Statistics

Probability and Statistics

- Statistics is the mathematical science behind the problem "what can I know about a population if I'm unable to reach every member?"

Probability and Statistics

- If we could measure the height of every resident of India, then we could make a statement about the average height of Indians at the time we took our measurement.
- This is where random sampling comes in.

Probability and Statistics

- If we take a reasonably sized random sample of Indians and measure their heights, we can form a statistical inference about the population of India.
- Probability helps us know how sure we are of our conclusions!

What is Data?

- Data =the collected observations we have about something.
- Data can be continuous:

  "What is the stock price?"

- or categorical:

  "What car has the best repair history?"

Why Data Matters

- Helps us understand things as they are:

  "What relationships if any exist between two events?"

  "Do people who eat an apple a day enjoy fewer doctor's visits than those who don't?"

Why Data Matters

- Helps us predict future behavior to guide business decisions:

  "Based on a user's click history which ad is more likely to bring them to our site?"

Visualizing Data

- Compare a table:

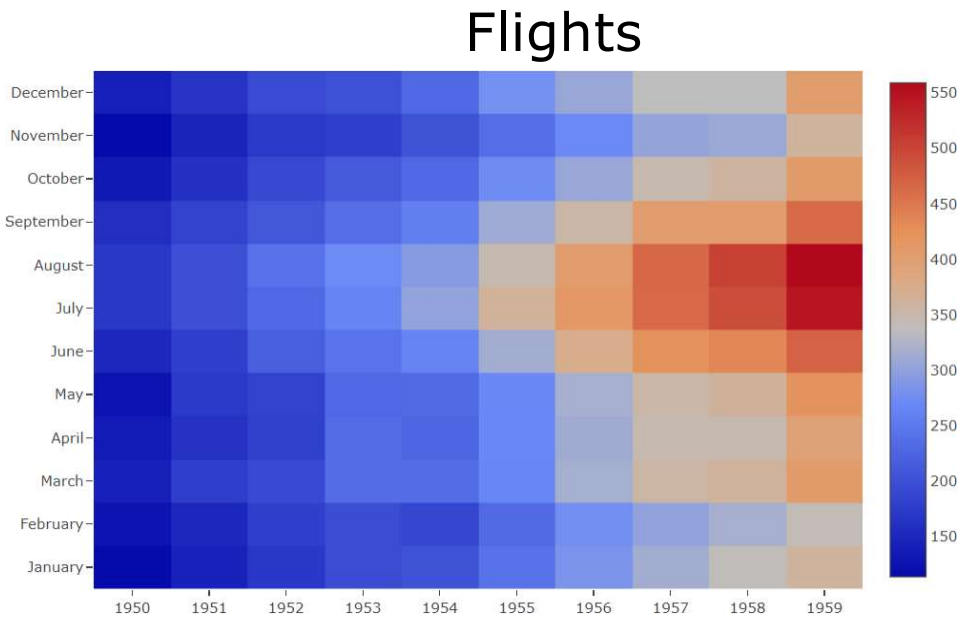
Flights

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	**year**	**month**	**passengers**		**year**	**month**	**passengers**		**year**	**month**	**passengers**		**year**	**month**	**passengers**
2	1950	January	115		1952	July	230		1955	January	242		1957	July	465
3	1950	February	126		1952	August	242		1955	February	233		1957	August	467
4	1950	March	141		1952	September	209		1955	March	267		1957	September	404
5	1950	April	135		1952	October	191		1955	April	269		1957	October	347
6	1950	May	125		1952	November	172		1955	May	270		1957	November	305
7	1950	June	149		1952	December	194		1955	June	315		1957	December	336
8	1950	July	170		1953	January	196		1955	July	364		1958	January	340
9	1950	August	170		1953	February	196		1955	August	347		1958	February	318
10	1950	September	158		1953	March	236		1955	September	312		1958	March	362
11	1950	October	133		1953	April	235		1955	October	274		1958	April	348
12	1950	November	114		1953	May	229		1955	November	237		1958	May	363
13	1950	December	140		1953	June	243		1955	December	278		1958	June	435
14	1951	January	145		1953	July	264		1956	January	284		1958	July	491
15	1951	February	150		1953	August	272		1956	February	277		1958	August	505
16	1951	March	178		1953	September	237		1956	March	317		1958	September	404
17	1951	April	163		1953	October	211		1956	April	313		1958	October	359
18	1951	May	172		1953	November	180		1956	May	318		1958	November	310

Not much can be gained by reading it.

Visualizing Data

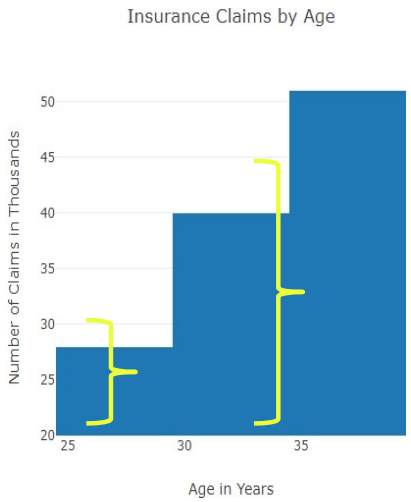
- Using a graph:

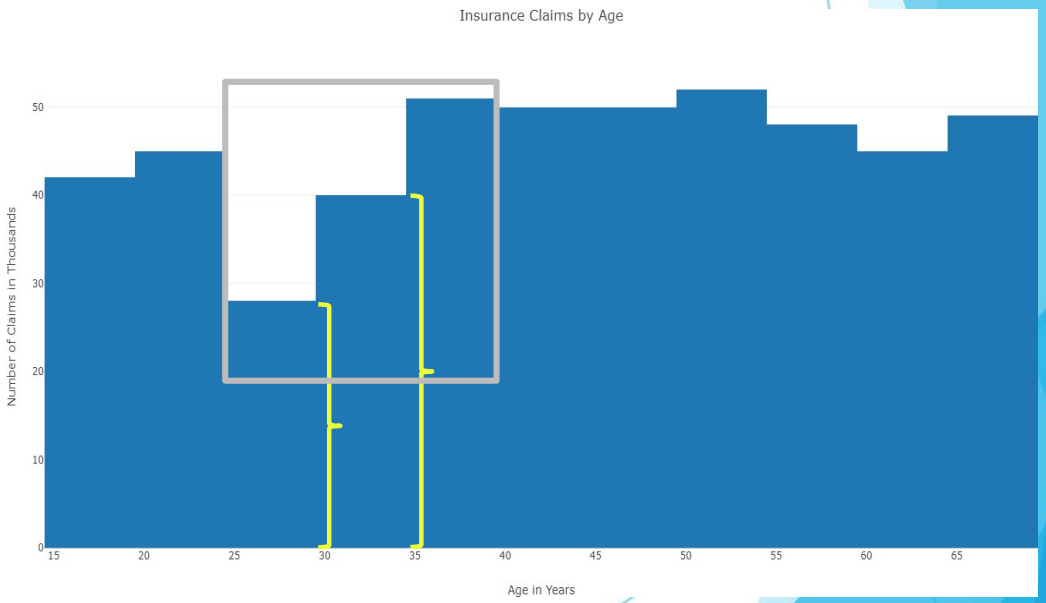


The graph uncovers two distinct trends - an increase in passengers flying over the years and a greater number of passengers flying in the summer months.

Analyze Visualizations Critically!

- Graphs can be misleading:





Levels of Measurement

Nominal

- Predetermined categories
- Can't be sorted

Animal classification (*mammal fish reptile*)

Political party (*BJP Congress JDS*)

Levels of Measurement

Ordinal

- Can be sorted
- Lacks scale

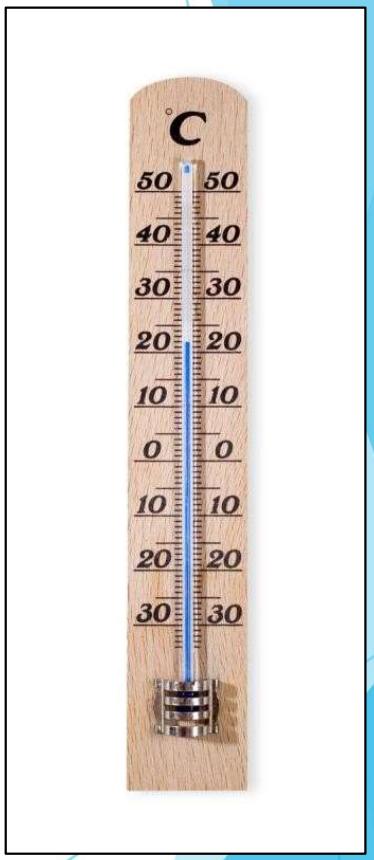
Survey responses

Levels of Measurement

Interval

- Provides scale
- Lacks a “zero”point

Temperature



Levels of Measurement

Ratio

- Values have a true zero point

Age, weight, salary

Population vs. Sample

- Population = every member of a group
- Sample = a subset of members that time and resources allow you to measure

Measurement Types
central tendency

Measurements of Data

- "What was the average return?"

  Measures of Central Tendency

- "How far from the average did individual values stray?"

