Obs	decade	Model	Dependent	Variable	DF	Estimate	StdErr	tValue	Probt	LowerCL	UpperCL
1	1870	MODEL1	log_wl	log_rra	1	1.94769	0.07626	25.54	<.0001	1.79578	2.09960
2	1880	MODEL1	log_wl	log_rra	1	1.81809	0.03650	49.81	<.0001	1.74600	1.89018
3	1890	MODEL1	log_wl	log_rra	1	2.04762	0.04431	46.21	<.0001	1.96000	2.13524
4	1900	MODEL1	log_wl	log_rra	1	1.75445	0.04108	42.71	<.0001	1.67329	1.83560
5	1910	MODEL1	log_wl	log_rra	1	1.92607	0.04399	43.79	<.0001	1.83925	2.01288
6	1920	MODEL1	log_wl	log_rra	1	1.91741	0.04154	46.15	<.0001	1.83536	1.99946
7	1930	MODEL1	log_wl	log_rra	1	1.97738	0.04498	43.96	<.0001	1.88854	2.06622
8	1940	MODEL1	log_wl	log_rra	1	1.79214	0.04186	42.81	<.0001	1.70946	1.87481
9	1950	MODEL1	log_wl	log_rra	1	1.81236	0.04572	39.64	<.0001	1.72206	1.90267
10	1960	MODEL1	log_wl	log_rra	1	1.90554	0.04396	43.34	<.0001	1.81884	1.99224
11	1970	MODEL1	log_wl	log_rra	1	1.73991	0.04245	40.99	<.0001	1.65630	1.82351
12	1980	MODEL1	log_wl	log_rra	1	1.93812	0.04885	39.68	<.0001	1.84193	2.03430
13	1990	MODEL1	log_wl	log_rra	1	1.88133	0.04398	42.78	<.0001	1.79475	1.96790
14	2000	MODEL1	log_wl	log_rra	1	1.94508	0.04248	45.79	<.0001	1.86148	2.02869
15	2010	MODEL1	log_wl	log_rra	1	1.74223	0.04283	40.68	<.0001	1.65785	1.82661
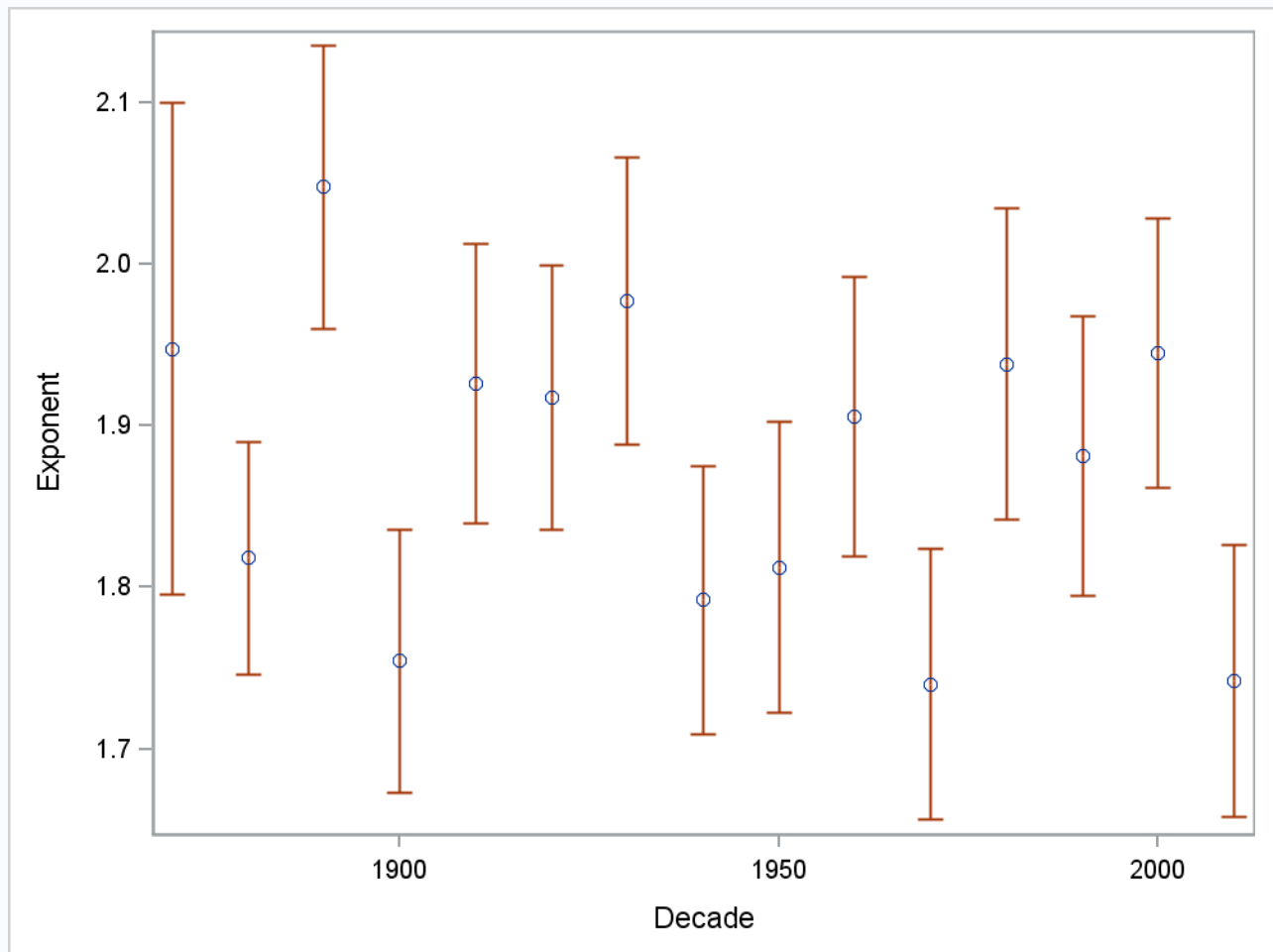
Error bars indicate the 95% CI for the exponent estimation. Notice no particular pattern throughout.

