it. Which one are you going to use an array or BST? and why?

Time Complexity	
BST	$O(h)$
Array	$O(\log n)$

- Q: When searching, we're traversing a path (since we're always moving to one of the children); since the length of the longest path is the height  $h$  of the binary search tree, then finding an element takes  $O(h)$ .
  - Since  $h = \log n$  (where  $n$  is the number of elements), then it's good! – right?
  - No, of course, it is wrong! Why?

A: The nodes could be arranged in linear sequence in BST, so the *height*  $h$  could be  $n$ . In worst case, it is  $O(n)$  instead of  $O(h)$ .



## Operations: growN() & trimN() for testing

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- It performs a user specified number of insertion(or grow) or deletion(or trim) of nodes in the tree.
- The function **growN()** inserts a user specified number N of nodes in the tree.
  - If it is an empty tree, the value of keys to add ranges from 0 to N-1.
  - If there are some existing nodes in the tree, the value of keys to add ranges from  $\text{max} + 1$  to  $\text{max} + 1 + N$ , where max is the maximum value of keys in the tree.
- This function growN() is provided for your reference^^.

## Operations: growN() & trimN() for testing

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- The function **trimN()** deletes N number of nodes in the tree.
  - The nodes to trim are **randomly** selected from the tree.
  - If N is less than the tree size (which is not N), you just trim N nodes.
  - If the N is larger than the tree size, set it to the tree size.
  - At any case, you should trim all nodes one by one, but randomly.
  - With an AVL tree, reconstruct it **after** trimming N nodes from BST.

Step 1: Get a list (vector) of all keys from the tree first.

Get the size of the tree using the size().

Use assert to check two sizes;

Step 2: Shuffle the vector with keys. – shuffle()

Step 3: Invoke trim() N times with a key from the vector in sequence.

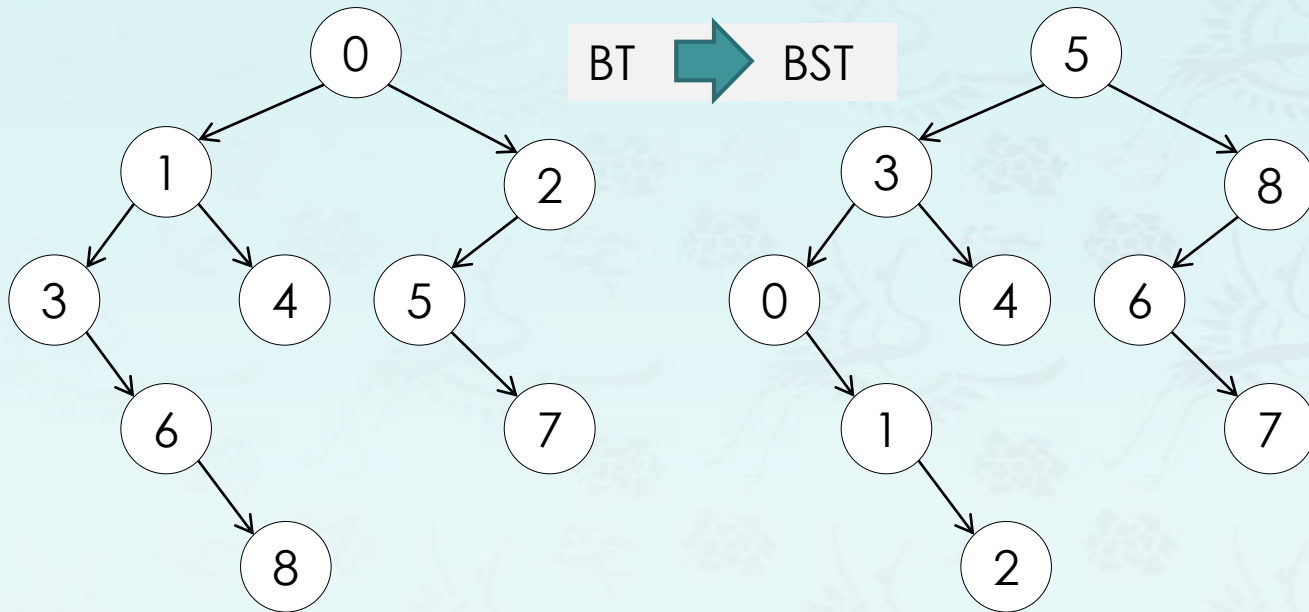
Inside a for loop, trim() may return a new root of the tree.