  Measures of Dispersion

Measures of Central Tendency (mean, median, mode)

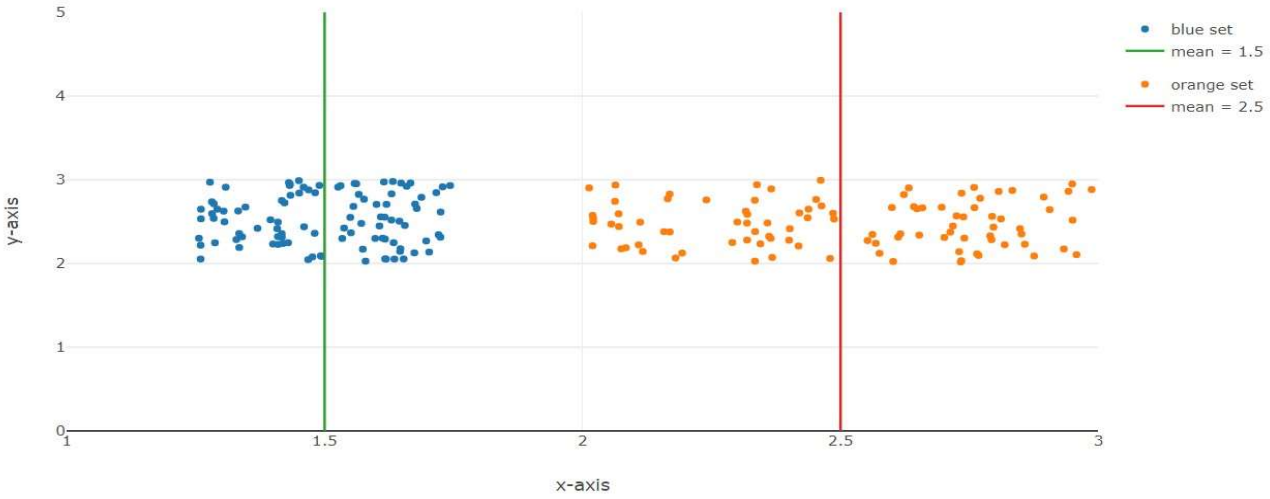
- Describe the "location" of the data
- Fail to describe the "shape" of the data

mean = "calculated average"

median = "middle value"

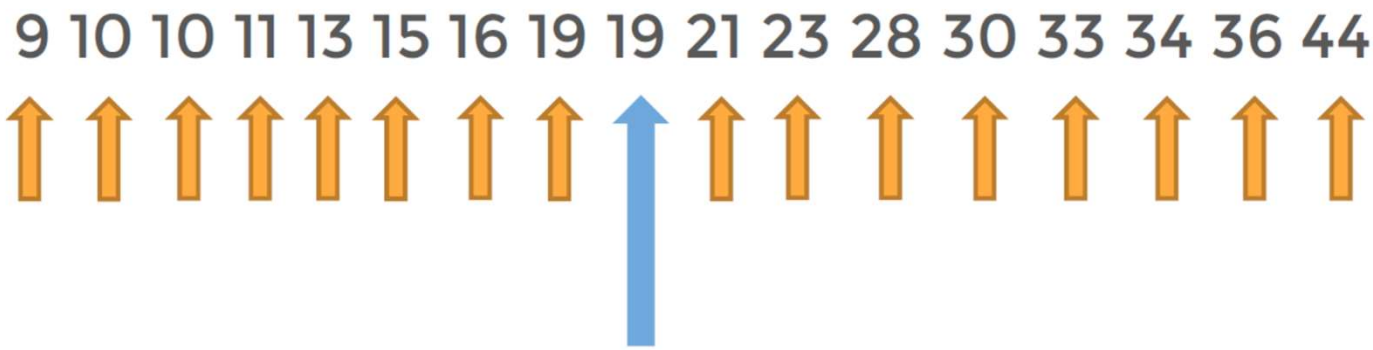
mode = "most occurring value"

Mean



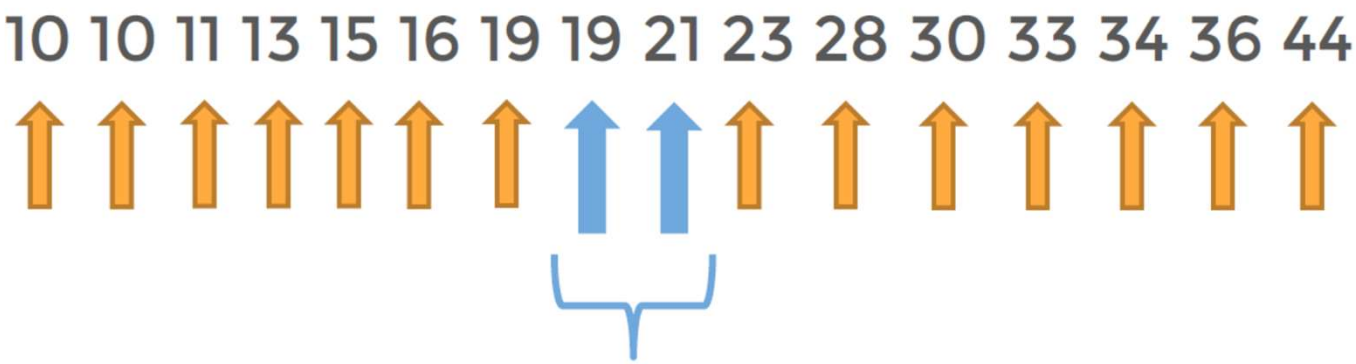
- Shows “location” but not “how spread out”

Median – odd number of values



$= 19$

Median - even number of values



$$\frac{19 + 21}{2} = 20$$

Mean vs. Median

- The mean can be influenced by *outliers*.
- The mean of $\{2, 3, 2, 3, 2, 12\}$ is 4
- The median is2.5
- The median is much closer to most of the values in the series!

Mode

10 10 11 13 15 16 16 16 21 23 28 30 33 34 36 44

= 16

Measurement Types: Dispersion

Measures of Dispersion (range, variance, standard deviation)

9 10 11 13 15 16 19 19 21 23 28 30 33 34 36 39

- In this sample the mean is 22.25
- How do we describe how "spread out" the sample is?

Range

9 10 11 13 15 16 19 19 21 23 28 30 33 34 36 39

$$\begin{aligned} Range &= max - min \\ &= 39 - 9 \\ &= 30 \end{aligned}$$

Variance

- Calculated as the sum of square distances from each point to the mean
- There's a difference between the SAMPLE variance and the POPULATION variance
- Subject to Bessel's correction ($\boldsymbol{n - 1}$)

Variance

SAMPLE VARIANCE: $s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$

POPULATION VARIANCE: $\sigma^2 = \frac{\Sigma(X-\mu)^2}{N}$

Sample Variance

$$s^2 = \frac{\Sigma(x - \bar{x})^2}{n - 1}$$

4 7 9 8 11

$$\bar{x} = \frac{4 + 7 + 9 + 8 + 11}{5} = \frac{39}{5} = 7.8$$ sample mean

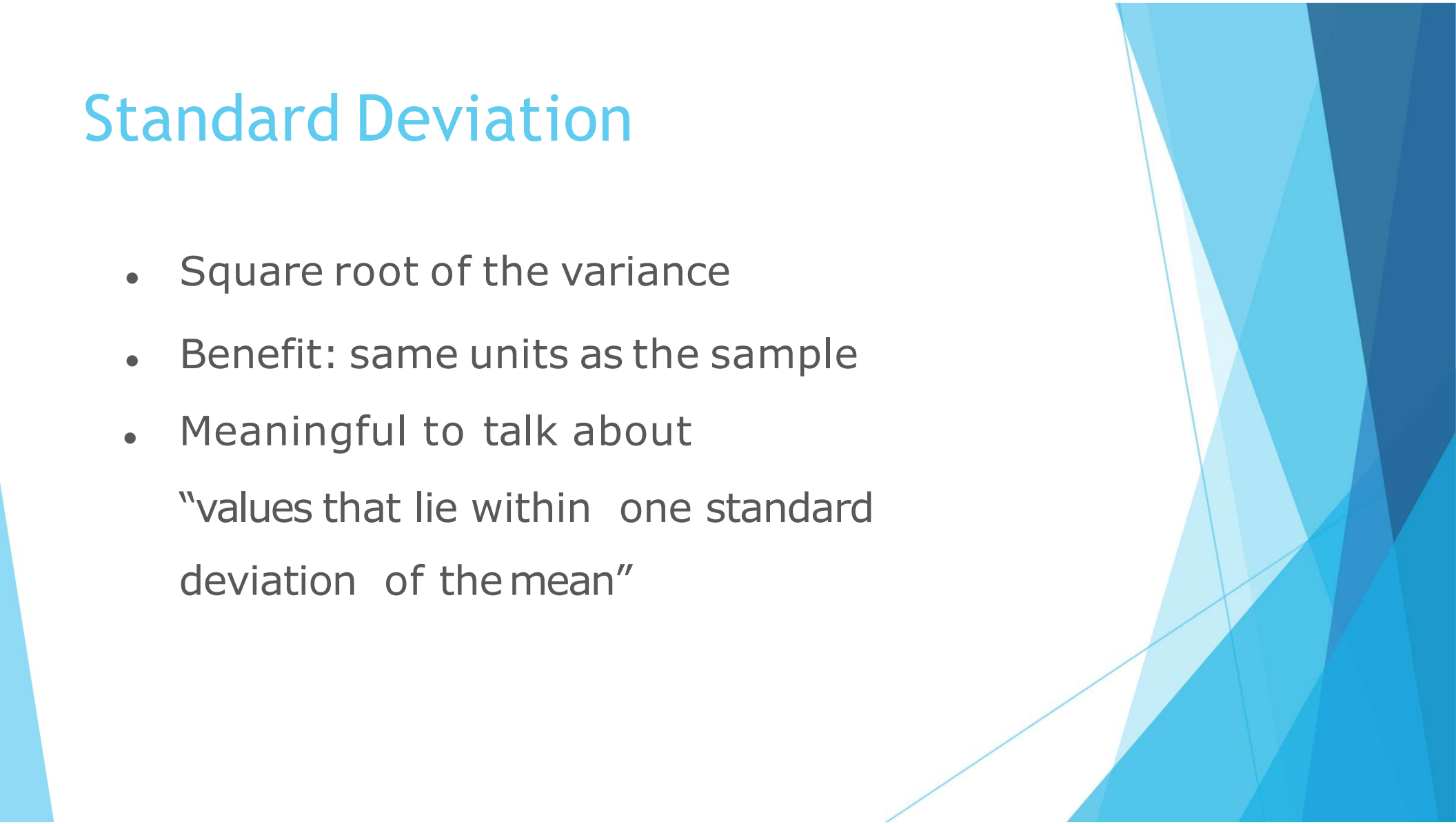
$$s^2 = \frac{(4-7.8)^2+(7-7.8)^2+(9-7.8)^2+(8-7.8)^2+(11-7.8)^2}{5-1}$$

$= 6.7$ sample variance

Standard Deviation

- Square root of the variance
- Benefit: same units as the sample
- Meaningful to talk about

  "values that lie within one standard deviation of the mean"



Sample standard deviation

$$s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}}$$

Sample:

4 7 9 8 11

$$\bar{x} = \frac{4 + 7 + 9 + 8 + 11}{5} = \frac{39}{5} = 7.8$$

sample mean

$$s = \sqrt{\frac{(4 - 7.8)^2 + (7 - 7.8)^2 + (9 - 7.8)^2 + (8 - 7.8)^2 + (11 - 7.8)^2}{5 - 1}}$$

$$= \sqrt{6.7} = 2.59$$

sample standard deviation

Population standard deviation

$$\sigma = \sqrt{\frac{\Sigma(X-\mu)^2}{N}}$$

Population:

4	7	9	8	11

$$\mu = \frac{4+7+9+8+11}{5} = \frac{39}{5} = 7.8$$

population mean

$$\sigma = \sqrt{\frac{(4-7.8)^2 + (7-7.8)^2 + (9-7.8)^2 + (8-7.8)^2 + (11-7.8)^2}{5}}$$

$$= \sqrt{5.36} = 2.32$$

population standard deviation

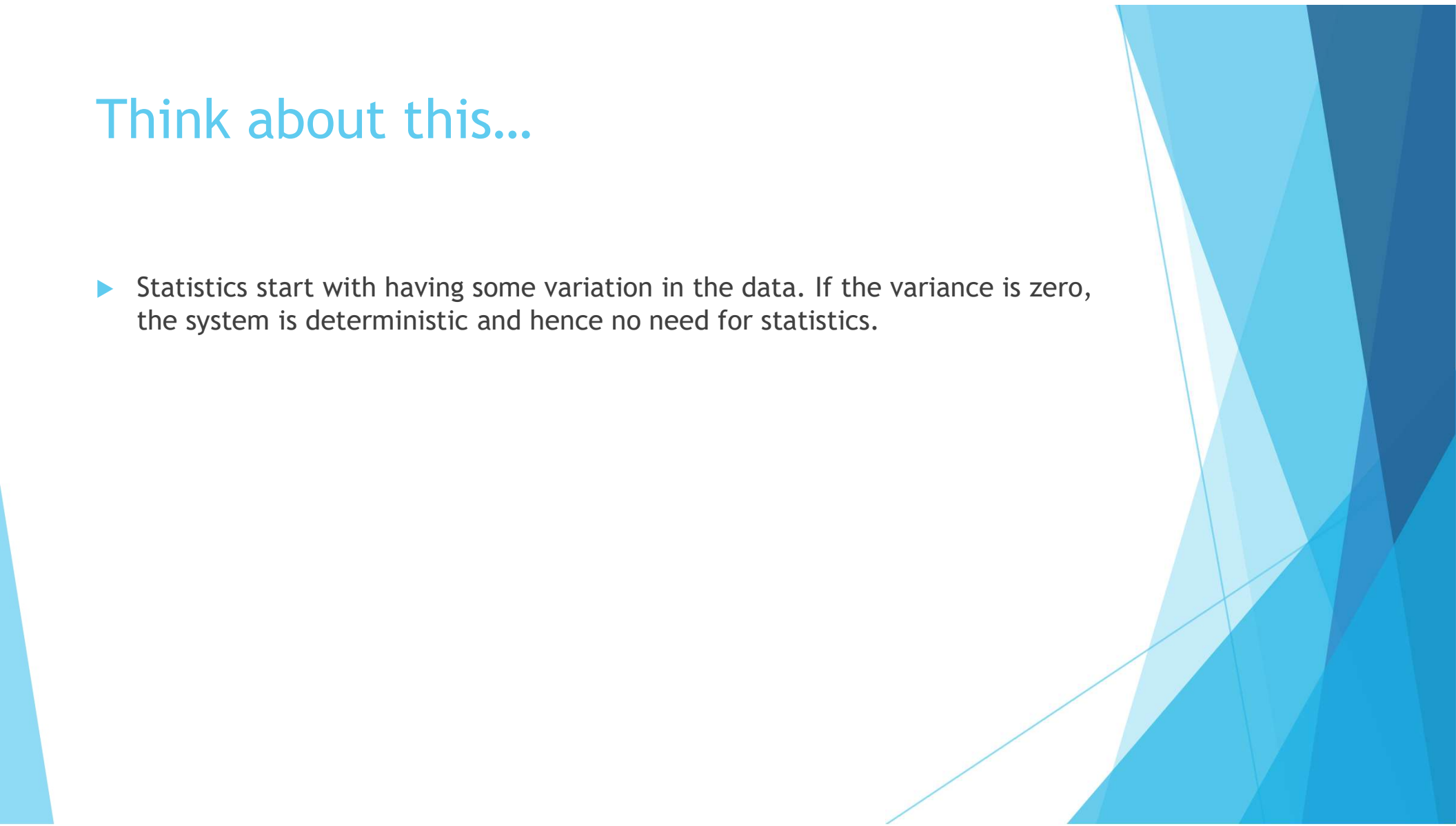
Practical usage

Consider stock returns. Say you've got the choice to buy stocks of two companies A and B. Upon analyzing the monthly returns the two stocks have given over two years, you find the mean monthly return for both stocks to be 10%. You calculate the variances of both stocks from the data sets and find that returns of company A have a greater variance than returns of company B, meaning there are larger deviations from the mean return in the data for A as compared to B.

Hence, company A will be riskier to invest in than company B for the same return, so you will invest in company B's stock (Oversimplified example obviously, but you get the drift). Variance/Standard deviation essentially measures variation from the expected value, or in other words, in the financial context, risk.

Think about this…

- Statistics start with having some variation in the data. If the variance is zero, the system is deterministic and hence no need for statistics.



Measurement types
quartiles

Quartiles and IQR

- Another way to describe data is through quartiles and the interquartile range (IQR)
- Has the advantage that every data point is considered, not aggregated!

Quartiles and iqr

- Consider the following series of 20 values:

9 10 10 11 13	15 16 19 19 21	23 28 30 33 34	36 44 45 47 60

1st quartile | 2nd quartile or median | 3rd quartile

1. Divide the series
2. Divide each subseries
3. These become quartiles

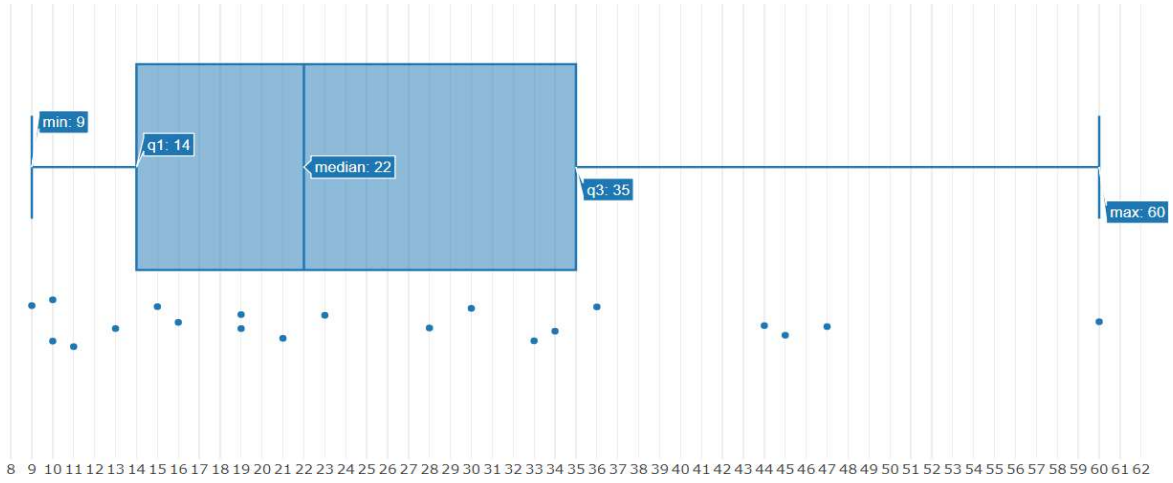
1st quartile = 14

2nd quartile = 22

3rd quartile = 35

Plot the Quartiles

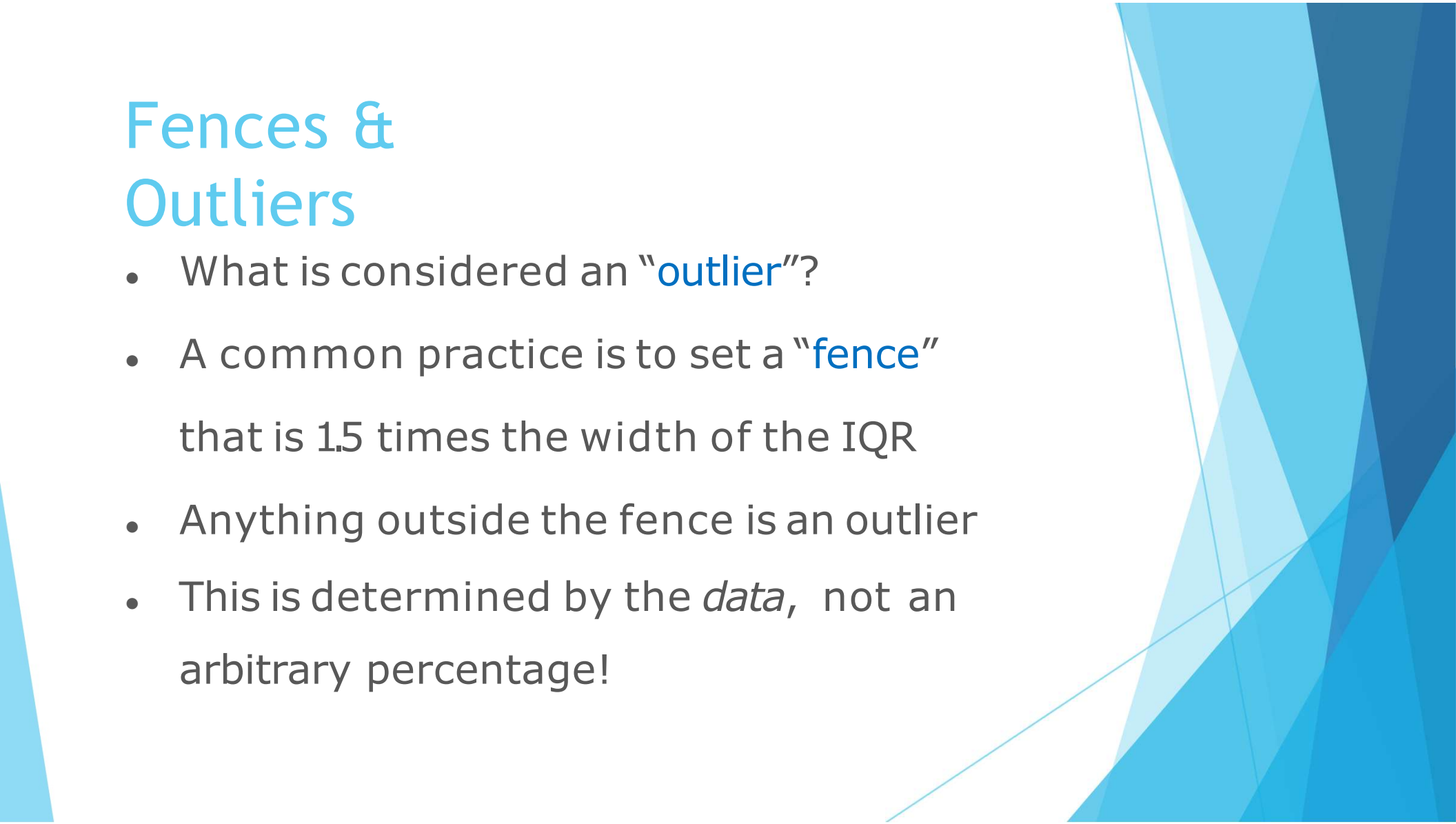
9 10 10 11 13	15 16 19 19 21	23 28 30 33 34	36 44 45 47 60



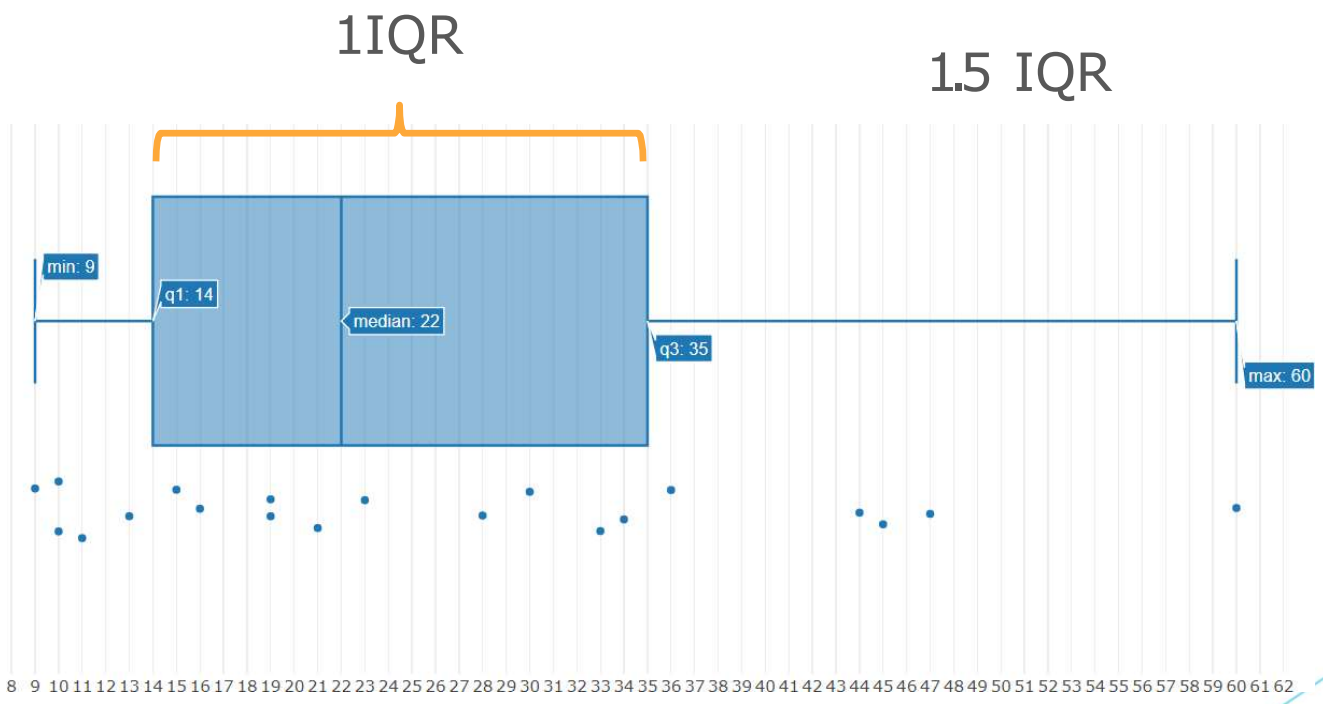
Quartile ranges are seldom the same size!

Fences & Outliers

- What is considered an "outlier"?
- A common practice is to set a "fence" that is 1.5 times the width of the IQR
- Anything outside the fence is an outlier
- This is determined by the *data*, not an arbitrary percentage!



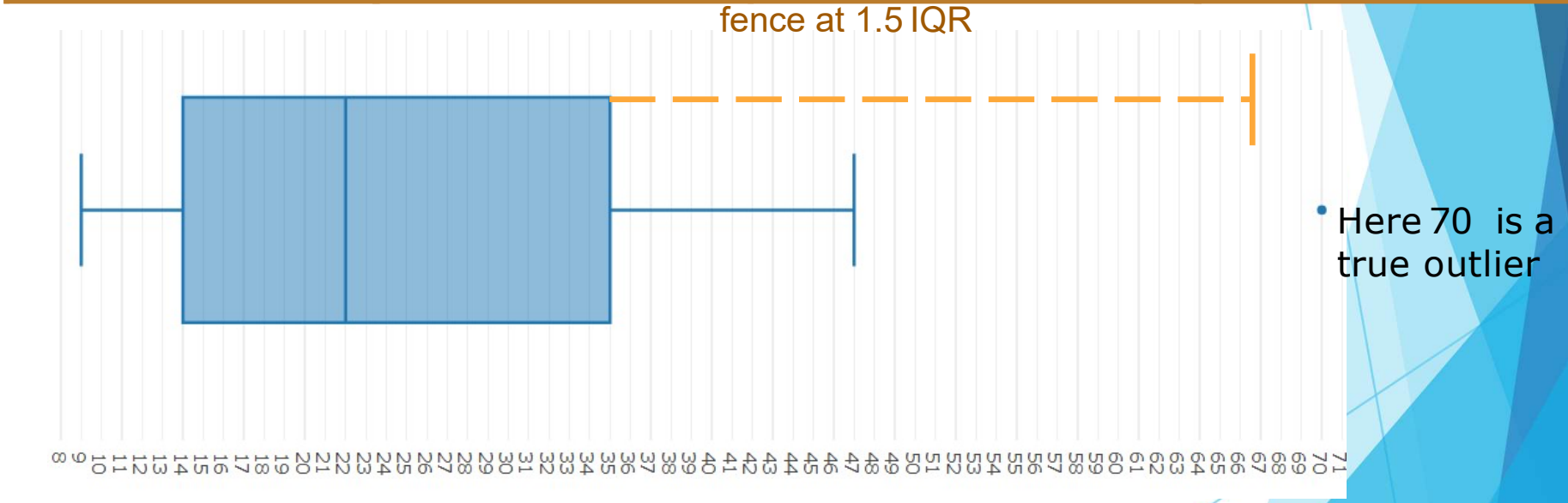
Fences AND Outliers



In this set, 60 is not an outlier, but 70 would be

Fences and outliers

9 10 10 11 13	15 16 19 19 21	23 28 30 33 34	36 44 45 47 70



- When drawing box plots, the whiskers are brought inward to the outermost values inside the fence.

Bivariate Data

Bivariate Data

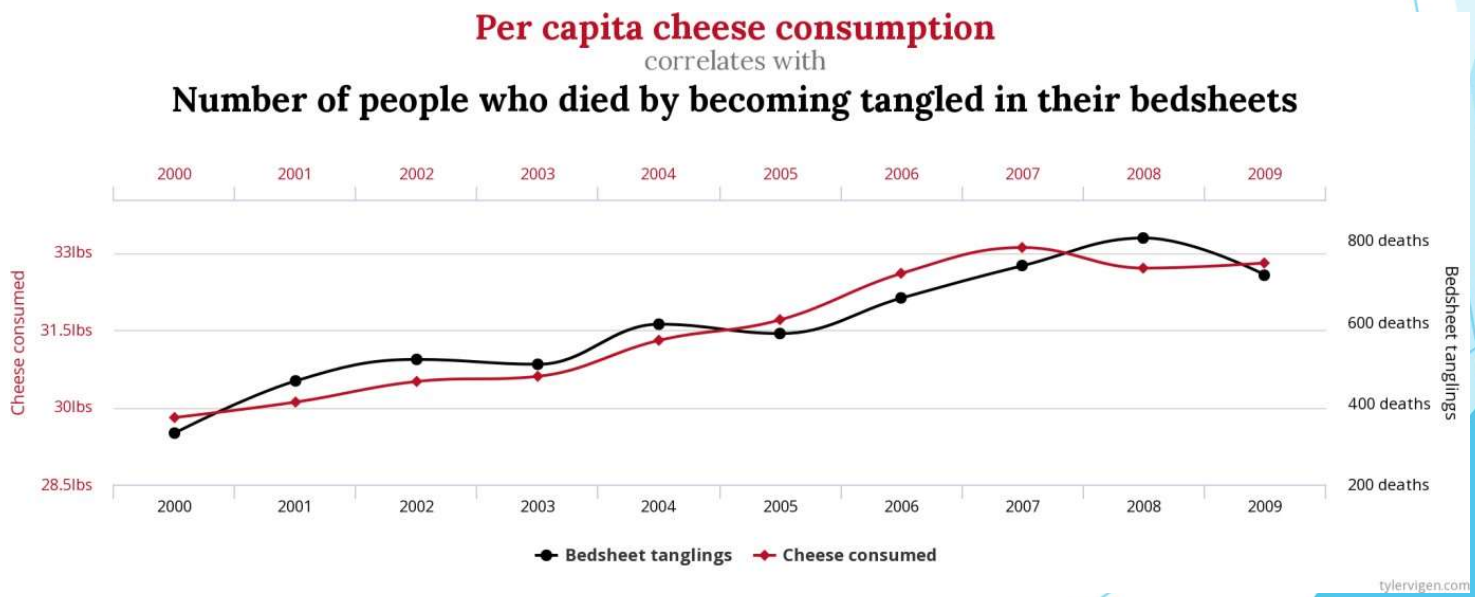
- Compares two variables
- By convention, the x-axis is set to the independent variable
- The y-axis is set to the dependent variable, or that which is being measured relative to x.

Bivariate Data

- Scatter plots may uncover a correlation between two variables
- They *can't* show causality!

Bivariate Data

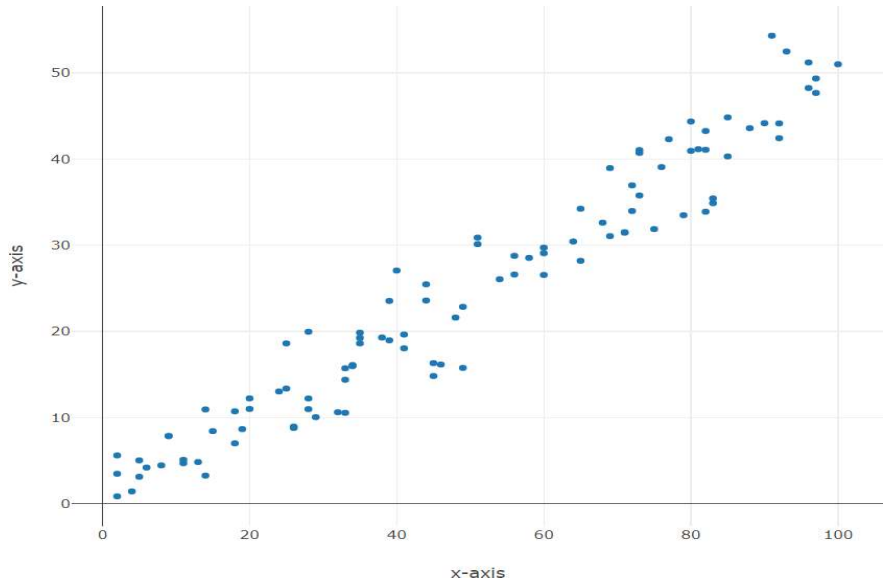
- Correlation between two variables
- Doesn't prove causality!



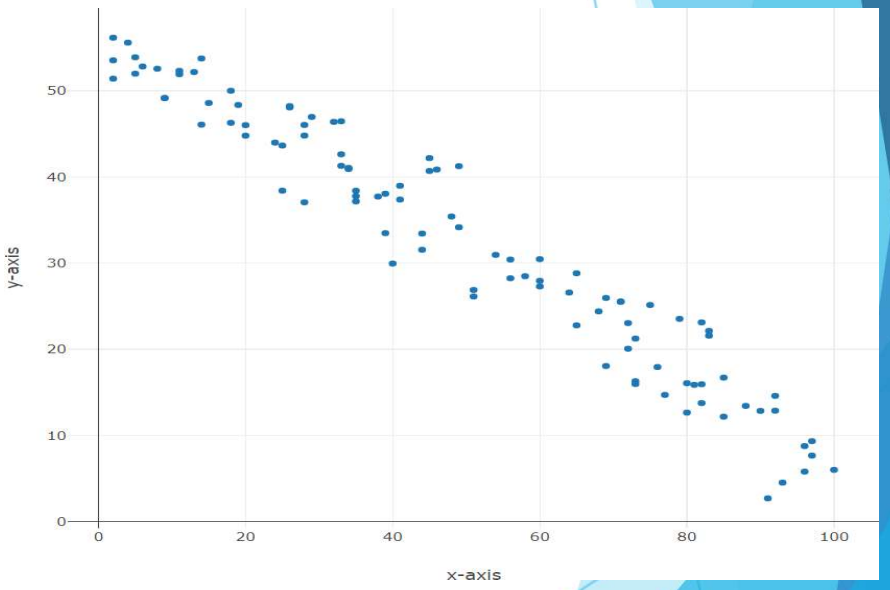
Bivariate Data

- More statistical analysis is needed to determine causality!
- For example: "Does increasing number of police officers decrease crime?"
- We would look at correlation, and do further analysis to understand causality.

Bivariate Data



Positive correlation



Negative or Inverse correlation

Covariance

- A common way to compare two variables is to compare their variances –how far from each item's mean do typical values fall?
- The first challenge is to match scale. Comparing height in inches to weight in pounds isn't meaningful unless we develop a standard score to normalize the data.

Covariance

- For simplicity, we'll consider the population covariance:

$$cov(X,Y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})$$

Covariance Exercise

- Consider the following two tables:

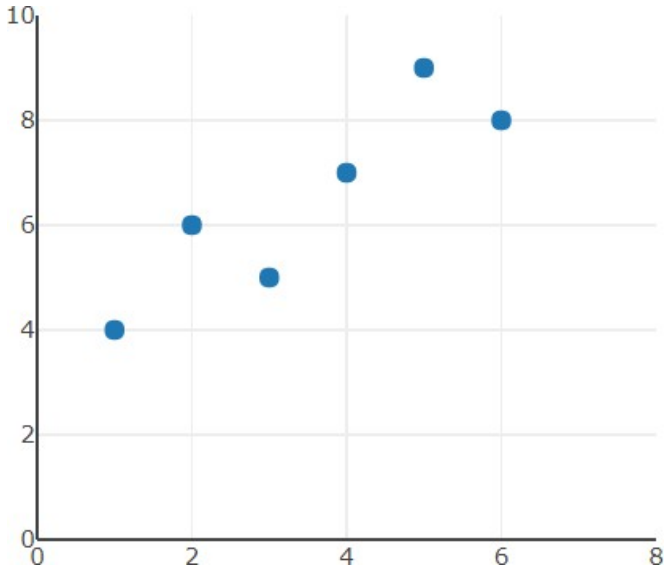
x	y
1	4
2	6
3	5
4	7
5	9
6	8

x	y
1	5
2	9
3	7
4	4
5	8
6	6

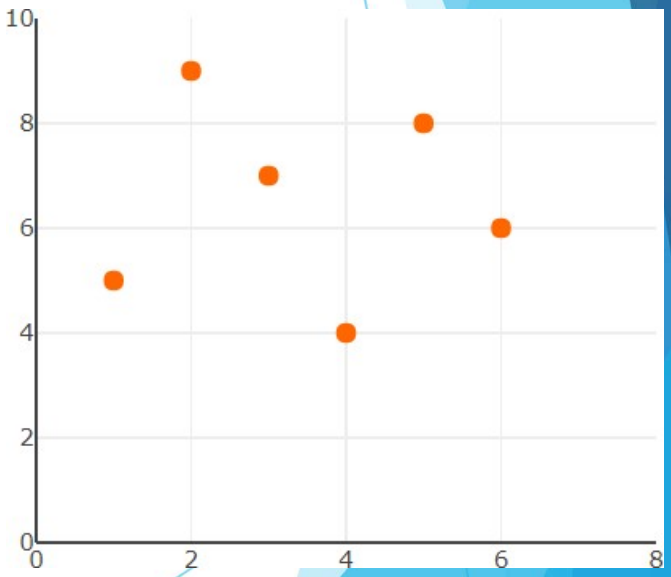
Covariance Exercise

- Plot them:

x	y
1	4
2	6
3	5
4	7
5	9
6	8



x	y
1	5
2	9
3	7
4	4
5	8
6	6



Covariance Exercise

$\bar{x} = 3.5, \quad \bar{y} = 6$

- Calculate mean values:

x	y
1	4
2	6
3	5
4	7
5	9
6	8

$$\bar{x} = \frac{1 + 2 + 3 + 4 + 5 + 6}{6} = 3.5$$

$$\bar{y} = \frac{4 + 6 + 5 + 7 + 9 + 8}{6} = 6.5$$

x	y
1	5
2	9
3	7
4	4
5	8
6	6

$$\bar{x} = \frac{1 + 2 + 3 + 4 + 5 + 6}{6} = 3.5$$

$$\bar{y} = \frac{5 + 9 + 7 + 4 + 8 + 6}{6} = 6.5$$

Covariance Exercise

$\bar{x} = 3.5, \quad \bar{y} = 6.5$

- Calculate $(x - \bar{x})$ and $(y - \bar{y})$:

x	y	$(x - \bar{x})$	$(y - \bar{y})$
1	4	-2.5	-2.5
2	6	-1.5	-0.5
3	5	-0.5	-1.5
4	7	0.5	0.5
5	9	1.5	2.5
6	8	2.5	1.5

x	y	$(x - \bar{x})$	$(y - \bar{y})$
1	5	-2.5	-1.5
2	9	-1.5	2.5
3	7	-0.5	0.5
4	4	0.5	-2.5
5	8	1.5	1.5
6	6	2.5	-0.5

