"And where a mathematical reasoning can be had, it is as great a folly to make use of any other, as to grope for a thing in the dark, when you have a candle standing by you" – John Arbuthnot, *On The Laws of Chance*

Definition 3.1: A *diagnostic problem* P is a 4-tuple <D, M, C, M$^+$> where $D = \{d_1, d_2, \ldots, d_n\}$ is a finite, non-empty set of objects, called disorders, $M = \{m_1, m_2, \ldots, m_n\}$ is a finite, non-empty set of objects called manifestations, and $C \subseteq D \times M$ is a relation with *domain*(C) = D and *range*(C) = M, called causation, and $M^+ \subseteq M$ is a distinguished subset of M which is said to be **present**.

Definition 3.2: For any element $d_i \in D$ and $m_j \in M$ in a diagnostic problem <D, M, C, M$^+$>, *effects*$(d_i) = \{m_j \mid \langle d_i, m_j \rangle \in C\}$, the set of objects directly caused by d_i, and *causes*$(m_j) = \{d_i \mid \langle d_i, m_j \rangle \in C\}$, the set of objects which can directly cause m_j

Definition 3.3: For any $D_I \subseteq D$ and $M_J \subseteq M$ in a diagnostic problem <D, M, C, M$^+$>, *effects*$(D_I) = \bigcup_{d_i \in D_I} effects(d_i)$, and *causes*$(M_J) = \bigcup_{m_j \in M_J} causes(m_j)$

Thus, for example, the effects of a set of disorders are just the union ("sum") of effects of individual disorders in the set.

Definition 3.4: The set $D_I \subseteq D$ is said to be a *cover* of $M_J \subseteq M$ if $M_J \subseteq effects(D_I)$

Definition 3.5: A set $E \subseteq D$ is said to be an *explanation* of M$^+$ for a problem P = <D, M, C, M$^+$>if E *covers* M$^+$ and E satisfies a given parsimony condition.

Definition 3.6:

1. A cover, D_I of M_J is said to be *minimum* if its cardinality is smallest among all covers of M_J.
2. A cover, D_I of M_J is said to be *irredundant* if none of its proper subsets is also a cover of M_J. It is said to be *redundant* otherwise.
3. A cover, D_I of M$^+$ is said to be *relevant* if it is a subset of *causes*(M^+); it is *irrelevant* otherwise.

Definition 3.7: The *solution* to a diagnostic problem P = <D, M, C, M$^+$>designated $Sol(P)$ is the set of all explanations of M$^+$.

Lemma 3.1: TBD

Lemma 3.2: TBD

Lemma 3.3: TBD

Thereom 3.4: TBD

Lemma 3.5: TBD

Lemma 3.6: TBD

Lemma 3.7: TBD

Thereom 3.8: TBD

Lemma 3.9: TBD

Lemma 3.10: TBD

Definition 3.8: TBD

Definition 3.9: TBD

Definition 3.10: Let $G_I = (g_1, g_2, \ldots, g_n)$ be a generator and let $H_1 \subseteq D$ where $H_1 \neq \theta$.
Then $Q = \{Q_k | Q_k$ is a generator $\}$ is a division of G_I by H_1 if for all k, $1 < k < n$, $Q_k = (\, q_{k1}, q_{k2}, \ldots, q_{kn})$ where

$$\begin{cases} g_j - H_1, \text{ if } j < k, \\ g_j \cap H_1, \text{ if } j = k, \\ g_j \text{ if } j > k \end{cases}$$

Definition 3.11:

Lemma 3.11:

Definition 3.12:

Lemma 3.12:

Definition 3.13:

Lemma 3.13:

Definition 3.14:

Lemma 3.14:

Definition 3.15:

Lemma 3.15:

Lemma 3.16:

Lemma 3.17:

Lemma 3.18:

Thereom 3.19:

Definition 3.16:

Lemma 3.20:

Definition 3.17:

Lemma 3.21:

Definition 3.18:

Lemma 3.22:

Thereom 3.23:

Definition 3.19:

Definition 3.20:

Lemma 3.24:

Thereom 3.25: