수치해석
(2019학년도 1학기)

[6주/1차시 학습내용]: Newton-Raphson 방법을 알아보자.

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Ch. 6. Open Methods Newton-Raphson

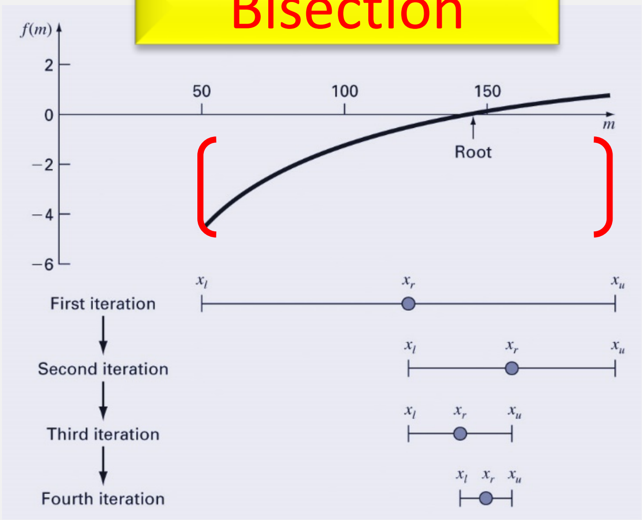
Problem of significant vertebrae injury if the free-fall velocity exceeds 36 m/s after 4 s of free fall.

수치 알고리즘의 근사화 전략

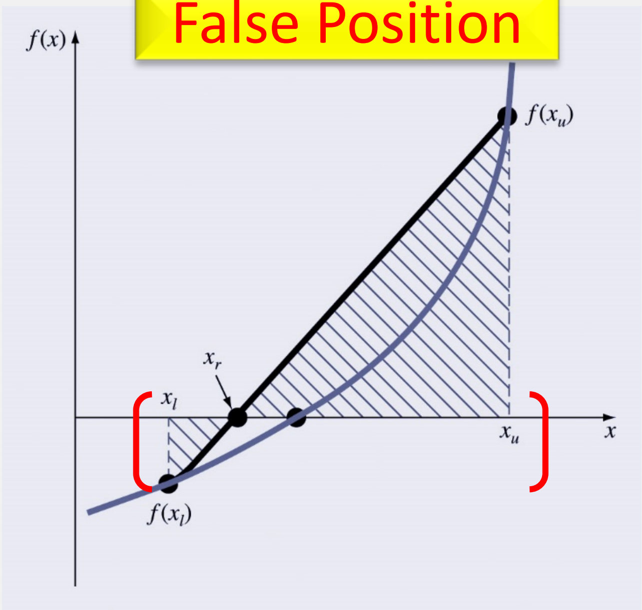
- trial and error의 반복을 통해서 참 값에 근사한다.
- Bracketing Method (구간법)
  - Graphical Method : 구간에 대한 정의 탄생
  - 증분법 : 증분점에 대한 fitting optimization 필요
  - 이분법 : 구간을 찾아내는 것이 아니라, 근을 찾아내려고 노력함
  - 가위치법 : 이분법과 같이 근을 찾아내려고 노력함
    - 가위치법은 선형보간법 (Linea Interpolation) 이라고도 함
- Open Method (개방법)
  - 구간법에서 꼭 필요한 구간을 더 이상 사용하지 않음
  - 어떤 점도 근이 될 수 있음
  - 구간법보다 빠르게 근을 찾는 알고리즘을 제공함
  - Newton-Raphson, Secant Method, Modified Secant Method

구간법(Bracketing)과 개방법(Open)

