Exercise 19.1

1. Find the mean of 8, 6, 10, 12, 1, 3, 4, 4
2. 5 people were asked about the time in a week they spend in doing social work in their community. They replied 10, 7, 13, 20 and 15 hours, respectively. Find the mean time in a week devoted by them for social work.
3. The enrolment of a school during six consecutive years was as follows:
   1620, 2060, 2540, 3250, 3500, 3710
   Find the mean enrolment.
4. Find the mean of the first twelve natural numbers.
5. (*i*) Find the mean of the first six prime numbers.
   (*ii*) Find the mean of the first seven odd prime numbers.
6. (*i*) The marks (out of 100) obtained by a group of students in a Mathematics test are 81, 72, 90, 90, 85, 86, 70, 93 and 71. Find the mean marks obtained by the group of students.
   (*ii*) The mean of the age of three students Vijay, Rahul and Rakhi is 15 years. If their ages are in the ratio 4 : 5 : 6 respectively, then find their ages.
7. The mean of 5 numbers is 20. If one number is excluded, mean of the remaining numbers becomes 23. Find the excluded number.
8. The mean of 25 observations is 27. If one observation is included, the mean still remains 27. Find the included observation.
9. The mean of 5 observations is 15. If the mean of first three observations is 14 and that of the last three is 17, find the third observation.
10. The mean of 8 variates is 10·5. If seven of them are 3, 15, 7, 19, 2, 17 and 8, then find the 8th variate.
11. The mean weight of 8 students is 45·5 kg. Two more students having weights 41·7 kg and 53·3 kg join the group. What is the new mean weight?
12. Mean of 9 observations was found to be 35. Later on, it was detected that an observation 81 was misread as 18. Find the correct mean of the observations.
13. A student scored the following marks in 11 questions of a question paper :
    7, 3, 4, 1, 5, 8, 2, 2, 5, 7, 6
    Find the median marks.
14. Calculate the mean and the median of the numbers :
    2, 3, 4, 3, 0, 5, 1, 1, 3, 2
15. A group of students was given a special test in Mathematics. The test was completed by the various students in the following time in (minutes) :
    24, 30, 28, 17, 22, 36, 30, 19, 32, 18, 20, 24
    Find the mean time and median time taken by the students to complete the test.
16. In a Science test given to a group of students, the marks scored by them (out of 100) are:
    41, 39, 52, 48, 54, 62, 46, 52, 40, 96, 42, 40, 98, 60, 52
    Find the mean and median of this data.
17. The points scored by a Kabaddi team in a series of matches are as follows :
    7, 17, 2, 5, 27, 15, 8, 14, 10, 48, 10, 7, 24, 8, 28, 18
    Find the mean and the median of the points scored by the Kabaddi team.

18. The following observations have been arranged in ascending order. If the median of the data is 47·5, find the value of x.

17, 21, 23, 29, 39, 40, x, 50, 51, 54, 59, 67, 91, 93

19. The following observations have been arranged in ascending order. If the median of the data is 13, find the value of x:

3, 6, 7, 10, x, $x + 4$, 19, 20, 25, 28

19.3 GROUPED OR CLASSIFIED DATA

Consider the following examples of grouped frequency distribution :

Example 1. Using class intervals 1 – 5, 6 – 10, 11 – 15, ... construct the frequency distribution for the following data:

13, 6, 12, 9, 11, 14, 2, 8, 18, 16, 9, 13, 17, 11, 19, 6, 7, 12, 22, 21, 18, 1, 8, 12, 18

Solution. The frequency distribution table for the given grouped data is:

Class-intervals	*Tally marks*	*Frequency*
1 – 5	||	2
6 – 10	𝍸 ||	7
11 – 15	𝍸 |||	8
16 – 20	𝍸 |	6
21 – 25	||	2
Total		25

Example 2. The following is the pocket money survey of 50 students in a school (pocket money in rupees per month):

49, 55, 22, 27, 30, 27, 25, 27, 30, 42, 40, 13, 24, 38, 10, 24, 30, 33, 17, 29, 10, 50, 18, 34, 15, 40, 13, 32, 36, 32, 27, 35, 17, 41, 18, 36, 20, 41, 35, 51, 29, 27, 44, 43, 15, 32, 29, 54, 14, 45

Form a frequency table with a grouping of 10 – 20, 20 – 30, 30 – 40 and so on (class 10 – 20 means including 10 but excluding 20, class 20 – 30 means including 20 but excluding 30).

Solution. The grouped frequency table for the given data is:

Classes	*Tally marks*	*Frequency*
10 – 20	𝍸 𝍸 |	11
20 – 30	𝍸 𝍸 |||	13
30 – 40	𝍸 𝍸 |||	13
40 – 50	𝍸 ||||	9
50 – 60	||||	4
Total		50

Class limits and types of frequency distribution

In example 1, for the class interval 1 – 5, 1 is the **lower limit** and 5 is the **upper limit**.

If x is a member of this class, then $1 \leq x \leq 5$. Similarly, 6 is the *lower limit* and 10 is the *upper limit* of the class 6 – 10. In this example, the classes are non-overlapping but discontinuous. Such a frequency distribution is called **discrete** (or **inclusive**) **distribution**. In this distribution, the upper limit of one class does not coincide with the lower limit of the next class.

In example 2, for the class 1 – 10, 1 is the **lower limit** and 10 is the **upper limit**. If x is a member of this class, then $1 \leq x < 10$. Similarly, 10 is the **lower limit** and 20 is the **upper limit** of the class 10 – 20. In this example, the classes are non-overlapping but continuous. Such a frequency distribution is called **continuous** (or **exclusive**) **distribution**. In this distribution, the upper limit of one class coincides with the lower limit of the next class.

Converting discrete distribution to continuous distribution

If we measure height, weight and time, there may be fractions of a metre, kilogram and hour respectively, therefore, we need continuous distribution.

To convert discrete classes into continuous classes, we require some *adjustment*.

$$\text{Adjustment factor} = \frac{\text{lower limit of one class} - \text{upper limit of previous class}}{2}$$

Subtract the adjustment factor from all the lower limits and add the adjustment factor to all the upper limits.

In example 1, adjustment factor $= \frac{6-5}{2} = \frac{1}{2} = 0{\cdot}5$

Continuous frequency distribution table for example 1 is:

Classes before adjustment	*Classes after adjustment*	*Tally marks*	*Frequency*
1 – 5	0·5 – 5·5	II	2
16 – 10	5·5 – 10·5	𝍸 II	7
11 – 15	10·5 – 15·5	𝍸 III	8
16 – 20	15·5 – 20·5	𝍸 I	6
21 – 25	20·5 – 25·5	II	2
Total			25

True class limits

In a continuous distribution, the class limits are called **true** or **actual class limits**. In a discrete distribution, the class limits obtained after adjustment are the *true* or *actual class limits*. The actual class limits are also called ***class boundaries***.

In discrete distribution, the original (given) class limits are called the ***stated class limits***.

Class size. *The difference between the actual upper limit and the actual lower limit of a class is called its* ***class size***.

In example 2, class size of the class 10 – 20 = 20 – 10 = 10

In example 1, class size of the class 1 – 5 = 5·5 – 0·5 = 5

Class mark. *The* ***class mark*** *of a class is the value midway between its actual lower limit and actual upper limit.*

In example 2, class mark of the class $10 - 20 = \frac{10+20}{2} = 15$.

In example 1, class mark of the class $1 - 5 = \frac{0.5+5.5}{2} = 3$.

Note

In discrete distribution, the class mark of a class is also the value mid-way between its stated class limits, thus in example 1, the class mark of the class $1 - 5 = \frac{1+5}{2} = 3$

Remarks

1. If the classes are of equal size, then class size = difference between two successive class marks.
2. If the classes are of equal size (width) and h is the size of each class and m is mid-value (class mark) of a class, then

   lower limit of the class = its mid-value – half the width of class

   $$= m - \frac{h}{2} \text{ and}$$

   upper limit of the class = its mid-value + half the width of class

   $$= m + \frac{h}{2}$$

Cumulative frequency and cumulative frequency table

The sum of frequencies of all the previous classes and that particular class is called the ***cumulative frequency*** of the class.

The cumulative frequency table for example 2 is:

Classes	*Tally marks*	*Frequency*	*Cumulative frequency*
10 – 20	𝍸 𝍸 I	11	11
20 – 30	𝍸 𝍸 III	13	24 (11 + 13)
30 – 40	𝍸 𝍸 III	13	37 (24 + 13)
40 – 50	𝍸 IIII	9	46 (37 + 9)
50 – 60	IIII	4	50 (46 + 4)

19.3.1 Formation of classes from a given raw data

We condense the given raw data into classes (or groups) as follows:

1. Find the *range i.e.* the difference between the maximum and minimum observations. Decide about the number of classes (usually between 5 to 10). In general, the size (width) is a convenient whole number immediately greater than the quotient obtained by dividing the range by the number of classes.
2. Classes should be non-overlapping and continuous.
3. There should be no gaps between classes.
4. As far as possible, classes should be of the same size.
5. Open ended classes such as less than 5 or greater than 9 should be avoided.
6. Limits of each class should be so chosen that there is no ambiguity as to which class a particular observation of the given data belongs to.

We illustrate the above procedure with the help of the following examples.

Illustrative Examples

Example 1. The subsidised electricity bills (in rupees) of 40 houses in a low-income locality are given below. Construct a grouped frequency distribution table:

78, 87, 81, 52, 59, 65, 101, 108, 115, 95, 98, 65, 62, 121, 128, 63, 76, 84, 89, 91, 65, 101, 95, 81, 87, 105, 129, 92, 75, 105, 78, 72, 107, 116, 127, 100, 80, 82, 61, 118

Solution. Here maximum = 129 and minimum = 52,

$\therefore$ range = 129 – 52 = 77

Let us form 8 classes each of size 10.

Since we want to include 129 in the last class, 130 is the upper limit of the last class, so the lower limit of the first class is 50.

The grouped frequency distribution table of the given data is:

Classes	*Tally marks*	*Frequency*
50 – 60	II	2
60 – 70	𝍸 I	6
70 – 80	𝍸	5
80 – 90	𝍸 III	8
90 – 100	𝍸	5
100 – 110	𝍸 II	7
110 – 120	III	3
120 – 130	IIII	4
Total		40

Example 2. The heights of 240 students of a school are measured and tabulated as below:

Height (in cm)	below 100	below 110	below 120	below 130	below 140	below 150
No. of students	12	30	65	180	218	240

Construct a frequency distribution table for the above data. Also answer the following:

(*i*) How many students have atleast 1 m height but less than 120 cm height?

(*ii*) How many students have atleast 130 cm height?

Solution. The frequency distribution table for the given data is:

Class intervals (Height in cm)	*Frequency (No. of students)*	
0 – 100	12	
100 – 110	18	(= 30 – 12)
110 – 120	35	(= 65 – 30)
120 – 130	115	(= 180 – 65)
130 – 140	38	(= 218 – 180)
140 – 150	22	(= 240 – 218)

(*i*) 53 (= sum of the frequencies of the class intervals 100 – 110 and 110 – 120)

(*ii*) 60 (= sum of the frequencies of the class intervals 130 – 140 and 140 – 150)

Example 3. If m is the mid-point and l is the upper limit of a class in a continuous frequency distribution, then what is the lower limit of the class?

Solution. As l is the upper limit and m is the mid-point of a class in a continuous frequency distribution,

so half of the width of the class = upper limit – mid-point = $l - m$

Let x be the lower limit of this class, then

$$x = \text{mid-point} - \frac{1}{2} \text{ of width of this class}$$

$$= m - (l - m) = 2m - l$$

Example 4. Prepare a continuous grouped frequency distribution from the following data:

Mid-points	5	15	25	35	45
Frequency	4	8	13	12	6

Also find the size of class intervals.

Solution. As the mid-points (class marks) of classes are 5, 15, 25, 35 and 45 which are at equal gaps, so the classes are of equal size.

Therefore, size of class = difference between two consecutive mid-points

$$= 15 - 5 = 10$$

$$\text{Half of class size} = \frac{10}{2} = 5$$

Therefore, the class limits of the first class are : lower limit = mid-point – half of class size and upper limit = mid-point + half of class size *i.e.* 5 – 5 and 5 + 5 *i.e.* 0 and 10. Similarly, we find the class limits for all other classes. Thus, the classes are 0 – 10, 10 – 20, 20 – 30, 30 – 40 and 40 – 50. The continuous distribution of the given data is:

Classes	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	4	8	13	12	6

Exercise 19.2

1. State which of the following variables are continuous and which are discrete:

(*i*) marks scored (out of 50) in a test

(*ii*) daily temperature of your city

(*iii*) sizes of shoes

(*iv*) distance travelled by a man

(*v*) time

2. Using class intervals 0 –4, 5 –9, 10 –14, ... construct the frequency distribution for the following data :

13, 6, 10, 5, 11, 14, 2, 8, 15, 16, 9, 13, 17, 11, 19, 5, 7, 12, 20, 21, 18, 1, 8, 12, 18

3. Given below are the marks obtained by 27 students in a test :

21, 3, 28, 38, 6, 40, 20, 26, 9, 8, 14, 18, 20, 16, 17, 10, 8, 5, 22, 27, 34, 2, 35, 31, 16, 28, 37

(*i*) Using the class intervals 1 – 10, 11 – 20 etc. construct a frequency table.

(*ii*) State the range of these marks.

(*iii*) State the class mark of the third class of your frequency table.

4. Explain the meaning of the following terms :

(*i*) variate (*ii*) class size (*iii*) class mark

(*iv*) class limits (*v*) true class limits (*vi*) frequency of a class

(*vii*) cumulative frequency of a class.

5. Fill in the blanks :

(*i*) The number of observations in a particular class is called ..... of the class.

(*ii*) The difference between the class marks of two consecutive classes is the ..... of the class.

(*iii*) The range of the data 16, 19, 23, 13, 11, 25, 18 is ..... .

(*iv*) The mid-point of the class interval is called its ..... .

(*v*) The class mark of the class 4 – 9 is ..... .

6. The marks obtained (out of 50) by 40 students in a test are given below:

28, 31, 45, 03, 05, 18, 35, 46, 49, 17, 10, 28, 31, 36, 40, 44, 47, 13, 19, 25, 24, 31, 38, 32, 27, 19, 25, 28, 48, 15, 18, 31, 37, 46, 06, 01, 20, 10, 45, 02.

(*i*) Taking class intervals 1 – 10, 11 – 20, ..., construct a tally chart and a frequency distribution table.

(*ii*) Convert the above distribution to continuous distribution.

(*iii*) State the true class limits of the third class.

(*iv*) State the class mark of the fourth class.

7. Use the adjoining table to find:

(*i*) upper and lower limits of fifth class

(*ii*) true class limits of the fifth class

(*iii*) class boundaries of the third class

(*iv*) class mark of the fourth class

(*v*) width of sixth class

Class	*Frequency*
28 – 32	5
33 – 37	8
38 – 42	13
43 – 47	9
48 – 52	7
53 – 57	5
58 – 62	2

8. The marks of 200 students in a test were recorded as follows:

Marks %	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89
No. of students	7	11	20	46	57	37	15	7

Draw the cumulative frequency table.

9. Given below are the marks secured by 35 students in a test:

41, 32, 35, 21, 11, 47, 42, 00, 05, 18, 25, 24, 29, 38, 30, 04, 14, 24, 34, 44, 48, 33, 36, 38, 41, 46, 08, 34, 39, 11, 13, 27, 26, 43, 03

Taking class intervals 0 – 10, 10 – 20, 20 – 30 ..., construct frequency as well as cumulative frequency distribution table. Find the number of students obtaining below 20 marks.

10. The marks out of 100 of 50 students in a test are given below:

5	35	6	35	18	36	12	36	85	32
20	36	22	38	24	50	22	39	74	31
25	54	25	64	25	70	28	66	58	25
29	72	31	82	31	84	31	82	37	21
32	84	32	92	35	95	34	92	35	5

(*i*) Taking a class interval of size 10, construct a frequency as well as cumulative frequency table for the given data.

(*ii*) Which class has the largest frequency?

(*iii*) How many students score less than 40 marks?

(*iv*) How many students score first division (60% or more) marks?

11. Construct the frequency distribution table from the following data:

Ages (in years)	below 4	below 7	below 10	below 13	below 16
No. of children	7	38	175	248	300

State the number of children in the age group 10 – 13.

12. Rewrite the following cumulative frequency distribution into frequency distribution:

less than or equal to 10	2
less than or equal to 20	7
less than or equal to 30	18
less than or equal to 40	32
less than or equal to 50	43
less than or equal to 60	50

13. The maximum temperatures (in degree celsius) for Delhi for the month of April, 2014, as reported by the Meteorological Department, are given below:

27·4, 28·3, 23·9, 23·6, 25·4, 27·5, 28·1, 28·4, 30·5, 29·7, 30·6, 31·7, 32·2, 32·6, 33·4, 35·7, 36·1, 37·2, 38·4, 40·1, 40·2, 40·5, 41·1, 42·0, 42·1, 42·3, 42·4, 42·9, 43·1, 43·2

Construct a frequency distribution table.

14. (*i*) The class marks of a distribution are 94, 104, 114, 124, 134, 144 and 154. Determine the class size and the class limits of the fourth class.

(*ii*) The class marks of a distribution are 9·5, 16·5, 23·5, 30·5, 37·5 and 44·5. Determine the class size and the class limits of the third class.

19.4 GRAPHICAL REPRESENTATION OF STATISTICAL DATA

Since pictures (or graphs) are good visual aids and leave more lasting effect on the mind of an observer, the information contained in a numerical data (or frequency distribution) can be easily understood if we represent it in the form of diagrams (graphs). It is well said that one picture is better than a thousand words. Usually, comparisons among the individual items are best shown by means of graphs.

There are various ways of representing numerical data (or frequency distribution) graphically. In the previous classes, you have learnt:

(*i*) Bar graphs (*ii*) Pie graphs and (*iii*) Broken line graphs

In this section, we will review bar graphs and will learn two more ways of representing grouped data (continuous frequency distribution) graphically :

(*i*) Histogram (*ii*) Frequency polygon

19.4.1 Bar graphs

In bar graphs,

(*i*) take the width of all bars (rectangles) equal.

(*ii*) space between consecutive bars should be equal. The bars can touch each other.

(*iii*) the height (or length) of a bar is equal (or proportional) to the frequency of the corresponding variate (observation).

(*iv*) the width of bars has no significance. In fact, the width of the bars is shown simply to make the representation more eye-catching.

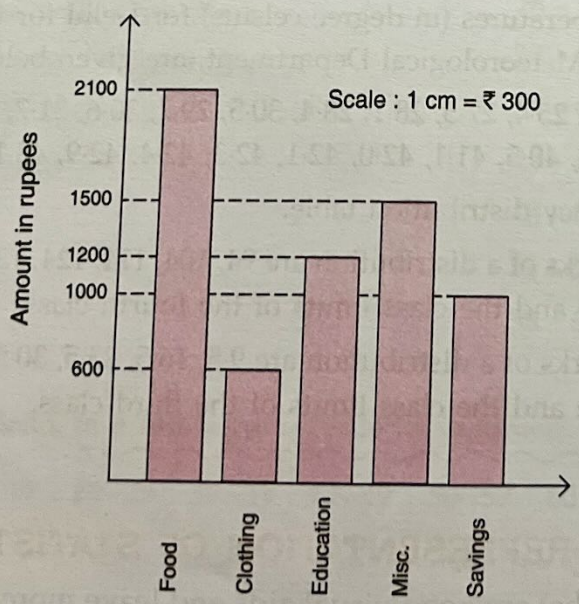
Illustrative Examples

Example 1. A man with a monthly salary of ₹ 6400 plans his budget for a month as given below:

Item	Food	Clothing	Education	Miscellaneous	Savings
Amount (in ₹)	2100	600	1200	1500	1000

Represent the above data by a bar graph.

Solution. The required bar graph is shown below:

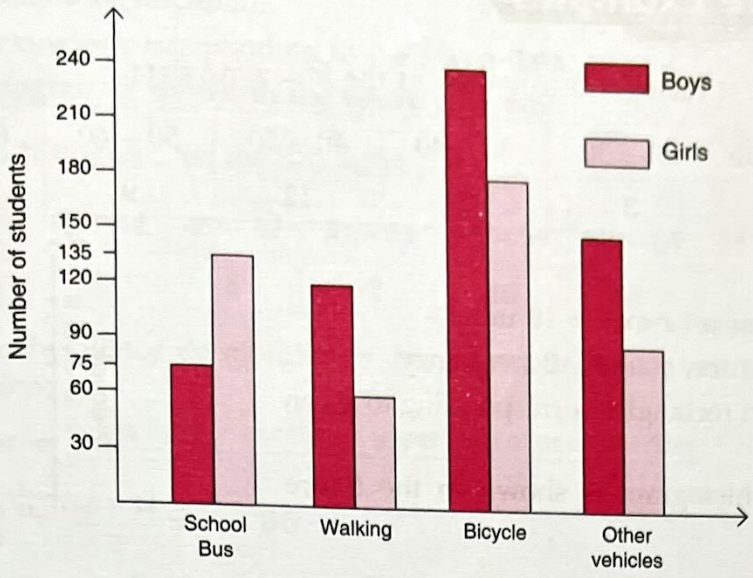


Example 2. Given below is the data of school going students (boys and girls):

Mode of transport	School bus	Walking	Bicycle	Other vehicles
Number of boys	75	120	240	150
Number of girls	135	60	180	90

Draw a bar graph to represent the above data.

Solution. The required bar graph is shown below:



19.4.2 Histogram

A histogram is used to represent continuous grouped data. It consists of adjacent rectangles.

Procedure to draw a histogram:

(*i*) Take the breadth of a rectangle equal to a class size and mark it along x-axis, the end points of which correspond to the class limits.

(*ii*) Take the length of a rectangle equal to the frequency of that class and mark it along y-axis.

(*iii*) Construct rectangles corresponding to each class with the help of steps (*i*) and (*ii*).

Remarks

- A histogram consists of a set of adjacent rectangles whose bases are equal to class sizes and heights are equal to class frequencies.
- In a bar graph, the breadth of a rectangle has no significance ; whereas in a histogram, the breadth of a rectangle is meaningful and it represents the class size.
- In a bar graph, there may be gaps between different bars (rectangles); whereas in a histogram, there is no gap between different rectangles.
- A bar graph represents discrete data, while a histogram represents a continuous grouped data.
- If the frequency distribution is discontinuous (inclusive), change it to continuous (exclusive) and then construct histogram. See example 4.
- The total area of the histogram = sum of areas of all rectangles. In particular, if the class intervals are of same size (width), then the area of histogram = Nk where k = size of a class and N = sum of frequencies of all classes.
- In a histogram, in fact, it is the area of a rectangle which represents the frequency of the class. Therefore, if the classes are of unequal size (width) then the heights of rectangles are not equal to the frequencies of the corresponding classes. However, we shall be dealing only with the problems in which class widths are equal.

Illustrative Examples

Example 1. Draw a histogram to represent the following data:

Class intervals	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	3	5	12	9	4

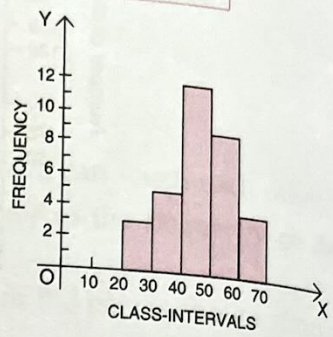
Solution. *Steps:*

(*i*) Take 1 cm on x-axis = 10 units

(*ii*) Take 1 cm on y-axis = 2 frequency

(*iii*) Construct rectangles corresponding to given data.

The required histogram is shown in the figure given alongside.



Example 2. Draw a histogram for the following data:

Height (in cm)	150 – 160	160 – 170	170 – 180	180 – 190	190 – 200
No. of students	8	3	4	10	2

Solution. *Steps:*

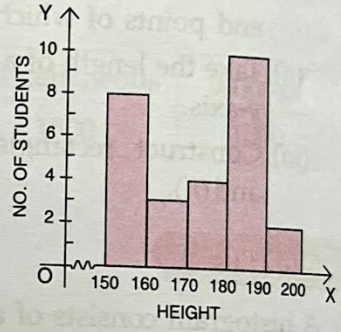
(*i*) Since the scale on x-axis starts at 150, a break (kink or zig-zag curve) is shown near the origin along x-axis to indicate that the graph is drawn to scale beginning at 150 and not at the origin itself.

(*ii*) Take 1 cm on x-axis = 10 cm (height)

(*iii*) Take 1 cm on y-axis = 2 (no. of students)

(*iv*) Construct rectangles corresponding to the given data.

The required histogram is shown in the adjoining diagram.

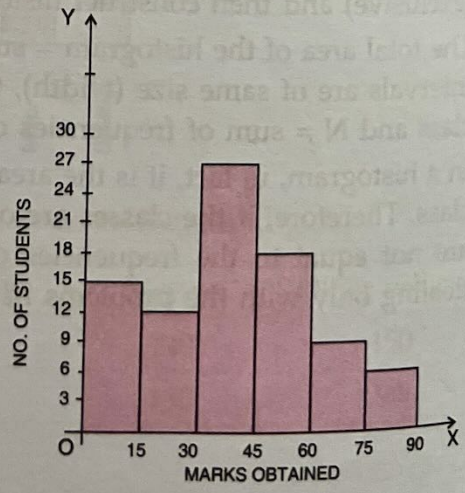


Example 3. Draw a histogram for the following frequency distribution:

Marks obtained	below 15	below 30	below 45	below 60	below 75	below 90
No. of students	15	27	54	72	81	87

Solution. The continuous frequency distribution table is:

Class interval (marks obtained)	*Frequency (no. of students)*
0 – 15	15
15 – 30	12
30 – 45	27
45 – 60	18
60 – 75	9
75 – 90	6



Take 1 cm on x-axis = 15 (marks).

Take 1 cm on y-axis = 6 (students).

Construct the rectangles corresponding to the given data.

The required histogram is shown in the above diagram.

Example 4. Draw a histogram for the following data:

Wt. (in kg)	40 – 44	45 – 49	50 – 54	55 – 59	60 – 64	65 – 69
No. of students	2	8	12	10	6	4

Solution. The given frequency distribution is discontinuous, to convert it into continuous frequency distribution,

$$\text{adjustment factor} = \frac{\text{lower limit of one class} - \text{upper limit of previous class}}{2}$$

$$= \frac{45 - 44}{2} = \frac{1}{2} = 0.5$$

Subtract the adjustment factor (0·5) from all the lower limits and add the adjustment factor (0·5) to all the upper limits.

Continuous frequency distribution for the given data is:

Classes before adjustment	*Classes after adjustment*	*Frequency*
40 – 44	39·5 – 44·5	2
45 – 49	44·5 – 49·5	8
50 – 54	49·5 – 54·5	12
55 – 59	54·5 – 59·5	10
60 – 64	59·5 – 64·5	6
65 – 69	64·5 – 69·5	4

To construct histogram:

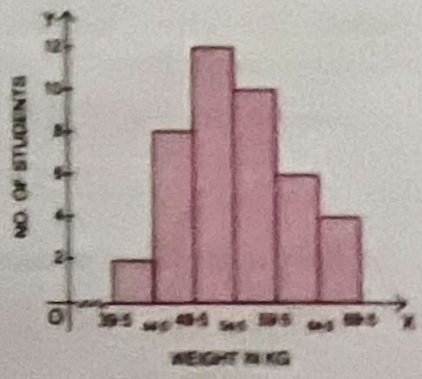
(*i*) Since the scale on x-axis starts at 39·5, a break (kink) is shown near the origin on x-axis to indicate that the graph is drawn to scale beginning at 39·5.

(*ii*) Take 1 cm along x-axis
= 5 kg (weight)

(*iii*) Take 1 cm along y-axis
= 2 (no. of students)

(*iv*) Construct rectangles corresponding to the continuous frequency distribution given in the above table.

The required histogram is shown in the adjoining figure.



19.4.3 Frequency polygon

Usually, in continuous frequency distributions, the frequencies are represented along y-axis and class intervals along x-axis. Find the mid-points of class intervals. Plot the points on the graph paper representing frequencies of different classes against the corresponding mid-points of class intervals. On joining consecutive points by line segments, we get *frequency graph (curve)*. In other words, on joining the mid-points of the upper bases of the adjacent rectangles of a histogram we get frequency curve. If the end points of *frequency graph* are

joined to the mid-points of immediate lower and upper class intervals outside the range with zero frequency, we get a closed figure. The figure so obtained is called a ***frequency polygon***.

Thus, a ***frequency polygon*** *is a closed curve representing continuous grouped frequency distribution.*

Histogram and frequency polygon can be constructed simultaneously, see example 3.

Illustrative Examples

Example 1. Draw a frequency polygon to represent the following data:

Class interval	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	5	10	19	24	18	6

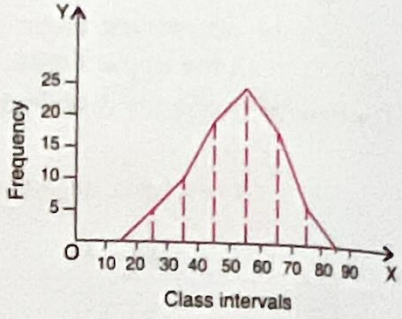
Solution. *Steps:*

(*i*) Let 1 cm on x-axis = 10 units and
1 cm on y-axis = 5 frequencies

(*ii*) Find mid-points of class intervals.

(*iii*) Find points corresponding to given frequencies of classes and the mid-points of class intervals, and plot them.

(*iv*) Join the consecutive points by line segments.



(*v*) Join the first end-point with mid-point of class 10 – 20 with zero frequency, and join the other end point with the mid-point of class 80 – 90 with zero frequency. The required frequency polygon has been shown above.

Example 2. 100 students in a school have heights as tabulated below:

Height (in cm)	121–130	131–140	141–150	151–160	161–170	171–180
No. of students	12	16	30	20	14	8

Draw frequency polygon of the above data.

Solution. The given frequency distribution is discontinuous; to convert it into a continuous distribution,

$$\text{adjustment factor} = \frac{\text{lower limit of one class} - \text{upper limit of previous class}}{2}$$

$$= \frac{131 - 130}{2} = \frac{1}{2} = 0{\cdot}5$$

Continuous frequency distribution for the given data is:

Classes before adjustment	*Classes after adjustment*	*Frequency*
121 – 130	120·5 – 130·5	12
131 – 140	130·5 – 140·5	16
141 – 150	140·5 – 150·5	30
151 – 160	150·5 – 160·5	20
161 – 170	160·5 – 170·5	14
171 – 180	170·5 – 180·5	8

Steps to draw frequency polygon:

(*i*) Since the scale on x-axis starts at 110·5, a kink is shown near the origin on x-axis to indicate that the graph is drawn to scale beginning at 110·5.

(*ii*) Take 1 cm along x-axis = 10 cm (height).

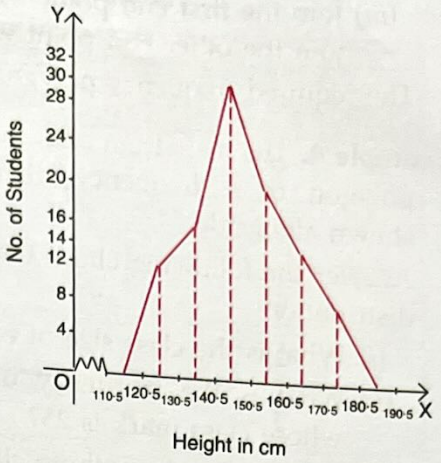
(*iii*) Take 1 cm along y-axis = 4 (no. of students).

(*iv*) Find mid-points of class intervals.

(*v*) Find points corresponding to given frequencies of classes and the mid-points of class intervals, and plot them.

(*vi*) Join consecutive points by line segments.

(*vii*) Join first end point with mid-point of class 110·5 – 120·5 with zero frequency and join the other end with mid-point of class 180·5 – 190·5 with zero frequency.



The required frequency polygon is shown alongside.

Example 3. The following is the pocket money survey of 50 students in a school (Pocket money in Rupees per day):

49	55	22	27	30	27	25	27	30	42
40	13	24	38	10	24	30	33	17	29
10	50	18	34	15	40	13	32	36	32
27	35	17	41	18	36	20	41	35	51
29	27	44	43	15	32	29	54	14	45

Form a frequency table with a grouping of 10 – 20, 20 – 30, 30 – 40, and so on. Construct a combined histogram and frequency polygon for the distribution.

Solution. The frequency distribution table for the given data is:

Classes	*Tally Marks*	*Frequency*
10 – 20	𝍸 𝍸 I	11
20 – 30	𝍸 𝍸 III	13
30 – 40	𝍸 𝍸 III	13
40 – 50	𝍸 IIII	9
50 – 60	IIII	4
Total		50

To construct histogram:

(*i*) Take 1 cm along x-axis = ₹10

(*ii*) Take 1 cm along y-axis = 2 students

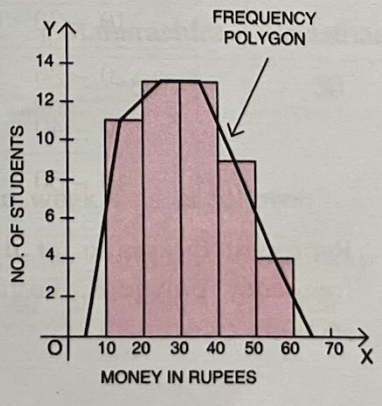
(*iii*) Construct rectangles corresponding to the above continuous frequency distribution table.

The required histogram is shown in the adjoining diagram.

To construct frequency polygon :

(*i*) Mark mid-points of upper bases of rectangles of the histogram.

(*ii*) Join the consecutive mid-points by line segments.



(*iii*) Join the first end point with the mid-point of class 0–10 with zero frequency, and join the other end point with the mid-point of class 60 – 70 with zero frequency.

The required frequency polygon is shown by *thick line segments* in the above diagram.

Example 4. The histogram and the frequency polygon of a frequency distribution is shown alongside.

Answer the following about the frequency distribution:

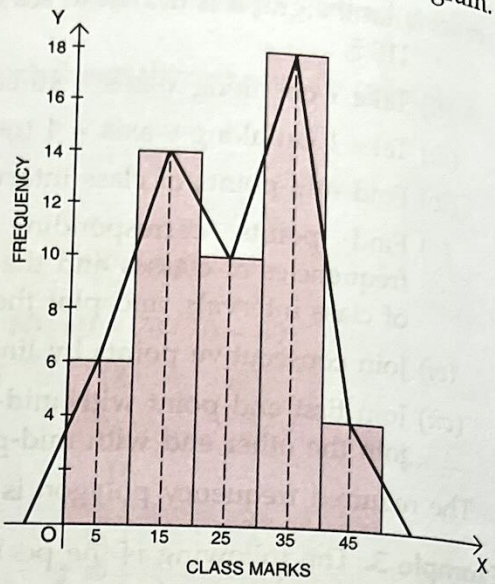
(*i*) What is the class size of each class?

(*ii*) What is the frequency of the class whose class mark is 25?

(*iii*) What is the class whose class mark is 15?

(*iv*) What is the class whose frequency is 18?

(*v*) Construct the frequency table for the given distribution.



Solution. (*i*) As the class marks are 5, 15, 25, 35 and 45 which are at equal gaps, so the classes are of equal size.

$\therefore$ Class size of each class = difference between two successive class marks

$= 15 - 5 = 10$

(*ii*) 10.

(*iii*) As the size of each class is 10 and the class mark is 15, so the class limits are 15 – 5 and 15 + 5 *i.e.* 10 and 20

Hence, the class is 10 – 20

(*iv*) The class whose frequency is 18 is 30 – 40

(*v*) The frequency distribution table is given below:

Classes	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	6	14	10	18	4

Example 5. The following table gives the distribution of students of two sections according to the marks obtained by them:

Section A		Section B	
Marks	Frequency	Marks	Frequency
0 – 10	3	0 – 10	5
10 – 20	9	10 – 20	19
20 – 30	17	20 – 30	15
30 – 40	12	30 – 40	10
40 – 50	9	40 – 50	1

Represent the marks of the students of both the sections on the same graph by two frequency polygons. From the two polygons compare the performance of the two sections.

Solution. We find the class marks and prepare a new table as under:

Class	*Class marks*	*Section A (No. of students)*	*Section B (No. of students)*
0 – 10	5	3	5
10 – 20	15	9	19
20 – 30	25	17	15
30 – 40	35	12	10
40 – 50	45	9	1

Step to draw frequency polygon:

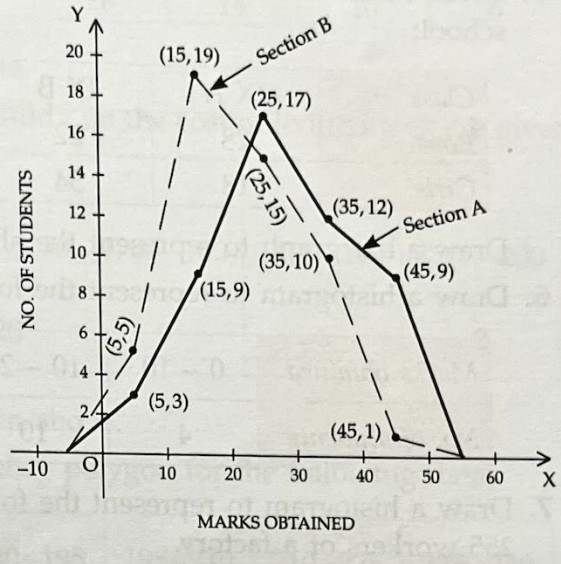
Take 1 cm on x-axis = 5 marks and 1 cm on y-axis = 2 students

For Section A

Plot the points (5, 3), (15, 9), (25, 17), (35, 12) and (45, 9).

Join the consecutive points by thick line segments.

Join the first end point with the mid-point of class (–10) – 0 with zero frequency, and join the other end point with the mid-point of class 50 – 60 with zero frequency. The required frequency polygon is shown by thick line segments in the adjoining figure.



For Section B

Plot the points (5, 5), (15, 19), (25, 15), (35, 10) and (45, 1).

Join the consecutive points with dotted line segments.

Join the first end point with the mid-point of class (–10) – 0 with zero frequency, and join the other end point with the mid-point of class 50 – 60 with zero frequency. The required frequency polygon is shown by dotted line segments in the same figure.

Performance of section A is better than performance of section B.

Exercise 19.3

1. The area under wheat cultivation last year in the following states, correct to the nearest lacs hectares was:

State	Punjab	Haryana	U.P.	M.P.	Maharashtra	Rajasthan
Cultivated area	220	120	100	40	80	30

Represent the above information by a bar graph.

2. The number of books sold by a shopkeeper in a certain week was as follows:

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
No. of books	420	180	230	340	160	120

Draw a bar graph for the above data.

3. Given below is the data of percentage of passes of a certain school in the ICSE for consecutive years:

Year	2000	2001	2002	2003	2004	2005	2006
% of passes	92	80	70	86	54	78	94

Draw a bar graph to represent the above data.

4. Birth rate per thousand of different countries over a certain period is:

Country	India	Pakistan	China	U.S.A.	France
Birth rate	36	45	12	18	20

Draw a horizontal bar graph to represent the above data.

5. Given below is the data of number of students (boys and girls) in class IX of a certain school:

Class	IX A	IX B	IX C	IX D
Boys	28	22	40	15
Girls	18	34	12	25

Draw a bar graph to represent the above data.

6. Draw a histogram to represent the following data:

Marks obtained	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of students	4	10	6	8	5	9

7. Draw a histogram to represent the following frequency distribution of daily wages of 255 workers of a factory.

Daily wages (in rupees)	850 – 950	950 – 1050	1050 – 1150	1150 – 1250	1250 – 1350
No. of workers	35	45	75	60	40

8. Draw a histogram for the following data:

Class marks	12·5	17·5	22·5	27·5	32·5	37·5
Frequency	7	12	20	28	8	11

9. Draw a histogram for the following frequency distribution:

Age (in years)	below 2	below 4	below 6	below 8	below 10	below 12
No. of children	12	15	36	45	72	90

10. Draw a histogram for the following data:

Classes	59 – 65	66 – 72	73 – 79	80 – 86	87 – 93	94 – 100
Frequency	10	5	25	15	30	10

11. Draw a frequency polygon for the following data:

Class intervals	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
Frequency	15	28	45	32	41	18

12. In a class of 60 students, the marks obtained in a monthly test were as under:

Marks	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Students	10	25	12	08	05

Draw a frequency polygon to represent the above data.

13. In a class of 90 students, the marks obtained in a weekly test were as under:

Marks	16 – 20	21 – 25	26 – 30	31 – 35	36 – 40	41 – 45	46 – 50
No. of students	4	12	18	26	14	10	6

Draw a frequency polygon for the above data.

14. In a city, the weekly observations made in a study on the cost of living index are given in the following table:

Cost of living index	140 – 150	150 – 160	160 – 170	170 – 180	180 – 190	190 – 200
Number of weeks	5	10	20	9	6	2

Draw a frequency polygon for the data given above.

15. Construct a combined histogram and frequency polygon for the following data:

Daily earnings (in rupees)	150–165	165–180	180–195	195–210	210–225	225–240
No. of workers	8	14	22	12	15	6

16. In a study of diabetic patients, the following data was obtained:

Age (in years)	10–20	20–30	30–40	40–50	50–60	60–70	70–80
No. of patients	3	8	30	36	27	15	6

Represent the above data by a histogram and a frequency polygon.

17. The electricity bills (in rupees) of 32 houses in a locality are given below :

300, 480, 520, 780, 1030, 850, 370, 940, 720, 730, 660, 520, 920, 650, 780, 810, 640, 600, 750, 780, 1080, 630, 710, 540, 590, 750, 1000, 1030, 350, 890, 950, 730

Taking class intervals 300 – 400, 400 – 500, 500 – 600, ..., form frequency distribution table. Construct a combined histogram and frequency polygon.

18. The number of matchsticks in 40 boxes on counting was found as given below:

44, 41, 42, 43, 47, 50, 51, 49, 43, 42, 40, 42, 44, 45, 49, 42, 46, 49, 45, 49, 45, 47, 48, 43, 43, 44, 48, 43, 46, 50, 43, 52, 46, 49, 52, 51, 47, 43, 43, 45

Taking classes 40 – 42, 42 – 44 ..., construct the frequency distribution table for the above data. Also draw a combined histogram and frequency polygon to represent the distribution.