Estimates of Pythagorean Exponent by Year
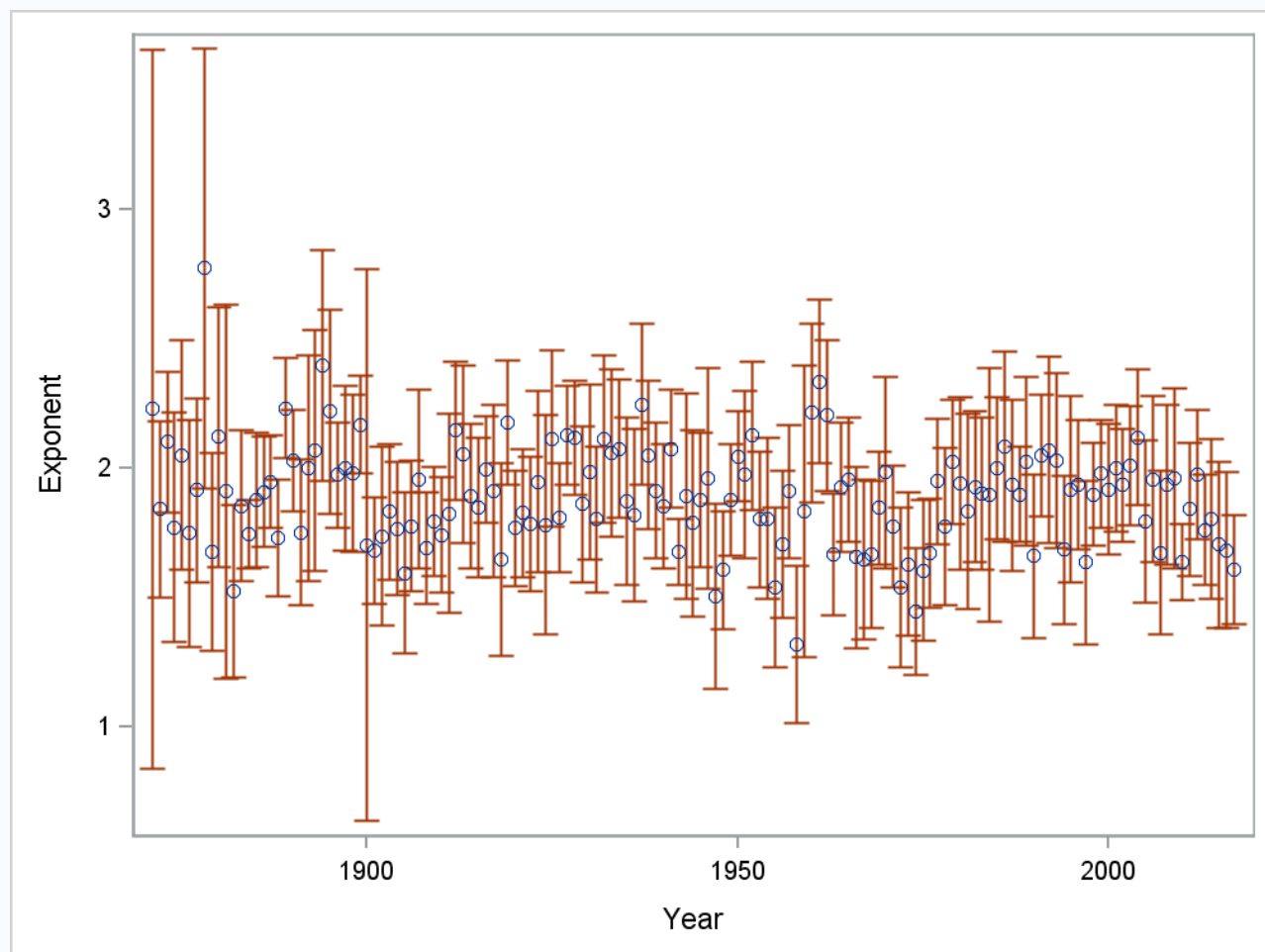
The REG Procedure
Model: MODEL1
Dependent Variable: Estimate Parameter Estimate

Number of Observations Read	147
Number of Observations Used	147

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	0.17392	0.17392	4.44	0.0368
Error	145	5.67803	0.03916		
Corrected Total	146	5.85195			

Root MSE	0.19789	R-Square	0.0297
Dependent Mean	1.89059	Adj R-Sq	0.0230
Coeff Var	10.46686		

Linear Regression of Pythagorean Exponent as a Function of Year

The REG Procedure
Model: MODEL1
Dependent Variable: Estimate Parameter Estimate

Parameter Estimates

Variable	Label	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	95% Confidence Limits	
Intercept	Intercept	1	3.46639	0.74789	4.63	<.0001	1.98821	4.94457
yearID		1	-0.00081059	0.00038463	-2.11	0.0368	-0.00157	-0.00005039

Fit Diagnostics for Estimate
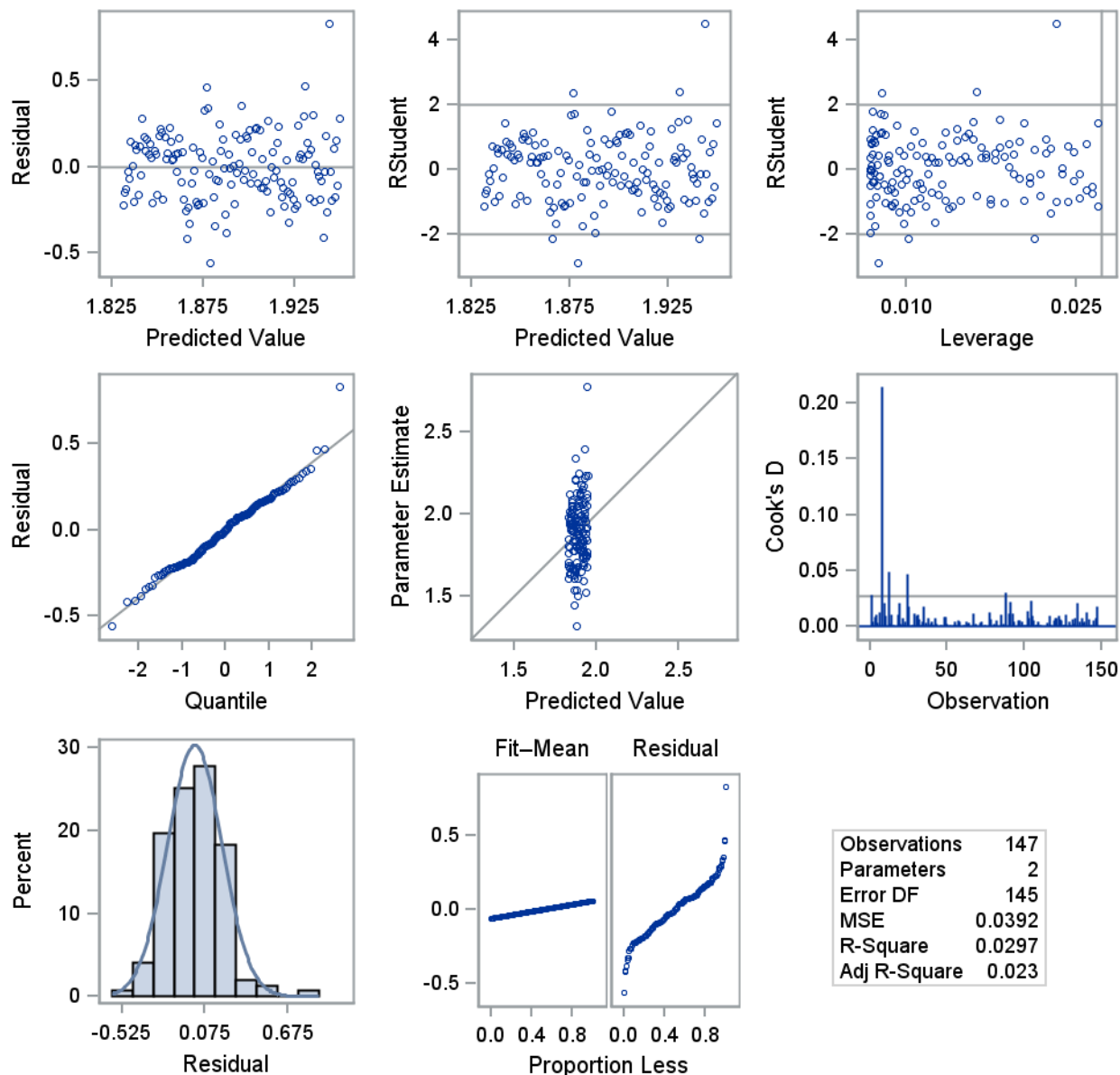
Observations	147
Parameters	2
Error DF	145
MSE	0.0392
R-Square	0.0297
Adj R-Square	0.023

Linear Regression of Pythagorean Exponent as a Function of Year

The REG Procedure
Model: MODEL1
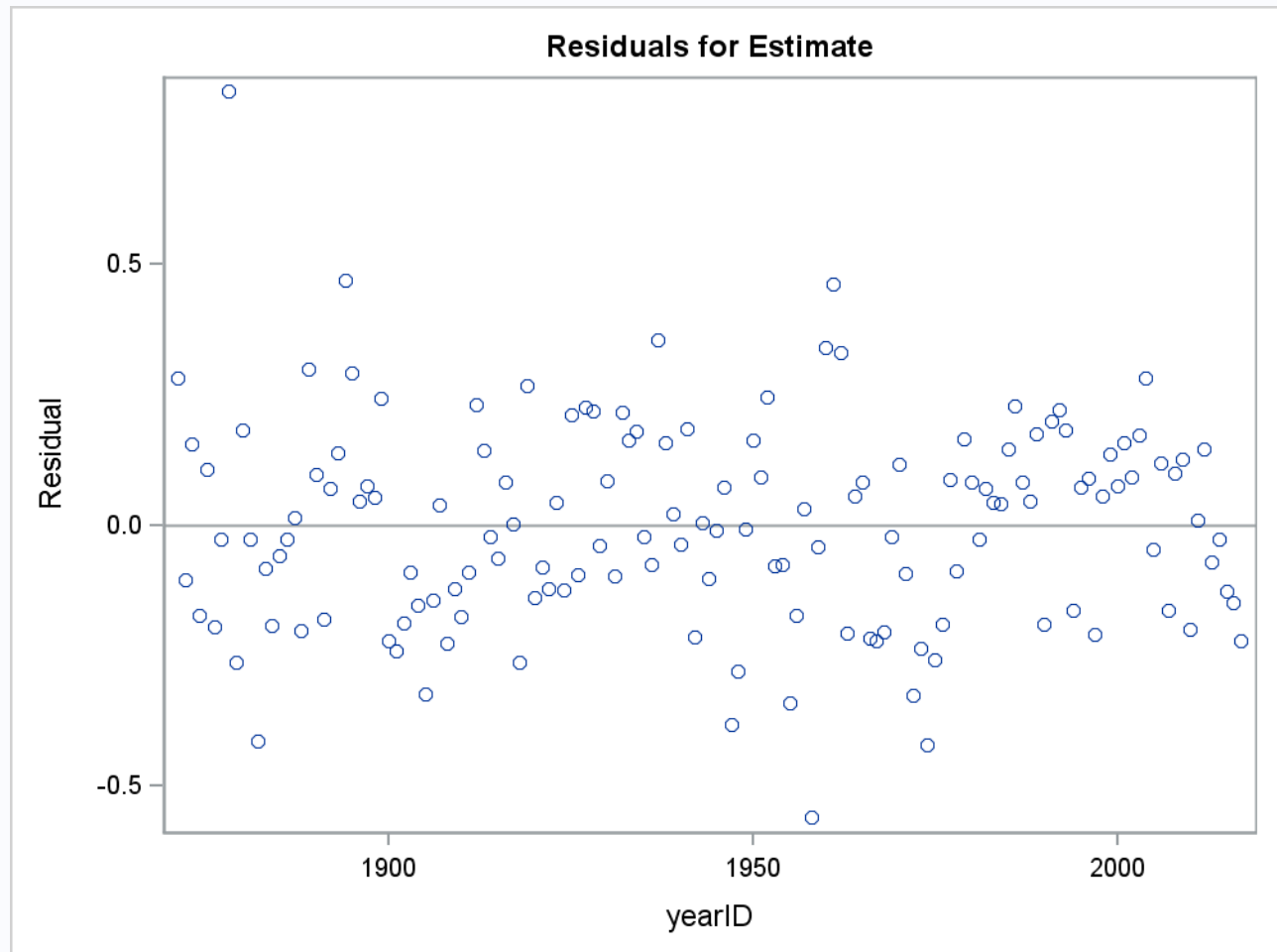**Dependent Variable: Estimate Parameter Estimate**

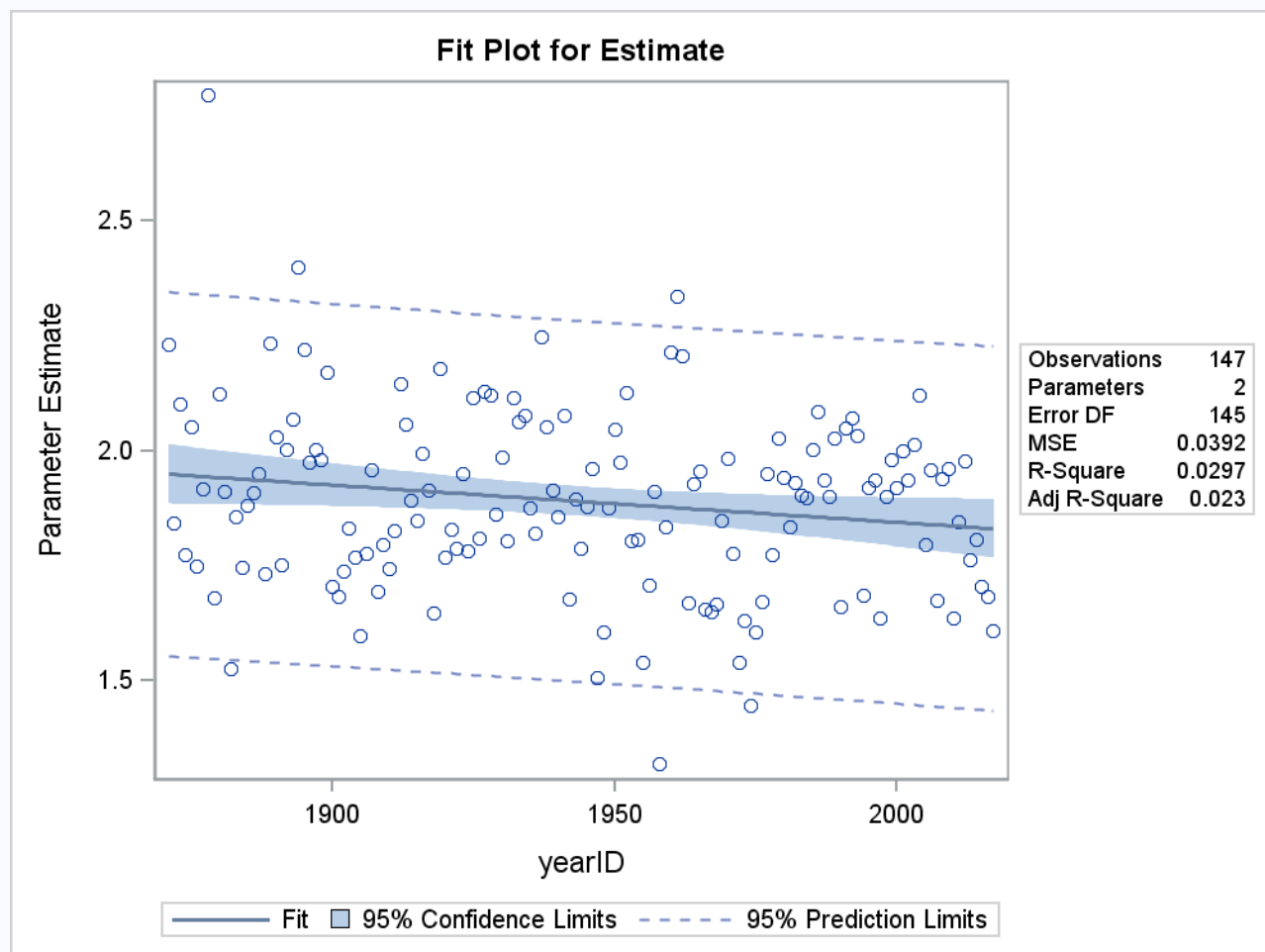Residuals for Estimate

Linear Regression of Pythagorean Exponent as a Function of Year

The REG Procedure
Model: MODEL1
Dependent Variable: Estimate Parameter Estimate

Fit Plot for Estimate

Observations	147
Parameters	2
Error DF	145
MSE	0.0392
R-Square	0.0297
Adj R-Square	0.023

Fit ☐ 95% Confidence Limits ‑ ‑ ‑ ‑ 95% Prediction Limits

The REG Procedure
Model: MODEL1
Dependent Variable: Estimate Parameter Estimate

Number of Observations Read	146
Number of Observations Used	146

Analysis of Variance					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
Model	1	0.09331	0.09331	2.70	0.1024
Error	144	4.97273	0.03453		
Corrected Total	145	5.06604			

Root MSE	0.18583	**R-Square**	0.0184
Dependent Mean	1.88454	**Adj R-Sq**	0.0116
Coeff Var	9.86075		

Linear Regression of Pythagorean Exponent as a Function of Year (without 1878 outlier)

The REG Procedure
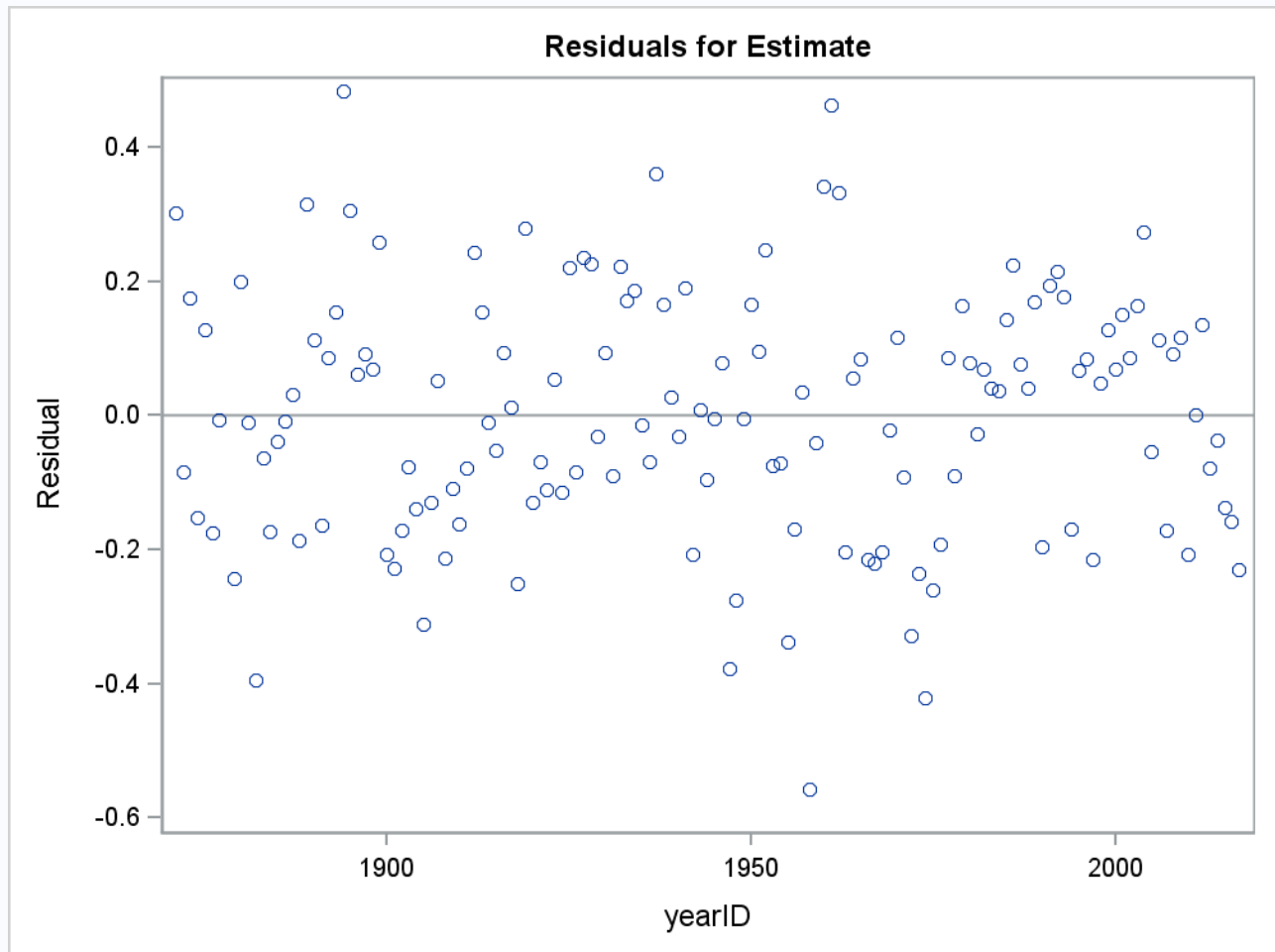### Model: MODEL1
Dependent Variable: Estimate Parameter Estimate

							95%	
Variable	**Label**	**DF**	**Parameter Estimate**	**Standard Error**	**t Value**	**Pr > |t|**	**Confidence Limits**	
Intercept	Intercept	1	3.04871	0.70838	4.30	<.0001	1.64854	4.44889
yearID		1	-0.00059871	0.00036422	-1.64	0.1024	-0.00132	0.00012120

Fit Diagnostics for Estimate
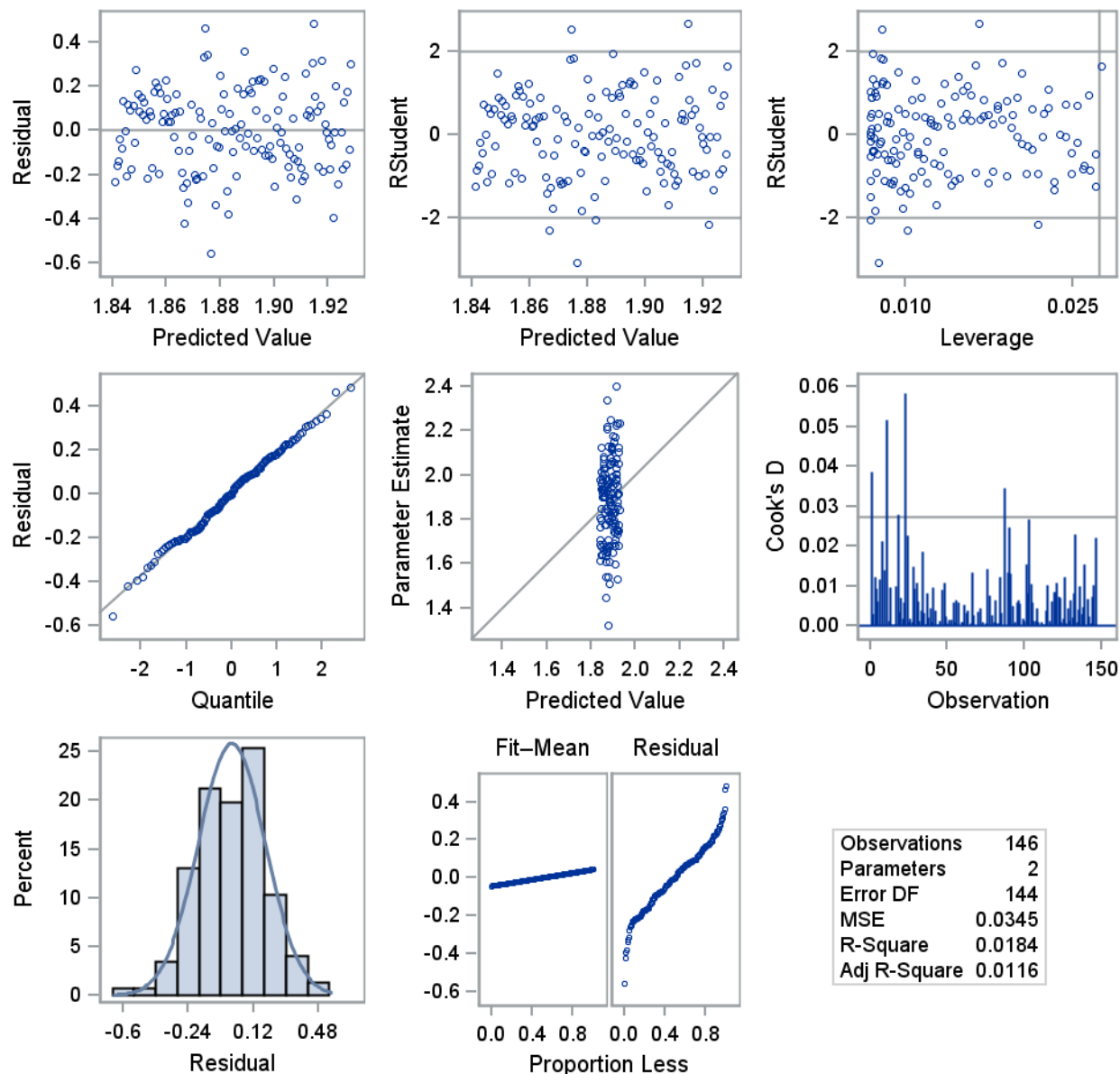
Observations	146
Parameters	2
Error DF	144
MSE	0.0345
R-Square	0.0184
Adj R-Square	0.0116

Linear Regression of Pythagorean Exponent as a Function of Year (without 1878 outlier)

The REG Procedure
Model: MODEL1
Dependent Variable: Estimate Parameter Estimate
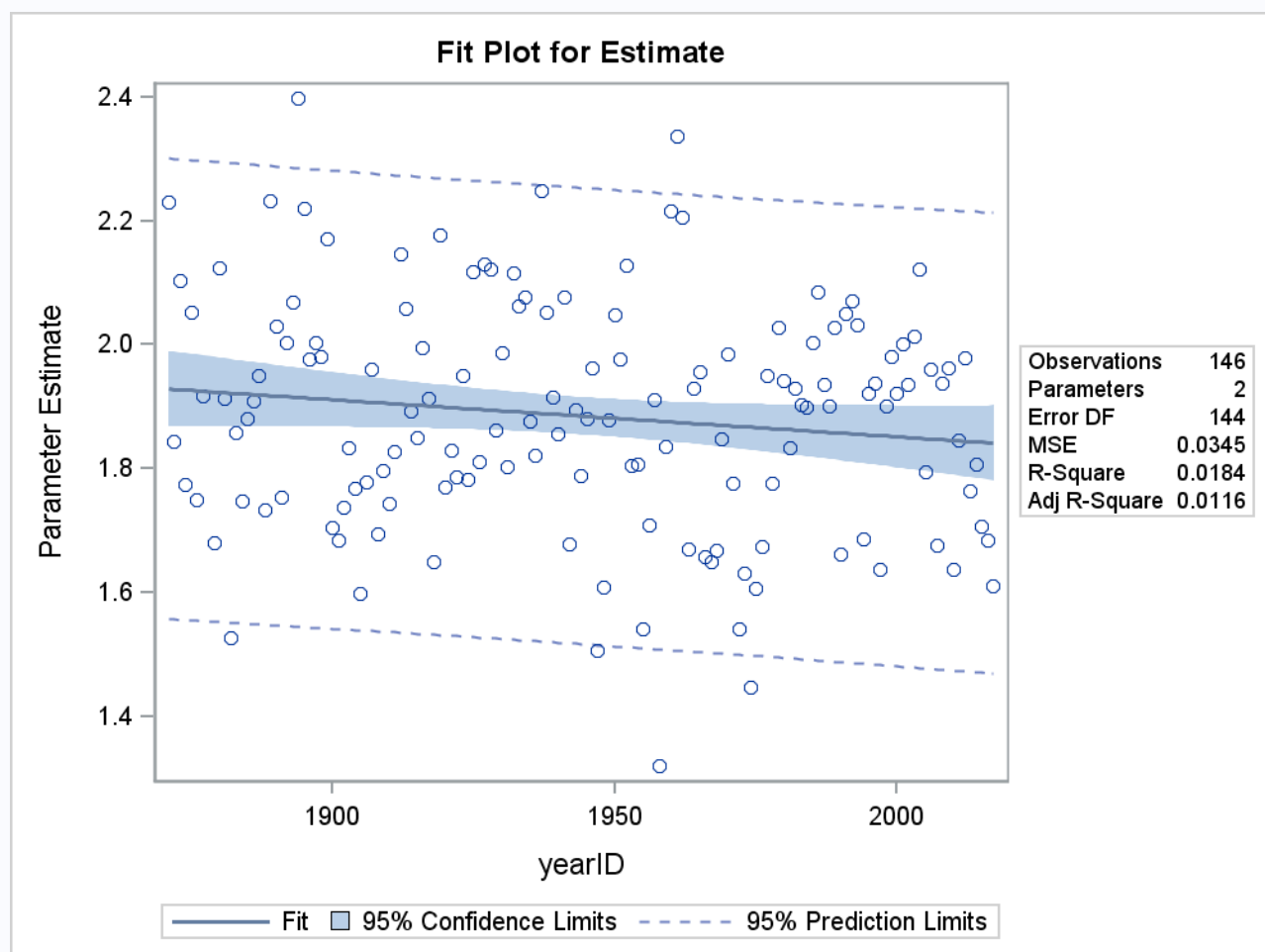

Residuals for Estimate

Linear Regression of Pythagorean Exponent as a Function of Year (without 1878 outlier)

The REG Procedure
Model: MODEL1
Dependent Variable: Estimate Parameter Estimate

Fit Plot for Estimate

Observations	146
Parameters	2
Error DF	144
MSE	0.0345
R-Square	0.0184
Adj R-Square	0.0116

When the 1878 outlier is removed, the slope is no longer significant. Safe to assume variation in exponent is due to error alone.

The REG Procedure
Model: MODEL1
Dependent Variable: log_wl

Number of Observations Read	2865
Number of Observations Used	2863
Number of Observations with Missing Values	2

Note: No intercept in model. R-Square is redefined.

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	79.01866	79.01866	27702.6	<.0001
Error	2862	8.16355	0.00285		
Uncorrected Total	2863	87.18221			

Root MSE	0.05341	R-Square	0.9064
Dependent Mean	-0.00363	Adj R-Sq	0.9063
Coeff Var	-1470.81390		

Linear Regression to get Overall Exponent

The REG Procedure
Model: MODEL1
Dependent Variable: log_wl

						95% Confidence Limits	
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|		
log_rra	1	1.87752	0.01128	166.44	<.0001	1.85540	1.89964

Make note of the differences in the overall exponent estimation from the linear regression method and the following nonlinear method, which iteratively determines the exponent as it belongs in the formula.

The NLIN Procedure
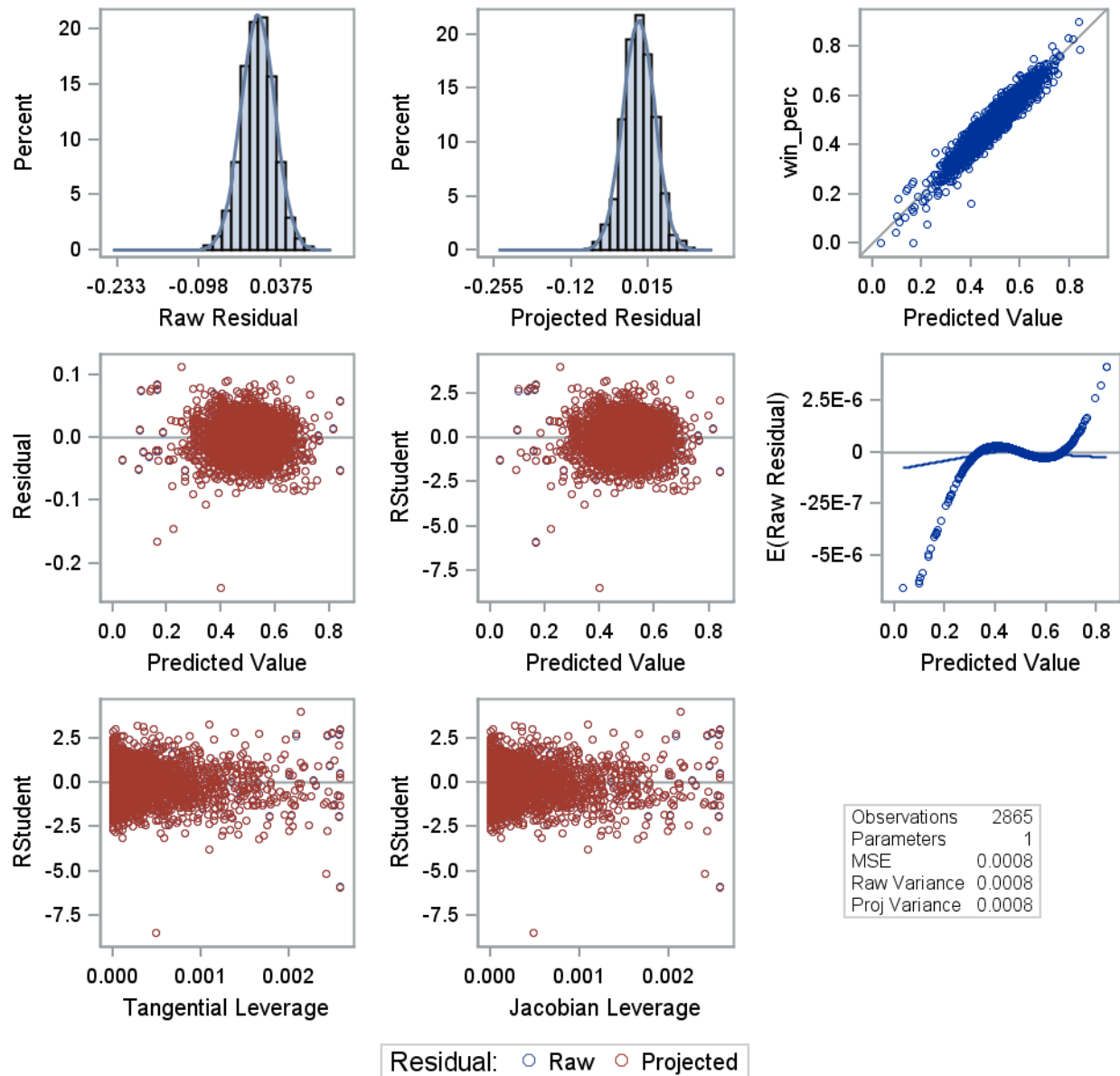**Dependent Variable win_perc**
Method: Gauss-Newton

Iterative Phase

Iter	x	Sum of Squares
0	2.0000	2.3651
1	1.8670	2.2699
2	1.8681	2.2699

NOTE: Convergence criterion met.