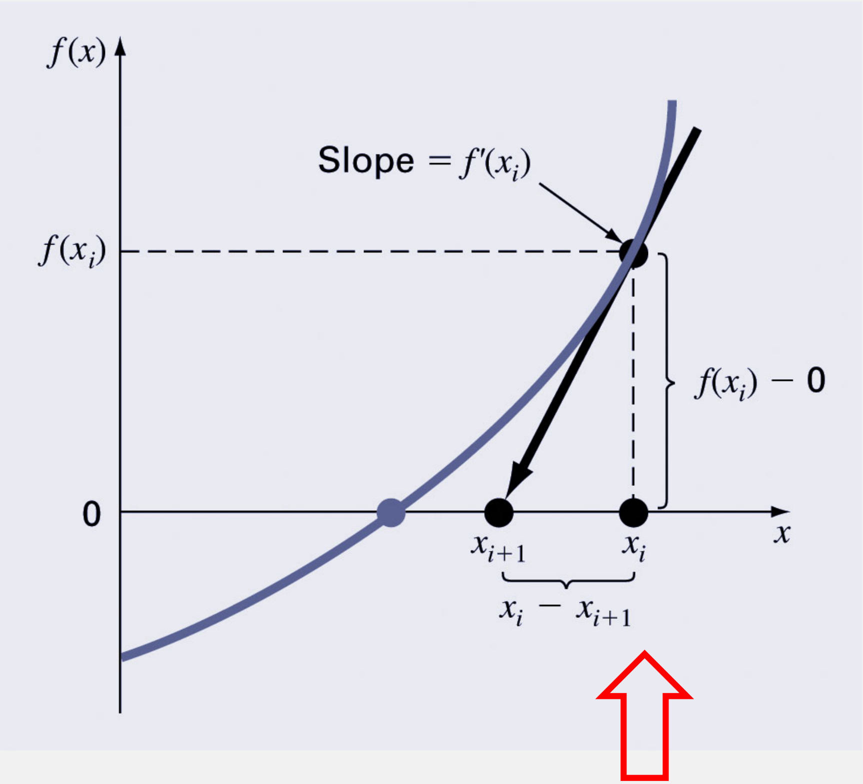
Bisection



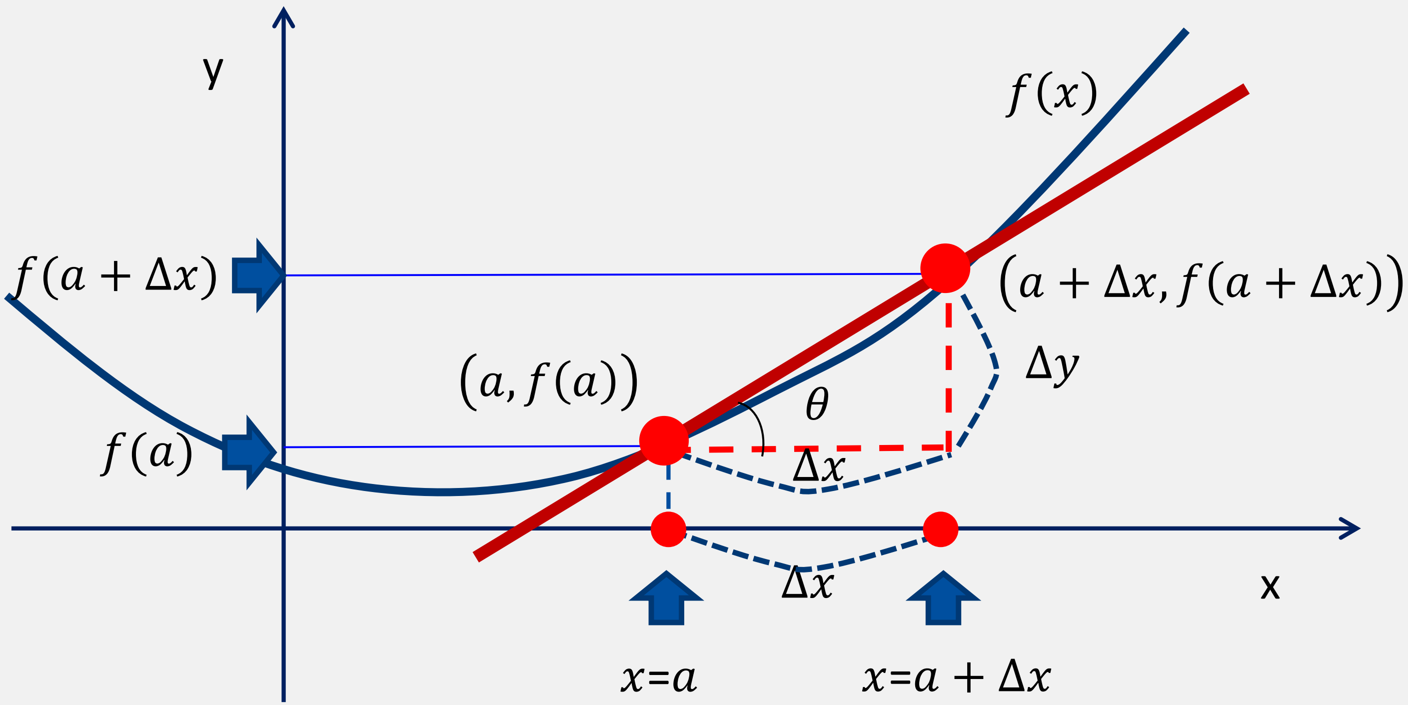
False Position



Newton Raphson



두 점을 지나는 직선의 기울기=평균 변화율



두 점을 지나는 직선 기울기 $= \dfrac{\Delta y}{\Delta x} = \dfrac{f(a+\Delta x)-f(a)}{(a+\Delta x)-a} = \dfrac{f(a+\Delta x)-f(a)}{\Delta x}$

= a 에 대한 평균 변화율

평균 변화율과 순간 변화율

- a 에 대한 **평균 변화율** = **두 점을 지나는 직선의 기울기**

$$\frac{\Delta y}{\Delta x} = \frac{f(a+\Delta x)-f(a)}{(a+\Delta x)-a} = \frac{f(a+\Delta x)-f(a)}{\Delta x}$$

- a 에 대한 **순간 변화율** = **미분계수**

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(a+\Delta x)-f(a)}{(a+\Delta x)-a} = \lim_{\Delta x \to 0} \frac{f(a+\Delta x)-f(a)}{\Delta x}$$

$$f'(a) = y'_{x=a} = \left[\frac{dy}{dx}\right]_{x=a}$$

a 에 대한 순간 변화율과 x 에 대한 순간 변화율

- $\boldsymbol{a}$ 에 대한 **순간 변화율** = **미분계수**

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{(a + \Delta x) - a} = \lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{\Delta x}$$

