**Exercise 1.13.** Prove that is the closest integer to , where . Hint: Let . Use induction and the definition of the Fibonacci numbers (see section 1.2.2) to prove that .

To prove is the closest integer to , let’s prove where by induction first.

Basis:

, and

. So is true for .

Induction:

Assume holds (for some unspecified value of n). It must then be shown that holds.

By definition:

Using the induction hypothesis that holds, the right-hand side can be rewritten to:

Since , so

The same for ,

So

Thereby holds.

Since both the basis and the inductive step have been proved, it has now been proved by mathematical induction that holds for all natural n.

Since ,

And

So is the closest integer to .

Q.E.D