Dynamic positioning capability assessment for ship design purposes

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Abstract. The article focuses on solving a problem of optimal thrust distribution over the actuators in a ship Dynamic Positioning, according to DNV-ST-0111 standard, *Level 1*. The classic Quadratic Programming approach is combined with the numerical solusion used to handle the propeller with the rudder constraints in the optimization task and the influence between thrusters and skeg. It is presented as an efficient method of minimizing the power consumption. The resulting tool for performing a Dynamic Positioning capability assessment allows for fast calculations to qualitatively compare different designs. The study has proven that the Quadratic Programming based method gives less optimistic solution in comparison with DNV tool and can be safely applied at an early design stage. Further validation of the tool with the time-domain simulations would contribute to increasing confidence in its application to the daily routine calculations.

Keywords: Optimal Thrust Allocation· Dynamic Positioning· Quadratic Programming

1 Introduction

Dynamic positioning (DP) is one of the ship's operational states in which its relative or absolute position and heading are automatically maintained at desired set-points, by using only its own, active thrusters without any mooring lines or other equipment. A DP control system (DPCS) that handles this task can be classified as an advanced, model-based and over-actuated closed-loop stabilizing with the aim of achieving high disturbance rejection capabilities.

A DP capability defines a ship ability to station keeping under given environmental conditions. The assessment of a vessel's capability to keep the position is critical for planning and executing safe and reliable dynamic positioning operations (DP). A leading classification society, Det Norske Veritas (DNV) has developed a method for DP station-keeping capability assessments provided in DNV-ST-0111 standard [5]. The results are documented by DP capability numbers (corresponding to the Beaufort scale) and capability plots (in polar form). The standard defines three different DP capability levels, each requiring a specific assessment method. The *Level 1* considered in this paper is specified for mono-hull ships. The calculation method at this level shall be based on a static balance of environmental and the vessel's actuator forces, assuming the same specified environmental data for all vessels. The static balance shall determine the thrust distribution among thrusters (both magnitude and direction) called thrust allocation. This task, however, has no unique solution since the DP-capable ships are over-actuated. Considering thrust x and y components as more than a number of equilibrium equations the task of DP capability assessment evolves to an optimal thrust allocation problem.

This paper focuses on an optimal thrust allocation problem based on its approximation to quadratic optimisation problem. The objective of this task is to minimize the ship's propulsion power consumption in DP operations. The effect of thruster failure can also be analyzed by using DP capability plot. The proposed approach meets DNV standards and can be easily adapted to mono - shaped ships for DP capability assesement. The results of DP capability were compared with existing tools offered by DNV.

1.1 Related works

A general overview of marine control systems and optimal thrust allocation problem is given in [6,8]. The applicable methods division into two categories may be observed in the literature, based on the task formulation and the used method. The first group of methods consists of gradient-based approaches. Among this group the most commonly used are the ones based on the Quadratic Programming (QP) or Lagrangian multipliers [4]. Both approaches assume that the objective function and constraints are smooth and they guarantee reaching a global minimum. In turn, the second group of methods consists of the so called non-gradient (derivative-free) methods. This group is represented by the so-called meta-heuristic algorithms, such as particle swarm optimization, genetic or evolutionary algorithms [11] or direct-search algorithms [2]. In both cases, if certain requirements are met, algorithms only tend to converge global optimum. Also, both groups of algorithms converge relatively slowly.

In addition to the mentioned above, a different allocation strategies without optimization are also presented. Those include deterministic and pseudo-inverse based matrix methods [14,18]. The model predictive control allocation [16] and adaptive control allocation [15,17] are quite complex and time-consuming, which allow to take into account the actuator dynamic and uncertainties in the calculations.

Literature that covers the subject of *optimal thrust allocation* using QP approximation, [4,9,7,10]. Other articles related to QP application in *optimal thrust allocation* are [12,13]. Related convex optimization problem is discussed in [3].

1.2 Motivation and contribution

DNV standard is a widely used method for DP capability assessment at an early stage of ship design, yet there is no proposition in the literature how to solve the thrust allocation problem with the conditions imposed by it. The method gives the procedure to calculate environmental and thruster forces as well as thruster losses but does not proposes a way of solving the over-actuated system problem. A typical ship design office uses a purchased input-output software with no awareness on the optimization applied. This paper presents a step by step Quadratic Programming application in DP capability assessment with DNV standard method which can be directly applied by a skilled engineer in the ship concept design process.

The quadratic programming algorithms can guarantee that an optimal solution will be found in the finite time (which can be very important for online applications) or that a solution does not exist, whereas no such guarantees can be given for nonlinear optimization techniques. The problem is simple to define in quadratic form by approximation techniques and relatively easy to use. The presented optimal thrust allocation method is build on foundation of the QP approach described in [4]. The azimuth and tunnel thruster constraints as well as objective function are adopted here accordingly. For propeller with the rudder a new approach was proposed to fulfill the DNV standard description.

The presented methodology allows for a complete (all kinds of thrusters, including propeller with the rudder) and fast in-house DP capability assessment for ship concept design.

1.3 Structure of the paper

The remaining section of this research work is organised in the following manner. In Section 2, the DP capability assessment problem has been formulated. Section 3, provides information on the implied method used to propose a solution to the formulated problem. The results obtained with comparison to available external tools are discussed in Section 4. Section 5, concludes the paper.

2 Problem definition

DP capability analysis can assist in determining the maximum environmental forces and moment that DP system can counteract for the given headings (from 0° to 360°). The environmental conditions are statically balanced by thrust forces and moment provided by the propulsion system. Considering a planar movement of the ship, the balance equations yield:

$$\begin{aligned} \sum_{i=1}^{N} T_{\mathrm{x}\,i} &= F_{\mathrm{env\,x}}, \\ \sum_{i=1}^{N} T_{\mathrm{y}\,i} &= F_{\mathrm{env\,y}}, \\ \sum_{i=1}^{N} (-T_{\mathrm{x}\,i}\, y_i + T_{\mathrm{y}\,i}\, x_i) &= M_{\mathrm{env\,z}}, \end{aligned} \tag{1}$$

where $F_{\mathrm{env\,x}}$, $F_{\mathrm{env\,y}}$ and $M_{\mathrm{env\,z}}$ denote x and y direction net force components and torques resulting from the environmental influences (wind, wave, current), expressed in [N] and [Nm], respectively, also considered as disturbance inputs; $T_{\mathrm{x}\,i}$ and $T_{\mathrm{y}\,i}$ are forces generated by the ith thruster in [N], considered as control inputs; x_i and y_i define position of the ith thruster in the ship centered coordinate frame [5]; N denotes the total number of thrusters.

Since DPCS belongs to a class of an over-actuated system, the equation (1) has no unique solution in terms of thruster generated forces $T_{\mathrm{x}\,i}$ and $T_{\mathrm{y}\,i}$, $\forall i$. Therefore, instead of solving (1), the thrust is to be allocated while optimizing the total power consumption, taking into account balance equation as a one of thrust constraints. To that goal, as indicated in the introductory part of the paper, the method applied is to utilize a constrained QP formulation of the problem under consideration.

For the mentioned purpose, the following set of assumptions is considered:

Assumption 1 *The DP capability is achieved at given operational conditions whenever the balance* (1) *holds.*

The Assumption 1 is a consequence of physical laws and does not introduce artificial limitations to the problem.

Assumption 2 *The environmental forces in* (1) *are assumed to be scenario driven.*

The static balance equation is used to calculate needed thrust for one scenario of waves, wind and current (e.g, by using Beaufort scale) to hold the ship in position. DP capability assessment needs to be studied various scenarios and environmental conditions. The results are given in the form of DP capability polar plot which presents the maximum weather condition that the ship can withstand for each environmental angle assuming the worst case scenario where the wind, current and waves are collinear.

Assumption 3 *The relation between power and thrust can be expressed as a quadratic function with a satisfactory accuracy.*

From [6] it is found that the physical relationship between produced thrust T and consumed power P can be given by non-linear relation $P = |T|^{3/2}$ but it can be effectively approximated by a function of at most second degree.

Assumption 4 *The thrust constraints imposed by its physical limits, the propulsion configuration and type can be approximated by a set of convex polygons.*

Different thruster types will have different thrust region shapes. In the general case, the constraints on the thrust form non-convex regions. By the virtue of Assumptions 3 and 4, the problem is reduced to convex. However, this simplification can lead to precision loss in the power assessment.

3 Methodology

In the following subsection, the QP approach to thrust allocation has been defined, including propeller thrust with rudder angle and thrust loss of spoiled zone of azimuth thruster.

3.1 Decision variables

Considering the alluded issue problem and (1), a vector of decision variables is defined in the following lines.

$$\boldsymbol{u} \stackrel{\text{def}}{=} \left[T_{\mathrm{x}\,1}, T_{\mathrm{y}\,1}, T_{\mathrm{x}\,2}, T_{\mathrm{y}\,2}, \ldots, T_{\mathrm{x}\,N}, T_{\mathrm{y}\,N}\right]^T . \tag{2}$$

The choice of thruster forces is not arbitrary but is subject to constraints resulting from propulsion type and respective mount points in the considered coordinate frame. Hence, the following constraints need to be taken into account.

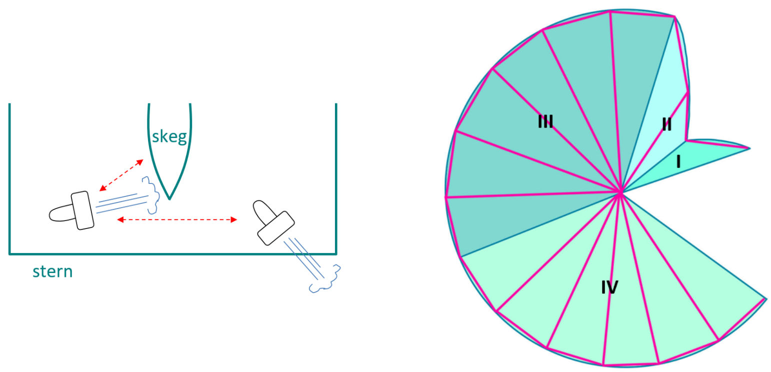
3.2 Constraints

In the described thruster allocation task, it can be observed that the direct formal description of the problem leads to nonlinear constraints. Invoking Assumption 4, by utilizing linearization mechanisms, with respect to the constraints resulting from the use of azimuth thrusters and propellers with rudder, allows one to preserve the linear nature of the problem constraints, as it has been shown in the following lines.

Azimuth thruster The constraints of the azimuth thruster arise from operational restrictions. First, flushing another operating thruster is forbidden (Fig. 1a). Second, the method and location of mounting face limitations on the generated thrust in some directions (Fig. 1a). The first case, introduces the so-called forbidden zones, where the thruster capacity is assumed to be equal to zero (Fig. 1b). The second one, introduces spoiled zones where the capacity of the thruster is reduced to a fraction of its maximum value (Fig. 1b).

An example of handling constraints for azimuth thruster is shown in Fig. 1b. In general, the approach is to divide the thrust constraint region into convex sets, in this case labeled I, II, III, IV, and then into polygons that can be defined by linear equations. From these considerations, the zone boundary and saturation inequality constraints, arise.

$T_{\mathrm{x}\,i}$, $T_{\mathrm{y}\,i}$ - thrust vectors on x and y direction
$\beta_{\mathrm{start}\,n}, \beta_{\mathrm{end}\,n}$ - angles at which the nth zone starts and ends



(a) Thruster flushing skeg and another working thruster

(b) Thruster capacity (background) and convex safe zones

Fig. 1: Forbidden, spoiled zones and safe zones of azimuth thruster

Zones boundary inequality constraint Following Assumption 4, boundary conditions are formulated as linear inequalities [4]:

$$\begin{aligned} T_{\mathrm{x}\,i}\,\sin(\beta_{\mathrm{start}\,n}) - T_{\mathrm{y}\,i}\,\cos(\beta_{\mathrm{start}\,n}) &\le 0, \\ -T_{\mathrm{x}\,i}\,\sin(\beta_{\mathrm{end}\,n}) + T_{\mathrm{y}\,i}\,\cos(\beta_{\mathrm{end}\,n}) &\le 0, \end{aligned} \tag{3}$$

where $\beta_{\mathrm{start}\,n}$ and $\beta_{\mathrm{end}\,n}$ denote angles at which the nth zone starts and ends, respectively.

Saturation inequality constraints For convenience, two mechanisms of description are provided. First, for the circle-shaped zones (e.g, Fig. 1b, zones III, IV) [4]:

$$T_{\mathrm{x}\,i}\,\cos(\varphi_j) + T_{\mathrm{y}\,i}\,\sin(\varphi_j) \le r_i, \tag{4}$$

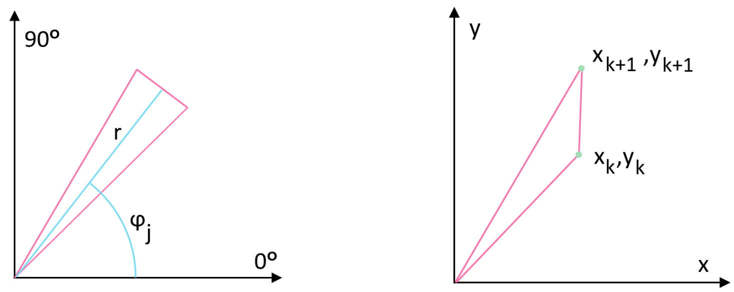
where r_i is the maximum effective thrust of the ith thruster and φ_j refers to each polygon middle angle. An illustration of the resulting polygon is provided in Fig. 2a. Second, for the spoiled zones (e.g, Fig. 1b, zones I, II) [4]:

$$T_{\mathrm{x}\,i}\,(y_{k+1} - y_k) + T_{\mathrm{y}\,i}\,(x_{k+1} - x_k) \le x_k\,y_{k+1} - x_{k+1}\,y_k, \tag{5}$$

where: x_k and y_k are coordinates of the first point while x_{k+1}, y_{k+1} are coordinates of the subsequent point. An illustration is provided in Fig. 2b.

In practice, the number of polygons, which the zone is divided to, determines the accuracy of the method and shall be chosen individually, however, an error of 1% is recommended [4].

Propeller with rudder In the case of a propeller with a rudder, specific coefficients for x and y thrust forces depending on the rudder angle are provided



(a) Saturation - circle shaped zone

(b) Saturation - spoiled zone

Fig. 2: Inequality constraints - azimuth thruster

in [5]. In the presented approach, rudder angle (α) is not a decision variable. Therefore, only the relation between x and y direction thrust components, is considered to complement the constraints set.

Remark 1. The consequence of the approach is that α remains a free variable to be handled outside the optimisation task.

Propeller equality constraints The forces produced by the sum of the propeller and the rudder are satisfied by equality constraints [5]:

$$T_{\mathrm{x}\,i} - \frac{a_{\mathrm{x}\,i}(\alpha)}{a_{\mathrm{y}\,i}(\alpha)} T_{\mathrm{y}\,i} = 0, \tag{6}$$

where $a_{x\,i}(\alpha)$ and $a_{y\,i}(\alpha)$ are coefficients dependant on the rudder angle α.

Propeller inequality constraints The propeller and rudder thrust inequality constraints yield [5]:

$$\begin{aligned} T_{\mathrm{x}\,i} &\leq T_{\max i}\, a_{\mathrm{x}\,i}(\alpha), \\ T_{\mathrm{y}\,i} &\leq T_{\max i}\, a_{\mathrm{y}\,i}(\alpha). \end{aligned} \tag{7}$$

3.3 Objective function

By the virtue of Assumption 3, the allocation of thrust is assessed by considering an objective function (P_{total}) which is given as a quadratic form and represents relation between thruster generated forces (control inputs) and total power consumption, which yields:

$$P_{\mathrm{total}}(\boldsymbol{u}) \overset{\mathrm{def}}{=} \boldsymbol{u}^T \boldsymbol{W} \boldsymbol{u}, \tag{8}$$

where $\boldsymbol{W} \overset{\mathrm{def}}{=} \mathrm{diag}\,(w_1,\, w_1,\, w_2,\, w_2,\, \ldots,\, w_N,\, w_N)$ is a diagonal matrix of weight coefficients w_i, $\forall i$, corresponding to each mounted thruster. Weight coefficients are determined according to [5] with the exception that the maximum thrust loss in case of spoiled zones is also taken into account.

Remark 2. The method used for weight selection is a type of worst-case approach, hence it introduces certain level of conservatism.