19. The histogram showing the weekly wages (in rupees) of workers in a factory is given alongside.

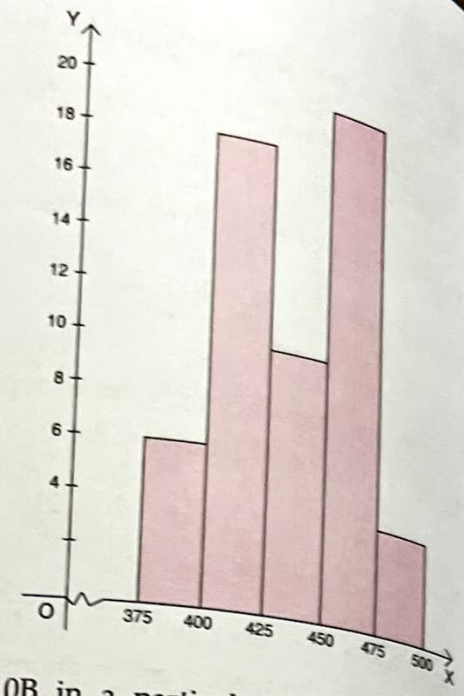
Answer the following about the frequency distribution:

(*i*) What is the frequency of the class 400 – 425?

(*ii*) What is the class having minimum frequency?

(*iii*) What is the cumulative frequency of the class 425 – 450?

(*iv*) Construct a frequency and cumulative frequency table for the given distribution.



20. Marks scored by students of class 10A and 10B in a particular class test are as follows:

Marks	1 – 5	6 – 10	11 – 15	16 – 20	21 – 25	26 – 30	31 – 35	36 – 40
No. of students of 10A	1	3	8	9	4	5	6	4
No. of students of 10B	2	6	3	10	7	6	4	2

Draw their frequency polygons on the same graph.

Multiple Choice Questions MCQs

Choose the correct answer from the given four options (1 to 17):

1. The marks obtained by 17 students in a mathematics test (out of 100) are given below:

91, 82, 100, 100, 96, 65, 82, 76, 79, 90, 46, 64, 72, 66, 68, 48, 49

The range of the data is

(*a*) 46 (*b*) 54 (*c*) 90 (*d*) 100

2. The class mark of the class 90–120 is

(*a*) 90 (*b*) 105 (*c*) 115 (*d*) 120

3. In a frequency distribution, the mid-value of a class is 10 and the width of the class is 6. The lower limit of the class is

(*a*) 6 (*b*) 7 (*c*) 8 (*d*) 12

4. The width of each of 5 continuous classes in a frequency distribution is 5 and the lower limit of the lowest class is 10. The upper limit of the highest class is

(*a*) 15 (*b*) 25 (*c*) 35 (*d*) 40

5. The class marks of a frequency distribution are given as follows:

15, 20, 25, ...

The class corresponding to the class mark 20 is

(*a*) 12·5 – 17·5 (*b*) 17·5 – 22·5 (*c*) 18·5 – 21·5 (*d*) 19·5 – 20·5

6. In the class intervals $10 - 20$, $20 - 30$, the number 20 is included in
(a) $10 - 20$ (b) $20 - 30$
(c) both the intervals (d) none of these intervals

7. A grouped frequency distribution table with class intervals of equal size using $250 - 270$ (270 not included in this interval) as one of the class intervals is constructed for the following data:
268, 220, 368, 258, 242, 310, 272, 342, 310, 290, 300, 320, 319, 304, 402, 318, 406, 292, 354, 278, 210, 240, 330, 316, 406, 215, 258, 236.
The frequency of the class $310 - 330$ is
(a) 4 (b) 5 (c) 6 (d) 7

8. The mean of $x - 1$, $x + 1$, $x + 3$ and $x + 5$ is
(a) $x + 1$ (b) $x + 2$ (c) $x + 3$ (d) $x + 4$

9. The mean of five numbers is 30. If one number is excluded, their mean becomes 28. The excluded number is
(a) 28 (b) 30 (c) 35 (d) 38

10. If the mean of x_1, x_2 is 7.5, and the mean of x_1, x_2, x_3 is 8, then the value of x_3 is
(a) 9 (b) 8 (c) 7·5 (d) 6

11. If each observation of the data is increased by 5, then their mean
(a) remains the same (b) becomes 5 times the original mean
(c) is decreased by 5 (d) is increased by 5

12. The mean of 100 observations is 50. If one of the observation which was 50 is replaced by 150, the resulting mean will be
(a) 50·5 (b) 51 (c) 51·5 (d) 52

13. For drawing a frequency polygon of a continuous frequency distribution, we plot the points whose ordinates are the frequencies of the respective classes and abscissae are respectively:
(a) upper limits of the classes (b) lower limits of the classes
(c) class marks of the classes (d) upper limits of preceding classes

14. Median of the numbers 4, 4, 5, 7, 6, 7, 7, 3, 12 is
(a) 4 (b) 5 (c) 6 (d) 7

15. The median of the data
78, 56, 22, 34, 45, 54, 39, 68, 54, 84 is
(a) 45 (b) 49·5 (c) 54 (d) 56

16. In a data, 10 numbers are arranged in ascending order. If the 8th entry is increased by 6, then the median increases by
(a) 0 (b) 2 (c) 3 (d) 6

17. Consider the following two statements:
Statement I: A histogram consists of a set of adjacent rectangles whose bases are equal to class size, and heights are equal to class frequencies.
Statement II: In a bar graph, the breadth of a rectangle has no significance, whereas in a histogram, the breadth of a rectangle is meaningful and it represents the class size.
Which of the following is valid?
(a) Both the Statements are true.
(b) Both the Statements are false.
(c) Statement I is true, and Statement II is false.
(d) Statement I is false, and Statement II is true.

ASSERTION-REASON TYPE QUESTIONS (SOLVED)

In these examples and following questions, read the given statements carefully and choose the correct option.

(*a*) Assertion (A) is true, Reason (R) is false.

(*b*) Assertion (A) is false, Reason (R) is true.

(*c*) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct reason for Assertion (A).

(*d*) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct reason (or explanation) for Assertion (A).

1. Assertion (A): Range of 6 observations 16, 19, 23, 12, 11, 25 is 15.

Reason (R): The difference between maximum and minimum values of a variable is called its range.

Sol. We know Reason (R) to be true.

Let us use this to calculate range of data in Assertion (A).

The given values in ascending order can be written as

11, 12, 16, 19, 23, 25.

Maximum value is 25 and minimum is 11.

$\therefore$ Range = 25 – 11 = 14.

Therefore, Assertion (A) that range is 15 is false.

Thus, Assertion (A) is false but Reason (R) is true.

$\therefore$ Correct answer is (*b*).

2. Assertion (A): Mean and median of data 10, 11, 12, 15, 16 is same.

Reason (R): Mean and median of a given set of data may or may not be same.

Sol. Values in Assertion (A) are already in ascending order.

Median is middle value, which is 12.

$$\text{Mean} = \frac{\text{Sum of observations}}{\text{Number of observations}} = \frac{10+11+12+15+16}{5} = \frac{64}{5} = 12{\cdot}8$$

Thus, mean $\neq$ median and Assertion (A) is false.

Now, we see above that mean $\neq$ median. However, we may have data like 10, 11, 12 in which mean = median.

$\therefore$ Reason (R) is true.

Thus, Assertion (A) is false but Reason (R) is true.

$\therefore$ Correct answer is (*b*).

ASSERTION-REASON TYPE QUESTIONS (UNSOLVED)

1. Assertion (A): Runs scored by batsman A are 55, 60, 30, 80 while runs scored by batsman B are 81, 87, 76, 92. Then B has higher range than A.

Reason (R): Range is the difference between maximum and minimum values of a variable.

2. Assertion (A): Mean of first 5 odd natural numbers is 5.

Reason (R): Mean is the middle value of a set of observations.

3. Assertion (A): If 6 students score 78, 62, 91, 37, 80 and 66 marks in a subject, then median score is 72.

Reason (R): If number of observations are

$$\text{Median} = \frac{\frac{n}{2}\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}$$

ascending or descending order.

Summary

- **Statistics** is that branch of mathematics which deals with the collection, presentation, analysis and interpretation of data.
- Facts and information, collected with a definite purpose, are called data.
- A quantity which is being measured in an experiment (or survey) is called a variable.
- The difference between the maximum and minimum value of a variable is called its **range**.
- A particular value of a variable is called **variate** (or **observation**).
- The number of times a particular observation occurs is called its **frequency**.
- A table showing the frequency of various observations is called **frequency distribution table**.
- **Mean** (or **arithmetic average**) of a number of observations is the sum of the values of all the observations divided by the total number of observations.

  Thus, the mean of n observations $x_1, x_2, x_3, \ldots x_n$ is given by

  $$\text{Mean} = \frac{\Sigma x_i}{n}, \text{ where } \Sigma x_i = x_1 + x_2 + x_3 + \ldots + x_n$$
- **Median** is the central value of a statistical data if it is arranged in ascending or descending order.

  If there are n observations $x_1, x_2, x_3, \ldots, x_n$ arranged in ascending or descending order, then

  $$\text{Median} = \begin{cases} \frac{n+1}{2}\text{th observation, if } n \text{ is odd} \\ \frac{\frac{n}{2}\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}, \text{if } n \text{ is even.} \end{cases}$$
- If the number of observations is large, then the grouping of data is done. The classes are formed. Usually, we convert the discrete distribution to continuous distribution. In a continuous distribution, the class limits are called **true** or **actual class limits**.
- The difference between the actual class limits of a class is called its **class size** or **class width**.
- The **class mark** of a class is the value midway between its actual lower limit and actual upper class limit.
- The (numerical) data can be represented graphically.
- In bar graphs, bar of equal width are drawn with equal spacing between them. The height of a bar represents the frequency of the corresponding variate (observation).
- Histograms are used to represent continuous grouped data. Histogram consists of a set of adjacent rectangles. The height of rectangles correspond to the frequency of the class and the breadth of the rectangle corresponds to the class size. There is no gap between different rectangles.

○ Continuous frequency distribution can also be represented by **frequency polygons**. In a frequency polygon, we plot points on the graph paper representing frequencies of different classes against the corresponding mid-points of class intervals. On joining consecutive points by line segments, we get **frequency curve** and if the end points of frequency graph are joined to the mid-points of immediate lower and upper class intervals outside the range with zero frequency, we get frequency polygon.

Chapter Test

1. Find the mean and the median of the following set of numbers:

   8, 0, 5, 3, 2, 9, 1, 5, 4, 7, 2, 5

2. Find the mean and the median of all the (positive) factors of 48.
3. The mean weight of 60 students of a class is 52·75 kg. If the mean weight of 35 of them is 54 kg, find the mean weight of the remaining students.
4. The mean age of 18 students of a class is 14·5 years. Two more students of age 15 years and 16 years join the class. What is the new mean age?
5. If the mean of the five observations $x + 1$, $x + 3$, $x + 5$, $2x + 2$, $3x + 3$ is 14, find the mean of first three observations.
6. The mean height of 36 students of a class is 150·5 cm. Later on, it was detected that the height of one student was wrongly copied as 165 cm instead of 156 cm. Find the correct mean height.
7. The mean of 40 items is 35. Later on, it was discovered that two items were misread as 36 and 29 instead of 63 and 22. Find the correct mean.
8. The following observations have been arranged in ascending order. If the median of the data is 63, find the value of x.

   29, 32, 48, 50, x, $x + 2$, 72, 75, 87, 91.

9. Draw a histogram showing marks obtained by the students of a school in a Mathematics paper carrying 60 marks.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Students	4	5	10	8	30	40

10. In a class of 60 students, the marks obtained in a surprise test were as under:

Marks	14–20	20–26	26–32	32–38	38–44	44–50	50–56	56–62
No. of students	4	10	9	15	12	5	3	2

Represent the above data by a histogram and a frequency polygon.

11. Construct a combined histogram and frequency polygon for the following distribution:

Classes	91–100	101–110	111–120	121–130	131–140	141–150	151–160
Frequency	16	28	44	20	32	12	4

Hint. Take 1 cm along y-axis = 8 (frequency).

12. The water bills (in rupees) of 40 houses in a locality are given below:

78	87	81	52	59	65	101	108	115	95
98	65	62	121	128	63	76	84	89	91
65	101	95	81	87	105	129	92	75	105
78	72	107	116	127	100	80	82	61	118

Form a frequency distribution table with a class size of 10. Also represent the above data with a histogram and frequency polygon.

13. The data given below represent the marks obtained by 35 students:

21	26	21	20	23	24	22	19	24
26	25	23	26	29	21	24	19	25
26	25	22	23	23	27	26	24	25
30	25	23	28	28	24	28	28	

Taking class intervals 19 – 20, 21 – 22 etc., make a frequency distribution for the above data.

Construct a combined histogram and frequency polygon for the distribution.

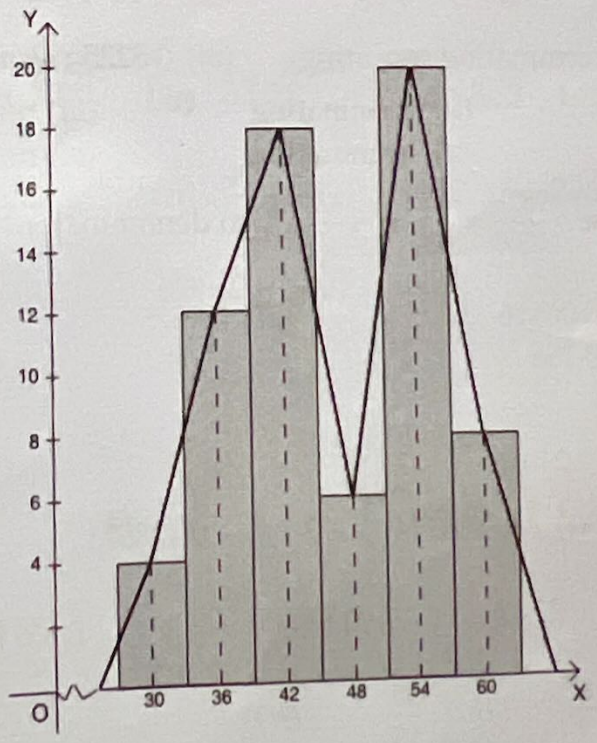
14. The given histogram and frequency polygon shows the ages of teachers in a school. Answer the following:

(*i*) What is the class size of each class?

(*ii*) What is the class whose class mark is 48?

(*iii*) What is the class whose frequency is maximum?

(*iv*) Construct a frequency table for the given distribution.



Answers

EXERCISE 1.1

1. $\frac{43}{144}; \frac{3}{8}, \frac{43}{144}, \frac{2}{9}$

2. $\frac{7}{24}, \frac{13}{48}; \frac{1}{4}, \frac{13}{48}, \frac{7}{24}, \frac{1}{3}$

3. $-\frac{11}{24}, -\frac{5}{12}; -\frac{1}{2}, -\frac{11}{24}, -\frac{5}{12}, -\frac{1}{3}$

4. $\frac{27}{60}, \frac{17}{30}, \frac{41}{60}; \frac{4}{5}, \frac{41}{60}, \frac{17}{30}, \frac{27}{60}, \frac{1}{3}$

5. (*i*) 3·1, 3·2, 3·3 (*ii*) 0·3, 0·31, 0·32, 0·34 (*iii*) 1·6, 1·61, 1·62, 1·63, 1·64

6. $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$

7. $\frac{19}{30}, \frac{2}{3}, \frac{7}{10}, \frac{11}{15}, \frac{23}{30}$

8. $-\frac{13}{35}, -\frac{12}{35}, -\frac{11}{35}, -\frac{2}{7}, -\frac{9}{35}, -\frac{8}{35}, -\frac{1}{5}, 0, \frac{1}{35}, \frac{2}{35}$

9. $\frac{11}{21}, \frac{23}{42}, \frac{4}{7}, \frac{25}{42}, \frac{13}{21}, \frac{9}{14}$

EXERCISE 1.3

2. (*i*) 0·36 ; terminating (*ii*) 4·125 ; terminating

(*iii*) $0.\overline{2}$; non-terminating repeating (*iv*) $0.\overline{18}$; non-terminating repeating

(*v*) $0.\overline{230769}$; non-terminating repeating (*vi*) 0·8225 ; terminating

3. (*i*) terminating (*ii*) terminating (*iii*) non-terminating repeating

(*iv*) terminating (*v*) terminating (*vi*) non-terminating repeating

4. terminating; because $\frac{987}{10500} = \frac{47}{500} = \frac{47}{2^2 \times 5^3}$, so denominator has only 2 and 5 as prime factors

5. (*i*) 2·125 (*ii*) 0·00416 (*iii*) 0·0875 (*iv*) 0·4

(*v*) 0·0448 (*vi*) 0·158

6. $5000 = 2^3 \times 5^4$; $\frac{257}{5000} = \frac{257}{2^3 \times 5^4}$; 0·0514

7. $\frac{1}{7} = 0.\overline{142857}, \frac{2}{7} = 2 \times \frac{1}{7} = 0.\overline{285714}, \frac{3}{7} = 3 \times \frac{1}{7} = 0.\overline{428571}$,

$\frac{4}{7} = 4 \times \frac{1}{7} = 0.\overline{571428}, \frac{5}{7} = 5 \times \frac{1}{7} = 0.\overline{714285}, \frac{6}{7} = 6 \times \frac{1}{7} = 0.\overline{857142}$

8. (*i*) $\frac{1}{3}$ (*ii*) $\frac{47}{9}$ (*iii*) $\frac{40}{99}$ (*iv*) $\frac{43}{90}$ (*v*) $\frac{133}{990}$ (*vi*) $\frac{1}{999}$

9. (*i*) and (*v*) are irrational; (*ii*), (*iii*), (*iv*) and (*vi*) are rational

10. (*i*) Rational; prime factors of q will be 2 or 5 or both only

(*ii*) Rational; prime factors of q will also have a prime factor other than 2 or 5

(*iii*) Not rational

(*iv*) Rational; prime factors of q will also have a prime factor other than 2 or 5

11. (*i*) 0·4141141114… (*ii*) 0·151551555… (*iii*) 0·070070007…

12. $\sqrt{5}, \sqrt{6}$ **13.** 0·5050050005...., 0·6060060006

15. 3·5, 3·6

Hint. $2\sqrt{3} = \sqrt{12}$; $12 < 12{\cdot}25 < 12{\cdot}96 < 15$

$\Rightarrow \sqrt{12} < \sqrt{12{\cdot}25} < \sqrt{12{\cdot}96} < \sqrt{15}$

16. $\sqrt{6}$ **17.** $\sqrt{5}, \sqrt{6}$

EXERCISE 1.4

1. (*i*) $\sqrt{5}$ (*ii*) $\frac{34\sqrt{3}}{3}$ (*iii*) 60 (*iv*) $4\sqrt{5}$ (*v*) $\frac{7\sqrt{6}}{12}$ (*vi*) $\frac{5\sqrt{2}}{4}$

2. (*i*) $10 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{35}$ (*ii*) 20 (*iii*) $7 + 2\sqrt{10}$

(*iv*) $10 - 2\sqrt{21}$ (*v*) $\sqrt{10} + \sqrt{14} + \sqrt{15} + \sqrt{21}$

(*vi*) $4\sqrt{3} - 4\sqrt{7} + \sqrt{15} - \sqrt{35}$

3. (*i*) 28·28 (*ii*) −36·764 **4.** (*i*) 8·66 (*ii*) 121·24

5. (*i*) $\sqrt{\frac{7}{25}}, \sqrt{\frac{16}{5}}$ (*ii*) $-\sqrt{\frac{2}{49}}, \sqrt{\frac{25}{3}}$ **6.** (*i*), (*iii*) and (*iv*)

7. (*i*), (*ii*), (*iii*), (*iv*), (*vi*) and (*viii*) **9.** (*i*) $\sqrt{15}; -\sqrt{7}$ (*ii*) $\frac{9}{\sqrt{5}}; -3\sqrt{2}$

10. (*i*) $2\sqrt{3}, \sqrt{15}, 4, 3\sqrt{2}$ (*ii*) $4, 3\sqrt{2}, 2\sqrt{8}, 4\sqrt{3}, \sqrt{50}$

11. (*i*) $4\sqrt{3}, \frac{9}{\sqrt{2}}, \frac{3}{2}\sqrt{5}, 3\sqrt{\frac{6}{5}}$ (*ii*) $3\sqrt{5}, 2\sqrt{7}, \frac{7}{3}\sqrt{2}, \frac{5}{\sqrt{3}}, -\sqrt{3}$

12. $\sqrt[3]{2}, \sqrt[6]{5}, \sqrt{3}$

EXERCISE 1.5

1. (*i*) $\frac{3\sqrt{5}}{20}$ (*ii*) $\frac{5\sqrt{21}}{3}$ (*iii*) $\frac{4+\sqrt{7}}{3}$ (*iv*) $3\sqrt{2} - 1$ (*v*) $\sqrt{41} + 5$

(*vi*) $\sqrt{7} + \sqrt{6}$ (*vii*) $\frac{\sqrt{5} - \sqrt{2}}{3}$ (*viii*) $-5 - 2\sqrt{6}$

2. (*i*) $\frac{47 + 21\sqrt{5}}{2}$ (*ii*) $17 - 12\sqrt{2}$ (*iii*) $\frac{-119 + 31\sqrt{14}}{7}$

3. 5 **4.** 1

5. (*i*) $a = \frac{9}{11}$ (*ii*) $a = 2, b = -\frac{5}{6}$ (*iii*) $a = 0, b = 1$ (*iv*) $a = -\frac{7}{2}, b = \frac{3}{4}$

6. $p = 0, q = 1$ **7.** (*i*) 0·414 (*ii*) 0·318 **8.** $2\sqrt{3}$ **9.** 16

10. 98 **11.** (*i*) −18 (*ii*) $8\sqrt{5}$ (*iii*) 322 (*iv*) $-144\sqrt{5}$

MULTIPLE CHOICE QUESTIONS

1. (*d*) **2.** (*c*) **3.** (*c*) **4.** (*d*) **5.** (*d*) **6.** (*c*) **7.** (*c*)

8. (*b*) **9.** (*d*) **10.** (*d*) **11.** (*c*) **12.** (*c*) **13.** (*b*) **14.** (*b*)

15. (*c*) **16.** (*c*) **17.** (*d*) **18.** (*d*) **19.** (*a*) **20.** (*b*) **21.** (*a*)

22. (*d*) **23.** (*c*) **24.** (*d*)

ASSERTION-REASON TYPE QUESTIONS

1. (*a*) **2.** (*c*) **3.** (*b*) **4.** (*a*) **5.** (*d*)

CHAPTER TEST

1. (*i*) Recurring decimal (*ii*) recurring decimal (*iii*) terminating decimal; 0·056
(*iv*) terminating decimal; −0·2875 (*v*) recurring decimal

2. (*i*) $\frac{74}{55}$ (*ii*) $\frac{2355}{999}$ **3.** $\frac{64}{117}, \frac{7}{13}, \frac{64}{117}, \frac{5}{9}$ **4.** $\frac{121}{150}, \frac{61}{75}, \frac{41}{50}, \frac{62}{75}$

9. (*i*) $2(2\sqrt{2} - \sqrt{3})$ (*ii*) $\frac{57}{15} - \frac{41}{30}\sqrt{6}$ (*iii*) $\frac{2 + \sqrt{6} - \sqrt{2}}{4}$

10. $p = 3, q = -\frac{2}{3}$ **11.** $-4\sqrt{2}$

12. (*i*) 2207 (*ii*) $20\frac{7}{9}$ (*iii*) 970

Hint. (*iii*) $x^3 + y^3 = (x + y)^3 - 3xy(x + y)$

13. $\frac{5}{2}\sqrt{3}, 3{\cdot}5, \sqrt{10}, 2\sqrt{2}, -\frac{5}{\sqrt{2}}$ **14.** $2; \frac{\sqrt{3} + \sqrt{5}}{2}$

15. $\sqrt{13}, \sqrt{15}, 3\sqrt{2}; 2\sqrt{5}, 3\sqrt{2}, \sqrt{15}, \sqrt{13}, 2\sqrt{3}$ **16.** (*i*) $\sqrt{2}, \sqrt{3}$ (*ii*) $\sqrt{2}, \sqrt{3}$

17. Take $a = 3\sqrt{2}$, $b = 5\sqrt{2}$

18. q can be expressed as $q = 2^m5^n$ where m and n are non-negative integers

19. (*i*) Rational; prime factors of q are 2 or 5 or both only
(*ii*) rational; q has a prime factor other than 2 and 5
(*iii*) irrational
(*iv*) rational; prime factors of q are 2 or 5 or both only
(*v*) rational; q has a prime factor other than 2 and 5
(*vi*) irrational

EXERCISE 2.1

1. ₹8820; ₹820 **2.** (*i*) ₹1875 (*ii*) ₹50700 (*iii*) ₹2028 **3.** ₹880; ₹10648

4. (*i*) ₹14080 (*ii*) ₹1408 (*iii*) ₹17036·80

5. (*i*) ₹5750 (*ii*) ₹710·70 **6.** (*i*) 12% (*ii*) ₹12544

7. (*i*) 6·5% p.a. (*ii*) ₹5671

8. (*i*) 12% (*ii*) ₹672 (*iii*) ₹7024·64 **9.** ₹2541; ₹541

10. ₹56243·20; ₹6243·20 **11.** ₹5724; ₹724

12. (*i*) ₹10560 (*ii*) ₹11616 (*iii*) ₹2016 (*iv*) ₹1056; ₹105·60 (*v*) ₹1161·60

13. ₹10648; ₹2648 **14.** ₹410 **15.** ₹4155 **16.** ₹3444, ₹244

17. ₹30000 **18.** ₹26450

EXERCISE 2.2

1. ₹5618; ₹618 **2.** ₹11712·80; ₹3712·80 **3.** ₹7774·625 **4.** ₹5788·12; ₹788·12

5. ₹103030·10; ₹3030·10 [**Hint.** $r = \frac{1}{4}$ of 4% = 1% and $n = \frac{9}{3} = 3$]

6. ₹12 **7.** ₹6·08 **8.** ₹2142·40; ₹142·40 **9.** ₹492·25 **10.** ₹8000

11. ₹7500 **12.** ₹125000 **13.** ₹70000 **14.** ₹80 **15.** $4\frac{1}{6}$% **16.** 25%

17. 5% **18.** 10% **19.** 40% p.a. **20.** 5%; ₹92610 **21.** 5%; ₹4800

22. 10%; ₹600 **23.** 3 years **24.** (*i*) 3 years (*ii*) 3 years **25.** $1\frac{1}{2}$ years **26.** ₹2500

27. (i) $6\frac{2}{3}\%$ (ii) ₹3375 (iii) ₹256 28. ₹10000 29. ₹6000 30. 10%
31. 4% p.a.; ₹1875 32. ₹21 33. ₹20000 34. ₹12000; 10%

EXERCISE 2.3

1. 253000 2. 1951 3. (i) 7311616 (ii) 6250000 4. ₹877·50 5. ₹20577
6. 1875 quintals 7. ₹480000. 8. 5% p.a. 9. 5% p.a. 10. 3 years
11. ₹387500 12. $95\frac{5}{16}\%$ 13. Profit ₹80420

MULTIPLE CHOICE QUESTIONS

1. (c) 2. (c) 3. (b) 4. (a) 5. (b) 6. (d) 7. (b)
8. (a)

ASSERTION-REASON TYPE QUESTIONS

1. (b) 2. (b)

CHAPTER TEST

1. ₹25 2. $6\frac{1}{4}\%$; ₹3468 3. ₹25000; ₹3090
4. (i) ₹20000 (each) (ii) ₹2050; ₹2472 5. ₹8000; 8%
6. ₹7500 7. (i) 6% p.a. (ii) ₹6000 8. 1 year 9. 20%
10. 397535 [**Hint.** Net growth rate = 9·2% − 1·7% = 7·5%]
11. (i) ₹493125 (ii) ₹161587·20 12. ₹43520

EXERCISE 3.1

1. (i) $4x^2 + 28xy + 49y^2$ (ii) $\frac{1}{4}x^2 + \frac{2}{3}xy + \frac{4}{9}y^2$
2. (i) $9x^2 + \frac{1}{4x^2} + 3$ (ii) $9x^4y^2 + 30x^2yz + 25z^2$
3. (i) $9x^2 + \frac{1}{4x^2} - 3$ (ii) $\frac{1}{4}x^2 - \frac{3}{2}xy + \frac{9}{4}y^2$
4. (i) $x^2 + 8x + 15$ (ii) $x^2 - 2x - 15$
   (iii) $x^2 + 2x - 63$ (iv) $x^2 - 5xy + 6y^2$
5. (i) $x^2 + 4y^2 + z^2 - 4xy + 4yz - 2zx$
   (ii) $4x^2 + 9y^2 + 16z^2 - 12xy - 24yz + 16zx$
6. (i) $4x^2 + \frac{9}{x^2} + 13 - \frac{6}{x} - 4x$ (ii) $\frac{4}{9}x^2 + \frac{9}{4x^2} - 1 - \frac{4}{3}x + \frac{3}{x}$
7. (i) $x^3 + 6x^2 + 12x + 8$ (ii) $8a^3 + b^3 + 12a^2b + 6ab^2$
8. (i) $27x^3 + \frac{1}{x^3} + 27x + \frac{9}{x}$ (ii) $8x^3 - 12x^2 + 6x - 1$
9. (i) $125x^3 - 27y^3 - 225x^2y + 135xy^2$ (ii) $8x^3 - \frac{1}{27y^3} - \frac{4x^2}{y} + \frac{2x}{3y^2}$
10. (i) $2\left(a^2 + \frac{1}{a^2}\right)$ (ii) 4 11. (i) $3 - 3x$ (ii) 0
12. (i) $49p^2 - 81q^2$ (ii) $4x^2 - \frac{9}{x^2}$
13. (i) $4x^2 - 4xy + y^2 - 9$ (ii) $9x^2 - 30x + 25 - y^2$

27. (i) $6\frac{2}{3}\%$ (ii) ₹3375 (iii) ₹256 28. ₹10000 29. ₹6000 30. 10%
31. 4% p.a.; ₹1875 32. ₹21 33. ₹20000 34. ₹12000; 10%

EXERCISE 2.3

1. 253000 2. 1951 3. (i) 7311616 (ii) 6250000 4. ₹877·50 5. ₹20577
6. 1875 quintals 7. ₹480000. 8. 5% p.a. 9. 5% p.a. 10. 3 years
11. ₹387500 12. $95\frac{5}{16}\%$ 13. Profit ₹80420

MULTIPLE CHOICE QUESTIONS

1. (c) 2. (c) 3. (b) 4. (a) 5. (b) 6. (d) 7. (b)
8. (a)

ASSERTION-REASON TYPE QUESTIONS

1. (b) 2. (b)

CHAPTER TEST

1. ₹25 2. $6\frac{1}{4}\%$; ₹3468 3. ₹25000; ₹3090
4. (i) ₹20000 (each) (ii) ₹2050; ₹2472 5. ₹8000; 8%
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EXERCISE 3.1

1. (i) $4x^2 + 28xy + 49y^2$ (ii) $\frac{1}{4}x^2 + \frac{2}{3}xy + \frac{4}{9}y^2$
2. (i) $9x^2 + \frac{1}{4x^2} + 3$ (ii) $9x^4y^2 + 30x^2yz + 25z^2$
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4. (i) $x^2 + 8x + 15$ (ii) $x^2 - 2x - 15$
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   (ii) $4x^2 + 9y^2 + 16z^2 - 12xy - 24yz + 16\,zx$
6. (i) $4x^2 + \frac{9}{x^2} + 13 - \frac{6}{x} - 4x$ (ii) $\frac{4}{9}x^2 + \frac{9}{4x^2} - 1 - \frac{4}{3}x + \frac{3}{x}$
7. (i) $x^3 + 6x^2 + 12x + 8$ (ii) $8a^3 + b^3 + 12a^2b + 6ab^2$
8. (i) $27x^3 + \frac{1}{x^3} + 27x + \frac{9}{x}$ (ii) $8x^3 - 12x^2 + 6x - 1$
9. (i) $125x^3 - 27y^3 - 225x^2y + 135xy^2$ (ii) $8x^3 - \frac{1}{27y^3} - \frac{4x^2}{y} + \frac{2x}{3y^2}$
10. (i) $2\left(a^2 + \frac{1}{a^2}\right)$ (ii) 4 11. (i) 3 − 3x (ii) 0
12. (i) $49p^2 - 81q^2$ (ii) $4x^2 - \frac{9}{x^2}$
13. (i) $4x^2 - 4xy + y^2 - 9$ (ii) $9x^2 - 30x + 25 - y^2$

14. (*i*) $x^2 - 6x + 9 - \frac{4}{x^2}$ (*ii*) $625 - 16x^4$

15. (*i*) $x^2 + 4xy + 4y^2 + 10x + 20y + 21$ (*ii*) $4x^2 + 4xy + y^2 - 8x - 4y - 45$

(*iii*) $x^2 - 4xy + 4y^2 - 2x + 4y - 15$ (*iv*) $9x^2 - 24xy + 16y^2 - 24x + 32y + 12$

16. (*i*) $8p^3 + 27q^3$ (*ii*) $x^3 + \frac{1}{x^3}$ **17.** (*i*) $27p^3 - 64q^3$ (*ii*) $x^3 - \frac{27}{x^3}$

18. $8x^3 + 27y^3 + 64z^3 - 72xyz$

19. (*i*) $x^3 + 6x^2 + 11x + 6$ (*ii*) $x^3 - x^2 - 14x + 24$

20. 0; – 37 **21.** 4

22. (*i*) 10201 (*ii*) 1006009 (*iii*) 104·04

23. (*i*) 9801 (*ii*) 994009 (*iii*) 96·04

24. (*i*) 1092727 (*ii*) 970299 (*iii*) 1030·301

25. Hint. $2a - b + c = 0 \Rightarrow 2a + c = b \Rightarrow (2a + c)^2 = b^2$

27. –98 **28.** 3 **29.** 0

30. (*i*) 13770 (*ii*) –16380 **31.** 100

EXERCISE 3.2

1. 74 **2.** 116 **3.** 25 **4.** 352

5. 68 **[Hint.** $2(x^2 + y^2) = (x + y)^2 + (x - y)^2]$ **6.** (*i*) ± 5 (*ii*) ± 1

7. (*i*) ± 8 (*ii*) ± 32 **8.** (*i*) ± 15 (*ii*) ± 135

9. (*i*) 26 (*ii*) 5 **[Hint.** (*ii*) $4xy = (x + y)^2 - (x - y)^2]$

10. 11 **[Hint.** $x - 3 = \frac{1}{x} \Rightarrow x - \frac{1}{x} = 3]$ **11.** (*i*) ± 7 (*ii*) $169\frac{1}{2}$ (*iii*) $310\frac{1}{2}$ or $254\frac{1}{2}$

12. 123 **13.** 63 **14.** 135 **15.** (*i*) 14 (*ii*) 194 (*iii*) 52 (*iv*) $\pm 2\sqrt{3}$

16. 727 **17.** (*i*) 7 (*ii*) ±3 (*iii*) ±18 **18.** (*i*) $\pm 4\sqrt{2}$ (*ii*) $\pm 24\sqrt{2}$

20. 45 **23.** ± 5 **24.** ± 445 **25.** ± 3 **26.** $\pm 22\frac{1}{2}$ **27.** (*i*) 7 (*ii*) 18

28. (*i*) 5 (*ii*) $\pm\sqrt{29}$ (*iii*) $\pm 5\sqrt{29}$ **[Hint.** $a = \frac{1}{a-5} \Rightarrow a^2 - 5a - 1 = 0]$

29. 0 **30.** $2\sqrt{3}$ **31.** 100 **32.** 22 **33.** ± 15 **34.** –2 **35.** 721

36. 10 **37.** 152

MULTIPLE CHOICE QUESTIONS

1. (*b*) **2.** (*d*) **3.** (*d*) **4.** (*a*) **5.** (*d*) **6.** (*b*) **7.** (*d*)

8. (*c*) **9.** (*d*) **10.** (*c*) **11.** (*b*)

ASSERTION-REASON TYPE QUESTIONS

1. (*d*) **2.** (*c*)

CHAPTER TEST

1. (*i*) $4x^2 + 9y^2 + 12xy - 25$

(*ii*) $36 + 16a^2 + 49b^2 - 48a + 56ab - 84b$

(*iii*) $343 - 27x^3y^3 - 441xy + 189x^2y^2$

(*iv*) $x^3 + y^3 + 3x^2y + 3xy^2 + 6x^2 + 6y^2 + 12xy + 12x + 12y + 8$

2. $x^8 - 256$ 3. 999996 5. 0 6. 27 7. $p^2 - q^2 = 4$

8. 104 9. (*i*) 4 (*ii*) 52 (*iii*) 2702 10. $27 + 19\sqrt{2}$ 11. $\pm 26\frac{26}{27}$

12. $-30\sqrt{3}$ 13. 29 14. (*i*) -10 (*ii*) 29

15. 6 **[Hint.** Given expression $= \{(x^2 + 1) + x\}^2 + \{(x^2 + 1) - x\}^2$
$= 2\{(x^2 + 1)^2 + x^2\}]$

16. **Hint.** $2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx = 0$
$\Rightarrow (x^2 + y^2 - 2xy) + (y^2 + z^2 - 2yz) + (2z^2 + x^2 - 2zx) = 0$
$\Rightarrow (x - y)^2 + (y - z)^2 + (z - x)^2 = 0$
$\Rightarrow x - y = 0, y - z = 0, z - x = 0$
$\Rightarrow x = y = z$

EXERCISE 4.1

1. (*i*) $4xy^2(2y + 3x)$ (*ii*) $3ax^2(5x - 3)$
2. (*i*) $7py(3y - 8)$ (*ii*) $2x^2(2x - 3)$
3. (*i*) $2\pi r\ (r - 2)$ (*ii*) $2(9m + 8n)$
4. (*i*) $5abc(5c - 3ab)$ (*ii*) $14pq^2r(2p - 3r)$
5. (*i*) $2x(4x^2 - 3x + 5)$ (*ii*) $2(7mn + 11m - 31p)$
6. (*i*) $6pq(3pq - 4q + 5p)$ (*ii*) $3a^2b^2(9ab - 6b + 25a)$
7. (*i*) $5(2p - 3q)(3a - 2b)$ (*ii*) $3(x^2 + y^2)(a + 2b)$
8. (*i*) $2(x + 2y)^2\ (3x + 6y + 4)$ (*ii*) $7(a - 3b)\ [2(a - 3b)^2 - 3p]$
9. (*i*) $5(2p + q)\ [2a(2p + q)^2 - 3b(2p + q) + 7]$ (*ii*) $(x^2 + y^2 - z^2)\ (x - y - z)$

EXERCISE 4.2

1. (*i*) $(x + y)\ (x - 1)$ (*ii*) $(y - z)\ (y - 5)$
2. (*i*) $(x - y)\ (5y - 7)$ (*ii*) $(5p - 8q)\ (p - 2)$
3. (*i*) $(a - b)\ (ab + 3)$ (*ii*) $(x - 3)\ (x^2 + 1)$
4. (*i*) $(2y - 1)\ (3xy - 5)$ (*ii*) $(x - 2y)\ (3a + 4b)$
5. (*i*) $(x - 2y)(5p + 4q)$ (*ii*) $3(a - 1)(3ay + 2)$
6. (*i*) $(1 - a)\ (1 - b)$ (*ii*) $(a - 2b)\ (a - c)$
7. (*i*) $(x + y)\ (x + y^2)$ (*ii*) $(y - x)\ (y + x^2)$
8. (*i*) $(b + 1)\ (ab - 1)$ (*ii*) $(a - 2b)\ (2 - x)$
9. (*i*) $(5 + 2r)\ (ph - 2qk)$ (*ii*) $(x - a)\ (x - 2b)$
10. (*i*) $(bx - ay)\ (ax - by)$ (*ii*) $(x^2 + y^2)\ (a^2 + b^2)$
11. (*i*) $(a - 2b)\ (a^2 + b)$ (*ii*) $3(x - 1)\ (xy + 4)$

Hint. (*ii*) 3 is a common factor. First take 3 outside.

12. $(a + b)\ (ab - bc + xy)$ 13. $(a - b)\ (x^2 + y^2 + z^2)$
14. $(x - 1)\ (2 - x + a)$

EXERCISE 4.3

1. (*i*) $(2x + 5y)\ (2x - 5y)$ (*ii*) $(3x + 1)\ (3x - 1)$
2. (*i*) $6(5 + a)\ (5 - a)$ (*ii*) $2(4x + 3y)\ (4x - 3y)$
3. (*i*) $(x - y + 3)\ (x - y - 3)$ (*ii*) $(4x + 3y)\ (2x + 3y)$
4. (*i*) $5(2x + 3y)\ (2x - 3y)$ (*ii*) $-(7x + 2y)\ (x + 2y)$
5. (*i*) $2(x + 3y)\ (x - 7y)$ (*ii*) $2x\ (8 - x)$

(*ii*) $\pi a\ (a^2 + \pi b)\ (a^2 - \pi b)$

7. (*i*) $8(2x + 1)(3x - 1)$ (*ii*) $(x + 1)(x - 1)$

Hint. (*ii*) $(x - 2)(x + 2) + 3 = (x^2 - 4) + 3 = x^2 - 1$

8. (*i*) $(x - 2y)(1 - x - 2y)$ (*ii*) $(2a + b)(2a - b + 1)$

9. (*i*) $(a - b)(a + b - 2)$ (*ii*) $(a - b)(a + b - 1)$

10. (*i*) $(3 + x - y)(3 - x + y)$ (*ii*) $(3x^2 + x + 1)(3x^2 - x - 1)$

11. (*i*) $(3x^2 + x + 6)(3x^2 - x - 6)$ (*ii*) $(x + 1)(x - 1)(x - 5)$

12. (*i*) $(a^2 + b^2 - 1)(a^2 - b^2 + 1)$ (*ii*) $x(x + 5)(x - 5)$

13. (*i*) $2(x^2 + 4)(x + 2)(x - 2)$ (*ii*) $(b + c)(a + b + c)(a - b - c)$

14. (*i*) $(a + b)(a + b + 1)(a + b - 1)$ (*ii*) $(x - y + a + b)(x - y - a - b)$

15. (*i*) $(ac - bd + bc + ad)(ac - bd - bc - ad)$ (*ii*) $(x - y)(4x + y + 2)$

Hint. (*i*) Given expression $= (ac - bd)^2 - (bc + ad)^2$

(*ii*) Given expression $= (x^2 - y^2) + (3x^2 - 3xy) + (2x - 2y)$

16. (*i*) $(x - \frac{1}{x} + 3)(x - \frac{1}{x} - 3)$ (*ii*) $(x^2 + x + 3)(x^2 - x + 3)$

17. (*i*) $\left(x - \frac{2}{x} + 1\right)\left(x - \frac{2}{x} - 1\right)$ (*ii*) $(x^2 + 11)(x^2 + 1)$

18. (*i*) $(a^2 + b^2 + 3ab)(a^2 + b^2 - 3ab)$ (*ii*) $(x^2 + 4x + 1)(x^2 - 4x + 1)$

19. (*i*) $(x^2 + 7)^2 - (5x)^2$ (*ii*) $(x^2 - 5x)^2 - 7^2$ (*iii*) $(x^2)^2 - (5x - 7)^2$

20. (*i*) 958000 (*ii*) 9980

EXERCISE 4.4

1. (*i*) $(x + 2)(x + 3)$ (*ii*) $(x - 1)(x - 7)$

2. (*i*) $(x + 7)(x - 1)$ (*ii*) $(y + 9)(y - 2)$

3. (*i*) $(y - 9)(y + 2)$ (*ii*) $(a + 6)(a - 9)$

4. (*i*) $(x - 2)(2x - 3)$ (*ii*) $(3x - 1)(2x + 5)$

5. (*i*) $(2x + 5)(3x - 2)$ (*ii*) $(3x + 1)(2x - 3)$

6. (*i*) $(x - 2)(2x + 3)$ (*ii*) $(1 - 21y)(1 + 3y)$

7. (*i*) $(y + 5)(2y - 9)$ (*ii*) $(1 - 2x)(5 + 6x)$

8. (*i*) $(4x + 5)(3x - 2)$ (*ii*) $(x - 8)(x - 2)$

Hint. (*i*) $x(12x + 7) - 10 = 12x^2 + 7x - 10$

(*ii*) $(4 - x)^2 - 2x = x^2 - 10x + 16$

9. (*i*) $10(2x - 3)(3x + 1)$ (*ii*) $(x + y)(x - 7y)$

10. (*i*) $(x + 8y)(2x - 3y)$ (*ii*) $(2x - 3y)(3x + 2y)$

11. (*i*) $(x + 4y)(5x - 3y)$ (*ii*) $(xy + 4)(xy - 12)$

12. (*i*) $(ab - 6)(2ab + 5)$ (*ii*) $(2a + b)(a - b)$

13. (*i*) $(x - y - 5)(x - y - 1)$ (*ii*) $(2x - y - 7)(2x - y - 4)$

14. (*i*) $(2a - 5)(2a - 1)$ (*ii*) $(1 - 3a - 3b)(1 + a + b)$

15. (*i*) $(1 - 3a - 3b)(3 + 4a + 4b)$ (*ii*) $(a^2 - 10)(a + 1)(a - 1)$

16. (*i*) $(x + y + 4)(x - 6y + 4)$ (*ii*) $(x + 2)(x - 4)(x + 3)(x - 5)$

Hint. (*i*) $(x + 4)^2 - 5xy - 20y - 6y^2 = (x + 4)^2 - 5y(x + 4) - 6y^2$

$= z^2 - 5yz - 6y^2$ where $z = x + 4$

$= (z + y)(z - 6y)$

17. $(3a - 4)(a - 5)$

Hint. Let $2a - 3 = x$ and $a - 1 = y$,

then given expression $= 4x^2 - 3xy - 7y^2 = (x + y)(4x - 7y)$.

18. $(x-1)(2x+7)(x+4)(2x-3)$

Hint. Let $2x^2+5x=y$, then

$(2x^2+5x)(2x^2+5x-19)+84 = y(y-19)+84$

$= y^2-19y+84 = (y-7)(y-12)$

EXERCISE 4.5

1. (*i*) $(2x+y)(4x^2-2xy+y^2)$ (*ii*) $(4x-5y)(16x^2+20xy+25y^2)$
2. (*i*) $(4x+1)(16x^2-4x+1)$ (*ii*) $7(a+2b)(a^2-2ab+4b^2)$
3. (*i*) $\left(\frac{x^2}{7}+\frac{7}{x^2}\right)\left(\frac{x^4}{49}-1+\frac{49}{x^4}\right)$ (*ii*) $\left(2x-\frac{1}{3y}\right)\left(4x^2+\frac{2x}{3y}+\frac{1}{9y^2}\right)$
4. (*i*) $x^2(1+x)(1-x+x^2)$ (*ii*) $4x(2x-5)(4x^2+10x+25)$
5. (*i*) $(3xy-2)(9x^2y^2+6xy+4)$ (*ii*) $(7x+y)(13x^2-4xy+19y^2)$

Hint. (*ii*) Given expression $= [3(x+y)+2(2x-y)][9(x+y)^2-3(x+y).2(2x-y)+4(2x-y)^2)$

6. (*i*) $(a+b)(a^2-ab+b^2+1)$ (*ii*) $(a-b)(a^2+ab+b^2-1)$
7. (*i*) $(x+1)(x^2-x+2)$ (*ii*) $(a-5)(a^2+5a+24)$ (*iii*) $(a-5)(a^2+5a+23)$

Hint. (*i*) $x^3+x+2 = (x^3+1)+(x+1)$

(*ii*) $a^3-a-120 = (a^3-125)-(a-5)$

(*iii*) $a^3-2a-115 = a^3-125-2a+10 = (a^3-125)-2(a-5)$

8. (*i*) $(x+4)(x^2+2x+4)$ (*ii*) $(a-2b)(a^2-ab+b^2)$
9. (*i*) $(a+2b)(2a^2-4ab+8b^2-5)$ (*ii*) $\left(a-\frac{1}{a}\right)\left(a^2+\frac{1}{a^2}-1\right)$
10. (*i*) $(a+b)(a-b)(a^2+ab+b^2)(a^2-ab+b^2)$

    (*ii*) $(x+1)(x-1)(x^2+x+1)(x^2-x+1)$
11. (*i*) $(2x+3y)(2x-3y)(4x^2+6xy+9y^2)(4x^2-6xy+9y^2)$

    (*ii*) $\frac{1}{x}(x-2)(x^2+2x+4)$
12. (*i*) $2(5a-5b+1)(25a^2-50ab+25b^2-5a+5b+1)$

    (*ii*) $(2a+b)(2a-b)(2x-y)(4x^2+2xy+y^2)$
13. (*i*) $(x+y)(x^2-xy+y^2)(x^6-x^3y^3+y^6)$

    (*ii*) $(x-2)(x^2+2x+4)(x+1)(x^2-x+1)$

MULTIPLE CHOICE QUESTIONS

1. (*c*)	2. (*b*)	3. (*c*)	4. (*b*)	5. (*d*)	6. (*c*)	7. (*b*)
8. (*d*)	9. (*b*)	10. (*a*)	11. (*b*)	12. (*a*)	13. (*c*)	14. (*d*)
15. (*c*)	16. (*d*)	17. (*c*)	18. (*b*)			

ASSERTION-REASON TYPE QUESTIONS

1. (*c*) 2. (*b*) 3. (*a*)

CHAPTER TEST

1. (*i*) $5(2x-3)[3(2x-3)^2-2]$ (*ii*) $(b-c)[a(b+c)+d]$
2. (*i*) $(x+1)(2a^2-b)$ (*ii*) $(p-a)(p-2b)$
3. (*i*) $(x-z)(xz+y^2)$ (*ii*) $5a(a-1)(a^2+6)$
4. (*i*) $(c-d)(bc-bd-a+3)$ (*ii*) $(x-1)(x^2-y+1)$
5. (*i*) $(x-y)(x+y+z)$ (*ii*) $a^4x^4(a^4+x^4)(a^2+x^2)(a+x)(a-x)$

6. (*i*) $(3x + 2 + 4y)(3x + 2 - 4y)$ (*ii*) $(x^2 + x + 2)(x^2 - x + 2)$

Hint. (*ii*) $x^4 + 3x^2 + 4 = (x^2 + 2)^2 - x^2$

7. (*i*) $(3x - 5y)(7x - 8y)$ (*ii*) $4xy(x - 4)(x - 7)$

8. (*i*) $(xy - 9)(xy + 8)$ (*ii*) $xy(x + 4y)(9x + 5y)$

9. (*i*) $(3a - 2b + 5)(3a - 2b - 2)$ (*ii*) $(x^2 - 3x + 5)(x - 1)(x - 2)$

Hint. (*ii*) Given expression $= y(y + 7) + 10$ where $y = x^2 - 3x$

$= y^2 + 7y + 10 = (y + 5)(y + 2)$

10. (*i*) $(x + 1)(x - 2)(4x^2 - 4x + 3)$ (*ii*) $(x^2 + 9y^2 + 3xy)(x^2 + 9y^2 - 3xy)$

Hint. (*ii*) $x^4 + 9x^2y^2 + 81y^4 = (x^4 + 18x^2y^2 + 81y^4) - 9x^2y^2$

$= (x^2 + 9y^2)^2 - (3xy)^2$

11. (*i*) $\left(\frac{2}{3}x - \frac{1}{2}y\right)\left(\frac{4}{9}x^2 + \frac{1}{3}xy + \frac{1}{4}y^2\right)$ (*ii*) $(x + 4)(x^2 - 4x + 16)(x - 1)(x^2 + x + 1)$

12. (*i*) $\left(x + \frac{1}{x}\right)\left(x^2 - 1 + \frac{1}{x^2} + x - \frac{1}{x}\right)$ (*ii*) $4x(x^2 + 3)(3x^2 + 1)$

Hint. (*ii*) Let $x + 1 = a$, $x - 1 = b$

Given expression $= a^6 - b^6 = (a^3 - b^3)(a^3 + b^3)$

$= (a - b)(a^2 + ab + b^2)(a + b)(a^2 - ab + b^2)$

13. $(x + 1)(2x + 1)$

15. 931

Hint. $a^4 + a^2b^2 + b^4 = (a^2 + b^2)^2 - a^2b^2$

EXERCISE 5.1

1. (*i*) $x = 9, y = 5$ (*ii*) $s = 9, t = 6$ (*iii*) $x = -21, y = 17$ (*iv*) $x = \frac{9}{13}, y = -\frac{5}{13}$

2. (*i*) $x = -\frac{3}{2}, y = -\frac{1}{2}$ (*ii*) $x = 2, y = -\frac{3}{2}$

3. (*i*) $x = 3, y = 4$ (*ii*) $x = 7\frac{27}{31}, y = 2\frac{13}{31}$

4. (*i*) $x = m + n, y = m - n$ (*ii*) $x = 2a, y = -2b$

5. $x = 15, y = 5; 3$ 6. $x = \frac{16}{5}, y = \frac{23}{5}; p = \frac{19}{8}$

EXERCISE 5.2

1. (*i*) $x = 2, y = 1$ (*ii*) $x = -8, y = -4$

2. (*i*) $x = 4, y = 3$ (*ii*) $x = \frac{21}{20}, y = -\frac{3}{10}$ 3. (*i*) $x = 5, y = -3$ (*ii*) $x = 2, y = 1$

4. (*i*) $x = 12, y = 8$ (*ii*) $x = \frac{26}{11}, y = -\frac{5}{11}$

5. (*i*) $x = \frac{26}{3}, y = -\frac{8}{3}$ (*ii*) $x = \frac{3}{4}, y = -\frac{9}{4}$

6. $x = 2, y = -1$

Hint. $x - 3y = 3x - 1 = 2x - y$

$\Rightarrow$ $x - 3y = 3x - 1, 3x - 1 = 2x - y$

7. (*i*) $x = 4, y = -4$ (*ii*) $x = 7, y = 13$ 8. (*i*) $x = \frac{1}{5}, y = -2$ (*ii*) $x = 2, y = 1$

9. (*i*) $x = 1, y = -1$ (*ii*) $x = a, y = b$

10. $x = 7, y = 9$; $x - 3y = -20$, $5y - 2x = 31$ 11. $a = 2, b = 3$

12. Yes; $x = 7, y = 2$

EXERCISE 5.3

1. (*i*) $x = -2, y = 5$ (*ii*) $x = -1, y = 1$ 2. (*i*) $x = 2, y = 1$ (*ii*) $x = -1, y = -1$

3. (*i*) $x = a, y = -b$ (*ii*) $x = 2a, y = -2b$

EXERCISE 5.4

1. (*i*) $x = 2, y = -1$ (*ii*) $x = \frac{1}{2}, y = \frac{1}{3}$

2. (*i*) $x = 1, y = 1$ (*ii*) $x = 0, y = 0; x = \frac{1}{2}, y = \frac{1}{3}$

3. (*i*) $x = 0, y = 0; x = 7, y = 1$ (*ii*) $x = 0, y = 0; x = \frac{11}{7}, y = \frac{11}{3}$

4. (*i*) $x = 4, y = 5$ (*ii*) $x = 2, y = 1$

5. (*i*) $x = 2, y = 1$ (*ii*) $x = \frac{1}{2}, y = \frac{5}{4}$

MULTIPLE CHOICE QUESTIONS

1. (*c*) 2. (*b*) 3. (*c*) 4. (*a*) 5. (*d*) 6. (*c*)

ASSERTION-REASON TYPE QUESTIONS

1. (*c*) 2. (*c*)

CHAPTER TEST

1. (*i*) $x = 3, y = 4$ (*ii*) $x = 14, y = 2$ 2. (*i*) $x = -2, y = 7$ (*ii*) $x = 2, y = -1$

3. (*i*) $x = 0, y = 0; x = \frac{1}{3}, y = \frac{1}{4}$ (*ii*) $x = 8, y = 3$

4. (*i*) $x = 1, y = -1$ (*ii*) $x = 2, y = 3$ 5. $x = 2, y = 2$ 6. $x = 3, y = -1; k = 2$

7. $x = 3, y = 2$; 14 8. Yes; $x = \frac{1}{3}, y = \frac{1}{2}$

EXERCISE 6

1. 33, 17 2. 11, −9 3. 33, 10 4. ₹1030

5. 16 kg and 20 kg 6. 10, 8

7. 20 paise 20 coins, 25 paise 18 coins 8. 200 rupee notes = 16, and 50 rupee notes = 12

9. 8 and 12 10. $\frac{2}{15}$ 11. $\frac{8}{15}$ 12. $\frac{6}{8}$ 13. 43

14. 34 15. 72 16. 84 17. 45 18. 842

19. 34 years, 14 years

20. ₹800 21. ₹5000 at 12% and ₹7000 at 10%

22. Table ₹600; chair ₹400 23. A has ₹500, B has ₹900

24. 60 25. 80 g 18-carat, 40 g 12-carat

26. A in 25 days, B in $37\frac{1}{2}$ days

Hint. Let A's one day work be x and B's one day work be y. Then

$$x = \frac{3}{2}y, x + y = \frac{1}{15}$$

27. 18 days

28. 360 km

Hint. Let the due speed of the train be x km/h and scheduled time by y hours, then length of journey $= xy$. Also $(x + 30)(y - 2) = xy$ and $(x - 15)(y + 2) = xy$

29. 15 km/h, 5 km/h **30.** 8 km/h, 3 km/h

31. 520 km/h, 40 km/h **32.** ₹1200, ₹70

MULTIPLE CHOICE QUESTIONS

1. (*a*) **2.** (*b*) **3.** (*c*) **4.** (*d*) **5.** (*d*) **6.** (*d*) **7.** (*d*)

8. (*c*) **9.** (*a*)

ASSERTION-REASON TYPE QUESTIONS

1. (*d*) **2.** (*a*)

CHAPTER TEST

1. Almonds 400 g, cashew kernel 300 g **2.** 8

3. ₹80 per regular hour and ₹100 per hour for overtime

Hint. Let Shikha's earning be ₹x per regular hour and ₹y per hour for overtime, then $40x + 7y = 3900$, $42x + 8y = 4160$

4. 16 **5.** Man's age 42 years, his son's age 12 years

6. 113 m^2 **7.** 40 metres **8.** 19·5 metres

9. Longer candle 27 cm, smaller candle 24 cm

Hint. Let the longer candle shorten at the rate of x cm/h when burning and the smaller candle shorten at the rate of y cm/h.

As the longer candle burns out completely in 6 hours and that the smaller candle in 4 hours, their lengths are $6x$ cm and $4y$ cm respectively.

According to given, $6x = 4y + 3$

At 9.30 p.m., the length of longer candle $= (6x - 4x)$ cm $= 2x$ cm and the length of the smaller candle $= \left(4y - \frac{5}{2}y\right)$ cm $= \frac{3}{2}y$ cm. As both the candles have same length at 9.30 p.m., $2x = \frac{3}{2}y \Rightarrow 4x = 3y$

EXERCISE 7

1. (*i*) $\frac{8}{27}$ (*ii*) $\frac{16}{49}$ **2.** (*i*) $8a^{-9}b^6$ (*ii*) $a + b$ **3.** (*i*) $\frac{1}{x+y}$ (*ii*) $3 \times (10)^{-8}$

4. (*i*) $5ab$ (*ii*) $2\frac{1}{4}$ **5.** (*i*) $2\frac{1}{2}$ (*ii*) $3\frac{1}{3}$ **6.** (*i*) 9 (*ii*) $1\frac{11}{16}$

7. (*i*) 3^{3n} (*ii*) $\frac{1}{6}$ **8.** (*i*) $\frac{1}{2}$ (*ii*) $-12\frac{3}{4}$ **9.** (*i*) $\frac{1}{27}$ (*ii*) $2x$

10. (*i*) 19 (*ii*) 231 **11.** (*i*) 12 (*ii*) 27 **12.** (*i*) $2\frac{1}{4}$ (*ii*) 19

13. (*i*) $\frac{1}{2}$ (*ii*) 243 **14.** (*i*) −2 (*ii*) $\frac{4}{21}$ **15.** (*i*) 4 (*ii*) $\frac{1}{25}$

16. (*i*) −42 (*ii*) 98·25 **17.** (*i*) 3 (*ii*) $x - \frac{1}{x}$ **18.** (*i*) 1 (*ii*) 1

19. (*i*) 1 (*ii*) 1 (*iii*) $x^2(a^3 + b^3 + c^3)$

20. (*i*) $\frac{ab}{b-a}$ (*ii*) 1

25. Hint. Let $2^x = 3^y = 12^z = k$, then $2 = k^{\frac{1}{x}}$, $3 = k^{\frac{1}{y}}$ and $12 = k^{\frac{1}{z}}$.

Now $12 = 2^2 \times 3 \Rightarrow k^{\frac{1}{z}} = \left(k^{\frac{1}{x}}\right)^2 \times k^{\frac{1}{y}}$

$\Rightarrow \frac{1}{z} = \frac{2}{x} + \frac{1}{y} \Rightarrow \frac{1}{z} - \frac{1}{y} = \frac{2}{x}$

26. 1, $\frac{1}{x^2y^2}$, $9a^6$

27. (*i*) $27\frac{1}{4}$ (*ii*) $-7\frac{8}{9}$

28. 3 **29.** 6 **30.** $x = 2, y = 3, z = 7$ **31.** $x = 2, y = -\frac{2}{3}$

33. 6 **34.** (*i*) $-\frac{3}{2}$ (*ii*) 4 (*iii*) -7 (*iv*) -4 **35.** (*i*) $\frac{3}{4}$ (*ii*) $\frac{5}{7}$

36. $x = 1, y = -3$ **38.** 1 **39.** 64 **40.** (*i*) $x = 3$ (*ii*) $x = 1, \ y = -1$

MULTIPLE CHOICE QUESTIONS

1. (*d*) **2.** (*c*) **3.** (*b*) **4.** (*a*) **5.** (*a*) **6.** (*c*) **7.** (*b*)

ASSERTION-REASON TYPE QUESTIONS

1. (*c*) **2.** (*b*) **3.** (*c*)

CHAPTER TEST

1. $x = 4, y = 3, z = 1$; $2\frac{1}{40}$ **2.** (*i*) $3\frac{26}{27}$ (*ii*) $9\frac{1}{8}$ **6.** (*i*) 1 (*ii*) 1 (*iii*) 1

7. Hint. $\frac{1}{1 + a^{y-x} + a^{z-x}} = \frac{1}{1 + \frac{a^y}{a^x} + \frac{a^z}{a^x}} = \frac{a^x}{a^x + a^y + a^z}$ etc.