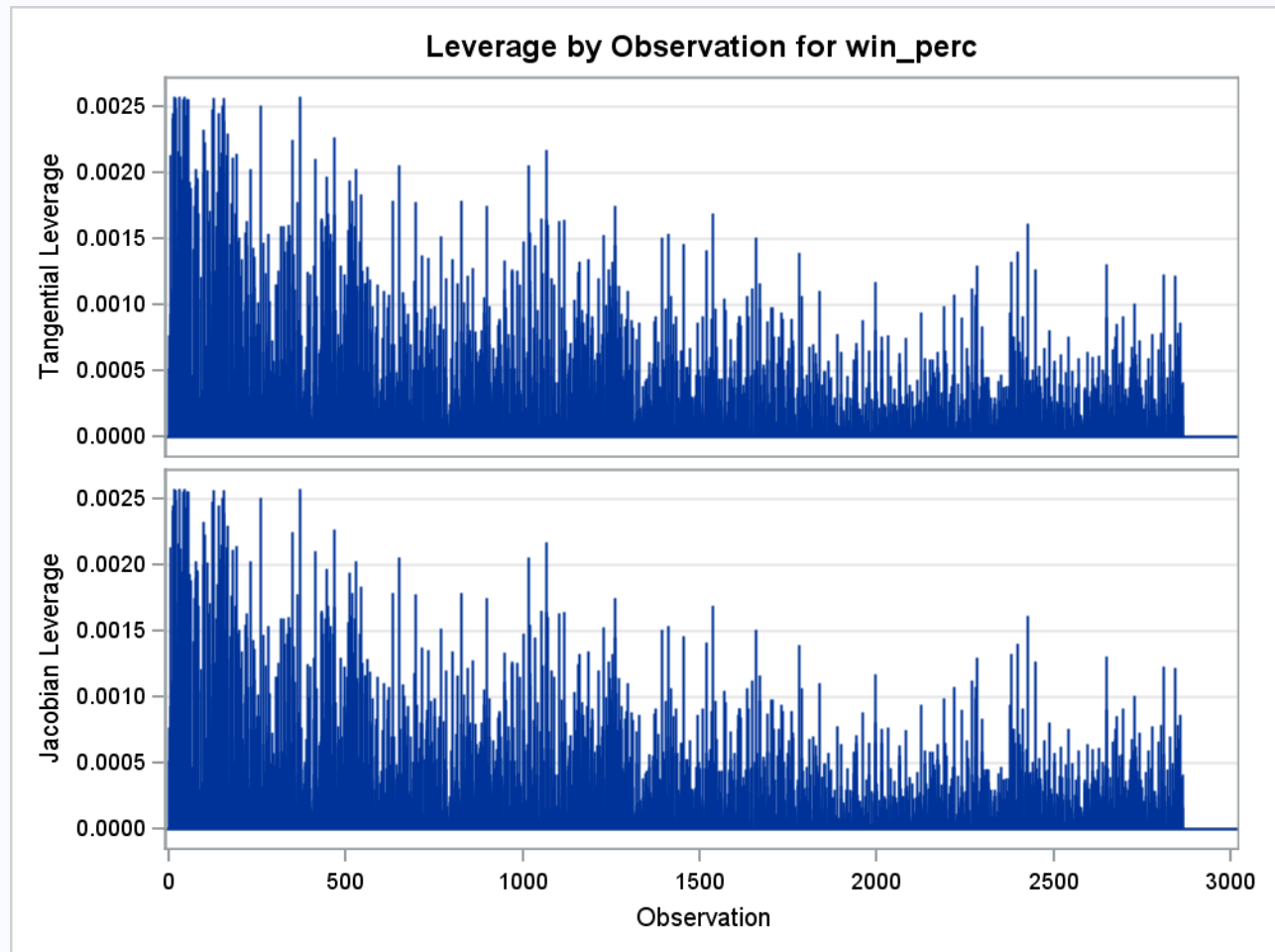# Estimating Pythagorean Exponent with Nonlinear Method