3.4 Optimization task

Taking into account the contribution of the Subsections 3.1 – 3.3, the optimal thrust allocation task yields:

$$\begin{aligned} \min_{\boldsymbol{u}} \quad & P_{\text{total}}(\boldsymbol{u}) \\ s.t. \quad & \boldsymbol{A}(\alpha)\,\boldsymbol{u} = \boldsymbol{b}\,, \\ & \boldsymbol{G}\,\boldsymbol{u} \leq \boldsymbol{h}(\alpha) \end{aligned} \tag{9}$$

where: $\boldsymbol{A}(\alpha)$ and $\boldsymbol{G}$ are the equality and inequality constraints matrices, respectively; $\boldsymbol{b}$ denotes a vector encompassing forces and torques; $\boldsymbol{h}(\alpha)$ represents the thrust saturation and limiting operation angle. The internal structure of vectors and matrices results directly from (3) – (7).

3.5 DP capability assessment

Finally, the ship's DP capability is assessed in the following manner. Considering Remark 1, the problem (9) is solved for discrete values of angle of environmental impact, from 0° to 360°, and rudder angle, which varies from ship to ship, to cover whole considered domain of interest. Consequently, a population of results is obtained for each considered environmental angle. In each case, the final result is obtained by applying the minimum operator. The results of DP capability are typically presented in a graphic form, as a polar plot, where each circle in the plot represents a DP number corresponding to specific weather condition [5].

4 Results

The case study is being used to illustrate the use of the proposed approach in assessing the DP capability of a rescue ship which total length can reach 96 m. The ship is equipped with five thrusters layout which has been illustrated in Fig. 3a. The crucial ship's parameters have been included in Table 1. The DP capability has been assessed using the approach described in Section 3 by the means of a dedicated tool developed using Python programming language. To improve legibility, first in Subsection 4.1, an elementary example for two arbitrarily selected angles is presented. Second, in Subsection 4.2 the DP capability assessment results are discussed. The obtained results have been compared with the results obtained using DNV tool available on-line[3].

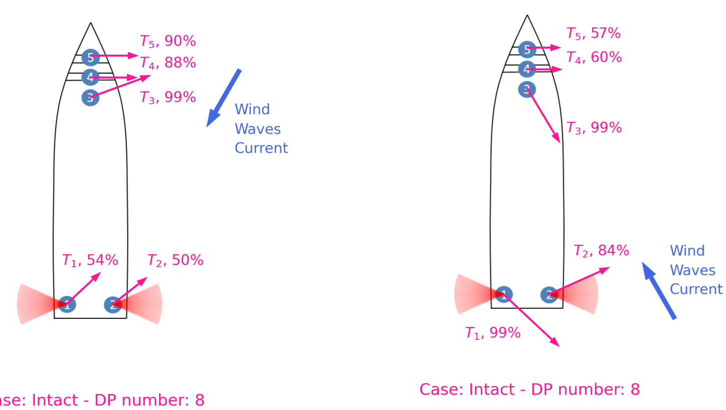
[3] The free version of the application is limited to analysis of maximum four thrusters and does not share detailed results, just the DP capability plot.

Table 1: Ship main data

Symbol	Value	Unit	Description
L_{pp}	86.6	m	Length between perpendiculars
B	18.8	m	Maximum breadth at waterline
T	5.0	m	Summer load line draft
$A_{\mathrm{F,\,wind}}$	392.0	m^2	Frontal projected wind area
$A_{\mathrm{L,\,wind}}$	1203.0	m^2	Longitudinal projected wind area
$A_{\mathrm{L,\,current}}$	441.0	m^2	Longitudinal projected submerged current area

4.1 Optimal thrust allocation

In Fig. 3 an example of optimal thrust allocation is presented. It results from solving (9), for two distinct angles, namely 30° and 150° indicated by blue arrow, has been presented. For the illustrated test, the environmental conditions were set to standardized level DP number 8 according to [5]. The mounting points of the thrusters have been indicated with blue numbered dots. The red areas near the rear thrusters indicate the forbidden zones. The result of thrust allocation has been indicated by magenta arrows attached to thrusters. For legibility of the result, the percentage of maximum thrust has been display to clearly characterize the magnitude of the thrust vectors.



