GARCH MODELS IN R

Markets take the stairs up, but the elevator down

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How?

Change the argument `distribution.model` of `ugarchspec()` from `"norm"` to `"sstd"`:

```
garchspec <- ugarchspec(
          mean.model=list(armaOrder=c(0,0)),
          variance.model=list(model="sGARCH"),
          distribution.model = "norm")
```

↓

```
garchspec <- ugarchspec(
          mean.model=list(armaOrder=c(0,0)),
          variance.model=list(model="sGARCH"),
          distribution.model = "sstd")
```

The normal GARCH model

Under the model assumptions

$$R_t = \mu_t + e_t$$
$$e_t \sim N(0, \sigma_t^2)$$

it follows that

$$\frac{R_t - \mu_t}{\sigma_t} \sim N(0,1)$$

Let's test

- Caveat: The normality of the standardized returns follows from an assumption
- Let's compute the standardized returns and test whether the assumption is correct.

Estimated standardized returns

- Formula

$$Z_t = \frac{R_t - \hat{\mu}_t}{\hat{\sigma}_t}$$

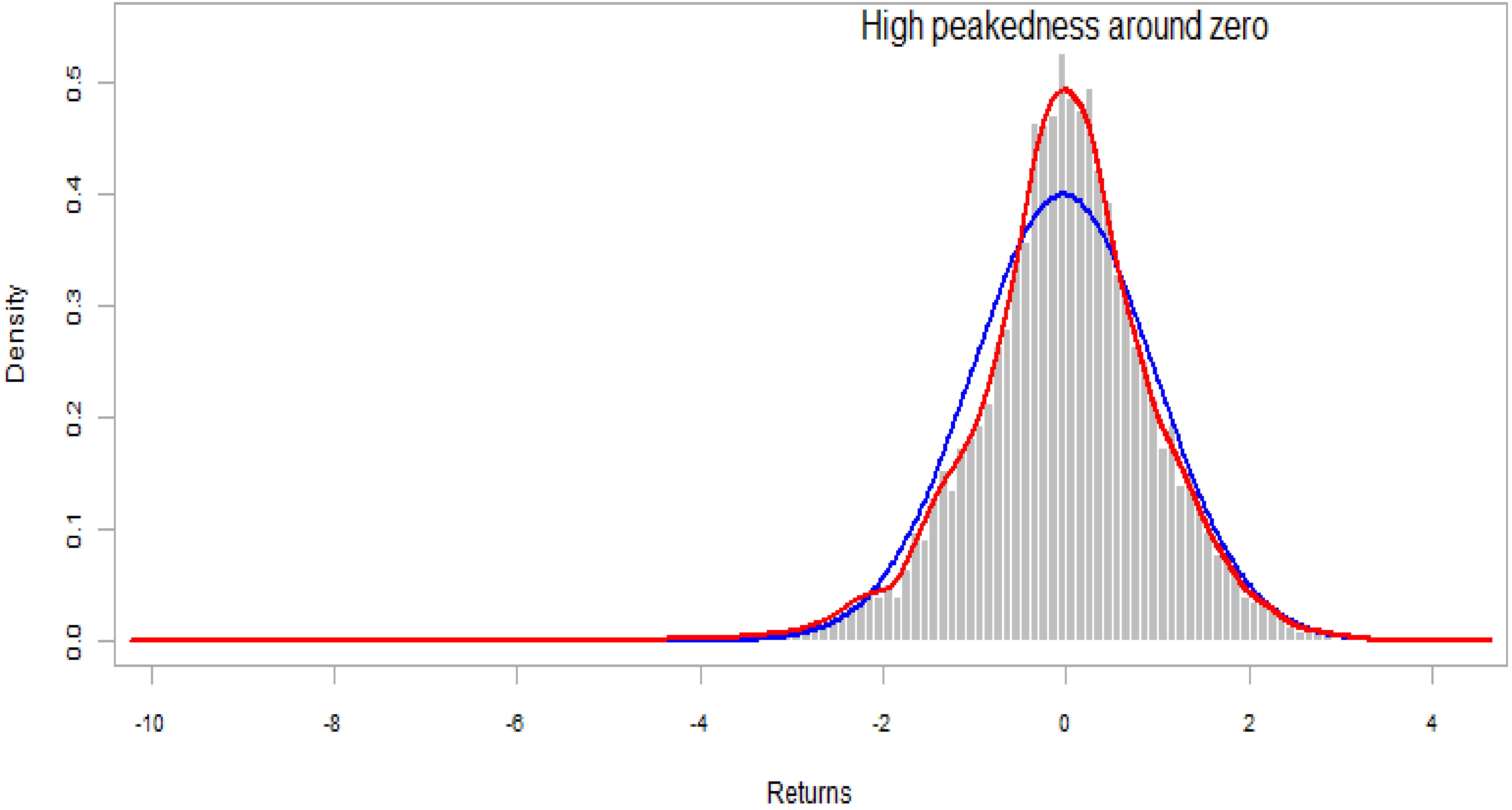
- Calculation in R

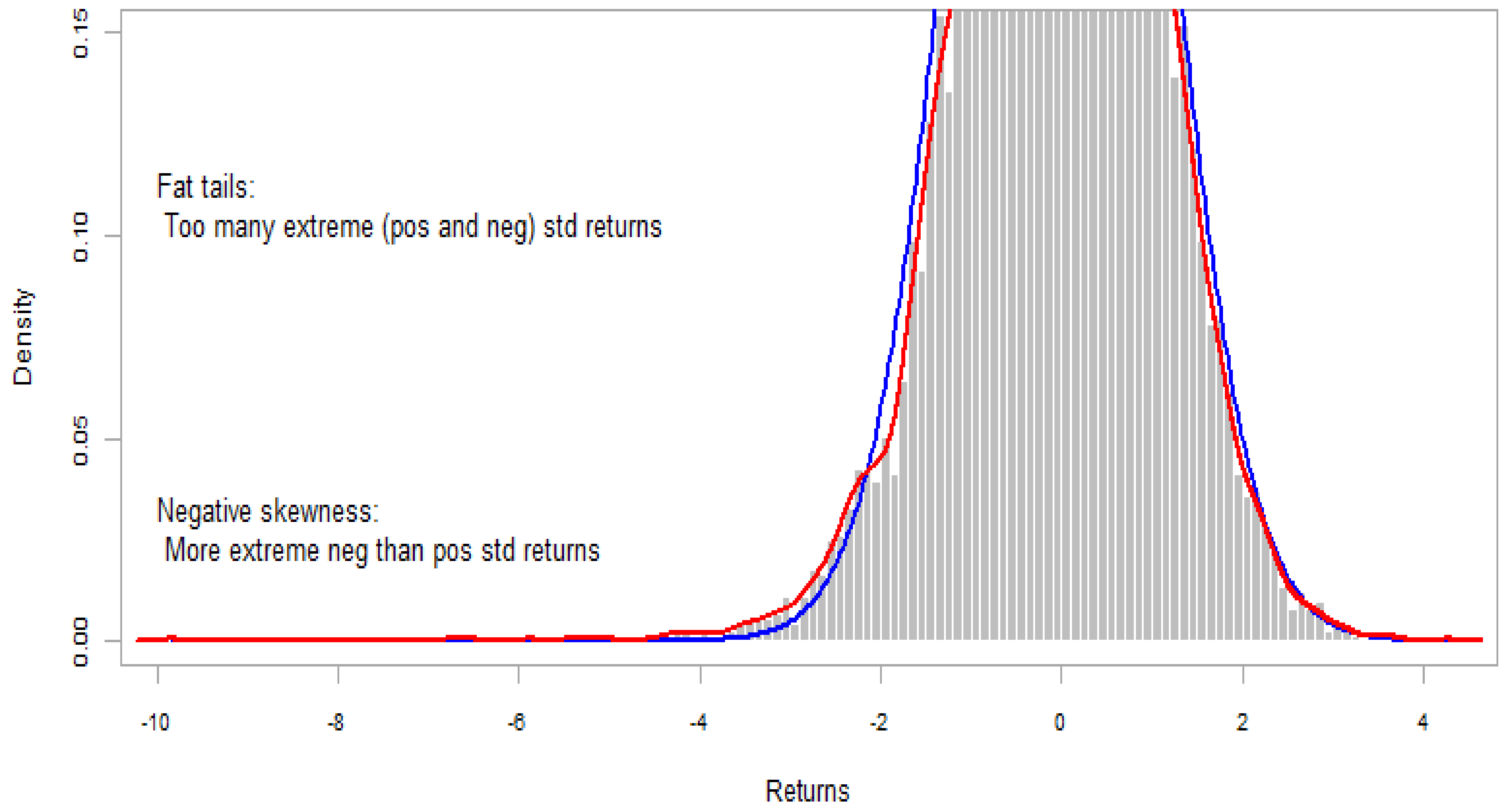
```
# obtain standardized returns
stdret <- residuals(garchfit, standardize = TRUE)
```

Testing the normality assumption

- Visual analysis

```
library(PerformanceAnalytics)
chart.Histogram(sp500ret, methods = c("add.normal", "add.density"),
          colorset=c("gray","red","blue"))
```


High peakedness around zero
Density
0.0
0.1
0.2
0.3
0.4
0.5
-10
-8
-6
-4
-2
0
2
4
Returns


Fat tails:
Too many extreme (pos and neg) std returns
Negative skewness:
More extreme neg than pos std returns
Density
0.00
0.05
0.10
0.15
-10
-8
-6
-4
-2
0
2
4
Returns

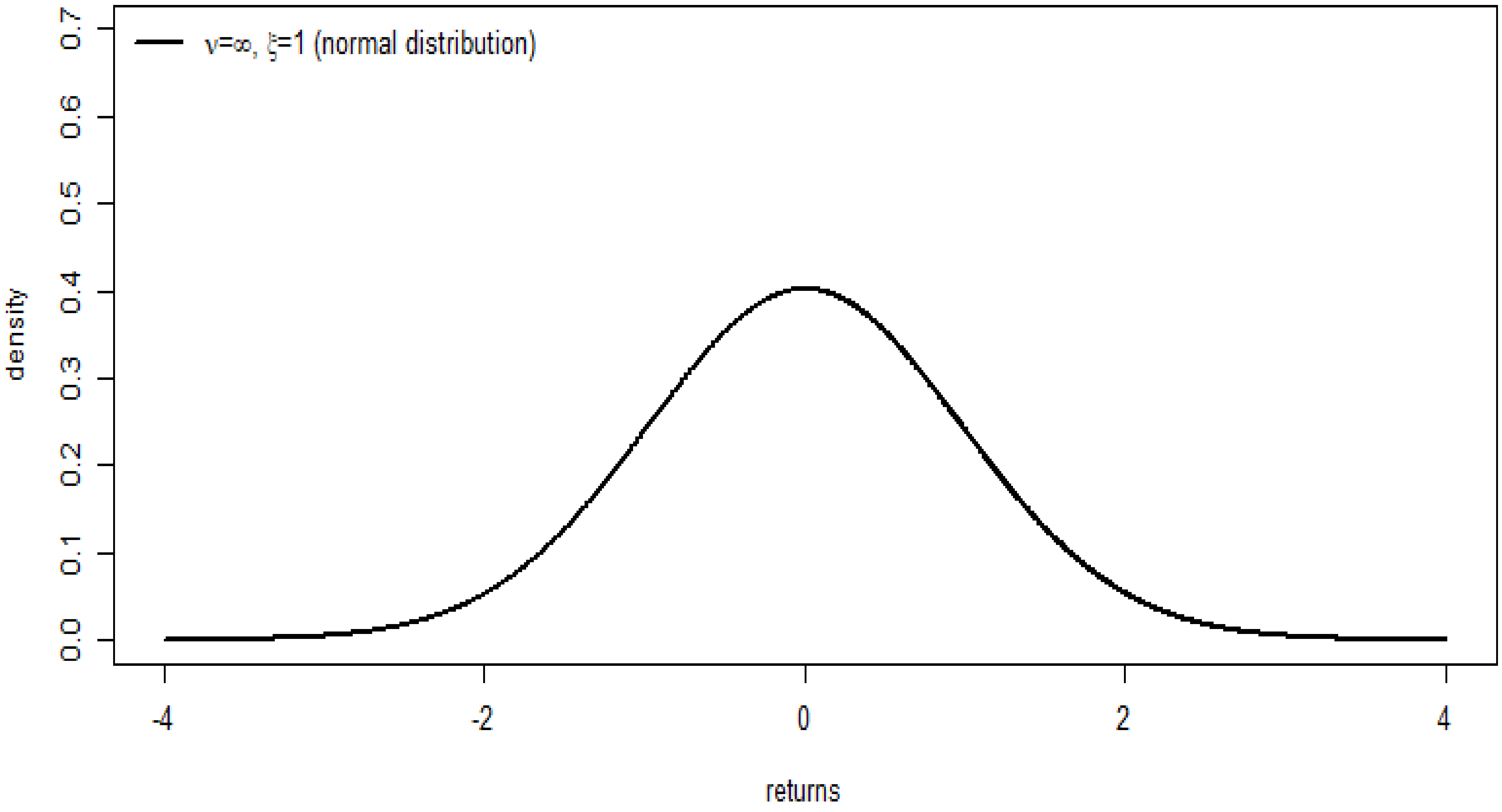
Solution

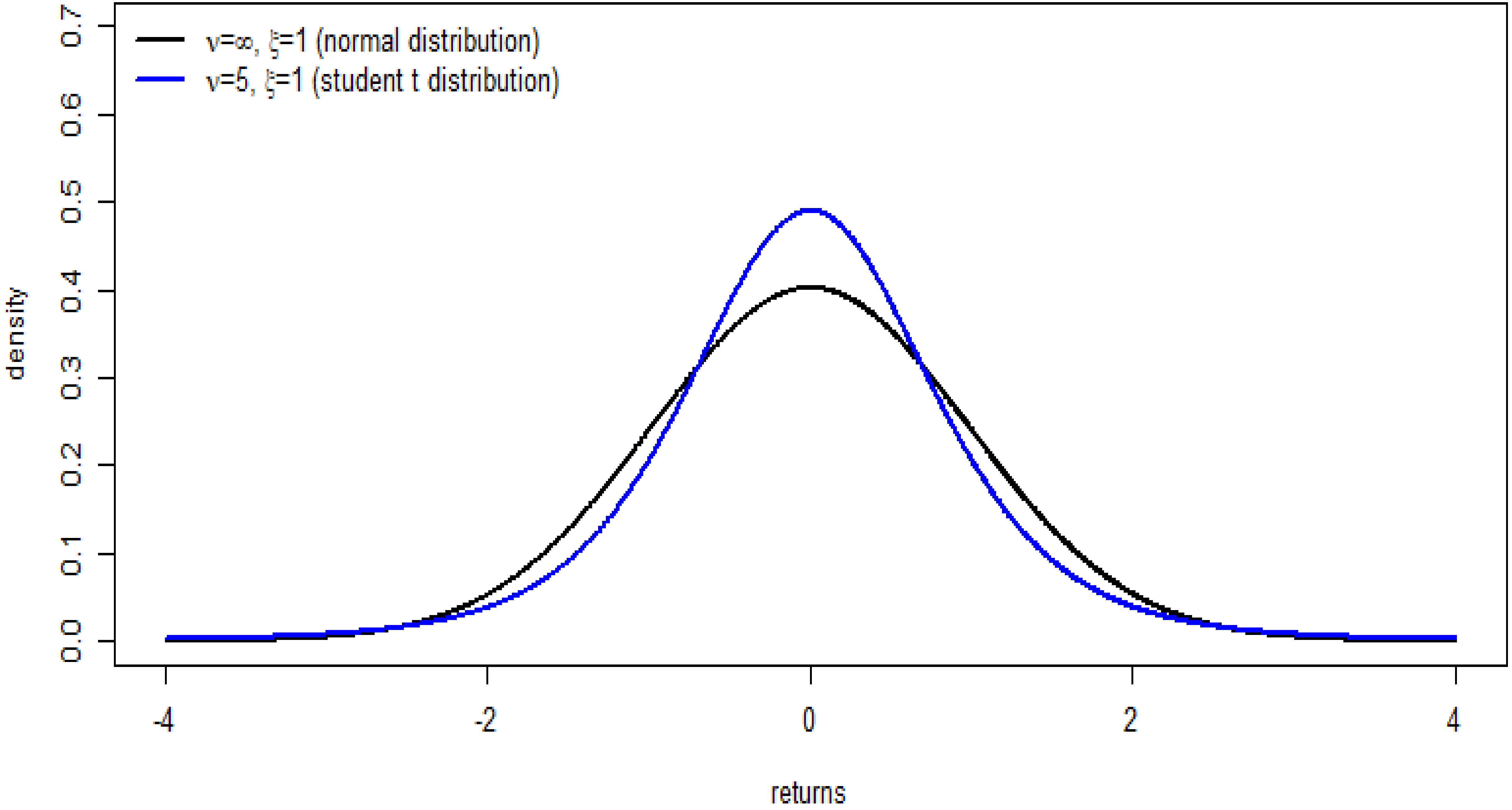
- A realistic distribution thus needs to accommodate the presence of
  - fat tails: higher probability to observe large (positive or negative) returns than under the normal distribution
  - skewness: asymmetry of the return distribution
- In `rugarch` this is possible with the **skewed student t** distribution:

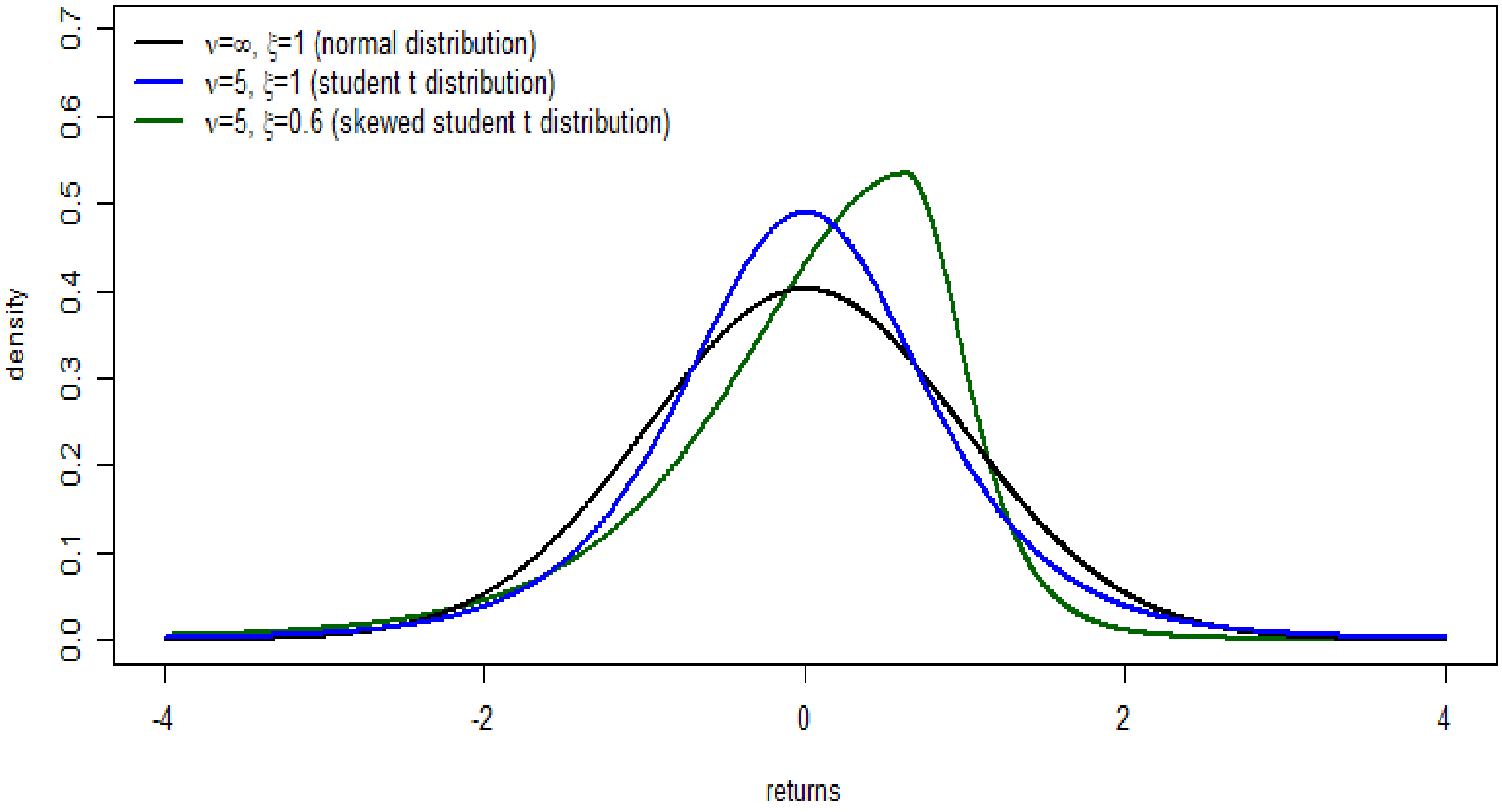
```
garchspec <- ugarchspec(distribution.model = "sstd")
```

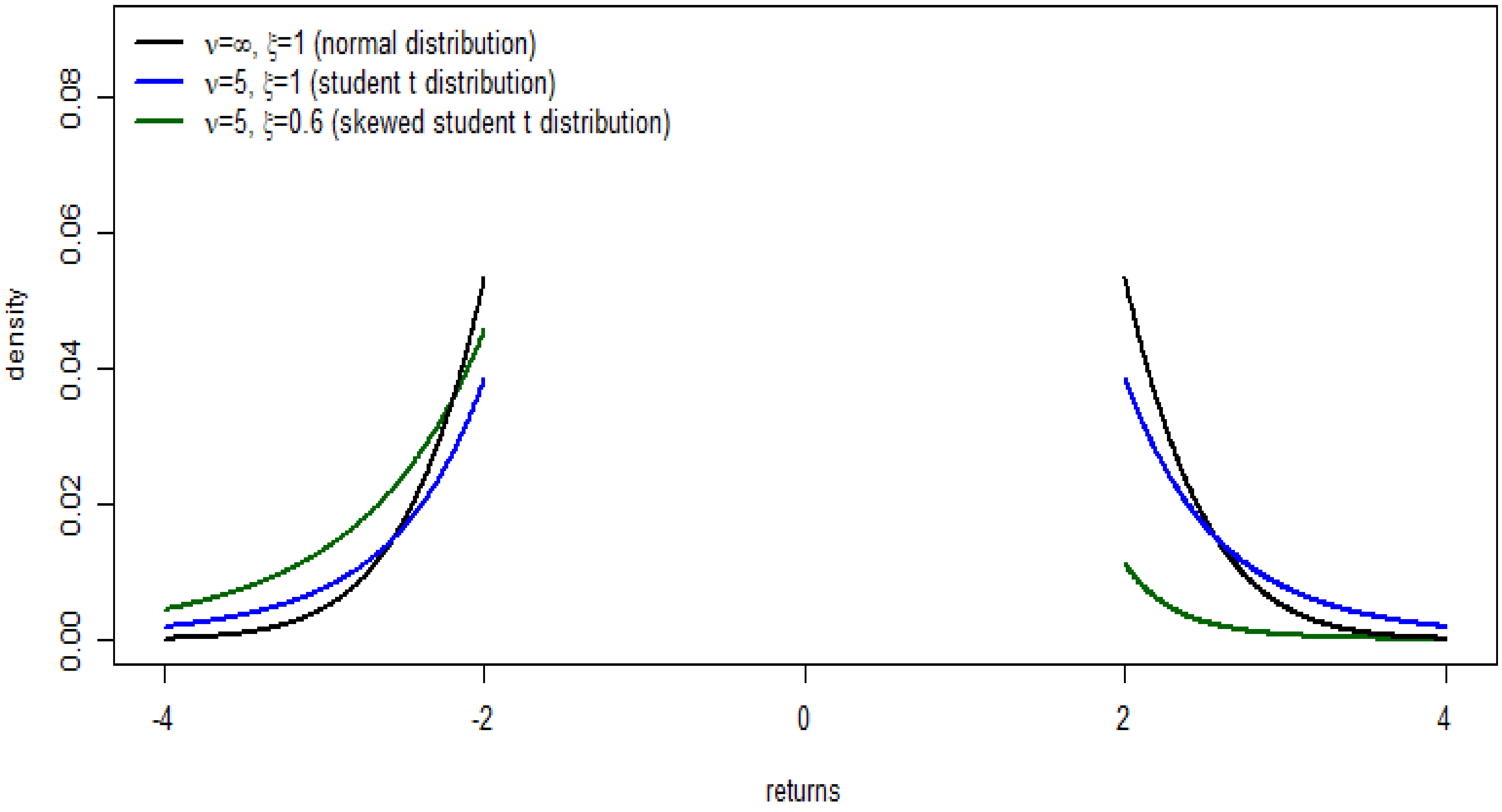
Parameters of the skewed student t distribution

- Compared to the normal distribution, the skewed student t distribution has two extra parameters:
  - Degrees of freedom parameter ν (in rugarch: `shape`): the lower is ν the fatter the tails.
  - Skewness parameter ξ (in rugarch: `skew`) : when $\xi = 1$: symmetry. When $\xi < 1$: negative skewness. For $\xi > 1$: positive skewness.
- Special cases:
  - When $\nu = \infty$ and $\xi = 1$: normal distribution.
  - When $\xi = 1$: student t distribution.


ν=∞, ξ=1 (normal distribution)
density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
-4
-2
0
2
4
returns


ν=∞, ξ=1 (normal distribution)
ν=5, ξ=1 (student t distribution)
density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
-4
-2
0
2
4
returns


ν=∞, ξ=1 (normal distribution)
ν=5, ξ=1 (student t distribution)
ν=5, ξ=0.6 (skewed student t distribution)
density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
-4
-2
0
2
4
returns


ν=∞, ξ=1 (normal distribution)
ν=5, ξ=1 (student t distribution)
ν=5, ξ=0.6 (skewed student t distribution)
density
0.00
0.02
0.04
0.06
0.08
-4
-2
0
2
4
returns

GARCH model estimation with skewed student t

- Set argument `distribution.model` to "sstd"

```
garchspec <- ugarchspec(mean.model = list(armaOrder = c(0,0)),
                        variance.model = list(model = "sGARCH"),
                        distribution.model = "sstd")
```

- Estimate the model

```
garchfit <- ugarchfit(data = sp500ret, spec = garchspec)
```

- We obtain

```
coef(garchfit)

          mu        omega       alpha1        beta1         skew        shape
5.669200e-04 6.281258e-07 7.462984e-02 9.223701e-01 9.436331e-01 6.318621e+00
```

GARCH MODELS IN R

Let's practice!

GARCH MODELS IN R

Size and sign of e_t matter for volatility prediction!

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Negative returns induce higher leverage

- $R_t < 0$
- $\downarrow$ market value
- $\uparrow$ leverage = debt / market value
- $\uparrow$ volatility

Two equations

- Separate equations for the variance following negative and positive unexpected return $e_t = R_t - \mu_t$:

$$\sigma_t^2 = \begin{cases} & : \ e_{t-1} \leq 0 \\ & : \ e_{t-1} > 0 \end{cases}$$

In case of a positive surprise

... we take the usual GARCH(1,1) equation:

$$\sigma_t^2 = \begin{cases} & : e_{t-1} \leq 0 \\ \omega + \alpha e_{t-1}^2 + \beta \sigma_{t-1}^2 & : e_{t-1} > 0 \end{cases}$$

In case of a negative surprise

- The predicted variance should be higher than after a positive surprise.
- This means a higher coefficient multiplying the squared prediction error, namely $\alpha + \gamma$ instead of α with $\gamma \geq 0$

$$\sigma_t^2 = \begin{cases} \omega + (\alpha + \gamma) e_{t-1}^2 + \beta \sigma_{t-1}^2 & : \; e_{t-1} \leq 0 \\ \omega + \alpha e_{t-1}^2 + \beta \sigma_{t-1}^2 & : \; e_{t-1} > 0 \end{cases}$$

with $\gamma \geq 0$

= **GJR model** proposed Glosten, Jagannathan and Runkle.

How?

Change the argument `variance.model` of `ugarchspec()` from `model="sGARCH"` to `model="gjrGARCH"`:

```
garchspec <- ugarchspec(
           mean.model=list(armaOrder=c(0,0)),
           variance.model=list(model="sGARCH"),
           distribution.model = "sstd")
```

↓

```
garchspec <- ugarchspec(
           mean.model=list(armaOrder=c(0,0)),
           variance.model=list(model="gjrGARCH"),
           distribution.model = "sstd")
```

Illustration on MSFT returns

- Estimate the model

```
garchfit <- ugarchfit(data = msftret, spec = garchspec)
```

- Inspect the GARCH coefficients

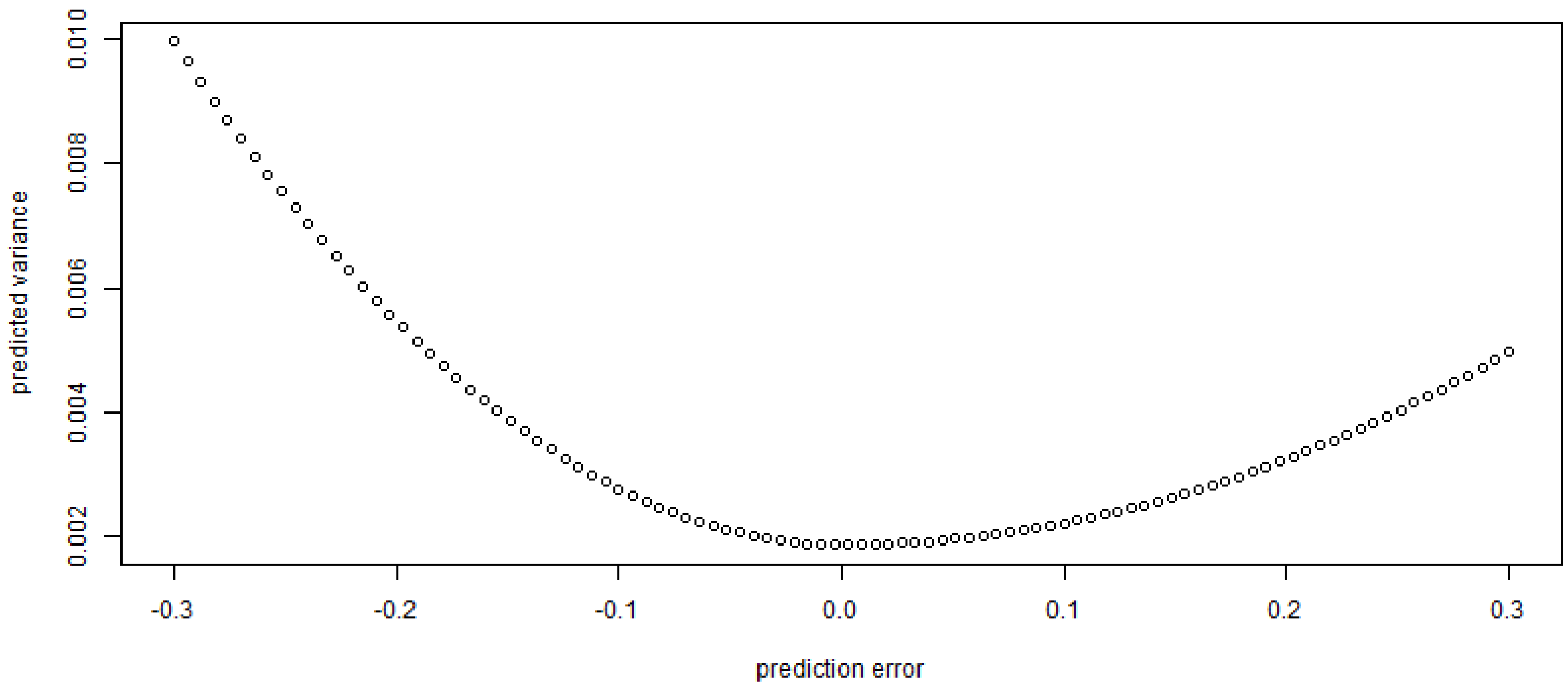
```
> coef(garchfit)[2:5]

       omega       alpha1        beta1       gamma1
2.007875e-06 3.423336e-02 9.363302e-01 5.531854e-02
```

$$\hat{\sigma}_t^2 = \begin{cases} 2.0 * 10^{-6} + 0.09 e_{t-1}^2 + 0.94 \sigma_{t-1}^2 & : \ e_{t-1} \leq 0 \\ 2.0 * 10^{-6} + 0.03 e_{t-1}^2 + 0.94 \sigma_{t-1}^2 & : \ e_{t-1} > 0 \end{cases}$$

Visualize volatility response using newsimpact()

```
out <- newsimpact(garchfit)
plot(out$zx, out$zy, xlab = "prediction error", ylab = "predicted variance")
```



GARCH MODELS IN R

Final Slide

GARCH MODELS IN R

No pain, no gain

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GARCH-in-mean model

- Quantify the risk-reward trade-off.
- Risk: σ_t^2. Reward: μ_t.
- GARCH-in-mean model:

$$\mu_t = \mu + \lambda\sigma_t^2$$

- $\lambda > 0$ is the risk/reward parameter indicating the increased in expected return per unit of variance risk.

How?

Change the argument `mean.model` in `ugarchspec()` from `list(armaOrder = c(0,0))` to `list(armaOrder = c(0,0), archm = TRUE, archpow = 2)`:

```
garchspec <- ugarchspec(
          mean.model = list(armaOrder = c(0,0)),
          variance.model = list(model = "gjrGARCH"),
          distribution.model = "sstd")
```

↓

```
garchspec <- ugarchspec(
          mean.model = list(armaOrder = c(0,0), archm = TRUE, archpow = 2),
          variance.model = list(model = "gjrGARCH"),
          distribution.model = "sstd")
```

Application to daily S&P 500 returns

- Estimation

```
garchfit <- ugarchfit( data = sp500ret , spec = garchspec)
```

- Inspection of estimated coefficients for the mean

```
round(coef(garchfit)[1:2],4)
    mu  archm
0.0002 1.9950
```

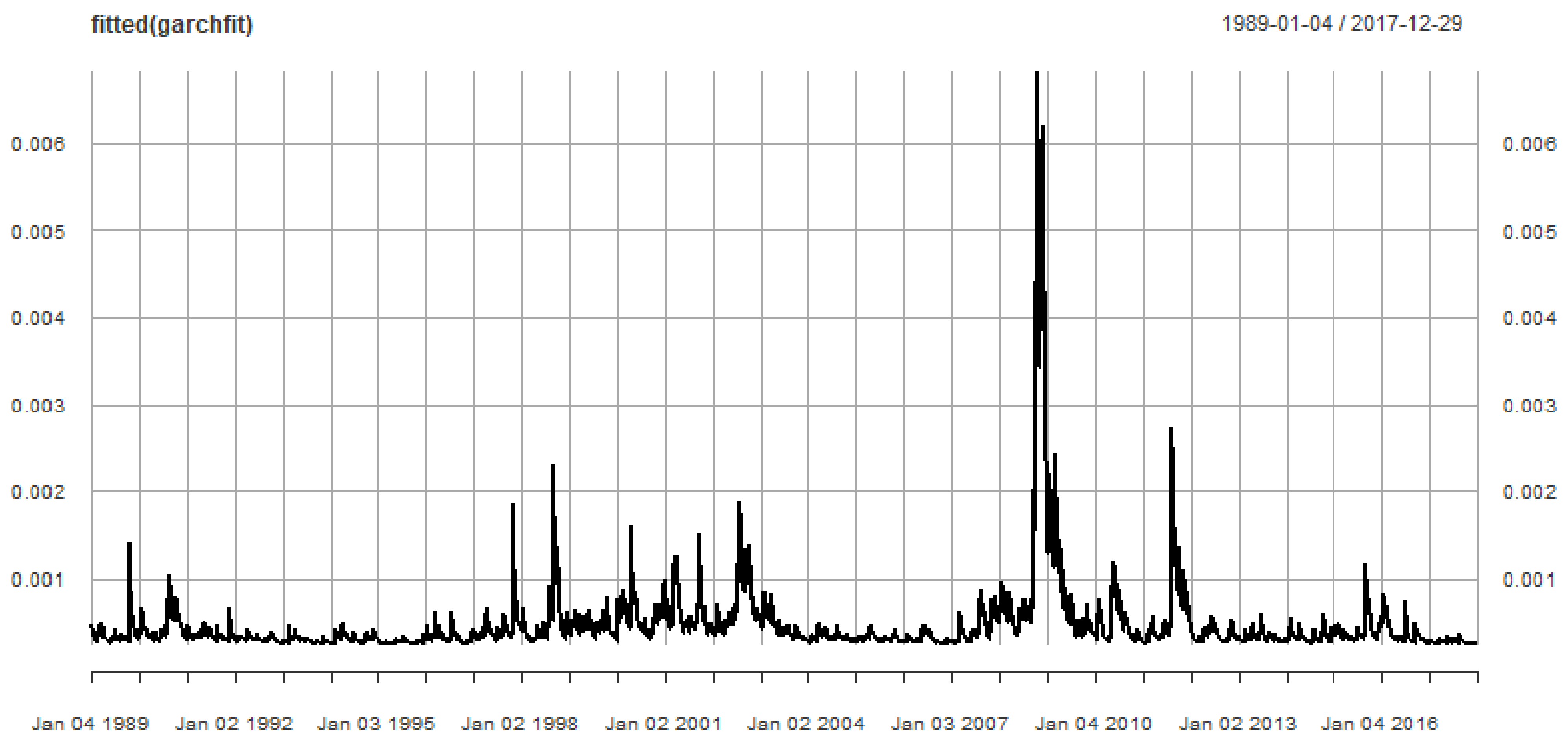
- Predicted mean returns

$$\hat{\mu}_t = 0.0002 + 1.9950\hat{\sigma}_t^2$$

Time series plot of predicted returns

- Plot them in R

```
plot(fitted(garchfit))
```



Today's return predicts tomorrow's return

- The GARCH-in-mean uses the financial theory of a risk-reward trade-off to build a conditional mean model
- Let's now use statistical theory to make a mean model that exploits the correlation between today's return and tomorrow's return.
- The most popular model is the AR(1) model:
  - AR(1) stands for autoregressive model of order 1
  - It predicts the next return using the deviation of the return from its long term mean value μ:

$$\mu_t = \mu + \rho (R_{t-1} - \mu)$$

A positive autoregressive coefficient

$$\mu_t = \mu + \rho\,(R_{t-1} - \mu)$$

- $\rho > 0$:
  - A higher (resp. lower) than average return is followed by a higher (resp. lower) than average return.
  - Possible explanation: markets underreact to news and hence there is momentum in returns.
- $|\rho| < 1$: Mean reversion: The deviations of R_t from μ are transitory.

A negative autoregressive coefficient

$$\mu_t = \mu + \rho \left(R_{t-1} - \mu\right)$$

- $\rho < 0$:
  - A higher (resp. lower) than average return is followed by a lower (resp. higher) than average return.
  - Possible explanation: markets overreact to news and hence there is reversal in returns.

Application to daily S&P 500 returns

- Specification and estimation of AR(1)-GJR GARCH with sst distribution

```
garchspec <- ugarchspec(
           mean.model = list(armaOrder = c(1,0)),
           variance.model = list(model = "gjrGARCH"),
           distribution.model = "sstd")
garchfit <- ugarchfit(data = sp500ret, spec = garchspec)
```

- Estimates of the AR(1) model

```
round(coef(garchfit)[1:2], 4)
```

```
     mu     ar1
 0.0003 -0.0292
```

MA(1) and ARMA(1,1) model

- The Moving Average model of order 1 uses the deviation of the return from its conditional mean:

$$\mu_t = \mu + \theta(R_{t-1} - \mu_{t-1})$$

- ARMA(1,1) combines AR(1) and MA(1):

$$\mu_t = \mu + \rho(R_{t-1} - \mu) + \theta(R_{t-1} - \mu_{t-1})$$

How?

- MA(1)

```
garchspec <- ugarchspec(
          mean.model = list(armaOrder = c(0,1)),
          variance.model = list(model = "gjrGARCH"),
          distribution.model = "sstd")
```

- ARMA(1,1)

```
garchspec <- ugarchspec(
          mean.model = list(armaOrder = c(1,1)),
          variance.model = list(model = "gjrGARCH"),
          distribution.model = "sstd")
```

GARCH MODELS IN R

Your turn to change the mean.model argument

GARCH MODELS IN R

Complexity has a price

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Avoid unneeded complexity

- If you know
  - The mean dynamics are negligible
  - There is no leverage effect in the variance
  - The distribution is symmetric and fat-tailed

Then a constant mean, standard GARCH(1,1) with student t distribution is an appropriate specification to use:

```
garchspec <- ugarchspec(mean.model = list(armaOrder = c(0,0)),
                        variance.model = list(model = "sGARCH"),
                        distribution.model = "std")
```

Restrict the parameter estimates

- If you know that the parameters
  - are equal to a certain value
  - or, are inside an interval
- Then you should impose this in the specification using the methods
  - `setfixed()`
  - `setbounds()`

Application to exchange rates

- Specification and estimation

```
garchspec <- ugarchspec(mean.model = list(armaOrder = c(0,0)),
                        variance.model = list(model = "sGARCH"),
                        distribution.model = "std")

garchfit <- ugarchfit(data = EURUSDret, spec = garchspec)
```

- Estimation results

```
coef(garchfit)

           mu         omega        alpha1         beta1         shape
-3.562136e-05  8.005123e-08  3.097322e-02  9.674496e-01  8.821902e+00
```

Example of setfixed()

- If you know `alpha1 = 0.05` and `shape = 6`: impose those values in the estimation.
- How? Use of `setfixed()` method on a `ugarchspec` object

```
setfixed(garchspec) <- list(alpha1 = 0.05, shape = 6)
```

- Result

```
garchfit <- ugarchfit(data = EURUSDret, spec = garchspec)
coef(garchfit)
           mu         omega        alpha1         beta1         shape
-4.142922e-05  2.061772e-07  5.000000e-02  9.489622e-01  6.000000e+00
```

Bounds on parameters

- The GARCH parameters can be restricted to an interval.
- Sometimes the interval of plausible values is large:
  - To ensure the variance is positive, we require e.g. that all variance parameters $(\omega, \alpha, \beta, \gamma)$ are positive.
- Sometimes the interval of plausible values is smaller:
  - Likely values of α are in between 0.05 and 0.2
  - Likely values of β are in between 0.7 and 0.95
- Such bound constraints on the parameters can be imposed using the `setbounds()` method.

Example of setbounds()

```
setbounds(garchspec) <- list(alpha1 = c(0.05,0.2), beta1 = c(0.8,0.95))
```

Use your intuition to avoid unneeded complexity.

Use the information you have:

- to build simple (and smart) models
- to fix parameter values or set bounds
- to make the GARCH dynamics realistic:
    - mean reversion of the volatility around the sample standard deviation

```
sd(EURUSDret)
```

```
0.006194049
```

Volatility clusters and mean reversion of volatility

Variance targeting

- Mathematically, this means that the unconditional variance implied by the GARCH models equals the sample variance $\hat{\sigma}^2$.
- How? By setting the argument `variance.targeting = TRUE` in `variance.model` of `ugarchspec()`:

```
garchspec <- ugarchspec(mean.model = list(armaOrder = c(0,0)),
                        variance.model = list(model = "sGARCH",
                        variance.targeting = TRUE),
                        distribution.model = "std")

garchfit <- ugarchfit(data = EURUSDret, spec = garchspec)
```

```
all.equal(uncvariance(garchfit), sd(EURUSDret)^2, tol = 1e-4)
```

```
TRUE
```

GARCH MODELS IN R

Let's impose restrictions on the GARCH model