Covariance exercise

$\bar{x} = 3.5, \bar{y} = 6.5$

- Calculate $(x - \bar{x})(y - \bar{y})$:

x	y	$(x - \bar{x})$	$(y - \bar{y})$	$(x - \bar{x})(y - \bar{y})$
1	4	-2.5	-2.5	6.25
2	6	-1.5	-0.5	0.75
3	5	-0.5	-1.5	0.75
4	7	0.5	0.5	0.25
5	9	1.5	2.5	3.75
6	8	2.5	1.5	3.75

x	y	$(x - \bar{x})$	$(y - \bar{y})$	$(x - \bar{x})(y - \bar{y})$
1	5	-2.5	-1.5	3.75
2	9	-1.5	2.5	-3.75
3	7	-0.5	0.5	-0.25
4	4	0.5	-2.5	-1.25
5	8	1.5	1.5	2.25
6	6	2.5	-0.5	-1.25

Covariance Exercise

$\bar{x} = 3.5, \bar{y} = 6.5$

- Calculate sums:

x	y	$(x - \bar{x})$	$(y - \bar{y})$	$(x - \bar{x})(y - \bar{y})$
1	4	-2.5	-2.5	6.25
2	6	-1.5	-0.5	0.75
3	5	-0.5	-1.5	0.75
4	7	0.5	0.5	0.25
5	9	1.5	2.5	3.75
6	8	2.5	1.5	3.75
			Σ	**15.5**

x	y	$(x - \bar{x})$	$(y - \bar{y})$	$(x - \bar{x})(y - \bar{y})$
1	5	-2.5	-1.5	3.75
2	9	-1.5	2.5	-3.75
3	7	-0.5	0.5	-0.25
4	4	0.5	-2.5	-1.25
5	8	1.5	1.5	2.25
6	6	2.5	-0.5	-1.25
			Σ	**-0.5**

Covariance Exercise

$\bar{x} = 3.5, \bar{y} = 6.5$

- Calculate covariance:

x	y
1	4
2	6
3	5
4	7
5	9
6	8

$$cov(X,Y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})$$

$$= \frac{15.5}{6} = \mathbf{2.583}$$

Σ	15.5

x	y
1	5
2	9
3	7
4	4
5	8
6	6

$$cov(X,Y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})$$

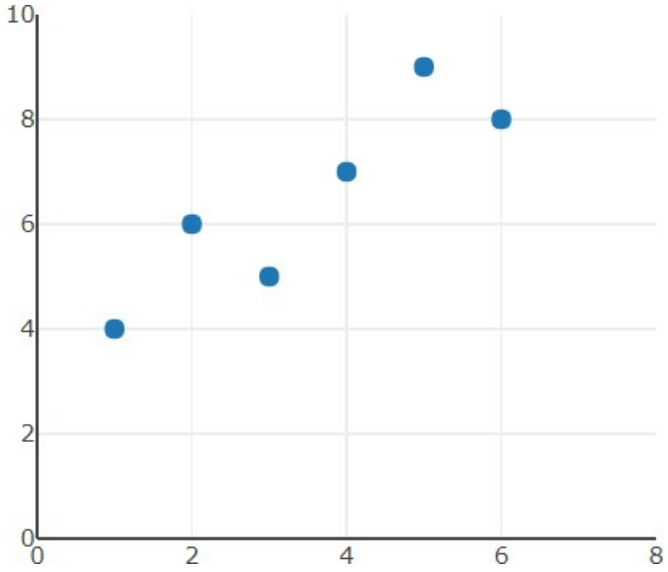
$$= \frac{-0.5}{6} = \mathbf{-0.083}$$

Σ	-0.5

Covariance Exercise

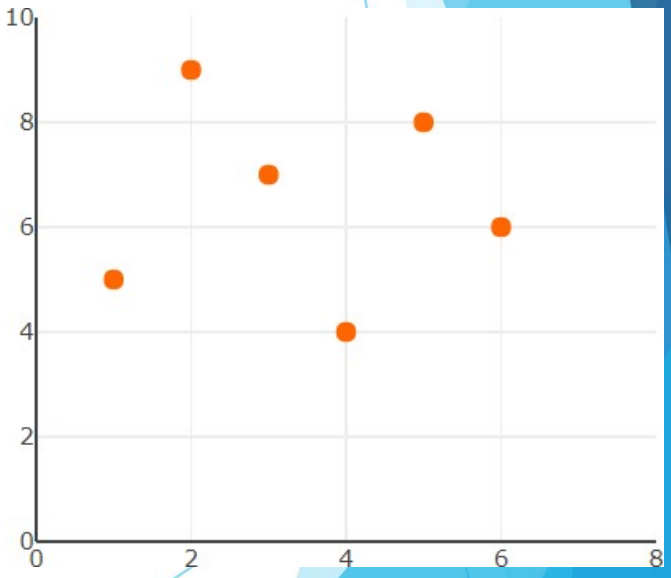
- Compare covariances:

x	y
1	4
2	6
3	5
4	7
5	9
6	8



cov(x,y) = 2.583

x	y
1	5
2	9
3	7
4	4
5	8
6	6



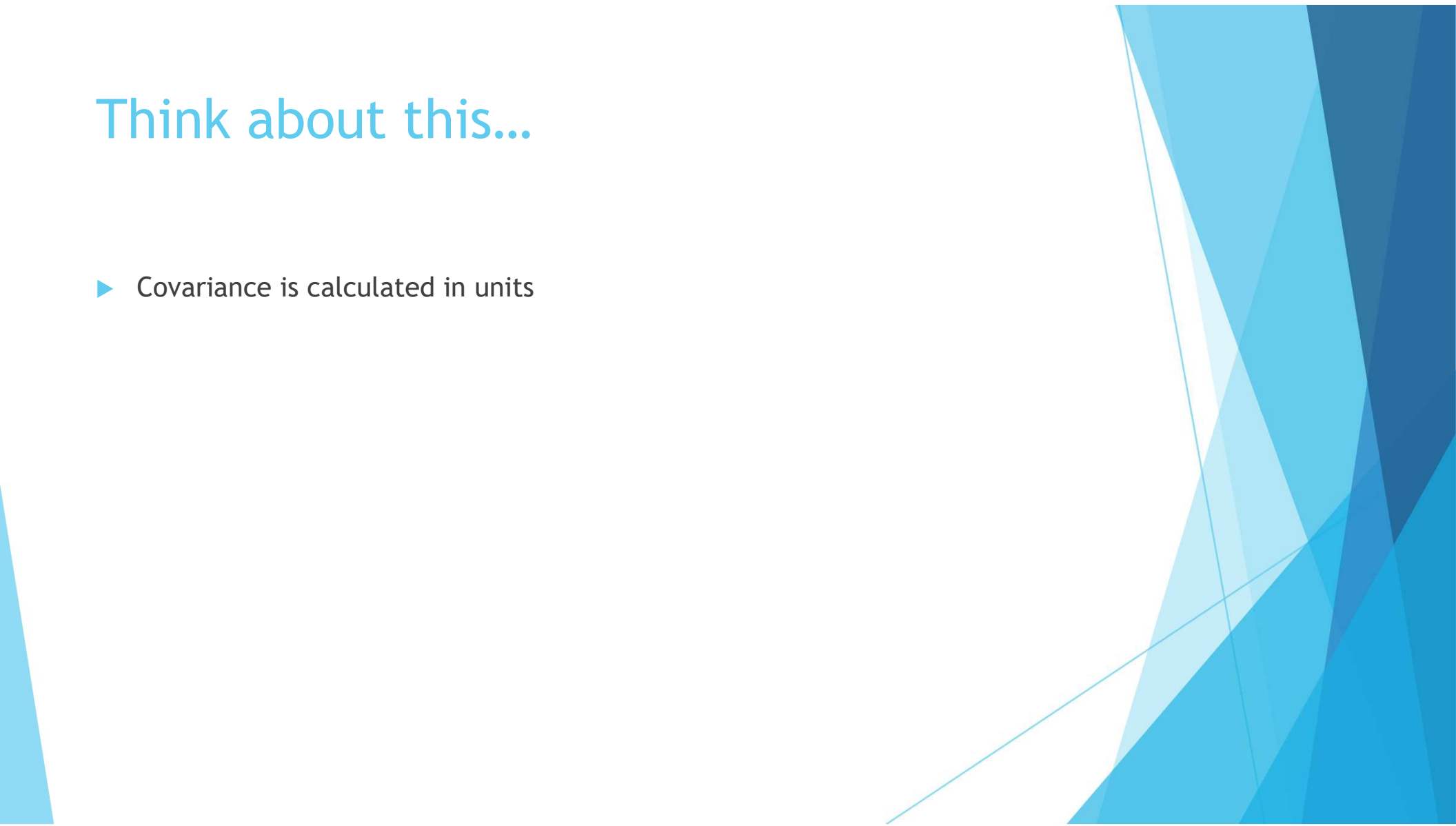
cov(x,y) = -0.083

Practical usage

- Raj is an investor. His portfolio primarily tracks the performance of the Nifty and Raj wants to add the stock of ABC Corp. Before adding the stock to his portfolio, he wants to assess the directional relationship between the stock and the Nifty.
- Raj does not want to increase the unsystematic risk of his portfolio. Thus, he is not interested in owning securities in the portfolio that tend to move in the same direction.
- If there is positive covariance, then it indicates that the price of the stock and the Nifty tend to move in the same direction.