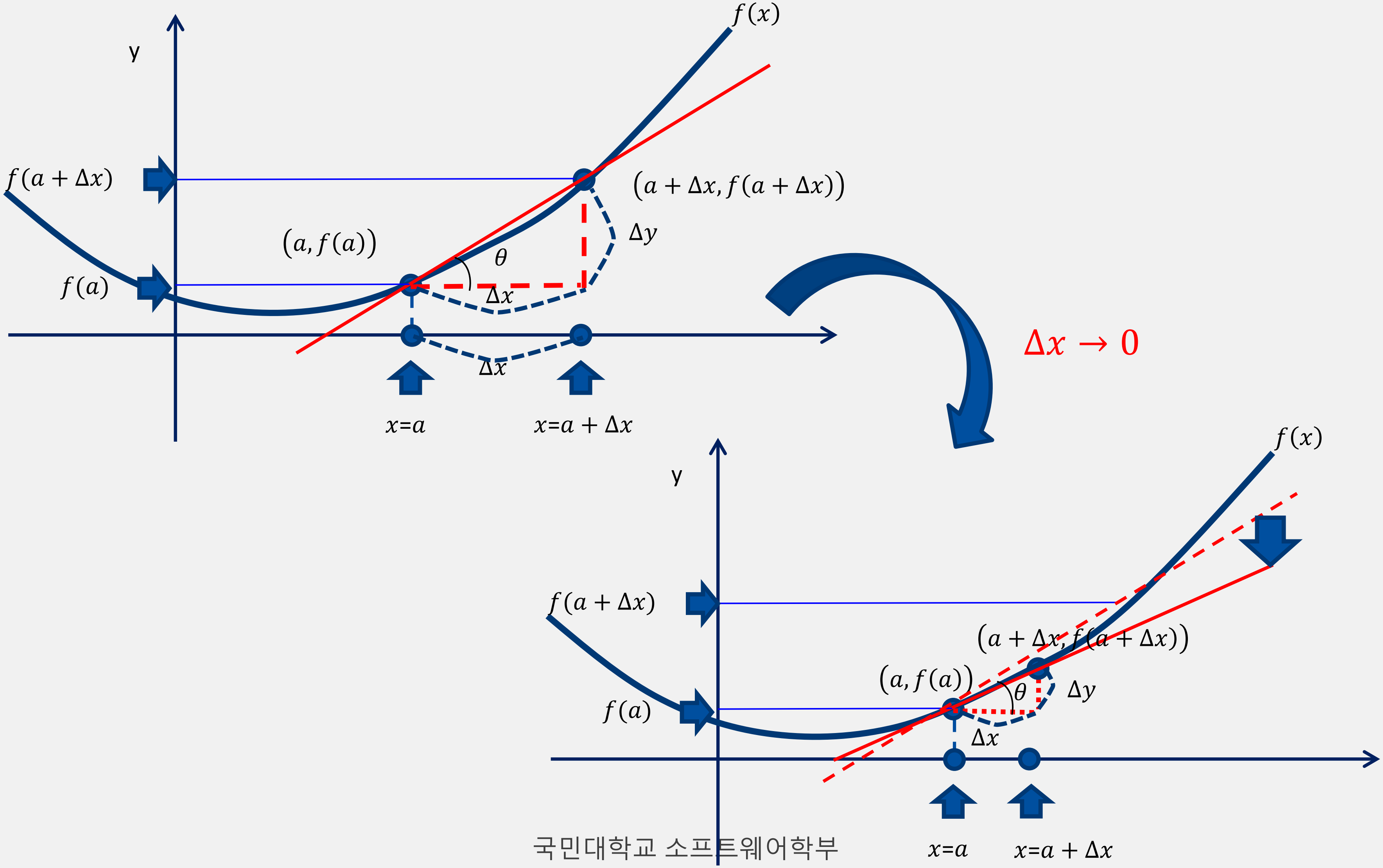
$$f'(a) = y'_{x=a} = \left[\frac{dy}{dx}\right]_{x=a}$$

- $\boldsymbol{x}$ 에 대한 **순간 변화율** = **도함수**

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$f'(x) = y' = \frac{dy}{dx} = \frac{df(x)}{dx} = \frac{d}{dx} f(x)$$

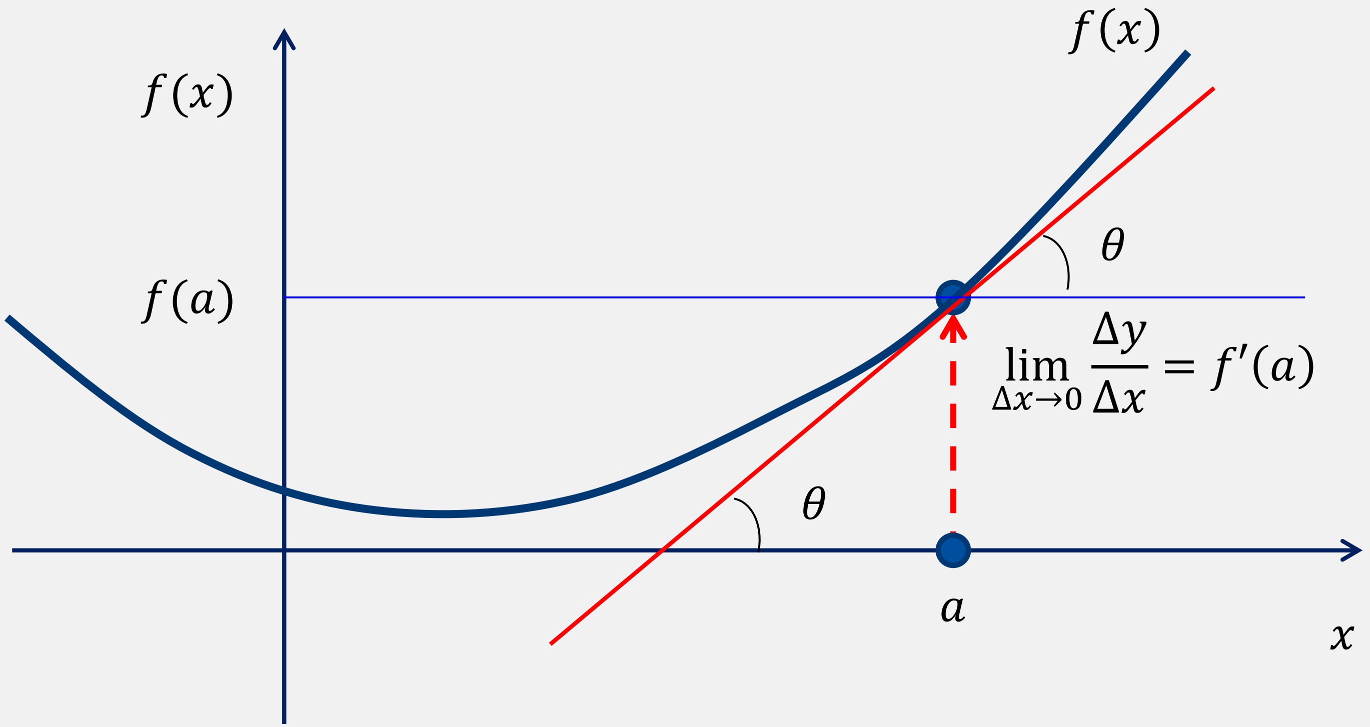
평균 변화율에서 순간 변화율(미분 계수)로



미분계수

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