



Hypergraph Contrastive Learning

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Content

☐ Background

☐ HyperGCL

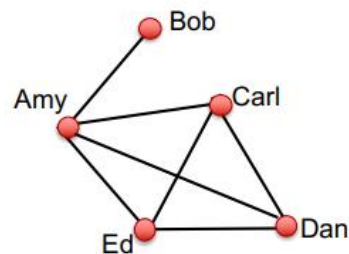
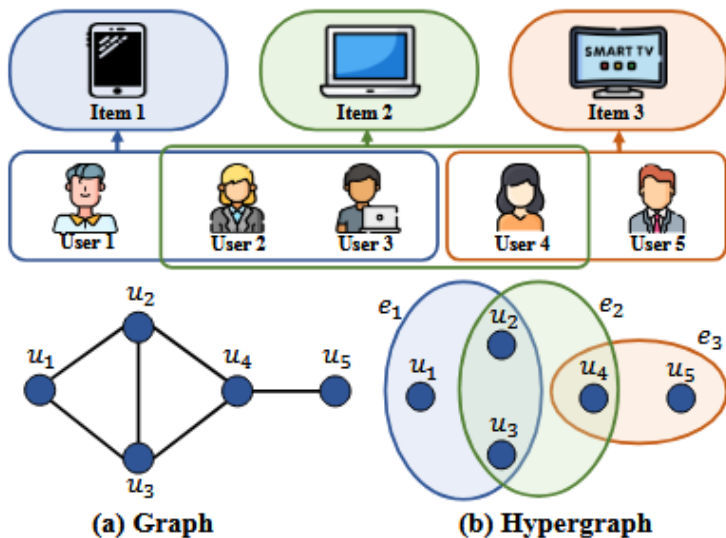
☐ TriCL

☐ CASH

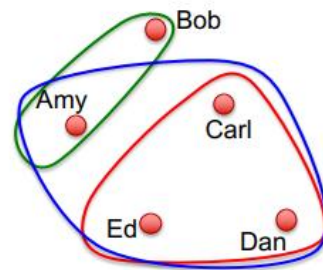
☐ Conclusion

□ Hypergraphs can naturally model group-wise relations as hyperedges

- Using graphs can incur information loss



(a) Graph model

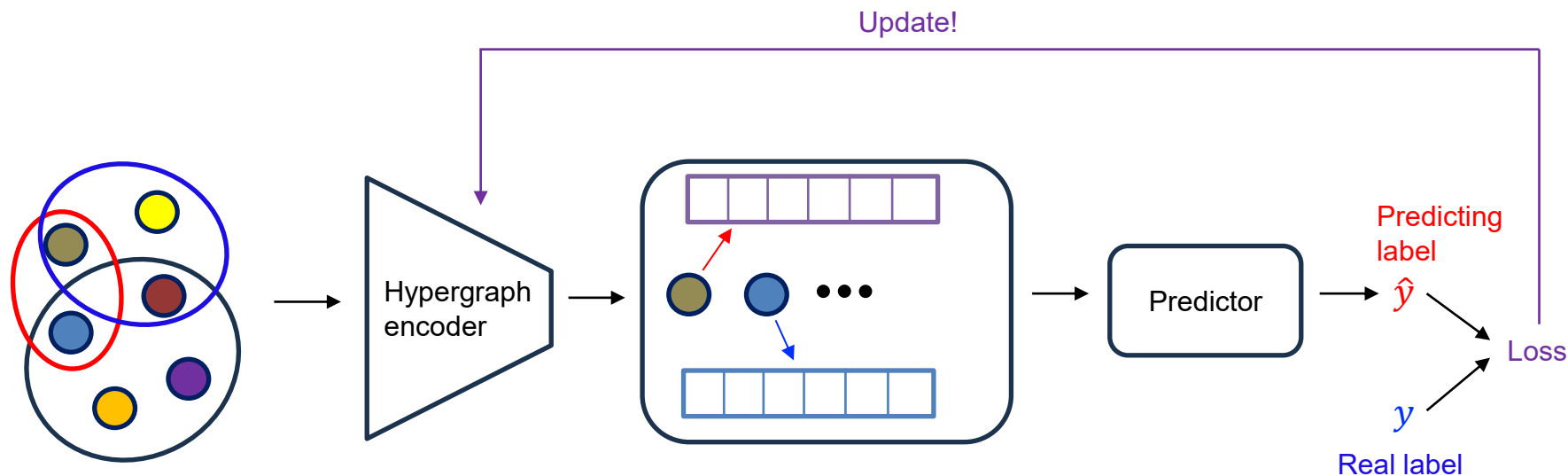


(b) Hypergraph model

Fig. 2. Graph and hypergraph representations of Fig. 1 data. Colored hyperedges correspond to different email messages.

Background

□ Hypergraph Neural Networks



Background

- Hypergraph data labeling is often **time, resource, and labor-intensive**



Background

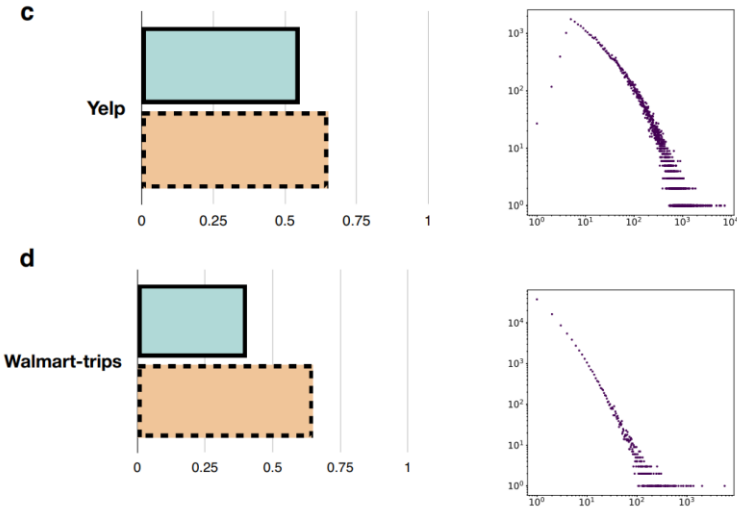
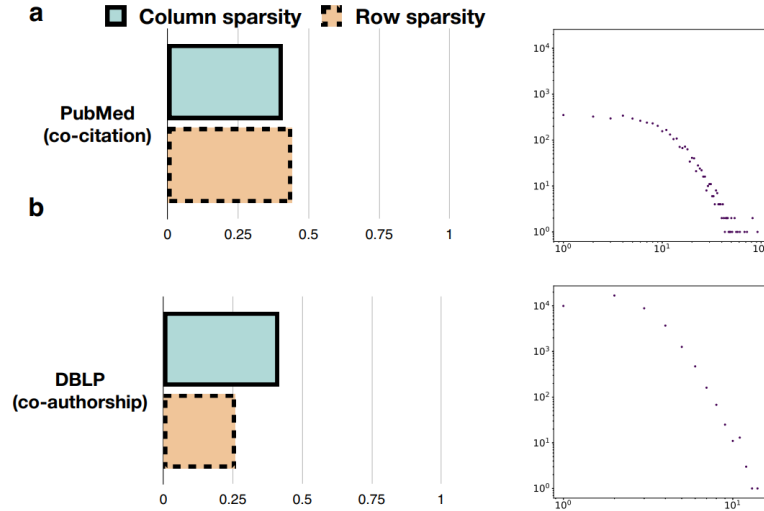
□ Real world (hyper)graphs are ‘sparse’

■ Most objects have only a few relationships

H

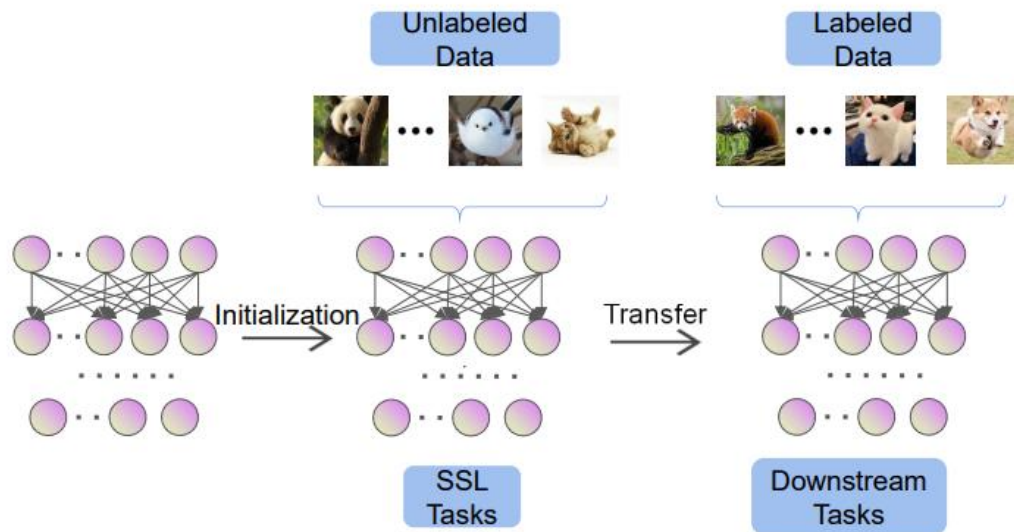
1			
1		1	
1	1	1	1
	1		1
		1	1

Incidence matrix



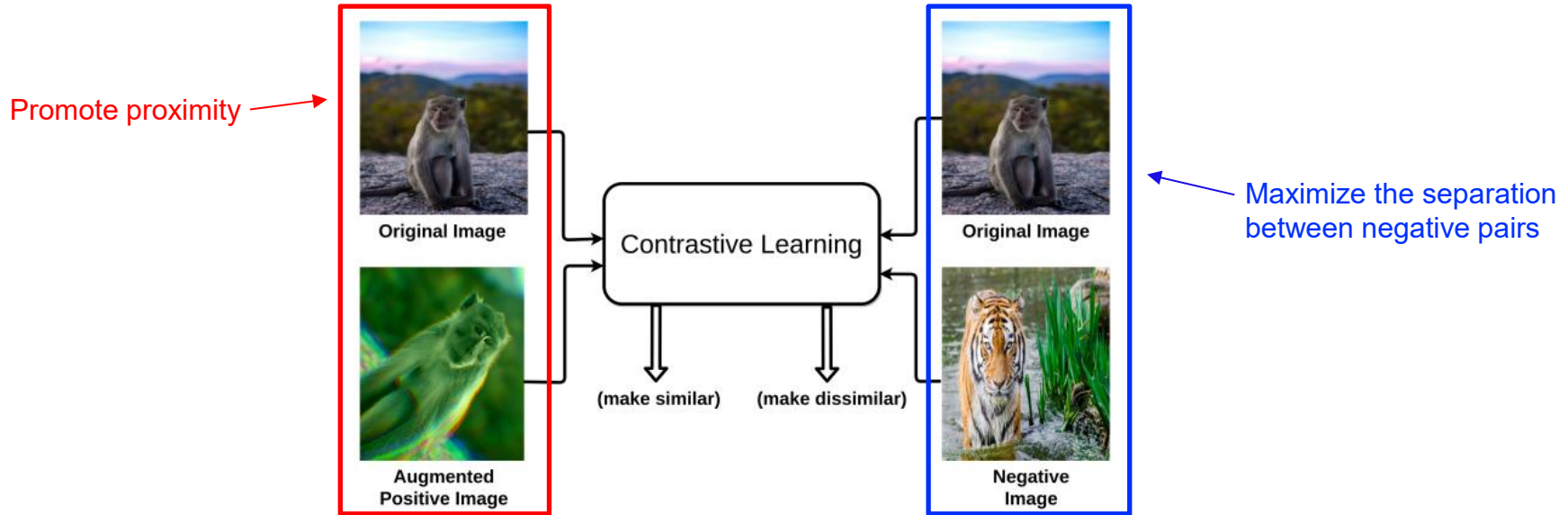
□ Self-Supervised Learning

- Learn discriminative features from vast quantities of unlabeled instances without relying on human annotations



□ Contrastive Learning

- Push original and augmented data closer, push original and negative data away



□ Graph Contrastive Learning

■ Graph data augmentation

- Dropping, perturbation and masking

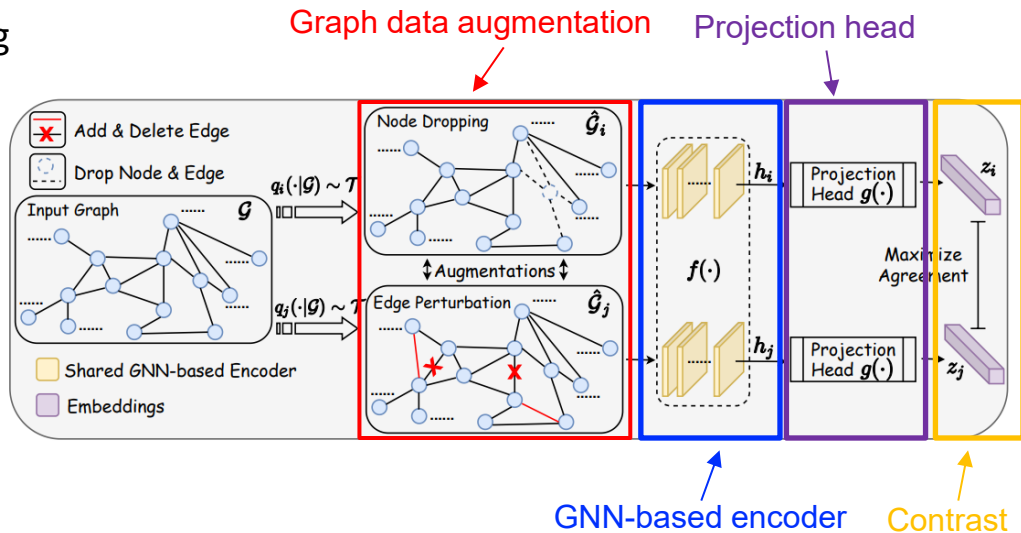
■ GNN-based encoder

- Extract graph-level representation vectors for augmented graphs

■ Projection head

■ Contrastive

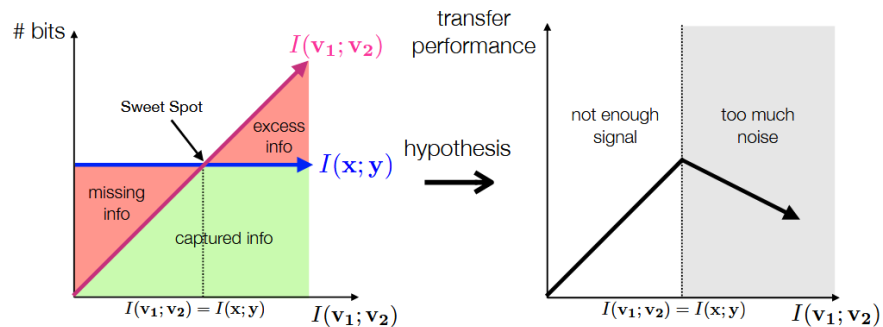
- Dependent on task, generally node-level contrast
- Maximize the consistency between positive pairs compared with negative pairs



Challenges

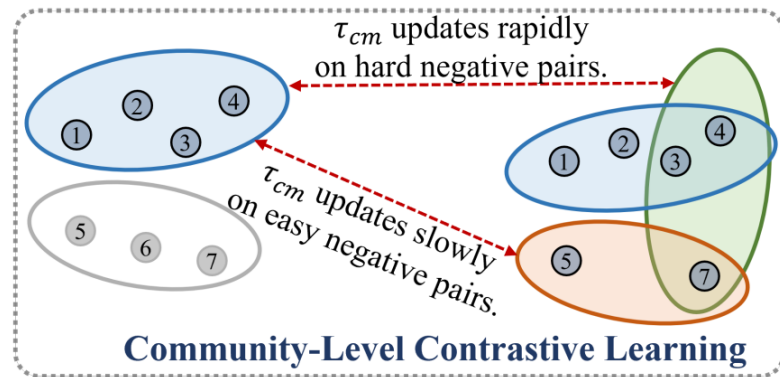
□ How to augment a hypergraph?

- The choice of views is what controls the information the representation captures



□ What to contrast?

- Node-only contrast cannot reflect higher-order information

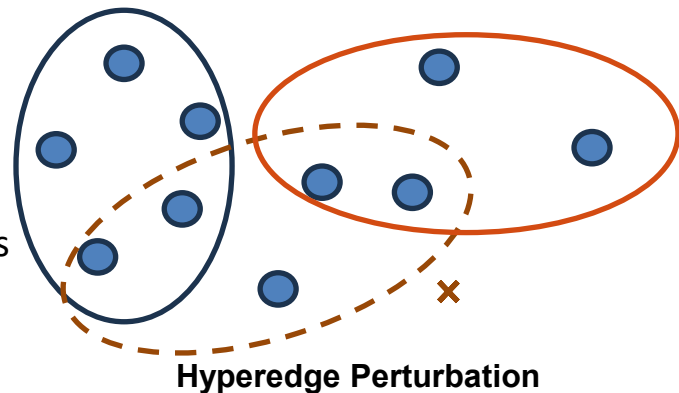


How to augment a hypergraph?

□ Augment Hyperedges

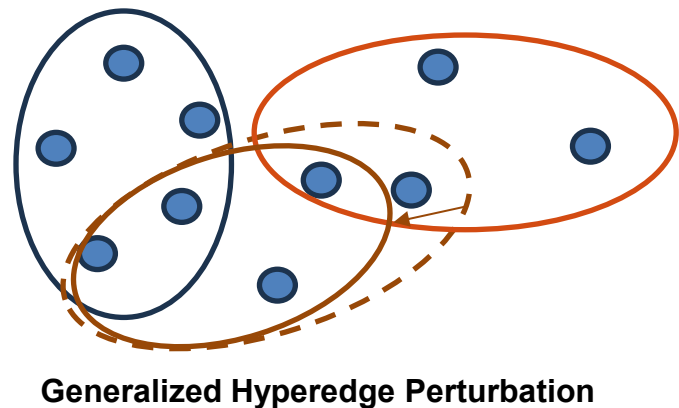
■ Hyperedge Perturbation

- Partially missing higher-order relations do not significantly affect the semantic meaning of hypergraphs



■ Generalized Hyperedge Perturbation

- Randomly kick out vertices from hyperedges



How to augment a hypergraph?

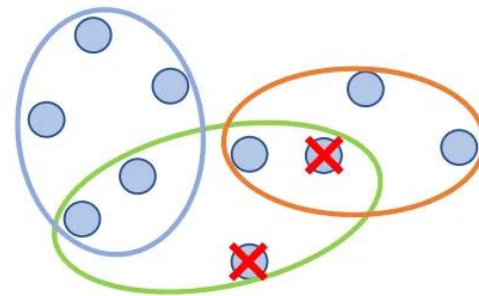
□ Augment Vertexes

■ Vertex dropping

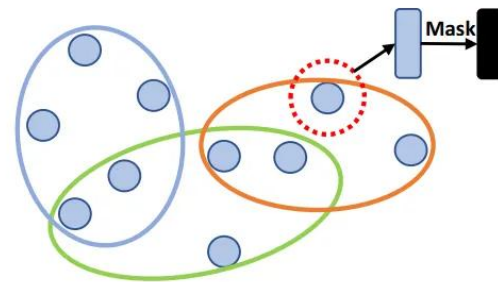
- Vertex missing does not alter semantics

■ Attribute masking

- Semantic robustness against losing partial attributes



A3: Drop Vertexes

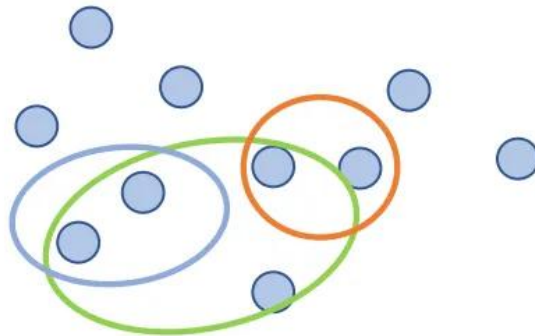


A4: Feature Mask

How to augment a hypergraph?

□ Subgraph

- Local structure can hint the full semantics
- Perform random walk to extract



A5: Subgraph



Augmentations in Hypergraph Contrastive Learning: Fabricated and Generative

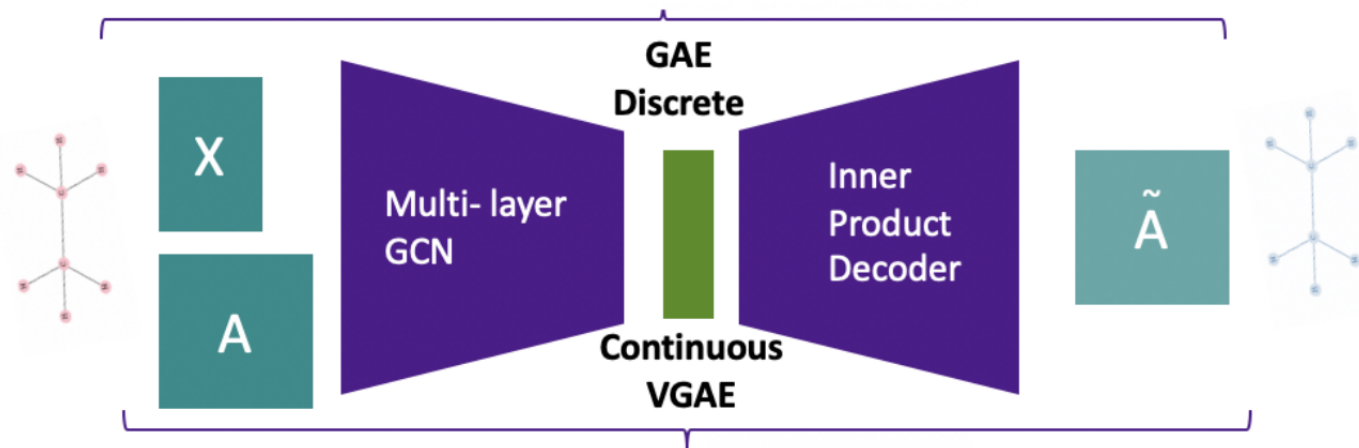
Tianxin Wei, Yuning You, Tianlong Chen, Yang Shen, Jingrui He,
Zhangyang Wang

University of Illinois Urbana-Champaign, Texas A&M University, University of Texas at Austin

22-NeurIPS

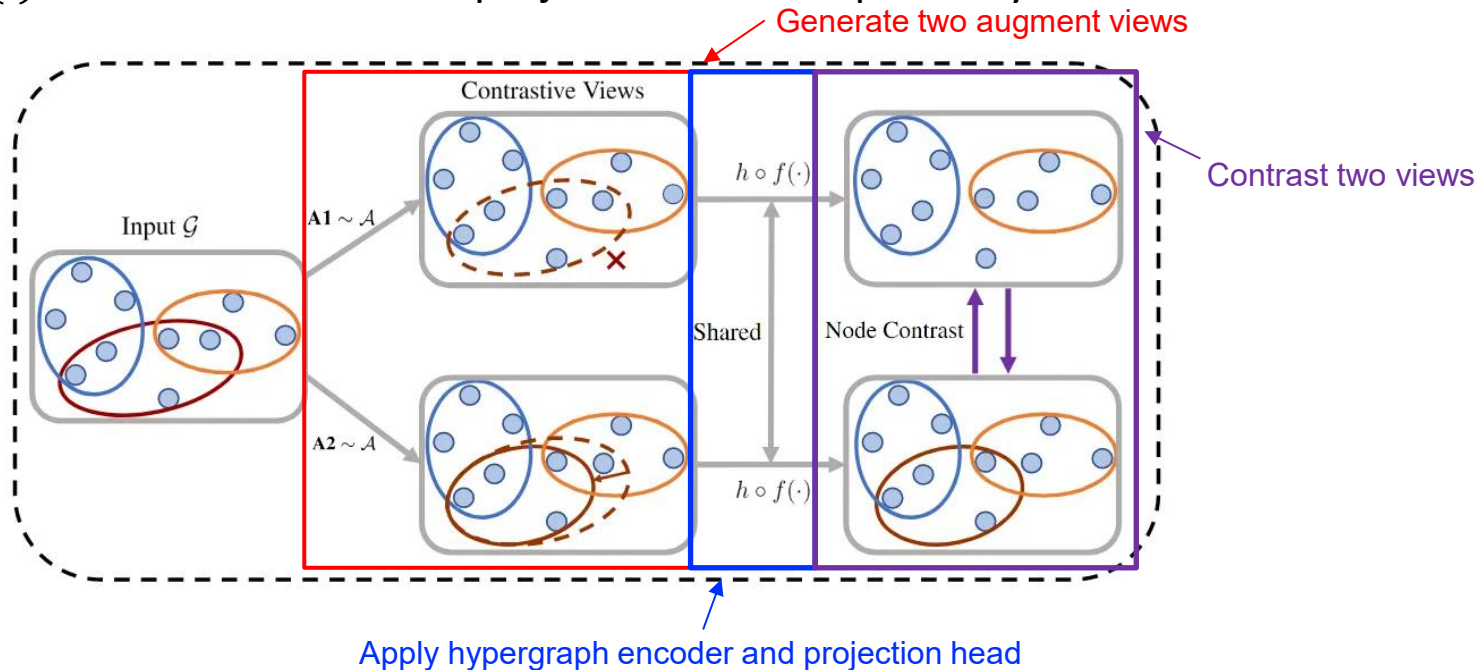
□ Motivation

- Augmentation of hypergraph can be learned *during contrastive learning!*
→ Leverage Variational Graph Auto-Encoder!



□ Overview

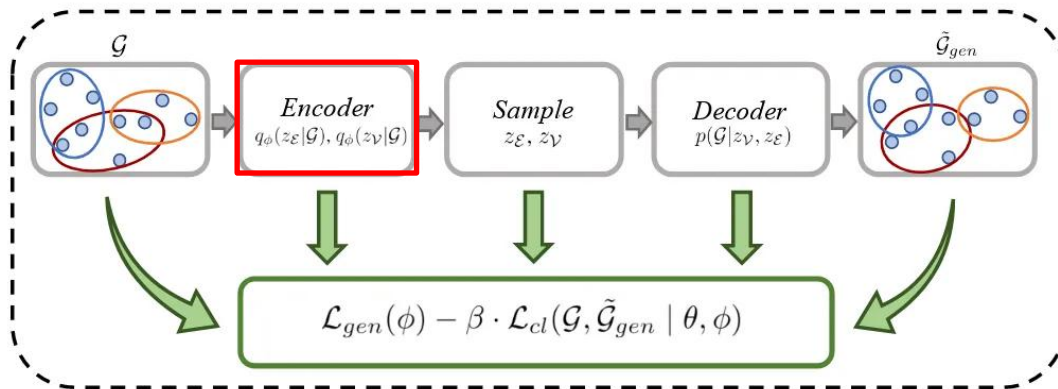
- $f(\cdot), h(\cdot)$: shared encoder and projection head respectively



□ Variational Hypergraph Auto-Encoder (VHGAE)

■ Embed hypergraphs into latent representations

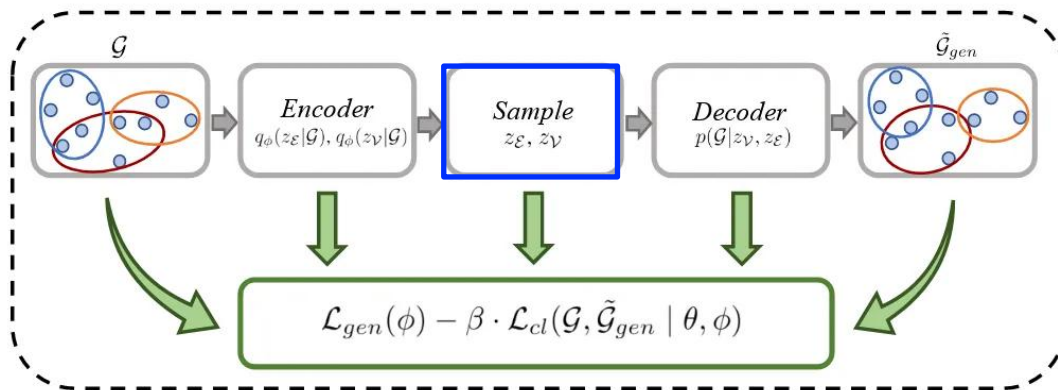
- $z_V \sim q_\phi(z_V|\mathcal{G}) = \mathcal{N}(\mu_V, \sigma_V^2); \mu_V = \text{HyperGNN}_\mu^V(\mathcal{G}), \log(\sigma_V) = \text{HyperGNN}_\sigma^V(\mathcal{G})$
- $z_\mathcal{E} \sim q_\phi(z_\mathcal{E}|\mathcal{G}) = \mathcal{N}(\mu_\mathcal{E}, \sigma_\mathcal{E}^2); \mu_\mathcal{E} = \text{HyperGNN}_\mu^\mathcal{E}(\mathcal{G}), \log(\sigma_\mathcal{E}) = \text{HyperGNN}_\sigma^\mathcal{E}(\mathcal{G})$



□ Variational Hypergraph Auto-Encoder (VHGAE)

■ Apply reparameterization trick

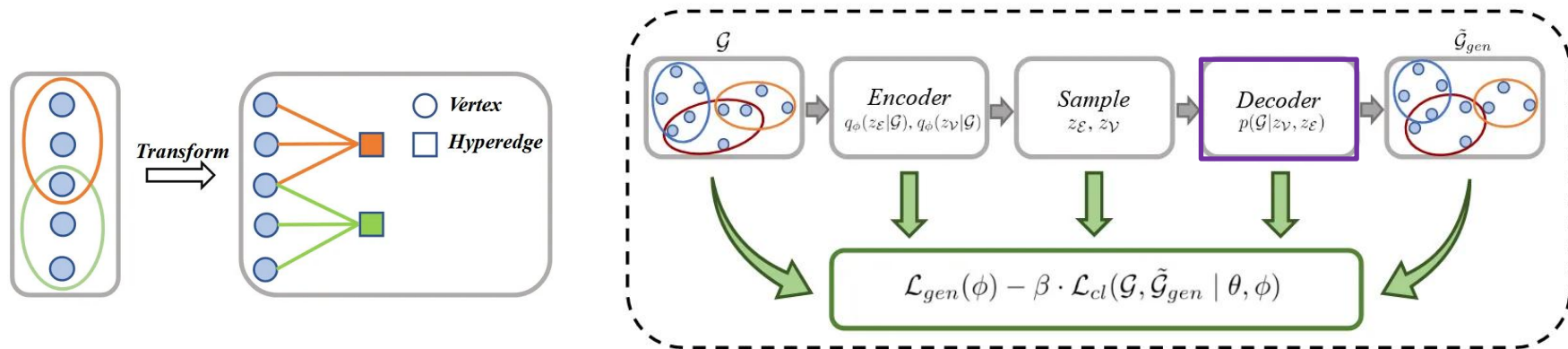
- $z_V = \mu_V + \sigma_V \odot \delta$
- $z_E = \mu_E + \sigma_E \odot \delta$
- $\delta \sim \mathcal{N}(0, I)$



□ Variational Hypergraph Auto-Encoder (VHGAE)

- Reconstruct the higher-order relations of hypergraphs
- Recover the relations on the converted bipartite graph $\tilde{\mathcal{G}} = \{\tilde{\mathcal{V}}, \tilde{\mathcal{E}}\}$

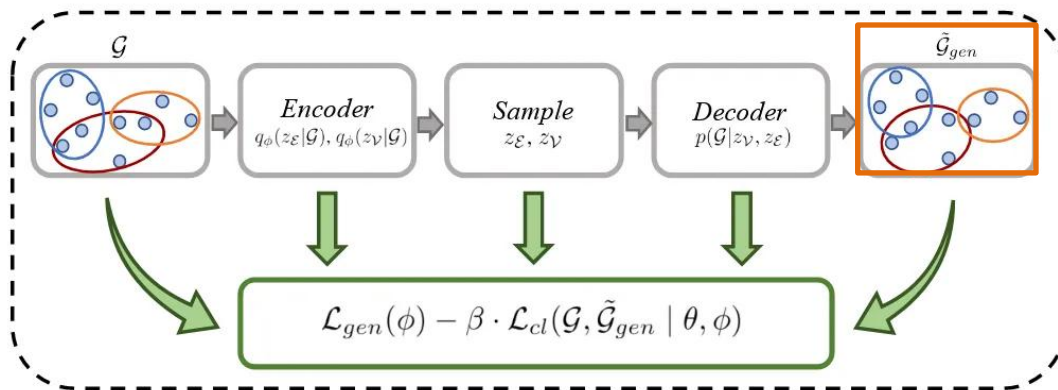
$$p(\mathcal{G}|z_{\mathcal{V}}, z_{\mathcal{E}}) \approx p(\tilde{\mathcal{G}}|z_{\mathcal{V}}, z_{\mathcal{E}}) = \prod_{e=1}^{|\mathcal{E}|} \prod_{v=1}^{|\mathcal{V}|} p(\tilde{\mathcal{E}}_{v,e}|z_v, z_e) = \prod_{e=1}^{|\mathcal{E}|} \prod_{v=1}^{|\mathcal{V}|} \text{Sigmoid}(z_v^T z_e),$$



□ Variational Hypergraph Auto-Encoder (VHGAE)

- Apply the Gumbel-Softmax trick for the hyperedge distribution

$$\begin{aligned}
 T(\mathcal{G}) &= \text{Gumbel-Softmax}(p(\mathcal{G} \mid z_{\mathcal{V}}, z_{\mathcal{E}})) \\
 &= \text{Sigmoid}((w_{\mathcal{V}\mathcal{E}} + \log(\delta) - \log(1 - \delta))/\tau) \\
 \tilde{\mathcal{G}}_{gen} &= T(\mathcal{G}) \circ \mathcal{G},
 \end{aligned}$$



Objective Function

Generator loss

$$\square \mathcal{L}_{gen}(\phi) = -\text{ELBO}$$

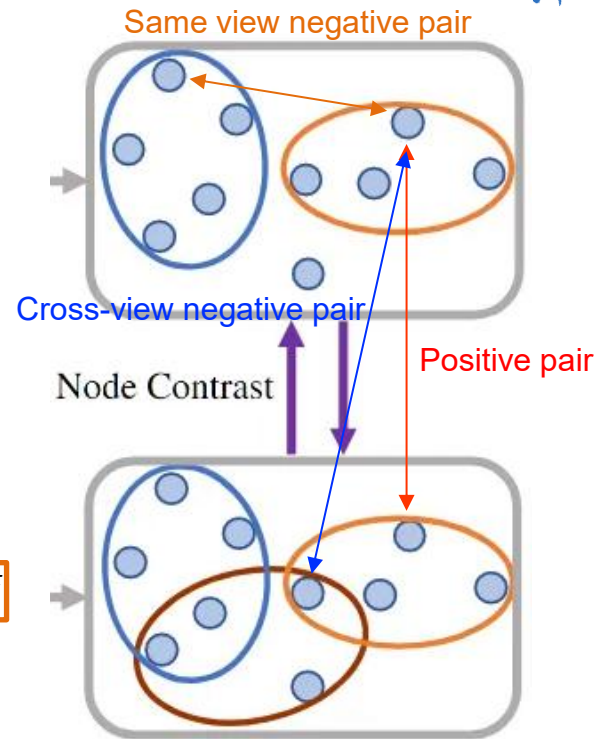
Contrastive loss

\square Node-level contrast

$$\square L_{cl}(\tilde{G}_1, \tilde{G}_2) = \frac{1}{2|\mathcal{V}|} \sum_{n=1}^{|\mathcal{V}|} (l(u_n, s_n) + l(s_n, u_n))$$

$$\square l(u_n, s_n) = -\log \frac{e^{\gamma(u_n, s_n)/\tau}}{e^{\gamma(u_n, s_n)/\tau} + \underbrace{\sum_{m \neq n} e^{\gamma(u_n, s_m)/\tau}}_{\text{blue box}} + \underbrace{\sum_{m \neq n} e^{\gamma(u_n, u_m)/\tau}}_{\text{orange box}}}$$

$$\rightarrow \min_{\phi} \mathcal{L}_{gen}(\phi) - \beta \cdot L_{cl}(\mathcal{G}, \tilde{\mathcal{G}}_{gen} | \theta, \phi)$$

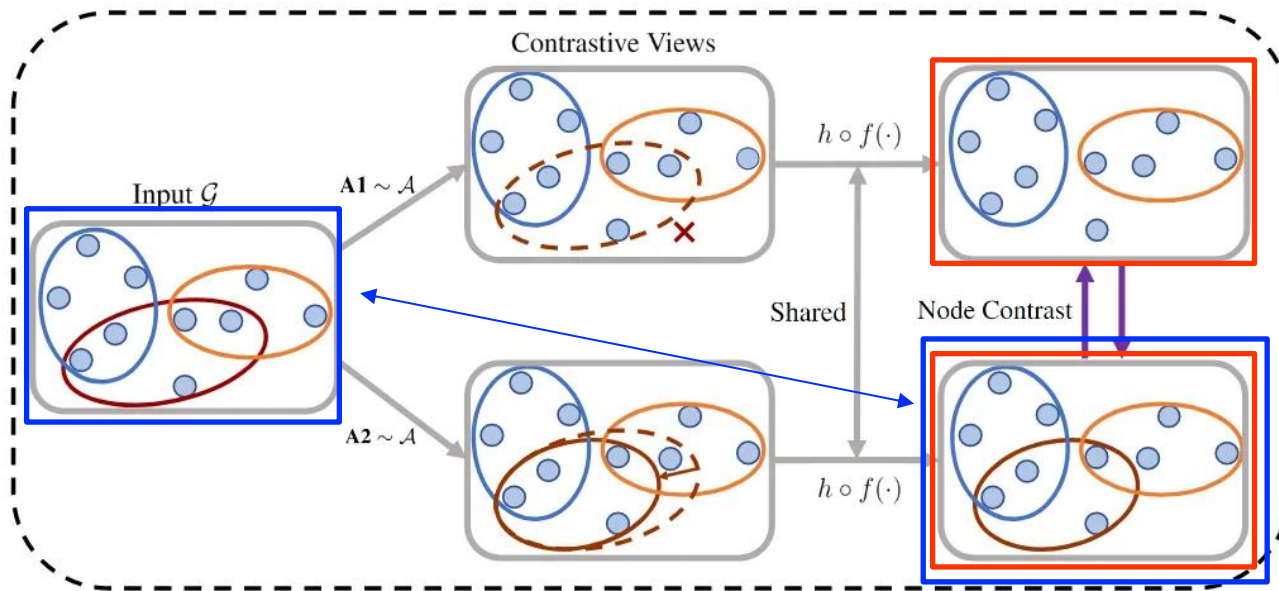


□ Training Pipeline

Algorithm 1 Hypergraph Contrastive Learning with Generative Augmentation (A6)

Input: Hypergraph \mathcal{G} ; HyperGNN and generator parameters θ and ϕ ; Multi-task training tradeoff parameters α, β

- 1: Randomly initialize θ and ϕ ;
- 2: **while** not converge **do**
- 3: Obtain view $\tilde{\mathcal{G}}_g$ via fabricated augmentation and view $\tilde{\mathcal{G}}_{gen}$ via generator ϕ ;
- 4: Define HyperGNN loss as: $\mathcal{L}_h = \mathcal{L}_{sup}(\theta) + \alpha \cdot \mathcal{L}_{cl}(\tilde{\mathcal{G}}_p, \tilde{\mathcal{G}}_{gen} | \theta, \phi)$;
- 5: Define generator loss as: $\mathcal{L}_g = \mathcal{L}_{gen}(\phi) - \beta \cdot \mathcal{L}_{cl}(\mathcal{G}, \tilde{\mathcal{G}}_{gen} | \theta, \phi)$;
- 6: Update HyperGNN θ to minimize \mathcal{L}_h ;
- 7: Update generator ϕ to minimize \mathcal{L}_g ;
- 8: **end while**



□ Datasets

- Split the data into training/validation/test samples using (10%/10%/80%) splitting percentages

	Cora	Citeseer	Pubmed	Cora-CA	DBLP-CA	Zoo	20News	Mushroom	NTU2012	ModelNet40	Yelp	House	Walmart
$ \mathcal{V} $	2708	3312	19717	2708	41302	101	16242	8124	2012	12311	50758	1290	88860
$ \mathcal{E} $	1579	1079	7963	1072	22363	43	100	298	2012	12311	679302	341	69906
# feature	1433	3703	500	1433	1425	16	100	22	100	100	1862	100	100
# class	7	6	3	7	6	7	4	2	67	40	9	2	11
max $ e $	5	26	171	43	202	93	2241	1808	5	5	2838	81	25
min $ e $	2	2	2	2	2	1	29	1	5	5	2	1	2
avg $ e $	3.03	3.2	4.35	4.28	4.45	39.93	654.51	136.31	5	5	6.66	34.72	6.59
med $ e $	3	2	3	3	3	40	537	72	5	5	3	40	5
max d_v	145	88	99	23	18	17	44	5	19	30	7855	44	5733
min d_v	0	0	0	0	1	17	1	5	1	1	1	0	0
avg d_v	1.77	1.04	1.76	1.69	2.41	17	4.03	5	5	5	89.12	9.18	5.18
med d_v	1	0	0	2	2	17	3	5	5	4	35	7	2
h_e	0.86	0.83	0.88	0.88	0.93	0.66	0.73	0.96	0.87	0.92	0.57	0.58	0.75
h_v	0.84	0.78	0.79	0.79	0.88	0.35	0.49	0.87	0.81	0.88	0.26	0.52	0.55

□ Comparison among different hypergraph augmentations

- Generalized hyperedge augmentation works the best among fabricated augmenting operators
- Generative augmentation usually performs the best in all the data sets

Table 3: Results on the test data sets: Mean accuracy (%) \pm standard deviation. Bold values indicate the best result. Underlined values indicate the second best. 10% of all vertices are used for training.

	Cora	Citeseer	Pubmed	Cora-CA	DBLP-CA	Zoo	20NewsGroups	Mushroom
SetGNN	67.93 \pm 1.27	63.53 \pm 1.32	84.33 \pm 0.36	72.21 \pm 1.51	89.51 \pm 0.18	65.06 \pm 12.82	79.37 \pm 0.35	99.75 \pm 0.11
Self	68.24 \pm 1.12	62.49 \pm 1.48	84.38 \pm 0.38	72.74 \pm 1.53	89.51 \pm 0.23	57.35 \pm 18.32	79.45 \pm 0.32	95.83 \pm 0.23
Con	68.89 \pm 1.80	62.82 \pm 1.21	84.56 \pm 0.34	73.22 \pm 1.65	89.59 \pm 0.13	61.05 \pm 14.54	79.49 \pm 0.45	95.85 \pm 0.31
A0	68.59 \pm 1.33	62.25 \pm 2.15	84.54 \pm 0.42	71.85 \pm 1.62	89.62 \pm 0.24	62.57 \pm 13.84	79.07 \pm 0.46	99.77 \pm 0.17
A1	72.39 \pm 1.34	66.28 \pm 1.27	85.17 \pm 0.37	75.45 \pm 1.54	89.83 \pm 0.21	65.80 \pm 13.31	79.47 \pm 0.32	99.80 \pm 0.14
A2	72.58 \pm 1.09	<u>66.40 \pm 1.35</u>	85.16 \pm 0.38	<u>75.62 \pm 1.42</u>	<u>90.22 \pm 0.23</u>	<u>66.35 \pm 13.26</u>	<u>79.56 \pm 0.42</u>	99.80 \pm 0.17
A3	72.33 \pm 1.23	65.79 \pm 1.18	<u>85.24 \pm 0.28</u>	75.34 \pm 1.40	89.85 \pm 0.16	65.79 \pm 14.05	79.47 \pm 0.34	<u>99.81 \pm 0.10</u>
A4	<u>72.95 \pm 1.19</u>	66.22 \pm 0.95	84.88 \pm 0.38	75.29 \pm 1.56	90.10 \pm 0.18	62.59 \pm 12.77	79.45 \pm 0.48	99.80 \pm 0.14
A5	67.96 \pm 0.99	63.21 \pm 1.25	84.48 \pm 0.40	72.61 \pm 1.86	89.75 \pm 0.24	62.47 \pm 12.39	79.42 \pm 0.52	99.79 \pm 0.10
A6	<u>73.12 \pm 1.48</u>	<u>66.94 \pm 1.00</u>	<u>85.72 \pm 0.38</u>	<u>76.21 \pm 1.26</u>	<u>90.28 \pm 0.19</u>	<u>66.89 \pm 12.44</u>	<u>79.78 \pm 0.40</u>	<u>99.86 \pm 0.10</u>
	NTU2012	ModelNet40	Yelp	House (0.6)	House (1.0)	Walmart (0.6)	Walmart (1.0)	Avg. Rank
SetGNN	73.86 \pm 1.62	95.85 \pm 0.38	28.78 \pm 1.51	68.54 \pm 1.89	58.34 \pm 2.25	74.97 \pm 0.22	59.13 \pm 0.20	7.71
Self	73.41 \pm 1.65	95.83 \pm 0.23	23.49 \pm 4.15	67.75 \pm 3.29	58.54 \pm 2.16	74.76 \pm 0.20	58.83 \pm 0.21	8.64
Con	73.27 \pm 1.53	95.85 \pm 0.31	26.14 \pm 1.86	68.50 \pm 2.52	58.56 \pm 2.42	75.17 \pm 0.21	59.39 \pm 0.20	7.07
A0	73.54 \pm 1.93	95.92 \pm 0.18	29.43 \pm 1.42	67.48 \pm 3.21	57.39 \pm 2.37	73.14 \pm 0.21	56.49 \pm 0.60	8.21
A1	74.71 \pm 1.81	95.87 \pm 0.27	27.18 \pm 0.71	68.64 \pm 2.99	58.10 \pm 3.22	75.42 \pm 0.13	60.09 \pm 0.25	4.50
A2	74.88 \pm 1.66	96.56 \pm 0.34	31.39 \pm 2.45	69.73 \pm 2.60	58.90 \pm 1.97	75.50 \pm 0.18	60.19 \pm 0.20	2.29
A3	74.68 \pm 1.74	96.48 \pm 0.29	27.57 \pm 1.00	67.88 \pm 2.90	58.51 \pm 2.22	75.29 \pm 0.23	60.19 \pm 0.20	4.71
A4	74.83 \pm 1.75	95.86 \pm 0.28	29.64 \pm 1.93	69.56 \pm 2.89	<u>58.91 \pm 2.69</u>	75.43 \pm 0.18	59.90 \pm 0.24	4.14
A5	74.41 \pm 1.86	96.46 \pm 0.33	29.24 \pm 1.42	68.14 \pm 2.97	57.70 \pm 2.98	75.26 \pm 0.18	59.81 \pm 0.22	6.71
A6	<u>75.34 \pm 1.91</u>	<u>96.93 \pm 0.33</u>	<u>34.64 \pm 0.39</u>	<u>70.96 \pm 2.27</u>	<u>59.93 \pm 1.99</u>	<u>75.62 \pm 0.16</u>	<u>60.46 \pm 0.20</u>	<u>1.00</u>

Table 4: Results on the test data sets: Mean accuracy (%) \pm standard deviation. Bold values indicate the best result. 1% of all vertices are used for training.

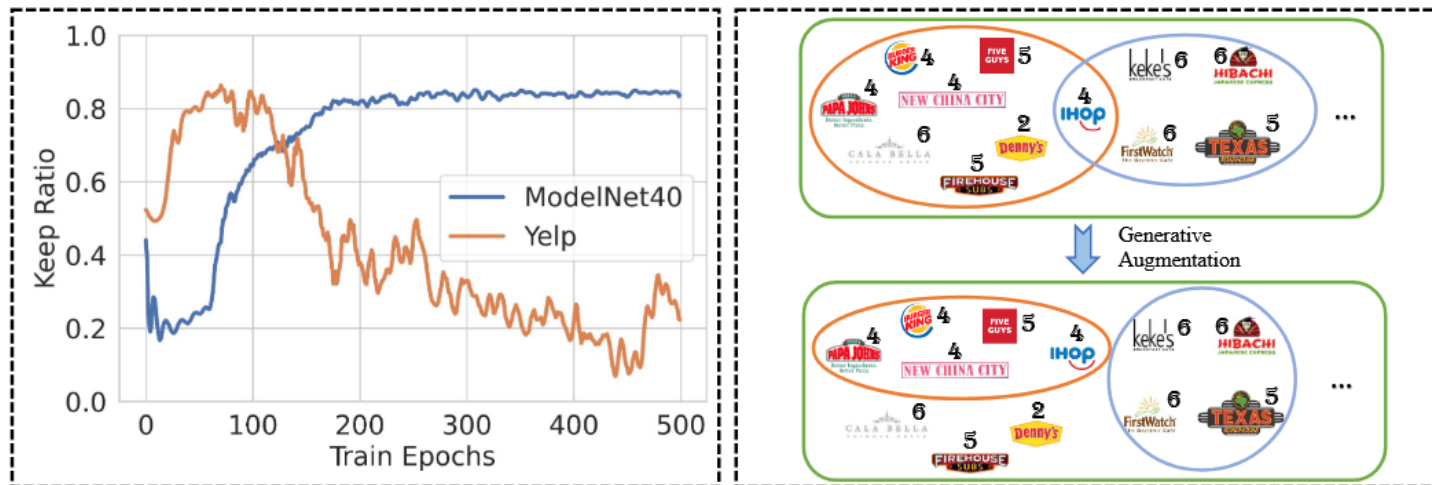
	Cora	Citeseer	Pubmed	Cora-CA	DBLP-CA	20NewsGroups	Mushroom
SetGNN	46.48 \pm 3.62	47.01 \pm 4.31	76.13 \pm 1.19	52.29 \pm 4.18	85.52 \pm 0.54	73.83 \pm 1.40	97.73 \pm 1.18
Self	45.79 \pm 5.34	44.22 \pm 4.43	76.71 \pm 0.90	51.64 \pm 5.37	84.42 \pm 0.37	73.91 \pm 0.90	92.25 \pm 0.89
Con	49.20 \pm 4.38	48.56 \pm 4.88	77.51 \pm 1.08	52.37 \pm 4.41	86.47 \pm 0.35	74.39 \pm 1.23	92.43 \pm 0.87
A0	48.50 \pm 4.77	46.43 \pm 4.24	78.83 \pm 1.79	49.87 \pm 5.08	87.34 \pm 0.73	74.43 \pm 1.11	97.32 \pm 1.33
A1	56.42 \pm 5.02	55.63 \pm 3.96	80.13 \pm 1.44	60.86 \pm 5.91	87.53 \pm 0.30	74.68 \pm 1.31	97.95 \pm 1.15
A2	56.81 \pm 4.49	56.10 \pm 2.86	80.22 \pm 1.24	<u>60.96 \pm 6.31</u>	88.10 \pm 0.35	74.72 \pm 1.16	<u>98.05 \pm 1.18</u>
A3	55.94 \pm 3.67	55.82 \pm 3.40	80.13 \pm 1.02	60.51 \pm 4.55	87.47 \pm 0.36	74.63 \pm 1.00	98.04 \pm 0.98
A4	<u>58.55 \pm 5.14</u>	<u>57.16 \pm 4.62</u>	<u>80.11 \pm 1.02</u>	60.91 \pm 5.15	<u>88.91 \pm 0.29</u>	74.67 \pm 1.39	97.72 \pm 1.12
A5	46.23 \pm 3.44	45.07 \pm 4.89	75.95 \pm 1.32	53.26 \pm 4.86	87.12 \pm 0.43	<u>74.81 \pm 1.04</u>	97.72 \pm 1.25
A6	<u>57.45 \pm 5.00</u>	<u>56.23 \pm 3.27</u>	<u>81.10 \pm 0.80</u>	<u>61.76 \pm 4.94</u>	<u>88.55 \pm 0.41</u>	<u>75.52 \pm 0.93</u>	<u>98.28 \pm 1.03</u>
	ModelNet40	Yelp	House (0.6)	House (1.0)	Walmart (0.6)	Walmart (1.0)	Avg. Rank (\downarrow)
SetGNN	88.34 \pm 2.69	27.64 \pm 1.10	53.69 \pm 2.20	51.85 \pm 1.64	65.48 \pm 0.45	51.15 \pm 0.52	7.62
Self	86.85 \pm 3.03	20.77 \pm 5.15	53.42 \pm 2.25	51.14 \pm 1.75	65.23 \pm 0.43	51.00 \pm 0.41	9.69
Con	87.00 \pm 2.99	24.23 \pm 0.43	53.58 \pm 3.04	51.96 \pm 1.87	65.47 \pm 0.44	51.13 \pm 0.46	7.31
A0	88.75 \pm 2.78	27.43 \pm 0.60	53.60 \pm 2.73	51.70 \pm 2.13	65.41 \pm 0.47	51.10 \pm 0.49	7.46
A1	89.34 \pm 2.66	26.18 \pm 0.51	54.12 \pm 3.29	52.23 \pm 2.46	65.96 \pm 0.36	51.22 \pm 0.35	4.08
A2	89.37 \pm 2.69	27.67 \pm 0.91	<u>54.42 \pm 2.83</u>	<u>52.31 \pm 1.44</u>	<u>66.01 \pm 0.41</u>	<u>51.32 \pm 0.30</u>	2.69
A3	89.31 \pm 2.62	26.98 \pm 0.66	53.71 \pm 2.71	52.11 \pm 2.24	65.88 \pm 0.50	51.35 \pm 0.53	4.38
A4	89.03 \pm 2.66	27.45 \pm 0.81	53.64 \pm 2.61	51.77 \pm 2.20	65.55 \pm 0.51	51.04 \pm 0.47	4.54
A5	<u>89.43 \pm 2.68</u>	<u>28.09 \pm 0.96</u>	54.07 \pm 3.09	51.94 \pm 1.84	65.52 \pm 0.39	50.97 \pm 0.47	6.00
A6	<u>90.22 \pm 2.72</u>	<u>29.61 \pm 0.71</u>	<u>56.27 \pm 4.18</u>	<u>52.55 \pm 2.18</u>	<u>66.42 \pm 0.40</u>	<u>51.82 \pm 0.39</u>	<u>1.23</u>

Experiments

□ Training dynamics of keep ratio

■ Highly related to the dataset homophily

- The homophily of ModelNet40 : (0.92/0.88)
- The homophily of Yelp : (0.57/0.26)





I'm Me, We're Us, and I'm Us: Tri-directional Contrastive Learning on Hypergraphs

Dongjin Lee and Kijung Shin

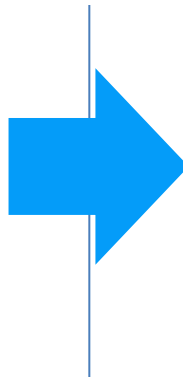
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□ Motivation

Node-only Contrast

- Cannot capture higher-order relations
- Lead to limited expressiveness



Tri-directional Contrast

- Node + Group + Membership contrast
- Preserve higher-order structural information
- Produce richer, more generalizable embeddings

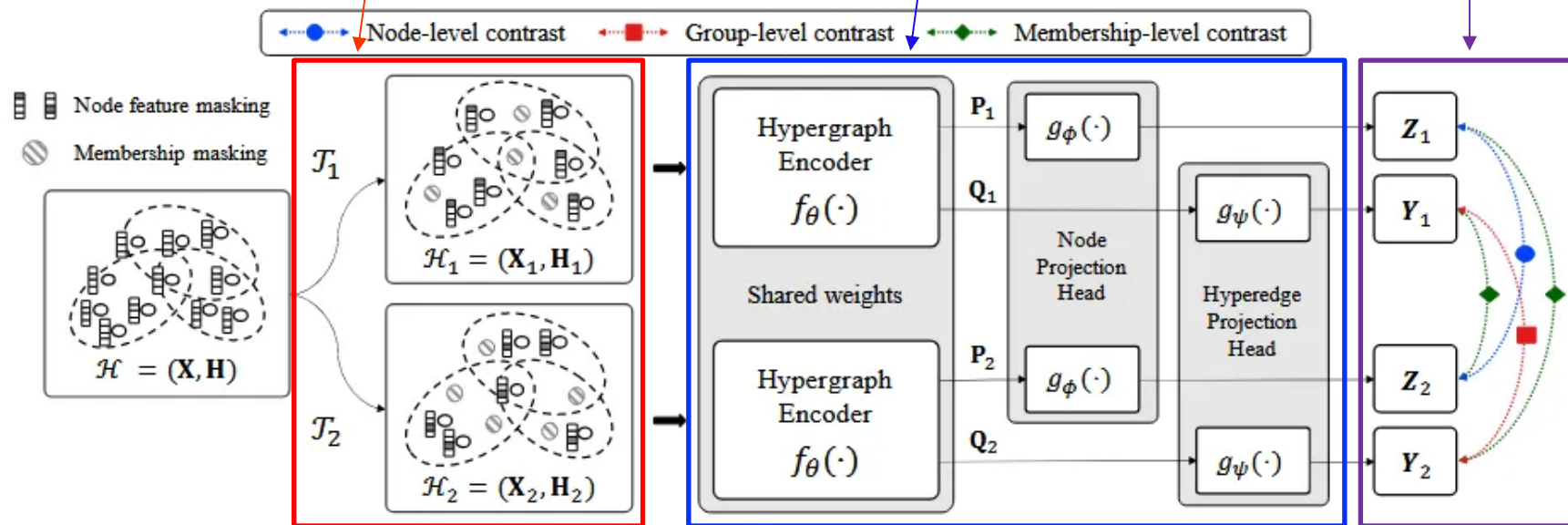
How to augment a hypergraph?	What to contrast?
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Overview

Generalized Hyperedge
Perturbation + Attribute masking

Form node and hyperedge
representations

Apply three forms of contrast

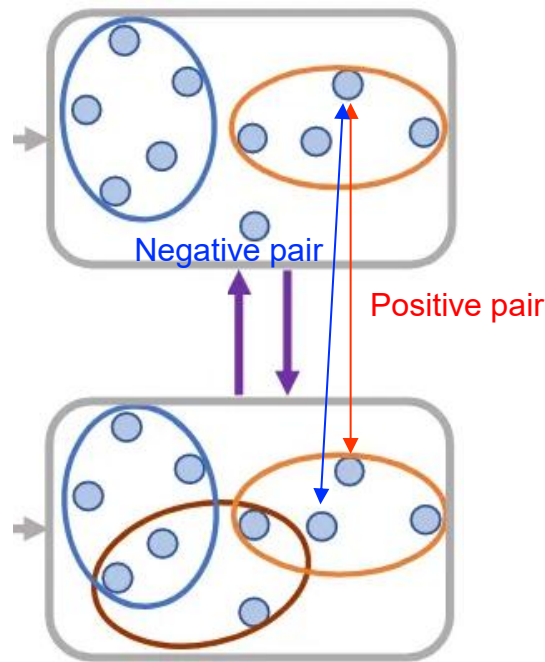


□ Node-level contrast

- Discriminate the representations of the same node in the two augmented views from other node representations

$$\ell_n(\mathbf{z}_{1,i}, \mathbf{z}_{2,i}) = -\log \frac{e^{s(\mathbf{z}_{1,i}, \mathbf{z}_{2,i})/\tau_n}}{\sum_{k=1}^{|V|} e^{s(\mathbf{z}_{1,i}, \mathbf{z}_{2,k})/\tau_n}},$$

$$\mathcal{L}_n = \frac{1}{2|V|} \sum_{i=1}^{|V|} \{\ell_n(\mathbf{z}_{1,i}, \mathbf{z}_{2,i}) + \ell_n(\mathbf{z}_{2,i}, \mathbf{z}_{1,i})\}.$$

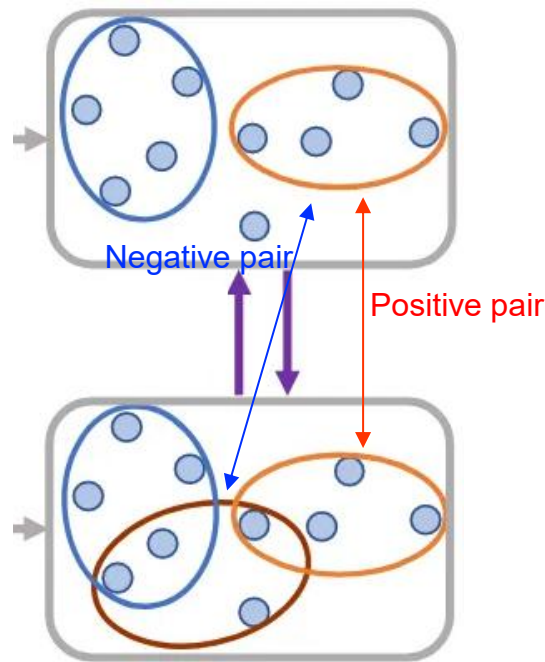


□ Group-level contrast

- Distinguish the representations of the same hyperedge in the two augmented views from other hyperedge representations

$$\ell_g(\mathbf{y}_{1,j}, \mathbf{y}_{2,j}) = -\log \frac{e^{s(\mathbf{y}_{1,j}, \mathbf{y}_{2,j})/\tau_g}}{\sum_{k=1}^{|E|} e^{s(\mathbf{y}_{1,j}, \mathbf{y}_{2,k})/\tau_g}},$$

$$\mathcal{L}_g = \frac{1}{2|E|} \sum_{j=1}^{|E|} \{\ell_g(\mathbf{y}_{1,j}, \mathbf{y}_{2,j}) + \ell_g(\mathbf{y}_{2,j}, \mathbf{y}_{1,j})\}.$$

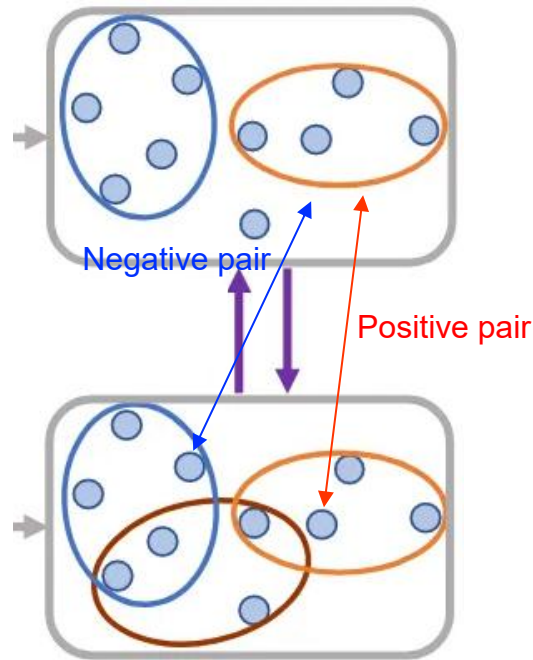


□ Membership-level contrast

- Learn to represent if relationships actually exist and to move away if they don't exist

$$\ell_m(\mathbf{z}_i, \mathbf{y}_j) = -\log \underbrace{\frac{e^{\mathcal{D}(\mathbf{z}_i, \mathbf{y}_j)/\tau_m}}{e^{\mathcal{D}(\mathbf{z}_i, \mathbf{y}_j)/\tau_m} + \sum_{k:i \notin k} e^{\mathcal{D}(\mathbf{z}_i, \mathbf{y}_k)/\tau_m}}}_{\text{when } \mathbf{z}_i \text{ is the anchor}} - \log \underbrace{\frac{e^{\mathcal{D}(\mathbf{z}_i, \mathbf{y}_j)/\tau_m}}{e^{\mathcal{D}(\mathbf{z}_i, \mathbf{y}_j)/\tau_m} + \sum_{k:k \notin j} e^{\mathcal{D}(\mathbf{z}_k, \mathbf{y}_j)/\tau_m}}}_{\text{when } \mathbf{y}_j \text{ is the anchor}},$$

$$\mathcal{L}_m = \frac{1}{2K} \sum_{i=1}^{|V|} \sum_{j=1}^{|E|} \mathbb{1}_{[h_{ij}=1]} \{ \ell_m(\mathbf{z}_{1,i}, \mathbf{y}_{2,j}) + \ell_m(\mathbf{z}_{2,i}, \mathbf{y}_{1,j}) \}.$$



□ Datasets

Table 6: Statistics of datasets used in our experiments.

	Cora-C	Citeseer	Pubmed	Cora-A	DBLP	Zoo	20News	Mushroom	NTU2012	ModelNet40
# Nodes	1,434	1,458	3,840	2,388	41,302	101	16,242	8,124	2,012	12,311
# Hyperedges	1,579	1,079	7,963	1,072	22,363	43	100	298	2,012	12,311
# Memberships	4,786	3,453	34,629	4,585	99,561	1,717	65,451	40,620	10,060	61,555
Avg. hyperedge size	3.03	3.20	4.35	4.28	4.45	39.93	654.51	136.31	5	5
Avg. node degree	3.34	2.37	9.02	1.92	2.41	17.00	4.03	5.00	5	5
Max. hyperedge size	5	26	171	43	202	93	2241	1808	5	5
Max. node degree	145	88	99	23	18	17	44	5	19	30
# Features	1,433	3,703	500	1,433	1,425	16	100	22	100	100
# Classes	7	6	3	7	6	7	4	2	67	40

□ Performance on Node Classification

- Graph contrastive learning methods show significantly lower accuracy compared to TriCL

Table 1: Node classification accuracy and standard deviations. Graph methods, marked as \star , are applied after converting hypergraphs to graphs via clique expansion. For each dataset, the best and the second-best performances are highlighted in **boldface** and underlined, respectively. A.R. denotes average rank, OOT denotes cases where results are not obtained within 24 hours, and OOM indicates out of memory on a 24GB GPU. In most cases, TriCL outperforms all others, including the supervised ones.

	Method	Cora-C	Citeseer	Pubmed	Cora-A	DBLP	Zoo	20News	Mushroom	NTU2012	ModelNet40	A.R.↓
Supervised	MLP	60.32 \pm 1.5	62.06 \pm 2.3	76.27 \pm 1.1	64.05 \pm 1.4	81.18 \pm 0.2	75.62 \pm 9.5	79.19 \pm 0.5	99.58 \pm 0.3	65.17 \pm 2.3	93.75 \pm 0.6	12.5
	GCN*	77.11 \pm 1.8	66.07 \pm 2.4	82.63 \pm 0.6	73.66 \pm 1.3	87.58 \pm 0.2	36.79 \pm 9.6	OOM	92.47 \pm 0.9	71.17 \pm 2.4	91.67 \pm 0.2	11.7
	GAT*	77.75 \pm 2.1	67.62 \pm 2.5	81.96 \pm 0.7	74.52 \pm 1.3	88.59 \pm 0.1	36.48 \pm 10.0	OOM	OOM	70.94 \pm 2.6	91.43 \pm 0.3	11
	HGNN	77.50 \pm 1.8	66.16 \pm 2.3	83.52 \pm 0.7	74.38 \pm 1.2	88.32 \pm 0.3	78.58 \pm 11.1	80.15 \pm 0.3	98.59 \pm 0.5	72.03 \pm 2.4	92.23 \pm 0.2	8.1
	HyperConv	76.19 \pm 2.1	64.12 \pm 2.6	83.42 \pm 0.6	73.52 \pm 1.0	88.83 \pm 0.2	62.53 \pm 14.5	79.83 \pm 0.4	97.56 \pm 0.6	72.62 \pm 2.6	91.84 \pm 0.1	9.8
	HNHN	76.21 \pm 1.7	67.28 \pm 2.2	80.97 \pm 0.9	74.88 \pm 1.6	86.71 \pm 1.2	78.89 \pm 10.2	79.51 \pm 0.4	99.78 \pm 0.1	71.45 \pm 3.2	92.96 \pm 0.2	8.9
	HyperGCN	64.11 \pm 7.4	59.92 \pm 9.6	78.40 \pm 9.2	60.65 \pm 9.2	76.59 \pm 7.6	40.86 \pm 2.1	77.31 \pm 6.0	48.26 \pm 0.3	46.05 \pm 3.9	69.23 \pm 2.8	15.1
	HyperSAGE	64.98 \pm 5.3	52.43 \pm 9.4	79.49 \pm 8.7	64.59 \pm 4.3	79.63 \pm 8.6	40.86 \pm 2.1	OOM	OOM	OOM	OOM	14.7
	UniGCN	77.91 \pm 1.9	66.40 \pm 1.9	84.08 \pm 0.7	77.30 \pm 1.4	90.31 \pm 0.2	72.10 \pm 12.1	80.24 \pm 0.4	98.84 \pm 0.5	73.27 \pm 2.7	94.62 \pm 0.2	5.9
	AllSet	76.21 \pm 1.7	67.83 \pm 1.8	82.85 \pm 0.9	76.94 \pm 1.3	90.07 \pm 0.3	72.72 \pm 11.8	79.90 \pm 0.4	99.78 \pm 0.1	75.09 \pm 2.5	96.85 \pm 0.2	6.2
Unsupervised	Node2vec*	70.99 \pm 1.4	53.85 \pm 1.9	78.75 \pm 0.9	58.50 \pm 2.1	72.09 \pm 0.3	17.02 \pm 4.1	63.35 \pm 1.7	88.16 \pm 0.8	67.72 \pm 2.1	84.94 \pm 0.4	15.6
	DGI*	78.17 \pm 1.4	68.81 \pm 1.8	80.83 \pm 0.6	76.94 \pm 1.1	88.00 \pm 0.2	36.54 \pm 9.7	OOM	OOM	72.01 \pm 2.5	92.18 \pm 0.2	9.3
	GRACE*	79.11 \pm 1.7	68.65 \pm 1.7	80.08 \pm 0.7	76.59 \pm 1.0	OOM	37.07 \pm 9.3	OOM	OOM	70.51 \pm 2.4	90.68 \pm 0.3	10.4
	S ² -HHGR	78.08 \pm 1.7	68.21 \pm 1.8	82.13 \pm 0.6	78.15 \pm 1.1	88.69 \pm 0.2	80.06 \pm 11.1	79.75 \pm 0.3	97.15 \pm 0.5	73.95 \pm 2.4	93.26 \pm 0.2	6.8
	Random-Init	63.62 \pm 3.1	60.44 \pm 2.5	67.49 \pm 2.2	66.27 \pm 2.2	76.57 \pm 0.6	78.43 \pm 11.0	77.14 \pm 0.6	97.40 \pm 0.6	74.39 \pm 2.6	96.29 \pm 0.3	11.9
	TriCL-N	80.23 \pm 1.2	70.28 \pm 1.5	83.44 \pm 0.6	81.94 \pm 1.1	90.88 \pm 0.1	79.94 \pm 11.1	<u>80.18 \pm 0.2</u>	99.76 \pm 0.2	75.20 \pm 2.6	97.01 \pm 0.2	3.4
	TriCL-NG	81.45 \pm 1.2	71.38 \pm 1.2	83.68 \pm 0.7	82.00 \pm 1.0	90.94 \pm 0.1	80.19 \pm 11.1	<u>80.18 \pm 0.2</u>	99.81 \pm 0.1	75.25 \pm 2.5	97.02 \pm 0.1	2
	TriCL	81.57 \pm 1.1	72.02 \pm 1.2	84.26 \pm 0.6	82.15 \pm 0.9	91.12 \pm 0.1	80.25 \pm 11.2	80.14 \pm 0.2	99.83 \pm 0.1	<u>75.23 \pm 2.4</u>	97.08 \pm 0.1	1.5

□ Performance on Node Classification

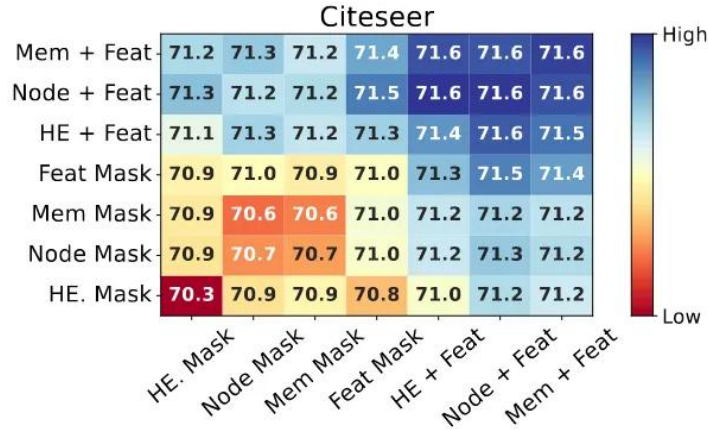
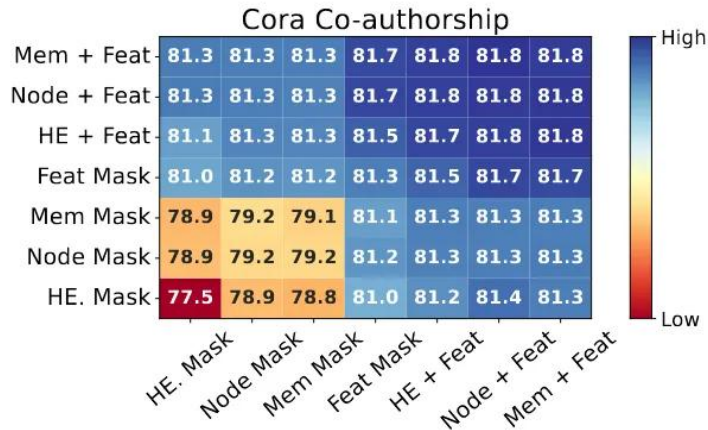
- Considering the different types of contrast together can help improve performance

Table 2: Comparison of node classification accuracy according to whether or not to use each type of contrast (i.e., \mathcal{L}_n , \mathcal{L}_g , and \mathcal{L}_m). Using all types of contrasts (i.e., node-, group-, and membership-level contrast) achieves the best performance in most cases as they are complementarily reinforcing each other.

\mathcal{L}_n	\mathcal{L}_g	\mathcal{L}_m	Cora-C	Citeseer	Pubmed	Cora-A	DBLP	Zoo	20News	Mushroom	NTU2012	ModelNet40	A.R.↓
✓	-	-	80.23 ± 1.2	70.28 ± 1.5*	83.44 ± 0.6	81.94 ± 1.1	90.88 ± 0.1	79.94 ± 11.1	80.18 ± 0.2	99.76 ± 0.2	75.20 ± 2.6	97.01 ± 0.2	3.8
-	✓	-	79.69 ± 1.6	71.02 ± 1.3*	80.20 ± 1.3	78.98 ± 1.4	88.60 ± 0.2	79.31 ± 10.7	79.35 ± 0.4	99.13 ± 0.3	74.41 ± 2.6	96.66 ± 0.2	5.7
-	-	✓	76.76 ± 1.8	63.98 ± 2.0	79.86 ± 0.9	76.77 ± 1.1	63.95 ± 7.2	79.80 ± 11.0	79.27 ± 0.3	94.87 ± 0.7	73.11 ± 2.8	96.57 ± 0.2	6.9
✓	✓	-	81.45 ± 1.2	71.38 ± 1.4	83.68 ± 0.7	82.00 ± 1.0	90.94 ± 0.1	80.19 ± 11.1	80.18 ± 0.2	99.81 ± 0.1	75.25 ± 2.5	97.02 ± 0.1	<u>2.3</u>
✓	-	✓	80.49 ± 1.3	70.46 ± 1.5	83.98 ± 0.7	81.62 ± 1.0	90.75 ± 0.1	80.19 ± 11.1	80.15 ± 0.2	99.74 ± 0.2	75.12 ± 2.5	97.03 ± 0.1	3.6
-	✓	✓	80.80 ± 1.1	71.73 ± 1.4	82.81 ± 0.7	80.24 ± 1.0	90.17 ± 0.1	80.20 ± 11.1	79.29 ± 0.2	99.82 ± 0.1	73.76 ± 2.5	96.74 ± 0.1	4.1
✓	✓	✓	81.57 ± 1.1	72.02 ± 1.4	84.26 ± 0.6	82.15 ± 0.9	91.12 ± 0.1	80.25 ± 11.2	80.14 ± 0.2	99.83 ± 0.1	<u>75.23 ± 2.4</u>	97.08 ± 0.1	1.4

□ Hypergraph augmentation

- Using the structural and attribute augmentations together always yields better performance than using just one





Enhancing Hyperedge Prediction with Context-Aware Self-Supervised Learning

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Overview

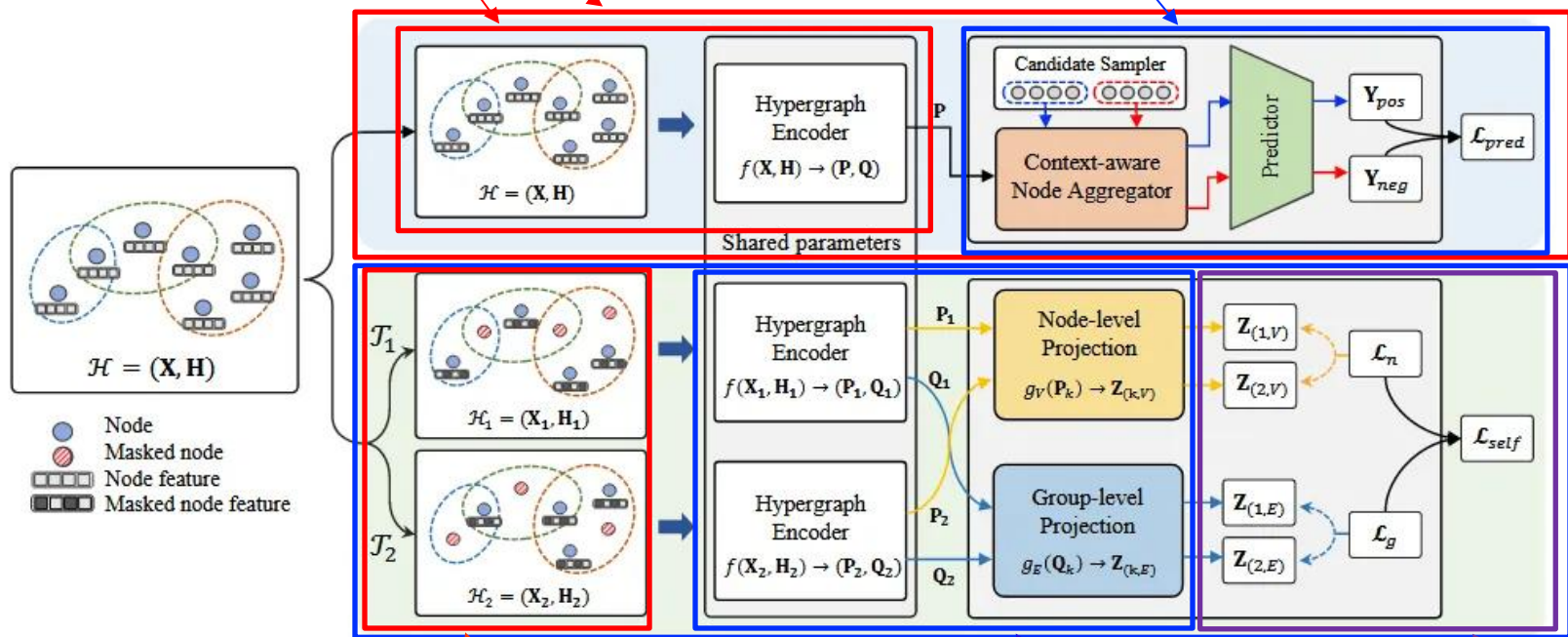


Fig. 2. The overview of CASH: (1) Context-aware hyperedge prediction (upper) and (2) Self-supervised contrastive hypergraph learning (lower).

Hyperedge-aware augmentation

Form node and hyperedge representations

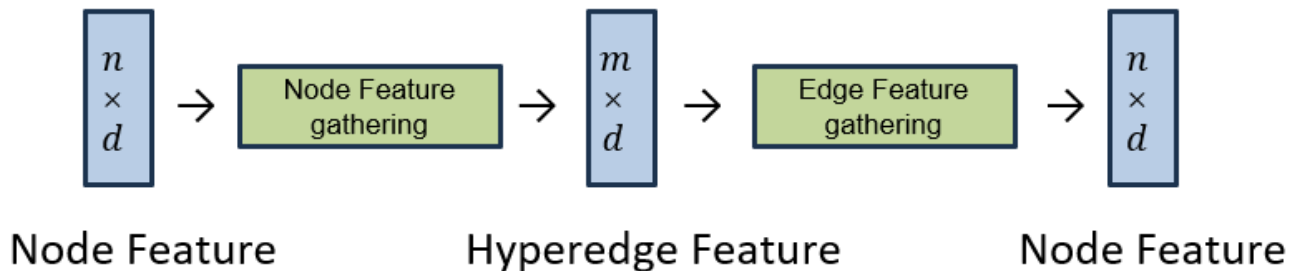
Self-supervised contrastive hypergraph learning

Apply dual contrast

□ Hypergraph encoding

- Produce the node and hyperedge embeddings via a 2-stage aggregation strategy

$$Q^{(k)} = \sigma \left(D_E^{-1} H^T P^{(k-1)} W_E^{(k)} + b_E^{(k)} \right)$$
$$P^{(k)} = \sigma \left(D_V^{-1} H Q^{(k)} W_V^{(k)} + b_V^{(k)} \right)$$

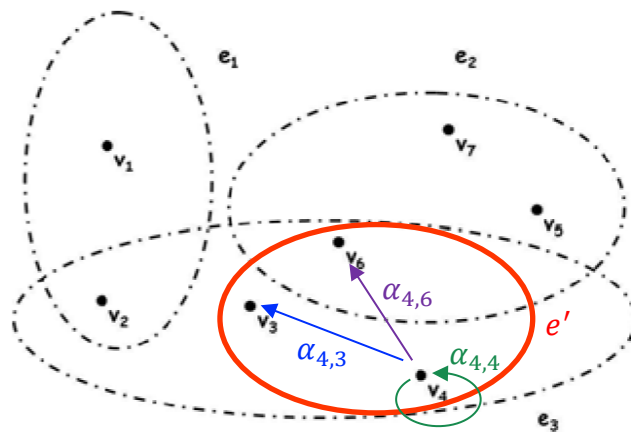


□ Context-Aware Hyperedge Prediction

■ Hyperedge candidate scoring

- Calculate the relative degrees of influences of nodes by using the attention mechanism

$$\alpha_{i,j} = \frac{\exp(p_{v'_i} \mathbf{W}_{agg}'' \cdot x^\top)}{\sum_{v'_j \in e'} \exp(p_{v'_j} \mathbf{W}_{agg}'' \cdot x^\top)}$$

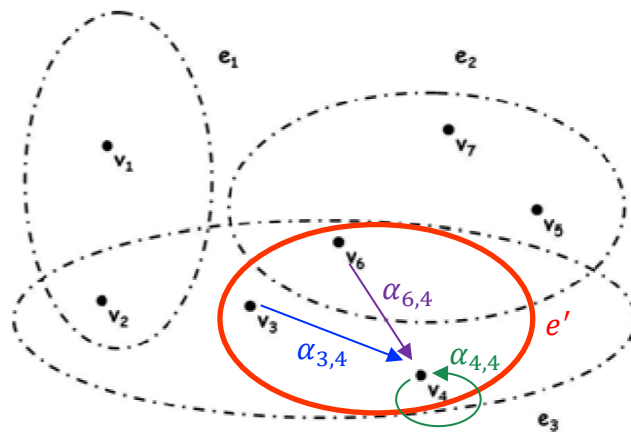


□ Context-Aware Hyperedge Prediction

■ Hyperedge candidate scoring

- Update each node embedding based on the relative degrees of influences

$$p_{v'_j}^* = \sum_{v'_i \in e'} \alpha_{i,j} \cdot p_{v'_i} \mathbf{W}'_{agg}$$

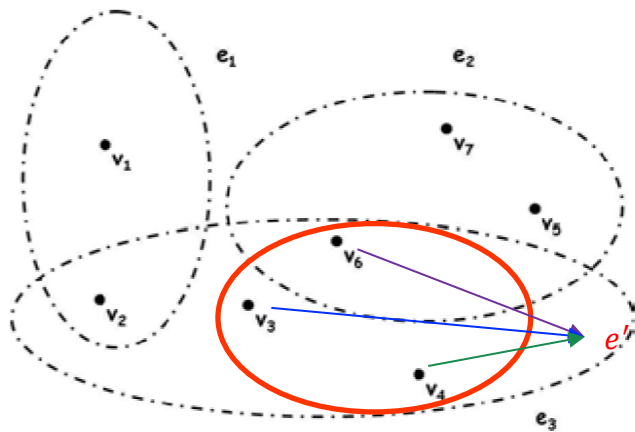


□ Context-Aware Hyperedge Prediction

■ Hyperedge candidate scoring

- Aggregate the influence-reflected embeddings of the nodes via *element-wise max pooling*
- $pred(\cdot)$: a hyperedge predictor (a fully-connected layer ($d \times 1$))

$$\hat{y}_{e'} = pred(q_{e'}^*), \quad q_{e'}^* = MaxPool(P^*[e', :])$$

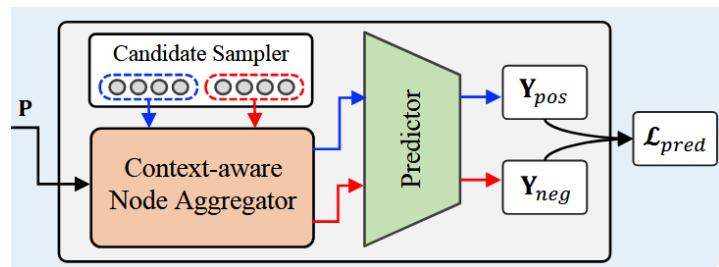


□ Context-Aware Hyperedge Prediction

■ Model training

□ Use heuristic negative sampling methods

- Sized NS (SNS) : sampling k random nodes (easy)
- Motif NS (MNS) : sampling a k -connected component in a clique-expanded hypergraph (difficult)
- Clique NS (CNS) : selecting a hyperedge e and replacing one of its incident nodes $u \in e$ with a node $v \notin e$ (most difficult)



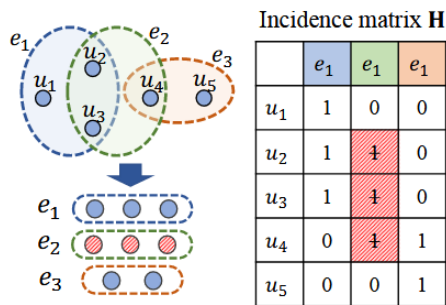
□ Prediction loss

$$\mathcal{L}_{pred} = -\frac{1}{|E'|} \sum_{e' \in E'} \underbrace{y_{e'} \cdot \log \hat{y}_{e'}}_{\text{positives}} + \underbrace{(1 - y_{e'}) \cdot \log (1 - \hat{y}_{e'})}_{\text{negatives}}$$

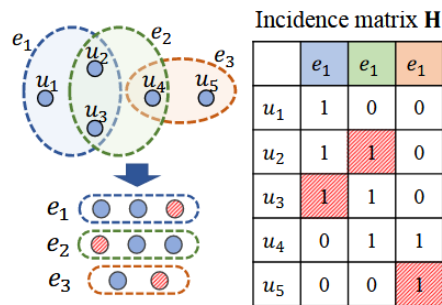
□ Self-Supervised Contrastive Hypergraph Learning

■ Hypergraph augmentation

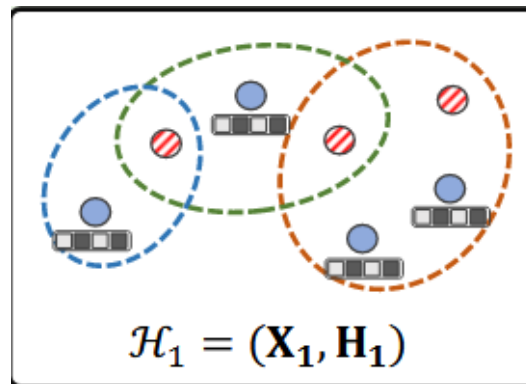
- Mask random $p_m\%$ members of each hyperedge *individually*
- Mask random $p_f\%$ dimensions of node features



(a) Random masking



(b) Hyperedge-aware masking



□ Self-Supervised Contrastive Hypergraph Learning

■ Contrastive loss

$$\mathcal{L}_{con} = \underbrace{-\log \text{sim}(\mathbf{Z}_{(1,V)}, \mathbf{Z}_{(2,V)})}_{\text{node-level contrast}} - \underbrace{\log \text{sim}(\mathbf{Z}_{(1,E)}, \mathbf{Z}_{(2,E)})}_{\text{group-level contrast}},$$

■ Unified loss of CASH

$$\mathcal{L} = \mathcal{L}_{pred} + \beta \mathcal{L}_{con}$$

□ Datasets

STATISTICS OF HYPERGRAPH DATASETS

Dataset	$ V $	$ E $	# Features	Type
Citeseer	1,457	1,078	3,703	Co-citation
Cora	1,434	1,579	1,433	Co-citation
Pubmed	3,840	7,962	500	Co-citation
Cora-A	2,388	1,072	1,433	Authorship
DBLP-A	39,283	16,483	4,543	Authorship
DBLP	15,639	22,964	4,543	Collaboration

Experiments

□ Hyperedge prediction accuracy

■ Consistently outperform all competitors

Dataset	Metric	AUROC					Average Precision (AP)				
		SNS	MNS	CNS	MIX	Average	SNS	MNS	CNS	MIX	Average
Citeseer	Expansion	0.663	0.781	0.331	0.588	0.591 ± 0.011	0.765	0.817	0.498	0.630	0.681 ± 0.001
	HyperSAGNN	0.540	0.410	0.473	0.478	0.475 ± 0.019	0.627	0.455	0.497	0.507	0.512 ± 0.015
	NHP	0.991	0.701	0.510	0.817	0.751 ± 0.009	0.990	0.731	0.520	0.768	0.751 ± 0.011
	AHP	0.943	0.881	0.651	0.820	0.824 ± 0.020	0.952	0.870	0.660	0.795	0.819 ± 0.022
	CASH	0.925	0.921	0.720	0.857	0.856 ± 0.011	0.928	0.919	0.701	0.831	0.845 ± 0.009
	Improvement (%)	-6.65%	+4.54%	+10.60%	+4.51%	+3.88%	-6.26%	5.63%	+6.21%	+4.53%	+3.17%
Cora	Expansion	0.470	0.707	0.256	0.476	0.477 ± 0.009	0.637	0.764	0.454	0.563	0.607 ± 0.009
	HyperSAGNN	0.617	0.527	0.494	0.540	0.545 ± 0.021	0.687	0.574	0.508	0.566	0.584 ± 0.019
	NHP	0.943	0.641	0.472	0.774	0.703 ± 0.015	0.949	0.678	0.509	0.744	0.718 ± 0.020
	AHP	0.964	0.860	0.572	0.799	0.799 ± 0.019	0.961	0.837	0.552	0.740	0.772 ± 0.035
	CASH	0.923	0.867	0.671	0.824	0.822 ± 0.011	0.915	0.854	0.644	0.789	0.801 ± 0.016
	Improvement (%)	-4.25%	+0.81%	+17.31%	+3.13%	+2.88%	-4.79%	+2.03%	+16.67%	+6.62%	+3.76%
Pubmed	Expansion	0.520	0.730	0.241	0.497	0.497 ± 0.015	0.675	0.755	0.440	0.565	0.612 ± 0.010
	HyperSAGNN	0.525	0.686	0.546	0.580	0.584 ± 0.066	0.534	0.680	0.529	0.561	0.576 ± 0.050
	NHP	0.973	0.694	0.524	0.745	0.733 ± 0.004	0.973	0.656	0.513	0.678	0.707 ± 0.004
	AHP	0.917	0.840	0.553	0.763	0.763 ± 0.009	0.918	0.834	0.526	0.717	0.749 ± 0.007
	CASH	0.805	0.871	0.640	0.772	0.772 ± 0.009	0.810	0.880	0.644	0.765	0.775 ± 0.008
	Improvement (%)	-17.26%	+3.69%	+15.73%	+1.18%	+1.18%	-16.75%	+5.52%	+21.74%	+6.69%	+3.47%
Core-A	Expansion	0.690	0.842	0.434	0.658	0.656 ± 0.011	0.690	0.876	0.577	0.672	0.706 ± 0.020
	HyperSAGNN	0.386	0.591	0.542	0.505	0.506 ± 0.019	0.532	0.643	0.545	0.563	0.571 ± 0.009
	NHP	0.909	0.672	0.550	0.773	0.723 ± 0.015	0.925	0.720	0.585	0.766	0.748 ± 0.019
	AHP	0.958	0.924	0.782	0.887	0.888 ± 0.014	0.957	0.898	0.796	0.878	0.882 ± 0.014
	CASH	0.971	0.975	0.833	0.931	0.927 ± 0.011	0.969	0.973	0.832	0.926	0.925 ± 0.011
	Improvement (%)	+1.36%	+5.52%	+6.52%	+4.96%	+4.39%	+1.25%	+8.35%	+4.52%	+5.47%	+4.88%
DBLP-A	Expansion	0.634	0.826	0.350	0.603	0.603 ± 0.006	0.730	0.852	0.512	0.641	0.687 ± 0.004
	HyperSAGNN	0.548	0.791	0.563	0.636	0.634 ± 0.007	0.686	0.805	0.552	0.655	0.675 ± 0.004
	NHP	0.966	0.623	0.555	0.721	0.716 ± 0.005	0.965	0.604	0.534	0.663	0.693 ± 0.007
	AHP	0.916	0.926	0.668	0.838	0.837 ± 0.004	0.928	0.928	0.707	0.836	0.850 ± 0.003
	CASH	0.929	0.957	0.747	0.877	0.877 ± 0.003	0.933	0.955	0.741	0.863	0.873 ± 0.005
	Improvement (%)	-3.83%	+3.35%	+11.83%	+4.65%	+4.78%	-3.32%	+2.91%	+4.81%	+3.23%	+2.71%
DBLP	Expansion	0.645	0.801	0.366	0.607	0.607 ± 0.005	0.751	0.856	0.518	0.655	0.698 ± 0.004
	HyperSAGNN	0.448	0.574	0.572	0.530	0.531 ± 0.018	0.562	0.602	0.586	0.577	0.582 ± 0.016
	NHP	0.663	0.540	0.503	0.572	0.569 ± 0.003	0.608	0.523	0.501	0.542	0.544 ± 0.002
	AHP	0.946	0.820	0.568	0.778	0.778 ± 0.002	0.947	0.815	0.561	0.735	0.764 ± 0.007
	CASH	0.875	0.836	0.708	0.807	0.807 ± 0.015	0.874	0.832	0.696	0.793	0.799 ± 0.011
	Improvement (%)	-7.50%	+1.95%	+23.78%	+3.73%	+3.73%	-7.70%	-2.80%	+18.77%	+7.89%	+4.58%

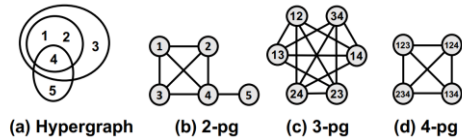


Figure 1: Hypergraph and its n -projected graphs (n -pgs). Edge weights, dropped for simplicity, reflect n -way interactions. For example, edge weight between the 3-pg nodes {1, 2} and {2, 3} is the number of times {1, 2, 3} interacted as a group.

□ Ablation study

- Each of strategies is always beneficial

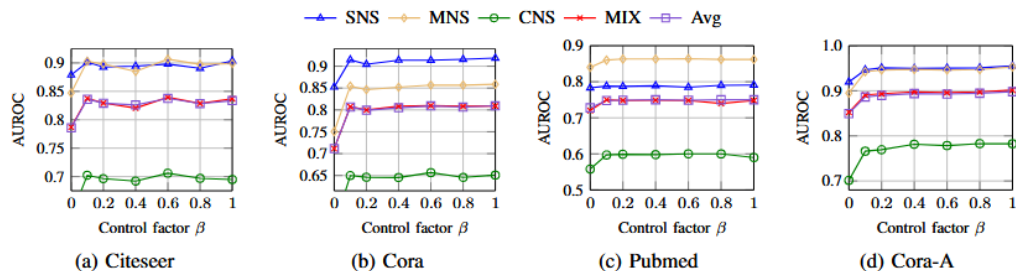
Dataset	Metric	AUROC					Average Precision (AP)				
	Test set	SNS	MNS	CNS	MIX	Average	SNS	MNS	CNS	MIX	Average
Citeseer	CASH-No	0.878	0.847	0.630	0.786	0.786 ± 0.003	0.890	0.841	0.653	0.775	0.790 ± 0.007
	CASH-CL	0.907	0.890	0.679	0.832	0.827 ± 0.019	0.905	0.874	0.675	0.815	0.817 ± 0.013
	CASH-HCL	0.908	0.897	0.691	0.839	0.833 ± 0.013	0.909	0.880	0.691	0.824	0.826 ± 0.005
	CASH-ALL	0.925	0.921	0.720	0.857	0.856 ± 0.011	0.928	0.919	0.701	0.831	0.845 ± 0.009
	Improvement (%)	+5.35%	+8.74%	+14.29%	+9.03%	+8.91%	+4.27%	+9.27%	+7.35%	+7.23%	+6.96%
Cora	CASH-No	0.852	0.750	0.532	0.711	0.712 ± 0.019	0.856	0.759	0.531	0.684	0.707 ± 0.021
	CASH-CL	0.895	0.837	0.600	0.782	0.779 ± 0.015	0.873	0.809	0.566	0.727	0.744 ± 0.019
	CASH-HCL	0.893	0.835	0.600	0.780	0.777 ± 0.017	0.879	0.816	0.565	0.730	0.747 ± 0.018
	CASH-ALL	0.923	0.867	0.671	0.824	0.822 ± 0.011	0.915	0.854	0.644	0.789	0.801 ± 0.016
	Improvement (%)	+8.33%	+15.60%	+26.13%	+15.89%	+15.45%	+6.89%	+12.52%	+21.28%	+15.35%	+13.30%
Pubmed	CASH-No	0.782	0.844	0.558	0.727	0.728 ± 0.007	0.802	0.852	0.555	0.708	0.730 ± 0.007
	CASH-CL	0.806	0.845	0.562	0.735	0.737 ± 0.010	0.817	0.847	0.552	0.708	0.731 ± 0.007
	CASH-HCL	0.814	0.848	0.562	0.739	0.741 ± 0.008	0.823	0.851	0.547	0.708	0.732 ± 0.006
	CASH-ALL	0.805	0.871	0.640	0.772	0.772 ± 0.009	0.810	0.880	0.644	0.765	0.775 ± 0.008
	Improvement (%)	+2.94%	+3.20%	+14.70%	+6.19%	+6.04%	+1.00%	+3.29%	+16.04%	+8.05%	+6.16%
Cora-A	CASH-No	0.949	0.894	0.701	0.852	0.849 ± 0.020	0.951	0.906	0.738	0.857	0.863 ± 0.017
	CASH-CL	0.943	0.934	0.756	0.883	0.879 ± 0.030	0.944	0.936	0.772	0.881	0.884 ± 0.026
	CASH-HCL	0.972	0.949	0.833	0.921	0.919 ± 0.008	0.972	0.919	0.845	0.908	0.911 ± 0.007
	CASH-ALL	0.971	0.975	0.833	0.931	0.927 ± 0.011	0.969	0.973	0.832	0.926	0.925 ± 0.011
	Improvement (%)	+2.32%	+9.06%	+18.83%	+9.27%	+9.19%	+1.89%	+7.40%	+12.74%	+8.05%	+7.18%

Experiments

Hyperparameter sensitivity

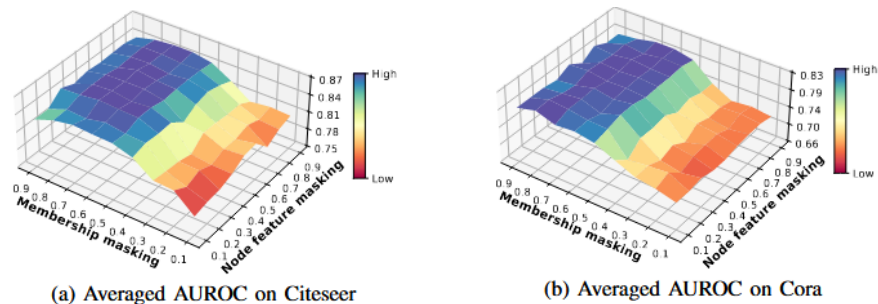
Control parameter β

Insensitive to its hyperparameter β



Augmentation parameter p_m, p_f

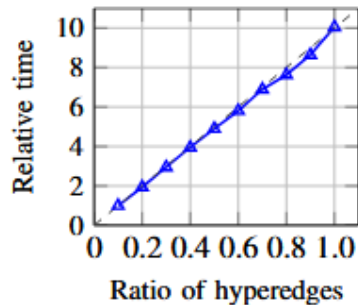
p_m is more important than p_f



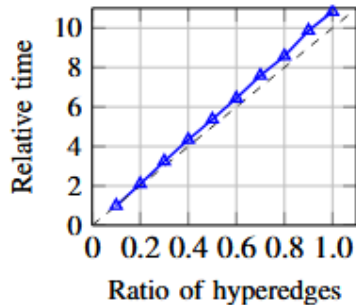
Experiments

□ Efficiency

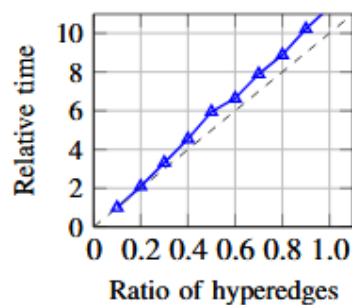
■ Scale up linearly



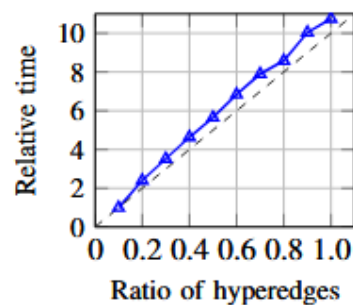
(a) Citeseer



(b) Cora



(c) DBLP-A



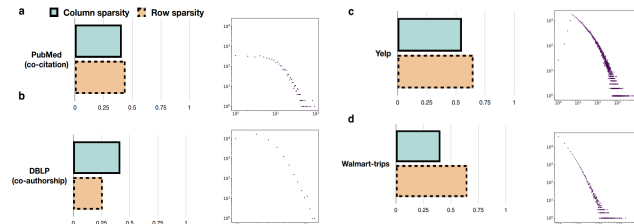
(d) DBLP

Conclusion

□ Hypergraph data labeling is difficult because of its sparsity

□ Graph Contrastive Learning

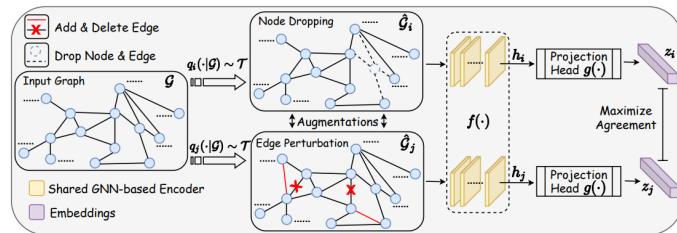
■ Maximize the agreement between the two views



□ Challenges

■ How to augment a hypergraph?

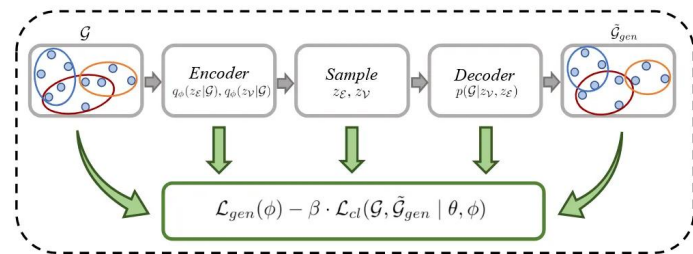
■ What to contrast?



Conclusion

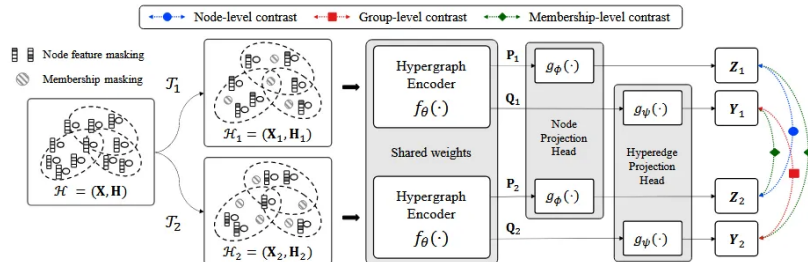
□ HyperGCL

- Augmentation of hypergraph can be learned during contrastive learning using variational hypergraph auto-encoder



□ TriCL

- Node + Group + Membership contrast



□ CASH

- Context-Aware Hyperedge Prediction
- Hyperedge-aware masking

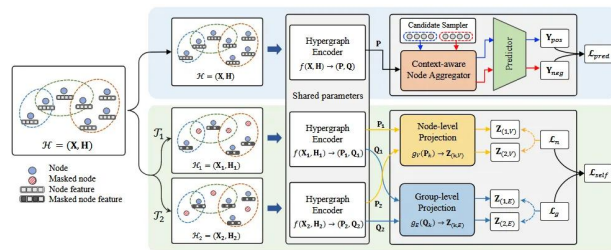


Fig. 2. The overview of CASH: (1) Context-aware hyperedge prediction (upper) and (2) Self-supervised contrastive hypergraph learning (lower).