9. (*i*) $x = 5$ (*ii*) $x = 6$ (*iii*) $x = \frac{3}{4}, y = -\frac{8}{3}$ (*iv*) $x = 3, y = \frac{5}{2}$ (*v*) $x = 3, y = 1$

EXERCISE 8.1

1. (*i*) $\log_5 25 = 2$ (*ii*) $\log_a 64 = 5$ (*iii*) $\log_7 100 = x$ (*iv*) $\log_9 1 = 0$

(*v*) $\log_6 6 = 1$ (*vi*) $\log_3 \frac{1}{9} = -2$ (*vii*) $\log_{10} 0{\cdot}01 = -2$ (*viii*) $\log_{81} 27 = \frac{3}{4}$

2. (*i*) $2^5 = 32$ (*ii*) $3^4 = 81$ (*iii*) $3^{-1} = \frac{1}{3}$ (*iv*) $(8)^{\frac{2}{3}} = 4$

(*v*) $(8)^{\frac{5}{3}} = 32$ (*vi*) $10^{-3} = 0{\cdot}001$ (*vii*) $2^{-2} = 0{\cdot}25$ (*viii*) $a^{-1} = \frac{1}{a}$

3. (*i*) 4 (*ii*) 3 (*iii*) $\frac{3}{2}$ (*iv*) $\frac{3}{2}$ (*v*) -2 (*vi*) -1 (*vii*) -8 (*viii*) -2

4. (*i*) 9 (*ii*) 5 (*iii*) 0·01 (*iv*) 2 (*v*) 11 (*vi*) 4 (*vii*) 729 (*viii*) 243

(*ix*) $\frac{1}{8}$ (*x*) 5 (*xi*) 10 (*xii*) 2 (*xiii*) $\frac{5}{2}$ (*xiv*) 2 (*xv*) ± 3 (*xvi*) $\frac{1}{10}$

(*xvii*) $6\frac{1}{2}$ (*xviii*) $\frac{1}{100}$, $1, \sqrt[3]{10}$

5. $\frac{a^2}{1000}$ **6.** (*i*) $\frac{x^2}{1000}$ (*ii*) $\frac{y^3}{10}$ (*iii*) $\frac{x^2\sqrt{y}}{z^3}$ **7.** 10^{a+b}

8. 10^{3m-2n} **9.** (*i*) $\sqrt{x}$ (*ii*) $10y^4$ (*iii*) $\frac{x^{\frac{3}{2}}}{y^4}$ **10.** y^3z^2 **11.** $\frac{x^4y^4}{100}$

EXERCISE 8.2

1. (*i*) log *a* (*ii*) $\frac{3}{2}$ (*iii*) 2 (*iv*) log 4 (*v*) 6 (*vi*) $\frac{1}{3}$

2. (*i*) $\frac{2}{3}$ (*ii*) $\frac{1}{2}$ (*iii*) 2 (*iv*) 1 (*v*) 2 (*vi*) 0 (*vii*) 2

3. (*i*) log 27 (*ii*) $\log_{10}8000$ (*iii*) log 256 (*iv*) $\log \frac{50}{9}$ (*v*) $\log \frac{9}{100}$

5. $1 - 4a + 2b - 3c$ **6.** (*i*) 10^a (*ii*) $\frac{2}{5}a$ (*iii*) $a + \log_{10} 3$ **7.** (*i*) 0 (*ii*) 1

8. (*i*) 0 (*ii*) 1 **9.** (*i*) 1 (*ii*) 7 **10.** $\frac{4}{3}\pi r^3$ **11.** 3 **12.** $\frac{100}{x^2}$

13. $\log_{10}20$ **14.** $\frac{\sqrt{x}}{\sqrt[3]{y}}$ **15.** $3x + y$ **16.** $1 - m + 3n$ **17.** 1

18. (*i*) $\frac{9}{5}$ (*ii*) 6 (*iii*) $\frac{3}{2}$ (*iv*) 1000 **19.** (*i*) 5 (*ii*) 1 **20.** $x = \frac{1}{25}, y = \frac{1}{2}$

22. (*i*) 2 (*ii*) 5 (*iii*) 4·5 (*iv*) 3 (*v*) 1 (*vi*) 14 (*vii*) 2

23. $1\frac{1}{40}$ **24.** 10 **27.** 3

30. $\frac{1}{\alpha + \beta + \gamma}$

Hint. $\frac{1}{\alpha} = \log_a x = \frac{\log x}{\log a} \Rightarrow \log a = \alpha \log x$ etc. and

$\log_{abc} x = \frac{\log x}{\log abc} = \frac{\log x}{\log a + \log b + \log c}$

31. (*i*) 3 (*ii*) 2

MULTIPLE CHOICE QUESTIONS

1. (*c*) **2.** (*d*) **3.** (*b*) **4.** (*a*) **5.** (*c*) **6.** (*d*) **7.** (*a*)

8. (*b*)

ASSERTION-REASON TYPE QUESTIONS

1. (*c*) **2.** (*c*) **3.** (*c*) **4.** (*c*)

CHAPTER TEST

1. $\frac{7}{3}\log_a x + \frac{8}{3}\log_a y - \frac{1}{12}\log_a z$ **2.** 5

7. (*i*) 7 (*ii*) $\sqrt{2}$ (*iii*) $\frac{1}{\sqrt{3}}$ (*iv*) $6\frac{1}{2}$ (*v*) 5, −5 (*vi*) 11, −11 (*vii*) 3, −4

(*viii*) 3, −4 (*ix*) 10

8. $x = 1000, y = 100$

Hint. $\frac{\log x}{3} = \frac{\log y}{2} \Rightarrow 2\log x - 3\log y = 0$

$\log(xy) = 5 \Rightarrow \log x + \log y = 5$ ...(*i*)

Multiplying (*ii*) by 3 and adding to (*i*), we get ...(*ii*)

$5\log x = 15 \Rightarrow \log x = 3 \Rightarrow x = 10^3 = 1000$

9. **Hint.** $a = 1 + \log_x yz = \log_x x + \log_x yz = \log_x xyz \Rightarrow \frac{1}{a} = \log_{xyz} x$ etc.

$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \log_{xyz} x + \log_{xyz} y + \log_{xyz} z = 1$

EXERCISE 9.1

1. No; because BC and QR are not corresponding sides. In fact, BC = PQ
2. No; angles must be included angles.
5. **Hint.** $\Delta ABD \cong \Delta BAC$ (SSS)
7. **Hint.** As AB || DC, $\angle ABD = \angle CDB$ (alt. $\angle$s)
   Show that $\Delta ABD \cong \Delta CDB$
8. **Hint.** Join DE
   Show that $\Delta ABC \cong \Delta ADE$
9. **Hint.** $AB = CD \Rightarrow AB + BC = CD + BC$
   $\Rightarrow$ $AC = BD$
13. $x = 15, y = 41$

EXERCISE 9.2

1. AB = QR, then $\Delta ABC \cong \Delta QRP$ by ASA congruence rule.
2. BC = RP, then $\Delta ABC \cong \Delta QRP$ by AAS congruence rule.
3. No; sides must be corresponding sides.
4. **Hint.** $\Delta BMD \cong \Delta CND$ by AAS
5. **Hint.** $\Delta BEM \cong \Delta DEN$ by AAS
7. **Hint.** $\Delta OBC \cong \Delta OAD$ by ASA
11. **Hint.** $BF = EC \Rightarrow BF + FC = EC + FC$
    $\Rightarrow$ $BC = FE$
    $\Delta ABC \cong \Delta DEF$ (RHS)
12. **Hint.** $\Delta ABX \cong \Delta BAY$ (RHS)
13. **Hint.** (*b*) Draw $AE \perp CD$, $BF \perp CD$
    AE = BF (distance between parallel lines).
    (*c*) In ΔBEG and ΔDCF, $\angle B = \angle D$ ($\because$ BA || DF, corres. $\angle$s are equal),
    $\angle E = \angle C$ and $BE = BC - EC = BC - BD = DC$
    $\Rightarrow \Delta BEG \cong \Delta DCF$
14. (*i*) $x = 13, y = 11$ (*ii*) $x = 8, y = 4$

EXERCISE 9.3

1. $\angle B = 45°, \angle C = 45°$
4. (*i*) 115 (*ii*) 68 (*iii*) 72°
5. (*i*) 42 (*ii*) 40 (*iii*) 93
6. (*a*) 95° (*b*) 74° (*c*) $x = 127°, y = 38°$

Hint. (*a*) Join BD

(*b*) $\angle ACD = 22°$ (why?), $\angle BCD + (90° - \frac{1}{2}\angle BCD) = 116°$ (why?)

7. $100°$

8. (*a*) $\angle ACE = 120°$, $\angle AEC = 30°$ (*c*) $x = 42°$, $y = 66°$, $z = 48°$

9. **Hint.** $\Delta BED \cong \Delta CFD$ (RHS)

$\Rightarrow$ $\angle B = \angle C \Rightarrow AC = AB$

13. $21°$

14. **Hint.** $AD = AE \Rightarrow \angle ADE = \angle AED$

$\Rightarrow$ $180° - \angle ADE = 180° - \angle AED$

$\Rightarrow$ $\angle ADB = \angle AEC$.

$\Delta ABD \cong \Delta ACE$ (SAS)

15. **Hint.** (*a*) $\angle ADE = \angle ADC + \angle CDE = 90° + 60° = 150°$. Similarly, $\angle BCE = 150°$

$\Rightarrow$ $\angle ADE = \angle BCE$. Also AD = BC and DE = EC

$\therefore$ $\Delta ADE \cong \Delta BCE$ (SAS) $\Rightarrow$ AE = BE (c.p.c.t.)

(*b*) $\angle OAD = \angle DAB - \angle OAB = 90° - 60° = 30°$. Similarly, $\angle OBC = 30°$

$\Rightarrow$ $\angle OAD = \angle OBC$. Also AD = BC and OA = OB

$\therefore$ $\Delta OAD \cong \Delta OBC$ (SAS) $\Rightarrow$ OD = OC (c.p.c.t.)

$\Rightarrow$ ΔOCD is an isosceles triangle.

(*c*) In ΔABC, $\angle A = 90°$ and AB = AC

$\Rightarrow$ $\angle B = \angle C$ but $\angle A + \angle B + \angle C = 180°$

$\Rightarrow$ $\angle B = 45°$.

As AD is bisector of $\angle A$, $\angle BAD = \angle CAD = 45°$

$\Delta ABD \cong \Delta ACD$ (SAS) $\Rightarrow$ BD = CD (c.p.c.t.)

In ΔABD, $\angle BAD = \angle ABD$ (each $= 45°$) $\Rightarrow$ AD = BD

$\therefore$ BC = BD + CD = BD + BD = 2BD = 2AD

EXERCISE 9.4

1. PR; because $\angle Q = 80°$ and the side opposite the greatest angle is longest 3. $\angle P$

4. (*i*) $\angle C$ (*ii*) $\angle A$ 5. BC, CA, AB 7. AB, DC, BD

8. **Hint.** (*b*) In Δ ACD, $\angle CAD = 30°$, AC > CD but AB = AC

10. (*i*) No; sum of the lengths of two sides = length of third side.

(*ii*) No; sum of the lengths of two sides is less than the length of third side.

(*iii*) Yes; because in this case the sum of lengths of two sides is greater than the length of third side.

MULTIPLE CHOICE QUESTIONS

1. (*c*)	2. (*a*)	3. (*b*)	4. (*c*)	5. (*c*)	6. (*d*)	7. (*b*)
8. (*b*)	9. (*c*)	10. (*a*)	11. (*a*)	12. (*d*)	13. (*d*)	14. (*c*)
15. (*b*)	16. (*d*)	17. (*a*)	18. (*b*)	19. (*d*)		

ASSERTION-REASON TYPE QUESTIONS

1. (*c*) 2. (*c*) 3. (*c*) 4. (*c*)

CHAPTER TEST

1. Two triangles need not be congruent, because AB and EF are not corresponding sides in two triangles.

3. **Hint.** $\Delta AOB \cong \Delta DOC$ (SAS)

5. **Hint.** $\angle DBC = \angle DCB \Rightarrow CD = BD$

$\Delta ABD \cong \Delta ACD$ by SSS rule of congruency.

8. **Hint.** $\angle BCE = \angle BCA + 90°$,

$\angle ACF = \angle BCA + 90°$

$\Rightarrow \angle BCE = \angle ACF$.

$\Delta BCE \cong \Delta FCA$ (SAS)

9. **Hint.** $\angle 1 = \angle 4$ (vert. opp. $\angle$s)

$\Rightarrow 2\angle 2 = 2\angle 3 \Rightarrow \angle 2 = \angle 3$

$TS = TR \Rightarrow \angle TRS = \angle TSR$

$\Rightarrow \angle TRS - \angle 2 = \angle TSR - \angle 3$

$\Rightarrow \angle ARB = \angle BSA$.

$\Delta RBT \cong \Delta SAT$ (ASA)

10. (*a*) 108° (*b*) (*i*) 34 (*ii*) 70 (*iii*) 44° (*c*) $x = 46°, y = 26°, z = 72°$

13. **Hint.** Take a point S on PR such that PS = PQ.

Join A and S. Mark angles as shown.

$\Delta APQ \cong \Delta APS$ (SAS)

$\Rightarrow \angle 1 = \angle 2$ (c.p.c.t.)

But $\angle 2 > \angle 3$ (ext. $\angle$ is greater than each int. opp. $\angle$)

$\Rightarrow \angle 1 > \angle 3$ ($\because \angle 1 = \angle 2$)

$\Rightarrow AR > AQ$

14. **Hint.** In ΔOBC, $OB + OC > BC$ ...(*i*)

($\because$ sum of any two sides of a triangle > third side)

Similarly $OC + OA > CA$ ...(*ii*)

and $OA + OB > AB$ ...(*iii*)

On adding (*i*), (*ii*) and (*iii*), we get

$2(OA + OB + OC) > AB + BC + CA$

EXERCISE 10

1. (*a*) (*i*) 13 cm (*ii*) 8·2 cm (*b*) (*i*) 2·8 cm (*ii*) 72° (*c*) 5·2

7. **Hint.** In ΔBCE, D is mid-point of BC and DF $\parallel$ BE, therefore, F is mi

(converse of mid-point theorem)

$\Rightarrow CF = \frac{1}{2} CE = \frac{1}{2} (\frac{1}{2} AC)$

9. **Hint.** $\Delta ABD \cong \Delta ACD$

10. (*c*) (*i*) 5 cm (*ii*) 10 cm

11. **Hint.** (*a*) PQ $\parallel$ AD and $PQ = \frac{1}{2} AD$, SR $\parallel$ AD and $SR = \frac{1}{2} AD$

(*b*) Diagonals of a kite intersect at right angles.

12. (*i*) 3 cm (*ii*) 4·6 cm (*iii*) 2·4 cm (*iv*) 2·2 cm

MULTIPLE CHOICE QUESTIONS

1. (*c*) **2.** (*b*) **3.** (*d*) **4.** (*c*) **5.** (*c*) **6.** (*d*) **7.** (*a*)

ASSERTION-REASON TYPE QUESTIONS

1. (*c*) **2.** (*a*) **3.** (*c*)

CHAPTER TEST

4. 17 cm

EXERCISE 11

1. (*i*) No (*ii*) Yes; 13 cm (*iii*) Yes; 5 cm **2.** 8 m **3.** $6\sqrt{7}$ m

4. 13 m **5.** 16 cm, 12 cm **7.** 8 km **8.** 10 m, 24 m, 26 m

12. $8\sqrt{2}$ cm; $32\sqrt{2}$ cm^2 **13.** 50 cm^2; $20\sqrt{2}$ cm

14. (*a*) 4 cm (*b*) $5\sqrt{2}$ cm; 25 cm^2

15. (*a*) 13 cm (*b*) 17 cm (*c*) $3\frac{1}{6}$ cm **16.** (*a*) 4 cm, 8 cm, 20 cm, $5\sqrt{17}$ cm

17. 13 cm **18.** 100 cm

19. (*a*) 12 cm (*b*) 26 cm (*c*) (*ii*) 24 cm^2; $14\sqrt{2}$ cm

Hint. (*c*) (*i*) $\angle AEH = 45°$, $\angle BEF = 45°$

22. Hint. From part (*iii*) of example 22, $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$

Similarly, $BC^2 + CA^2 = 2BE^2 + \frac{1}{2}CA^2$, $CA^2 + AB^2 = 2CF^2 + \frac{1}{2}AB^2$. Add all these.

MULTIPLE CHOICE QUESTIONS

1. (*b*) **2.** (*a*) **3.** (*b*) **4.** (*b*) **5.** (*c*) **6.** (*b*) **7.** (*c*)

8. (*d*)

ASSERTION-REASON TYPE QUESTIONS

1. (*c*) **2.** (*a*) **3.** (*c*)

CHAPTER TEST

1. (*a*) 28 cm (*b*) (*i*) 10 cm (*ii*) 24 cm (*iii*) 96 cm^2 (*c*) (*i*) 12 cm (*ii*) 13·5 cm^2

4. Hint. RT = TS = SQ = x (say)

6. $\frac{bc}{\sqrt{b^2 + c^2}}$ **7.** $36\sqrt{2}$ cm

EXERCISE 12.1

1. 90°, 120° **3.** 72°, 108°, 72°, 108°

4. (*a*) $\angle CDB = 30°$, $\angle ADB = 80°$

(*b*) $\angle OAD = 35°$, $\angle AOD = 68°$, $\angle ADO = 77°$ (*c*) 54

5. (*a*) $x = 4$, $y = 5$ (*b*) $x = 30°$, $y = 95°$ (*c*) $x = 6$, $y = 21$ **6.** 80°

7. (*a*) 45° (*b*) $\angle OAB = \angle OBA = 34°$, $\angle AOB = 112°$

(*c*) $\angle OAD = 54°$, $\angle ADO = 36°$, $\angle AOD = 90°$

8. (*a*) $x = 50°$, $y = 88°$ (*b*) $x = 36°$, $y = 108°$ (*c*) $\angle ODC = 58°$, $\angle OBA = 34°$

13. Hint. Show that $\Delta OAP \cong \Delta OCQ$

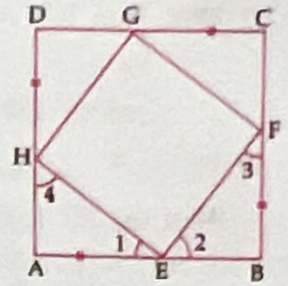
15. Hint. $\Delta ABP \cong \Delta DAQ \Rightarrow \angle BAP = \angle ADQ$. But $\angle BAD = 90°$

$\Rightarrow \angle PAD + \angle ADQ = 90°$

16. Hint. $\Delta ABP \cong \Delta CDQ$

21. Hint. Given $AE = BF = CG = DH$

$\Rightarrow$ $EB = FC = GD = HA$



In Δs AEH and BFE,

$AE = BF$, $AH = EB$,

$\angle A = \angle B$ (each $\angle = 90°$)

$\therefore$ $\Delta AEH \cong \Delta BFE$

$\Rightarrow$ $EH = EF$ and $\angle 4 = \angle 2$

But $\angle 1 + \angle 4 = 90° \Rightarrow \angle 1 + \angle 2 = 90°$

$\Rightarrow$ $\angle HEF = 90°$ ($\because \angle 4 = \angle 2$)

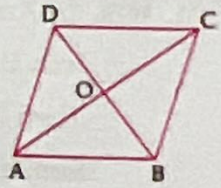
22. Hint. (*a*) $AB \parallel DC$, $AB \parallel FE \Rightarrow DC \parallel FE$, $AB = DC$, $AB = FE \Rightarrow DC = FE$

(*b*) $\Delta AFG \cong \Delta BCA$, for, $AF = BC$, $AG = AB$,

$\angle FAG = 360° - 90° - 90° - \angle A = 180° - \angle A = \angle B$

23. $\sqrt{3} : 1$

Hint. As $\angle A = 60°$, ABD is an equilateral triangle.



Let $AB = a$, then $BD = a \Rightarrow OB = \frac{a}{2}$. In ΔAOB, $\angle AOB = 90°$.

By Pythagoras theorem, $AO^2 = AB^2 - OB^2 = a^2 - \left(\frac{1}{2}a\right)^2 = \frac{3}{4}a^2$

$\Rightarrow$ $AO = \frac{\sqrt{3}}{2}a \Rightarrow AC = \sqrt{3}\,a$

EXERCISE 12.2

1. 63°

6. Hint. Draw BC = 6·2 cm. At B, construct $\angle CBP = 60°$. From BP, cut off AB = 5 cm
Through A, draw a line parallel to BC

9. 7·2 cm **10.** 5·6 cm **11.** 6·1 cm **12.** 3·1 cm **13.** 61° **17.** 3·5 cm

18. 106° **19.** 78°

23. Hint. Draw a line PQ parallel to AB at a distance = 2·6 cm

MULTIPLE CHOICE QUESTIONS

1. (*d*) **2.** (*d*) **3.** (*c*) **4.** (*b*) **5.** (*c*) **6.** (*c*) **7.** (*d*)

8. (*a*) **9.** (*c*) **10.** (*d*) **11.** (*c*) **12.** (*d*) **13.** (*d*)

ASSERTION-REASON TYPE QUESTIONS

1. (*c*) **2.** (*a*) **3.** (*c*)

CHAPTER TEST

4. (*i*) $x = 29°$ (*ii*) $x = 39°$, $y = 111°$ (*iii*) $x = 64°$, $y = 96°$

5. (*i*) $x = 37°$, $y = 106°$, $z = 37°$ (*ii*) $x = 110°$ (*iii*) $x = 70°$, $y = 120°$, $z = 85°$

6. (*i*) 17° (*ii*) 73° (*iii*) 45° (*iv*) 73°

7. 8 cm, $8\sqrt{3}$ cm

EXERCISE 13

3. Hint. (*b*) (*i*) Area of ΔDEC = area of ΔDEB, add area of ΔADE to both sides.

4. Hint. (*b*) (*i*) Through O, draw a straight line parallel to AB.

5. Hint. Join HF. $AH = \frac{1}{2}AD$ and $BF = \frac{1}{2}BC \Rightarrow AH = BF$ and $AH \parallel BF$, so ABFH is a $\parallel$gm.

$\therefore$ Area of $\Delta EFH = \frac{1}{2}$ area of $\parallel$ gm ABFH

9. (*b*) 5 : 9 (*c*) (*i*) 2 : 1 (*ii*) 1 : 6

Hint. (*b*) $PC = 2BP \Rightarrow PC = \frac{2}{3}BC \Rightarrow$ area of $\Delta APC = \frac{2}{3}$ area of ΔABC. $QA = 5PQ$

$\Rightarrow AQ = \frac{5}{6}AP \Rightarrow$ area of $\Delta AQC = \frac{5}{6}$ area of ΔAPC

$= \frac{5}{6}.\frac{2}{3}$ area of $\Delta ABC = \frac{5}{9}$ area of ΔABC.

(*c*) AD is median of $\Delta ABC \Rightarrow$ area of ABD = area of ΔADC

Given area of ΔADP : area of $\Delta ABD = 2 : 3$

$\Rightarrow$ area of ΔADP : area of $\Delta ADC = 2 : 3 \Rightarrow AP : AC = 2 : 3$

$\Rightarrow$ $AP : PC = 2 : 1$

10. (*a*) 5 cm (*b*) 6 units (*c*) (*i*) 18 cm^2 (*ii*) AEFD

11. Hint. (*b*) Area of ΔACE = area of ΔACD

(*c*) Join BD. Let diagonals AC and BD of $\parallel$ gm ABCD meet at P.

Then AP is median of $\Delta ABD \Rightarrow$ area of ΔABP = area of ΔADP.

Similarly, area of ΔPBO = area of ΔPDO

12. Hint. (*a*) Join BG. Area of $\Delta ABG = \frac{1}{2}$(area of $\parallel$ gm ABCD);

area of $\Delta ABG = \frac{1}{2}$(area of $\parallel$ gm AEFG)

(*b*) Join AC and EF. Area of ΔCAF = area of ΔEAF

14. Hint. $\Delta APD \cong \Delta PQB$

15. (*a*) AB = 12 cm, BC = 9 cm (*b*) BC = 12 cm, CA = 10 cm, AB = 15 cm

(*c*) (*i*) 80 cm^2 (*ii*) 5 : 3

Hint. (*a*) Let $AB = p$, then $BC = \frac{1}{2}.42 - p$. Also $6.p = 8.(21 - p)$

(*b*) Let $BC = p$, $CA = q$, then $AB = 37 - p - q$

Also $\frac{1}{2}p.5x = \frac{1}{2}q.6x = \frac{1}{2}(37 - p - q).4x$

16. 25 sq. units

Hint. Let EF meet AC at G. As E is mid-point of AB and EF $\parallel$ BC, G is mid-point of AC *i.e.* AG = GC

Also $\angle EAG = \angle GCF$ and $\angle EGA = \angle CGF$

$\Rightarrow \Delta AEG \cong \Delta CFG$

17. Hint. (*a*) Join CE. Area of ΔABC = area of ΔEBC. Area of ΔBCD = area of ΔECD.

(*b*) (*ii*) Area of $\Delta BCF = \frac{1}{2}$ area of square ABFG.

MULTIPLE CHOICE QUESTIONS

1. (*a*) 2. (*b*) 3. (*b*) 4. (*a*) 5. (*c*) 6. (*a*) 7. (*b*)
8. (*c*) 9. (*a*)

ASSERTION-REASON TYPE QUESTIONS

1. (*c*) 2. (*c*) 3. (*b*)

CHAPTER TEST

1. (*a*) (*i*) 24 cm^2 (*ii*) 12 cm^2

3. **Hint.** As Δs LZX and MZX are on the same base ZX and between same parallels XZ and LM, area of ΔLZX = area of ΔMZX.

   Add same area of ΔXZY on both sides.

5. **Hint.** Let ABCD be a quadrilateral such that each diagonal divide it into triangles of equal areas, then area of $\Delta ABC = \frac{1}{2}$ (area of quad. ABCD) and area of $\Delta ABD = \frac{1}{2}$ (area of quad. ABCD). So, area of Δ ABC = area of ΔABD. By Theorem 13.6, DC $\|$ AB.

6. 12 cm^2

   Hint. Join AC and DE. ACED is a parallelogram (why?). Diagonals AE and DC of $\|$gm ACED bisect each other, so F is mid-point of DC.

   Area of ΔBFC = area of ΔDBF = 3 cm^2 (given)

7. **Hint.** Show that $\Delta ABE \cong \Delta ADF$

   $\Rightarrow$ AE = AF

   Now, show that $\Delta AER \cong \Delta AFR$

8. **Hint.** Since X and Y are mid-points of AC and AB, YX $\|$ BC. But QP $\|$ BC, so YX $\|$ QP. In ΔBAP, Y is mid-point of AB and YX $\|$ AP, so X is mid-point of BP and $YX = \frac{1}{2}$ AP

   Similarly, Y is mid-point of CQ and $YX = \frac{1}{2}$ QA $\Rightarrow$ AP = QA

   Thus, ABP and ACQ are two triangles on equal bases AP, QA and are between same parallels BC and QP, therefore,

   area of ΔABP = area of ΔACQ

EXERCISE 14.1

1. 10 cm 2. 7 cm 3. 5 cm 4. 16 cm 5. 12 cm

6. (*i*) 1 cm (*ii*) 7 cm 7. (*a*) (*i*) 13 cm (*ii*) 10 cm (*b*) $4\frac{1}{6}$ cm

8. 21 cm 9. $2\sqrt{29}$ cm 10. 15 cm 11. $2\sqrt{3}$ cm
12. 24 cm 13. 12 cm^2 14. 28 cm

17. **Hint.** Prove that medians are perpendicular bisectors of the sides of triangle.

18. **Hint.** (*a*) OD becomes the line joining the mid-points of the sides BC and AB of ΔABC

    (*b*) Prove that $\Delta MPB \cong \Delta NQC$

19. **Hint.** (*a*) Draw OM $\perp l$

    (*b*) Draw OM $\perp$ AB and ON $\perp$ CD. $\Delta OME \cong \Delta ONE$ (A.A.S. rule of congruency)
    $\Rightarrow$ OM = ON

20. **Hint.** (*a*) raw OM ⊥ AB and ON ⊥ CD, $\Delta OAM \cong \Delta ODN$

(*b*) Draw OM ⊥ AB and ON ⊥ CD, $\Delta OME \cong \Delta ONE$

EXERCISE 14.2

1. 1 : 1

4. **Hint.** Minor arc AB = minor arc CD. Subtract minor arc BD from both sides.

MULTIPLE CHOICE QUESTIONS

1. (*c*) 2. (*d*) 3. (*a*) 4. (*d*) 5. (*c*) 6. (*a*) 7. (*a*)

ASSERTION-REASON TYPE QUESTIONS

1. (*c*) 2. (*c*) 3. (*c*)

CHAPTER TEST

1. (*i*) 24 cm (*ii*) $12\sqrt{5}$ cm (*iii*) $6\sqrt{5}$ cm 2. 9 cm 3. 24 cm

4. (*a*) 6.65 cm

Hint. (*b*) From C, D draw CM, DN perpendiculars to AB. Then MCDN is a rectangle, so

$$MN = CD.\ MN = MP + PN = \frac{1}{2}AP + \frac{1}{2}PB = \frac{1}{2}AB$$

6. **Hint.** As the chords AB and AC are equidistant from the centre, so AB = AC

Since angle in a semicircle is 90°,

$\angle B = \angle C$ (each = 90°)

$\Delta ABD \cong \Delta ACD$ (Why?

EXERCISE 15.1

1. 12 cm^2 2. (*i*) 6 cm^2 (*ii*) 210 cm^2 (*iii*) 34·56 cm^2

3. 336 cm^2; 33·6 cm 4. ₹ 4,72,50,000 5. 30 cm^2, 30 cm 6. 27·71 m^2

7. 54 cm 8. 62·4 cm^2; 10·4 cm 9. (*i*) 96 cm^2 (*ii*) 2598 m^2

10. 8 cm^2; 2·82 cm

Hint. By Pythagoras theorem, $BC^2 = AB^2 + AC^2 = 4^2 + 4^2 = 32 \Rightarrow BC = 4\sqrt{2}$ cm

11. 34·86 cm^2 12. 12 cm^2 13. 17 cm

14. 40 cm 15. 150 cm^2 16. Base = 16 cm, each of other sides = 12 cm

17. 24 cm or 10 cm 18. 1200 m, 400 m 19. $375\sqrt{15}$ m^2; ₹49400

20. $1000\sqrt{6}$ cm^2; $1000\sqrt{6}$ cm^2

21. (*a*) 19·3 cm^2 (*b*) 12·5 cm^2

Hint. (*b*) Area of $\Delta ADE = \frac{1}{2} \times 3 \times 3\ cm^2 = \frac{9}{2}\ cm^2$

ΔDBG is an isosceles triangle with BG = DG = x cm (say)

From ΔDBG, by Pythagoras theorem, $x^2 + x^2 = 4^2 \Rightarrow x^2 = 8$

Area of ΔDBG = area of $\Delta EFC = \frac{1}{2}x \times x\ cm^2 = \frac{1}{2} \times 8\ cm^2 = 4\ cm^2$

EXERCISE 15.2

1. (*i*) 240 cm^2 (*ii*) 108 cm^2 2. 306 m^2 3. 92·35 cm^2 4. 84 cm^2

5. 30 cm, 540 cm^2 6. 98 m 7. (*a*) 400 m^2 (*b*) 28 cm^2

8. 160 m^2 **9.** (*i*) 26 m, 10 m (*ii*) 100 m^2 **10.** 3 m

11. 8000 m^2 **12.** 960; $\frac{7}{20}$ **13.** $16x = 5y$; 60 m^2

14. $x^2 - 13x + 30 = 0$, $x = 2$ metres **15.** 75·6 m^2 **16.** 278 m^2

17. (*a*) 23 m, 17 m^2 (*b*) 50 m, 51 m^2 (*c*) 50 cm^2, 54 cm

18. 8 cm **19.** 22 m, 31·11 m **20.** 196 m^2, 19·80 m

21. (*i*) 81 cm^2 (*ii*) 80 cm^2 **22.** 120 cm^2, 12 cm

23. 119·8 cm^2, 11·98 cm

[Hint. Find area of ΔABC. Area of $\parallel$ gm ABCD = 2 area of ΔABC]

24. 18 cm^2 **[Hint.** Area of $\Delta AOD = \frac{1}{4}$ (area of parallelogram ABCD)]

25. 96 cm^2

Hint. If diagonals intersect at O, find area of ΔOAB.

Area of parallelogram ABCD = 4 (area of ΔOAB)

26. $\frac{pq}{p+qr}$ cm **27.** 96 cm^2 **28.** 14 cm **29.** 90 cm^2

30. 7·5 cm^2 **31.** 20 cm **32.** (*i*) 6 cm (*ii*) 24 cm^2

33. (*a*) 5 cm, 26 cm^2 (*b*) 8 units, 40 sq. units (*c*) 2·4 m

34. 216 cm^2

Hint. Let ABCD be the given trapezium in which AB $\parallel$ DC. Let E and F be mid-points of sides AD and BC respectively, then EF = 18 cm

Area of trap. ABCD = $\frac{1}{2}$(AB + DC) × height = EF × height

35. 35 cm, 25 cm **36.** 40 cm **37.** Base = 14 cm, altitude = 7 cm

38. 24 m, 12 m **39.** 240 m^2 **40.** 96 cm

41. 48 cm

Hint. Area of square = 144 cm^2

Let breadth of rectangle be x cm, then $x(x + 4) = 144 - 4$

42. $(130 + 32\sqrt{3})$ cm^2, $4(8 + 5\sqrt{2})$ cm

Hint. Let AE = EB = x cm, then from right angled ΔAEB,

$x^2 + x^2 = 10^2 \Rightarrow x^2 = 50 \Rightarrow x = 5\sqrt{2}$

Area of ΔAEB = area of $\Delta DCG = \frac{1}{2} x \times x$ cm^2 = 25 cm^2

Area of ΔBFC = area of $\Delta HAD = \frac{\sqrt{3}}{4} \times 8^2$ cm^2 = $16\sqrt{3}$ cm^2

Area of shaded region = area of rect. ABCD + 2 × area of ΔAEB + 2 × area of ΔBFC.

Perimeter of the figure = AE + EB + BF + FC + CG + GD + DH + HA

= 4 AE + 4 BF = $(4 \times 5\sqrt{2} + 4 \times 8)$ cm

43. (*a*) 51·59 cm^2 (*b*) 32 cm^2 (*c*) 192 cm^2

44. (*a*) 4·5 m^2 (*b*) 1087·5 m^2 (*c*) 72 cm^2 **45.** 40 metres

46. 12 cm **47.** $\frac{105}{x}$ cm, $44 = 2\left(x + \frac{105}{x}\right)$; 15 cm, 7 cm

48. $(90 - x)$ m ; $x(90 - x) = 1800$; 60 m, 30 m

EXERCISE 15.3

1. 14 cm **2.** 9 cm; $254\frac{4}{7}$ cm^2 **3.** 19·8 cm **4.** 12 cm; $75\frac{3}{7}$ cm

5. 88 [**Hint.** First we have to cut squares of side 0·5 cm]

6. 36 cm 7. (*a*) (*i*) 28 cm (*ii*) 88 cm (*b*) 100·48 cm^2 ; 57·12 cm

8. 40 9. 48 km/h 10. (*i*) 19·6 cm (*ii*) 1207 cm^2 (*iii*) 4 times

11. (*a*) 14 cm (*b*) 17·5 cm 12. ₹38500 13. 26 cm, 18 cm

14. 44 cm 15. 154 cm^2 16. 346·5 cm^2 17. (*a*) 88 cm (*b*) 19·35 cm^2

18. 567 cm^2

19. (*a*) 42 cm^2; 72 cm (*b*) 42 cm^2 (*c*) 98 cm^2 20. (*a*) 231 cm^2; 80 cm (*b*) 42 cm^2

21. (*a*) 28·5 m^2 (*b*) 24·5 cm^2 22. 346·5 cm^2 ; 283·5 cm^2

23. 12 cm^2 24. (*a*) 72·75 cm^2 (*b*) 14 cm^2 25. 22050 m^2

26. (*i*) $1 : \sqrt{2}$ (*ii*) 1 : 2

Hint. Let a side of the square be $2a$ units, then radius of incircle = a units and radius of circumcircle = $\sqrt{2}\,a$ units (why?)

27. (*a*) (*i*) 17850 m^2 (*ii*) 664 m (*b*) $636\frac{4}{7}$ m^2

28. (*a*) 122·57 cm^2 (*b*) 42 cm^2 29. (*a*) 4032 m^2 (*b*) 228 cm^2

30. (*a*) 59·5 cm^2 (*b*) 283.97 cm^2 31. (*a*) 504 cm^2 (*b*) (*i*) 31·4 cm (*ii*) 39·25 cm^2

32. (*a*) 31·4 cm; 31·4 cm^2 (*b*) 9·72 cm^2

33. (*a*) 40·25 cm^2 (*b*) 1404 cm^2 34. (*a*) (*i*) 44 cm (*ii*) 86·625 cm^2 (*b*) 6·125 cm^2

35. (*a*) 428·75 cm^2 (*b*) 2·58 cm^2 36. (*i*) $\frac{132}{7}$ cm (*ii*) $\frac{198}{7}$ cm^2

37. 78·5 cm^2; 285·5 cm^2

Hint. Area of minor segment = area of sector OACB – area of ΔOAB

EXERCISE 15.4

1. 294 cm^2, 343 cm^3 2. 94 m^2; 60 m^3; 7·07 m 3. 14 cm

4. 40 ; 6 5. (*a*) 12 cm (*b*) 150 cm^2, 125 cm^3 6. 7

7. 70 cm 8. 486 cm^2, 15·57 cm 9. 9 cm

10. 504000 cm^3; 38200 cm^2 11. 120

12. 60 [**Hint.** Since height of box is 7 cm, only 3 cubes can be put height-wise]

13. 0·729 m^3; 0·9 m 14. 7·39 cm 15. 38 cm^3

16. 1440 cm^2 17. 3 cm 18. ₹12480

19. 13·95 cm 20. $\frac{8}{13}$ metre 21. ₹5400 22. ₹7800

23. 75 24. 6 25. 8·4 litres

26. (*a*) 80 cm^3 (*b*) (*i*) 16·5 m^2 (*ii*) 6600 m^3 (*c*) 1000 m^3 27. 2250 m^3

MULTIPLE CHOICE QUESTIONS

1. (*b*)	2. (*c*)	3. (*a*)	4. (*c*)	5. (*d*)	6. (*b*)	7. (*c*)
8. (*c*)	9. (*a*)	10. (*c*)	11. (*a*)	12. (*b*)	13. (*d*)	14. (*d*)
15. (*b*)	16. (*c*)	17. (*d*)	18. (*a*)	19. (*c*)	20. (*a*)	21. (*a*)
22. (*b*)	23. (*c*)	24. (*b*)	25. (*c*)			

ASSERTION-REASON TYPE QUESTIONS

1. (*c*) 2. (*b*) 3. (*c*)

CHAPTER TEST

1. (*a*) 54 cm^2 (*b*) 32 cm **2.** (*a*) 51 cm^2 (*b*) 144 cm^2 (*c*) 54 m^2

3. 19·3 cm^2 **4.** 31·4 cm **5.** 196 cm^2 **6.** (*a*) 300; 6450 cm^2 (*b*) 36 cm

7. ₹ 83160 **8.** 1480·5 m^2

9. 59·4 cm; 61·1 cm^2 [**Hint.** Angle in a semicircle is a right angle]

10. (*a*) 12·5 cm (*b*) (*i*) $\sqrt{10}$ cm (*ii*) 11·4 cm^2

Hint. (*b*) (*i*) radius of the circle = $\sqrt{3^2+1}$ cm = $\sqrt{10}$ cm

11. (*a*) 32·5 cm^2; 23 cm (*b*) 5 cm

13. 154 cm^2 **14.** 7 cm; 38·5 cm^2 **15.** 1008 cm^2 **16.** No

17. 378 km **18.** 75 cm **19.** (*i*) 8 cm (*ii*) 448 cm^3

21. 152000 cm^3; 16800 cm^2

Hint. Volume = $(50 \times 40 \times 12 + 50 \times 40 \times 40 + 50 \times 40 \times 24)\,cm^3$

Surface area = (area of front + area of back) + area of the vertical faces + area of top faces = $2(50 \times 12 + 50 \times 40 + 50 \times 24)\,cm^2$

$+ (12 \times 40 + 28 \times 40 + 16 \times 40 + 24 \times 40)\,cm^2 + 3\,(50 \times 40)\,cm^2$

22. (*i*) 564000 cm^3 (*ii*) 69864 cm^3 (*iii*) 55·9 kg

23. (*i*) 450 (*ii*) ₹54000 **24.** ₹756

EXERCISE 16

1. (*a*) (*i*) $\frac{3}{5}$ (*ii*) $\frac{4}{5}$ (*iii*) $\frac{3}{4}$ (*iv*) $\frac{4}{3}$ (*v*) $\frac{5}{4}$ (*vi*) $\frac{5}{3}$

(*b*) (*i*) $\frac{5}{13}$ (*ii*) $\frac{12}{13}$ (*iii*) 1 (*iv*) 1

2. (*a*) (*i*) $\frac{4}{5}$ (*ii*) $\frac{4}{5}$ (*iii*) $\frac{7}{5}$ (*iv*) 1 (*b*) (*i*) $\frac{5}{12}$ (*ii*) $\frac{4}{5}$ (*iii*) 1 (*iv*) 14

3. (*a*) $\frac{5}{4}$ (*b*) (*i*) $\frac{4}{5}$ (*ii*) $\frac{3}{4}$ (*iii*) −1 (*iv*) $\frac{3}{2\sqrt{2}}$ (*v*) −1

4. (*a*) (*i*) $\frac{2}{5}$ (*ii*) $\frac{2}{5}$ (*iii*) $\frac{6}{5}$ (*iv*) $\frac{14}{5}$ (*b*) AB = 10 cm, AC = 26 cm

5. (*i*) $\frac{12}{13}$ (*ii*) $\frac{1}{5}$ **6.** (*a*) (*i*) $\frac{7}{25}$ (*ii*) $\frac{24}{7}$ (*b*) (*i*) $\frac{9}{41}$ (*ii*) $\frac{40}{41}$ (*iii*) $\frac{40}{9}$

7. (*i*) $\frac{3}{5}$ (*ii*) $\frac{4}{5}$

Hint. Draw AD perpendicular to BC, then D is mid-point of BC, so BD = 9 cm. By Pythagoras theorem, AD = 12 cm.

8. (*a*) (*i*) $\frac{4}{5}$ (*ii*) $\frac{4}{3}$ (*iii*) $\frac{7}{12}$ (*b*) $4\frac{4}{5}$

(*c*) (*i*) AD = 20 cm, AB = 25 cm, DC = 20 cm, AC = $20\sqrt{2}$ cm

Hint. (*a*) Draw AD perpendicular to BC, then BD = DC = 3 cm and AD= 4 cm.