Step 4: The function is called with AVLtree = true, then reconstruct the tree.



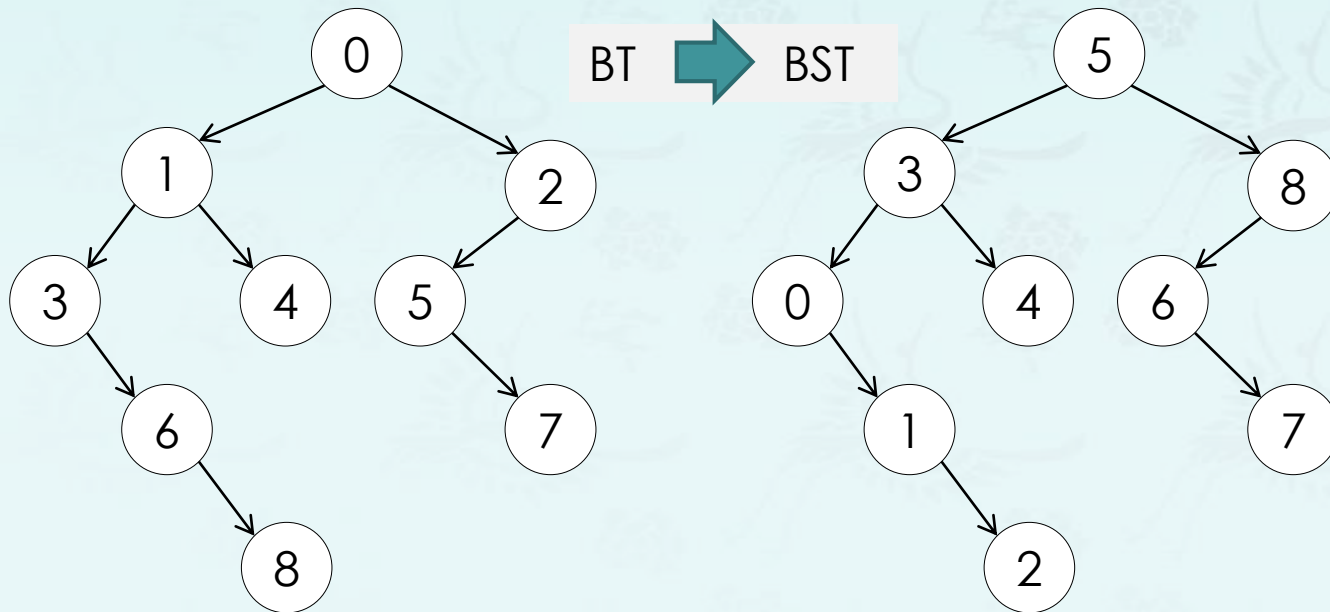
## Convert BT to BST in-place

- Convert a binary tree to a binary search tree while keeping its tree structure as it is.
- For example:



# Convert BT to BST in-place

- Convert a binary tree to a binary search tree while keeping its tree structure as it is.
- Algorithm:
  - Step 1** – store keys of a binary tree into a container like **vector** or **set**. (**Do not use an array.**)
  - Step 2** – sort the keys in vector. Skip this step if set is used since it is already sorted.
  - Step 3** – Now, do the **inorder** traversal of the tree and copy back the elements of the container into the nodes of the tree one by one.



- (1) Retrieve the keys from BT:  
3 6 8 1 4 0 5 7 2 // if in-order used
- (2) Sort keys in the container:  
0 1 2 3 4 5 6 7 8
- (3) Replace keys in BT with sorted keys while in-order traversal.

```
void inorder(tree root) {
    if (root == nullptr) return;

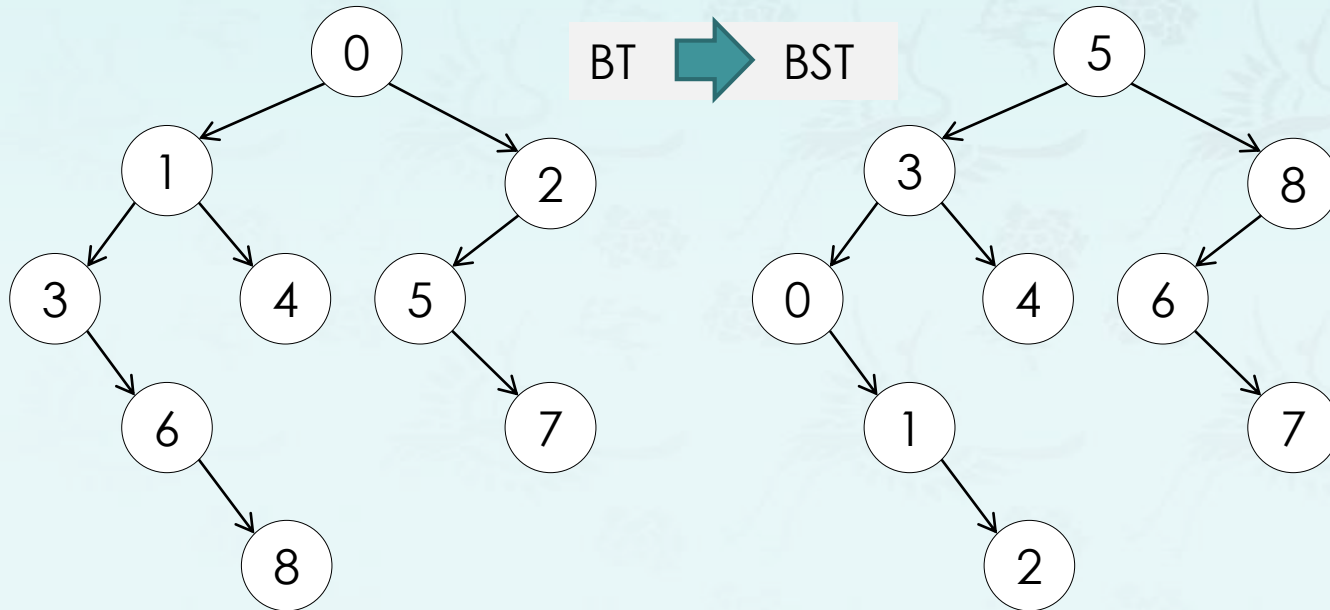
    inorder(root->left);
    cout << root->key;
    inorder(root->right);
}
```

**L**  
**V**  
**R**

# Convert BT to BST in-place

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3 6 8 1 4 0 5 7 2 // if in-order used  
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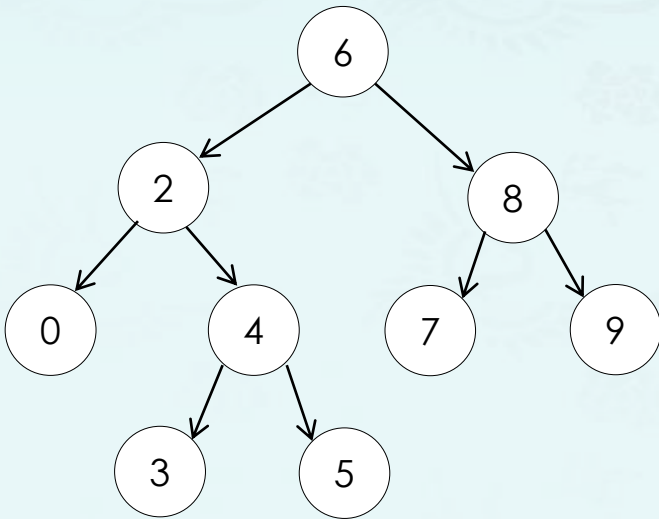
```
void get_keys(tree root, set<int> &keys) {
    if (root == nullptr) return;
    keys.insert(root->key);
    get_keys(root->left, keys);
    get_keys(root->right, keys);
}
```

```
void put_keys(tree root,
               set<int>::iterator& it) {
}
```

```
void BTtoBST(tree root) {
    set<int> keys;
    get_keys(root, keys);
    assert(
    set<int>::iterator it = keys.begin();
    put_keys(root, it);
}
```

# Operations: LCA in BST

- Find the lowest common ancestor(LCA) of two given nodes, given in BST.
  - The LCA is defined between two nodes p and q as the lowest node in T that has both p and q as descendants (where we allow a node to be a descendant of itself)."
  - In BST, all of the nodes' values will be unique.  
Two nodes given, p and q, are different and both values will exist in the BST.



For example:

2, 8 -> 6

2, 5 -> 2

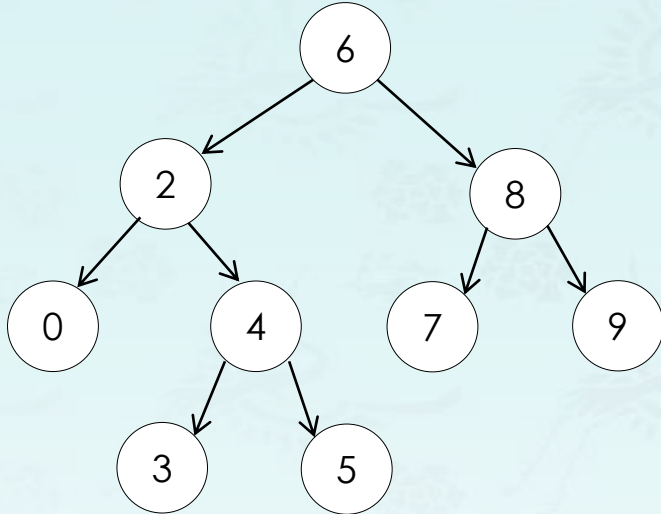
9, 5 -> 6

8, 7 -> 8

0, 5 -> 2

## Operations: LCA(**iteration**) in BST

- **Intuition (Iteration):** Traverse down the tree iteratively to **find the split point**. The point from where p and q won't be part of the same subtree or when one is the parent of the other.



For example:

2, 5 -> 2

9, 7 -> 8

0, 4 -> 2

0, 5 -> 2

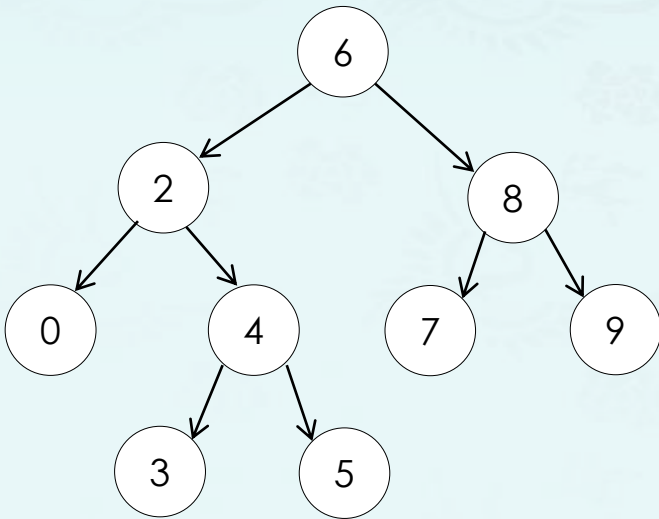
2, 7 -> 6

```
int LCAiteration(tree node, tree p, tree q) {  
    while (node != nullptr) {  
        if (both p & q > node)  
            node move to right to search  
        else if (both p & q < node)  
            node moves to left to search  
        else  
            return node->key    // found  
    }  
    return 0;    // not found  
} // iteration solution
```

# Operations: LCA(**recursion**) in BST

## ■ Algorithm: (Recursion)

1. Start traversing the tree from the root node.
2. If both the nodes p and q are in the right subtree, then continue the search with right subtree starting step 1.
3. If both the nodes p and q are in the left subtree, then continue the search with left subtree starting step 1.
4. If both step 2 and step 3 are **not true**, this means we have **found** the node which is common to node p's and q's subtrees. Hence we return this common node as the LCA.



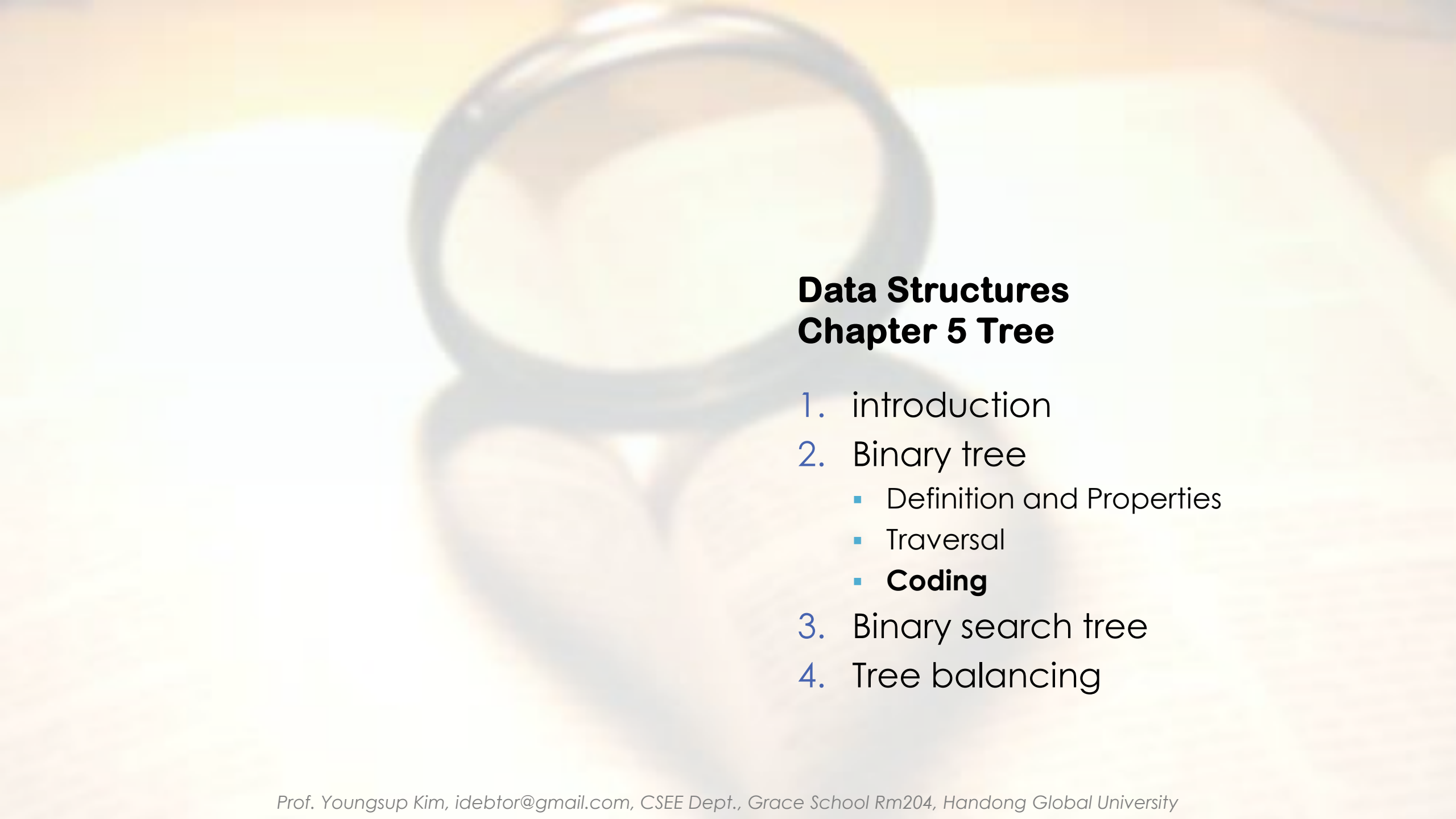
```
tree LCA(tree root, tree p, tree q) {  
  
    // your code here  
  
} // recursive solution
```



# Operations: LCA in BST

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- Recursion Algorithm
  - Time Complexity:  $O(N)$ , where  $N$  is the number of nodes in the BST. In the worst case we might be visiting all the nodes of the BST.
  - Space Complexity:  $O(N)$ . This is because the maximum amount of space utilized by the recursion stack would be  $N$  since the height of a skewed BST could be  $N$ .
- Iteration Algorithm
  - Time Complexity :  $O(N)$ , where  $N$  is the number of nodes in the BST. In the worst case we might be visiting all the nodes of the BST.
  - Space Complexity :  $O(1)$ .



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