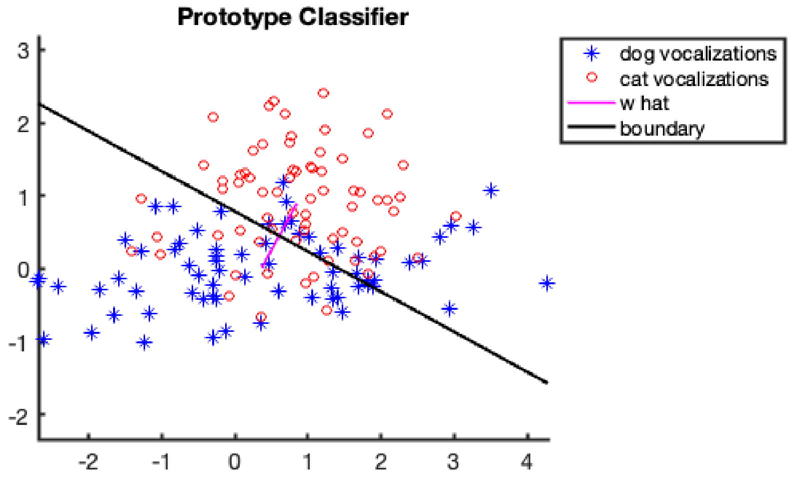
Contents

```
clear; close all; clc;
```

a)

```
load('fisherData.mat')
mean_data1 = mean(data1);
mean_data2 = mean(data2);
w = mean_data2 - mean_data1;
w_norm = sqrt(sum(w.^2));
w_hat = w./w_norm;
midpoint_data = (mean_data1 + mean_data2)/2;

figure();
scatter(data1(:, 1), data1(:, 2), 'b*', 'DisplayName', 'dog vocalizations');
hold on;
scatter(data2(:, 1), data2(:, 2), 'ro', 'DisplayName', 'cat vocalizations');

plot([midpoint_data(1) - w_hat(1)/2, midpoint_data(1) + w_hat(1)/2], ...
    [midpoint_data(2) - w_hat(2)/2, midpoint_data(2) + w_hat(2)/2], ...
    'm-', 'DisplayName', 'w hat', 'LineWidth', 1.5)
axis equal;
x_dec_boundary = xlim;
y_dec_boundary = decision_boundary(x_dec_boundary, midpoint_data, w_hat);

plot(x_dec_boundary, y_dec_boundary, 'k-', 'LineWidth', 2, ...
    'DisplayName', 'boundary')
set(gca, 'FontSize', 14)
set(gca, 'LineWidth', 2)
xlim(x_dec_boundary)
title('Prototype Classifier')
legend('Location', 'northeastoutside');

frac_correctly_classified = classification_performance(data1, ...
    data2, x_dec_boundary, y_dec_boundary, w_hat)
```

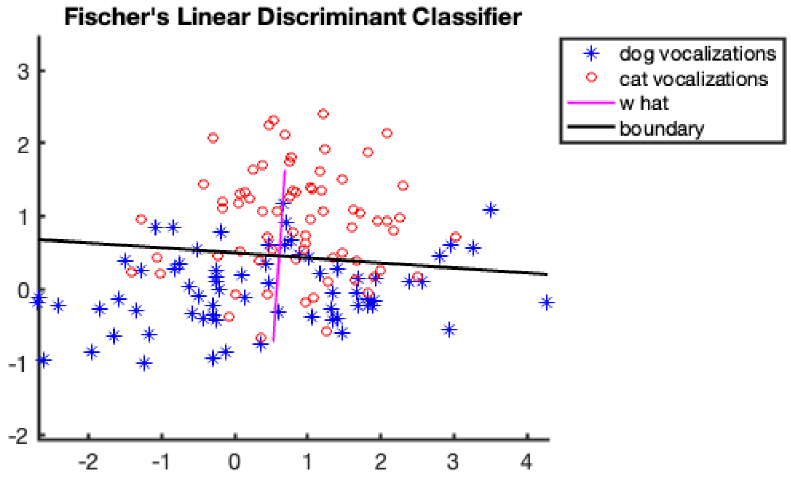
```
frac_correctly_classified =

    0.7357
```