(a)$$

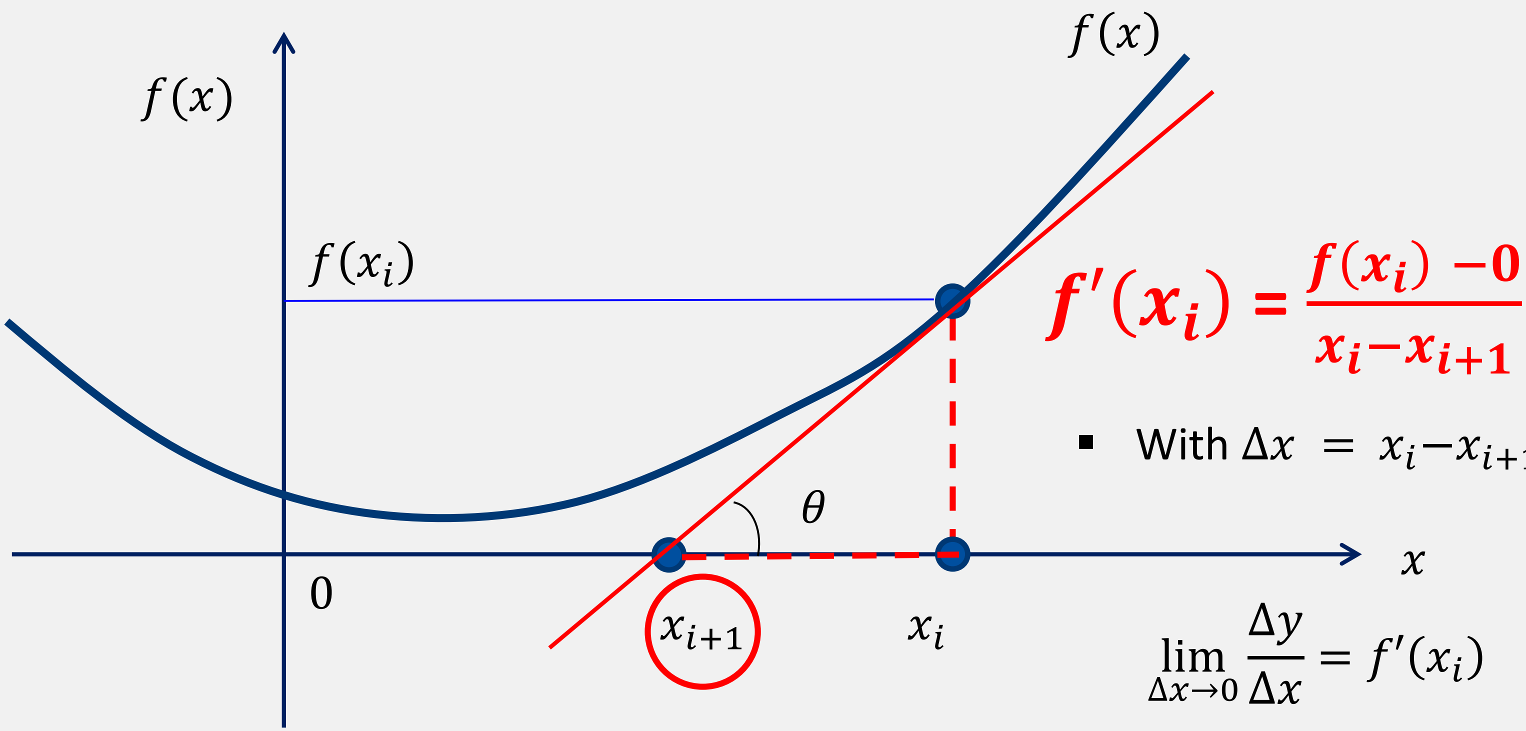
Δx should not be 0 since it is denominator