9. (*i*) $\frac{4}{5}$ (*ii*) $\frac{3}{4}$ **10.** $\frac{5}{13}; \frac{12}{13}$ **11.** $1\frac{11}{20}$ **12.** $1\frac{2}{5}$

13. $\frac{2(\sqrt{5}-1)}{\sqrt{5}}$ **14.** $\frac{p+\sqrt{q^2-p^2}}{q}$ **15.** $3\frac{31}{120}$ **16.** $\frac{12}{65}$ **17.** 8

18. $\frac{3}{5}$ **19.** $2\frac{3}{7}$ **20.** $\frac{17}{19}$ **21.** $\frac{p^2-q^2}{p^2+q^2}$ **22.** $\frac{1}{9}$

23. (i) $\frac{17}{19}$ (ii) 3

Hint (i) $5\cos\theta - 12\sin\theta = 0 \Rightarrow 5\cos\theta = 12\sin\theta \Rightarrow \cot\theta = \frac{12}{5}$

(ii) $\cot^2\theta = \operatorname{cosec}^2\theta - 1 = \left(\frac{13}{12}\right)^2 - 1 = \frac{169}{144} - 1 = \frac{25}{144} \Rightarrow \cot\theta = \frac{5}{12}$

24. $\frac{1}{4}$ 25. $\frac{3}{2}$

28. (a) $\frac{5}{12}$ (b) (i) $\frac{12}{13}$ (ii) $\frac{5}{12}$ [**Hint** (b) $\angle CBD = \angle A$ and $\angle ABD = \angle C$]

29. $\frac{150}{\sqrt{13}}$ cm ; $103\frac{11}{13}$ cm^2

30. (a) (i) $\frac{5}{13}$ (ii) $\frac{4}{3}$ (b) $\frac{12}{\sin\theta}$ or $\frac{9}{\cos\theta}$

[**Hint** (b) CD = 5. Draw DE perpendicular to AB, BE = 5, EA = 9]

32. $\cot\theta$ 33. 2

34. **Hint** Square and add. Use $\sin^2\theta + \cos^2\theta = 1$

MULTIPLE CHOICE QUESTIONS

1. (b) 2. (c) 3. (a) 4. (d) 5. (a) 6. (b) 7. (b)
8. (a) 9. (b) 10. (a) 11. (a) 12. (b) 13. (c) 14. (b)

ASSERTION-REASON TYPE QUESTIONS

1. (c) 2. (a) 3. (c)

CHAPTER TEST

1. (a) $\sin A = \frac{2}{3}$, $\cos A = \frac{\sqrt{5}}{3}$, $\tan A = \frac{2}{\sqrt{5}}$, $\cot A = \frac{\sqrt{5}}{2}$, $\sec A = \frac{3}{\sqrt{5}}$, $\operatorname{cosec} A = \frac{3}{2}$; $\sin C = \frac{\sqrt{5}}{3}$, $\cos C = \frac{2}{3}$, $\tan C = \frac{\sqrt{5}}{2}$, $\cot C = \frac{2}{\sqrt{5}}$, $\sec C = \frac{3}{2}$, $\operatorname{cosec} C = \frac{3}{\sqrt{5}}$

(b) $x = 10\cot\theta$, $y = 10\operatorname{cosec}\theta$

2. (a) (i) $\frac{4}{5}$ (ii) $3\frac{2}{15}$ (b) (i) 3 (ii) $5\frac{1}{4}$ (iii) $-4\frac{3}{4}$ 3. $\frac{q^2 - 2p^2}{p\sqrt{q^2 - p^2}}$

4. (i) $\frac{3}{5}$ (ii) $\frac{4}{5}$ (iii) -1 6. $\frac{3}{5}$ 7. $9\frac{1}{9}$ 8. 55 cm

9. $12\frac{8}{15}$ m [**Hint** $\sin\alpha = \frac{3}{5} \Rightarrow \tan\alpha = \frac{3}{4}$. Also $\cos\beta = \frac{12}{13} \Rightarrow \tan\beta = \frac{5}{12}$]

EXERCISE 17.1

1. (i) $\frac{7}{4}$ (ii) 2 (iii) $\frac{3}{2}$ (iv) $\frac{1}{4}$ 2. (i) $\frac{1}{3}$ (ii) $\frac{3}{2}$ (iii) $\frac{25}{36}$

3. (i) 0 (ii) 3 (iii) $-5\frac{17}{20}$ 6. (i) $\frac{1}{\sqrt{3}}$ (ii) $\frac{1}{3}$ 8. $2 : \sqrt{3}$

9. **Hint** Take A = 30° and B = 60°

11. (*i*) 30° (*ii*) 10° (*iii*) 60°

12. 45°; $\frac{3}{2}$

13. (*i*) $\sqrt{3}$ (*ii*) 60 (*iii*) $\frac{1}{2}$ (*iv*) 2

14. (*i*) 20° (*ii*) 15°

15. 15°

16. (*i*) 60 (*ii*) 1 (*iii*) $-\frac{1}{2}$

17. (*i*) 45° (*ii*) 45°

18. (*i*) $\frac{1}{2}$ (*ii*) $\frac{1}{2}$ (*iii*) -1

19. $\frac{1}{2}$

20. 60°

21. A = 45°, B = 15°

22. 8 cm, $8\sqrt{3}$ cm

23. BC = $2\sqrt{3}$ cm, AB = $4\sqrt{3}$ cm

24. $12\sqrt{3}$ m

Hint. Let AB = d metres, then PC = d metres. From right-angled ΔPCQ,

$$\tan 30° = \frac{CQ}{PC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{13{\cdot}8 - 1{\cdot}8}{d}$$

25. $4(3 - \sqrt{3})$ m

Hint. In ΔABD, $\tan 45° = \frac{AD}{BD}$

$$\Rightarrow \quad 1 = \frac{AD}{BD} \Rightarrow BD = AD.$$

$$\text{In } \Delta ADC, \tan 60° = \frac{AD}{DC} \Rightarrow \sqrt{3} = \frac{AD}{DC} \Rightarrow DC = \frac{AD}{\sqrt{3}}$$

$$\text{But } BD + DC = BC = 8 \text{ m} \Rightarrow AD + \frac{AD}{\sqrt{3}} = 8$$

EXERCISE 17.2

1. (*i*) 1 (*ii*) 1 (*iii*) 1 **2.** (*i*) $\frac{1}{2}$ (*ii*) 2 (*iii*) 0 (*iv*) 1 (*v*) 1

3. (*i*) 0 (*ii*) 0 **4.** (*i*) 2 (*ii*) 3 **5.** (*i*) 5 (*ii*) 0

6. (*i*) cot 9° + sin 18° (*ii*) tan 41° + sec 3° **12.** (*i*) 0 (*ii*) 1 (*iii*) 0

15. (*i*) $\sin^2\theta$ (*ii*) 2 **17.** (*i*) 24° (*ii*) 36° (*iii*) 39° **18.** (*i*) 27° (*ii*) 55°

MULTIPLE CHOICE QUESTIONS

1. (*d*) **2.** (*b*) **3.** (*c*) **4.** (*b*) **5.** (*b*) **6.** (*a*) **7.** (*d*)

8. (*c*) **9.** (*d*) **10.** (*b*) **11.** (*d*) **12.** (*d*) **13.** (*a*) **14.** (*b*)

15. (*a*) **16.** (*a*)

ERTION-REASON TYPE QUESTIONS

3. (*b*) **6.** 10°, $22\frac{1}{2}°$

(*iv*) 2

11. (*i*) 30° (*ii*) 10° (*iii*) 60° 12. 45°; $\frac{3}{2}$

13. (*i*) $\sqrt{3}$ (*ii*) 60 (*iii*) $\frac{1}{2}$ (*iv*) 2 14. (*i*) 20° (*ii*) 15° 15. 15°

16. (*i*) 60 (*ii*) 1 (*iii*) $-\frac{1}{2}$ 17. (*i*) 45° (*ii*) 45°

18. (*i*) $\frac{1}{2}$ (*ii*) $\frac{1}{2}$ (*iii*) -1 19. $\frac{1}{2}$ 20. 60°

21. A = 45°, B = 15° 22. 8 cm, $8\sqrt{3}$ cm 23. BC = $2\sqrt{3}$ cm, AB = $4\sqrt{3}$ cm

24. $12\sqrt{3}$ m

Hint. Let AB = d metres, then PC = d metres. From right-angled ΔPCQ,

$$\tan 30° = \frac{CQ}{PC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{13.8 - 1.8}{d}$$

25. $4(3 - \sqrt{3})$ m

Hint. In ΔABD, $\tan 45° = \frac{AD}{BD}$

$$\Rightarrow \quad 1 = \frac{AD}{BD} \Rightarrow BD = AD.$$

$$\text{In } \Delta ADC, \tan 60° = \frac{AD}{DC} \Rightarrow \sqrt{3} = \frac{AD}{DC} \Rightarrow DC = \frac{AD}{\sqrt{3}}$$

$$\text{But } BD + DC = BC = 8 \text{ m} \Rightarrow AD + \frac{AD}{\sqrt{3}} = 8$$

EXERCISE 17.2

1. (*i*) 1 (*ii*) 1 (*iii*) 1 2. (*i*) $\frac{1}{2}$ (*ii*) 2 (*iii*) 0 (*iv*) 1 (*v*) 1

3. (*i*) 0 (*ii*) 0 4. (*i*) 2 (*ii*) 3 5. (*i*) 5 (*ii*) 0

6. (*i*) cot 9° + sin 18° (*ii*) tan 41° + sec 3° 12. (*i*) 0 (*ii*) 1 (*iii*) 0

15. (*i*) $\sin^2\theta$ (*ii*) 2 17. (*i*) 24° (*ii*) 36° (*iii*) 39° 18. (*i*) 27° (*ii*) 55°

MULTIPLE CHOICE QUESTIONS

1. (*d*) 2. (*b*) 3. (*c*) 4. (*b*) 5. (*b*) 6. (*a*) 7. (*d*)
8. (*c*) 9. (*d*) 10. (*b*) 11. (*d*) 12. (*d*) 13. (*a*) 14. (*b*)
15. (*a*) 16. (*a*)

ASSERTION-REASON TYPE QUESTIONS

1. (*c*) 2. (*a*) 3. (*b*)

CHAPTER TEST

1. (*i*) $1\frac{1}{4}$ (*ii*) 1 (*iii*) $3\frac{1}{2}$ 5. $\frac{4}{3}$ 6. $10°, 22\frac{1}{2}°$

7. A = $52\frac{1}{2}°$, B = $7\frac{1}{2}°$ 8. (*i*) 0 (*ii*) 4 (*iii*) 1 (*iv*) 2

10. (*i*) 18° (*ii*) 15°

Hint. (*i*) $\sin 3A = \cos 2A \Rightarrow \sin 3A = \sin(90° - 2A) \Rightarrow 3A = 90° - 2A$

11. (*i*) 27° (*ii*) 22°

12. $6\sqrt{3}$ cm

Hint. In ΔABC, $\tan 30° = \frac{BC}{AB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{2\sqrt{3}\text{ cm}}{AB}$

$\Rightarrow AB = 6$ cm.

For ΔABE, $90° = 30° + \angle ABE$ ($\because$ ext $\angle$ = sum of two opp. int. $\angle$s)

$\Rightarrow \quad \angle ABE = 60°$.

In ΔABD, $\tan 60° = \frac{AD}{AB} \Rightarrow \sqrt{3} = \frac{AD}{6\text{ cm}}$

EXERCISE 18.1

1. (*i*) (3, −4) (*ii*) $\left(-\frac{3}{2}, 5\right)$ (*iii*) $\left(-1\frac{2}{3}, -2\frac{1}{4}\right)$ (*iv*) (− 2, 5)

(*v*) (−2, 0) (*vi*) $\left(0, \frac{3}{2}\right)$

2. II, IV, *x*-axis, I, III respectively 3. (*i*) C, F, G (*ii*) A, D, E

5. A(2, 2), B(−3, 0), C(−2, −4), D(3, −1), E(−4, 4), F(0, −2), G(2, −3), H(0, 3)

6. A lies in the first quadrant, B lies on *x*-axis, C lies in the third quadrant and D in the fourth quadrant

9. (*i*) collinear (*ii*) non-collinear (*iii*) collinear

10. M (−3, 0) and N (0, 4)

11. Rectangle; 15 sq. units

12. Rhombus; 12 sq. units

13. (2, −2); 25 square units

14. (0, 0), (6, 0), (6, 4), (0, 4)

15. A(−3, 0), B(3, 0), C(3, 3), D(−3, 3)

16. O(0, 0), A(a, $\sqrt{3}\,a$), B($2a$, 0)

17. $(2\sqrt{3}, 0)$

EXERCISE 18.2

3. 15 sq. units 4. 6 sq. units 5. (*i*) −7 (*ii*) 3 6. 2 7. $a = 0$, $b = 1$

EXERCISE 18.3

1. $x = -2, y = -5$ 2. $x = -4, y = -2$ 3. $x = 2, y = -2$ 4. $x = -1, y = 0$

5. $x = 2, y = 2$ 6. $x = 2, y = 1$ 7. (*ii*) (−2, −1) (*iii*) 4·5 cm

8. $x = 2, y = -1$ [**Hint.** $2x - 3y + 2 = 4x + 1$, $4x + 1 = 3x - y + 2$]

9. (*ii*) (2, 4) ; $6\frac{2}{3}$ sq. units

10. $x = 1, y = -2$; $\frac{3}{2}$ sq. units

11. $x = 2, y = 1$; 4 sq. units; $\frac{4}{3}$ sq. units

12. (−3, 2), (−3, 6), (3, 2)

13. (0, 0), (5, 0), (2, 2) ; 5 sq. units

EXERCISE 18.4

1. (*i*) $2\sqrt{2}$ units (*ii*) 39 units (*iii*) $2\sqrt{a^2 + b^2}$ units 2. 5 units 3. −3

4. (*i*) (2, 0) or (8, 0) (*ii*) (0, 2) or (0, 14) (*iii*) (1, 2) or (3, 6)

Hint. (*iii*) The point (points) are of the form $(k, 2k)$

5. (−7, 0) 6. 5 or −3 7. 4 or −4 8. $x - y = 2$

9. (16, 8) [**Hint.** The point P is of the form $(2k, k)$]

10. 2 [**Hint.** AC = BC] 11. 1

13. $2, -\frac{46}{13}$

Hint. CA = radius of circle = $\frac{1}{2} \times 20$ units = 10 units

$$\Rightarrow \sqrt{(2\alpha - 1 + 3)^2 + (3\alpha + 1 + 1)^2} = 10$$

$$\Rightarrow (2(\alpha + 1))^2 + (3\alpha + 2)^2 = 100$$

$$\Rightarrow 4(\alpha^2 + 2\alpha + 1) + 9\alpha^2 + 12\alpha + 4 - 100 = 0$$

$$\Rightarrow 13\alpha^2 + 20\alpha - 92 = 0 \Rightarrow (\alpha - 2)(13\alpha + 46) = 0$$

15. Yes 16. Scalene triangle 19. 1 20. 40 sq. units

21. 24 sq. units [**Hint.** Show that PQ = QR = RS = SP and PR ≠ QS]

23. (*i*) square (*ii*) parallelogram 24. (5, 2); 5 units

MULTIPLE CHOICE QUESTIONS

1. (*b*)	2. (*c*)	3. (*b*)	4. (*c*)	5. (*d*)	6. (*d*)	7. (*c*)
8. (*b*)	9. (*b*)	10. (*d*)	11. (*d*)	12. (*b*)	13. (*c*)	14. (*c*)
15. (*b*)	16. (*b*)	17. (*c*)	18. (*b*)	19. (*c*)	20. (*b*)	21. (*a*)
22. (*c*)	23. (*b*)	24. (*b*)	25. (*a*)	26. (*a*)		

ASSERTION-REASON TYPE QUESTIONS

1. (*d*) 2. (*b*) 3. (*d*) 4. (*c*) 5. (*d*)

CHAPTER TEST

1. (4, −1); (*i*) (3, 7) (*ii*) (4, 3) (*iii*) (3, 3); 16 sq. units

2. (−1, 3); (−1, 0); 24 sq. units

Hint. Height of the parallelogram corresponding to the side CD is 4 units

4. (*i*) −6 (*ii*) 8 5. $x = 5, y = 1$

6. $x = 2, y = -1$ 7. (4, 5); 12·5 sq. units 8. (−3, −2), (5, 1), (1, 6)

9. (2, −1), (3, 2), (−4, 2); 10·5 sq. units 10. 6 or −10

11. −7 14. (*i*) 8 sq. units (*ii*) 45 sq. units; No

15. 6 or −6

Hint. $\sqrt{2}$ (length of side) = length of diagonal = $\sqrt{(p+2)^2 + (2-p)^2}$

$\Rightarrow$ 2(length of side)^2 = $2p^2 + 8$ $\Rightarrow$ (length of side)^2 = $p^2 + 4$

$\Rightarrow$ area of square = $p^2 + 4 = 40$ (given)

16. Rectangle 17. (2, −4)

EXERCISE 19.1

1. 6 **2.** 13 hours **3.** 2780 **4.** 6·5 **5.** (*i*) $\frac{41}{6}$ (*ii*) $10\frac{5}{7}$

6. (*i*) 82 (*ii*) Vijay : 12 years, Rahul : 15 years; Rakhi : 18 years **7.** 8 **8.** 27

9. 18 **10.** 13 **11.** 45·9 kg **12.** 42 **13.** 5

14. Mean = 2·4, median = 2·5

15. Mean time = 25 minutes, median time = 24 minutes

16. Mean = 54·8, median = 52 **17.** Mean = 15·5 points, median = 12 points

18. 45 **19.** 11

EXERCISE 19.2

1. (*i*) Discrete (*ii*) continuous (*iii*) discrete (*iv*) continuous (*v*) continuous

2.

Classes	0 – 4	5 – 9	10 – 14	15 – 19	20 – 24
Frequency	2	7	8	6	2

3. (*i*)

Classes	1 – 10	11 – 20	21 – 30	31 – 40
Frequency	8	7	6	6

(*ii*) Range = 38 (*iii*) 25·5

5. (*i*) frequency (*ii*) size (*iii*) 14 (*iv*) class mark (*v*) 6·5

6. (*i*)

Classes	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50
Frequency	7	8	7	10	8

(*ii*)

Classes	0·5 – 10·5	10·5 – 20·5	20·5 – 30·5	30·5 – 40·5	40·5 – 50·5
Frequency	7	8	7	10	8

(*iii*) lower limit = 20·5; upper limit = 30·5 (*iv*) 35·5

7. (*i*) upper limit = 52; lower limit = 48

(*ii*) upper limit = 52·5; lower limit = 47·5

(*iii*) 37·5 and 42·5 (*iv*) 45 (*v*) 5

8.

Marks % (classes)	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69	70 – 79	80 – 89
Frequency	7	11	20	46	57	37	15	7
C. frequency	7	18	38	84	141	178	193	200

9.

Classes	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	5	5	7	10	8
C. frequency	5	10	17	27	35

10. (*i*)

Classes	Tally marks	Frequency	Cumulative frequency
0 – 10	III	3	3
10 – 20	II	2	5
20 – 30	𝍸 𝍸 I	11	16
30 – 40	𝍸 𝍸 𝍸 III	18	34
40 – 50	–	–	34
50 – 60	III	3	37
60 – 70	II	2	39
70 – 80	III	3	42
80 – 90	𝍸	5	47
90 – 100	III	3	50

(*ii*) 30 – 40 (*iii*) 34 (*iv*) 13.

11.

Classes	0 – 4	4 – 7	7 – 10	10 – 13	13 – 16
Frequency	7	31	137	73	52

Number of children in the age group 10 – 13 = 73

12.

Classes	0 – 10	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60
Frequency	2	5	11	14	11	7

13.

Classes	23·5 – 27·5	27·5 – 31·5	31·5 – 35·5	35·5 – 39·5	39·5 – 43·5
Frequency	4	7	4	4	11

14. (*i*) 10; lower limit = 119; upper limit = 129

(*ii*) 7; lower limit = 20; upper limit = 27

EXERCISE 19.3

8. Hint. Classes are 10 – 15, 15 – 20, 20 – 25, 25 – 30, 30 – 35, 35 – 40

17.

Class intervals	300–400	400–500	500–600	600–700	700–800	800–900	900–1000	1000–1100
No. of houses	3	1	4	5	9	3	3	4

18.

Class intervals	40–42	42–44	44–46	46–48	48–50	50–52	52–54
Frequency	2	12	7	6	7	4	2

19. (*i*) 18 (*ii*) 475 – 500 (*iii*) 34

(*iv*)

Classes	Frequency	Cumulative frequency
375 – 400	6	6
400 – 425	18	24
425 – 450	10	34
450 – 475	20	54
475 – 500	4	58

MULTIPLE CHOICE QUESTIONS

1. (*b*) **2.** (*b*) **3.** (*b*) **4.** (*c*) **5.** (*b*) **6.** (*b*) **7.** (*c*)
8. (*b*) **9.** (*d*) **10.** (*a*) **11.** (*d*) **12.** (*b*) **13.** (*c*) **14.** (*c*)
15. (*c*) **16.** (*a*) **17.** (*a*)

ASSERTION-REASON TYPE QUESTIONS

1. (*b*) **2.** (*a*) **3.** (*c*)

CHAPTER TEST

1. Mean = 4·25, median = 4·5 **2.** Mean = 12·4, median = 7
3. 51 kg **4.** 14·6 years **5.** 10 **6.** 150·25 cm
7. 35·5 **8.** 62

12.

Class intervals	50–60	60–70	70–80	80–90	90–100	100–110	110–120	120–130
No. of houses	2	6	5	8	5	7	3	4

13.

Class intervals	19 – 20	21 – 22	23 – 24	25 – 26	27 – 28	29 – 30
No. of students	3	5	10	10	5	2

14. (*i*) 6 (*ii*) 45 – 51 (*iii*) 51 – 57

Classes	27 – 33	33 – 39	39 – 45	45 – 51	51 – 57	57 – 63
Frequency	4	12	18	6	20	8