b)

```
%cov_data1 = (data1' * data1)./(size(data1 , 1) - 1);
%cov_data2 = (data2' * data2)./(size(data2, 1) - 1);
cov_data1 = cov(data1); cov_data2 = cov(data2);
cov_combined = (cov_data1 + cov_data2)/2;
w_hat_fisch = cov_combined \ w';% \ cov_combined;

figure();
scatter(data1(:, 1), data1(:, 2), 'b*', 'DisplayName', 'dog vocalizations');
hold on;
scatter(data2(:, 1), data2(:, 2), 'ro', 'DisplayName', 'cat vocalizations');

plot([midpoint_data(1) - w_hat_fisch(1)/2, midpoint_data(1) + w_hat_fisch(1)/2], ...
    [midpoint_data(2) - w_hat_fisch(2)/2, midpoint_data(2) + w_hat_fisch(2)/2], ...
    'm-', 'DisplayName', 'w hat', 'LineWidth', 1.5)
axis equal;

x_dec_boundary = xlim;
y_dec_boundary = decision_boundary(x_dec_boundary, midpoint_data, w_hat_fisch);

plot(x_dec_boundary, y_dec_boundary, 'k-', 'LineWidth', 2, ...
    'DisplayName', 'boundary')
set(gca, 'FontSize', 14)
set(gca, 'LineWidth', 2)
xlim(x_dec_boundary)
title("Fischer's Linear Discriminant Classifier")
legend('Location', 'northeastoutside')

frac_correctly_classified = classification_performance(data1, ...
    data2, x_dec_boundary, y_dec_boundary, w_hat_fisch)
```

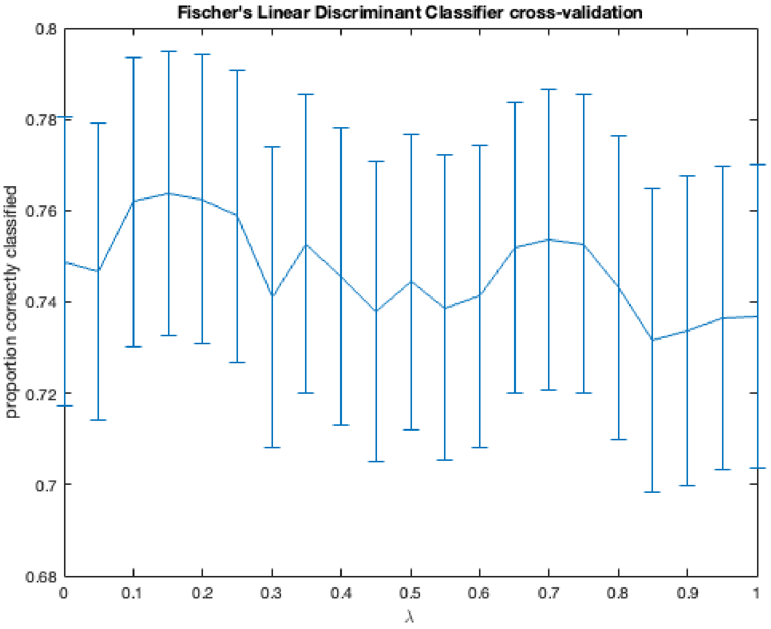
```
frac_correctly_classified =

    0.7571
```



c)

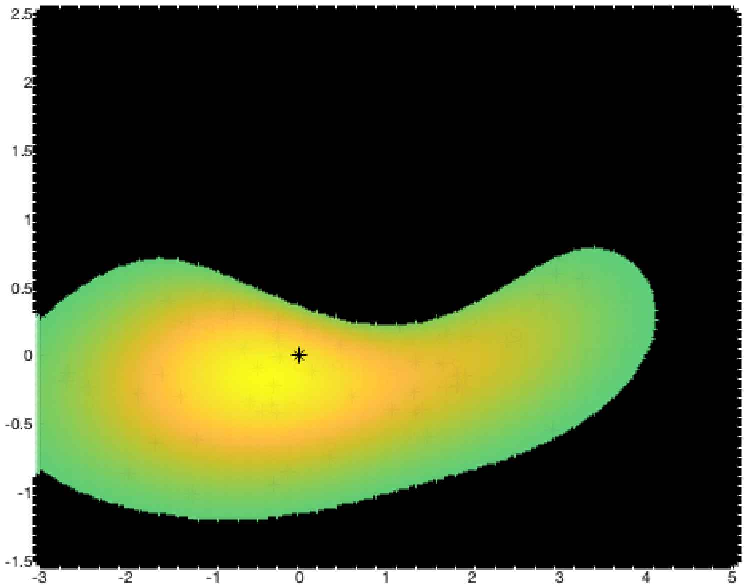
```
lambdas = 0:0.05:1;
runs = 1e3;

train_size = floor(0.95 * size(data1, 1));
frac_correctly_classified_cv = zeros(length(lambdas), runs);
for run = 1:runs
    for ll = 1:length(lambdas)
        lambda = lambdas(ll);

        train1_indices = randperm(size(data1, 1), train_size);
        test1_indices = setdiff(1:size(data1, 1), train1_indices);
        train2_indices = randperm(size(data2, 1), train_size);
        test2_indices = setdiff(1:size(data2, 1), train2_indices);

        train1 = data1(train1_indices, :);
        train2 = data2(train2_indices, :);
        test1 = data1(test1_indices, :);
        test2 = data2(test2_indices, :);

        mean_train1 = mean(train1);
        mean_train2 = mean(train2);
        w = mean_train2 - mean_train1;
        w = w./sqrt(sum(w.^2));

        midpoint_train = (mean_train1 + mean_train2)/2;

        %cov_train1 = (train1' * train1)./(size(train1, 1) - 1);
        %cov_train2 = (train2' * train2)./(size(train2, 1) - 1);
        cov_train1 = cov(train1); cov_train2 = cov(train2);
        cov_combined = (cov_train1 + cov_train2)/2;

        cov_estimated = (1 - lambda) .* cov_combined + lambda .* eye(2);
        w_hat_estim = cov_estimated \ w';% \ cov_estimated;

        x_dec_boundary = xlim;
        y_dec_boundary = decision_boundary(x_dec_boundary, midpoint_train, ...
            w_hat_estim);

        frac_correctly_classified_cv(ll, run) = classification_performance(test1, ...
            test2, x_dec_boundary, y_dec_boundary, w_hat_estim);
    end
end

frac_correctly_classified_mean = mean(frac_correctly_classified_cv, 2);
frac_correctly_classified_stderror = std(frac_correctly_classified_cv, 0, 2)...
```

```
    ./sqrt(size(frac_correctly_classified_mean, 1));

figure()
errorbar(lambdas, frac_correctly_classified_mean, frac_correctly_classified_stderror)
xlabel('\lambda')
ylabel('proportion correctly classified')
title("Fischer's Linear Discriminant Classifier cross-validation")
```