Think about this…

- Covariance is calculated in units



Pearson correlation coefficient

Pearson Correlation Coefficient

- In order to normalize values coming from two different distributions, we use:

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{\frac{1}{n}\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}\sqrt{\frac{\Sigma(y-\bar{y})^2}{n}}} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$$

ρ = Greek letter "rho"

cov = covariance

σ = standard deviation

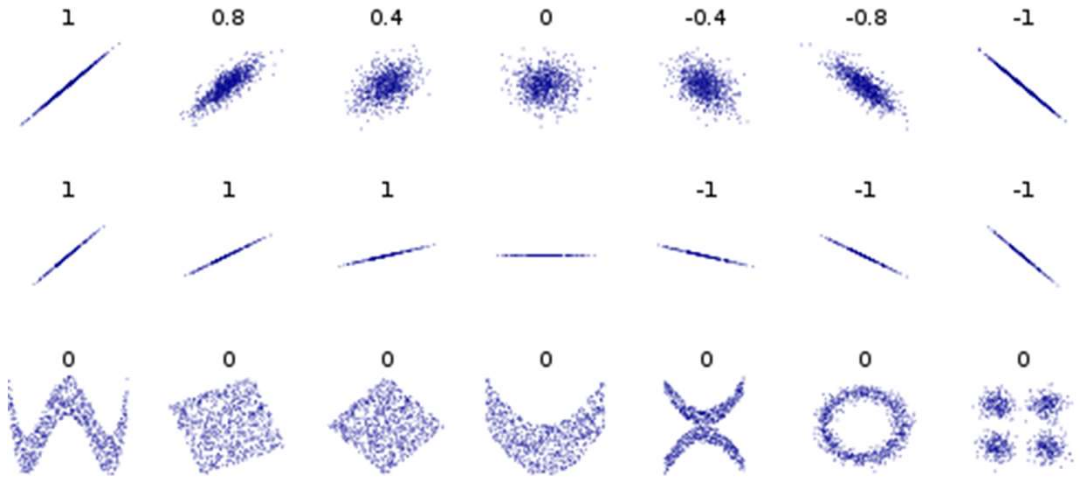
$\bar{x}$ = mean of X

Pearson Correlation Coefficient

- Values fall between +1 and −1, where
  - 1 = total positive linear correlation
  - 0 = no linear correlation
  - −1 = total negative linear correlation

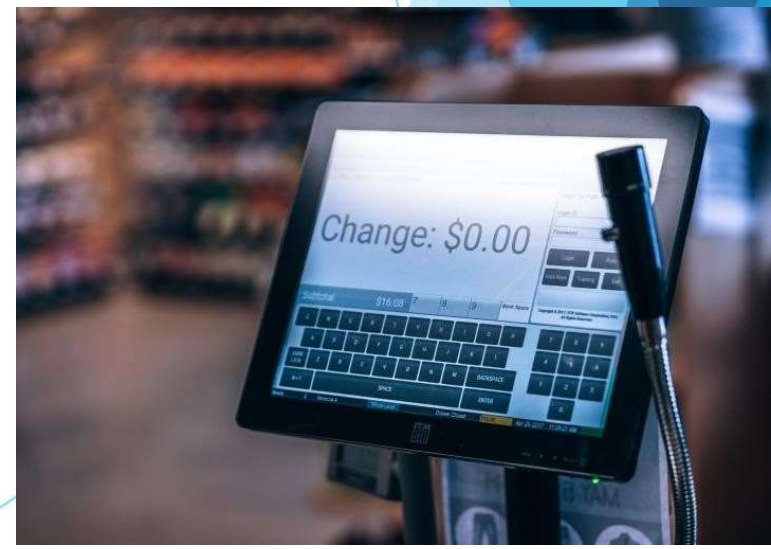
Pearson Correlation Coefficient

- Several sets of (x, y) points, with the correlation coefficient for each set:



Correlation Exercise

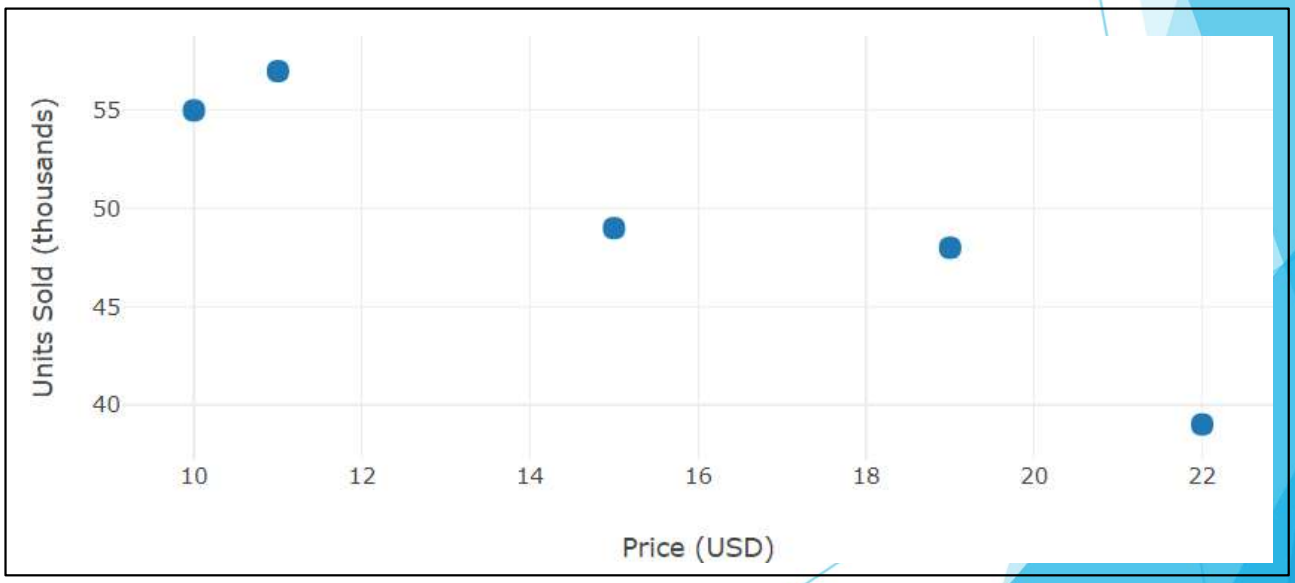
- A company decides to test sales of a new product in five separate markets, to determine the best price point.
- They set a different price in each market and record sales volume over the same 30 dayperiod.



Correlation Exercise

- These are theresults
- Plot theresults

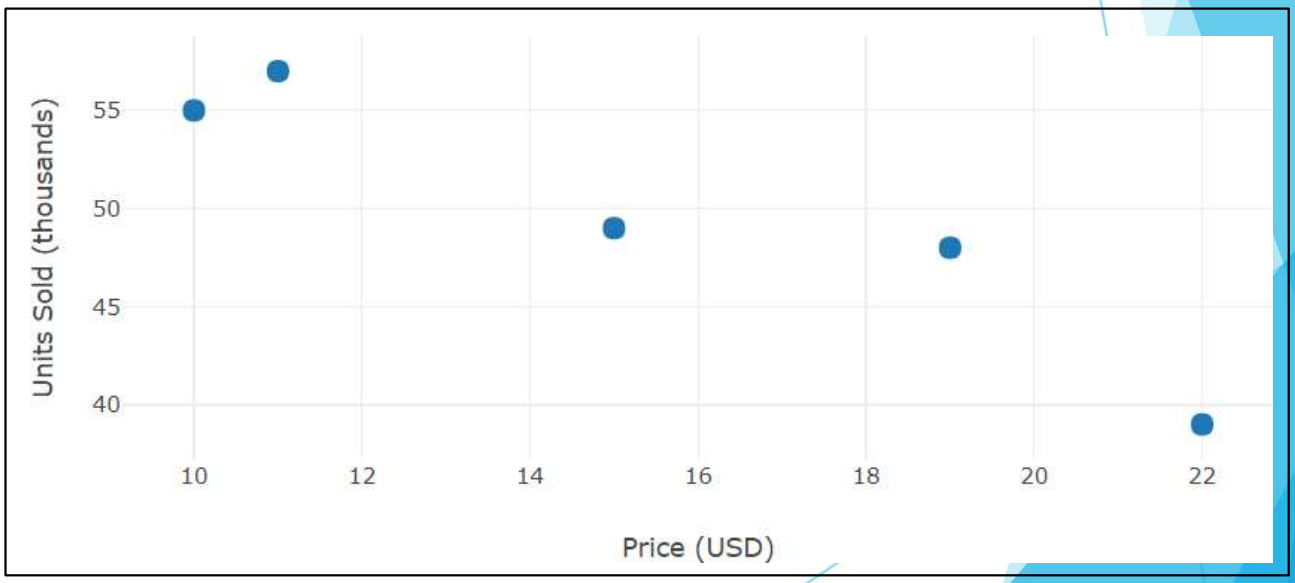
Price (USD)	Units Sold (thousands)
10	55
11	57
15	49
19	48
22	39



Correlation Exercise

- There appears to be a strong correlation, but how strong?

Price (USD)	Units Sold (thousands)
10	55
11	57
15	49
19	48
22	39



Correlation Exercise

1. Recall the simplified correlation formula:

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2}\sqrt{\sum(y-\bar{y})^2}}$$

Price (USD)	Units Sold (thousands)
10	55
11	57
15	49
19	48
22	39

2. Find the mean of x and y:

$$\bar{x} = \frac{10+11+15+19+22}{5} = 15.4$$

$$\bar{y} = \frac{55+57+49+48+39}{5} = 49.6$$

Correlation Exercise

$$\rho_{X,Y} = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sqrt{\sum(x - \bar{x})^2}\sqrt{\sum(y - \bar{y})^2}}$$

$$\bar{x} = 15.4 \quad \bar{y} = 49.6$$

3. Calculate $(x - \bar{x})$ and $(y - \bar{y})$:

Price (USD)	Units Sold (thousands)	$(x - \bar{x})$	$(y - \bar{y})$
10	55	-5.4	5.4
11	57	-4.4	7.4
15	49	-0.4	-0.6
19	48	3.6	-1.6
22	39	6.6	-10.6

Correlation Exercise

$$\rho_{X,Y} = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sqrt{\sum(x - \bar{x})^2}\sqrt{\sum(y - \bar{y})^2}}$$

$$\bar{x} = 15.4 \quad \bar{y} = 49.6$$

4. Calculate $(x - \bar{x})(y - \bar{y})$:

Price (USD)	Units Sold (thousands)	$(x - \bar{x})$	$(y - \bar{y})$	$(x - \bar{x})(y - \bar{y})$
10	55	-5.4	5.4	-29.16
11	57	-4.4	7.4	-32.56
15	49	-0.4	-0.6	0.24
19	48	3.6	-1.6	-5.76
22	39	6.6	-10.6	-69.96