(a) Environmental angle 30°

(b) Environmental angle 150°

Fig. 3: Optimal thrust allocation

As a result of the analysis, one is able to clearly identify that the environmental impact from the direction coming from the bow (30°) causes higher engagement of the fore thrusters. In turn, the environmental impact from the direction from the stern (150°) engages stern thrusters more. This directly relates to the ship's geometry, both under and above water level, and exerted environmental forces and torques that depend on it. Therefore, this tendency will vary from ship to ship and is considered as crucial during DP ship design.

4.2 DP capability assessment

In Fig. 4 the DP capability assessment results have been presented. Two DP operation modes have been investigated. First, 'intact', where all the thrusters are considered operational. Second, single failure, where one of the thrusters failed. In the later case, two scenarios were explored. One considering failure of thruster 2 and second of thruster 5. In both failure scenarios, the DP capability is significantly reduced. In case of starboard azimuth thruster's failure, the capability the capability plot is no longer symmetric. This is an indication of the effective thrust reduction due to flushing a dead thruster by the port azimuth thruster. When azimuth thruster angle is within a spoiled zone, the efficiency of a generated thrust decreases.

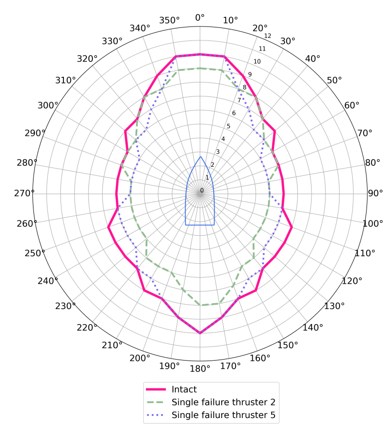
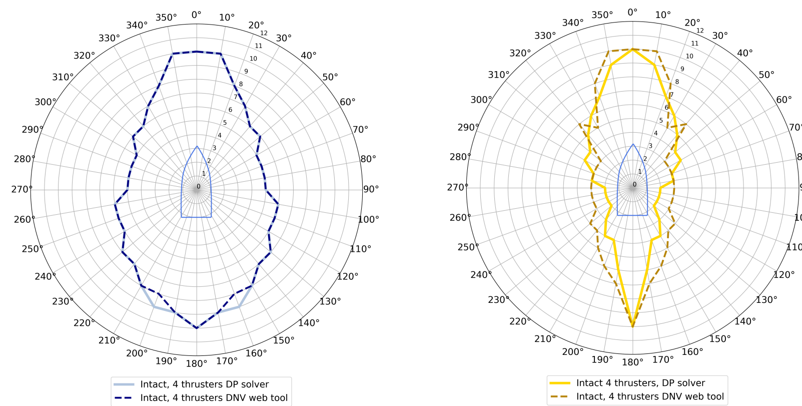


Fig. 4: DP capability plot

The results from the developed tool were compared to the ones obtained obtained using the on-line application by DNV (Fig. 5). It has been found that in case of using azimuth thrusters as main propulsion, the results obtained from both tools are very similar (Fig. 5a). The only difference is at the environmental angles of 160° and 200°, respectively. Since the information on the optimization algorithm used by DNV free web application is not available, it is hypothesized that this may be the consequence of different handling of the spoiled zones due to skeg. In turn, the results, obtained considering propellers with the rudder as main thruster, vary significantly (Fig. 5b). In general, the methodology presented in this work shows a more pessimistic outcome in comparison to the DNV web application. Further investigation of the discrepancies is required to gain better understanding of the matter.



(a) 4 thrusters - two aft propulsors: azimuth thrusters

(b) 4 thrusters - two aft propulsors: propellers with rudders

Fig. 5: Comparison of DP capability plots

5 Conclusions

The recently increasing need for DP assessment tools, both fast rough calculations as well as time-domain simulations for the early stage of the design, was the motivation for the study. The Quadratic Programming method used in *optimal thrust allocation* proved relevant results when applied to DNV rules (DNV-ST-0111 standard). In particular, it was evident while concerning handling the influences between thrusters and skeg. The presented method could be effectively used by design offices around the world for rough initial calculation without a need for investment in expensive software. An important element of future research will be the evaluation of the accuracy of the method based on time - domain simulation and model tests. The verification of simulation is planned on the basis of experiments on dedicated test stand of Maritime Advanced Research Centre, using the physical model of the ship, equipped with DP system.

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