The NLIN Procedure
Dependent Variable win_perc
Method: Gauss-Newton

Fit Diagnostics for win_perc

Estimating Pythagorean Exponent with Nonlinear Method

The NLIN Procedure
Dependent Variable win_perc
Method: Gauss-Newton

Leverage by Observation for win_perc

Estimating Pythagorean Exponent with Nonlinear Method

The NLIN Procedure
Dependent Variable win_perc
Method: Gauss-Newton

Local Influence by Observation for win_perc

Raw and Projected Residuals by Regressors for win_perc

Residual: ○ Raw   ○ Projected

Estimating Pythagorean Exponent with Nonlinear Method

The NLIN Procedure
Dependent Variable win_perc
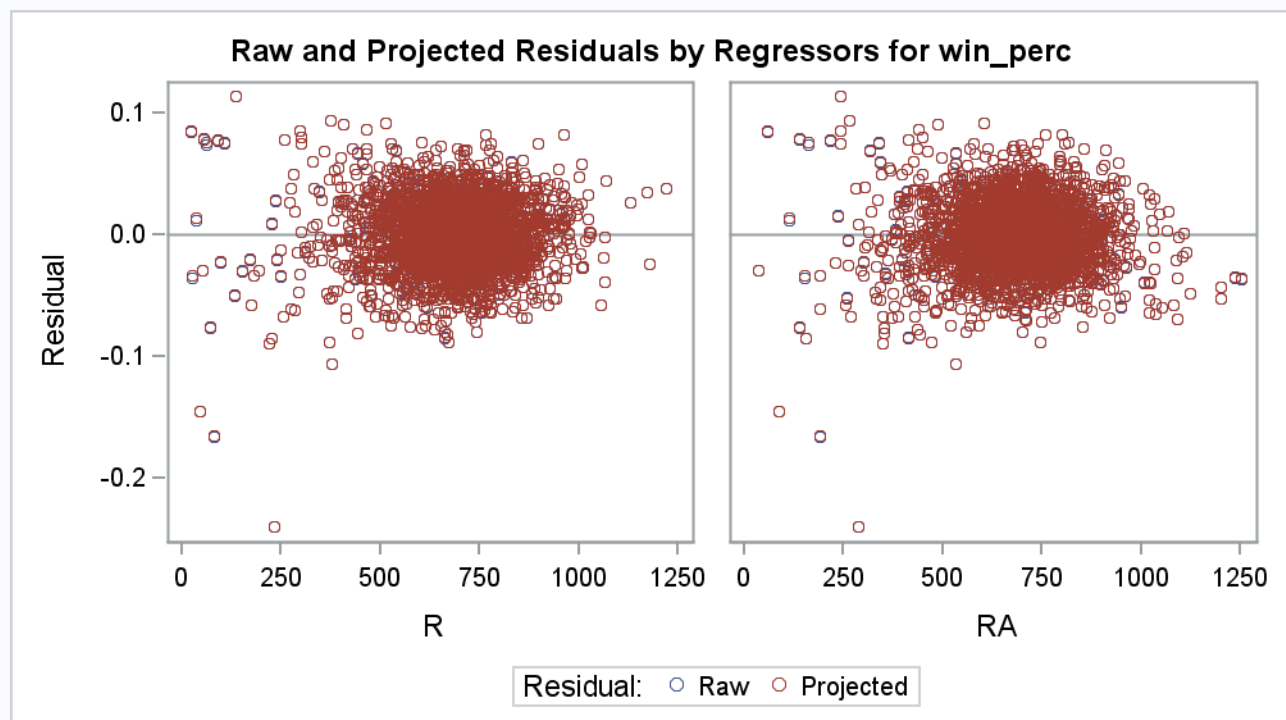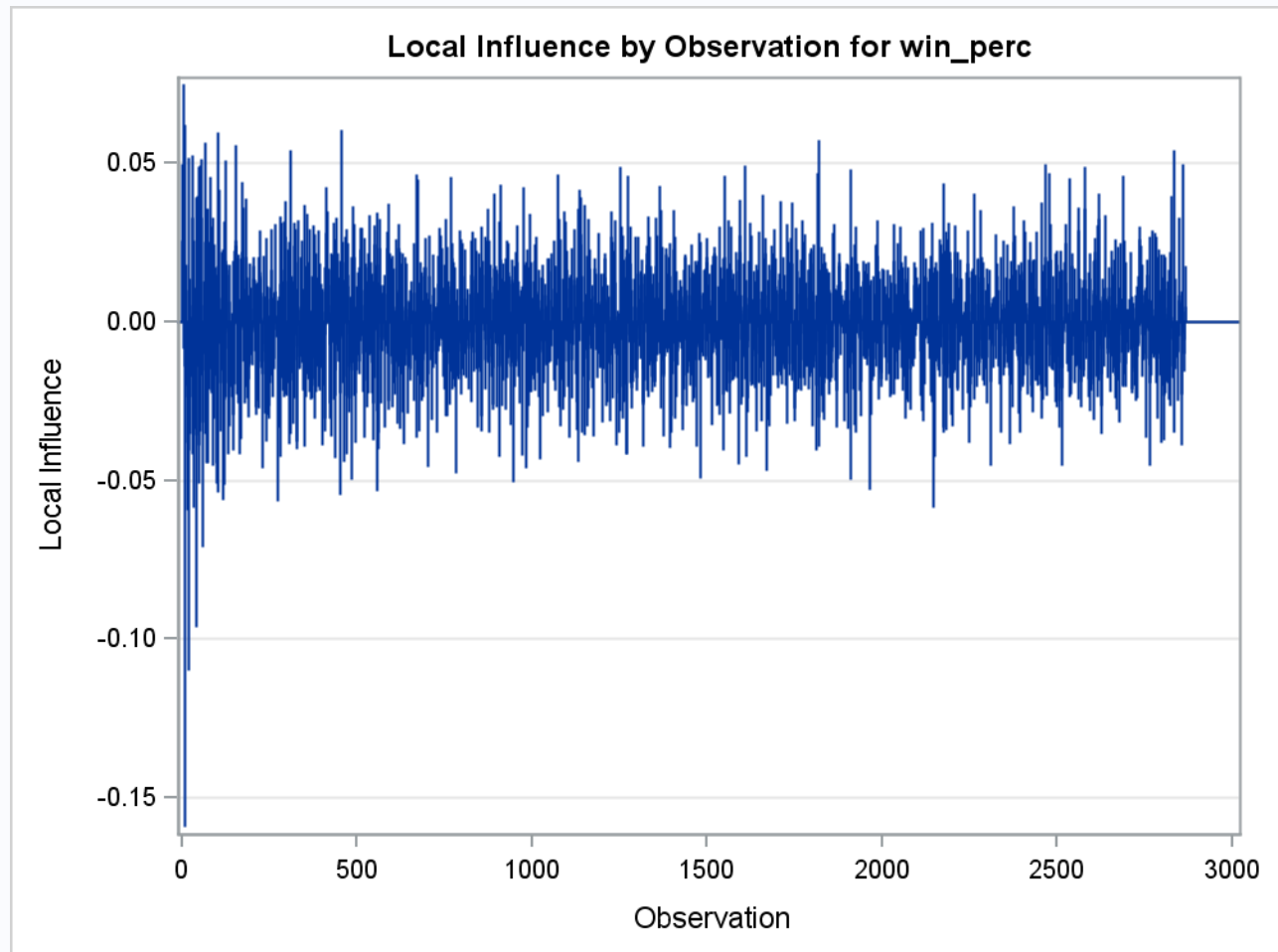**Method: Gauss-Newton**

Fit Plot for win_perc

Observations   2865
Parameters   1
MSE   0.0008
Raw Variance 0.0008
Proj Variance 0.0008

Estimation Summary	
Method	Gauss-Newton
Iterations	2
R	6.073E-7
PPC(x)	2.077E-7
RPC(x)	0.000588
Object	2.955E-6
Objective	2.269891
Observations Read	2865
Observations Used	2865
Observations Missing	0

Note: An intercept was not specified for this model.

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Model	1	734.9	734.9	927306	<.0001
Error	2864	2.2699	0.000793		
Uncorrected Total	2865	737.2			

Estimating Pythagorean Exponent with Nonlinear Method

The NLIN Procedure

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits		Skewness
x	1.8681	0.0119	1.8447	1.8915	0.00431

Approximate Correlation Matrix	
	x
x	1.0000000

The NLIN Procedure
Dependent Variable win_perc
Method: Gauss-Newton

Iterative Phase			
Iter	x	y	Sum of Squares
0	2.0000	2.0000	2.3651
1	1.8670	1.8674	2.2690
2	1.8681	1.8684	2.2690

NOTE: Convergence criterion met.

Estimation Summary	
Method	Gauss-Newton
Iterations	2
R	3.348E-6
PPC(y)	4.427E-7
RPC(y)	0.000567
Object	2.881E-6
Objective	2.26896
Observations Read	2865
Observations Used	2865
Observations Missing	0

Note: An intercept was not specified for this model.

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Model	2	734.9	367.5	463682	<.0001
Error	2863	2.2690	0.000793		
Uncorrected Total	2865	737.2			

Estimating Different Exponents Within the Formula

The NLIN Procedure

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits		Skewness
x	1.8681	0.0119	1.8447	1.8915	0.00430
y	1.8684	0.0119	1.8450	1.8919	0.00432

Approximate Correlation Matrix		
	x	y
x	1.0000000	0.9996106
y	0.9996106	1.0000000