Correlation Exercise

$$\rho_{X,Y} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2}\sqrt{\sum(y-\bar{y})^2}}$$

$$\bar{x} = 15.4 \quad \bar{y} = 49.6$$

5. Calculate $(x-\bar{x})^2$ and $(y-\bar{y})^2$:

Price (USD)	Units Sold (thousands)	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})(y-\bar{y})$	$(x-\bar{x})^2$	$(y-\bar{y})^2$
10	55	-5.4	5.4	-29.16	29.16	29.16
11	57	-4.4	7.4	-32.56	19.36	54.76
15	49	-0.4	-0.6	0.24	0.16	0.36
19	48	3.6	-1.6	-5.76	12.96	2.56
22	39	6.6	-10.6	-69.96	43.56	112.36

Correlation Exercise

$$\rho_{X,Y} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$$

$$\bar{x} = 15.4 \quad \bar{y} = 49.6$$

6. Compute the sums:

Price (USD)	Units Sold (thousands)	$(x - \bar{x})$	$(y - \bar{y})$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$	$(y - \bar{y})^2$
10	55	-5.4	5.4	-29.16	29.16	29.16
11	57	-4.4	7.4	-32.56	19.36	54.76
15	49	-0.4	-0.6	0.24	0.16	0.36
19	48	3.6	-1.6	-5.76	12.96	2.56
22	39	6.6	-10.6	-69.96	43.56	112.36
			Σ	**-137.2**	**105.2**	**199.2**

Correlation Exercise

$$\rho_{X,Y} = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\sqrt{\Sigma(x - \bar{x})^2}\sqrt{\Sigma(y - \bar{y})^2}}$$

$$\bar{x} = 15.4 \quad \bar{y} = 49.6$$

7. Plug these into the original formula:

Price (USD)	Units Sold (thousands)	$(x - \bar{x})$	$(y - \bar{y})$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$	$(y - \bar{y})^2$
10	55	-5.4	5.4	-29.16	29.16	29.16
11	57	-4.4	7.4	-32.56	19.36	54.76
15	49	-0.4	-0.6	0.24	0.16	0.36
19	48	3.6	-1.6	-5.76	12.96	2.56
22	39	6.6	-10.6	-69.96	43.56	112.36
			Σ	**-137.2**	**105.2**	**199.2**

Correlation Exercise

$$\rho_{X,Y} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$$

$$\bar{x} = 15.4 \quad \bar{y} = 49.6$$

7. Plug these into the original formula:

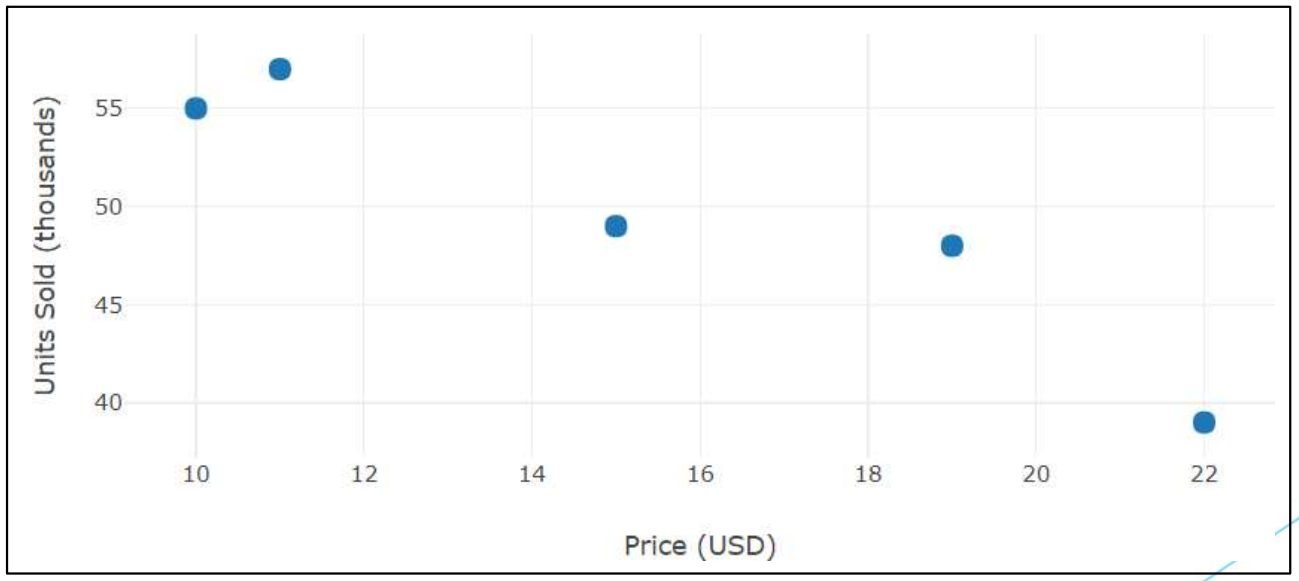
$$\rho_{X,Y} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}} = \frac{-137.2}{\sqrt{105.2}\sqrt{199.2}}$$

$$= \frac{-137.2}{10.26 \times 14.11} = \frac{-137.2}{144.8} = \mathbf{-0.948}$$

Σ	-137.2	105.2	199.2

Correlation Exercise

- $\rho_{X,Y} = -0.948$ shows a *very* strong negative correlation!

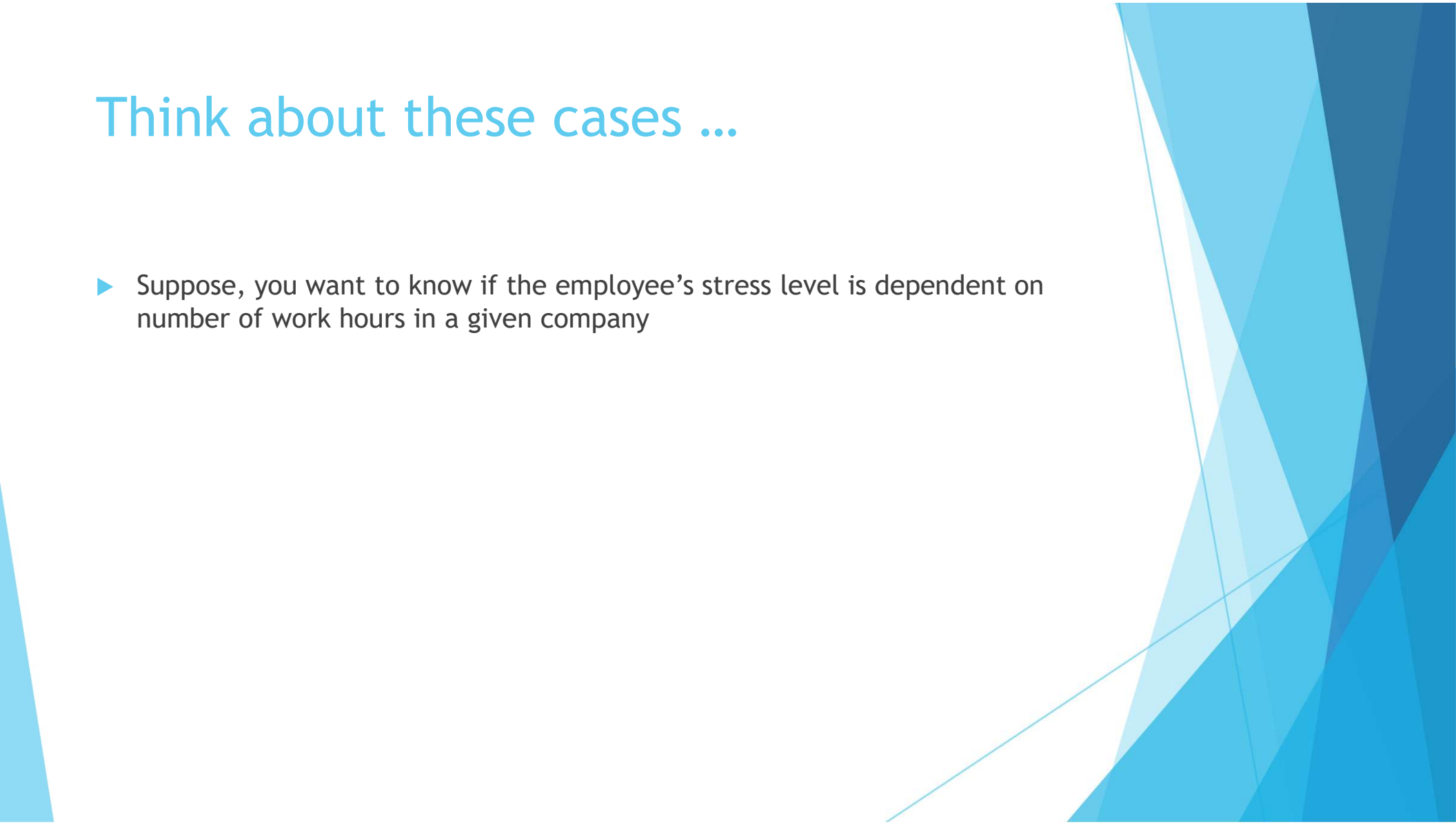


Practical example

- Let's say you were analyzing the relationship between your participants' age and reported level of income. You're curious as to if there is a positive or negative relationship between someone's age and their income level. After conducting the test, your Pearson correlation coefficient value is +0.20. Therefore, you would have a slightly positive correlation between the two variables, so the strength of the relationship is also positive and considered strong. You could confidently conclude there is a strong relationship and positive correlation between one's age and their income. In other words, as people grow older, their income tends to increase as well.

Think about these cases …

- Suppose, you want to know if the employee's stress level is dependent on number of work hours in a given company



Python Statistics Module

- This module provides functions for calculating mathematical statistics of numeric data
- All the functions defined can be found in the link:
- https://docs.python.org/3/library/statistics.html