d)

```
figure();
scatter(data1(:, 1), data1(:, 2), 'b*', 'DisplayName', 'dog vocalizations');
hold on;
scatter(data2(:, 1), data2(:, 2), 'ro', 'DisplayName', 'cat vocalizations');

xx_ = xlim;
yy_ = ylim;
pps = 4*1e2;
xx = linspace(xx_(1), xx_(2), pps);
yy = linspace(yy_(1), yy_(2), pps);
[X, Y] = meshgrid(xx, yy);
XY = [X(:) Y(:)];
p1 = mvnpdf(XY, mean_data1, cov_data1);
p2 = mvnpdf(XY, mean_data2, cov_data2);
diff_p = p1 - p2;
scatter(XY(:, 1), XY(:, 2), [], diff_p, 'MarkerFaceAlpha', 0.2, ...
    'MarkerEdgeAlpha', 0.2)

dec_x = zeros(length(diff_p), 1);
dec_y = zeros(length(diff_p), 1);
for i = 1:length(diff_p)
    if diff_p(i) <= 0.01
        dec_x(i) = XY(i, 1);
        dec_y(i) = XY(i, 2);
    end
end
plot(dec_x, dec_y, 'k*')
```



```
p1_data1 = mvnpdf(data1, mean_data1, cov_data1);
p2_data1 = mvnpdf(data1, mean_data2, cov_data2);
p1_data2 = mvnpdf(data2, mean_data1, cov_data1);
p2_data2 = mvnpdf(data2, mean_data2, cov_data2);

correct_data1 = sum(p1_data1 > p2_data1);
correct_data2 = sum(p2_data2 > p1_data2);

frac_correctly_classified = (correct_data1 + correct_data2)./...
    (2 * size(data1, 1))
```

```
frac_correctly_classified =

    0.8143
```

```
function samples = nRandn(mean, cov, num)
    % The function draws samples from an N-dimensional normal distribution.
    % It does so my first drawing samples from an N-dimensional normal
    % distribution of 0 mean and identity covariance matrix. It then
    % transforms the data using a matrix M to have the covariance "cov".
    % Lastly, it translates the data to have the mean "mean".

    N = length(mean);
    [V, D] = eig(cov);  % Eigen value decomposition of covariance matrix
    M = V * sqrt(D); % Transformation matrix is given as product of V and square-root of eigenvalues
    samp = randn(num, N); % Drawing samples from N-dimensional normal distribution with mean 0 and identity covariance matrix
    samples = repmat(mean, num , 1) + samp * M';
end

function y_dec_boundary = decision_boundary(x_dec_boundary, ...
    midpoint_data, w_hat)

    dec_boundary_slope = -(w_hat(1))/(w_hat(2));
    y_dec_boundary(1) = midpoint_data(2) + dec_boundary_slope * ...
        (x_dec_boundary(1) - midpoint_data(1));
    y_dec_boundary(2) = midpoint_data(2) + dec_boundary_slope * ...
        (x_dec_boundary(2) - midpoint_data(1));
end

function frac_correctly_classified = classification_performance(data1, ...
    data2, x_dec_boundary, y_dec_boundary, w_hat)

    dec_boundary_slope = -(w_hat(1))/(w_hat(2));
    x_ones = ones(size(data1(:, 1))) .* x_dec_boundary(1);
    y_ones = ones(size(data1(:, 1))) .* y_dec_boundary(1);
    y_data1 = y_ones + dec_boundary_slope * (data1(:, 1) - x_ones);
    y_data2 = y_ones + dec_boundary_slope * (data2(:, 1) - x_ones);
    correct_data1 = sum(y_data1 >= data1(:, 2));
    correct_data2 = sum(y_data2 <= data2(:, 2));
    frac_correctly_classified = (correct_data1 + correct_data2)./...
        (2 * size(data1, 1));
end
```

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