Newton-Raphson Algorithm

- 미분계수 $f'(x\_i)$ 는 접점 x_i 에서의 접선의 기울기를 충실히 따르는 알고리즘
- 구하고자 하는 것은 $x\_(i+1)$

$$f'(x_i) = \frac{f(x_i) - 0}{x_i - x_{i+1}}$$



- With $\Delta x = x_i - x_{i+1} \to 0$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(x_i)$$

Calculate $x^{new} = x_{i+1}$

$$f'(x_i) = \frac{f(x_i) - 0}{x_i - x_{i+1}}$$

$$(x_i - x_{i+1}) \cdot f'(x_i) = f(x_i)$$

$$x_i \cdot f'(x_i) - x_{i+1} \cdot f'(x_i) = f(x_i)$$

$$x_i \cdot f'(x_i) - f(x_i) = x_{i+1} \cdot f'(x_i)$$

$$x_i \cdot f'(x_i) - f(x_i) = x_{i+1} \cdot f'(x_i)$$

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

Newton-Raphson Algorithm

- $f'(x_i)$를 구해야 하는 문제 발생, 200kg에서 시작한다면?

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}, x_1 = 200 \text{ (초기값)}$$

$$x_2 = 200 - \frac{f(200)}{f'(200)} \text{ 계산 필요}$$

$$f(200) = \sqrt{\frac{gm}{c_d}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - 36$$

$$f(200) = \sqrt{\frac{9.81 \times 200}{0.25}} \cdot tanh\left(\sqrt{\frac{9.81 \times 0.25}{200}} \cdot 4\right) - 36$$

$$f'(200) = ?$$

$$f'(200) = \left[\sqrt{\frac{9.81 \times 200}{0.25}} \cdot tanh\left(\sqrt{\frac{9.81 \times 0.25}{200}} \cdot 4\right) - 36\right]' \ ?$$

$f'(x_i)$ 구하기, 200kg에서 시작한다면?

$$f(m) = \underbrace{\sqrt{\frac{gm}{c_d}}}_{f_1(m)} \cdot \tanh\underbrace{\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)}_{f_2(m)} - 36$$

$$f(m) = f_1(m) \cdot f_2(m)$$

$$f'(m) = f_1{}'(m) \cdot f_2(m) + f_1(m) \cdot f_2{}'(m)$$

$$f_2(m) = \tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)$$

$$f_2{}'(m) = \tanh'\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) \cdot \left(\sqrt{\frac{gc_d}{m}} \cdot t\right)'$$

use $y(x) = f(g(x))$

$$y'(x) = f'(g(x)) \cdot g'(x)$$

$$f_1(m) = \sqrt{\frac{gm}{c_d}} = \sqrt{\frac{g}{c_d}} \cdot (m)^{\frac{1}{2}}$$

$$f_1{}'(m) = \left(\sqrt{\frac{g}{c_d}} \cdot m^{\frac{1}{2}}\right)'$$

$$f_1{}'(m) = \sqrt{\frac{g}{c_d}} \cdot \left(m^{\frac{1}{2}}\right)' = \sqrt{\frac{g}{c_d}} \cdot \frac{1}{2} \cdot m^{\frac{1}{2}-1}$$

use $y(x) = x^n$

$$y'(x) = n \cdot x^{n-1}$$

$$f(m) = \sqrt{\frac{gm}{c_d}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - 36$$

$$f'(m) = f_1{}'(m) \cdot f_2(m) + f_1(m) \cdot f_2{}'(m)$$

$$f_1{}'(m) = \sqrt{\frac{g}{c_d}} \cdot \left(m^{\frac{1}{2}}\right)' = \sqrt{\frac{g}{c_d}} \cdot \frac{1}{2} \cdot m^{-\frac{1}{2}} \qquad f_2{}'(m) = tanh'\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) \cdot \left(\sqrt{\frac{gc_d}{m}} \cdot t\right)'$$

$$f'(m) = \sqrt{\frac{g}{c_d}} \cdot \frac{1}{2} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) + \sqrt{\frac{gm}{c_d}} \cdot tanh'\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) \cdot \left(\sqrt{\frac{gc_d}{m}} \cdot t\right)'$$

$$y = tanh(x) \Rightarrow y' = sech^2(x)$$

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{c_d}} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) + \sqrt{\frac{gm}{c_d}} \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) \cdot \left(\sqrt{gc_d} \cdot t \cdot m^{-\frac{1}{2}}\right)'$$

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{c_d}} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) + \sqrt{\frac{gm}{c_d}} \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) \cdot \left(-\frac{1}{2} \cdot m^{-\frac{1}{2}-1} \cdot \sqrt{gc_d} \cdot t\right)$$

$f'(x_i)$ 구하기

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{c_d}} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) + \sqrt{\frac{gm}{c_d}} \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) \cdot \left(-\frac{1}{2} \cdot m^{-\frac{1}{2}-1} \cdot \sqrt{gc_d} \cdot t\right)$$

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{c_d}} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) + \sqrt{\frac{gm}{c_d}} \cdot \left(-\frac{1}{2}\right) \cdot \sqrt{gc_d} \cdot m^{-\frac{3}{2}} \cdot t \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)$$

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{c_d}} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - \frac{1}{2} \cdot \sqrt{\frac{gm}{c_d}} \cdot \sqrt{gc_d} \cdot \frac{1}{\sqrt{m}} \cdot \frac{1}{m} \cdot t \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)$$

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{c_d}} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - \frac{1}{2} \cdot \frac{\sqrt{g}\cancel{\sqrt{m}}}{\cancel{\sqrt{c_d}}} \cdot \sqrt{g} \cdot \cancel{\sqrt{c_d}} \cdot \frac{1}{\cancel{\sqrt{m}}} \cdot \frac{1}{m} \cdot t \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)$$

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{c_d}} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - \frac{1}{2} \cdot g \cdot \frac{t}{m} \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)$$

$f'(x_i)$ 구하기

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{c_d}} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - \frac{gt}{2m} \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)$$

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{mc_d}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - \frac{gt}{2m} \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)$$

$$f'(m) = \frac{1}{2} \cdot \sqrt{\frac{g}{mc_d}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - \frac{gt}{2m} \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)$$

$$f(m) = \sqrt{\frac{gm}{c_d}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - 36$$